Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

6-Hyperbolic-functions/6.1-Hyperbolic-sine/160-6.1.1-c+d-x-^m-a+b-sinh-^n

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [502]. This is test number [160].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	98.80 (496)	1.20 (6)
Fricas	92.63 (465)	7.37 (37)
Rubi	89.24 (448)	10.76 (54)
Maple	67.73 (340)	32.27 (162)
Maxima	57.57 (289)	42.43 (213)
Mupad	41.63 (209)	58.37 (293)
Giac	39.44 (198)	60.56 (304)
Sympy	23.90 (120)	76.10 (382)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

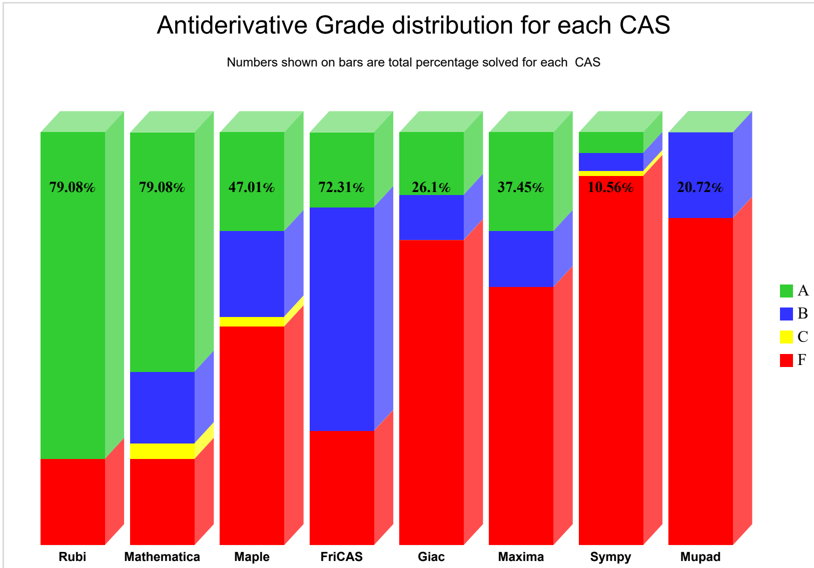
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

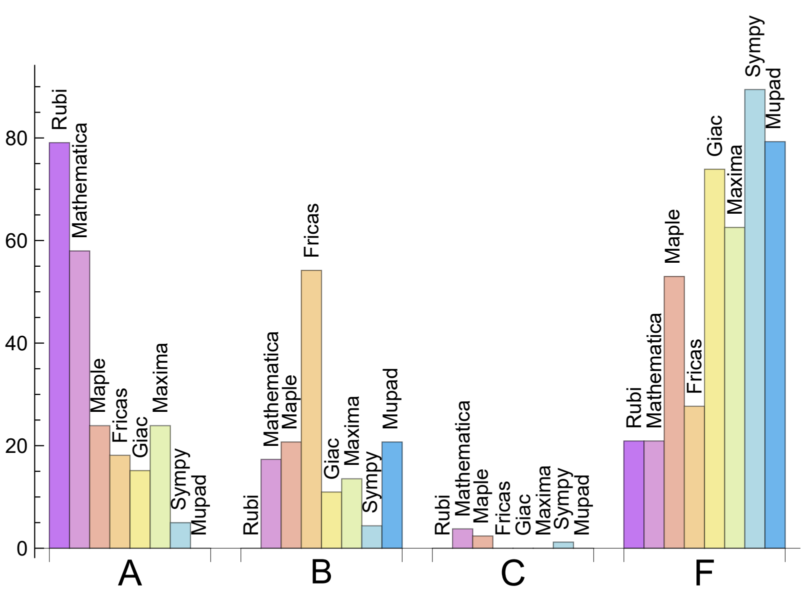
System	% A grade	% B grade	% C grade	% F grade
Mathematica	57.968	17.331	3.785	20.916
Rubi	47.211	0.000	21.116	31.673
Maple	23.904	20.717	2.390	52.988
Maxima	23.904	13.546	0.000	62.550
Fricas	18.127	54.183	0.000	27.689
Giac	15.139	10.956	0.000	73.904
Sympy	4.980	4.382	1.195	89.442
Mupad	0.000	20.717	0.000	79.283

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	6	0.00	100.00	0.00
Fricas	37	24.32	2.70	72.97
Rubi	54	100.00	0.00	0.00
Maple	162	99.38	0.62	0.00
Maxima	213	91.55	0.00	8.45
Giac	304	62.50	36.84	0.66
Mupad	293	0.00	100.00	0.00
Sympy	382	57.59	38.74	3.66

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.40
Fricas	0.80
Rubi	1.04
Mupad	1.96
Giac	1.99
Maple	6.81
Mathematica	10.34
Sympy	10.72

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	130.37	1.77	38.00	1.12
Giac	166.67	1.76	100.00	1.34
Sympy	173.32	2.59	43.50	1.26
Rubi	209.83	1.01	125.50	1.00
Maxima	268.32	5.75	167.00	1.76
Maple	364.26	1.69	112.50	1.24
Mathematica	605.70	1.54	144.00	1.07
Fricas	1827.51	5.15	454.00	2.87

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

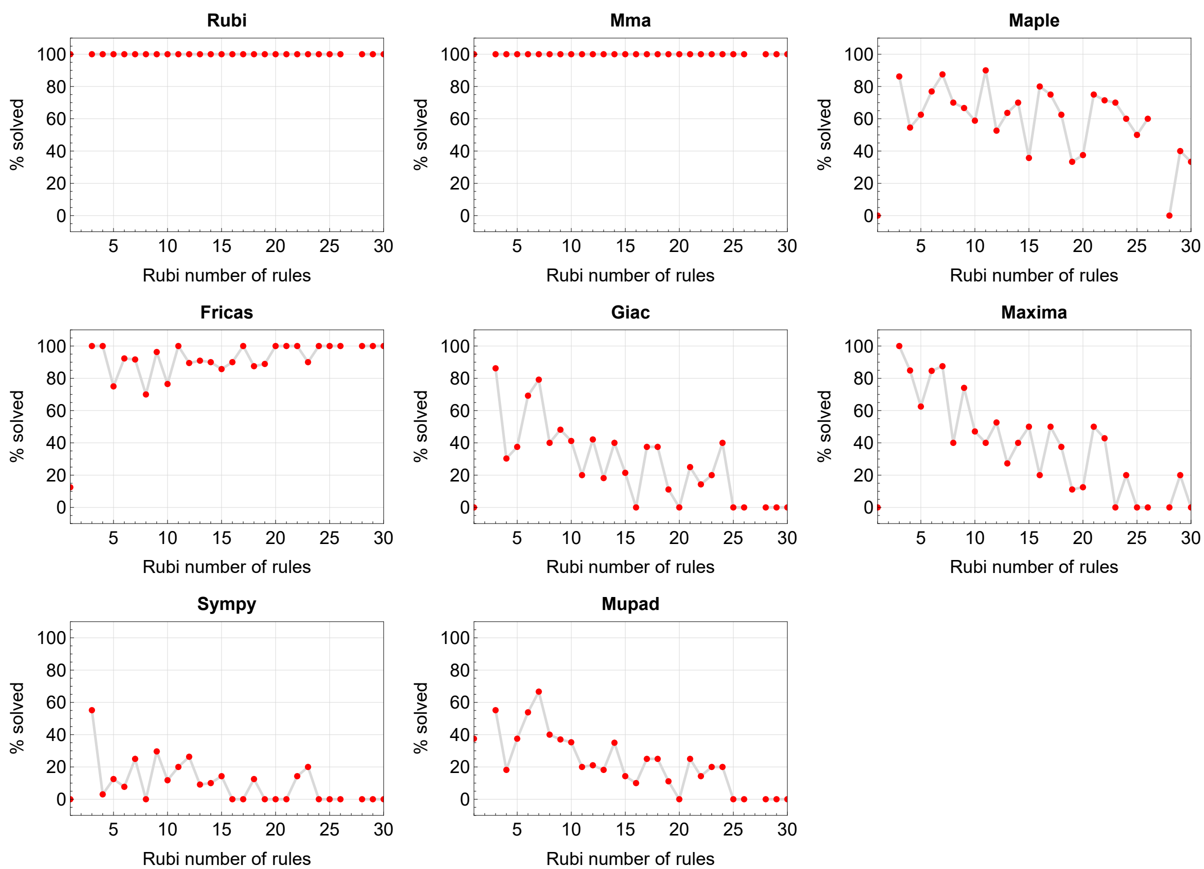


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

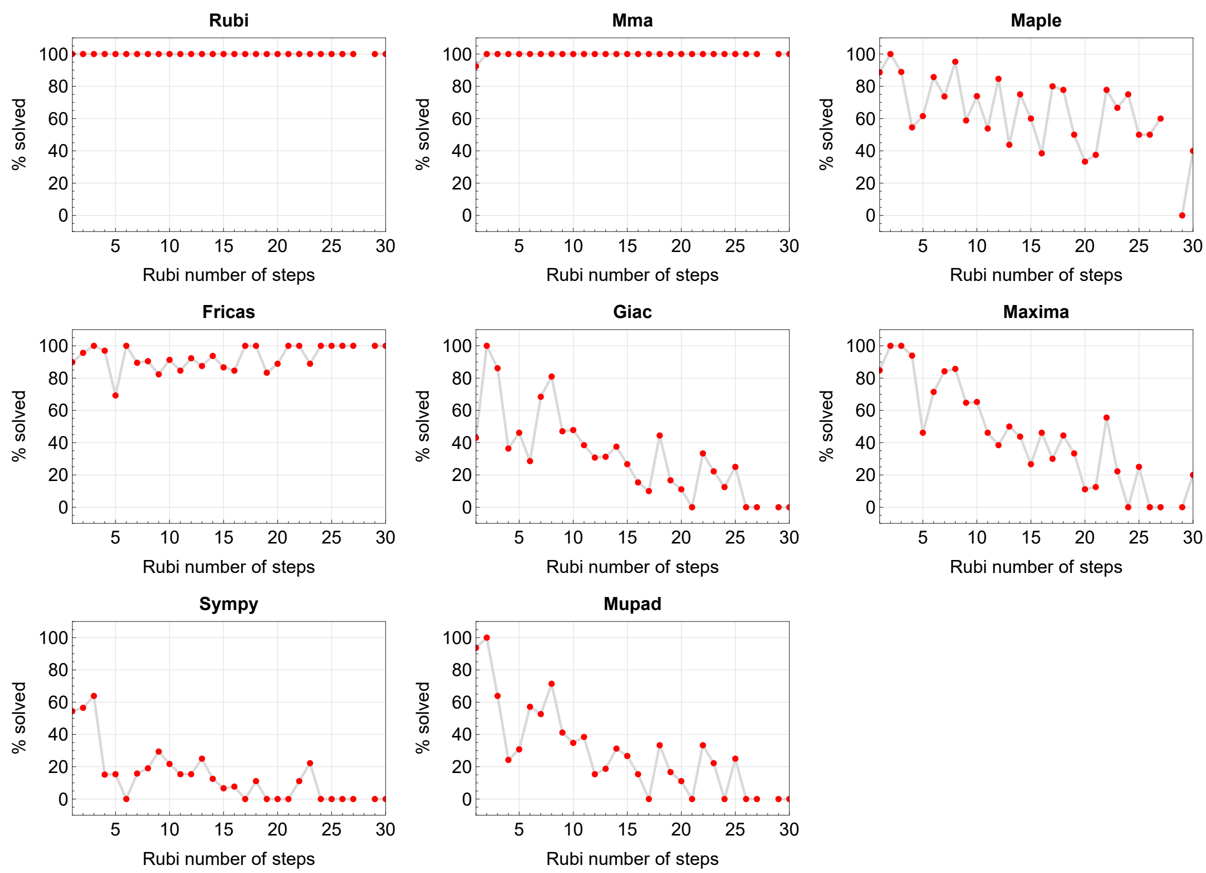


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

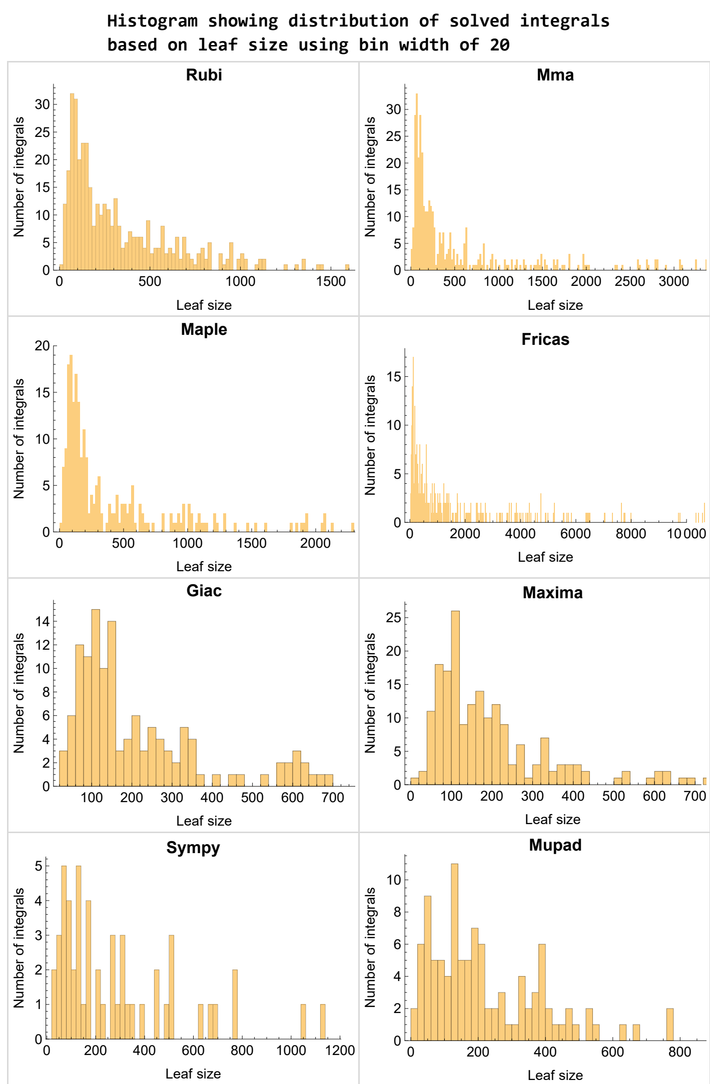


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

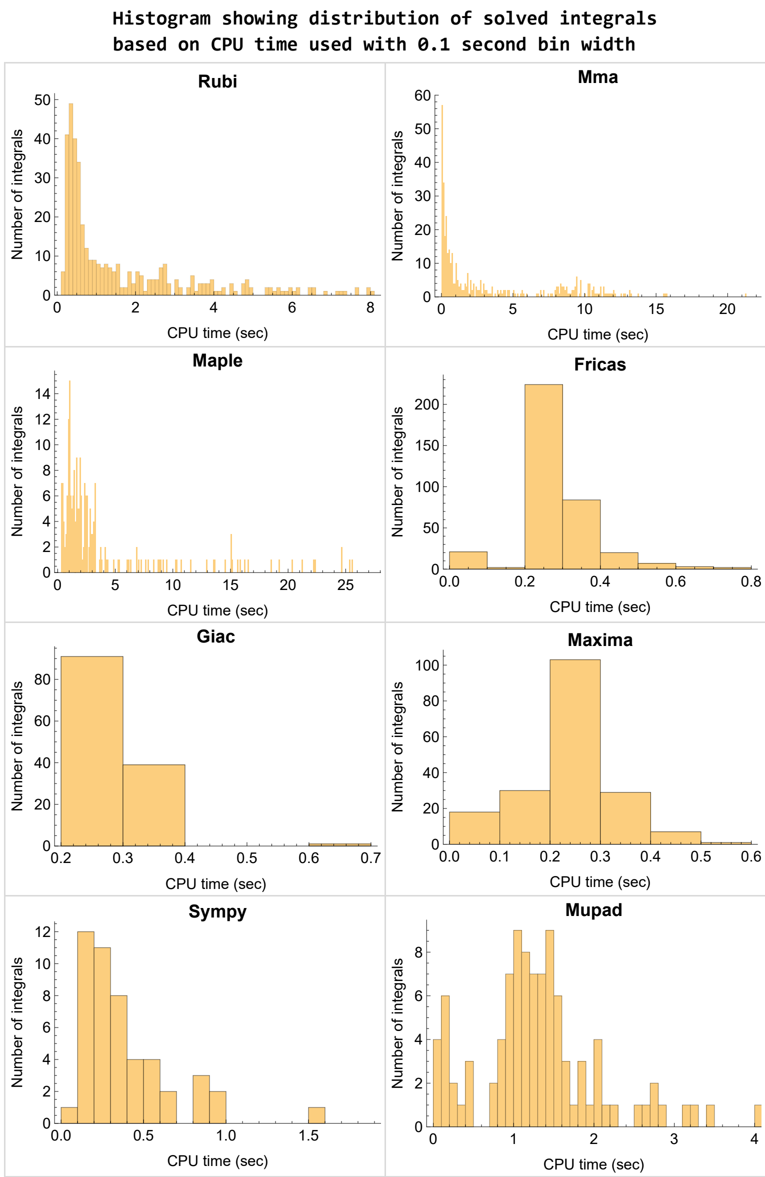


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

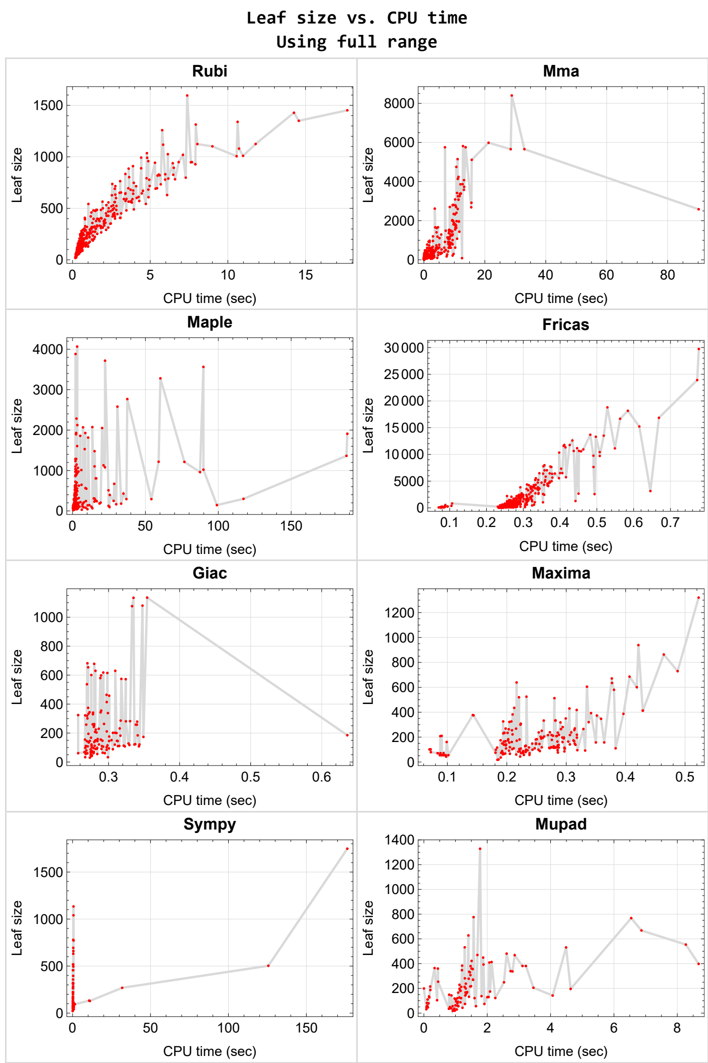


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{26, 27, 31, 32, 36, 37, 65, 66, 67, 72, 76, 77, 111, 112, 116, 117, 139, 140, 144, 145, 149, 150, 151, 155, 156, 172, 173, 176, 177, 179, 180, 181, 185, 186, 191, 192, 197, 198, 203, 204, 209, 210, 215, 216, 221, 222, 227, 232, 237, 242, 247, 252, 257, 258, 275, 276, 281, 282, 287, 288, 293, 298, 303, 308, 313, 317, 318, 319, 320, 337, 342, 347, 352, 357, 361, 366, 371, 376, 381, 386, 390, 395, 400, 405, 410, 415, 419, 424, 429, 434, 439, 444, 448, 453, 458, 463, 468, 472, 475, 480, 485, 490, 495, 499, 502}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {226, 231, 236, 241, 246, 251, 297, 312, 321, 324, 327, 330, 341, 356, 370, 385, 399, 414, 428, 457, 484}

Mathematica {211, 217, 218, 243, 248, 277, 283, 284, 314, 343, 358, 372, 373, 387, 391, 401, 416, 436, 445, 469, 473, 481, 488, 491, 492, 493, 496, 497, 500}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

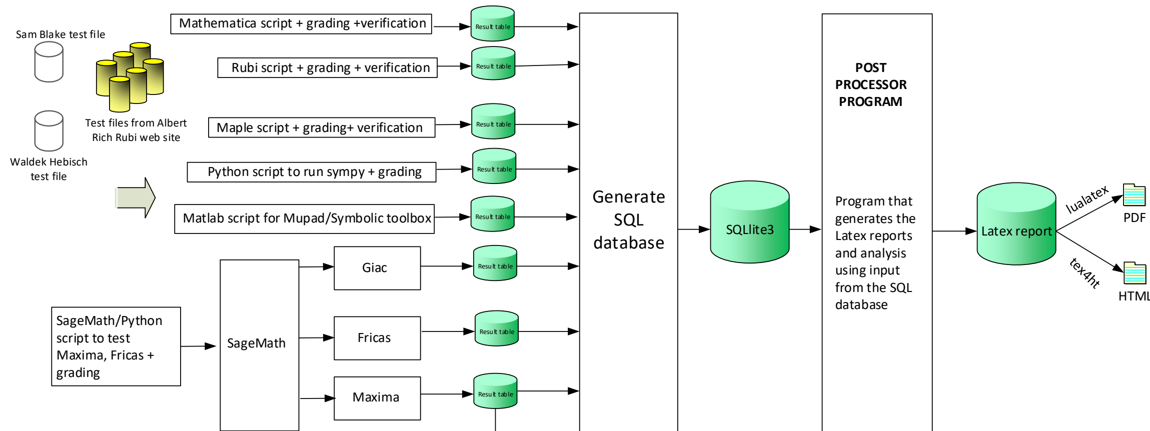
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 8, 9, 10, 11, 12, 14, 45, 46, 47, 48, 50, 52, 68, 69, 70, 71, 74, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174, 175, 178, 182, 183, 184, 187, 188, 189, 190, 193, 194, 195, 196, 201, 202, 205, 206, 207, 208, 213, 214, 220, 223, 224, 225, 253, 254, 255, 256, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 277, 278, 279, 280, 284, 285, 286, 289, 290, 291, 292, 297, 301, 302, 304, 305, 306, 307, 309, 310, 311, 312, 314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 335, 336, 345, 346, 348, 349, 350, 351, 353, 354, 355, 358, 359, 360, 364, 365, 375, 377, 378, 379, 380, 382, 383, 384, 385, 387, 388, 389, 393, 394, 404, 408, 409, 418, 423, 433, 435, 436, 437, 438, 440, 441, 442, 445, 446, 447, 452, 462, 464, 465, 466, 467, 469, 471, 473, 474, 479, 489, 491, 492, 493, 494, 500, 501 }

B grade { }

C grade { 1, 2, 3, 4, 5, 6, 7, 13, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 28, 29, 30, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 49, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 73, 75, 78, 79, 80, 81, 82, 83, 84, 226, 228, 229, 230, 231, 235, 236, 238, 239, 240, 241, 243, 244, 245, 246, 251, 294, 295, 296, 299, 300, 333, 334, 338, 339, 340, 341, 356, 362, 363, 369, 370, 398, 399, 414, 420, 421, 422, 427, 428, 432, 443, 449, 450, 451, 457, 470, 477, 478, 484, 497, 498 }

F normal fail { 16, 130, 199, 200, 211, 212, 217, 218, 219, 233, 234, 248, 249, 250, 283, 343, 344, 367, 368, 372, 373, 374, 391, 392, 396, 397, 401, 402, 403, 406, 407, 411, 412, 413, 416, 417, 425, 426, 430, 431, 454, 455, 456, 459, 460, 461, 476, 481, 482, 483, 486, 487, 488, 496 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 28, 29, 30, 33, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 68, 69, 70, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174, 175, 178, 182, 183, 184, 187, 188, 190, 194, 195, 196, 199, 201, 202, 205, 206, 208, 214, 220, 223, 224, 225, 226, 228, 229, 230, 231, 233, 234, 235, 236, 238, 239, 240, 241, 244, 245, 246, 250, 251, 253, 254, 255, 256, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 278, 279, 280, 286, 289, 290, 291, 292, 294, 295, 296, 301, 302, 306, 307, 309, 310, 312, 315, 321, 322, 323, 324, 325, 326, 327, 330, 335, 336, 340, 341, 345, 346, 350, 353, 354, 356, 359, 364, 365, 367, 368, 369, 370, 374, 375, 379, 382, 383, 385, 388, 393, 394, 399, 402, 404, 408, 411, 412, 414, 417, 423, 425, 426, 427, 428, 432, 433, 437, 440, 441, 446, 447, 452, 455, 456, 457, 461, 462, 466, 467, 471, 473, 479, 483, 484, 488, 489, 493, 494, 496, 498, 500 }

B grade { 25, 34, 35, 110, 130, 135, 189, 193, 200, 207, 211, 212, 213, 217, 218, 219, 243, 248, 249, 262, 273, 277, 283, 284, 285, 299, 300, 304, 305, 314, 328, 329, 331, 332, 333, 334, 338, 339, 343, 344, 348, 349, 358, 362, 363, 372, 373, 377, 378, 387, 391, 392, 396, 397, 398, 401, 403, 406, 407, 416, 420, 421, 422, 430, 431, 435, 436, 443, 445, 449, 450, 451, 454, 459, 460, 464, 465, 469, 476, 477, 478, 481, 482, 486, 487, 491, 492 }

C grade { 71, 297, 311, 316, 351, 355, 360, 380, 384, 389, 409, 413, 418, 438, 442, 470, 474, 497, 501 }

F normal fail { }

F(-1) timedout fail { 221, 222, 276, 281, 282, 288 }

F(-2) exception fail { }

2.1.3 Maple

A grade { 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 25, 30, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 110, 114, 115, 118, 119, 120, 121, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 189, 190, 195, 196, 201, 202, 207, 208, 214, 220, 226, 231, 235, 236, 241, 246, 251, 256, 261, 262, 263, 264, 266, 267, 268, 269, 270, 273, 274, 278, 279, 280, 285, 286, 292, 297, 302, 307, 312, 316, 336, 341, 346, 351, 356, 360, 365, 370, 375, 380, 385, 389, 394, 399, 404, 409, 414, 418, 423, 428, 438, 443, 447, 452, 457, 467, 471, 474, 479, 484, 489, 494, 498, 501 }

B grade { 1, 7, 14, 15, 22, 23, 24, 28, 29, 33, 34, 35, 101, 107, 108, 109, 113, 162, 168, 171, 175, 178, 187, 188, 193, 194, 199, 200, 205, 206, 211, 212, 213, 217, 218, 219, 225, 230, 240, 245, 250, 253, 254, 255, 259, 260, 265, 271, 272, 277, 283, 284, 291, 296, 301, 306, 311, 315, 321, 322, 324, 325, 327, 328, 330, 331, 335, 340, 345, 350, 355, 359, 364, 369, 374, 379, 384, 388, 393, 398, 403, 408, 413, 417, 422, 427, 432, 433, 437, 442, 446, 451, 456, 461, 462, 466, 470, 473, 478, 483, 488, 493, 497, 500 }

C grade { 60, 61, 62, 63, 64, 78, 79, 80, 81, 82, 83, 84 }

F normal fail { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 68, 69, 70, 71, 73, 74, 75, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 169, 170, 174, 182, 183, 184, 223, 224, 228, 229, 233, 234, 238, 239, 243, 244, 248, 249, 289, 290, 294, 295, 299, 300, 304, 305, 309, 310, 314, 323, 326, 329, 332, 333, 334, 338, 339, 343, 344, 348, 349, 353, 354, 358, 362, 363, 367, 368, 372, 373, 377, 378, 382, 383, 387, 391, 392, 396, 397, 401, 402, 406, 407, 411, 412, 416, 420, 421, 425, 426, 430, 431, 435, 436, 440, 441, 445, 449, 450, 454, 455, 459, 460, 464, 465, 469, 476, 477, 481, 482, 486, 487, 491, 492, 496 }

F(-1) timedout fail { 288 }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 9, 10, 11, 12, 18, 19, 20, 41, 48, 56, 62, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 104, 105, 106, 110, 115, 152, 153, 154, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 182, 183, 184, 189, 190, 195, 196, 201, 202, 208, 255, 256, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 279, 280, 306, 307, 348, 349, 350, 351, 377, 378, 379, 380, 423, 438 }

B grade { 6, 7, 8, 13, 14, 15, 16, 17, 21, 22, 23, 24, 25, 28, 29, 30, 33, 34, 35, 38, 39, 40, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 63, 64, 69, 96, 97, 102, 103, 107, 108, 109, 113, 114, 162, 168, 169, 170, 171, 174, 175, 178, 187, 188, 193, 194, 199, 200, 205, 206, 207, 211, 212, 213, 214, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 230, 231, 233, 234, 235, 236, 238, 239, 240, 241, 243, 244, 245, 246, 248, 249, 250, 251, 253, 254, 259, 260, 271, 272, 273, 274, 277, 278, 283, 284, 285, 286, 289, 290, 291, 292, 294, 295, 296, 297, 299, 300, 301, 302, 304, 305, 309, 310, 311, 312, 314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 343, 344, 345, 346, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 365, 367, 368, 369, 370, 372, 373, 374, 375, 382, 383, 384, 385, 387, 388, 389, 391, 392, 393, 394, 396, 397, 398, 399, 401, 402, 403, 404, 406, 407, 408, 409, 411, 412, 413, 414, 416, 417, 418, 420, 421, 422, 425, 426, 427, 428, 430, 431, 432, 433, 435, 436, 437, 440, 441, 442, 443, 445, 446, 447, 449, 450, 451, 452, 454, 455, 456, 457, 459, 460, 461, 462, 464, 465, 466, 467, 469, 470, 471, 473, 474, 476, 477, 478, 479, 481, 482, 483, 484, 486, 487, 488, 489, 491, 492, 493, 494, 496, 497, 498, 500, 501 }

C grade { }

F normal fail { 136, 137, 138, 141, 142, 143, 146, 147, 148 }

F(-1) timedout fail { 502 }

F(-2) exception fail { 67, 68, 70, 71, 92, 93, 94, 95, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 150 }

2.1.5 Maxima

A grade { 5, 6, 7, 10, 11, 12, 13, 14, 15, 20, 21, 22, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 63, 64, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 98, 99, 100, 101, 104, 105, 106, 107, 110, 152, 153, 154, 159, 160, 161, 162, 164, 165, 166, 167, 168, 182, 183, 184, 189, 190, 196, 202, 206, 208, 220, 226, 231, 236, 241, 246, 251, 254, 256, 263, 264, 268, 271, 272, 277, 279, 292, 297, 307, 312, 316, 341, 351, 356, 360, 370, 380, 385, 389, 399, 409, 414, 418, 428, 438, 443, 447, 457, 467, 471, 474, 494, 498 }

B grade { 1, 2, 3, 4, 8, 9, 16, 17, 18, 19, 23, 24, 28, 30, 33, 34, 38, 39, 40, 41, 60, 61, 62, 96, 97, 102, 103, 108, 113, 115, 157, 158, 163, 187, 193, 195, 205, 211, 212, 214, 217, 218, 253, 259, 260, 261, 262, 274, 280, 302, 321, 324, 327, 330, 336, 346, 365, 375, 394, 404, 423, 433, 452, 462, 479, 484, 489, 501 }

C grade { }

F normal fail { 25, 29, 35, 68, 69, 70, 71, 92, 93, 94, 95, 109, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 169, 170, 171, 174, 175, 178, 188, 194, 207, 213, 219, 223, 224, 225, 228, 229, 230, 233, 234, 235, 238, 239, 240, 243, 244, 245, 248, 249, 250, 255, 273, 278, 289, 290, 291, 294, 295, 296, 299, 300, 301, 304, 305, 306, 309, 310, 311, 314, 315, 322, 323, 325, 326, 328, 329, 331, 332, 333, 334, 335, 338, 339, 340, 343, 344, 345, 348, 349, 350, 353, 354, 355, 358, 359, 362, 363, 364, 367, 368, 369, 372, 373, 374, 377, 378, 379, 382, 383, 384, 387, 388, 391, 392, 393, 396, 397, 398, 401, 402, 403, 406, 407, 408, 411, 412, 413, 416, 417, 420, 421, 422, 425, 426, 427, 430, 431, 432, 435, 436, 437, 440, 441, 442, 445, 446, 449, 450, 451, 454, 455, 456, 459, 460, 461, 464, 465, 466, 469, 470, 473, 476, 477, 478, 481, 482, 483, 486, 487, 488, 491, 492, 493, 496, 497, 500 }

F(-1) timedout fail { }

F(-2) exception fail { 90, 91, 199, 200, 201, 203, 204, 265, 266, 267, 269, 270, 283, 284, 285, 286, 287, 288 }

2.1.6 Giac

A grade { 4, 5, 10, 11, 12, 19, 20, 38, 39, 40, 41, 48, 60, 61, 62, 98, 99, 104, 105, 110, 115, 159, 160, 165, 166, 189, 190, 196, 202, 208, 214, 220, 226, 231, 236, 241, 246, 251, 256, 261, 263, 267, 268, 269, 279, 280, 292, 297, 302, 307, 312, 336, 341, 346, 351, 356, 365, 370, 375, 380, 385, 394, 399, 404, 409, 414, 423, 428, 433, 438, 443, 457, 467, 471, 484, 498 }

B grade { 1, 2, 3, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 21, 22, 30, 96, 97, 100, 101, 102, 103, 106, 107, 157, 158, 161, 162, 163, 164, 167, 168, 195, 201, 259, 260, 262, 264, 265, 266, 270, 274, 286, 316, 360, 389, 418, 447, 452, 462, 474, 479, 489, 494, 501 }

C grade { }

F normal fail { 23, 24, 25, 28, 29, 33, 34, 35, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 63, 64, 68, 69, 70, 71, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 108, 109, 113, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 169, 170, 171, 174, 175, 178, 182, 183, 184, 187, 188, 193, 194, 199, 200, 205, 206, 207, 211, 212, 213, 217, 218, 219, 223, 224, 225, 228, 229, 230, 233, 234, 235, 240, 253, 254, 255, 271, 272, 273, 277, 278, 283, 284, 285, 289, 290, 291, 294, 295, 296, 299, 300, 301, 304, 305, 306, 311, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 338, 339, 340, 343, 344, 345, 362, 363, 364, 367, 368, 369, 372, 373, 374, 391, 392, 393, 396, 397, 398, 401, 402, 403, 420, 421, 422, 435, 436, 437, 482 }

F(-1) timedout fail { 37, 210, 215, 216, 221, 222, 238, 239, 242, 243, 244, 245, 247, 248, 249, 250, 252, 282, 288, 309, 310, 313, 314, 315, 317, 348, 349, 350, 352, 353, 354, 355, 357, 358, 359, 361, 377, 378, 379, 381, 382, 383, 384, 386, 387, 388, 390, 406, 407, 408, 410, 411, 412, 413, 415, 416, 417, 419, 425, 426, 427, 429, 431, 432, 434, 440, 441, 442, 444, 445, 446, 448, 449, 450, 451, 453, 454, 455, 456, 458, 459, 460, 461, 463, 464, 465, 466, 468, 469, 470, 472, 473, 475, 476, 477, 478, 480, 481, 483, 485, 486, 487, 488, 490, 491, 492, 493, 495, 497, 499, 500, 502 }

F(-2) exception fail { 430, 496 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 30, 68, 69, 70, 96, 97, 98, 102, 103, 104, 110, 115, 118, 119, 120, 121, 157, 158, 159, 163, 164, 165, 189, 190, 195, 196, 201, 202, 208, 214, 220, 226, 231, 236, 241, 246, 251, 256, 259, 260, 261, 262, 265, 266, 267, 268, 274, 279, 280, 286, 292, 297, 302, 307, 312, 316, 321, 324, 336, 341, 346, 351, 356, 360, 365, 370, 375, 380, 385, 389, 394, 399, 404, 409, 414, 418, 423, 428, 433, 443, 452, 457, 462, 467, 471, 474, 479, 484, 489, 494, 498, 501 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 28, 29, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 71, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 105, 106, 107, 108, 109, 113, 114, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 160, 161, 162, 166, 167, 168, 169, 170, 171, 174, 175, 178, 182, 183, 184, 187, 188, 193, 194, 199, 200, 205, 206, 207, 211, 212, 213, 217, 218, 219, 223, 224, 225, 228, 229, 230, 233, 234, 235, 238, 239, 240, 243, 244, 245, 248, 249, 250, 253, 254, 255, 263, 264, 269, 270, 271, 272, 273, 277, 278, 283, 284, 285, 289, 290, 291, 294, 295, 296, 299, 300, 301, 304, 305, 306, 309, 310, 311, 314, 315, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 338, 339, 340, 343, 344, 345, 348, 349, 350, 353, 354, 355, 358, 359, 362, 363, 364, 367, 368, 369, 372, 373, 374, 377, 378, 379, 382, 383, 384, 387, 388, 391, 392, 393, 396, 397, 398, 401, 402, 403, 406, 407, 408, 411, 412, 413, 416, 417, 420, 421, 422, 425, 426, 427, 430, 431, 432, 435, 436, 437, 438, 440, 441, 442, 445, 446, 447, 449, 450, 451, 454, 455, 456, 459, 460, 461, 464, 465, 466, 469, 470, 473, 476, 477, 478, 481, 482, 483, 486, 487, 488, 491, 492, 493, 496, 497, 500 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 4, 19, 96, 97, 98, 102, 103, 104, 110, 115, 159, 165, 189, 190, 195, 196, 201, 202, 256, 259, 260, 261, 262, 365, 394 }

B grade { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 157, 158, 163, 164, 231, 265, 266, 267, 268, 292, 297, 336 }

C grade { 60, 61, 62, 63, 64, 226 }

F normal fail { 5, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 28, 29, 30, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 68, 69, 70, 71, 73, 74, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 105, 106, 107, 108, 109, 113, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 133, 136, 137, 138, 141, 142, 143, 160, 166, 167, 168, 169, 170, 171, 187, 188, 193, 194, 199, 200, 205, 206, 207, 208, 211, 212, 213, 214, 218, 219, 220, 223, 224, 225, 238, 239, 240, 241, 243, 244, 245, 246, 249, 250, 251, 253, 254, 255, 263, 269, 271, 272, 273, 274, 277, 278, 279, 280, 283, 284, 285, 286, 290, 291, 304, 305, 306, 307, 309, 310, 311, 312, 314, 315, 316, 348, 349, 350, 351, 353, 354, 355, 356, 358, 359, 360, 377, 378, 379, 380, 382, 383, 384, 385, 387, 388, 389, 406, 407, 408, 409, 411, 412, 413, 414, 416, 417, 418, 420, 421, 422, 423, 425, 426, 427, 428, 430, 431, 432, 433, 438, 443, 449, 450, 451, 452, 454, 455, 456, 457, 459, 460, 461, 462, 467, 477, 478, 479, 482, 483, 484, 486, 487, 488, 489 }

F(-1) timedout fail { 6, 7, 52, 100, 101, 116, 117, 130, 131, 132, 134, 135, 146, 147, 148, 149, 151, 156, 161, 162, 174, 175, 176, 177, 178, 179, 180, 181, 198, 203, 204, 216, 217, 222, 227, 228, 229, 230, 232, 233, 234, 235, 236, 237, 248, 264, 270, 289, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 435, 436, 437, 440, 441, 442, 444, 445, 446, 447, 448, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 481, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502 }

F(-2) exception fail { 75, 78, 79, 80, 81, 82, 83, 84, 152, 153, 154, 182, 183, 184 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	116	76	185	326	169	311	324	215
N.S.	1	1.27	0.84	2.03	3.58	1.86	3.42	3.56	2.36
time (sec)	N/A	0.593	0.183	0.988	0.208	0.256	0.352	0.257	0.201

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	92	61	126	222	109	202	204	143
N.S.	1	1.31	0.87	1.80	3.17	1.56	2.89	2.91	2.04
time (sec)	N/A	0.456	0.125	1.136	0.210	0.242	0.270	0.283	0.871

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	61	44	91	134	62	112	112	82
N.S.	1	1.24	0.90	1.86	2.73	1.27	2.29	2.29	1.67
time (sec)	N/A	0.344	0.097	0.993	0.198	0.242	0.210	0.280	0.096

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	37	27	37	68	29	46	46	35
N.S.	1	1.32	0.96	1.32	2.43	1.04	1.64	1.64	1.25
time (sec)	N/A	0.228	0.082	0.582	0.197	0.233	0.156	0.276	0.791

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	61	49	82	57	94	0	57	0
N.S.	1	1.20	0.96	1.61	1.12	1.84	0.00	1.12	0.00
time (sec)	N/A	0.402	0.057	0.793	0.238	0.234	0.000	0.268	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	84	65	133	80	148	0	615	0
N.S.	1	1.18	0.92	1.87	1.13	2.08	0.00	8.66	0.00
time (sec)	N/A	0.513	0.167	0.880	0.232	0.243	0.000	0.298	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	121	88	277	94	254	0	301	0
N.S.	1	1.16	0.85	2.66	0.90	2.44	0.00	2.89	0.00
time (sec)	N/A	0.643	0.335	0.905	0.235	0.245	0.000	0.270	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	167	132	145	382	312	660	374	334
N.S.	1	1.03	0.81	0.90	2.36	1.93	4.07	2.31	2.06
time (sec)	N/A	0.513	0.366	1.020	0.210	0.241	0.511	0.271	1.429

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	129	104	121	263	209	456	243	229
N.S.	1	0.96	0.78	0.90	1.96	1.56	3.40	1.81	1.71
time (sec)	N/A	0.360	0.258	0.986	0.203	0.250	0.361	0.279	1.124

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	97	75	79	165	123	264	136	127
N.S.	1	1.02	0.79	0.83	1.74	1.29	2.78	1.43	1.34
time (sec)	N/A	0.319	0.187	0.883	0.197	0.265	0.258	0.272	0.175

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	52	64	88	64	126	63	60
N.S.	1	1.00	0.95	1.16	1.60	1.16	2.29	1.15	1.09
time (sec)	N/A	0.212	0.181	0.493	0.192	0.241	0.196	0.275	0.106

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	66	97	72	104	0	68	0
N.S.	1	1.00	0.85	1.24	0.92	1.33	0.00	0.87	0.00
time (sec)	N/A	0.360	0.161	3.106	0.228	0.260	0.000	0.266	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	94	75	152	88	166	0	574	0
N.S.	1	1.16	0.93	1.88	1.09	2.05	0.00	7.09	0.00
time (sec)	N/A	0.547	0.276	2.960	0.227	0.249	0.000	0.318	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	147	102	299	99	280	0	330	0
N.S.	1	1.31	0.91	2.67	0.88	2.50	0.00	2.95	0.00
time (sec)	N/A	0.484	0.608	3.221	0.243	0.254	0.000	0.278	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	170	123	555	110	411	0	537	0
N.S.	1	1.05	0.76	3.43	0.68	2.54	0.00	3.31	0.00
time (sec)	N/A	0.761	0.582	3.114	0.247	0.268	0.000	0.270	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	B	B	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	0	150	181	639	528	772	654	532
N.S.	1	0.00	0.67	0.80	2.84	2.35	3.43	2.91	2.36
time (sec)	N/A	0.000	0.619	1.895	0.216	0.246	0.671	0.271	1.290

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	246	127	142	435	345	495	414	364
N.S.	1	1.41	0.73	0.81	2.49	1.97	2.83	2.37	2.08
time (sec)	N/A	1.142	0.612	1.657	0.213	0.243	0.501	0.297	0.338

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	150	86	106	269	199	284	230	184
N.S.	1	1.22	0.70	0.86	2.19	1.62	2.31	1.87	1.50
time (sec)	N/A	0.633	0.249	1.613	0.198	0.234	0.358	0.286	1.146

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	88	59	63	141	97	126	98	79
N.S.	1	1.17	0.79	0.84	1.88	1.29	1.68	1.31	1.05
time (sec)	N/A	0.368	0.207	1.410	0.199	0.244	0.241	0.267	0.149

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	133	102	166	117	188	0	113	0
N.S.	1	1.10	0.84	1.37	0.97	1.55	0.00	0.93	0.00
time (sec)	N/A	0.493	0.263	2.220	0.237	0.244	0.000	0.276	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	156	160	271	145	301	0	1076	0
N.S.	1	1.08	1.10	1.87	1.00	2.08	0.00	7.42	0.00
time (sec)	N/A	0.455	0.730	2.367	0.249	0.251	0.000	0.333	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	264	220	562	145	529	0	601	0
N.S.	1	1.43	1.20	3.05	0.79	2.88	0.00	3.27	0.00
time (sec)	N/A	0.997	0.567	2.381	0.255	0.251	0.000	0.276	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	173	168	541	333	396	0	0	0
N.S.	1	1.16	1.13	3.63	2.23	2.66	0.00	0.00	0.00
time (sec)	N/A	0.688	0.202	1.865	0.262	0.249	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	115	118	306	195	242	0	0	0
N.S.	1	1.16	1.19	3.09	1.97	2.44	0.00	0.00	0.00
time (sec)	N/A	0.475	0.139	1.680	0.261	0.255	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	61	116	89	0	119	0	0	0
N.S.	1	1.22	2.32	1.78	0.00	2.38	0.00	0.00	0.00
time (sec)	N/A	0.289	0.128	0.876	0.000	0.273	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	18
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.29
time (sec)	N/A	0.205	14.735	0.341	0.474	0.238	0.368	0.359	0.807

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	27	14	16	18
N.S.	1	1.00	1.14	1.00	1.14	1.93	1.00	1.14	1.29
time (sec)	N/A	0.205	15.046	0.355	0.458	0.252	0.507	2.132	0.802

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	146	185	473	320	1159	0	0	0
N.S.	1	1.42	1.80	4.59	3.11	11.25	0.00	0.00	0.00
time (sec)	N/A	0.707	0.718	2.080	0.338	0.276	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	106	137	240	0	623	0	0	0
N.S.	1	1.43	1.85	3.24	0.00	8.42	0.00	0.00	0.00
time (sec)	N/A	0.498	0.530	1.765	0.000	0.244	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	33	52	56	91	166	0	80	49
N.S.	1	1.14	1.79	1.93	3.14	5.72	0.00	2.76	1.69
time (sec)	N/A	0.265	0.171	1.315	0.204	0.271	0.000	0.269	0.083

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	157	18	14	18	18
N.S.	1	1.00	1.12	1.00	9.81	1.12	0.88	1.12	1.12
time (sec)	N/A	0.215	22.428	0.227	0.366	0.241	0.381	0.289	0.840

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	237	29	15	18	18
N.S.	1	1.00	1.12	1.00	14.81	1.81	0.94	1.12	1.12
time (sec)	N/A	0.212	22.681	0.240	0.368	0.247	0.531	0.334	0.831

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	294	440	876	605	4008	0	0	0
N.S.	1	1.15	1.72	3.42	2.36	15.66	0.00	0.00	0.00
time (sec)	N/A	1.190	2.763	1.952	0.335	0.321	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	184	420	444	393	2218	0	0	0
N.S.	1	1.19	2.73	2.88	2.55	14.40	0.00	0.00	0.00
time (sec)	N/A	0.733	6.723	1.759	0.342	0.256	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	108	271	197	0	1026	0	0	0
N.S.	1	1.17	2.95	2.14	0.00	11.15	0.00	0.00	0.00
time (sec)	N/A	0.417	0.224	1.355	0.000	0.262	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	433	18	14	18	18
N.S.	1	1.00	1.12	1.00	27.06	1.12	0.88	1.12	1.12
time (sec)	N/A	0.220	65.117	0.233	0.525	0.257	0.371	2.498	0.853

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	545	29	15	0	18
N.S.	1	1.00	1.12	1.00	34.06	1.81	0.94	0.00	1.12
time (sec)	N/A	0.222	69.345	0.216	0.666	0.249	0.541	0.000	0.908

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	204	108	0	308	521	0	232	0
N.S.	1	1.19	0.63	0.00	1.80	3.05	0.00	1.36	0.00
time (sec)	N/A	0.832	0.049	0.000	0.224	0.258	0.000	0.316	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	146	175	106	0	268	385	0	202	0
N.S.	1	1.20	0.73	0.00	1.84	2.64	0.00	1.38	0.00
time (sec)	N/A	0.650	0.062	0.000	0.216	0.241	0.000	0.306	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	141	104	0	230	301	0	168	0
N.S.	1	1.15	0.85	0.00	1.87	2.45	0.00	1.37	0.00
time (sec)	N/A	0.509	0.050	0.000	0.194	0.268	0.000	0.275	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	112	104	0	181	122	0	90	0
N.S.	1	1.08	1.00	0.00	1.74	1.17	0.00	0.87	0.00
time (sec)	N/A	0.394	0.028	0.000	0.221	0.264	0.000	0.282	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	139	120	0	103	339	0	0	0
N.S.	1	1.18	1.02	0.00	0.87	2.87	0.00	0.00	0.00
time (sec)	N/A	0.512	0.099	0.000	0.209	0.253	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	176	161	0	114	532	0	0	0
N.S.	1	1.18	1.08	0.00	0.77	3.57	0.00	0.00	0.00
time (sec)	N/A	0.633	0.468	0.000	0.281	0.269	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	174	207	168	0	114	855	0	0	0
N.S.	1	1.19	0.97	0.00	0.66	4.91	0.00	0.00	0.00
time (sec)	N/A	0.771	0.367	0.000	0.271	0.274	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	239	246	137	0	281	1001	0	0	0
N.S.	1	1.03	0.57	0.00	1.18	4.19	0.00	0.00	0.00
time (sec)	N/A	0.719	0.385	0.000	0.294	0.257	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	218	137	0	239	755	0	0	0
N.S.	1	1.03	0.65	0.00	1.13	3.58	0.00	0.00	0.00
time (sec)	N/A	0.620	0.219	0.000	0.273	0.267	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	129	0	189	590	0	0	0
N.S.	1	1.00	0.78	0.00	1.14	3.55	0.00	0.00	0.00
time (sec)	N/A	0.490	0.380	0.000	0.296	0.257	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	142	0	107	155	0	115	0
N.S.	1	1.00	1.02	0.00	0.77	1.12	0.00	0.83	0.00
time (sec)	N/A	0.421	0.087	0.000	0.297	0.271	0.000	0.299	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	160	152	0	116	571	0	0	0
N.S.	1	1.13	1.07	0.00	0.82	4.02	0.00	0.00	0.00
time (sec)	N/A	0.557	0.390	0.000	0.287	0.274	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	174	221	156	0	118	864	0	0	0
N.S.	1	1.27	0.90	0.00	0.68	4.97	0.00	0.00	0.00
time (sec)	N/A	0.603	0.744	0.000	0.283	0.263	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	220	243	204	0	118	1352	0	0	0
N.S.	1	1.10	0.93	0.00	0.54	6.15	0.00	0.00	0.00
time (sec)	N/A	0.791	0.575	0.000	0.282	0.277	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	251	304	222	0	118	1827	0	0	0
N.S.	1	1.21	0.88	0.00	0.47	7.28	0.00	0.00	0.00
time (sec)	N/A	0.826	0.359	0.000	0.259	0.280	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	381	565	194	0	513	2090	0	0	0
N.S.	1	1.48	0.51	0.00	1.35	5.49	0.00	0.00	0.00
time (sec)	N/A	2.100	0.471	0.000	0.280	0.268	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	325	485	211	0	430	1543	0	0	0
N.S.	1	1.49	0.65	0.00	1.32	4.75	0.00	0.00	0.00
time (sec)	N/A	1.672	0.376	0.000	0.305	0.274	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	291	209	0	333	1216	0	0	0
N.S.	1	1.06	0.76	0.00	1.21	4.42	0.00	0.00	0.00
time (sec)	N/A	0.703	0.237	0.000	0.281	0.255	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	228	240	191	0	178	252	0	0	0
N.S.	1	1.05	0.84	0.00	0.78	1.11	0.00	0.00	0.00
time (sec)	N/A	0.601	0.140	0.000	0.276	0.249	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	246	265	245	0	197	1346	0	0	0
N.S.	1	1.08	1.00	0.00	0.80	5.47	0.00	0.00	0.00
time (sec)	N/A	0.636	0.789	0.000	0.293	0.260	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	277	422	253	0	196	2059	0	0	0
N.S.	1	1.52	0.91	0.00	0.71	7.43	0.00	0.00	0.00
time (sec)	N/A	1.242	2.019	0.000	0.318	0.297	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	331	480	376	0	197	3286	0	0	0
N.S.	1	1.45	1.14	0.00	0.60	9.93	0.00	0.00	0.00
time (sec)	N/A	1.453	1.223	0.000	0.296	0.313	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	B	B	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	140	50	132	175	189	133	146	0
N.S.	1	1.26	0.45	1.19	1.58	1.70	1.20	1.32	0.00
time (sec)	N/A	0.542	0.013	0.501	0.190	0.251	10.630	0.274	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	B	B	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	110	49	120	149	137	99	108	0
N.S.	1	1.20	0.53	1.30	1.62	1.49	1.08	1.17	0.00
time (sec)	N/A	0.418	0.012	0.422	0.192	0.257	0.901	0.271	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	B	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	85	47	71	116	58	70	61	0
N.S.	1	1.10	0.61	0.92	1.51	0.75	0.91	0.79	0.00
time (sec)	N/A	0.328	0.008	0.355	0.184	0.253	0.537	0.257	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	A	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	108	49	120	74	137	94	0	0
N.S.	1	1.24	0.56	1.38	0.85	1.57	1.08	0.00	0.00
time (sec)	N/A	0.416	0.015	0.503	0.193	0.255	1.512	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	A	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	141	84	132	57	178	129	0	0
N.S.	1	1.24	0.74	1.16	0.50	1.56	1.13	0.00	0.00
time (sec)	N/A	0.523	0.059	0.491	0.229	0.260	11.085	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	18
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	1.12
time (sec)	N/A	0.204	27.995	0.487	0.820	0.232	0.608	0.313	0.784

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	18
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	1.12
time (sec)	N/A	0.212	36.874	0.401	1.075	0.254	0.485	0.278	0.817

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	10	0	10	10	10
N.S.	1	1.00	1.20	0.80	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.322	4.864	0.202	0.319	0.000	4.454	0.293	0.725

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0	38
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	1.90
time (sec)	N/A	0.189	0.134	0.000	0.000	0.000	0.000	0.000	0.865

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	22	0	0	108	0	0	40
N.S.	1	1.00	0.92	0.00	0.00	4.50	0.00	0.00	1.67
time (sec)	N/A	0.197	0.075	0.000	0.000	0.248	0.000	0.000	0.835

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	47	47	33	0	0	0	0	0	111
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	2.36
time (sec)	N/A	0.223	0.111	0.000	0.000	0.000	0.000	0.000	1.005

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	68	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.249	1.007	0.000	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11
time (sec)	N/A	0.225	2.226	0.444	0.346	0.238	12.871	0.503	0.855

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	237	249	206	0	161	340	0	0	0
N.S.	1	1.05	0.87	0.00	0.68	1.43	0.00	0.00	0.00
time (sec)	N/A	0.536	0.149	0.000	0.099	0.087	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	131	0	102	241	0	0	0
N.S.	1	1.00	0.91	0.00	0.71	1.67	0.00	0.00	0.00
time (sec)	N/A	0.424	0.120	0.000	0.071	0.079	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	118	101	0	79	168	0	0	0
N.S.	1	1.07	0.92	0.00	0.72	1.53	0.00	0.00	0.00
time (sec)	N/A	0.308	0.037	0.000	0.072	0.077	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	18
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.29
time (sec)	N/A	0.203	11.187	0.296	0.308	0.247	0.312	0.272	0.763

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12
time (sec)	N/A	0.225	3.061	0.340	0.326	0.244	0.325	0.282	0.842

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	67	54	73	55	86	0	0	0
N.S.	1	1.14	0.92	1.24	0.93	1.46	0.00	0.00	0.00
time (sec)	N/A	0.280	0.024	0.413	0.087	0.076	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	67	53	73	55	86	0	0	0
N.S.	1	1.14	0.90	1.24	0.93	1.46	0.00	0.00	0.00
time (sec)	N/A	0.278	0.021	0.384	0.088	0.078	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	67	54	73	55	86	0	0	0
N.S.	1	1.14	0.92	1.24	0.93	1.46	0.00	0.00	0.00
time (sec)	N/A	0.277	0.017	0.370	0.094	0.076	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	67	53	73	55	78	0	0	0
N.S.	1	1.14	0.90	1.24	0.93	1.32	0.00	0.00	0.00
time (sec)	N/A	0.270	0.014	0.324	0.091	0.070	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	57	49	67	43	78	0	0	0
N.S.	1	1.16	1.00	1.37	0.88	1.59	0.00	0.00	0.00
time (sec)	N/A	0.258	0.016	0.381	0.098	0.084	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	63	51	67	55	86	0	0	0
N.S.	1	1.15	0.93	1.22	1.00	1.56	0.00	0.00	0.00
time (sec)	N/A	0.271	0.015	0.385	0.097	0.075	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	67	54	71	55	86	0	0	0
N.S.	1	1.14	0.92	1.20	0.93	1.46	0.00	0.00	0.00
time (sec)	N/A	0.274	0.017	0.396	0.102	0.077	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	71	136	0	0	0
N.S.	1	1.00	0.92	0.00	0.83	1.58	0.00	0.00	0.00
time (sec)	N/A	0.366	0.092	0.000	0.088	0.078	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	78	0	71	136	0	0	0
N.S.	1	1.00	0.92	0.00	0.84	1.60	0.00	0.00	0.00
time (sec)	N/A	0.340	0.091	0.000	0.083	0.075	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	71	136	0	0	0
N.S.	1	1.00	0.92	0.00	0.83	1.58	0.00	0.00	0.00
time (sec)	N/A	0.337	0.096	0.000	0.094	0.077	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	71	122	0	0	0
N.S.	1	1.00	0.89	0.00	0.84	1.44	0.00	0.00	0.00
time (sec)	N/A	0.328	0.078	0.000	0.084	0.075	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	63	0	55	117	0	0	0
N.S.	1	1.00	0.88	0.00	0.76	1.62	0.00	0.00	0.00
time (sec)	N/A	0.320	0.055	0.000	0.094	0.077	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	72	0	0	136	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.341	0.077	0.000	0.000	0.081	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	136	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.337	0.083	0.000	0.000	0.074	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.238	0.153	0.000	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.252	0.146	0.000	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	47	47	45	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	0.088	0.000	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	67	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.314	0.000	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	128	112	235	284	517	262	196
N.S.	1	1.00	1.31	1.14	2.40	2.90	5.28	2.67	2.00
time (sec)	N/A	0.351	0.386	1.021	0.195	0.245	0.353	0.280	1.127

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	88	85	141	172	314	150	118
N.S.	1	1.00	1.19	1.15	1.91	2.32	4.24	2.03	1.59
time (sec)	N/A	0.297	0.298	0.945	0.195	0.248	0.276	0.282	0.970

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	48	66	81	162	69	56
N.S.	1	1.00	0.96	0.96	1.32	1.62	3.24	1.38	1.12
time (sec)	N/A	0.245	0.112	0.750	0.205	0.243	0.186	0.279	0.908

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	60	96	71	79	0	69	0
N.S.	1	1.00	0.86	1.37	1.01	1.13	0.00	0.99	0.00
time (sec)	N/A	0.359	0.464	0.980	0.238	0.268	0.000	0.275	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	83	153	88	134	0	630	0
N.S.	1	1.00	0.87	1.61	0.93	1.41	0.00	6.63	0.00
time (sec)	N/A	0.395	0.467	1.000	0.256	0.255	0.000	0.282	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	109	303	99	222	0	322	0
N.S.	1	1.00	0.83	2.31	0.76	1.69	0.00	2.46	0.00
time (sec)	N/A	0.459	0.605	1.074	0.253	0.260	0.000	0.267	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	232	220	191	525	596	1134	580	393
N.S.	1	0.95	0.90	0.78	2.14	2.43	4.63	2.37	1.60
time (sec)	N/A	0.542	0.937	1.191	0.233	0.259	0.617	0.288	1.883

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	189	141	326	351	694	333	217
N.S.	1	1.00	1.09	0.81	1.87	2.02	3.99	1.91	1.25
time (sec)	N/A	0.429	0.523	1.029	0.194	0.261	0.454	0.272	1.491

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	102	86	84	167	164	357	155	104
N.S.	1	0.84	0.70	0.69	1.37	1.34	2.93	1.27	0.85
time (sec)	N/A	0.307	12.522	0.812	0.198	0.250	0.322	0.267	0.418

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	143	117	193	150	149	0	135	0
N.S.	1	0.96	0.79	1.30	1.01	1.00	0.00	0.91	0.00
time (sec)	N/A	0.581	0.770	2.357	0.249	0.253	0.000	0.272	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	173	214	313	183	266	0	1134	0
N.S.	1	1.02	1.26	1.84	1.08	1.56	0.00	6.67	0.00
time (sec)	N/A	0.566	0.804	2.410	0.251	0.248	0.000	0.335	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	327	198	625	205	454	0	682	0
N.S.	1	1.39	0.84	2.65	0.87	1.92	0.00	2.89	0.00
time (sec)	N/A	0.895	2.062	2.530	0.263	0.262	0.000	0.270	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	146	206	435	238	364	0	0	0
N.S.	1	1.11	1.56	3.30	1.80	2.76	0.00	0.00	0.00
time (sec)	N/A	0.793	0.994	1.445	0.309	0.254	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	115	152	227	0	201	0	0	0
N.S.	1	1.14	1.50	2.25	0.00	1.99	0.00	0.00	0.00
time (sec)	N/A	0.604	0.723	1.338	0.000	0.236	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	65	185	66	75	60	56	67	56
N.S.	1	1.03	2.94	1.05	1.19	0.95	0.89	1.06	0.89
time (sec)	N/A	0.363	0.378	1.526	0.181	0.237	0.111	0.267	1.091

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	102	126	117	23	24
N.S.	1	1.00	1.09	0.91	4.43	5.48	5.09	1.00	1.04
time (sec)	N/A	0.237	26.868	0.343	0.298	0.249	3.311	0.268	0.942

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	158	204	187	23	24
N.S.	1	1.00	1.09	0.91	6.87	8.87	8.13	1.00	1.04
time (sec)	N/A	0.235	26.722	0.342	0.333	0.257	10.174	0.317	0.995

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	308	473	723	635	917	0	0	0
N.S.	1	1.01	1.55	2.37	2.08	3.01	0.00	0.00	0.00
time (sec)	N/A	1.281	2.885	1.941	0.377	0.263	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	269	374	0	482	0	0	0
N.S.	1	1.00	1.12	1.55	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	0.978	1.688	1.411	0.000	0.237	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	154	241	113	255	160	167	195	160
N.S.	1	0.97	1.53	0.72	1.61	1.01	1.06	1.23	1.01
time (sec)	N/A	0.532	1.282	1.205	0.209	0.245	0.217	0.272	1.346

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	606	793	0	23	24
N.S.	1	1.00	1.09	0.91	26.35	34.48	0.00	1.00	1.04
time (sec)	N/A	0.242	25.917	0.509	0.527	0.259	0.000	0.286	1.061

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	723	965	0	23	24
N.S.	1	1.00	1.09	0.91	31.43	41.96	0.00	1.00	1.04
time (sec)	N/A	0.236	26.605	0.501	0.725	0.257	0.000	0.390	1.165

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	204	141	174	0	0	0	0	149
N.S.	1	1.13	0.78	0.96	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.773	0.266	0.901	0.000	0.000	0.000	0.000	0.799

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	164	125	151	0	0	0	0	126
N.S.	1	1.21	0.92	1.11	0.00	0.00	0.00	0.00	0.93
time (sec)	N/A	0.648	2.143	0.546	0.000	0.000	0.000	0.000	1.121

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	130	105	128	0	0	0	0	92
N.S.	1	1.17	0.95	1.15	0.00	0.00	0.00	0.00	0.83
time (sec)	N/A	0.527	0.764	0.467	0.000	0.000	0.000	0.000	1.072

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	90	87	105	0	0	0	0	80
N.S.	1	1.36	1.32	1.59	0.00	0.00	0.00	0.00	1.21
time (sec)	N/A	0.375	0.361	0.483	0.000	0.000	0.000	0.000	1.058

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	89	96	0	0	0	0	0	0
N.S.	1	0.71	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.503	0.312	0.000	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	122	133	0	0	0	0	0	0
N.S.	1	0.82	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.609	0.479	0.000	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	204	159	170	0	0	0	0	0	0
N.S.	1	0.78	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.744	0.587	0.000	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	377	398	269	0	0	0	0	0	0
N.S.	1	1.06	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.462	7.661	0.000	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	303	281	173	0	0	0	0	0	0
N.S.	1	0.93	0.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.881	7.465	0.000	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	185	176	138	0	0	0	0	0	0
N.S.	1	0.95	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.558	3.405	0.000	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	261	149	146	0	0	0	0	0	0
N.S.	1	0.57	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.538	1.721	0.000	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	302	184	243	0	0	0	0	0	0
N.S.	1	0.61	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.525	2.023	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	638	0	2918	0	0	0	0	0	0
N.S.	1	0.00	4.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	15.617	0.000	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	506	460	300	0	0	0	0	0	0
N.S.	1	0.91	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.289	9.753	0.000	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	312	262	218	0	0	0	0	0	0
N.S.	1	0.84	0.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.687	9.433	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	403	205	242	0	0	0	0	0	0
N.S.	1	0.51	0.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.635	4.701	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	444	240	347	0	0	0	0	0	0
N.S.	1	0.54	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.617	5.362	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	536	408	4751	0	0	0	0	0	0
N.S.	1	0.76	8.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.856	10.656	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	493	288	331	0	0	0	0	0	0
N.S.	1	0.58	0.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.902	0.811	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	349	210	276	0	0	0	0	0	0
N.S.	1	0.60	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.675	0.604	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	134	178	0	0	0	0	0	0
N.S.	1	0.65	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.429	0.448	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	19	40	19	19	20
N.S.	1	1.00	1.10	0.81	0.90	1.90	0.90	0.90	0.95
time (sec)	N/A	0.247	3.740	0.175	0.336	0.245	2.573	0.425	1.116

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	19	44	20	19	20
N.S.	1	1.00	1.10	0.81	0.90	2.10	0.95	0.90	0.95
time (sec)	N/A	0.256	3.883	0.175	0.340	0.252	4.811	0.433	1.108

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	807	475	546	0	0	0	0	0	0
N.S.	1	0.59	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.601	2.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	506	320	384	0	0	0	0	0	0
N.S.	1	0.63	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.996	1.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	288	215	258	0	0	0	0	0	0
N.S.	1	0.75	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.596	0.766	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	19	209	19	19	20
N.S.	1	1.00	1.10	0.81	0.90	9.95	0.90	0.90	0.95
time (sec)	N/A	0.261	21.864	0.180	0.351	0.244	28.433	0.455	1.665

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	19	209	20	19	20
N.S.	1	1.00	1.10	0.81	0.90	9.95	0.95	0.90	0.95
time (sec)	N/A	0.257	23.838	0.178	0.357	0.253	52.714	0.445	1.649

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1016	737	1200	0	0	0	0	0	0
N.S.	1	0.73	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.711	2.993	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	689	484	482	0	0	0	0	0	0
N.S.	1	0.70	0.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.598	1.477	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	416	299	337	0	0	0	0	0	0
N.S.	1	0.72	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.792	1.056	0.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	19	396	0	19	20
N.S.	1	1.00	1.10	0.81	0.90	18.86	0.00	0.90	0.95
time (sec)	N/A	0.257	36.544	0.193	0.354	0.251	0.000	0.490	2.069

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	19	0	17	19	20
N.S.	1	1.00	1.10	0.81	0.90	0.00	0.81	0.90	0.95
time (sec)	N/A	0.245	4.245	0.168	0.354	0.000	2.278	0.361	0.910

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	48	0	23	24
N.S.	1	1.00	1.09	0.91	1.00	2.09	0.00	1.00	1.04
time (sec)	N/A	0.240	3.029	0.391	0.352	0.266	0.000	0.362	1.065

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	410	394	410	0	375	374	0	0	0
N.S.	1	0.96	1.00	0.00	0.91	0.91	0.00	0.00	0.00
time (sec)	N/A	0.861	1.044	0.000	0.144	0.105	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	262	229	0	210	259	0	0	0
N.S.	1	0.98	0.85	0.00	0.78	0.97	0.00	0.00	0.00
time (sec)	N/A	0.629	0.525	0.000	0.090	0.094	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	207	0	101	134	0	0	0
N.S.	1	1.00	1.53	0.00	0.75	0.99	0.00	0.00	0.00
time (sec)	N/A	0.378	0.385	0.000	0.072	0.087	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	86	20	23	24
N.S.	1	1.00	1.09	0.91	1.00	3.74	0.87	1.00	1.04
time (sec)	N/A	0.237	3.559	0.305	0.289	0.264	11.915	0.272	0.967

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	696	0	23	24
N.S.	1	1.00	1.09	0.91	1.00	30.26	0.00	1.00	1.04
time (sec)	N/A	0.231	13.664	0.556	0.310	0.254	0.000	0.285	1.154

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	123	110	234	168	264	258	187
N.S.	1	1.00	1.38	1.24	2.63	1.89	2.97	2.90	2.10
time (sec)	N/A	0.340	0.260	1.238	0.196	0.248	0.295	0.269	0.204

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	83	83	139	102	151	146	110
N.S.	1	1.00	1.24	1.24	2.07	1.52	2.25	2.18	1.64
time (sec)	N/A	0.292	0.189	1.279	0.195	0.244	0.216	0.284	0.117

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	46	65	51	68	64	49
N.S.	1	1.00	0.96	1.02	1.44	1.13	1.51	1.42	1.09
time (sec)	N/A	0.231	0.127	0.968	0.195	0.234	0.169	0.292	0.822

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	57	94	71	111	0	68	0
N.S.	1	1.00	0.89	1.47	1.11	1.73	0.00	1.06	0.00
time (sec)	N/A	0.334	0.102	1.006	0.224	0.248	0.000	0.278	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	71	149	88	162	0	630	0
N.S.	1	1.00	0.82	1.71	1.01	1.86	0.00	7.24	0.00
time (sec)	N/A	0.375	0.272	1.097	0.232	0.245	0.000	0.309	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	95	296	99	274	0	319	0
N.S.	1	1.00	0.77	2.41	0.80	2.23	0.00	2.59	0.00
time (sec)	N/A	0.437	0.408	1.079	0.232	0.251	0.000	0.270	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	237	235	213	520	418	779	598	481
N.S.	1	0.95	0.94	0.85	2.08	1.67	3.12	2.39	1.92
time (sec)	N/A	0.542	0.692	1.482	0.220	0.256	0.427	0.289	2.613

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	249	157	322	247	456	344	281
N.S.	1	1.00	1.37	0.86	1.77	1.36	2.51	1.89	1.54
time (sec)	N/A	0.436	0.416	1.354	0.204	0.249	0.328	0.273	1.295

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	113	98	111	164	128	219	159	135
N.S.	1	0.97	0.84	0.96	1.41	1.10	1.89	1.37	1.16
time (sec)	N/A	0.315	4.399	1.053	0.195	0.245	0.226	0.285	0.161

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	134	201	148	232	0	144	0
N.S.	1	1.00	0.86	1.29	0.95	1.49	0.00	0.92	0.00
time (sec)	N/A	0.550	0.223	4.197	0.243	0.259	0.000	0.270	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	232	319	181	357	0	1135	0
N.S.	1	1.00	1.27	1.74	0.99	1.95	0.00	6.20	0.00
time (sec)	N/A	0.593	0.444	4.282	0.249	0.258	0.000	0.354	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	395	626	203	590	0	678	0
N.S.	1	1.00	1.63	2.59	0.84	2.44	0.00	2.80	0.00
time (sec)	N/A	0.719	0.615	4.362	0.257	0.249	0.000	0.280	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	404	377	318	0	0	1004	0	0	0
N.S.	1	0.93	0.79	0.00	0.00	2.49	0.00	0.00	0.00
time (sec)	N/A	1.673	0.184	0.000	0.000	0.278	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	296	283	233	0	0	708	0	0	0
N.S.	1	0.96	0.79	0.00	0.00	2.39	0.00	0.00	0.00
time (sec)	N/A	1.230	0.093	0.000	0.000	0.263	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	189	142	393	0	455	0	0	0
N.S.	1	1.01	0.76	2.10	0.00	2.43	0.00	0.00	0.00
time (sec)	N/A	0.753	0.028	1.281	0.000	0.263	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	27	17	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.35	0.85	1.10	1.10
time (sec)	N/A	0.237	0.892	0.518	0.308	0.246	10.270	0.286	0.877

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	51	19	22	22
N.S.	1	1.00	1.10	1.00	1.10	2.55	0.95	1.10	1.10
time (sec)	N/A	0.235	0.830	0.575	0.365	0.236	59.085	0.468	0.911

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	549	520	428	0	0	3957	0	0	0
N.S.	1	0.95	0.78	0.00	0.00	7.21	0.00	0.00	0.00
time (sec)	N/A	2.683	1.043	0.000	0.000	0.329	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	263	194	519	0	1717	0	0	0
N.S.	1	1.04	0.76	2.04	0.00	6.76	0.00	0.00	0.00
time (sec)	N/A	1.060	0.617	1.527	0.000	0.274	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	406	55	0	22	22
N.S.	1	1.00	1.10	1.00	20.30	2.75	0.00	1.10	1.10
time (sec)	N/A	0.238	26.611	0.575	0.620	0.244	0.000	0.592	1.001

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	606	96	0	22	22
N.S.	1	1.00	1.10	1.00	30.30	4.80	0.00	1.10	1.10
time (sec)	N/A	0.229	26.977	0.501	0.857	0.257	0.000	1.487	1.128

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	544	834	773	1232	0	6396	0	0	0
N.S.	1	1.53	1.42	2.26	0.00	11.76	0.00	0.00	0.00
time (sec)	N/A	3.631	5.029	2.046	0.000	0.354	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	1651	83	0	22	22
N.S.	1	1.00	1.10	1.00	82.55	4.15	0.00	1.10	1.10
time (sec)	N/A	0.232	46.764	0.745	2.412	0.264	0.000	1.471	0.980

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	2122	141	0	22	22
N.S.	1	1.00	1.10	1.00	106.10	7.05	0.00	1.10	1.10
time (sec)	N/A	0.236	46.118	0.852	3.622	0.288	0.000	60.711	1.092

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	0	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.00	1.10	1.10
time (sec)	N/A	0.235	3.174	0.435	0.343	0.261	0.000	0.408	0.979

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	543	543	448	0	377	829	0	0	0
N.S.	1	1.00	0.83	0.00	0.69	1.53	0.00	0.00	0.00
time (sec)	N/A	1.101	1.107	0.000	0.143	0.107	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	281	281	254	0	208	517	0	0	0
N.S.	1	1.00	0.90	0.00	0.74	1.84	0.00	0.00	0.00
time (sec)	N/A	0.622	0.464	0.000	0.088	0.088	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	118	0	101	249	0	0	0
N.S.	1	1.00	0.90	0.00	0.77	1.90	0.00	0.00	0.00
time (sec)	N/A	0.357	0.122	0.000	0.070	0.094	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	1.10
time (sec)	N/A	0.225	1.180	0.393	0.279	0.247	1.227	0.276	0.860

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	38	19	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.90	0.95	1.10	1.10
time (sec)	N/A	0.224	4.845	0.610	0.304	0.246	27.363	0.326	0.931

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	168	214	579	318	456	0	0	0
N.S.	1	1.03	1.31	3.55	1.95	2.80	0.00	0.00	0.00
time (sec)	N/A	0.949	1.609	1.984	0.290	0.262	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	137	187	321	0	261	0	0	0
N.S.	1	1.05	1.44	2.47	0.00	2.01	0.00	0.00	0.00
time (sec)	N/A	0.721	1.087	1.665	0.000	0.265	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	87	239	86	108	96	73	111	74
N.S.	1	0.97	2.66	0.96	1.20	1.07	0.81	1.23	0.82
time (sec)	N/A	0.473	0.752	1.888	0.223	0.252	0.150	0.268	1.298

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	38	61	28	36	33	24	33	27
N.S.	1	1.09	1.74	0.80	1.03	0.94	0.69	0.94	0.77
time (sec)	N/A	0.262	0.155	0.960	0.190	0.244	0.076	0.267	0.937

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	116	154	425	29	30
N.S.	1	1.00	1.07	0.93	4.00	5.31	14.66	1.00	1.03
time (sec)	N/A	0.226	37.760	0.463	0.264	0.263	12.079	0.297	1.006

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	194	232	631	29	30
N.S.	1	1.00	1.07	0.93	6.69	8.00	21.76	1.00	1.03
time (sec)	N/A	0.229	29.545	0.464	0.296	0.237	22.541	0.372	1.019

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	266	2619	699	671	823	0	0	0
N.S.	1	1.10	10.87	2.90	2.78	3.41	0.00	0.00	0.00
time (sec)	N/A	1.967	3.576	2.896	0.377	0.265	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	204	260	385	0	474	0	0	0
N.S.	1	1.11	1.41	2.09	0.00	2.58	0.00	0.00	0.00
time (sec)	N/A	1.474	1.952	2.503	0.000	0.263	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	130	238	134	238	172	224	251	143
N.S.	1	1.09	2.00	1.13	2.00	1.45	1.88	2.11	1.20
time (sec)	N/A	0.880	2.454	2.062	0.254	0.248	0.277	0.280	1.307

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	50	59	60	74	69	99	61	59
N.S.	1	0.96	1.13	1.15	1.42	1.33	1.90	1.17	1.13
time (sec)	N/A	0.364	0.174	1.177	0.190	0.243	0.140	0.279	1.063

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	177	203	950	31	32
N.S.	1	1.00	1.06	0.94	5.71	6.55	30.65	1.00	1.03
time (sec)	N/A	0.256	21.378	1.082	0.327	0.232	74.643	0.307	1.086

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	270	281	0	31	32
N.S.	1	1.00	1.06	0.94	8.71	9.06	0.00	1.00	1.03
time (sec)	N/A	0.255	22.822	1.062	0.364	0.257	0.000	0.414	1.098

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	0	376	1006	0	1044	0	0	0
N.S.	1	0.00	0.96	2.56	0.00	2.66	0.00	0.00	0.00
time (sec)	N/A	0.000	4.700	2.530	0.000	0.310	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	B	F(-2)	B	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	0	1661	560	0	595	0	0	0
N.S.	1	0.00	5.79	1.95	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	0.000	3.722	2.425	0.000	0.255	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	197	325	197	0	230	396	341	215
N.S.	1	1.13	1.86	1.13	0.00	1.31	2.26	1.95	1.23
time (sec)	N/A	1.368	3.196	2.026	0.000	0.254	0.406	0.278	1.464

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	87	109	95	98	96	175	87	94
N.S.	1	1.05	1.31	1.14	1.18	1.16	2.11	1.05	1.13
time (sec)	N/A	0.331	0.127	1.362	0.182	0.258	0.195	0.275	1.019

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	0	249	0	31	32
N.S.	1	1.00	1.06	0.94	0.00	8.03	0.00	1.00	1.03
time (sec)	N/A	0.252	32.676	0.898	0.000	0.253	0.000	0.339	1.086

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	0	327	0	31	32
N.S.	1	1.00	1.06	0.94	0.00	10.55	0.00	1.00	1.03
time (sec)	N/A	0.249	15.866	0.907	0.000	0.255	0.000	0.482	1.207

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	325	341	1034	580	1000	0	0	0
N.S.	1	1.04	1.09	3.30	1.85	3.19	0.00	0.00	0.00
time (sec)	N/A	1.956	2.588	1.981	0.380	0.271	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	236	275	573	347	559	0	0	0
N.S.	1	1.05	1.23	2.56	1.55	2.50	0.00	0.00	0.00
time (sec)	N/A	1.392	1.867	1.777	0.359	0.263	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	132	257	211	0	211	0	0	0
N.S.	1	1.05	2.04	1.67	0.00	1.67	0.00	0.00	0.00
time (sec)	N/A	0.705	2.225	1.689	0.000	0.258	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	50	52	36	62	57	0	48	56
N.S.	1	1.22	1.27	0.88	1.51	1.39	0.00	1.17	1.37
time (sec)	N/A	0.319	0.051	1.060	0.194	0.246	0.000	0.275	1.643

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	169	235	39	29	32
N.S.	1	1.00	1.07	0.93	5.83	8.10	1.34	1.00	1.10
time (sec)	N/A	0.242	35.039	0.780	0.374	0.254	14.904	17.160	1.346

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	277	341	71	0	32
N.S.	1	1.00	1.07	0.93	9.55	11.76	2.45	0.00	1.10
time (sec)	N/A	0.232	43.145	0.862	0.461	0.272	168.504	0.000	1.707

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	B	B	B	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD
size	419	0	1205	1604	939	2562	0	0	0
N.S.	1	0.00	2.88	3.83	2.24	6.11	0.00	0.00	0.00
time (sec)	N/A	0.000	8.704	3.171	0.421	0.296	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	B	B	B	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	0	803	889	601	1355	0	0	0
N.S.	1	0.00	2.71	3.00	2.03	4.58	0.00	0.00	0.00
time (sec)	N/A	0.000	8.081	2.523	0.419	0.275	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	175	366	334	0	506	0	0	0
N.S.	1	1.07	2.25	2.05	0.00	3.10	0.00	0.00	0.00
time (sec)	N/A	1.194	3.076	2.074	0.000	0.271	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	59	61	63	109	146	0	90	122
N.S.	1	1.04	1.07	1.11	1.91	2.56	0.00	1.58	2.14
time (sec)	N/A	0.439	0.164	1.602	0.199	0.255	0.000	0.290	2.255

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	330	342	41	0	32
N.S.	1	1.00	1.06	0.94	10.65	11.03	1.32	0.00	1.03
time (sec)	N/A	0.253	70.539	0.689	0.512	0.268	15.969	0.000	1.280

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	477	500	0	0	32
N.S.	1	1.00	1.06	0.94	15.39	16.13	0.00	0.00	1.03
time (sec)	N/A	0.253	82.980	0.849	0.706	0.284	0.000	0.000	1.471

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	B	B	B	F(-1)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD
size	546	0	2585	2124	1320	4252	0	0	0
N.S.	1	0.00	4.73	3.89	2.42	7.79	0.00	0.00	0.00
time (sec)	N/A	0.000	90.159	3.278	0.523	0.315	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	B	B	B	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD
size	368	0	1496	1147	863	2215	0	0	0
N.S.	1	0.00	4.07	3.12	2.35	6.02	0.00	0.00	0.00
time (sec)	N/A	0.000	8.799	2.778	0.464	0.282	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	B	F	B	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	0	461	441	0	817	0	0	0
N.S.	1	0.00	2.15	2.06	0.00	3.82	0.00	0.00	0.00
time (sec)	N/A	0.000	3.545	2.377	0.000	0.283	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	97	90	91	156	234	0	97	132
N.S.	1	1.11	1.03	1.05	1.79	2.69	0.00	1.11	1.52
time (sec)	N/A	0.583	0.355	1.361	0.204	0.261	0.000	0.284	1.424

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	0	29	775	903	41	0	32
N.S.	1	1.00	0.00	0.94	25.00	29.13	1.32	0.00	1.03
time (sec)	N/A	0.255	0.000	0.913	0.816	0.292	42.900	0.000	4.639

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	0	29	975	1144	0	0	32
N.S.	1	1.00	0.00	0.94	31.45	36.90	0.00	0.00	1.03
time (sec)	N/A	0.262	0.000	0.953	1.184	0.323	0.000	0.000	7.164

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	453	399	608	0	0	1112	0	0	0
N.S.	1	0.88	1.34	0.00	0.00	2.45	0.00	0.00	0.00
time (sec)	N/A	1.740	1.087	0.000	0.000	0.278	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	337	305	367	0	0	782	0	0	0
N.S.	1	0.91	1.09	0.00	0.00	2.32	0.00	0.00	0.00
time (sec)	N/A	1.352	0.771	0.000	0.000	0.256	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	211	189	440	0	500	0	0	0
N.S.	1	0.96	0.86	2.00	0.00	2.27	0.00	0.00	0.00
time (sec)	N/A	0.885	0.567	1.525	0.000	0.270	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	C	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	61	64	82	85	186	269	84	121
N.S.	1	1.13	1.19	1.52	1.57	3.44	4.98	1.56	2.24
time (sec)	N/A	0.313	0.301	0.812	0.297	0.253	31.846	0.297	1.602

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	88	34	0	28	28
N.S.	1	1.00	1.08	1.00	3.38	1.31	0.00	1.08	1.08
time (sec)	N/A	0.227	2.479	0.365	0.318	0.249	0.000	0.409	0.947

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	551	501	979	0	0	2612	0	0	0
N.S.	1	0.91	1.78	0.00	0.00	4.74	0.00	0.00	0.00
time (sec)	N/A	2.855	1.895	0.000	0.000	0.341	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	407	376	453	0	0	1689	0	0	0
N.S.	1	0.92	1.11	0.00	0.00	4.15	0.00	0.00	0.00
time (sec)	N/A	2.116	1.881	0.000	0.000	0.301	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	258	261	510	0	946	0	0	0
N.S.	1	0.98	0.99	1.93	0.00	3.58	0.00	0.00	0.00
time (sec)	N/A	1.282	2.748	1.889	0.000	0.270	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	B	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	79	74	121	119	331	1748	111	166
N.S.	1	1.11	1.04	1.70	1.68	4.66	24.62	1.56	2.34
time (sec)	N/A	0.425	1.111	0.922	0.275	0.261	176.255	0.315	1.086

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	158	36	0	30	30
N.S.	1	1.00	1.07	1.00	5.64	1.29	0.00	1.07	1.07
time (sec)	N/A	0.259	9.307	0.848	0.344	0.237	0.000	0.455	0.998

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	712	0	1407	0	0	5191	0	0	0
N.S.	1	0.00	1.98	0.00	0.00	7.29	0.00	0.00	0.00
time (sec)	N/A	0.000	4.151	0.000	0.000	0.360	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	522	0	740	0	0	3247	0	0	0
N.S.	1	0.00	1.42	0.00	0.00	6.22	0.00	0.00	0.00
time (sec)	N/A	0.000	4.102	0.000	0.000	0.325	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	325	306	589	0	1727	0	0	0
N.S.	1	0.97	0.91	1.76	0.00	5.16	0.00	0.00	0.00
time (sec)	N/A	1.743	5.507	1.734	0.000	0.309	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	124	101	191	164	601	0	151	212
N.S.	1	1.16	0.94	1.79	1.53	5.62	0.00	1.41	1.98
time (sec)	N/A	0.644	3.127	1.074	0.303	0.256	0.000	0.304	1.180

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	236	36	0	30	30
N.S.	1	1.00	1.07	1.00	8.43	1.29	0.00	1.07	1.07
time (sec)	N/A	0.264	11.113	0.664	0.405	0.263	0.000	0.531	1.079

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	605	558	734	0	0	1645	0	0	0
N.S.	1	0.92	1.21	0.00	0.00	2.72	0.00	0.00	0.00
time (sec)	N/A	2.453	1.734	0.000	0.000	0.292	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	433	406	454	0	0	1096	0	0	0
N.S.	1	0.94	1.05	0.00	0.00	2.53	0.00	0.00	0.00
time (sec)	N/A	1.925	1.233	0.000	0.000	0.287	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	258	267	532	0	649	0	0	0
N.S.	1	0.99	1.02	2.04	0.00	2.49	0.00	0.00	0.00
time (sec)	N/A	1.202	2.062	2.254	0.000	0.278	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	70	82	63	112	223	0	102	347
N.S.	1	1.09	1.28	0.98	1.75	3.48	0.00	1.59	5.42
time (sec)	N/A	0.365	0.753	1.000	0.272	0.276	0.000	0.297	1.202

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	34	22	0	30
N.S.	1	1.00	1.08	1.00	1.08	1.31	0.85	0.00	1.15
time (sec)	N/A	0.237	4.598	0.908	0.436	0.263	5.750	0.000	0.993

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	745	715	1493	0	0	6416	0	0	0
N.S.	1	0.96	2.00	0.00	0.00	8.61	0.00	0.00	0.00
time (sec)	N/A	4.426	8.539	0.000	0.000	0.386	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	535	523	920	0	0	3805	0	0	0
N.S.	1	0.98	1.72	0.00	0.00	7.11	0.00	0.00	0.00
time (sec)	N/A	2.991	7.987	0.000	0.000	0.327	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	303	350	626	0	1830	0	0	0
N.S.	1	0.99	1.14	2.05	0.00	5.98	0.00	0.00	0.00
time (sec)	N/A	1.641	5.912	1.625	0.000	0.312	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	89	113	97	137	479	0	123	360
N.S.	1	1.11	1.41	1.21	1.71	5.99	0.00	1.54	4.50
time (sec)	N/A	0.515	1.820	0.767	0.287	0.283	0.000	0.338	0.442

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	304	36	24	0	30
N.S.	1	1.00	1.07	1.00	10.86	1.29	0.86	0.00	1.07
time (sec)	N/A	0.265	92.953	0.848	0.568	0.296	6.033	0.000	1.150

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1053	0	2801	0	0	18159	0	0	0
N.S.	1	0.00	2.66	0.00	0.00	17.25	0.00	0.00	0.00
time (sec)	N/A	0.000	9.459	0.000	0.000	0.584	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	725	0	1532	0	0	10341	0	0	0
N.S.	1	0.00	2.11	0.00	0.00	14.26	0.00	0.00	0.00
time (sec)	N/A	0.000	8.242	0.000	0.000	0.398	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	0	617	861	0	4720	0	0	0
N.S.	1	0.00	1.47	2.05	0.00	11.24	0.00	0.00	0.00
time (sec)	N/A	0.000	8.416	1.953	0.000	0.350	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	138	167	142	211	1203	0	176	776
N.S.	1	1.22	1.48	1.26	1.87	10.65	0.00	1.56	6.87
time (sec)	N/A	0.903	2.437	0.878	0.278	0.299	0.000	0.301	1.569

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	757	36	24	0	30
N.S.	1	1.00	1.07	1.00	27.04	1.29	0.86	0.00	1.07
time (sec)	N/A	0.256	77.769	0.566	0.957	0.303	17.855	0.000	1.473

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	118	647	264	299	0	0	0
N.S.	1	1.00	0.85	4.65	1.90	2.15	0.00	0.00	0.00
time (sec)	N/A	0.717	0.053	6.173	0.291	0.250	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	107	94	405	164	184	0	0	0
N.S.	1	1.01	0.89	3.82	1.55	1.74	0.00	0.00	0.00
time (sec)	N/A	0.561	0.031	2.900	0.285	0.255	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	76	66	188	0	91	0	0	0
N.S.	1	1.04	0.90	2.58	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.374	0.022	1.832	0.000	0.252	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	24	23	38	20	23	22	30	19
N.S.	1	1.04	1.00	1.65	0.87	1.00	0.96	1.30	0.83
time (sec)	N/A	0.197	0.012	0.965	0.186	0.240	0.104	0.274	0.974

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	48	40	39	29	30
N.S.	1	1.00	1.07	0.93	1.66	1.38	1.34	1.00	1.03
time (sec)	N/A	0.228	6.357	0.506	0.270	0.241	4.972	0.293	1.027

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	75	64	71	29	30
N.S.	1	1.00	1.07	0.93	2.59	2.21	2.45	1.00	1.03
time (sec)	N/A	0.225	26.406	0.489	0.294	0.245	22.514	0.353	1.101

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	111	106	266	373	263	518	355	269
N.S.	1	1.03	0.98	2.46	3.45	2.44	4.80	3.29	2.49
time (sec)	N/A	0.623	1.371	15.665	0.352	0.253	0.362	0.290	1.554

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	80	78	160	270	160	318	208	167
N.S.	1	0.98	0.95	1.95	3.29	1.95	3.88	2.54	2.04
time (sec)	N/A	0.482	0.945	6.809	0.314	0.234	0.290	0.281	1.314

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	70	188	76	167	96	87
N.S.	1	1.00	1.02	1.25	3.36	1.36	2.98	1.71	1.55
time (sec)	N/A	0.345	2.255	3.081	0.258	0.237	0.199	0.279	1.154

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	139	40	44	40	78	41	36
N.S.	1	1.00	6.32	1.82	2.00	1.82	3.55	1.86	1.64
time (sec)	N/A	0.208	0.121	2.845	0.223	0.235	0.130	0.276	0.964

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	62	103	76	79	0	76	0
N.S.	1	1.00	0.82	1.36	1.00	1.04	0.00	1.00	0.00
time (sec)	N/A	0.551	0.432	12.949	0.286	0.246	0.000	0.277	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	101	85	164	92	129	0	572	0
N.S.	1	0.98	0.83	1.59	0.89	1.25	0.00	5.55	0.00
time (sec)	N/A	0.669	0.545	29.965	0.332	0.254	0.000	0.324	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	223	134	429	0	405	1040	618	449
N.S.	1	0.97	0.58	1.86	0.00	1.75	4.50	2.68	1.94
time (sec)	N/A	1.182	1.857	34.932	0.000	0.248	0.595	0.293	1.873

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	154	99	241	0	227	631	338	271
N.S.	1	0.90	0.58	1.41	0.00	1.33	3.69	1.98	1.58
time (sec)	N/A	0.820	1.310	16.567	0.000	0.231	0.440	0.299	1.552

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	94	60	113	0	92	321	138	144
N.S.	1	0.96	0.61	1.15	0.00	0.94	3.28	1.41	1.47
time (sec)	N/A	0.549	2.714	6.805	0.000	0.238	0.304	0.280	1.313

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	27	28	30	60	49	133	55	29
N.S.	1	0.79	0.82	0.88	1.76	1.44	3.91	1.62	0.85
time (sec)	N/A	0.216	0.043	6.341	0.216	0.241	0.165	0.284	1.039

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	128	112	180	0	127	0	144	0
N.S.	1	0.98	0.85	1.37	0.00	0.97	0.00	1.10	0.00
time (sec)	N/A	1.061	0.516	33.333	0.000	0.241	0.000	0.281	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	187	212	299	0	226	0	1080	0
N.S.	1	1.04	1.18	1.66	0.00	1.26	0.00	6.00	0.00
time (sec)	N/A	1.391	0.695	117.283	0.000	0.245	0.000	0.348	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	442	828	1080	685	1461	0	0	0
N.S.	1	0.95	1.79	2.33	1.48	3.16	0.00	0.00	0.00
time (sec)	N/A	2.900	8.072	22.217	0.407	0.262	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	252	530	550	387	805	0	0	0
N.S.	1	0.94	1.98	2.05	1.44	3.00	0.00	0.00	0.00
time (sec)	N/A	1.672	5.797	9.474	0.396	0.250	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	151	400	218	0	352	0	0	0
N.S.	1	0.94	2.48	1.35	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	0.853	2.236	5.384	0.000	0.249	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	47	30	64	87	102	0	102	74
N.S.	1	1.12	0.71	1.52	2.07	2.43	0.00	2.43	1.76
time (sec)	N/A	0.249	0.036	3.741	0.222	0.260	0.000	0.276	1.910

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	327	391	39	29	32
N.S.	1	1.00	1.07	0.93	11.28	13.48	1.34	1.00	1.10
time (sec)	N/A	0.228	84.097	0.699	0.489	0.271	4.151	1.990	1.160

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	0	27	436	511	71	29	32
N.S.	1	1.00	0.00	0.93	15.03	17.62	2.45	1.00	1.10
time (sec)	N/A	0.236	0.000	1.038	0.614	0.257	30.129	32.952	1.211

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	450	443	1078	1021	730	1405	0	0	0
N.S.	1	0.98	2.40	2.27	1.62	3.12	0.00	0.00	0.00
time (sec)	N/A	2.818	8.475	89.800	0.487	0.266	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	310	575	510	0	714	0	0	0
N.S.	1	0.95	1.77	1.57	0.00	2.20	0.00	0.00	0.00
time (sec)	N/A	1.930	4.610	24.633	0.000	0.253	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	151	194	143	251	201	0	260	205
N.S.	1	0.96	1.23	0.91	1.59	1.27	0.00	1.65	1.30
time (sec)	N/A	0.970	2.755	11.562	0.259	0.258	0.000	0.278	3.458

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	43	104	54	0	59	43
N.S.	1	1.00	1.00	0.91	2.21	1.15	0.00	1.26	0.91
time (sec)	N/A	0.297	0.046	10.224	0.221	0.235	0.000	0.270	1.218

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	0	29	632	768	41	31	32
N.S.	1	1.00	0.00	0.94	20.39	24.77	1.32	1.00	1.03
time (sec)	N/A	0.258	0.000	1.090	0.657	0.260	4.187	53.274	1.392

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	0	29	762	919	73	0	32
N.S.	1	1.00	0.00	0.94	24.58	29.65	2.35	0.00	1.03
time (sec)	N/A	0.253	0.000	0.893	0.902	0.254	31.370	0.000	1.430

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	B	F(-2)	B	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD
size	667	0	2008	1909	0	3854	0	0	0
N.S.	1	0.00	3.01	2.86	0.00	5.78	0.00	0.00	0.00
time (sec)	N/A	0.000	9.268	188.518	0.000	0.292	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	423	400	1363	961	0	2066	0	0	0
N.S.	1	0.95	3.22	2.27	0.00	4.88	0.00	0.00	0.00
time (sec)	N/A	2.831	8.179	87.451	0.000	0.287	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	219	617	396	0	917	0	0	0
N.S.	1	0.94	2.65	1.70	0.00	3.94	0.00	0.00	0.00
time (sec)	N/A	1.150	4.016	25.566	0.000	0.266	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	93	101	125	0	287	0	173	137
N.S.	1	1.02	1.11	1.37	0.00	3.15	0.00	1.90	1.51
time (sec)	N/A	0.282	0.080	24.693	0.000	0.248	0.000	0.349	1.819

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	29	0	1783	41	31	32
N.S.	1	1.00	1.06	0.94	0.00	57.52	1.32	1.00	1.03
time (sec)	N/A	0.257	51.852	0.782	0.000	0.305	8.770	150.912	1.911

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	F(-1)	F(-2)	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	0	0	0	2026	73	0	32
N.S.	1	1.00	0.00	0.00	0.00	65.35	2.35	0.00	1.03
time (sec)	N/A	0.267	0.000	180.000	0.000	0.318	31.246	0.000	2.049

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	356	358	329	0	0	882	0	0	0
N.S.	1	1.01	0.92	0.00	0.00	2.48	0.00	0.00	0.00
time (sec)	N/A	1.398	0.118	0.000	0.000	0.268	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	264	264	244	0	0	609	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	2.31	0.00	0.00	0.00
time (sec)	N/A	1.013	0.107	0.000	0.000	0.242	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	157	412	0	380	0	0	0
N.S.	1	1.00	0.92	2.42	0.00	2.24	0.00	0.00	0.00
time (sec)	N/A	0.617	0.029	1.470	0.000	0.260	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	44	41	33	18
N.S.	1	1.00	1.00	1.06	1.00	2.44	2.28	1.83	1.00
time (sec)	N/A	0.198	0.007	0.434	0.184	0.234	0.825	0.299	0.912

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	93	34	0	28	28
N.S.	1	1.00	1.08	1.00	3.58	1.31	0.00	1.08	1.08
time (sec)	N/A	0.220	5.340	0.306	0.302	0.245	0.000	0.350	0.999

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	527	501	933	0	0	2020	0	0	0
N.S.	1	0.95	1.77	0.00	0.00	3.83	0.00	0.00	0.00
time (sec)	N/A	2.529	1.551	0.000	0.000	0.290	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	389	376	447	0	0	1313	0	0	0
N.S.	1	0.97	1.15	0.00	0.00	3.38	0.00	0.00	0.00
time (sec)	N/A	1.870	1.409	0.000	0.000	0.272	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	258	258	901	0	742	0	0	0
N.S.	1	1.02	1.02	3.58	0.00	2.94	0.00	0.00	0.00
time (sec)	N/A	1.115	1.606	2.427	0.000	0.274	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	B	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	71	458	122	116	259	503	110	121
N.S.	1	1.04	6.74	1.79	1.71	3.81	7.40	1.62	1.78
time (sec)	N/A	0.439	1.047	1.466	0.274	0.245	125.598	0.291	1.174

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	166	36	0	30	30
N.S.	1	1.00	1.07	1.00	5.93	1.29	0.00	1.07	1.07
time (sec)	N/A	0.264	9.765	0.604	0.332	0.243	0.000	0.427	0.983

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	642	591	1977	0	0	4371	0	0	0
N.S.	1	0.92	3.08	0.00	0.00	6.81	0.00	0.00	0.00
time (sec)	N/A	3.547	10.712	0.000	0.000	0.310	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	477	428	1253	0	0	2726	0	0	0
N.S.	1	0.90	2.63	0.00	0.00	5.71	0.00	0.00	0.00
time (sec)	N/A	2.336	9.311	0.000	0.000	0.296	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	274	418	975	0	1416	0	0	0
N.S.	1	0.92	1.40	3.27	0.00	4.75	0.00	0.00	0.00
time (sec)	N/A	1.374	3.565	4.116	0.000	0.263	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	53	53	53	127	327	0	92	120
N.S.	1	0.90	0.90	0.90	2.15	5.54	0.00	1.56	2.03
time (sec)	N/A	0.261	0.045	2.859	0.231	0.241	0.000	0.284	1.055

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	251	36	0	30	30
N.S.	1	1.00	1.07	1.00	8.96	1.29	0.00	1.07	1.07
time (sec)	N/A	0.254	31.584	0.724	0.499	0.238	0.000	0.540	1.026

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	786	690	3078	0	0	1718	0	0	0
N.S.	1	0.88	3.92	0.00	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	2.933	11.354	0.000	0.000	0.293	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	558	501	1639	0	0	1094	0	0	0
N.S.	1	0.90	2.94	0.00	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	2.068	10.638	0.000	0.000	0.277	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	318	520	954	0	588	0	0	0
N.S.	1	0.95	1.56	2.86	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	1.265	2.790	3.250	0.000	0.264	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	74	114	88	95	92	0	121	129
N.S.	1	1.07	1.65	1.28	1.38	1.33	0.00	1.75	1.87
time (sec)	N/A	0.247	0.078	2.572	0.319	0.246	0.000	0.276	2.001

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	34	22	28	30
N.S.	1	1.00	1.08	1.00	1.08	1.31	0.85	1.08	1.15
time (sec)	N/A	0.239	12.588	0.776	0.435	0.274	1.598	1.523	1.048

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	780	654	1072	0	0	6367	0	0	0
N.S.	1	0.84	1.37	0.00	0.00	8.16	0.00	0.00	0.00
time (sec)	N/A	3.222	8.016	0.000	0.000	0.378	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	548	469	633	0	0	3582	0	0	0
N.S.	1	0.86	1.16	0.00	0.00	6.54	0.00	0.00	0.00
time (sec)	N/A	2.428	4.326	0.000	0.000	0.337	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	273	333	1928	0	1296	0	0	0
N.S.	1	0.93	1.13	6.54	0.00	4.39	0.00	0.00	0.00
time (sec)	N/A	1.494	2.482	8.737	0.000	0.308	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	75	104	90	115	353	0	108	413
N.S.	1	0.97	1.35	1.17	1.49	4.58	0.00	1.40	5.36
time (sec)	N/A	0.364	0.320	7.609	0.297	0.250	0.000	0.343	2.138

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	391	36	24	0	30
N.S.	1	1.00	1.07	1.00	13.96	1.29	0.86	0.00	1.07
time (sec)	N/A	0.256	65.101	0.495	0.552	0.279	1.634	0.000	1.353

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	928	764	3368	0	0	10642	0	0	0
N.S.	1	0.82	3.63	0.00	0.00	11.47	0.00	0.00	0.00
time (sec)	N/A	3.255	11.817	0.000	0.000	0.437	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	485	832	2051	0	4729	0	0	0
N.S.	1	0.87	1.49	3.66	0.00	8.44	0.00	0.00	0.00
time (sec)	N/A	1.940	8.530	20.306	0.000	0.339	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	158	131	205	216	893	0	282	381
N.S.	1	1.33	1.10	1.72	1.82	7.50	0.00	2.37	3.20
time (sec)	N/A	0.354	0.351	19.282	0.269	0.255	0.000	0.330	3.112

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	1100	36	24	0	30
N.S.	1	1.00	1.07	1.00	39.29	1.29	0.86	0.00	1.07
time (sec)	N/A	0.254	65.558	0.869	1.843	3.894	2.965	0.000	3.748

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	22	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	1.08
time (sec)	N/A	0.228	10.441	0.684	0.388	0.232	2.706	0.379	1.041

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	22	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	1.08
time (sec)	N/A	0.231	8.762	0.434	0.389	0.230	1.376	0.341	0.943

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	205	24	20	24	24
N.S.	1	1.00	1.09	1.00	9.32	1.09	0.91	1.09	1.09
time (sec)	N/A	0.209	2.942	0.197	1.005	0.237	0.785	0.369	0.946

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	72	78	164	157	411	0	0	199
N.S.	1	0.97	1.05	2.22	2.12	5.55	0.00	0.00	2.69
time (sec)	N/A	0.329	1.051	3.095	0.364	0.240	0.000	0.000	1.322

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	224	202	491	0	1378	0	0	0
N.S.	1	0.96	0.86	2.10	0.00	5.89	0.00	0.00	0.00
time (sec)	N/A	0.964	1.106	3.239	0.000	0.266	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	348	318	368	0	0	2420	0	0	0
N.S.	1	0.91	1.06	0.00	0.00	6.95	0.00	0.00	0.00
time (sec)	N/A	1.380	1.053	0.000	0.000	0.294	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	72	78	164	157	411	0	0	199
N.S.	1	0.97	1.05	2.22	2.12	5.55	0.00	0.00	2.69
time (sec)	N/A	0.321	0.170	2.963	0.350	0.249	0.000	0.000	0.002

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	224	202	491	0	1378	0	0	0
N.S.	1	0.96	0.86	2.10	0.00	5.89	0.00	0.00	0.00
time (sec)	N/A	0.975	0.725	3.217	0.000	0.273	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	348	318	368	0	0	2420	0	0	0
N.S.	1	0.91	1.06	0.00	0.00	6.95	0.00	0.00	0.00
time (sec)	N/A	1.391	0.249	0.000	0.000	0.288	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	116	112	308	413	1230	0	0	0
N.S.	1	1.04	1.00	2.75	3.69	10.98	0.00	0.00	0.00
time (sec)	N/A	0.455	1.390	15.046	0.429	0.272	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	301	623	805	0	5233	0	0	0
N.S.	1	0.98	2.04	2.63	0.00	17.10	0.00	0.00	0.00
time (sec)	N/A	1.255	10.282	16.253	0.000	0.332	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	631	561	5753	0	0	11757	0	0	0
N.S.	1	0.89	9.12	0.00	0.00	18.63	0.00	0.00	0.00
time (sec)	N/A	2.878	13.702	0.000	0.000	0.426	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	116	112	308	413	1230	0	0	0
N.S.	1	1.04	1.00	2.75	3.69	10.98	0.00	0.00	0.00
time (sec)	N/A	0.457	0.635	14.547	0.429	0.260	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	301	623	805	0	5233	0	0	0
N.S.	1	0.98	2.04	2.63	0.00	17.10	0.00	0.00	0.00
time (sec)	N/A	1.264	6.681	15.877	0.000	0.340	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	631	561	5753	0	0	11757	0	0	0
N.S.	1	0.89	9.12	0.00	0.00	18.63	0.00	0.00	0.00
time (sec)	N/A	2.887	6.909	0.000	0.000	0.411	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	448	452	2809	0	0	1976	0	0	0
N.S.	1	1.01	6.27	0.00	0.00	4.41	0.00	0.00	0.00
time (sec)	N/A	2.276	9.733	0.000	0.000	0.293	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	330	334	1301	0	0	1265	0	0	0
N.S.	1	1.01	3.94	0.00	0.00	3.83	0.00	0.00	0.00
time (sec)	N/A	1.640	9.480	0.000	0.000	0.287	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	209	377	483	0	692	0	0	0
N.S.	1	0.99	1.78	2.28	0.00	3.26	0.00	0.00	0.00
time (sec)	N/A	0.981	2.630	2.553	0.000	0.261	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	30	33	33	83	132	65	60	31
N.S.	1	0.88	0.97	0.97	2.44	3.88	1.91	1.76	0.91
time (sec)	N/A	0.252	0.032	1.199	0.219	0.246	0.870	0.270	0.079

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	164	40	0	34	34
N.S.	1	1.00	1.06	1.00	5.12	1.25	0.00	1.06	1.06
time (sec)	N/A	0.245	21.620	0.518	0.423	0.240	0.000	0.390	1.071

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	696	641	1994	0	0	3847	0	0	0
N.S.	1	0.92	2.86	0.00	0.00	5.53	0.00	0.00	0.00
time (sec)	N/A	3.630	9.612	0.000	0.000	0.340	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	510	484	1188	0	0	2410	0	0	0
N.S.	1	0.95	2.33	0.00	0.00	4.73	0.00	0.00	0.00
time (sec)	N/A	2.739	4.447	0.000	0.000	0.308	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	324	582	1012	0	1284	0	0	0
N.S.	1	0.99	1.78	3.09	0.00	3.93	0.00	0.00	0.00
time (sec)	N/A	1.523	1.885	6.047	0.000	0.276	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	107	109	167	160	446	0	155	212
N.S.	1	1.13	1.15	1.76	1.68	4.69	0.00	1.63	2.23
time (sec)	N/A	0.536	0.252	3.816	0.294	0.266	0.000	0.289	1.286

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	243	42	0	36	36
N.S.	1	1.00	1.06	1.00	7.15	1.24	0.00	1.06	1.06
time (sec)	N/A	0.282	12.148	0.654	0.381	0.246	0.000	0.532	1.371

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	864	0	5656	0	0	7980	0	0	0
N.S.	1	0.00	6.55	0.00	0.00	9.24	0.00	0.00	0.00
time (sec)	N/A	0.000	32.984	0.000	0.000	0.357	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	636	0	1961	0	0	4887	0	0	0
N.S.	1	0.00	3.08	0.00	0.00	7.68	0.00	0.00	0.00
time (sec)	N/A	0.000	11.524	0.000	0.000	0.359	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	360	604	1102	0	2465	0	0	0
N.S.	1	0.90	1.51	2.76	0.00	6.16	0.00	0.00	0.00
time (sec)	N/A	2.250	1.703	15.085	0.000	0.300	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	75	75	80	183	652	0	145	180
N.S.	1	0.88	0.88	0.94	2.15	7.67	0.00	1.71	2.12
time (sec)	N/A	0.317	0.106	8.980	0.202	0.288	0.000	0.301	1.160

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	348	42	0	36	36
N.S.	1	1.00	1.06	1.00	10.24	1.24	0.00	1.06	1.06
time (sec)	N/A	0.288	38.207	0.853	0.491	0.257	0.000	0.523	2.756

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1021	886	3078	0	0	1715	0	0	0
N.S.	1	0.87	3.01	0.00	0.00	1.68	0.00	0.00	0.00
time (sec)	N/A	3.894	11.334	0.000	0.000	0.321	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	716	633	1640	0	0	1083	0	0	0
N.S.	1	0.88	2.29	0.00	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	2.978	10.377	0.000	0.000	0.281	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	421	390	521	1287	0	589	0	0	0
N.S.	1	0.93	1.24	3.06	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	1.822	2.978	2.477	0.000	0.279	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	71	71	97	95	92	0	121	130
N.S.	1	1.03	1.03	1.41	1.38	1.33	0.00	1.75	1.88
time (sec)	N/A	0.255	0.067	1.065	0.285	0.271	0.000	0.287	2.044

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	34	22	0	28
N.S.	1	1.00	1.08	1.00	1.08	1.31	0.85	0.00	1.08
time (sec)	N/A	0.226	11.387	1.014	0.409	0.283	1.373	0.000	1.205

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	917	792	1144	0	0	6503	0	0	0
N.S.	1	0.86	1.25	0.00	0.00	7.09	0.00	0.00	0.00
time (sec)	N/A	5.064	7.717	0.000	0.000	0.400	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	648	573	633	0	0	3662	0	0	0
N.S.	1	0.88	0.98	0.00	0.00	5.65	0.00	0.00	0.00
time (sec)	N/A	3.626	4.485	0.000	0.000	0.332	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	313	331	1858	0	1338	0	0	0
N.S.	1	0.93	0.99	5.55	0.00	3.99	0.00	0.00	0.00
time (sec)	N/A	2.052	2.653	5.269	0.000	0.312	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	84	104	101	117	350	0	106	170
N.S.	1	1.08	1.33	1.29	1.50	4.49	0.00	1.36	2.18
time (sec)	N/A	0.386	0.377	2.849	0.285	0.258	0.000	0.292	1.369

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	386	40	29	0	36
N.S.	1	1.00	1.06	1.00	12.06	1.25	0.91	0.00	1.12
time (sec)	N/A	0.262	71.447	0.690	0.488	0.300	2.198	0.000	1.549

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1176	958	3390	0	0	11164	0	0	0
N.S.	1	0.81	2.88	0.00	0.00	9.49	0.00	0.00	0.00
time (sec)	N/A	5.093	12.129	0.000	0.000	0.446	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	711	600	821	2074	0	4993	0	0	0
N.S.	1	0.84	1.15	2.92	0.00	7.02	0.00	0.00	0.00
time (sec)	N/A	2.892	9.119	13.606	0.000	0.377	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	145	130	212	218	926	0	286	337
N.S.	1	1.19	1.07	1.74	1.79	7.59	0.00	2.34	2.76
time (sec)	N/A	0.381	0.467	7.812	0.288	0.271	0.000	0.317	2.797

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	1104	42	31	0	36
N.S.	1	1.00	1.06	1.00	32.47	1.24	0.91	0.00	1.06
time (sec)	N/A	0.276	75.403	0.724	1.964	4.088	3.698	0.000	3.852

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	606	591	2684	0	0	3891	0	0	0
N.S.	1	0.98	4.43	0.00	0.00	6.42	0.00	0.00	0.00
time (sec)	N/A	3.271	15.548	0.000	0.000	0.326	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	449	428	1453	0	0	2414	0	0	0
N.S.	1	0.95	3.24	0.00	0.00	5.38	0.00	0.00	0.00
time (sec)	N/A	2.452	9.368	0.000	0.000	0.289	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	274	483	565	0	1248	0	0	0
N.S.	1	0.99	1.74	2.03	0.00	4.49	0.00	0.00	0.00
time (sec)	N/A	1.507	0.340	8.821	0.000	0.282	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	48	49	54	119	309	87	88	46
N.S.	1	0.87	0.89	0.98	2.16	5.62	1.58	1.60	0.84
time (sec)	N/A	0.277	0.047	3.295	0.205	0.241	0.882	0.306	0.128

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	235	42	0	36	36
N.S.	1	1.00	1.06	1.00	6.91	1.24	0.00	1.06	1.06
time (sec)	N/A	0.283	27.098	0.770	0.399	0.258	0.000	0.495	1.442

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	897	0	1667	0	0	7042	0	0	0
N.S.	1	0.00	1.86	0.00	0.00	7.85	0.00	0.00	0.00
time (sec)	N/A	0.000	4.637	0.000	0.000	0.364	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	649	0	966	0	0	4311	0	0	0
N.S.	1	0.00	1.49	0.00	0.00	6.64	0.00	0.00	0.00
time (sec)	N/A	0.000	2.975	0.000	0.000	0.334	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	391	626	1128	0	2195	0	0	0
N.S.	1	0.97	1.55	2.80	0.00	5.45	0.00	0.00	0.00
time (sec)	N/A	2.256	2.521	21.237	0.000	0.309	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	163	123	244	209	745	0	211	278
N.S.	1	1.16	0.87	1.73	1.48	5.28	0.00	1.50	1.97
time (sec)	N/A	1.107	0.539	9.181	0.284	0.264	0.000	0.298	1.416

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	334	44	0	38	38
N.S.	1	1.00	1.06	1.00	9.28	1.22	0.00	1.06	1.06
time (sec)	N/A	0.340	11.016	0.769	0.391	0.238	0.000	0.520	1.184

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1123	0	8401	0	0	12603	0	0	0
N.S.	1	0.00	7.48	0.00	0.00	11.22	0.00	0.00	0.00
time (sec)	N/A	0.000	28.848	0.000	0.000	0.433	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	819	0	5113	0	0	7645	0	0	0
N.S.	1	0.00	6.24	0.00	0.00	9.33	0.00	0.00	0.00
time (sec)	N/A	0.000	15.705	0.000	0.000	0.376	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	499	0	904	1217	0	3795	0	0	0
N.S.	1	0.00	1.81	2.44	0.00	7.61	0.00	0.00	0.00
time (sec)	N/A	0.000	1.880	59.061	0.000	0.319	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	100	98	97	234	1069	0	202	238
N.S.	1	0.88	0.87	0.86	2.07	9.46	0.00	1.79	2.11
time (sec)	N/A	0.360	0.169	25.339	0.201	0.271	0.000	0.309	1.324

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	442	44	0	38	38
N.S.	1	1.00	1.06	1.00	12.28	1.22	0.00	1.06	1.06
time (sec)	N/A	0.327	30.220	0.870	0.528	0.247	0.000	0.756	1.203

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1218	1036	3251	0	0	1962	0	0	0
N.S.	1	0.85	2.67	0.00	0.00	1.61	0.00	0.00	0.00
time (sec)	N/A	5.141	11.151	0.000	0.000	0.319	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	861	748	1759	0	0	1248	0	0	0
N.S.	1	0.87	2.04	0.00	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	4.110	10.573	0.000	0.000	0.324	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	516	471	578	3882	0	682	0	0	0
N.S.	1	0.91	1.12	7.52	0.00	1.32	0.00	0.00	0.00
time (sec)	N/A	2.550	3.915	2.073	0.000	0.293	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	85	78	132	110	111	0	121	174
N.S.	1	1.15	1.05	1.78	1.49	1.50	0.00	1.64	2.35
time (sec)	N/A	0.340	0.078	1.096	0.383	0.280	0.000	0.336	2.090

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	252	40	29	0	34
N.S.	1	1.00	1.06	1.00	7.88	1.25	0.91	0.00	1.06
time (sec)	N/A	0.254	15.538	0.468	0.506	0.304	1.417	0.000	1.439

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1118	949	1147	0	0	6494	0	0	0
N.S.	1	0.85	1.03	0.00	0.00	5.81	0.00	0.00	0.00
time (sec)	N/A	6.361	8.175	0.000	0.000	0.406	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	772	670	633	0	0	3661	0	0	0
N.S.	1	0.87	0.82	0.00	0.00	4.74	0.00	0.00	0.00
time (sec)	N/A	4.790	5.061	0.000	0.000	0.343	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	353	331	1928	0	1337	0	0	0
N.S.	1	0.92	0.86	5.01	0.00	3.47	0.00	0.00	0.00
time (sec)	N/A	2.554	3.060	2.497	0.000	0.313	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	88	106	103	115	351	0	108	422
N.S.	1	0.98	1.18	1.14	1.28	3.90	0.00	1.20	4.69
time (sec)	N/A	0.511	0.384	0.909	0.278	0.258	0.000	0.323	1.507

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	391	36	24	0	30
N.S.	1	1.00	1.07	1.00	13.96	1.29	0.86	0.00	1.07
time (sec)	N/A	0.252	52.789	0.619	0.548	0.289	1.228	0.000	1.377

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1256	1026	3390	0	0	10934	0	0	0
N.S.	1	0.82	2.70	0.00	0.00	8.71	0.00	0.00	0.00
time (sec)	N/A	6.381	12.072	0.000	0.000	0.463	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	760	646	823	2068	0	4903	0	0	0
N.S.	1	0.85	1.08	2.72	0.00	6.45	0.00	0.00	0.00
time (sec)	N/A	3.564	9.271	7.204	0.000	0.371	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	146	130	209	219	917	0	281	339
N.S.	1	1.21	1.07	1.73	1.81	7.58	0.00	2.32	2.80
time (sec)	N/A	0.412	0.275	2.862	0.318	0.284	0.000	0.324	2.738

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	1102	42	31	0	38
N.S.	1	1.00	1.06	1.00	32.41	1.24	0.91	0.00	1.12
time (sec)	N/A	0.276	68.834	0.688	1.941	4.074	2.907	0.000	3.318

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	792	0	5656	0	0	7020	0	0	0
N.S.	1	0.00	7.14	0.00	0.00	8.86	0.00	0.00	0.00
time (sec)	N/A	0.000	28.493	0.000	0.000	0.358	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	578	0	1961	0	0	4263	0	0	0
N.S.	1	0.00	3.39	0.00	0.00	7.38	0.00	0.00	0.00
time (sec)	N/A	0.000	11.409	0.000	0.000	0.329	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	333	452	671	0	2129	0	0	0
N.S.	1	0.96	1.30	1.93	0.00	6.12	0.00	0.00	0.00
time (sec)	N/A	2.169	0.918	28.602	0.000	0.299	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	66	66	65	171	602	105	117	63
N.S.	1	0.87	0.87	0.86	2.25	7.92	1.38	1.54	0.83
time (sec)	N/A	0.287	0.079	13.595	0.217	0.270	0.918	0.329	0.997

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	326	42	0	36	36
N.S.	1	1.00	1.06	1.00	9.59	1.24	0.00	1.06	1.06
time (sec)	N/A	0.275	35.624	0.878	0.433	0.259	0.000	0.552	2.770

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1038	0	5984	0	0	10658	0	0	0
N.S.	1	0.00	5.76	0.00	0.00	10.27	0.00	0.00	0.00
time (sec)	N/A	0.000	21.236	0.000	0.000	0.451	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	755	0	3579	0	0	6459	0	0	0
N.S.	1	0.00	4.74	0.00	0.00	8.55	0.00	0.00	0.00
time (sec)	N/A	0.000	13.140	0.000	0.000	0.372	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	455	1627	1213	0	3228	0	0	0
N.S.	1	0.96	3.43	2.56	0.00	6.81	0.00	0.00	0.00
time (sec)	N/A	2.873	4.223	76.737	0.000	0.325	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	222	153	293	257	1134	0	258	330
N.S.	1	1.21	0.83	1.59	1.40	6.16	0.00	1.40	1.79
time (sec)	N/A	1.519	1.560	36.969	0.314	0.265	0.000	0.340	1.453

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	417	44	0	38	38
N.S.	1	1.00	1.06	1.00	11.58	1.22	0.00	1.06	1.06
time (sec)	N/A	0.343	15.783	0.886	0.438	0.252	0.000	0.784	1.184

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1443	0	5147	0	0	18801	0	0	0
N.S.	1	0.00	3.57	0.00	0.00	13.03	0.00	0.00	0.00
time (sec)	N/A	0.000	11.067	0.000	0.000	0.529	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1049	0	1811	0	0	11318	0	0	0
N.S.	1	0.00	1.73	0.00	0.00	10.79	0.00	0.00	0.00
time (sec)	N/A	0.000	9.732	0.000	0.000	0.415	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	B	F	B	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	641	0	2704	1363	0	5548	0	0	0
N.S.	1	0.00	4.22	2.13	0.00	8.66	0.00	0.00	0.00
time (sec)	N/A	0.000	8.518	187.964	0.000	0.334	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	125	123	144	300	1660	0	258	307
N.S.	1	0.89	0.87	1.02	2.13	11.77	0.00	1.83	2.18
time (sec)	N/A	0.374	0.265	99.050	0.223	0.282	0.000	0.338	1.470

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	551	44	0	38	38
N.S.	1	1.00	1.06	1.00	15.31	1.22	0.00	1.06	1.06
time (sec)	N/A	0.332	28.684	0.799	0.559	0.244	0.000	0.576	1.292

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1519	0	4239	0	0	4522	0	0	0
N.S.	1	0.00	2.79	0.00	0.00	2.98	0.00	0.00	0.00
time (sec)	N/A	0.000	11.355	0.000	0.000	0.365	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1067	0	3418	0	0	2775	0	0	0
N.S.	1	0.00	3.20	0.00	0.00	2.60	0.00	0.00	0.00
time (sec)	N/A	0.000	10.878	0.000	0.000	0.342	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	631	570	637	4066	0	1410	0	0	0
N.S.	1	0.90	1.01	6.44	0.00	2.23	0.00	0.00	0.00
time (sec)	N/A	3.597	8.635	3.147	0.000	0.312	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	97	91	169	147	288	0	145	249
N.S.	1	1.09	1.02	1.90	1.65	3.24	0.00	1.63	2.80
time (sec)	N/A	0.405	0.147	1.602	0.314	0.270	0.000	0.313	2.529

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	322	42	31	0	36
N.S.	1	1.00	1.06	1.00	9.47	1.24	0.91	0.00	1.06
time (sec)	N/A	0.289	39.989	1.193	0.514	0.317	2.043	0.000	2.219

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1294	0	1111	0	0	7331	0	0	0
N.S.	1	0.00	0.86	0.00	0.00	5.67	0.00	0.00	0.00
time (sec)	N/A	0.000	5.130	0.000	0.000	0.403	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	904	0	665	0	0	4196	0	0	0
N.S.	1	0.00	0.74	0.00	0.00	4.64	0.00	0.00	0.00
time (sec)	N/A	0.000	2.924	0.000	0.000	0.366	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	0	358	1897	0	1571	0	0	0
N.S.	1	0.00	0.79	4.18	0.00	3.46	0.00	0.00	0.00
time (sec)	N/A	0.000	5.672	2.356	0.000	0.306	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	127	96	125	141	459	0	128	468
N.S.	1	1.05	0.79	1.03	1.17	3.79	0.00	1.06	3.87
time (sec)	N/A	0.692	0.813	1.099	0.306	0.260	0.000	0.328	2.868

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	411	42	31	0	36
N.S.	1	1.00	1.06	1.00	12.09	1.24	0.91	0.00	1.06
time (sec)	N/A	0.291	37.650	0.577	0.498	0.307	2.111	0.000	3.394

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1479	0	3368	0	0	10574	0	0	0
N.S.	1	0.00	2.28	0.00	0.00	7.15	0.00	0.00	0.00
time (sec)	N/A	0.000	11.934	0.000	0.000	0.457	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	894	0	834	2284	0	4729	0	0	0
N.S.	1	0.00	0.93	2.55	0.00	5.29	0.00	0.00	0.00
time (sec)	N/A	0.000	9.102	2.710	0.000	0.372	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	146	152	210	217	896	0	279	381
N.S.	1	1.22	1.27	1.75	1.81	7.47	0.00	2.32	3.18
time (sec)	N/A	0.382	0.352	1.428	0.306	0.283	0.000	0.340	3.221

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	1095	36	24	0	30
N.S.	1	1.00	1.07	1.00	39.11	1.29	0.86	0.00	1.07
time (sec)	N/A	0.252	60.757	0.661	1.936	4.200	1.759	0.000	3.818

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	451	532	1914	0	0	1228	0	0	0
N.S.	1	1.18	4.24	0.00	0.00	2.72	0.00	0.00	0.00
time (sec)	N/A	2.304	9.793	0.000	0.000	0.289	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	325	397	1296	0	0	813	0	0	0
N.S.	1	1.22	3.99	0.00	0.00	2.50	0.00	0.00	0.00
time (sec)	N/A	1.844	5.700	0.000	0.000	0.271	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	263	431	451	0	476	0	0	0
N.S.	1	1.28	2.10	2.20	0.00	2.32	0.00	0.00	0.00
time (sec)	N/A	1.138	2.244	1.930	0.000	0.305	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	53	75	67	0	61	254
N.S.	1	1.00	0.82	1.56	2.21	1.97	0.00	1.79	7.47
time (sec)	N/A	0.220	0.026	1.126	0.227	0.273	0.000	0.295	0.450

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	34	22	28	28
N.S.	1	1.00	1.08	1.00	1.08	1.31	0.85	1.08	1.08
time (sec)	N/A	0.224	10.619	0.796	0.454	0.285	2.559	0.641	1.114

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	638	0	781	0	0	1470	0	0	0
N.S.	1	0.00	1.22	0.00	0.00	2.30	0.00	0.00	0.00
time (sec)	N/A	0.000	1.086	0.000	0.000	0.323	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	462	0	490	0	0	992	0	0	0
N.S.	1	0.00	1.06	0.00	0.00	2.15	0.00	0.00	0.00
time (sec)	N/A	0.000	0.713	0.000	0.000	0.283	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	368	298	970	0	598	0	0	0
N.S.	1	1.29	1.04	3.39	0.00	2.09	0.00	0.00	0.00
time (sec)	N/A	2.185	1.359	1.953	0.000	0.310	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	80	94	109	126	209	0	113	384
N.S.	1	1.13	1.32	1.54	1.77	2.94	0.00	1.59	5.41
time (sec)	N/A	0.558	0.315	1.542	0.279	0.292	0.000	0.321	1.208

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	167	40	29	0	34
N.S.	1	1.00	1.06	1.00	5.22	1.25	0.91	0.00	1.06
time (sec)	N/A	0.256	8.266	0.560	0.400	0.282	2.904	0.000	1.312

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F	F(-2)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	656	0	3089	0	0	3344	0	0	0
N.S.	1	0.00	4.71	0.00	0.00	5.10	0.00	0.00	0.00
time (sec)	N/A	0.000	10.255	0.000	0.000	0.340	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	486	0	1462	0	0	2101	0	0	0
N.S.	1	0.00	3.01	0.00	0.00	4.32	0.00	0.00	0.00
time (sec)	N/A	0.000	8.327	0.000	0.000	0.321	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	422	487	932	0	1108	0	0	0
N.S.	1	1.31	1.51	2.89	0.00	3.44	0.00	0.00	0.00
time (sec)	N/A	2.910	3.950	3.235	0.000	0.290	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	55	48	133	130	203	0	94	360
N.S.	1	0.96	0.84	2.33	2.28	3.56	0.00	1.65	6.32
time (sec)	N/A	0.308	0.068	2.345	0.232	0.266	0.000	0.301	1.507

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	249	42	31	0	36
N.S.	1	1.00	1.06	1.00	7.32	1.24	0.91	0.00	1.06
time (sec)	N/A	0.280	44.853	1.041	0.548	0.275	5.532	0.000	2.321

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1049	910	3862	0	0	2448	0	0	0
N.S.	1	0.87	3.68	0.00	0.00	2.33	0.00	0.00	0.00
time (sec)	N/A	4.034	12.012	0.000	0.000	0.357	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	734	651	3730	0	0	1535	0	0	0
N.S.	1	0.89	5.08	0.00	0.00	2.09	0.00	0.00	0.00
time (sec)	N/A	3.125	13.223	0.000	0.000	0.315	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	439	402	788	1065	0	808	0	0	0
N.S.	1	0.92	1.79	2.43	0.00	1.84	0.00	0.00	0.00
time (sec)	N/A	1.939	8.367	3.666	0.000	0.306	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	104	92	108	138	134	0	147	0
N.S.	1	1.16	1.02	1.20	1.53	1.49	0.00	1.63	0.00
time (sec)	N/A	0.365	0.102	1.707	0.316	0.304	0.000	0.291	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	34	40	29	34	38
N.S.	1	1.00	1.06	1.00	1.06	1.25	0.91	1.06	1.19
time (sec)	N/A	0.258	19.563	0.942	0.557	0.832	67.651	6.619	1.637

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1164	991	1441	0	0	9707	0	0	0
N.S.	1	0.85	1.24	0.00	0.00	8.34	0.00	0.00	0.00
time (sec)	N/A	4.926	9.074	0.000	0.000	0.508	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	795	695	928	0	0	5562	0	0	0
N.S.	1	0.87	1.17	0.00	0.00	7.00	0.00	0.00	0.00
time (sec)	N/A	3.744	8.304	0.000	0.000	0.397	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	442	388	428	1815	0	2176	0	0	0
N.S.	1	0.88	0.97	4.11	0.00	4.92	0.00	0.00	0.00
time (sec)	N/A	2.180	7.942	10.704	0.000	0.324	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	126	233	106	168	581	0	146	668
N.S.	1	1.12	2.06	0.94	1.49	5.14	0.00	1.29	5.91
time (sec)	N/A	0.489	1.109	4.800	0.313	0.329	0.000	0.312	6.868

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	463	42	0	0	38
N.S.	1	1.00	1.06	1.00	13.62	1.24	0.00	0.00	1.12
time (sec)	N/A	0.273	49.809	0.806	0.645	0.569	0.000	0.000	5.660

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1185	993	4072	0	0	16670	0	0	0
N.S.	1	0.84	3.44	0.00	0.00	14.07	0.00	0.00	0.00
time (sec)	N/A	4.521	13.163	0.000	0.000	0.563	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	746	627	1080	2580	0	7645	0	0	0
N.S.	1	0.84	1.45	3.46	0.00	10.25	0.00	0.00	0.00
time (sec)	N/A	2.603	10.908	30.793	0.000	0.491	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	181	196	230	265	1279	0	343	0
N.S.	1	1.13	1.22	1.44	1.66	7.99	0.00	2.14	0.00
time (sec)	N/A	0.441	0.538	15.007	0.329	0.442	0.000	0.289	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	1214	42	0	0	38
N.S.	1	1.00	1.06	1.00	35.71	1.24	0.00	0.00	1.12
time (sec)	N/A	0.278	102.423	0.910	2.430	19.432	0.000	0.000	12.251

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	601	681	2598	0	0	4313	0	0	0
N.S.	1	1.13	4.32	0.00	0.00	7.18	0.00	0.00	0.00
time (sec)	N/A	3.979	11.106	0.000	0.000	0.356	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	419	492	1735	0	0	2528	0	0	0
N.S.	1	1.17	4.14	0.00	0.00	6.03	0.00	0.00	0.00
time (sec)	N/A	2.706	10.383	0.000	0.000	0.316	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	304	621	528	0	1221	0	0	0
N.S.	1	1.25	2.56	2.17	0.00	5.02	0.00	0.00	0.00
time (sec)	N/A	1.599	8.150	2.331	0.000	0.288	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	49	50	35	110	211	0	110	409
N.S.	1	0.98	1.00	0.70	2.20	4.22	0.00	2.20	8.18
time (sec)	N/A	0.275	0.039	0.674	0.212	0.283	0.000	0.313	2.066

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	316	40	29	0	36
N.S.	1	1.00	1.06	1.00	9.88	1.25	0.91	0.00	1.12
time (sec)	N/A	0.243	76.912	0.692	0.521	0.314	5.559	0.000	1.823

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	721	0	1490	0	0	4612	0	0	0
N.S.	1	0.00	2.07	0.00	0.00	6.40	0.00	0.00	0.00
time (sec)	N/A	0.000	7.174	0.000	0.000	0.369	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	517	0	917	0	0	2729	0	0	0
N.S.	1	0.00	1.77	0.00	0.00	5.28	0.00	0.00	0.00
time (sec)	N/A	0.000	6.907	0.000	0.000	0.333	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	0	347	1017	0	1338	0	0	0
N.S.	1	0.00	1.18	3.46	0.00	4.55	0.00	0.00	0.00
time (sec)	N/A	0.000	3.279	2.442	0.000	0.292	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	88	112	105	134	360	0	120	380
N.S.	1	1.14	1.45	1.36	1.74	4.68	0.00	1.56	4.94
time (sec)	N/A	0.663	0.537	1.285	0.286	0.268	0.000	0.309	1.474

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	311	36	24	0	30
N.S.	1	1.00	1.07	1.00	11.11	1.29	0.86	0.00	1.07
time (sec)	N/A	0.253	67.271	0.685	0.532	0.301	2.524	0.000	1.357

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	718	0	2696	0	0	5829	0	0	0
N.S.	1	0.00	3.75	0.00	0.00	8.12	0.00	0.00	0.00
time (sec)	N/A	0.000	10.565	0.000	0.000	0.361	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	518	0	1806	0	0	3506	0	0	0
N.S.	1	0.00	3.49	0.00	0.00	6.77	0.00	0.00	0.00
time (sec)	N/A	0.000	10.065	0.000	0.000	0.321	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	0	504	938	0	1735	0	0	0
N.S.	1	0.00	1.56	2.90	0.00	5.35	0.00	0.00	0.00
time (sec)	N/A	0.000	7.165	3.019	0.000	0.287	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	52	128	131	299	0	121	356
N.S.	1	1.00	0.88	2.17	2.22	5.07	0.00	2.05	6.03
time (sec)	N/A	0.323	0.067	1.689	0.232	0.264	0.000	0.344	1.542

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	353	42	31	0	36
N.S.	1	1.00	1.06	1.00	10.38	1.24	0.91	0.00	1.06
time (sec)	N/A	0.283	54.219	0.813	0.621	0.449	5.161	0.000	3.912

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1428	1259	4139	0	0	9763	0	0	0
N.S.	1	0.88	2.90	0.00	0.00	6.84	0.00	0.00	0.00
time (sec)	N/A	6.186	11.132	0.000	0.000	0.491	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	982	889	2323	0	0	5664	0	0	0
N.S.	1	0.91	2.37	0.00	0.00	5.77	0.00	0.00	0.00
time (sec)	N/A	4.685	10.202	0.000	0.000	0.379	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	591	529	864	1529	0	2593	0	0	0
N.S.	1	0.90	1.46	2.59	0.00	4.39	0.00	0.00	0.00
time (sec)	N/A	2.844	8.954	8.332	0.000	0.354	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	118	160	140	173	441	0	200	142
N.S.	1	1.13	1.54	1.35	1.66	4.24	0.00	1.92	1.37
time (sec)	N/A	0.367	0.425	3.765	0.312	0.324	0.000	0.311	4.065

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	472	42	0	0	38
N.S.	1	1.00	1.06	1.00	13.88	1.24	0.00	0.00	1.12
time (sec)	N/A	0.283	41.564	0.866	0.804	6.704	0.000	0.000	3.662

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	914	818	2029	0	0	10432	0	0	0
N.S.	1	0.89	2.22	0.00	0.00	11.41	0.00	0.00	0.00
time (sec)	N/A	5.683	9.328	0.000	0.000	0.508	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	B	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	499	444	1295	3717	0	4086	0	0	0
N.S.	1	0.89	2.60	7.45	0.00	8.19	0.00	0.00	0.00
time (sec)	N/A	3.019	8.795	22.306	0.000	0.368	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	152	139	208	1040	0	185	768
N.S.	1	1.00	1.06	0.97	1.44	7.22	0.00	1.28	5.33
time (sec)	N/A	0.544	1.696	10.363	0.283	0.331	0.000	0.636	6.546

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	626	44	0	0	38
N.S.	1	1.00	1.06	1.00	17.39	1.22	0.00	0.00	1.06
time (sec)	N/A	0.318	52.114	0.816	0.766	0.700	0.000	0.000	6.309

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	978	803	1437	3280	0	15223	0	0	0
N.S.	1	0.82	1.47	3.35	0.00	15.57	0.00	0.00	0.00
time (sec)	N/A	3.816	11.642	60.273	0.000	0.615	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	207	227	249	350	2568	0	458	398
N.S.	1	1.15	1.26	1.38	1.94	14.27	0.00	2.54	2.21
time (sec)	N/A	0.470	0.800	28.203	0.299	0.494	0.000	0.301	8.673

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	1586	44	0	0	38
N.S.	1	1.00	1.06	1.00	44.06	1.22	0.00	0.00	1.06
time (sec)	N/A	0.315	109.979	0.804	2.436	157.055	0.000	0.000	20.549

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	752	0	3254	0	0	11595	0	0	0
N.S.	1	0.00	4.33	0.00	0.00	15.42	0.00	0.00	0.00
time (sec)	N/A	0.000	11.719	0.000	0.000	0.409	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	502	565	1816	0	0	6479	0	0	0
N.S.	1	1.13	3.62	0.00	0.00	12.91	0.00	0.00	0.00
time (sec)	N/A	3.733	10.325	0.000	0.000	0.344	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	350	713	649	0	2899	0	0	0
N.S.	1	1.17	2.39	2.18	0.00	9.73	0.00	0.00	0.00
time (sec)	N/A	2.166	8.402	2.163	0.000	0.320	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	68	60	54	161	545	0	145	470
N.S.	1	0.94	0.83	0.75	2.24	7.57	0.00	2.01	6.53
time (sec)	N/A	0.289	0.084	0.647	0.208	0.255	0.000	0.284	1.690

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	684	42	31	0	36
N.S.	1	1.00	1.06	1.00	20.12	1.24	0.91	0.00	1.06
time (sec)	N/A	0.283	142.621	0.593	0.782	0.769	16.470	0.000	1.877

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1038	0	2799	0	0	13504	0	0	0
N.S.	1	0.00	2.70	0.00	0.00	13.01	0.00	0.00	0.00
time (sec)	N/A	0.000	9.494	0.000	0.000	0.519	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	714	0	1530	0	0	7726	0	0	0
N.S.	1	0.00	2.14	0.00	0.00	10.82	0.00	0.00	0.00
time (sec)	N/A	0.000	8.338	0.000	0.000	0.374	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	0	615	1284	0	3585	0	0	0
N.S.	1	0.00	1.49	3.11	0.00	8.68	0.00	0.00	0.00
time (sec)	N/A	0.000	8.832	1.960	0.000	0.344	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	136	167	140	217	892	0	182	628
N.S.	1	1.23	1.50	1.26	1.95	8.04	0.00	1.64	5.66
time (sec)	N/A	1.046	2.093	1.192	0.275	0.291	0.000	0.317	1.406

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	762	42	31	0	38
N.S.	1	1.00	1.06	1.00	22.41	1.24	0.91	0.00	1.12
time (sec)	N/A	0.280	70.685	0.595	0.998	0.342	15.211	0.000	1.600

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	972	0	3868	0	0	13683	0	0	0
N.S.	1	0.00	3.98	0.00	0.00	14.08	0.00	0.00	0.00
time (sec)	N/A	0.000	12.727	0.000	0.000	0.482	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	689	0	2403	0	0	7775	0	0	0
N.S.	1	0.00	3.49	0.00	0.00	11.28	0.00	0.00	0.00
time (sec)	N/A	0.000	10.697	0.000	0.000	0.355	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD
size	435	0	766	1098	0	3547	0	0	0
N.S.	1	0.00	1.76	2.52	0.00	8.15	0.00	0.00	0.00
time (sec)	N/A	0.000	8.631	1.979	0.000	0.309	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	76	64	143	173	617	0	184	1329
N.S.	1	0.95	0.80	1.79	2.16	7.71	0.00	2.30	16.61
time (sec)	N/A	0.285	0.095	1.416	0.202	0.272	0.000	0.341	1.774

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	751	36	24	0	30
N.S.	1	1.00	1.07	1.00	26.82	1.29	0.86	0.00	1.07
time (sec)	N/A	0.247	139.403	0.653	0.960	0.900	4.902	0.000	1.985

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1795	1596	5813	0	0	23903	0	0	0
N.S.	1	0.89	3.24	0.00	0.00	13.32	0.00	0.00	0.00
time (sec)	N/A	8.159	12.835	0.000	0.000	0.773	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1219	1118	2784	0	0	13309	0	0	0
N.S.	1	0.92	2.28	0.00	0.00	10.92	0.00	0.00	0.00
time (sec)	N/A	6.158	10.382	0.000	0.000	0.497	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	762	673	1009	1478	0	5731	0	0	0
N.S.	1	0.88	1.32	1.94	0.00	7.52	0.00	0.00	0.00
time (sec)	N/A	4.094	9.431	15.114	0.000	0.416	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	144	164	186	236	1035	0	263	196
N.S.	1	1.11	1.26	1.43	1.82	7.96	0.00	2.02	1.51
time (sec)	N/A	0.411	0.308	7.069	0.286	0.353	0.000	0.294	4.634

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	912	42	0	0	38
N.S.	1	1.00	1.06	1.00	26.82	1.24	0.00	0.00	1.12
time (sec)	N/A	0.284	112.918	0.724	1.261	37.808	0.000	0.000	11.508

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1245	0	2346	0	0	29722	0	0	0
N.S.	1	0.00	1.88	0.00	0.00	23.87	0.00	0.00	0.00
time (sec)	N/A	0.000	9.786	0.000	0.000	0.777	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	B	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	699	611	798	2767	0	11126	0	0	0
N.S.	1	0.87	1.14	3.96	0.00	15.92	0.00	0.00	0.00
time (sec)	N/A	4.390	9.436	37.578	0.000	0.549	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	231	212	183	334	2653	0	224	531
N.S.	1	1.12	1.03	0.89	1.62	12.88	0.00	1.09	2.58
time (sec)	N/A	0.627	3.371	18.533	0.282	0.450	0.000	0.282	4.489

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	1426	44	0	0	38
N.S.	1	1.00	1.06	1.00	39.61	1.22	0.00	0.00	1.06
time (sec)	N/A	0.329	79.402	0.608	1.389	1.889	0.000	0.000	18.741

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1122	943	1552	3563	0	16848	0	0	0
N.S.	1	0.84	1.38	3.18	0.00	15.02	0.00	0.00	0.00
time (sec)	N/A	5.599	10.268	89.783	0.000	0.669	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	234	237	292	418	3148	0	464	554
N.S.	1	1.11	1.12	1.38	1.98	14.92	0.00	2.20	2.63
time (sec)	N/A	0.514	0.590	54.093	0.318	0.645	0.000	0.297	8.269

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	1950	0	0	0	38
N.S.	1	1.00	1.06	1.00	54.17	0.00	0.00	0.00	1.06
time (sec)	N/A	0.324	139.069	0.764	3.034	0.000	0.000	0.000	19.629

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [17] had the largest ratio of [1.37500000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	C	15	15	1.27	14	1.071
2	C	11	11	1.31	14	0.786
3	C	9	9	1.24	14	0.643
4	C	5	5	1.32	12	0.417
5	C	8	8	1.20	14	0.571
6	C	10	10	1.18	14	0.714
7	C	14	14	1.16	14	1.000
8	A	14	14	1.03	16	0.875
9	A	9	9	0.96	16	0.562
10	A	9	9	1.02	16	0.562
11	A	4	4	1.00	14	0.286
12	A	4	4	1.00	16	0.250
13	C	12	12	1.16	16	0.750
14	A	9	9	1.31	16	0.562
15	C	17	17	1.05	16	1.062
16	F	0	0	N/A	0.000	N/A
17	C	22	22	1.41	16	1.375
18	C	16	15	1.22	16	0.938
19	C	9	9	1.17	14	0.643
20	C	4	4	1.10	16	0.250
21	C	4	4	1.08	16	0.250
22	C	14	14	1.43	16	0.875

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	C	8	7	1.16	14	0.500
24	C	7	6	1.16	14	0.429
25	C	6	5	1.22	12	0.417
26	N/A	3	0	1.00	14	0.000
27	N/A	3	0	1.00	14	0.000
28	C	12	11	1.42	16	0.688
29	C	11	10	1.43	16	0.625
30	C	7	7	1.14	14	0.500
31	N/A	3	0	1.00	16	0.000
32	N/A	3	0	1.00	16	0.000
33	C	14	13	1.15	16	0.812
34	C	12	11	1.19	16	0.688
35	C	10	9	1.17	14	0.643
36	N/A	3	0	1.00	16	0.000
37	N/A	3	0	1.00	16	0.000
38	C	16	15	1.19	16	0.938
39	C	13	12	1.20	16	0.750
40	C	10	9	1.15	16	0.562
41	C	7	6	1.08	16	0.375
42	C	10	9	1.18	16	0.562
43	C	13	12	1.18	16	0.750
44	C	16	15	1.19	16	0.938
45	A	9	9	1.03	18	0.500
46	A	9	9	1.03	18	0.500
47	A	4	4	1.00	18	0.222
48	A	4	4	1.00	18	0.222
49	C	11	10	1.13	18	0.556
50	A	9	9	1.27	18	0.500
51	C	16	15	1.10	18	0.833
52	A	14	14	1.21	18	0.778
53	C	22	21	1.48	18	1.167
54	C	19	18	1.49	18	1.000
55	C	4	4	1.06	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	C	4	4	1.05	18	0.222
57	C	4	4	1.08	18	0.222
58	C	13	12	1.52	18	0.667
59	C	16	15	1.45	18	0.833
60	C	13	12	1.26	12	1.000
61	C	10	9	1.20	12	0.750
62	C	7	6	1.10	12	0.500
63	C	10	9	1.24	12	0.750
64	C	13	12	1.24	12	1.000
65	N/A	3	0	1.00	16	0.000
66	N/A	3	0	1.00	16	0.000
67	N/A	4	0	1.00	10	0.000
68	A	1	1	1.00	18	0.056
69	A	1	1	1.00	20	0.050
70	A	1	1	1.00	20	0.050
71	A	1	1	1.00	22	0.045
72	N/A	2	0	1.00	18	0.000
73	C	4	4	1.05	16	0.250
74	A	4	4	1.00	16	0.250
75	C	4	4	1.07	14	0.286
76	N/A	3	0	1.00	14	0.000
77	N/A	3	0	1.00	16	0.000
78	C	4	4	1.14	12	0.333
79	C	4	4	1.14	12	0.333
80	C	4	4	1.14	12	0.333
81	C	4	4	1.14	10	0.400
82	C	4	4	1.16	12	0.333
83	C	4	4	1.15	12	0.333
84	C	4	4	1.14	12	0.333
85	A	4	4	1.00	14	0.286
86	A	4	4	1.00	14	0.286
87	A	4	4	1.00	14	0.286
88	A	4	4	1.00	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
89	A	4	4	1.00	14	0.286
90	A	4	4	1.00	14	0.286
91	A	4	4	1.00	14	0.286
92	A	1	1	1.00	20	0.050
93	A	1	1	1.00	20	0.050
94	A	1	1	1.00	20	0.050
95	A	1	1	1.00	24	0.042
96	A	3	3	1.00	21	0.143
97	A	3	3	1.00	21	0.143
98	A	3	3	1.00	19	0.158
99	A	3	3	1.00	21	0.143
100	A	3	3	1.00	21	0.143
101	A	3	3	1.00	21	0.143
102	A	3	3	0.95	23	0.130
103	A	3	3	1.00	23	0.130
104	A	3	3	0.84	21	0.143
105	A	5	5	0.96	23	0.217
106	A	5	5	1.02	23	0.217
107	A	7	7	1.39	23	0.304
108	A	16	15	1.11	23	0.652
109	A	15	14	1.14	23	0.609
110	A	10	10	1.03	21	0.476
111	N/A	2	0	1.00	23	0.000
112	N/A	2	0	1.00	23	0.000
113	A	17	16	1.01	23	0.696
114	A	17	16	1.00	23	0.696
115	A	10	10	0.97	21	0.476
116	N/A	2	0	1.00	23	0.000
117	N/A	2	0	1.00	23	0.000
118	A	16	16	1.13	21	0.762
119	A	14	14	1.21	21	0.667
120	A	10	10	1.17	21	0.476
121	A	8	8	1.36	19	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
122	A	9	9	0.71	21	0.429
123	A	13	13	0.82	21	0.619
124	A	15	15	0.78	21	0.714
125	A	23	23	1.06	21	1.095
126	A	15	14	0.93	21	0.667
127	A	10	10	0.95	19	0.526
128	A	5	5	0.57	21	0.238
129	A	5	5	0.61	21	0.238
130	F	0	0	N/A	0.000	N/A
131	A	19	18	0.91	21	0.857
132	A	12	12	0.84	19	0.632
133	A	5	5	0.51	21	0.238
134	A	5	5	0.54	21	0.238
135	A	7	7	0.76	21	0.333
136	A	9	8	0.58	21	0.381
137	A	8	7	0.60	21	0.333
138	A	7	6	0.65	19	0.316
139	N/A	2	0	1.00	21	0.000
140	N/A	2	0	1.00	21	0.000
141	A	13	12	0.59	21	0.571
142	A	11	10	0.63	21	0.476
143	A	9	8	0.75	19	0.421
144	N/A	2	0	1.00	21	0.000
145	N/A	2	0	1.00	21	0.000
146	A	20	19	0.73	21	0.905
147	A	16	15	0.70	21	0.714
148	A	11	10	0.72	19	0.526
149	N/A	2	0	1.00	21	0.000
150	N/A	2	0	1.00	21	0.000
151	N/A	2	0	1.00	23	0.000
152	A	7	7	0.96	23	0.304
153	A	5	5	0.98	23	0.217
154	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
155	N/A	2	0	1.00	23	0.000
156	N/A	2	0	1.00	23	0.000
157	A	3	3	1.00	18	0.167
158	A	3	3	1.00	18	0.167
159	A	3	3	1.00	16	0.188
160	A	3	3	1.00	18	0.167
161	A	3	3	1.00	18	0.167
162	A	3	3	1.00	18	0.167
163	A	3	3	0.95	20	0.150
164	A	3	3	1.00	20	0.150
165	A	3	3	0.97	18	0.167
166	A	3	3	1.00	20	0.150
167	A	3	3	1.00	20	0.150
168	A	3	3	1.00	20	0.150
169	A	11	10	0.93	20	0.500
170	A	10	9	0.96	20	0.450
171	A	9	8	1.01	18	0.444
172	N/A	2	0	1.00	20	0.000
173	N/A	2	0	1.00	20	0.000
174	A	16	15	0.95	20	0.750
175	A	13	12	1.04	18	0.667
176	N/A	2	0	1.00	20	0.000
177	N/A	2	0	1.00	20	0.000
178	A	20	19	1.53	18	1.056
179	N/A	2	0	1.00	20	0.000
180	N/A	2	0	1.00	20	0.000
181	N/A	2	0	1.00	20	0.000
182	A	3	3	1.00	20	0.150
183	A	3	3	1.00	20	0.150
184	A	3	3	1.00	18	0.167
185	N/A	2	0	1.00	20	0.000
186	N/A	2	0	1.00	20	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
187	A	18	17	1.03	29	0.586
188	A	17	16	1.05	29	0.552
189	A	12	12	0.97	27	0.444
190	A	5	5	1.09	22	0.227
191	N/A	1	0	1.00	29	0.000
192	N/A	1	0	1.00	29	0.000
193	A	30	29	1.10	31	0.935
194	A	27	26	1.11	31	0.839
195	A	18	18	1.09	29	0.621
196	A	9	9	0.96	24	0.375
197	N/A	1	0	1.00	31	0.000
198	N/A	1	0	1.00	31	0.000
199	F	0	0	N/A	0.000	N/A
200	F	0	0	N/A	0.000	N/A
201	A	23	23	1.13	29	0.793
202	A	7	7	1.05	24	0.292
203	N/A	1	0	1.00	31	0.000
204	N/A	1	0	1.00	31	0.000
205	A	23	22	1.04	29	0.759
206	A	21	20	1.05	29	0.690
207	A	16	15	1.05	27	0.556
208	A	8	8	1.22	22	0.364
209	N/A	1	0	1.00	29	0.000
210	N/A	1	0	1.00	29	0.000
211	F	0	0	N/A	0.000	N/A
212	F	0	0	N/A	0.000	N/A
213	A	24	23	1.07	29	0.793
214	A	15	14	1.04	24	0.583
215	N/A	1	0	1.00	31	0.000
216	N/A	1	0	1.00	31	0.000
217	F	0	0	N/A	0.000	N/A
218	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
219	F	0	0	N/A	0.000	N/A
220	A	20	19	1.11	24	0.792
221	N/A	1	0	1.00	31	0.000
222	N/A	1	0	1.00	31	0.000
223	A	13	12	0.88	26	0.462
224	A	12	11	0.91	26	0.423
225	A	11	10	0.96	24	0.417
226	C	8	7	1.13	19	0.368
227	N/A	1	0	1.00	26	0.000
228	C	25	24	0.91	28	0.857
229	C	22	21	0.92	28	0.750
230	C	17	16	0.98	26	0.615
231	C	13	12	1.11	21	0.571
232	N/A	1	0	1.00	28	0.000
233	F	0	0	N/A	0.000	N/A
234	F	0	0	N/A	0.000	N/A
235	C	22	21	0.97	26	0.808
236	C	13	12	1.16	21	0.571
237	N/A	1	0	1.00	28	0.000
238	C	15	14	0.92	26	0.538
239	C	15	14	0.94	26	0.538
240	C	14	13	0.99	24	0.542
241	C	11	10	1.09	19	0.526
242	N/A	1	0	1.00	26	0.000
243	C	27	26	0.96	28	0.929
244	C	26	25	0.98	28	0.893
245	C	22	21	0.99	26	0.808
246	C	15	14	1.11	21	0.667
247	N/A	1	0	1.00	28	0.000
248	F	0	0	N/A	0.000	N/A
249	F	0	0	N/A	0.000	N/A
250	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
251	C	19	18	1.22	21	0.857
252	N/A	1	0	1.00	28	0.000
253	A	7	6	1.00	29	0.207
254	A	6	5	1.01	29	0.172
255	A	5	4	1.04	27	0.148
256	A	4	3	1.04	22	0.136
257	N/A	1	0	1.00	29	0.000
258	N/A	1	0	1.00	29	0.000
259	A	13	13	1.03	31	0.419
260	A	11	11	0.98	31	0.355
261	A	7	7	1.00	29	0.241
262	A	3	3	1.00	24	0.125
263	A	10	10	1.00	31	0.323
264	A	12	12	0.98	31	0.387
265	A	23	23	0.97	31	0.742
266	A	14	14	0.90	31	0.452
267	A	12	12	0.96	29	0.414
268	A	4	3	0.79	24	0.125
269	A	18	18	0.98	31	0.581
270	A	24	24	1.04	31	0.774
271	A	22	21	0.95	29	0.724
272	A	17	16	0.94	29	0.552
273	A	12	11	0.94	27	0.407
274	A	5	4	1.12	22	0.182
275	N/A	1	0	1.00	29	0.000
276	N/A	1	0	1.00	29	0.000
277	A	23	22	0.98	31	0.710
278	A	22	21	0.95	31	0.677
279	A	14	14	0.96	29	0.483
280	A	6	5	1.00	24	0.208
281	N/A	1	0	1.00	31	0.000
282	N/A	1	0	1.00	31	0.000
283	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
284	A	24	23	0.95	31	0.742
285	A	14	13	0.94	29	0.448
286	A	5	4	1.02	24	0.167
287	N/A	1	0	1.00	31	0.000
288	N/A	1	0	1.00	31	0.000
289	A	7	6	1.01	26	0.231
290	A	6	5	1.00	26	0.192
291	A	5	4	1.00	24	0.167
292	A	4	3	1.00	19	0.158
293	N/A	1	0	1.00	26	0.000
294	C	23	22	0.95	28	0.786
295	C	20	19	0.97	28	0.679
296	C	15	14	1.02	26	0.538
297	A	10	9	1.04	21	0.429
298	N/A	1	0	1.00	28	0.000
299	C	30	29	0.92	28	1.036
300	C	20	19	0.90	28	0.679
301	A	17	16	0.92	26	0.615
302	A	6	5	0.90	21	0.238
303	N/A	1	0	1.00	28	0.000
304	A	10	9	0.88	26	0.346
305	A	9	8	0.90	26	0.308
306	A	8	7	0.95	24	0.292
307	A	8	7	1.07	19	0.368
308	N/A	1	0	1.00	26	0.000
309	A	14	13	0.84	28	0.464
310	A	13	12	0.86	28	0.429
311	A	12	11	0.93	26	0.423
312	A	9	8	0.97	21	0.381
313	N/A	1	0	1.00	28	0.000
314	A	10	9	0.82	28	0.321
315	A	9	8	0.87	26	0.308
316	A	7	6	1.33	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
317	N/A	1	0	1.00	28	0.000
318	N/A	1	0	1.00	24	0.000
319	N/A	1	0	1.00	24	0.000
320	N/A	1	0	1.00	22	0.000
321	A	6	5	0.97	24	0.208
322	A	10	9	0.96	26	0.346
323	A	11	10	0.91	26	0.385
324	A	6	5	0.97	24	0.208
325	A	10	9	0.96	26	0.346
326	A	11	10	0.91	26	0.385
327	A	10	9	1.04	24	0.375
328	A	14	13	0.98	26	0.500
329	A	17	16	0.89	26	0.615
330	A	10	9	1.04	24	0.375
331	A	14	13	0.98	26	0.500
332	A	17	16	0.89	26	0.615
333	C	20	19	1.01	32	0.594
334	C	15	14	1.01	32	0.438
335	A	12	11	0.99	30	0.367
336	A	8	7	0.88	25	0.280
337	N/A	1	0	1.00	32	0.000
338	C	30	29	0.92	34	0.853
339	C	27	26	0.95	34	0.765
340	C	19	18	0.99	32	0.562
341	C	11	10	1.13	27	0.370
342	N/A	1	0	1.00	34	0.000
343	F	0	0	N/A	0.000	N/A
344	F	0	0	N/A	0.000	N/A
345	A	26	25	0.90	32	0.781
346	A	8	7	0.88	27	0.259
347	N/A	1	0	1.00	34	0.000
348	A	14	13	0.87	26	0.500
349	A	14	13	0.88	26	0.500

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
350	A	13	12	0.93	24	0.500
351	A	10	9	1.03	19	0.474
352	N/A	1	0	1.00	26	0.000
353	A	25	24	0.86	32	0.750
354	A	23	22	0.88	32	0.688
355	A	19	18	0.93	30	0.600
356	C	10	9	1.08	25	0.360
357	N/A	1	0	1.00	32	0.000
358	A	18	17	0.81	34	0.500
359	A	16	15	0.84	32	0.469
360	A	10	9	1.19	27	0.333
361	N/A	1	0	1.00	34	0.000
362	C	31	30	0.98	34	0.882
363	C	21	20	0.95	34	0.588
364	A	18	17	0.99	32	0.531
365	A	8	7	0.87	27	0.259
366	N/A	1	0	1.00	34	0.000
367	F	0	0	N/A	0.000	N/A
368	F	0	0	N/A	0.000	N/A
369	C	24	23	0.97	34	0.676
370	C	22	21	1.16	29	0.724
371	N/A	1	0	1.00	36	0.000
372	F	0	0	N/A	0.000	N/A
373	F	0	0	N/A	0.000	N/A
374	F	0	0	N/A	0.000	N/A
375	A	7	6	0.88	29	0.207
376	N/A	1	0	1.00	36	0.000
377	A	20	19	0.85	32	0.594
378	A	21	20	0.87	32	0.625
379	A	20	19	0.91	30	0.633
380	A	7	6	1.15	25	0.240
381	N/A	1	0	1.00	32	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
382	A	31	30	0.85	28	1.071
383	A	29	28	0.87	28	1.000
384	A	23	22	0.92	26	0.846
385	A	14	13	0.98	21	0.619
386	N/A	1	0	1.00	28	0.000
387	A	26	25	0.82	34	0.735
388	A	21	20	0.85	32	0.625
389	A	11	10	1.21	27	0.370
390	N/A	1	0	1.00	34	0.000
391	F	0	0	N/A	0.000	N/A
392	F	0	0	N/A	0.000	N/A
393	A	24	23	0.96	32	0.719
394	A	8	7	0.87	27	0.259
395	N/A	1	0	1.00	34	0.000
396	F	0	0	N/A	0.000	N/A
397	F	0	0	N/A	0.000	N/A
398	C	27	26	0.96	34	0.765
399	C	25	24	1.21	29	0.828
400	N/A	1	0	1.00	36	0.000
401	F	0	0	N/A	0.000	N/A
402	F	0	0	N/A	0.000	N/A
403	F	0	0	N/A	0.000	N/A
404	A	8	7	0.89	29	0.241
405	N/A	1	0	1.00	36	0.000
406	F	0	0	N/A	0.000	N/A
407	F	0	0	N/A	0.000	N/A
408	A	31	30	0.90	32	0.938
409	A	10	9	1.09	27	0.333
410	N/A	1	0	1.00	34	0.000
411	F	0	0	N/A	0.000	N/A
412	F	0	0	N/A	0.000	N/A

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
413	F	0	0	N/A	0.000	N/A
414	C	18	17	1.05	27	0.630
415	N/A	1	0	1.00	34	0.000
416	F	0	0	N/A	0.000	N/A
417	F	0	0	N/A	0.000	N/A
418	A	9	8	1.22	21	0.381
419	N/A	1	0	1.00	28	0.000
420	C	13	12	1.18	26	0.462
421	C	13	12	1.22	26	0.462
422	C	12	11	1.28	24	0.458
423	A	7	6	1.00	19	0.316
424	N/A	1	0	1.00	26	0.000
425	F	0	0	N/A	0.000	N/A
426	F	0	0	N/A	0.000	N/A
427	C	25	24	1.29	30	0.800
428	C	15	14	1.13	25	0.560
429	N/A	1	0	1.00	32	0.000
430	F	0	0	N/A	0.000	N/A
431	F	0	0	N/A	0.000	N/A
432	C	30	29	1.31	32	0.906
433	A	8	7	0.96	27	0.259
434	N/A	1	0	1.00	34	0.000
435	A	16	15	0.87	32	0.469
436	A	16	15	0.89	32	0.469
437	A	15	14	0.92	30	0.467
438	A	8	7	1.16	25	0.280
439	N/A	1	0	1.00	32	0.000
440	A	19	18	0.85	34	0.529
441	A	18	17	0.87	34	0.500
442	A	15	14	0.88	32	0.438
443	C	4	4	1.12	27	0.148
444	N/A	1	0	1.00	34	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
445	A	15	14	0.84	34	0.412
446	A	12	11	0.84	32	0.344
447	A	8	7	1.13	27	0.259
448	N/A	1	0	1.00	34	0.000
449	C	21	20	1.13	32	0.625
450	C	20	19	1.17	32	0.594
451	C	17	16	1.25	30	0.533
452	A	8	7	0.98	25	0.280
453	N/A	1	0	1.00	32	0.000
454	F	0	0	N/A	0.000	N/A
455	F	0	0	N/A	0.000	N/A
456	F	0	0	N/A	0.000	N/A
457	C	18	17	1.14	21	0.810
458	N/A	1	0	1.00	28	0.000
459	F	0	0	N/A	0.000	N/A
460	F	0	0	N/A	0.000	N/A
461	F	0	0	N/A	0.000	N/A
462	A	7	6	1.00	27	0.222
463	N/A	1	0	1.00	34	0.000
464	A	21	20	0.88	34	0.588
465	A	21	20	0.91	34	0.588
466	A	18	17	0.90	32	0.531
467	A	7	6	1.13	27	0.222
468	N/A	1	0	1.00	34	0.000
469	A	30	29	0.89	36	0.806
470	C	24	23	0.89	34	0.676
471	A	4	4	1.00	29	0.138
472	N/A	1	0	1.00	36	0.000
473	A	15	14	0.82	34	0.412
474	A	8	7	1.15	29	0.241
475	N/A	1	0	1.00	36	0.000
476	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
477	C	29	28	1.13	34	0.824
478	C	23	22	1.17	32	0.688
479	A	8	7	0.94	27	0.259
480	N/A	1	0	1.00	34	0.000
481	F	0	0	N/A	0.000	N/A
482	F	0	0	N/A	0.000	N/A
483	F	0	0	N/A	0.000	N/A
484	C	22	21	1.23	27	0.778
485	N/A	1	0	1.00	34	0.000
486	F	0	0	N/A	0.000	N/A
487	F	0	0	N/A	0.000	N/A
488	F	0	0	N/A	0.000	N/A
489	A	7	6	0.95	21	0.286
490	N/A	1	0	1.00	28	0.000
491	A	24	23	0.89	34	0.676
492	A	24	23	0.92	34	0.676
493	A	21	20	0.88	32	0.625
494	A	8	7	1.11	27	0.259
495	N/A	1	0	1.00	34	0.000
496	F	0	0	N/A	0.000	N/A
497	C	27	26	0.87	34	0.765
498	C	4	4	1.12	29	0.138
499	N/A	1	0	1.00	36	0.000
500	A	26	25	0.84	34	0.735
501	A	8	7	1.11	29	0.241
502	N/A	1	0	1.00	36	0.000

CHAPTER 3

Listing of integrals

3.1 $\int (c+dx)^4 \sinh(a+bx)\,dx$

3.1.1 Optimal result

Integrand size = 14, antiderivative size = 91

$$\int (c+dx)^4 \sinh(a+bx)\,dx = \frac{24d^4\cosh(a+bx)}{b^5}+\frac{12d^2(c+dx)^2\cosh(a+bx)}{b^3}+\frac{(c+dx)^4\cosh(a+bx)}{b}-\frac{24d^3(c+dx)\sinh(a+bx)}{b^4}-\frac{4d(c+dx)^3\sinh(a+bx)}{b^2}$$

output
```
24*d^4*cosh(b*x+a)/b^5+12*d^2*(d*x+c)^2*cosh(b*x+a)/b^3+(d*x+c)^4*cosh(b*x
+a)/b-24*d^3*(d*x+c)*sinh(b*x+a)/b^4-4*d*(d*x+c)^3*sinh(b*x+a)/b^2
```

3.1.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int (c+dx)^4 \sinh(a+bx)\,dx = \frac{(24d^4+12b^2d^2(c+dx)^2+b^4(c+dx)^4)\cosh(a+bx)-4bd(c+dx)\left(6d^2+b^2(c+dx)^2\right)\sinh(a+bx)}{b^5}$$

input
```
Integrate[(c + d*x)^4*Sinh[a + b*x],x]
```

output
```
((24*d^4 + 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cosh[a + b*x] - 4*b*d
*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Sinh[a + b*x])/b^5
```

3.1.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sinh(a+bx) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(c+dx)^4 \sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)^4 \sin(ia+ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^4 \cosh(a+bx)}{b}-\frac{4id\int (c+dx)^3 \cosh(a+bx)dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^4 \cosh(a+bx)}{b}-\frac{4id\int (c+dx)^3 \sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^4 \cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3 \sinh(a+bx)}{b}-\frac{3id\int -i(c+dx)^2 \sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i(c+dx)^4 \cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3 \sinh(a+bx)}{b}-\frac{3d\int (c+dx)^2 \sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^4 \cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3 \sinh(a+bx)}{b}-\frac{3d\int -i(c+dx)^2 \sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\int(c+dx)^2\sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \; 3777$$

$$-i\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int(c+dx)\cosh(a+bx)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \; 3042$$

$$-i\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \; 3777$$

$$-i\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{id\int-i\sinh(a+bx)dx}{b}\right)}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int\sinh(a+bx)dx}{b}\right)}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int-i\sin(ia+ibx)dx}{b}\right)}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}+\frac{id\int\sin(ia+ibx)dx}{b}\right)}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3118}$$

$$-i\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)$$

input
```
Int[(c + d*x)^4*Sinh[a + b*x],x]
```

output
```
(-I)*((I*(c + d*x)^4*Cosh[a + b*x])/b - ((4*I)*d*(((c + d*x)^3*Sinh[a + b*
x])/b + ((3*I)*d*((I*(c + d*x)^2*Cosh[a + b*x])/b - ((2*I)*d*(-((d*Cosh[a
+ b*x])/b^2) + ((c + d*x)*Sinh[a + b*x])/b))/b))/b))/b)
```

3.1.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.1.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(91) = 182$.

Time = 0.99 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.03

method	result
parallelrisch	$\frac{-4\left(\left(\frac{1}{2}d^2x^2+cdx+c^2\right)b^2+6d^2\right)xd\,b^2\left(\frac{dx}{2}+c\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2+8d\left((dx+c)^2b^2+6d^2\right)b(dx+c)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+\left(-d^4x^4-4c\right.}{b^5\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)}$
risch	$\frac{\left(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2-4b^3d^4x^3+4b^4c^3dx-12b^3c\,d^3x^2+b^4c^4-12b^3c^2d^2x+12b^2d^4x^2-4b^3c^3d+24b^2c\,d^3x+12b\right.}{2b^5}$
parts	$\frac{\cosh(bx+a)d^4x^4}{b}+\frac{4\cosh(bx+a)c\,d^3x^3}{b}+\frac{6\cosh(bx+a)c^2d^2x^2}{b}+\frac{4\cosh(bx+a)c^3dx}{b}+\frac{\cosh(bx+a)c^4}{b}-\frac{4d\left(\frac{d^3}{}\right.}{}$
meijerg	$-\frac{16id^4\sinh(a)\sqrt{\pi}\left(-\frac{ixb\left(\frac{5b^2x^2}{2}+15\right)\cosh(bx)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8}x^4b^4+\frac{15}{2}b^2x^2+15\right)\sinh(bx)}{10\sqrt{\pi}}\right)}{b^5}-\frac{16d^4\cosh(a)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}x^4b^4+\right.}{}\right.}{}$
derivativedivides	$\frac{d^4\left((bx+a)^4\cosh(bx+a)-4(bx+a)^3\sinh(bx+a)+12(bx+a)^2\cosh(bx+a)-24(bx+a)\sinh(bx+a)+24\cosh(bx+a)\right)}{b^4}+\frac{d^4a^4\cosh(bx+a)}{b^4}-$
default	$\frac{d^4\left((bx+a)^4\cosh(bx+a)-4(bx+a)^3\sinh(bx+a)+12(bx+a)^2\cosh(bx+a)-24(bx+a)\sinh(bx+a)+24\cosh(bx+a)\right)}{b^4}+\frac{d^4a^4\cosh(bx+a)}{b^4}-$

input
```
int((d*x+c)^4*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(-4*((1/2*d^2*x^2+c*d*x+c^2)*b^2+6*d^2)*x*d*b^2*(1/2*d*x+c)*tanh(1/2*b*x+1
/2*a)^2+8*d*((d*x+c)^2*b^2+6*d^2)*b*(d*x+c)*tanh(1/2*b*x+1/2*a)+(-d^4*x^4-
4*c*d^3*x^3-6*c^2*d^2*x^2-4*c^3*d*x-2*c^4)*b^4+12*(-d^4*x^2-2*c*d^3*x-2*c^
2*d^2)*b^2-48*d^4)/b^5/(tanh(1/2*b*x+1/2*a)^2-1)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.86

$$\int (c+dx)^4\sinh(a+bx)\,dx$$
$$=\frac{(b^4d^4x^4+4\,b^4cd^3x^3+b^4c^4+12\,b^2c^2d^2+24\,d^4+6\,(b^4c^2d^2+2\,b^2d^4)x^2+4\,(b^4c^3d+6\,b^2cd^3)x)\cosh\left(bx+a\right.}{b^5}$$

input
```
integrate((d*x+c)^4*sinh(b*x+a),x, algorithm="fricas")
```

output
```
((b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 + 12*b^2*c^2*d^2 + 24*d^4 + 6*(b
^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d + 6*b^2*c*d^3)*x)*cosh(b*x + a)
 - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d + 6*b*c*d^3 + 3*(b^3*c^2*d
^2 + 2*b*d^4)*x)*sinh(b*x + a))/b^5
```

3.1.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(92) = 184$.

Time = 0.35 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.42

$$\int (c+dx)^4 \sinh(a+bx)\,dx$$
$$= \begin{cases} \frac{c^4 \cosh{(a+bx)}}{b} + \frac{4c^3dx \cosh{(a+bx)}}{b} + \frac{6c^2d^2x^2 \cosh{(a+bx)}}{b} + \frac{4cd^3x^3 \cosh{(a+bx)}}{b} + \frac{d^4x^4 \cosh{(a+bx)}}{b} - \frac{4c^3d \sinh{(a+bx)}}{b^2} - 12c^2d^2 \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right) \sinh{(a)} \end{cases}$$

input
```
integrate((d*x+c)**4*sinh(b*x+a),x)
```

output
```
Piecewise((c**4*cosh(a + b*x)/b + 4*c**3*d*x*cosh(a + b*x)/b + 6*c**2*d**2
*x**2*cosh(a + b*x)/b + 4*c*d**3*x**3*cosh(a + b*x)/b + d**4*x**4*cosh(a +
 b*x)/b - 4*c**3*d*sinh(a + b*x)/b**2 - 12*c**2*d**2*x*sinh(a + b*x)/b**2
- 12*c*d**3*x**2*sinh(a + b*x)/b**2 - 4*d**4*x**3*sinh(a + b*x)/b**2 + 12*
c**2*d**2*cosh(a + b*x)/b**3 + 24*c*d**3*x*cosh(a + b*x)/b**3 + 12*d**4*x*
*2*cosh(a + b*x)/b**3 - 24*c*d**3*sinh(a + b*x)/b**4 - 24*d**4*x*sinh(a +
b*x)/b**4 + 24*d**4*cosh(a + b*x)/b**5, Ne(b, 0)), ((c**4*x + 2*c**3*d*x**
2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sinh(a), True))
```

3.1.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(91) = 182$.

Time = 0.21 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.58

$$\begin{aligned}\int (c+dx)^4 \sinh(a+bx)\,dx =\ & \frac{c^4e^{(bx+a)}}{2\,b} + \frac{2\,(bxe^a - e^a)c^3de^{(bx)}}{b^2} + \frac{c^4e^{(-bx-a)}}{2\,b} \\ & + \frac{2\,(bx+1)c^3de^{(-bx-a)}}{b^2} + \frac{3\,(b^2x^2e^a - 2\,bxe^a + 2\,e^a)c^2d^2e^{(bx)}}{b^3} \\ & + \frac{3\,(b^2x^2 + 2\,bx + 2)c^2d^2e^{(-bx-a)}}{b^3} \\ & + \frac{2\,(b^3x^3e^a - 3\,b^2x^2e^a + 6\,bxe^a - 6\,e^a)cd^3e^{(bx)}}{b^4} \\ & + \frac{2\,(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6)cd^3e^{(-bx-a)}}{b^4} \\ & + \frac{(b^4x^4e^a - 4\,b^3x^3e^a + 12\,b^2x^2e^a - 24\,bxe^a + 24\,e^a)d^4e^{(bx)}}{2\,b^5} \\ & + \frac{(b^4x^4 + 4\,b^3x^3 + 12\,b^2x^2 + 24\,bx + 24)d^4e^{(-bx-a)}}{2\,b^5}\end{aligned}$$

input
```
integrate((d*x+c)^4*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*c^4*e^(b*x + a)/b + 2*(b*x*e^a - e^a)*c^3*d*e^(b*x)/b^2 + 1/2*c^4*e^(-
b*x - a)/b + 2*(b*x + 1)*c^3*d*e^(-b*x - a)/b^2 + 3*(b^2*x^2*e^a - 2*b*x*e
^a + 2*e^a)*c^2*d^2*e^(b*x)/b^3 + 3*(b^2*x^2 + 2*b*x + 2)*c^2*d^2*e^(-b*x
- a)/b^3 + 2*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*c*d^3*e^(b*
x)/b^4 + 2*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*c*d^3*e^(-b*x - a)/b^4 + 1/2*
(b^4*x^4*e^a - 4*b^3*x^3*e^a + 12*b^2*x^2*e^a - 24*b*x*e^a + 24*e^a)*d^4*e
^(b*x)/b^5 + 1/2*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*d^4*e^(-
b*x - a)/b^5
```

3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(91) = 182$.

Time = 0.26 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.56

$$\int (c+dx)^4 \sinh(a+bx)\,dx$$
$$= \frac{(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 - 4\,b^3d^4x^3 + 4\,b^4c^3dx - 12\,b^3cd^3x^2 + b^4c^4 - 12\,b^3c^2d^2x + 12\,b^2d^4x^2 - 4\,b}{2\,b^5}$$
$$+ \frac{(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 + 4\,b^3d^4x^3 + 4\,b^4c^3dx + 12\,b^3cd^3x^2 + b^4c^4 + 12\,b^3c^2d^2x + 12\,b^2d^4x^2 +}{2\,b^5}$$

input
```
integrate((d*x+c)^4*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 4
*b^4*c^3*d*x - 12*b^3*c*d^3*x^2 + b^4*c^4 - 12*b^3*c^2*d^2*x + 12*b^2*d^4*
x^2 - 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*
d^3 + 24*d^4)*e^(b*x + a)/b^5 + 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4
*c^2*d^2*x^2 + 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4
+ 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^2 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^
2*c^2*d^2 + 24*b*d^4*x + 24*b*c*d^3 + 24*d^4)*e^(-b*x - a)/b^5
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.36

$$\int (c+dx)^4 \sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)\,\left(b^4\,c^4+12\,b^2\,c^2\,d^2+24\,d^4\right)}{b^5} - \frac{4\sinh(a+b\,x)\,\left(b^2\,c^3\,d+6\,c\,d^3\right)}{b^4} + \frac{d^4\,x^4\cosh(a+b\,x)}{b} + \frac{4\,x\cosh(a+b\,x)\,\left(b^2\,c^3\,d+6\,c\,d^3\right)}{b^3} - \frac{4\,d^4\,x^3\sinh(a+b\,x)}{b^2} - \frac{12\,x\sinh(a+b\,x)\,\left(b^2\,c^2\,d^2+2\,d^4\right)}{b^4} + \frac{6\,x^2\cosh(a+b\,x)\,\left(b^2\,c^2\,d^2+2\,d^4\right)}{b^3} + \frac{4\,c\,d^3\,x^3\cosh(a+b\,x)}{b} - \frac{12\,c\,d^3\,x^2\sinh(a+b\,x)}{b^2}$$

input
```
int(sinh(a + b*x)*(c + d*x)^4,x)
```

output
```
(cosh(a + b*x)*(24*d^4 + b^4*c^4 + 12*b^2*c^2*d^2))/b^5 - (4*sinh(a + b*x)
*(6*c*d^3 + b^2*c^3*d))/b^4 + (d^4*x^4*cosh(a + b*x))/b + (4*x*cosh(a + b*
x)*(6*c*d^3 + b^2*c^3*d))/b^3 - (4*d^4*x^3*sinh(a + b*x))/b^2 - (12*x*sinh
(a + b*x)*(2*d^4 + b^2*c^2*d^2))/b^4 + (6*x^2*cosh(a + b*x)*(2*d^4 + b^2*c
^2*d^2))/b^3 + (4*c*d^3*x^3*cosh(a + b*x))/b - (12*c*d^3*x^2*sinh(a + b*x)
)/b^2
```

3.2 $\int (c+dx)^3 \sinh(a+bx)\,dx$

3.2.1 Optimal result

Integrand size = 14, antiderivative size = 70

$$\int (c+dx)^3 \sinh(a+bx)\,dx = \frac{6d^2(c+dx)\cosh(a+bx)}{b^3} + \frac{(c+dx)^3\cosh(a+bx)}{b} - \frac{6d^3\sinh(a+bx)}{b^4} - \frac{3d(c+dx)^2\sinh(a+bx)}{b^2}$$

output
```
6*d^2*(d*x+c)*cosh(b*x+a)/b^3+(d*x+c)^3*cosh(b*x+a)/b-6*d^3*sinh(b*x+a)/b^
4-3*d*(d*x+c)^2*sinh(b*x+a)/b^2
```

3.2.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int (c+dx)^3 \sinh(a+bx)\,dx = \frac{b(c+dx)\left(6d^2+b^2(c+dx)^2\right)\cosh(a+bx) - 3d\left(2d^2+b^2(c+dx)^2\right)\sinh(a+bx)}{b^4}$$

input
```
Integrate[(c + d*x)^3*Sinh[a + b*x],x]
```

output
```
(b*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] - 3*d*(2*d^2 + b^2*(c
 + d*x)^2)*Sinh[a + b*x])/b^4
```

3.2.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(c+dx)^3 \sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)^3 \sin(ia+ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\int (c+dx)^2\cosh(a+bx)dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\int (c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2id\int -i(c+dx)\sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int (c+dx)\sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int -i(c+dx)\sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \color{blue}{26}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\int(c+dx)\sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \color{blue}{3777}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\cosh(a+bx)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \color{blue}{3042}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)}{b}\right)$$

$$\Big\downarrow \color{blue}{3117}$$

$$-i\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)$$

input
```
Int[(c + d*x)^3*Sinh[a + b*x],x]
```

output
```
(-I)*((I*(c + d*x)^3*Cosh[a + b*x])/b - ((3*I)*d*(((c + d*x)^2*Sinh[a + b*
x])/b + ((2*I)*d*((I*(c + d*x)*Cosh[a + b*x])/b - (I*d*Sinh[a + b*x])/b^2)
)/b))/b)
```

3.2. $\int(c+dx)^3\sinh(a+bx)\,dx$

3.2.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.2.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.80

method	result
parallelrisch	$\frac{-3xd\left(\left(\frac{1}{3}d^2x^2+cdx+c^2\right)b^2+2d^2\right)b\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2+6d\left((dx+c)^2b^2+2d^2\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-2b\left(\frac{dx}{2}+c\right)\left(\left(d^2x^2+cdx+c^2\right)b^2\right.}{b^4\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)}$
risch	$\frac{\left(d^3x^3b^3+3b^3c\,d^2x^2+3b^3c^2dx-3b^2d^3x^2+b^3c^3-6b^2c\,d^2x-3b^2c^2d+6b\,d^3x+6bc\,d^2-6d^3\right)\mathrm{e}^{bx+a}}{2b^4}+\frac{\left(d^3x^3b^3+3b^3c\,d^2x^2+\right.}{}$
parts	$\frac{\cosh(bx+a)d^3x^3}{b}+\frac{3\cosh(bx+a)c\,d^2x^2}{b}+\frac{3\cosh(bx+a)dx\,c^2}{b}+\frac{\cosh(bx+a)c^3}{b}-\frac{3d\left(\frac{d^2\left((bx+a)^2\sinh(bx+a)-2(bx\right.}{b}\right.}{}$
derivativedivides	$\frac{\frac{d^3\left((bx+a)^3\cosh(bx+a)-3(bx+a)^2\sinh(bx+a)+6(bx+a)\cosh(bx+a)-6\sinh(bx+a)\right)}{b^3}-\frac{3d^3a\left((bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(bx+\right.}{b^3}}{}$
default	$\frac{\frac{d^3\left((bx+a)^3\cosh(bx+a)-3(bx+a)^2\sinh(bx+a)+6(bx+a)\cosh(bx+a)-6\sinh(bx+a)\right)}{b^3}-\frac{3d^3a\left((bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(bx+\right.}{b^3}}{}$
meijerg	$\frac{8d^3\sinh(a)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(\frac{3b^2x^2}{2}+3\right)\cosh(bx)}{4\sqrt{\pi}}+\frac{xb\left(\frac{b^2x^2}{2}+3\right)\sinh(bx)}{4\sqrt{\pi}}\right)}{b^4}-\frac{8id^3\cosh(a)\sqrt{\pi}\left(\frac{ixb\left(\frac{5b^2x^2}{2}+15\right)\cosh(bx)}{20\sqrt{\pi}}-\frac{i\left(\right.}{}\right.}{b^4}$

input
```
int((d*x+c)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(-3*x*d*((1/3*d^2*x^2+c*d*x+c^2)*b^2+2*d^2)*b*tanh(1/2*b*x+1/2*a)^2+6*d*((
d*x+c)^2*b^2+2*d^2)*tanh(1/2*b*x+1/2*a)-2*b*(1/2*d*x+c)*((d^2*x^2+c*d*x+c^
2)*b^2+6*d^2))/b^4/(tanh(1/2*b*x+1/2*a)^2-1)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.56

$$\int (c+dx)^3 \sinh(a+bx)\,dx$$
$$= \frac{(b^3d^3x^3 + 3\,b^3cd^2x^2 + b^3c^3 + 6\,bcd^2 + 3\,(b^3c^2d + 2\,bd^3)x)\cosh{(bx+a)} - 3\,(b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d + 2}{b^4}$$

input
```
integrate((d*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

output
```
((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 6*b*c*d^2 + 3*(b^3*c^2*d + 2*b
*d^3)*x)*cosh(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + 2*d^
3)*sinh(b*x + a))/b^4
```

3.2.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(70) = 140$.

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.89

$$\int (c+dx)^3 \sinh(a+bx)\,dx$$
$$= \begin{cases} \frac{c^3\cosh{(a+bx)}}{b} + \frac{3c^2dx\cosh{(a+bx)}}{b} + \frac{3cd^2x^2\cosh{(a+bx)}}{b} + \frac{d^3x^3\cosh{(a+bx)}}{b} - \frac{3c^2d\sinh{(a+bx)}}{b^2} - \frac{6cd^2x\sinh{(a+bx)}}{b^2} - \frac{3d^3x^2\sin}{b} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sinh{(a)} \end{cases}$$

input
```
integrate((d*x+c)**3*sinh(b*x+a),x)
```

output
```
Piecewise((c**3*cosh(a + b*x)/b + 3*c**2*d*x*cosh(a + b*x)/b + 3*c*d**2*x*
*2*cosh(a + b*x)/b + d**3*x**3*cosh(a + b*x)/b - 3*c**2*d*sinh(a + b*x)/b*
*2 - 6*c*d**2*x*sinh(a + b*x)/b**2 - 3*d**3*x**2*sinh(a + b*x)/b**2 + 6*c*
d**2*cosh(a + b*x)/b**3 + 6*d**3*x*cosh(a + b*x)/b**3 - 6*d**3*sinh(a + b*
x)/b**4, Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4
)*sinh(a), True))
```

3.2.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(70) = 140$.

Time = 0.21 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.17

$$\int (c+dx)^3\sinh(a+bx)\,dx = \frac{c^3e^{(bx+a)}}{2\,b} + \frac{3\,(bxe^a - e^a)c^2de^{(bx)}}{2\,b^2} + \frac{c^3e^{(-bx-a)}}{2\,b} + \frac{3\,(bx+1)c^2de^{(-bx-a)}}{2\,b^2} + \frac{3\,(b^2x^2e^a - 2\,bxe^a + 2\,e^a)cd^2e^{(bx)}}{2\,b^3} + \frac{3\,(b^2x^2 + 2\,bx + 2)cd^2e^{(-bx-a)}}{2\,b^3} + \frac{(b^3x^3e^a - 3\,b^2x^2e^a + 6\,bxe^a - 6\,e^a)d^3e^{(bx)}}{2\,b^4} + \frac{(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6)d^3e^{(-bx-a)}}{2\,b^4}$$

input
```
integrate((d*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*c^3*e^(b*x + a)/b + 3/2*(b*x*e^a - e^a)*c^2*d*e^(b*x)/b^2 + 1/2*c^3*e^
(-b*x - a)/b + 3/2*(b*x + 1)*c^2*d*e^(-b*x - a)/b^2 + 3/2*(b^2*x^2*e^a - 2
*b*x*e^a + 2*e^a)*c*d^2*e^(b*x)/b^3 + 3/2*(b^2*x^2 + 2*b*x + 2)*c*d^2*e^(-
b*x - a)/b^3 + 1/2*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*d^3*e
^(b*x)/b^4 + 1/2*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*d^3*e^(-b*x - a)/b^4
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.91

$$\int (c+dx)^3\sinh(a+bx)\,dx = \frac{(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx - 3\,b^2d^3x^2 + b^3c^3 - 6\,b^2cd^2x - 3\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 - 6\,d^3)e^{(bx+a)}}{2\,b^4} + \frac{(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + 3\,b^2d^3x^2 + b^3c^3 + 6\,b^2cd^2x + 3\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 + 6\,d^3)e^{(-bx-a)}}{2\,b^4}$$

input
```
integrate((d*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x - 3*b^2*d^3*x^2 + b^3*c
^3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 6*d^3)*e^(b*x +
 a)/b^4 + 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*b^2*d^3*x
^2 + b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 6*d^3
)*e^(-b*x - a)/b^4
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.04

$$\int (c+dx)^3 \sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)\,\left(b^2\,c^3+6\,c\,d^2\right)}{b^3} - \frac{3\sinh(a+b\,x)\,\left(b^2\,c^2\,d+2\,d^3\right)}{b^4} + \frac{d^3\,x^3\cosh(a+b\,x)}{b} - \frac{3\,d^3\,x^2\sinh(a+b\,x)}{b^2} + \frac{3\,x\cosh(a+b\,x)\,\left(b^2\,c^2\,d+2\,d^3\right)}{b^3} - \frac{6\,c\,d^2\,x\sinh(a+b\,x)}{b^2} + \frac{3\,c\,d^2\,x^2\cosh(a+b\,x)}{b}$$

input
```
int(sinh(a + b*x)*(c + d*x)^3,x)
```

output
```
(cosh(a + b*x)*(6*c*d^2 + b^2*c^3))/b^3 - (3*sinh(a + b*x)*(2*d^3 + b^2*c^
2*d))/b^4 + (d^3*x^3*cosh(a + b*x))/b - (3*d^3*x^2*sinh(a + b*x))/b^2 + (3
*x*cosh(a + b*x)*(2*d^3 + b^2*c^2*d))/b^3 - (6*c*d^2*x*sinh(a + b*x))/b^2
+ (3*c*d^2*x^2*cosh(a + b*x))/b
```

3.3 $\int (c+dx)^2 \sinh(a+bx)\,dx$

3.3.1 Optimal result

Integrand size = 14, antiderivative size = 49

$$\int (c+dx)^2 \sinh(a+bx)\,dx = \frac{2d^2\cosh(a+bx)}{b^3} + \frac{(c+dx)^2\cosh(a+bx)}{b} - \frac{2d(c+dx)\sinh(a+bx)}{b^2}$$

output
```
2*d^2*cosh(b*x+a)/b^3+(d*x+c)^2*cosh(b*x+a)/b-2*d*(d*x+c)*sinh(b*x+a)/b^2
```

3.3.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int (c+dx)^2 \sinh(a+bx)\,dx = \frac{(2d^2+b^2(c+dx)^2)\cosh(a+bx) - 2bd(c+dx)\sinh(a+bx)}{b^3}$$

input
```
Integrate[(c + d*x)^2*Sinh[a + b*x],x]
```

output
```
((2*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] - 2*b*d*(c + d*x)*Sinh[a + b*x])/
b^3
```

3.3.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sinh(a+bx) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(c+dx)^2 \sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)^2 \sin(ia+ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int (c+dx)\cosh(a+bx)dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int (c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{id\int -i\sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int \sinh(a+bx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int -i\sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \ 26$$

$$-i\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}+\frac{id\int\sin(ia+ibx)dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \ 3118$$

$$-i\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}\right)}{b}\right)$$

input
```
Int[(c + d*x)^2*Sinh[a + b*x],x]
```

output
```
(-I)*((I*(c + d*x)^2*Cosh[a + b*x])/b - ((2*I)*d*(-((d*Cosh[a + b*x])/b^2)
 + ((c + d*x)*Sinh[a + b*x])/b))/b)
```

3.3.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.3.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.86

method	result
parallelrisch	$\frac{-2xd\,b^{2}\left(\frac{dx}{2}+c\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+4bd(dx+c)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+\left(-d^{2}x^{2}-2cdx-2c^{2}\right)b^{2}-4d^{2}}{b^{3}\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)}$
parts	$\frac{\cosh(bx+a)d^{2}x^{2}}{b}+\frac{2\cosh(bx+a)cdx}{b}+\frac{\cosh(bx+a)c^{2}}{b}-\frac{2d\left(\frac{d((bx+a)\sinh(bx+a)-\cosh(bx+a))}{b}-\frac{da\sinh(bx+a)}{b}+c\sinh\right.}{b^{2}}$
risch	$\frac{\left(b^{2}d^{2}x^{2}+2b^{2}cdx+b^{2}c^{2}-2b\,d^{2}x-2bcd+2d^{2}\right)\mathrm{e}^{bx+a}}{2b^{3}}+\frac{\left(b^{2}d^{2}x^{2}+2b^{2}cdx+b^{2}c^{2}+2b\,d^{2}x+2bcd+2d^{2}\right)\mathrm{e}^{-bx-a}}{2b^{3}}$
derivativedivides	$\frac{\frac{d^{2}\left((bx+a)^{2}\cosh(bx+a)-2(bx+a)\sinh(bx+a)+2\cosh(bx+a)\right)}{b^{2}}-\frac{2d^{2}a((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b^{2}}+\frac{2dc((bx+a)\cosh(bx+a)-\mathrm{si}}{b}}{b}$
default	$\frac{\frac{d^{2}\left((bx+a)^{2}\cosh(bx+a)-2(bx+a)\sinh(bx+a)+2\cosh(bx+a)\right)}{b^{2}}-\frac{2d^{2}a((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b^{2}}+\frac{2dc((bx+a)\cosh(bx+a)-\mathrm{si}}{b}}{b}$
meijerg	$\frac{4id^{2}\sinh(a)\sqrt{\pi}\left(\frac{ibx\cosh(bx)}{2\sqrt{\pi}}-\frac{i\left(\frac{3b^{2}x^{2}}{2}+3\right)\sinh(bx)}{6\sqrt{\pi}}\right)}{b^{3}}+\frac{4d^{2}\cosh(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{b^{2}x^{2}}{2}+1\right)\cosh(bx)}{2\sqrt{\pi}}-\frac{xb\sinh(bx)}{2\sqrt{\pi}}\right)}{b^{3}}$

input
```
int((d*x+c)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(-2*x*d*b^2*(1/2*d*x+c)*tanh(1/2*b*x+1/2*a)^2+4*b*d*(d*x+c)*tanh(1/2*b*x+1
/2*a)+(-d^2*x^2-2*c*d*x-2*c^2)*b^2-4*d^2)/b^3/(tanh(1/2*b*x+1/2*a)^2-1)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int (c+dx)^2 \sinh(a+bx)\,dx$$
$$=\frac{\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}+2\,d^{2}\right)\cosh\left(bx+a\right)-2\left(bd^{2}x+bcd\right)\sinh\left(bx+a\right)}{b^{3}}$$

input
```
integrate((d*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

output
```
((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*d^2)*cosh(b*x + a) - 2*(b*d^2*x
+ b*c*d)*sinh(b*x + a))/b^3
```

3.3.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(48) = 96$.

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int (c+dx)^2 \sinh(a+bx)\,dx = \begin{cases} \frac{c^2\cosh(a+bx)}{b} + \frac{2cdx\cosh(a+bx)}{b} + \frac{d^2x^2\cosh(a+bx)}{b} - \frac{2cd\sinh(a+bx)}{b^2} - \frac{2d^2x\sinh(a+bx)}{b^2} + \frac{2d^2\cosh(a+bx)}{b^3} & \text{for } b \neq 0 \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sinh(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)**2*sinh(b*x+a),x)
```

output
```
Piecewise((c**2*cosh(a + b*x)/b + 2*c*d*x*cosh(a + b*x)/b + d**2*x**2*cosh
(a + b*x)/b - 2*c*d*sinh(a + b*x)/b**2 - 2*d**2*x*sinh(a + b*x)/b**2 + 2*d
**2*cosh(a + b*x)/b**3, Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sinh
(a), True))
```

3.3.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(49) = 98$.

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.73

$$\int (c+dx)^2 \sinh(a+bx)\,dx = \frac{c^2e^{(bx+a)}}{2\,b} + \frac{(bxe^a - e^a)cde^{(bx)}}{b^2} + \frac{c^2e^{(-bx-a)}}{2\,b} + \frac{(bx+1)cde^{(-bx-a)}}{b^2} + \frac{(b^2x^2e^a - 2\,bxe^a + 2\,e^a)d^2e^{(bx)}}{2\,b^3} + \frac{(b^2x^2 + 2\,bx + 2)d^2e^{(-bx-a)}}{2\,b^3}$$

input
```
integrate((d*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*c^2*e^(b*x + a)/b + (b*x*e^a - e^a)*c*d*e^(b*x)/b^2 + 1/2*c^2*e^(-b*x
- a)/b + (b*x + 1)*c*d*e^(-b*x - a)/b^2 + 1/2*(b^2*x^2*e^a - 2*b*x*e^a + 2
*e^a)*d^2*e^(b*x)/b^3 + 1/2*(b^2*x^2 + 2*b*x + 2)*d^2*e^(-b*x - a)/b^3
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(49) = 98$.

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int (c+dx)^2 \sinh(a+bx)\,dx = \frac{(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - 2\,bd^2x - 2\,bcd + 2\,d^2)e^{(bx+a)}}{2\,b^3} + \frac{(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + 2\,bd^2x + 2\,bcd + 2\,d^2)e^{(-bx-a)}}{2\,b^3}$$

input
```
integrate((d*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b*d^2*x - 2*b*c*d + 2*d^2)*e^
(b*x + a)/b^3 + 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*b*d^2*x + 2*b
*c*d + 2*d^2)*e^(-b*x - a)/b^3
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int (c+dx)^2 \sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)\,\left(b^2\,c^2+2\,d^2\right)}{b^3} + \frac{d^2\,x^2\,\cosh(a+b\,x)}{b} - \frac{2\,c\,d\sinh(a+b\,x)}{b^2} - \frac{2\,d^2\,x\sinh(a+b\,x)}{b^2} + \frac{2\,c\,d\,x\cosh(a+b\,x)}{b}$$

input
```
int(sinh(a + b*x)*(c + d*x)^2,x)
```

output
```
(cosh(a + b*x)*(2*d^2 + b^2*c^2))/b^3 + (d^2*x^2*cosh(a + b*x))/b - (2*c*d
*sinh(a + b*x))/b^2 - (2*d^2*x*sinh(a + b*x))/b^2 + (2*c*d*x*cosh(a + b*x)
)/b
```

3.4 $\int (c+dx)\sinh(a+bx)\,dx$

3.4.1 Optimal result

Integrand size = 12, antiderivative size = 28

$$\int (c+dx)\sinh(a+bx)\,dx=\frac{(c+dx)\cosh(a+bx)}{b}-\frac{d\sinh(a+bx)}{b^2}$$

output
```
(d*x+c)*cosh(b*x+a)/b-d*sinh(b*x+a)/b^2
```

3.4.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int (c+dx)\sinh(a+bx)\,dx=\frac{b(c+dx)\cosh(a+bx)-d\sinh(a+bx)}{b^2}$$

input
```
Integrate[(c + d*x)*Sinh[a + b*x],x]
```

output
```
(b*(c + d*x)*Cosh[a + b*x] - d*Sinh[a + b*x])/b^2
```

3.4.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sinh(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -i(c+dx)\sin(ia+ibx)dx$$
$$\Big\downarrow \text{26}$$
$$-i\int (c+dx)\sin(ia+ibx)dx$$
$$\Big\downarrow \text{3777}$$
$$-i\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int \cosh(a+bx)dx}{b}\right)$$
$$\Big\downarrow \text{3042}$$
$$-i\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int \sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)$$
$$\Big\downarrow \text{3117}$$
$$-i\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)$$

input
```
Int[(c + d*x)*Sinh[a + b*x],x]
```

output
```
(-I)*((I*(c + d*x)*Cosh[a + b*x])/b - (I*d*Sinh[a + b*x])/b^2)
```

3.4.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.4.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

method	result
parts	$\frac{\cosh(bx+a)dx}{b}+\frac{\cosh(bx+a)c}{b}-\frac{d\sinh(bx+a)}{b^2}$
risch	$\frac{(bdx+bc-d)e^{bx+a}}{2b^2}+\frac{(bdx+bc+d)e^{-bx-a}}{2b^2}$
derivativedivides	$\frac{\frac{d((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b}-\frac{da\cosh(bx+a)}{b}+c\cosh(bx+a)}{b}$
default	$\frac{\frac{d((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b}-\frac{da\cosh(bx+a)}{b}+c\cosh(bx+a)}{b}$
parallelrisch	$\frac{-x\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2bd+2d\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-2b\left(\frac{dx}{2}+c\right)}{b^2\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)}$
meijerg	$-\frac{2d\sinh(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(bx)}{2\sqrt{\pi}}-\frac{xb\sinh(bx)}{2\sqrt{\pi}}\right)}{b^2}-\frac{d\cosh(a)(-\cosh(bx)bx+\sinh(bx))}{b^2}+\frac{c\sinh(a)\sinh(bx)}{b}-$ c cosl

input
```
int((d*x+c)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
cosh(b*x+a)/b*d*x+cosh(b*x+a)/b*c-d*sinh(b*x+a)/b^2
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int (c+dx)\sinh(a+bx)\,dx = \frac{(bdx+bc)\cosh(bx+a) - d\sinh(bx+a)}{b^2}$$

input
```
integrate((d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
((b*d*x + b*c)*cosh(b*x + a) - d*sinh(b*x + a))/b^2
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int (c+dx)\sinh(a+bx)\,dx = \begin{cases} \frac{c\cosh(a+bx)}{b} + \frac{dx\cosh(a+bx)}{b} - \frac{d\sinh(a+bx)}{b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\sinh(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*sinh(b*x+a),x)
```

output
```
Piecewise((c*cosh(a + b*x)/b + d*x*cosh(a + b*x)/b - d*sinh(a + b*x)/b**2,
 Ne(b, 0)), ((c*x + d*x**2/2)*sinh(a), True))
```

3.4.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(28) = 56.

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int (c+dx)\sinh(a+bx)\,dx = \frac{ce^{(bx+a)}}{2\,b} + \frac{(bxe^a - e^a)de^{(bx)}}{2\,b^2} + \frac{ce^{(-bx-a)}}{2\,b} + \frac{(bx+1)de^{(-bx-a)}}{2\,b^2}$$

input
```
integrate((d*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*c*e^(b*x + a)/b + 1/2*(b*x*e^a - e^a)*d*e^(b*x)/b^2 + 1/2*c*e^(-b*x -
a)/b + 1/2*(b*x + 1)*d*e^(-b*x - a)/b^2
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int (c+dx)\sinh(a+bx)\,dx = \frac{(bdx+bc-d)e^{(bx+a)}}{2\,b^2} + \frac{(bdx+bc+d)e^{(-bx-a)}}{2\,b^2}$$

input
```
integrate((d*x+c)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b*d*x + b*c - d)*e^(b*x + a)/b^2 + 1/2*(b*d*x + b*c + d)*e^(-b*x - a)
/b^2
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int (c+dx)\sinh(a+bx)\,dx = \frac{c\cosh(a+b\,x)+d\,x\cosh(a+b\,x)}{b} - \frac{d\sinh(a+b\,x)}{b^2}$$

input
```
int(sinh(a + b*x)*(c + d*x),x)
```

output
```
(c*cosh(a + b*x) + d*x*cosh(a + b*x))/b - (d*sinh(a + b*x))/b^2
```

3.5 $\int \frac{\sinh(a+bx)}{c+dx} dx$

3.5.1 Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx = \frac{\operatorname{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{d} + \frac{\cosh\left(a-\frac{bc}{d}\right)\operatorname{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

output
```
cosh(a-b*c/d)*Shi(b*c/d+b*x)/d+Chi(b*c/d+b*x)*sinh(a-b*c/d)/d
```

3.5.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx = \frac{\operatorname{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)+\cosh\left(a-\frac{bc}{d}\right)\operatorname{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Integrate[Sinh[a + b*x]/(c + d*x),x]
```

output
```
(CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d] + Cosh[a - (b*c)/d]*SinhInt
egral[(b*c)/d + b*x])/d
```

3.5.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ia+ibx)}{c+dx}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ia+ibx)}{c+dx}dx$$

$$\Big\downarrow \text{3784}$$

$$-i\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int \frac{i\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+i\cosh\left(a-\frac{bc}{d}\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(a-\frac{bc}{d}\right)\int -\frac{i\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)$$

$$\Big\downarrow \text{3779}$$

$$-i\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)$$

$$\Big\downarrow \text{3782}$$

$$-i\left(\frac{i\sinh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{i\cosh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)$$

input
```
Int[Sinh[a + b*x]/(c + d*x),x]
```

output
```
(-I)*((I*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/d + (I*Cosh[a - (b
*c)/d]*SinhIntegral[(b*c)/d + b*x])/d)
```

3.5.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

3.5.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.61

method	result	size
risch	$\frac{\mathrm{e}^{-\frac{ad-bc}{d}} \operatorname{Ei}_1\left(bx+a-\frac{ad-bc}{d}\right)}{2d}-\frac{\mathrm{e}^{\frac{ad-bc}{d}} \operatorname{Ei}_1\left(-bx-a-\frac{-ad+bc}{d}\right)}{2d}$	82

input
```
int(sinh(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-1/2/d*exp((a*d-b*c)/d)*Ei(
1,-b*x-a-(-a*d+b*c)/d)
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.84

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx = \frac{\left(\operatorname{Ei}\left(\frac{bdx+bc}{d}\right)-\operatorname{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)+\left(\operatorname{Ei}\left(\frac{bdx+bc}{d}\right)+\operatorname{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\sinh\left(-\frac{bc-ad}{d}\right)}{2\,d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
1/2*((Ei((b*d*x + b*c)/d) - Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) + (
Ei((b*d*x + b*c)/d) + Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/d
```

3.5.6 Sympy [F]

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx = \int \frac{\sinh(a+bx)}{c+dx}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x+c),x)
```

output
```
Integral(sinh(a + b*x)/(c + d*x), x)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx = \frac{e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{2\,d} - \frac{e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{2\,d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d - 1/2*e^(a - b*c/d)*
exp_integral_e(1, -(d*x + c)*b/d)/d
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx = \frac{\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} - \mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}}{2\,d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
1/2*(Ei((b*d*x + b*c)/d)*e^(a - b*c/d) - Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/
d))/d
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx = \int \frac{\sinh(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(sinh(a + b*x)/(c + d*x),x)
```

output
```
int(sinh(a + b*x)/(c + d*x), x)
```

3.6 $\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx$

3.6.1 Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx = \frac{b\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\sinh(a+bx)}{d(c+dx)}+\frac{b\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{bc}{d}+bx\right)}{d^2}$$

output
```
b*Chi(b*c/d+b*x)*cosh(a-b*c/d)/d^2+b*Shi(b*c/d+b*x)*sinh(a-b*c/d)/d^2-sinh
(b*x+a)/d/(d*x+c)
```

3.6.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx = \frac{b\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(b\left(\frac{c}{d}+x\right)\right)-\frac{d\sinh(a+bx)}{c+dx}+b\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(b\left(\frac{c}{d}+x\right)\right)}{d^2}$$

input
```
Integrate[Sinh[a + b*x]/(c + d*x)^2,x]
```

output
```
(b*Cosh[a - (b*c)/d]*CoshIntegral[b*(c/d + x)] - (d*Sinh[a + b*x])/(c + d*
x) + b*Sinh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)])/d^2
```

3.6.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ia+ibx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ia+ibx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3778}$$

$$-i\left(\frac{ib\int \frac{\cosh(a+bx)}{c+dx}dx}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ib\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{c+dx}dx}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{3784}$$

$$-i\left(\frac{ib\left(\cosh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx-i\sinh\left(a-\frac{bc}{d}\right)\int \frac{i\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{ib\left(\sinh\left(a-\frac{bc}{d}\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ib\left(\sinh\left(a-\frac{bc}{d}\right)\int -\frac{i\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{ib\left(\cosh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx-i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{3779}$$

$$-i\left(\frac{ib\left(\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}+\cosh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{3782}$$

$$-i\left(\frac{ib\left(\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)$$

input
```
Int[Sinh[a + b*x]/(c + d*x)^2,x]
```

output
```
(-I)*(((-I)*Sinh[a + b*x])/(d*(c + d*x)) + (I*b*((Cosh[a - (b*c)/d]*CoshIn
tegral[(b*c)/d + b*x])/d + (Sinh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])
/d))/d)
```

3.6. $\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx$

3.6.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

3.6.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.87

method	result	size
risch	$\frac{b\,\mathrm{e}^{-bx-a}}{2d(bdx+bc)}-\frac{b\,\mathrm{e}^{-\frac{ad-bc}{d}}\,\mathrm{Ei}_1\left(bx+a-\frac{ad-bc}{d}\right)}{2d^2}-\frac{b\,\mathrm{e}^{bx+a}}{2d^2\left(\frac{bc}{d}+bx\right)}-\frac{b\,\mathrm{e}^{\frac{ad-bc}{d}}\,\mathrm{Ei}_1\left(-bx-a-\frac{-ad+bc}{d}\right)}{2d^2}$	133

input
```
int(sinh(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*exp(-b*x-a)/d/(b*d*x+b*c)-1/2*b/d^2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*
d-b*c)/d)-1/2*b/d^2*exp(b*x+a)/(b*c/d+b*x)-1/2*b/d^2*exp((a*d-b*c)/d)*Ei(1
,-b*x-a-(-a*d+b*c)/d)
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(71) = 142$.

Time = 0.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.08

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx = \frac{\left((bdx+bc)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)-2\,d\sinh\left(bx+a\right)+\left((bdx+bc)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right.\right.}{2\left(d^3x+cd^2\right)}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(((b*d*x + b*c)*Ei((b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-(b*d*x + b*c)/
d))*cosh(-(b*c - a*d)/d) - 2*d*sinh(b*x + a) + ((b*d*x + b*c)*Ei((b*d*x +
b*c)/d) - (b*d*x + b*c)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/(d^3*x
 + c*d^2)
```

3.6.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)**2,x)
```

output
```
Timed out
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx = -\frac{b\left(\frac{e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{d}+\frac{e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{d}\right)}{2\,d} - \frac{\sinh\left(bx+a\right)}{(dx+c)d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*b*(e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d + e^(a - b*c/d)*
exp_integral_e(1, -(d*x + c)*b/d)/d)/d - sinh(b*x + a)/((d*x + c)*d)
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(71) = 142$.

Time = 0.30 (sec) , antiderivative size = 615, normalized size of antiderivative = 8.66

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx$$

$$= \frac{\Bigg((dx+c)\big(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\big)b^2\mathrm{Ei}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}+b^3c\mathrm{Ei}\Bigg(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-}{d}}{2\,\big((dx+c)\big(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\big)}$$

$$+\frac{\Bigg((dx+c)\big(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\big)b^2\mathrm{Ei}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}+b^3c\mathrm{Ei}\Bigg(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-}{d}}{2\,\big((dx+c)\big(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(-((d*x + c)*(b -
 b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) + b^3*c*
Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c
 - a*d)/d) - a*b^2*d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) +
b*c - a*d)/d)*e^((b*c - a*d)/d) + b^2*d*e^(-(d*x + c)*(b - b*c/(d*x + c) +
 a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^
4 + b*c*d^4 - a*d^5)*b) + 1/2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c
))*b^2*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e
^(-(b*c - a*d)/d) + b^3*c*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)
) + b*c - a*d)/d)*e^(-(b*c - a*d)/d) - a*b^2*d*Ei(((d*x + c)*(b - b*c/(d*x
 + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-(b*c - a*d)/d) - b^2*d*e^((d*x
+ c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x
 + c) + a*d/(d*x + c))*d^4 + b*c*d^4 - a*d^5)*b)
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sinh(a+b\,x)}{(c+d\,x)^2}\,dx$$

input
```
int(sinh(a + b*x)/(c + d*x)^2,x)
```

output
```
int(sinh(a + b*x)/(c + d*x)^2, x)
```

3.7 $\int \frac{\sinh(a+bx)}{(c+dx)^3} \, dx$

3.7.1 Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{\sinh(a+bx)}{(c+dx)^3} \, dx = -\frac{b\cosh(a+bx)}{2d^2(c+dx)} + \frac{b^2\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{2d^3} - \frac{\sinh(a+bx)}{2d(c+dx)^2} + \frac{b^2\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{2d^3}$$

output
```
-1/2*b*cosh(b*x+a)/d^2/(d*x+c)+1/2*b^2*cosh(a-b*c/d)*Shi(b*c/d+b*x)/d^3+1/
2*b^2*Chi(b*c/d+b*x)*sinh(a-b*c/d)/d^3-1/2*sinh(b*x+a)/d/(d*x+c)^2
```

3.7.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{\sinh(a+bx)}{(c+dx)^3} \, dx = \frac{b^2\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right)\sinh\left(a-\frac{bc}{d}\right) - \frac{d(b(c+dx)\cosh(a+bx)+d\sinh(a+bx))}{(c+dx)^2} + b^2\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

input
```
Integrate[Sinh[a + b*x]/(c + d*x)^3,x]
```

output
```
(b^2*CoshIntegral[b*(c/d + x)]*Sinh[a - (b*c)/d] - (d*(b*(c + d*x)*Cosh[a
+ b*x] + d*Sinh[a + b*x]))/(c + d*x)^2 + b^2*Cosh[a - (b*c)/d]*SinhIntegra
l[b*(c/d + x)])/(2*d^3)
```

3.7.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 26, 3778, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ia+ibx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ia+ibx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3778}$$

$$-i\left(\frac{ib\int \frac{\cosh(a+bx)}{(c+dx)^2}dx}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ib\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{(c+dx)^2}dx}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3778}$$

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}+\frac{ib\int -\frac{i\sinh(a+bx)}{c+dx}dx}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{ib\left(\frac{b\int \frac{\sinh(a+bx)}{c+dx}dx}{d}-\frac{\cosh(a+bx)}{d(c+dx)}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}+\frac{b\int -\frac{i\sin(ia+ibx)}{c+dx}dx}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\int \frac{\sin(ia+ibx)}{c+dx}dx}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3784}$$

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int \frac{i\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+i\cosh\left(a-\frac{bc}{d}\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(a-\frac{bc}{d}\right)\int -\frac{i\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{26}$$

3.7. $\int \frac{\sinh(a+bx)}{(c+dx)^3}\,dx$

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3779}$$

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3782}$$

$$-i\left(\frac{ib\left(-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(\frac{i\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}\right)}{2d}-\frac{i\sinh(a+bx)}{2d(c+dx)^2}\right)$$

input
```
Int[Sinh[a + b*x]/(c + d*x)^3,x]
```

output
```
(-I)*(((-1/2*I)*Sinh[a + b*x])/(d*(c + d*x)^2) + ((I/2)*b*(-(Cosh[a + b*x]
/(d*(c + d*x))) - (I*b*((I*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/
d + (I*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d))/d))/d)
```

3.7.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

3.7.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(96) = 192$.

Time = 0.90 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.66

method	result
risch	$-\frac{b^3 e^{-bx-a} x}{4d(b^2d^2x^2+2b^2cdx+b^2c^2)}-\frac{b^3 e^{-bx-a} c}{4d^2(b^2d^2x^2+2b^2cdx+b^2c^2)}+\frac{b^2 e^{-bx-a}}{4d(b^2d^2x^2+2b^2cdx+b^2c^2)}+\frac{b^2 e^{-\frac{ad-bc}{d}} \mathrm{Ei}_1\left(bx+a-\frac{ad-bc}{d}\right)}{4d^3}-$

input
```
int(sinh(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^3*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*x-1/4*b^3*exp(-b*
x-a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c+1/4*b^2*exp(-b*x-a)/d/(b^2*d^
2*x^2+2*b^2*c*d*x+b^2*c^2)+1/4*b^2/d^3*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b
*c)/d)-1/4*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)^2-1/4*b^2/d^3*exp(b*x+a)/(b*c/d+
b*x)-1/4*b^2/d^3*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/d)
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(96) = 192$.

Time = 0.24 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.44

$$\int \frac{\sinh(a+bx)}{(c+dx)^3}\,dx = -\frac{2\,d^2\sinh\left(bx+a\right)+2\left(bd^2x+bcd\right)\cosh\left(bx+a\right)-\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-\left(b^2d^2x^2+2\,b\right.\right.}{}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*d^2*sinh(b*x + a) + 2*(b*d^2*x + b*c*d)*cosh(b*x + a) - ((b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) - ((b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*
x + c^2*d^3)
```

3.7.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)**3,x)
```

output
```
Timed out
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{\sinh(a+bx)}{(c+dx)^3}\,dx = -\frac{b\left(\frac{e^{\left(-a+\frac{bc}{d}\right)}E_2\left(\frac{(dx+c)b}{d}\right)}{(dx+c)d}+\frac{e^{\left(a-\frac{bc}{d}\right)}E_2\left(-\frac{(dx+c)b}{d}\right)}{(dx+c)d}\right)}{4\,d}-\frac{\sinh\left(bx+a\right)}{2\left(dx+c\right)^2d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*b*(e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d) + e^
(a - b*c/d)*exp_integral_e(2, -(d*x + c)*b/d)/((d*x + c)*d))/d - 1/2*sinh(
b*x + a)/((d*x + c)^2*d)
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(96) = 192$.

Time = 0.27 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.89

$$\int \frac{\sinh(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{b^2d^2x^2\text{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} - b^2d^2x^2\text{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)} + 2\,b^2cdx\text{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} - 2\,b^2cdx\text{Ei}\left(-\frac{bdx+bc}{d}\right)}{}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(b^2*d^2*x^2*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) - b^2*d^2*x^2*Ei(-(b*d*
x + b*c)/d)*e^(-a + b*c/d) + 2*b^2*c*d*x*Ei((b*d*x + b*c)/d)*e^(a - b*c/d)
 - 2*b^2*c*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + b^2*c^2*Ei((b*d*x + b
*c)/d)*e^(a - b*c/d) - b^2*c^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - b*d^2
*x*e^(b*x + a) - b*d^2*x*e^(-b*x - a) - b*c*d*e^(b*x + a) - b*c*d*e^(-b*x
- a) - d^2*e^(b*x + a) + d^2*e^(-b*x - a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sinh(a+b\,x)}{{(c+d\,x)}^3}\,dx$$

input
```
int(sinh(a + b*x)/(c + d*x)^3,x)
```

output
```
int(sinh(a + b*x)/(c + d*x)^3, x)
```

3.8 $\int (c+dx)^4 \sinh^2(a+bx)\,dx$

3.8.1 Optimal result

Integrand size = 16, antiderivative size = 162

$$\begin{aligned}\int (c+dx)^4 \sinh^2(a+bx)\,dx = & -\frac{3d^4x}{4b^4} - \frac{d(c+dx)^3}{2b^2} - \frac{(c+dx)^5}{10d} \\ & + \frac{3d^4\cosh(a+bx)\sinh(a+bx)}{4b^5} \\ & + \frac{3d^2(c+dx)^2\cosh(a+bx)\sinh(a+bx)}{2b^3} \\ & + \frac{(c+dx)^4\cosh(a+bx)\sinh(a+bx)}{2b} \\ & - \frac{3d^3(c+dx)\sinh^2(a+bx)}{2b^4} - \frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}\end{aligned}$$

output
```
-3/4*d^4*x/b^4-1/2*d*(d*x+c)^3/b^2-1/10*(d*x+c)^5/d+3/4*d^4*cosh(b*x+a)*si
nh(b*x+a)/b^5+3/2*d^2*(d*x+c)^2*cosh(b*x+a)*sinh(b*x+a)/b^3+1/2*(d*x+c)^4*
cosh(b*x+a)*sinh(b*x+a)/b-3/2*d^3*(d*x+c)*sinh(b*x+a)^2/b^4-d*(d*x+c)^3*si
nh(b*x+a)^2/b^2
```

3.8.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.81

$$\int (c+dx)^4 \sinh^2(a+bx)\,dx$$
$$=\frac{-8b^5x(5c^4+10c^3dx+10c^2d^2x^2+5cd^3x^3+d^4x^4)-20bd(c+dx)\left(3d^2+2b^2(c+dx)^2\right)\cosh(2(a+bx))+}{80b^5}$$

input
```
Integrate[(c + d*x)^4*Sinh[a + b*x]^2,x]
```

output
```
(-8*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) -
20*b*d*(c + d*x)*(3*d^2 + 2*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)] + 10*(3*d^4
 + 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sinh[2*(a + b*x)])/(80*b^5)
```

3.8.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {3042, 25, 3792, 17, 25, 3042, 25, 3792, 17, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sinh^2(a+bx)\,dx$$
$$\Big\downarrow \ 3042$$
$$\int -(c+dx)^4 \sin(ia+ibx)^2dx$$
$$\Big\downarrow \ 25$$
$$-\int (c+dx)^4 \sin(ia+ibx)^2dx$$
$$\Big\downarrow \ 3792$$
$$-\frac{3d^2\int -(c+dx)^2\sinh^2(a+bx)dx}{b^2}-\frac{1}{2}\int (c+dx)^4dx-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}$$
$$\Big\downarrow \ 17$$

$$-\frac{3d^2\int -(c+dx)^2\sinh^2(a+bx)dx}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{25}$$

$$\frac{3d^2\int (c+dx)^2\sinh^2(a+bx)dx}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d^2\int -(c+dx)^2\sin(ia+ibx)^2dx}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3d^2\int (c+dx)^2\sin(ia+ibx)^2dx}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{3792}$$

$$-\frac{3d^2\left(\frac{d^2\int -\sinh^2(a+bx)dx}{2b^2}+\frac{1}{2}\int (c+dx)^2dx+\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{17}$$

$$-\frac{3d^2\left(\frac{d^2\int -\sinh^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3d^2\left(-\frac{d^2\int \sinh^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{3042}$$

3.8. $\int (c+dx)^4\sinh^2(a+bx)\,dx$

$$-\frac{3d^2\left(-\frac{d^2\int -\sin(ia+ibx)^2dx}{2b^2}+\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \; 25$$

$$-\frac{3d^2\left(\frac{d^2\int \sin(ia+ibx)^2dx}{2b^2}+\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \; 3115$$

$$-\frac{3d^2\left(\frac{d^2\left(\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b^2}+\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \; 24$$

$$-\frac{3d^2\left(\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}+\frac{d^2\left(\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^5}{10d}$$

input
```
Int[(c + d*x)^4*Sinh[a + b*x]^2,x]
```

output
```
-1/10*(c + d*x)^5/d + ((c + d*x)^4*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (d
*(c + d*x)^3*Sinh[a + b*x]^2)/b^2 - (3*d^2*((c + d*x)^3/(6*d) - ((c + d*x)
^2*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) + (d*(c + d*x)*Sinh[a + b*x]^2)/(2*b
^2) + (d^2*(x/2 - (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/(2*b^2)))/b^2
```

3.8.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.8.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{\left(2(dx+c)^4b^4+6d^2(dx+c)^2b^2+3d^4\right)\sinh(2bx+2a)-4\left(d\left((dx+c)^2b^2+\frac{3d^2}{2}\right)(dx+c)\cosh(2bx+2a)+x\left(\frac{1}{5}d^4x^4+c\,d^3x^3+2c\right.\right.}{8b^5}$
risch	$-\frac{d^4x^5}{10}-\frac{d^3c\,x^4}{2}-d^2c^2x^3-c^3d\,x^2-\frac{x\,c^4}{2}-\frac{c^5}{10d}+\frac{(2d^4x^4b^4+8b^4c\,d^3x^3+12b^4c^2d^2x^2-4b^3d^4x^3+8b^4c^3dx}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^4*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*((2*(d*x+c)^4*b^4+6*d^2*(d*x+c)^2*b^2+3*d^4)*sinh(2*b*x+2*a)-4*(d*((d*
x+c)^2*b^2+3/2*d^2)*(d*x+c)*cosh(2*b*x+2*a)+x*(1/5*d^4*x^4+c*d^3*x^3+2*c^2
*d^2*x^2+2*c^3*d*x+c^4)*b^4-b^2*c^3*d-3/2*d^3*c)*b)/b^5
```

3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(148) = 296$.

Time = 0.24 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.93

$$\int (c+dx)^4 \sinh^2(a+bx)\,dx =$$
$$-\frac{2\,b^5d^4x^5+10\,b^5cd^3x^4+20\,b^5c^2d^2x^3+20\,b^5c^3dx^2+10\,b^5c^4x+5\,(2\,b^3d^4x^3+6\,b^3cd^3x^2+2\,b^3c^3d+3\,bcd^3}{}$$

input
```
integrate((d*x+c)^4*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/20*(2*b^5*d^4*x^5 + 10*b^5*c*d^3*x^4 + 20*b^5*c^2*d^2*x^3 + 20*b^5*c^3*
d*x^2 + 10*b^5*c^4*x + 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d +
3*b*c*d^3 + 3*(2*b^3*c^2*d^2 + b*d^4)*x)*cosh(b*x + a)^2 - 5*(2*b^4*d^4*x^
4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 + 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2
 + b^2*d^4)*x^2 + 4*(2*b^4*c^3*d + 3*b^2*c*d^3)*x)*cosh(b*x + a)*sinh(b*x
+ a) + 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d + 3*b*c*d^3 + 3*(2
*b^3*c^2*d^2 + b*d^4)*x)*sinh(b*x + a)^2)/b^5
```

3.8.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(156) = 312$.

Time = 0.51 (sec) , antiderivative size = 660, normalized size of antiderivative = 4.07

$$\int (c+dx)^4 \sinh^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c^4x\sinh^2(a+bx)}{2} - \frac{c^4x\cosh^2(a+bx)}{2} + c^3dx^2\sinh^2(a+bx) - c^3dx^2\cosh^2(a+bx) + c^2d^2x^3\sinh^2(a+bx) - c^2 \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sinh^2(a) \end{cases}$$

input
```
integrate((d*x+c)**4*sinh(b*x+a)**2,x)
```

output
```
Piecewise((c**4*x*sinh(a + b*x)**2/2 - c**4*x*cosh(a + b*x)**2/2 + c**3*d*
x**2*sinh(a + b*x)**2 - c**3*d*x**2*cosh(a + b*x)**2 + c**2*d**2*x**3*sinh
(a + b*x)**2 - c**2*d**2*x**3*cosh(a + b*x)**2 + c*d**3*x**4*sinh(a + b*x)
**2/2 - c*d**3*x**4*cosh(a + b*x)**2/2 + d**4*x**5*sinh(a + b*x)**2/10 - d
**4*x**5*cosh(a + b*x)**2/10 + c**4*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 2*
c**3*d*x*sinh(a + b*x)*cosh(a + b*x)/b + 3*c**2*d**2*x**2*sinh(a + b*x)*co
sh(a + b*x)/b + 2*c*d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/b + d**4*x**4*si
nh(a + b*x)*cosh(a + b*x)/(2*b) - c**3*d*cosh(a + b*x)**2/b**2 - 3*c**2*d*
*2*x*sinh(a + b*x)**2/(2*b**2) - 3*c**2*d**2*x*cosh(a + b*x)**2/(2*b**2) -
 3*c*d**3*x**2*sinh(a + b*x)**2/(2*b**2) - 3*c*d**3*x**2*cosh(a + b*x)**2/
(2*b**2) - d**4*x**3*sinh(a + b*x)**2/(2*b**2) - d**4*x**3*cosh(a + b*x)**
2/(2*b**2) + 3*c**2*d**2*sinh(a + b*x)*cosh(a + b*x)/(2*b**3) + 3*c*d**3*x
*sinh(a + b*x)*cosh(a + b*x)/b**3 + 3*d**4*x**2*sinh(a + b*x)*cosh(a + b*x
)/(2*b**3) - 3*c*d**3*cosh(a + b*x)**2/(2*b**4) - 3*d**4*x*sinh(a + b*x)**
2/(4*b**4) - 3*d**4*x*cosh(a + b*x)**2/(4*b**4) + 3*d**4*sinh(a + b*x)*cos
h(a + b*x)/(4*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**
3 + c*d**3*x**4 + d**4*x**5/5)*sinh(a)**2, True))
```

3.8.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(148) = 296$.

Time = 0.21 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.36

$$\int (c+dx)^4\sinh^2(a+bx)\,dx$$
$$= -\frac{1}{4}\left(4\,x^2 - \frac{\left(2\,bxe^{(2\,a)} - e^{(2\,a)}\right)e^{(2\,bx)}}{b^2} + \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{b^2}\right)c^3d$$
$$-\frac{1}{8}\left(8\,x^3 - \frac{3\left(2\,b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)}}{b^3} + \frac{3\left(2\,b^2x^2 + 2\,bx + 1\right)e^{(-2\,bx-2\,a)}}{b^3}\right)c^2d^2$$
$$-\frac{1}{8}\left(4\,x^4 - \frac{\left(4\,b^3x^3e^{(2\,a)} - 6\,b^2x^2e^{(2\,a)} + 6\,bxe^{(2\,a)} - 3\,e^{(2\,a)}\right)e^{(2\,bx)}}{b^4} + \frac{\left(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3\right)e^{(-2\,bx-2\,a}}{b^4}\right.$$
$$-\frac{1}{80}\left(8\,x^5 - \frac{5\left(2\,b^4x^4e^{(2\,a)} - 4\,b^3x^3e^{(2\,a)} + 6\,b^2x^2e^{(2\,a)} - 6\,bxe^{(2\,a)} + 3\,e^{(2\,a)}\right)e^{(2\,bx)}}{b^5} + \frac{5\left(2\,b^4x^4 + 4\,b^3x^3 + 6\right.}{}\right.$$
$$-\frac{1}{8}\,c^4\left(4\,x - \frac{e^{(2\,bx+2\,a)}}{b} + \frac{e^{(-2\,bx-2\,a)}}{b}\right)$$

input
```
integrate((d*x+c)^4*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(4*x^2 - (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + (2*b*x + 1)*e^(-2*
b*x - 2*a)/b^2)*c^3*d - 1/8*(8*x^3 - 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a)
+ e^(2*a))*e^(2*b*x)/b^3 + 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)
*c^2*d^2 - 1/8*(4*x^4 - (4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(
2*a) - 3*e^(2*a))*e^(2*b*x)/b^4 + (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-
2*b*x - 2*a)/b^4)*c*d^3 - 1/80*(8*x^5 - 5*(2*b^4*x^4*e^(2*a) - 4*b^3*x^3*e
^(2*a) + 6*b^2*x^2*e^(2*a) - 6*b*x*e^(2*a) + 3*e^(2*a))*e^(2*b*x)/b^5 + 5*
(2*b^4*x^4 + 4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^5)*d^4
- 1/8*c^4*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)
```

3.8.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(148) = 296$.

Time = 0.27 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.31

$$\int (c+dx)^4 \sinh^2(a+bx)\,dx = -\frac{1}{10}\,d^4x^5 - \frac{1}{2}\,cd^3x^4 - c^2d^2x^3 - c^3dx^2 - \frac{1}{2}\,c^4x$$
$$+ \frac{(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 - 4\,b^3d^4x^3 + 8\,b^4c^3dx - 12\,b^3cd^3x^2 + 2\,b^4c^4 - 12\,b^3c^2d^2x + 6\,b^2d^4x^2}{16\,b^5}$$
$$- \frac{(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 + 4\,b^3d^4x^3 + 8\,b^4c^3dx + 12\,b^3cd^3x^2 + 2\,b^4c^4 + 12\,b^3c^2d^2x + 6\,b^2d^4x^2}{16\,b^5}$$

input
```
integrate((d*x+c)^4*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/10*d^4*x^5 - 1/2*c*d^3*x^4 - c^2*d^2*x^3 - c^3*d*x^2 - 1/2*c^4*x + 1/16
*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 8
*b^4*c^3*d*x - 12*b^3*c*d^3*x^2 + 2*b^4*c^4 - 12*b^3*c^2*d^2*x + 6*b^2*d^4
*x^2 - 4*b^3*c^3*d + 12*b^2*c*d^3*x + 6*b^2*c^2*d^2 - 6*b*d^4*x - 6*b*c*d^
3 + 3*d^4)*e^(2*b*x + 2*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 1
2*b^4*c^2*d^2*x^2 + 4*b^3*d^4*x^3 + 8*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + 2*b
^4*c^4 + 12*b^3*c^2*d^2*x + 6*b^2*d^4*x^2 + 4*b^3*c^3*d + 12*b^2*c*d^3*x +
 6*b^2*c^2*d^2 + 6*b*d^4*x + 6*b*c*d^3 + 3*d^4)*e^(-2*b*x - 2*a)/b^5
```

3.8.9 Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.06

$$\int (c+dx)^4\sinh^2(a+bx)\,dx = \frac{c^4\sinh(2a+2bx)}{4b} - \frac{d^4x^5}{10} - c^3dx^2 - \frac{cd^3x^4}{2} - \frac{c^4x}{2} + \frac{3d^4\sinh(2a+2bx)}{8b^5} - c^2d^2x^3 - \frac{c^3d\cosh(2a+2bx)}{2b^2} - \frac{3cd^3\cosh(2a+2bx)}{4b^4} - \frac{3d^4x\cosh(2a+2bx)}{4b^4} + \frac{3c^2d^2\sinh(2a+2bx)}{4b^3} - \frac{d^4x^3\cosh(2a+2bx)}{2b^2} + \frac{d^4x^4\sinh(2a+2bx)}{4b} + \frac{3d^4x^2\sinh(2a+2bx)}{4b^3} + \frac{3c^2d^2x^2\sinh(2a+2bx)}{2b} + \frac{c^3dx\sinh(2a+2bx)}{b} + \frac{3cd^3x\sinh(2a+2bx)}{2b^3} - \frac{3c^2d^2x\cosh(2a+2bx)}{2b^2} - \frac{3cd^3x^2\cosh(2a+2bx)}{2b^2} + \frac{cd^3x^3\sinh(2a+2bx)}{b}$$

input
```
int(sinh(a + b*x)^2*(c + d*x)^4,x)
```

output
```
(c^4*sinh(2*a + 2*b*x))/(4*b) - (d^4*x^5)/10 - c^3*d*x^2 - (c*d^3*x^4)/2 -
 (c^4*x)/2 + (3*d^4*sinh(2*a + 2*b*x))/(8*b^5) - c^2*d^2*x^3 - (c^3*d*cosh
(2*a + 2*b*x))/(2*b^2) - (3*c*d^3*cosh(2*a + 2*b*x))/(4*b^4) - (3*d^4*x*co
sh(2*a + 2*b*x))/(4*b^4) + (3*c^2*d^2*sinh(2*a + 2*b*x))/(4*b^3) - (d^4*x^
3*cosh(2*a + 2*b*x))/(2*b^2) + (d^4*x^4*sinh(2*a + 2*b*x))/(4*b) + (3*d^4*
x^2*sinh(2*a + 2*b*x))/(4*b^3) + (3*c^2*d^2*x^2*sinh(2*a + 2*b*x))/(2*b) +
 (c^3*d*x*sinh(2*a + 2*b*x))/b + (3*c*d^3*x*sinh(2*a + 2*b*x))/(2*b^3) - (
3*c^2*d^2*x*cosh(2*a + 2*b*x))/(2*b^2) - (3*c*d^3*x^2*cosh(2*a + 2*b*x))/(
2*b^2) + (c*d^3*x^3*sinh(2*a + 2*b*x))/b
```

3.9 $\int (c+dx)^3 \sinh^2(a+bx)\,dx$

3.9.1 Optimal result

Integrand size = 16, antiderivative size = 134

$$\begin{aligned}\int (c+dx)^3 \sinh^2(a+bx)\,dx = & -\frac{3cd^2x}{4b^2}-\frac{3d^3x^2}{8b^2}-\frac{(c+dx)^4}{8d} \\ & +\frac{3d^2(c+dx)\cosh(a+bx)\sinh(a+bx)}{4b^3} \\ & +\frac{(c+dx)^3\cosh(a+bx)\sinh(a+bx)}{2b} \\ & -\frac{3d^3\sinh^2(a+bx)}{8b^4}-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}\end{aligned}$$

output
```
-3/4*c*d^2*x/b^2-3/8*d^3*x^2/b^2-1/8*(d*x+c)^4/d+3/4*d^2*(d*x+c)*cosh(b*x+
a)*sinh(b*x+a)/b^3+1/2*(d*x+c)^3*cosh(b*x+a)*sinh(b*x+a)/b-3/8*d^3*sinh(b*
x+a)^2/b^4-3/4*d*(d*x+c)^2*sinh(b*x+a)^2/b^2
```

3.9.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.78

$$\int (c+dx)^3 \sinh^2(a+bx)\,dx$$
$$= \frac{-2b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)-3d(d^2+2b^2(c+dx)^2)\cosh(2(a+bx))+2b(c+dx)\left(3d^2+2b^2(c-\right.}{16b^4}$$

input
```
Integrate[(c + d*x)^3*Sinh[a + b*x]^2,x]
```

output
```
(-2*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(d^2 + 2*b^2*(
c + d*x)^2)*Cosh[2*(a + b*x)] + 2*b*(c + d*x)*(3*d^2 + 2*b^2*(c + d*x)^2)*
Sinh[2*(a + b*x)])/(16*b^4)
```

3.9.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 25, 3792, 17, 25, 3042, 25, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^3 \sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^3 \sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{3d^2 \int -((c+dx)\sinh^2(a+bx))\,dx}{2b^2}-\frac{1}{2}\int (c+dx)^3dx-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{3d^2 \int -((c+dx)\sinh^2(a+bx))\,dx}{2b^2}-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^4}{8d}$$

$$\Big\downarrow \text{25}$$

$$\frac{3d^2 \int (c+dx)\sinh^2(a+bx)dx}{2b^2}-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^4}{8d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d^2\int -\left((c+dx)\sin(ia+ibx)^2\right)dx}{2b^2}-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^4}{8d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3d^2\int (c+dx)\sin(ia+ibx)^2dx}{2b^2}-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^4}{8d}$$

$$\Big\downarrow \text{3791}$$

$$-\frac{3d^2\left(\frac{1}{2}\int (c+dx)dx+\frac{d\sinh^2(a+bx)}{4b^2}-\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b^2}-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^4}{8d}$$

$$\Big\downarrow \text{17}$$

$$-\frac{3d^2\left(\frac{d\sinh^2(a+bx)}{4b^2}-\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^4}{8d}$$

input
```
Int[(c + d*x)^3*Sinh[a + b*x]^2,x]
```

output
```
-1/8*(c + d*x)^4/d + ((c + d*x)^3*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (3*
d*(c + d*x)^2*Sinh[a + b*x]^2)/(4*b^2) - (3*d^2*((c + d*x)^2/(4*d) - ((c +
 d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) + (d*Sinh[a + b*x]^2)/(4*b^2)))/(
2*b^2)
```

3.9.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.9.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{4b\left(\left(dx+c\right)^2b^2+\frac{3d^2}{2}\right)\sinh(2bx+2a)(dx+c)-6d\left(\left(dx+c\right)^2b^2+\frac{d^2}{2}\right)\cosh(2bx+2a)+(-2d^3x^4-8d^2c\,x^3-12c^2d\,x^2-8c^3x)b^{4}}{16b^4}$
risch	$-\frac{d^3x^4}{8}-\frac{d^2c\,x^3}{2}-\frac{3c^2d\,x^2}{4}-\frac{c^3x}{2}-\frac{c^4}{8d}+\frac{(4d^3x^3b^3+12b^3c\,d^2x^2+12b^3c^2dx-6b^2d^3x^2+4b^3c^3-12b^2c\,d^2x-6b^2c^{2}}{32b^4}$
derivativedivides	$\frac{d^3\left(\frac{(bx+a)^3\cosh(bx+a)\sinh(bx+a)}{2}-\frac{(bx+a)^4}{8}-\frac{3(bx+a)^2\cosh(bx+a)^2}{4}+\frac{3(bx+a)\cosh(bx+a)\sinh(bx+a)}{4}+\frac{3(bx+a)^2}{8}-\frac{3\cosh(bx+a}{8}\right.}{b^3}$
default	$\frac{d^3\left(\frac{(bx+a)^3\cosh(bx+a)\sinh(bx+a)}{2}-\frac{(bx+a)^4}{8}-\frac{3(bx+a)^2\cosh(bx+a)^2}{4}+\frac{3(bx+a)\cosh(bx+a)\sinh(bx+a)}{4}+\frac{3(bx+a)^2}{8}-\frac{3\cosh(bx+a}{8}\right.}{b^3}$

input
```
int((d*x+c)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*b*((d*x+c)^2*b^2+3/2*d^2)*sinh(2*b*x+2*a)*(d*x+c)-6*d*((d*x+c)^2*b
^2+1/2*d^2)*cosh(2*b*x+2*a)+(-2*d^3*x^4-8*c*d^2*x^3-12*c^2*d*x^2-8*c^3*x)*
b^4+6*b^2*c^2*d+3*d^3)/b^4
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.56

$$\int (c+dx)^3 \sinh^2(a+bx)\,dx =$$
$$-\frac{2\,b^4d^3x^4 + 8\,b^4cd^2x^3 + 12\,b^4c^2dx^2 + 8\,b^4c^3x + 3\,(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d + d^3)\cosh{(bx+a)}^2 - 4\,(\ldots}{\ldots}$$

input
```
integrate((d*x+c)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/16*(2*b^4*d^3*x^4 + 8*b^4*c*d^2*x^3 + 12*b^4*c^2*d*x^2 + 8*b^4*c^3*x +
3*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*cosh(b*x + a)^2 - 4*
(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d
+ b*d^3)*x)*cosh(b*x + a)*sinh(b*x + a) + 3*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x
 + 2*b^2*c^2*d + d^3)*sinh(b*x + a)^2)/b^4
```

3.9.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(131) = 262$.

Time = 0.36 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.40

$$\int (c+dx)^3 \sinh^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c^3x\sinh^2{(a+bx)}}{2} - \frac{c^3x\cosh^2{(a+bx)}}{2} + \frac{3c^2dx^2\sinh^2{(a+bx)}}{4} - \frac{3c^2dx^2\cosh^2{(a+bx)}}{4} + \frac{cd^2x^3\sinh^2{(a+bx)}}{2} - \frac{cd^2x^3\cosh^2{(a+bx)}}{2} + \ldots \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sinh^2{(a)} \end{cases}$$

input
```
integrate((d*x+c)**3*sinh(b*x+a)**2,x)
```

output
```
Piecewise((c**3*x*sinh(a + b*x)**2/2 - c**3*x*cosh(a + b*x)**2/2 + 3*c**2*
d*x**2*sinh(a + b*x)**2/4 - 3*c**2*d*x**2*cosh(a + b*x)**2/4 + c*d**2*x**3
*sinh(a + b*x)**2/2 - c*d**2*x**3*cosh(a + b*x)**2/2 + d**3*x**4*sinh(a +
b*x)**2/8 - d**3*x**4*cosh(a + b*x)**2/8 + c**3*sinh(a + b*x)*cosh(a + b*x
)/(2*b) + 3*c**2*d*x*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 3*c*d**2*x**2*sin
h(a + b*x)*cosh(a + b*x)/(2*b) + d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/(2*
b) - 3*c**2*d*cosh(a + b*x)**2/(4*b**2) - 3*c*d**2*x*sinh(a + b*x)**2/(4*b
**2) - 3*c*d**2*x*cosh(a + b*x)**2/(4*b**2) - 3*d**3*x**2*sinh(a + b*x)**2
/(8*b**2) - 3*d**3*x**2*cosh(a + b*x)**2/(8*b**2) + 3*c*d**2*sinh(a + b*x)
*cosh(a + b*x)/(4*b**3) + 3*d**3*x*sinh(a + b*x)*cosh(a + b*x)/(4*b**3) -
3*d**3*cosh(a + b*x)**2/(8*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 +
c*d**2*x**3 + d**3*x**4/4)*sinh(a)**2, True))
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(120) = 240$.

Time = 0.20 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.96

$$\int (c+dx)^3 \sinh^2(a+bx)\,dx$$
$$= -\frac{3}{16}\left(4\,x^2 - \frac{\left(2\,bxe^{(2\,a)} - e^{(2\,a)}\right)e^{(2\,bx)}}{b^2} + \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{b^2}\right)c^2d$$
$$-\frac{1}{16}\left(8\,x^3 - \frac{3\left(2\,b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)}}{b^3} + \frac{3\left(2\,b^2x^2 + 2\,bx + 1\right)e^{(-2\,bx-2\,a)}}{b^3}\right)cd^2$$
$$-\frac{1}{32}\left(4\,x^4 - \frac{\left(4\,b^3x^3e^{(2\,a)} - 6\,b^2x^2e^{(2\,a)} + 6\,bxe^{(2\,a)} - 3\,e^{(2\,a)}\right)e^{(2\,bx)}}{b^4} + \frac{\left(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3\right)e^{(-2\,bx-2}}{b^4}\right.$$
$$-\frac{1}{8}\,c^3\left(4\,x - \frac{e^{(2\,bx+2\,a)}}{b} + \frac{e^{(-2\,bx-2\,a)}}{b}\right)$$

input
```
integrate((d*x+c)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-3/16*(4*x^2 - (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + (2*b*x + 1)*e^(-2
*b*x - 2*a)/b^2)*c^2*d - 1/16*(8*x^3 - 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a
) + e^(2*a))*e^(2*b*x)/b^3 + 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^
3)*c*d^2 - 1/32*(4*x^4 - (4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^
(2*a) - 3*e^(2*a))*e^(2*b*x)/b^4 + (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(
-2*b*x - 2*a)/b^4)*d^3 - 1/8*c^3*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*
a)/b)
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(120) = 240$.

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.81

$$\int (c+dx)^3 \sinh^2(a+bx)\,dx = -\frac{1}{8}d^3x^4 - \frac{1}{2}cd^2x^3 - \frac{3}{4}c^2dx^2 - \frac{1}{2}c^3x + \frac{(4b^3d^3x^3 + 12b^3cd^2x^2 + 12b^3c^2dx - 6b^2d^3x^2 + 4b^3c^3 - 12b^2cd^2x - 6b^2c^2d + 6bd^3x + 6bcd^2 - 3d^3)e^{(}}{32b^4} - \frac{(4b^3d^3x^3 + 12b^3cd^2x^2 + 12b^3c^2dx + 6b^2d^3x^2 + 4b^3c^3 + 12b^2cd^2x + 6b^2c^2d + 6bd^3x + 6bcd^2 + 3d^3)e^{(}}{32b^4}$$

input
```
integrate((d*x+c)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/8*d^3*x^4 - 1/2*c*d^2*x^3 - 3/4*c^2*d*x^2 - 1/2*c^3*x + 1/32*(4*b^3*d^3
*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x - 6*b^2*d^3*x^2 + 4*b^3*c^3 - 12*
b^2*c*d^2*x - 6*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 3*d^3)*e^(2*b*x + 2*a)
/b^4 - 1/32*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 6*b^2*d^3
*x^2 + 4*b^3*c^3 + 12*b^2*c*d^2*x + 6*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 +
3*d^3)*e^(-2*b*x - 2*a)/b^4
```

3.9.9 Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.71

$$\int (c+dx)^3 \sinh^2(a+bx)\,dx = -\frac{\frac{3d^3\cosh(2a+2bx)}{2} + 4b^4c^3x - 2b^3c^3\sinh(2a+2bx) + b^4d^3x^4 + 3b^2c^2d\cosh(2a+2bx) + 6b^4c^2dx^2 -}{}$$

input
```
int(sinh(a + b*x)^2*(c + d*x)^3,x)
```

output
```
-((3*d^3*cosh(2*a + 2*b*x))/2 + 4*b^4*c^3*x - 2*b^3*c^3*sinh(2*a + 2*b*x)
+ b^4*d^3*x^4 + 3*b^2*c^2*d*cosh(2*a + 2*b*x) + 6*b^4*c^2*d*x^2 + 4*b^4*c*
d^2*x^3 + 3*b^2*d^3*x^2*cosh(2*a + 2*b*x) - 2*b^3*d^3*x^3*sinh(2*a + 2*b*x
) - 3*b*c*d^2*sinh(2*a + 2*b*x) - 3*b*d^3*x*sinh(2*a + 2*b*x) + 6*b^2*c*d^
2*x*cosh(2*a + 2*b*x) - 6*b^3*c^2*d*x*sinh(2*a + 2*b*x) - 6*b^3*c*d^2*x^2*
sinh(2*a + 2*b*x))/(8*b^4)
```

3.10 $\int (c+dx)^2 \sinh^2(a+bx)\,dx$

3.10.1 Optimal result

Integrand size = 16, antiderivative size = 95

$$\int (c+dx)^2 \sinh^2(a+bx)\,dx = -\frac{d^2x}{4b^2} - \frac{(c+dx)^3}{6d} + \frac{d^2\cosh(a+bx)\sinh(a+bx)}{4b^3} + \frac{(c+dx)^2\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{d(c+dx)\sinh^2(a+bx)}{2b^2}$$

output
```
-1/4*d^2*x/b^2-1/6*(d*x+c)^3/d+1/4*d^2*cosh(b*x+a)*sinh(b*x+a)/b^3+1/2*(d*
x+c)^2*cosh(b*x+a)*sinh(b*x+a)/b-1/2*d*(d*x+c)*sinh(b*x+a)^2/b^2
```

3.10.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

$$\int (c+dx)^2 \sinh^2(a+bx)\,dx = \frac{-4b^3x(3c^2+3cdx+d^2x^2) - 6bd(c+dx)\cosh(2(a+bx)) + 3(d^2+2b^2(c+dx)^2)\sinh(2(a+bx))}{24b^3}$$

input
```
Integrate[(c + d*x)^2*Sinh[a + b*x]^2,x]
```

output
```
(-4*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 6*b*d*(c + d*x)*Cosh[2*(a + b*x)]
+ 3*(d^2 + 2*b^2*(c + d*x)^2)*Sinh[2*(a + b*x)])/(24*b^3)
```

3.10.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 25, 3792, 17, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sinh^2(a+bx) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^2 \sin(ia+ibx)^2 dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^2 \sin(ia+ibx)^2 dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{d^2 \int -\sinh^2(a+bx)dx}{2b^2} - \frac{1}{2}\int (c+dx)^2 dx - \frac{d(c+dx)\sinh^2(a+bx)}{2b^2} + \frac{(c+dx)^2 \sinh(a+bx)\cosh(a+bx)}{2b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{d^2 \int -\sinh^2(a+bx)dx}{2b^2} - \frac{d(c+dx)\sinh^2(a+bx)}{2b^2} + \frac{(c+dx)^2 \sinh(a+bx)\cosh(a+bx)}{2b} - \frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \text{25}$$

$$\frac{d^2 \int \sinh^2(a+bx)dx}{2b^2} - \frac{d(c+dx)\sinh^2(a+bx)}{2b^2} + \frac{(c+dx)^2 \sinh(a+bx)\cosh(a+bx)}{2b} - \frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2 \int -\sin(ia+ibx)^2dx}{2b^2} - \frac{d(c+dx)\sinh^2(a+bx)}{2b^2} + \frac{(c+dx)^2 \sinh(a+bx)\cosh(a+bx)}{2b} - \frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{d^2 \int \sin(ia+ibx)^2dx}{2b^2} - \frac{d(c+dx)\sinh^2(a+bx)}{2b^2} + \frac{(c+dx)^2 \sinh(a+bx)\cosh(a+bx)}{2b} - \frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{d^2\left(\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b^2}-\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^3}{6d}$$

input
```
Int[(c + d*x)^2*Sinh[a + b*x]^2,x]
```

output
```
-1/6*(c + d*x)^3/d + ((c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (d*
(c + d*x)*Sinh[a + b*x]^2)/(2*b^2) - (d^2*(x/2 - (Cosh[a + b*x]*Sinh[a + b
*x])/(2*b)))/(2*b^2)
```

3.10.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.10.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{\left(2(dx+c)^2b^2+d^2\right)\sinh(2bx+2a)-4b\left(\frac{d(dx+c)\cosh(2bx+2a)}{2}+x\left(\frac{1}{3}d^2x^2+cdx+c^2\right)b^2-\frac{cd}{2}\right)}{8b^3}$
risch	$-\frac{x^3d^2}{6}-\frac{dcx^2}{2}-\frac{c^2x}{2}-\frac{c^3}{6d}+\frac{\left(2b^2d^2x^2+4b^2cdx+2b^2c^2-2b\,d^2x-2bcd+d^2\right)e^{2bx+2a}}{16b^3}-\frac{(2b^2d^2x^2+4b^2cdx+2b^2c$
derivativedivides	$\frac{d^2\left(\frac{(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{2}-\frac{(bx+a)^3}{6}-\frac{(bx+a)\cosh(bx+a)^2}{2}+\frac{\cosh(bx+a)\sinh(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b^2}-\frac{2d^2a\left(\frac{(bx+a)\cosh(bx+}{2}\right.}{}$
default	$\frac{d^2\left(\frac{(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{2}-\frac{(bx+a)^3}{6}-\frac{(bx+a)\cosh(bx+a)^2}{2}+\frac{\cosh(bx+a)\sinh(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b^2}-\frac{2d^2a\left(\frac{(bx+a)\cosh(bx+}{2}\right.}{}$

input
```
int((d*x+c)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*((2*(d*x+c)^2*b^2+d^2)*sinh(2*b*x+2*a)-4*b*(1/2*d*(d*x+c)*cosh(2*b*x+2
*a)+x*(1/3*d^2*x^2+c*d*x+c^2)*b^2-1/2*c*d))/b^3
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.29

$$\int (c+dx)^2\sinh^2(a+bx)\,dx =$$
$$-\frac{2\,b^3d^2x^3+6\,b^3cdx^2+6\,b^3c^2x+3\left(bd^2x+bcd\right)\cosh\left(bx+a\right)^2-3\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2+d^2\right)\cosh}{12\,b^3}$$

input
```
integrate((d*x+c)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/12*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*b^3*c^2*x + 3*(b*d^2*x + b*c*d)*c
osh(b*x + a)^2 - 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cosh(b*
x + a)*sinh(b*x + a) + 3*(b*d^2*x + b*c*d)*sinh(b*x + a)^2)/b^3
```

3.10.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(85) = 170$.

Time = 0.26 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.78

$$\int (c+dx)^2 \sinh^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c^2x\sinh^2(a+bx)}{2} - \frac{c^2x\cosh^2(a+bx)}{2} + \frac{cdx^2\sinh^2(a+bx)}{2} - \frac{cdx^2\cosh^2(a+bx)}{2} + \frac{d^2x^3\sinh^2(a+bx)}{6} - \frac{d^2x^3\cosh^2(a+bx)}{6} + \frac{c^2\sinh}{} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sinh^2(a) \end{cases}$$

input
```
integrate((d*x+c)**2*sinh(b*x+a)**2,x)
```

output
```
Piecewise((c**2*x*sinh(a + b*x)**2/2 - c**2*x*cosh(a + b*x)**2/2 + c*d*x**
2*sinh(a + b*x)**2/2 - c*d*x**2*cosh(a + b*x)**2/2 + d**2*x**3*sinh(a + b*
x)**2/6 - d**2*x**3*cosh(a + b*x)**2/6 + c**2*sinh(a + b*x)*cosh(a + b*x)/
(2*b) + c*d*x*sinh(a + b*x)*cosh(a + b*x)/b + d**2*x**2*sinh(a + b*x)*cosh
(a + b*x)/(2*b) - c*d*cosh(a + b*x)**2/(2*b**2) - d**2*x*sinh(a + b*x)**2/
(4*b**2) - d**2*x*cosh(a + b*x)**2/(4*b**2) + d**2*sinh(a + b*x)*cosh(a +
b*x)/(4*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sinh(a)**2, T
rue))
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.74

$$\int (c+dx)^2 \sinh^2(a+bx)\,dx$$
$$= -\frac{1}{8}\left(4x^2 - \frac{(2bxe^{(2a)} - e^{(2a)})e^{(2bx)}}{b^2} + \frac{(2bx+1)e^{(-2bx-2a)}}{b^2}\right)cd$$
$$- \frac{1}{48}\left(8x^3 - \frac{3(2b^2x^2e^{(2a)} - 2bxe^{(2a)} + e^{(2a)})e^{(2bx)}}{b^3} + \frac{3(2b^2x^2 + 2bx + 1)e^{(-2bx-2a)}}{b^3}\right)d^2$$
$$- \frac{1}{8}c^2\left(4x - \frac{e^{(2bx+2a)}}{b} + \frac{e^{(-2bx-2a)}}{b}\right)$$

input
```
integrate((d*x+c)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*(4*x^2 - (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + (2*b*x + 1)*e^(-2*
b*x - 2*a)/b^2)*c*d - 1/48*(8*x^3 - 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) +
 e^(2*a))*e^(2*b*x)/b^3 + 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)*
d^2 - 1/8*c^2*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.43

$$\int (c+dx)^2\sinh^2(a+bx)\,dx = -\frac{1}{6}\,d^2x^3-\frac{1}{2}\,cdx^2-\frac{1}{2}\,c^2x+\frac{(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-2\,bd^2x-2\,bcd+d^2)e^{(2\,bx+2\,a)}}{16\,b^3} - \frac{(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2+2\,bd^2x+2\,bcd+d^2)e^{(-2\,bx-2\,a)}}{16\,b^3}$$

input
```
integrate((d*x+c)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/6*d^2*x^3 - 1/2*c*d*x^2 - 1/2*c^2*x + 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x
 + 2*b^2*c^2 - 2*b*d^2*x - 2*b*c*d + d^2)*e^(2*b*x + 2*a)/b^3 - 1/16*(2*b^
2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*b*d^2*x + 2*b*c*d + d^2)*e^(-2*b*x
 - 2*a)/b^3
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.34

$$\int (c+dx)^2\sinh^2(a+bx)\,dx = \frac{c^2\sinh(2\,a+2\,b\,x)}{4\,b}-\frac{d^2\,x^3}{6}-\frac{c^2\,x}{2}+\frac{d^2\sinh(2\,a+2\,b\,x)}{8\,b^3} - \frac{c\,d\,x^2}{2}-\frac{d^2\,x\cosh(2\,a+2\,b\,x)}{4\,b^2}+\frac{d^2\,x^2\sinh(2\,a+2\,b\,x)}{4\,b} - \frac{c\,d\cosh(2\,a+2\,b\,x)}{4\,b^2}+\frac{c\,d\,x\sinh(2\,a+2\,b\,x)}{2\,b}$$

input
```
int(sinh(a + b*x)^2*(c + d*x)^2,x)
```

output
```
(c^2*sinh(2*a + 2*b*x))/(4*b) - (d^2*x^3)/6 - (c^2*x)/2 + (d^2*sinh(2*a +
2*b*x))/(8*b^3) - (c*d*x^2)/2 - (d^2*x*cosh(2*a + 2*b*x))/(4*b^2) + (d^2*x
^2*sinh(2*a + 2*b*x))/(4*b) - (c*d*cosh(2*a + 2*b*x))/(4*b^2) + (c*d*x*sin
h(2*a + 2*b*x))/(2*b)
```

3.10. $\int (c+dx)^2 \sinh^2(a+bx)\,dx$

3.11 $\int (c+dx)\sinh^2(a+bx)\,dx$

3.11.1 Optimal result

Integrand size = 14, antiderivative size = 55

$$\int (c+dx)\sinh^2(a+bx)\,dx = -\frac{cx}{2}-\frac{dx^2}{4}+\frac{(c+dx)\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{d\sinh^2(a+bx)}{4b^2}$$

output
```
-1/2*c*x-1/4*d*x^2+1/2*(d*x+c)*cosh(b*x+a)*sinh(b*x+a)/b-1/4*d*sinh(b*x+a)
^2/b^2
```

3.11.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int (c+dx)\sinh^2(a+bx)\,dx = \frac{-d\cosh(2(a+bx))+2b(-2ac-bx(2c+dx)+(c+dx)\sinh(2(a+bx)))}{8b^2}$$

input
```
Integrate[(c + d*x)*Sinh[a + b*x]^2,x]
```

output
```
(-(d*Cosh[2*(a + b*x)]) + 2*b*(-2*a*c - b*x*(2*c + d*x) + (c + d*x)*Sinh[2
*(a + b*x)]))/(8*b^2)
```

3.11.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\left((c+dx)\sin(ia+ibx)^2\right)dx$$

$$\Big\downarrow \; 25$$

$$-\int (c+dx)\sin(ia+ibx)^2dx$$

$$\Big\downarrow \; 3791$$

$$-\frac{1}{2}\int (c+dx)dx-\frac{d\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}$$

$$\Big\downarrow \; 17$$

$$-\frac{d\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^2}{4d}$$

input
```
Int[(c + d*x)*Sinh[a + b*x]^2,x]
```

output
```
-1/4*(c + d*x)^2/d + ((c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (d*Si
nh[a + b*x]^2)/(4*b^2)
```

3.11.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.11.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{d\,x^2}{4}-\frac{cx}{2}+\frac{(2bdx+2bc-d)\,\mathrm{e}^{2bx+2a}}{16b^2}-\frac{(2bdx+2bc+d)\,\mathrm{e}^{-2bx-2a}}{16b^2}$
derivativedivides	$\frac{\frac{d\left(\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{2}-\frac{(bx+a)^2}{4}-\frac{\cosh(bx+a)^2}{4}\right)}{b}-\frac{da\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}\right)}{b}+c\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\right.}{b}$
default	$\frac{\frac{d\left(\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{2}-\frac{(bx+a)^2}{4}-\frac{\cosh(bx+a)^2}{4}\right)}{b}-\frac{da\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}\right)}{b}+c\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\right.}{b}$

input
```
int((d*x+c)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*d*x^2-1/2*c*x+1/16*(2*b*d*x+2*b*c-d)/b^2*exp(2*b*x+2*a)-1/16*(2*b*d*x
+2*b*c+d)/b^2*exp(-2*b*x-2*a)
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int (c+dx)\sinh^2(a+bx)\,dx = -\frac{2\,b^2dx^2+4\,b^2cx+d\cosh\left(bx+a\right)^2-4\left(bdx+bc\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)+d\sinh\left(bx+a\right)^2}{8\,b^2}$$

input
```
integrate((d*x+c)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*b^2*d*x^2 + 4*b^2*c*x + d*cosh(b*x + a)^2 - 4*(b*d*x + b*c)*cosh(b
*x + a)*sinh(b*x + a) + d*sinh(b*x + a)^2)/b^2
```

3.11.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(49) = 98$.

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.29

$$\int (c+dx)\sinh^2(a+bx)\,dx$$
$$= \begin{cases} \frac{cx\sinh^2(a+bx)}{2} - \frac{cx\cosh^2(a+bx)}{2} + \frac{dx^2\sinh^2(a+bx)}{4} - \frac{dx^2\cosh^2(a+bx)}{4} + \frac{c\sinh(a+bx)\cosh(a+bx)}{2b} + \frac{dx\sinh(a+bx)\cosh(a+bx}{2b} \\ \left(cx+\frac{dx^2}{2}\right)\sinh^2(a) \end{cases}$$

input
```
integrate((d*x+c)*sinh(b*x+a)**2,x)
```

output
```
Piecewise((c*x*sinh(a + b*x)**2/2 - c*x*cosh(a + b*x)**2/2 + d*x**2*sinh(a
 + b*x)**2/4 - d*x**2*cosh(a + b*x)**2/4 + c*sinh(a + b*x)*cosh(a + b*x)/(
2*b) + d*x*sinh(a + b*x)*cosh(a + b*x)/(2*b) - d*cosh(a + b*x)**2/(4*b**2)
, Ne(b, 0)), ((c*x + d*x**2/2)*sinh(a)**2, True))
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.60

$$\int (c+dx)\sinh^2(a+bx)\,dx$$
$$= -\frac{1}{16}\left(4x^2 - \frac{(2bxe^{(2a)} - e^{(2a)})e^{(2bx)}}{b^2} + \frac{(2bx+1)e^{(-2bx-2a)}}{b^2}\right)d - \frac{1}{8}c\left(4x - \frac{e^{(2bx+2a)}}{b} + \frac{e^{(-2bx-2a)}}{b}\right)$$

input
```
integrate((d*x+c)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/16*(4*x^2 - (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + (2*b*x + 1)*e^(-2
*b*x - 2*a)/b^2)*d - 1/8*c*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int (c+dx)\sinh^2(a+bx)\,dx = -\frac{1}{4}\,dx^2 - \frac{1}{2}\,cx + \frac{(2\,bdx + 2\,bc - d)e^{(2\,bx+2\,a)}}{16\,b^2} - \frac{(2\,bdx + 2\,bc + d)e^{(-2\,bx-2\,a)}}{16\,b^2}$$

input
```
integrate((d*x+c)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/4*d*x^2 - 1/2*c*x + 1/16*(2*b*d*x + 2*b*c - d)*e^(2*b*x + 2*a)/b^2 - 1/
16*(2*b*d*x + 2*b*c + d)*e^(-2*b*x - 2*a)/b^2
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int (c+dx)\sinh^2(a+bx)\,dx = -\frac{\frac{d\cosh(2\,a+2\,b\,x)}{2} + b^2\,d\,x^2 - b\,c\sinh(2\,a+2\,b\,x) + 2\,b^2\,c\,x - b\,d\,x\sinh(2\,a+2\,b\,x)}{4\,b^2}$$

input
```
int(sinh(a + b*x)^2*(c + d*x),x)
```

output
```
-((d*cosh(2*a + 2*b*x))/2 + b^2*d*x^2 - b*c*sinh(2*a + 2*b*x) + 2*b^2*c*x
- b*d*x*sinh(2*a + 2*b*x))/(4*b^2)
```

3.12 $\int \frac{\sinh^2(a+bx)}{c+dx}\,dx$

3.12.1 Optimal result

Integrand size = 16, antiderivative size = 78

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx = \frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d} - \frac{\log(c+dx)}{2d} + \frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

output
```
1/2*Chi(2*b*c/d+2*b*x)*cosh(2*a-2*b*c/d)/d-1/2*ln(d*x+c)/d+1/2*Shi(2*b*c/d
+2*b*x)*sinh(2*a-2*b*c/d)/d
```

3.12.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx = \frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2b(c+dx)}{d}\right) - \log(c+dx) + \sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2b(c+dx)}{d}\right)}{2d}$$

input
```
Integrate[Sinh[a + b*x]^2/(c + d*x),x]
```

output
```
(Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*(c + d*x))/d] - Log[c + d*x] + Si
nh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/(2*d)
```

3.12.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{\sin(ia+ibx)^2}{c+dx}dx$$
$$\Big\downarrow \; 25$$
$$-\int \frac{\sin(ia+ibx)^2}{c+dx}dx$$
$$\Big\downarrow \; 3793$$
$$-\int \left(\frac{1}{2(c+dx)}-\frac{\cosh(2a+2bx)}{2(c+dx)}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\log(c+dx)}{2d}$$

input
```
Int[Sinh[a + b*x]^2/(c + d*x),x]
```

output
```
(Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*c)/d + 2*b*x])/(2*d) - Log[c + d*
x]/(2*d) + (Sinh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.12.4 Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.24

method	result	size
risch	$-\frac{\ln(dx+c)}{2d}-\frac{\mathrm{e}^{-\frac{2(ad-bc)}{d}}\operatorname{Ei}_1\left(2bx+2a-\frac{2(ad-bc)}{d}\right)}{4d}-\frac{\mathrm{e}^{\frac{2ad-2bc}{d}}\operatorname{Ei}_1\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)}{4d}$	97

input
```
int(sinh(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(d*x+c)/d-1/4/d*exp(-2*(a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1
/4/d*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.33

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx = \frac{\left(\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)+\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{2\,(bc-ad)}{d}\right)+\left(\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)-\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{2\,(bc-ad)}{d}\right.}{4\,d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
1/4*((Ei(2*(b*d*x + b*c)/d) + Ei(-2*(b*d*x + b*c)/d))*cosh(-2*(b*c - a*d)/
d) + (Ei(2*(b*d*x + b*c)/d) - Ei(-2*(b*d*x + b*c)/d))*sinh(-2*(b*c - a*d)/
d) - 2*log(d*x + c))/d
```

3.12.6 Sympy [F]

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx = \int \frac{\sinh^2{(a+bx)}}{c+dx}\,dx$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(sinh(a + b*x)**2/(c + d*x), x)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx = -\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_1\left(\frac{2\,(dx+c)b}{d}\right)}{4\,d} - \frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_1\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,d} - \frac{\log{(dx+c)}}{2\,d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
-1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(1, 2*(d*x + c)*b/d)/d - 1/4*e^(2*a
- 2*b*c/d)*exp_integral_e(1, -2*(d*x + c)*b/d)/d - 1/2*log(d*x + c)/d
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx = \frac{\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} + \mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)} - 2\,\log{(dx+c)}}{4\,d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
1/4*(Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + Ei(-2*(b*d*x + b*c)/d)*e^(-
2*a + 2*b*c/d) - 2*log(d*x + c))/d
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx=\int \frac{{\mathrm{sinh}\left(a+b\,x\right)}^2}{c+d\,x} \,d x$$

input
```
int(sinh(a + b*x)^2/(c + d*x),x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x), x)
```

3.13 $\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx$

3.13.1 Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx = \frac{b\,\mathrm{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{d^2}-\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{b\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

output
```
b*cosh(2*a-2*b*c/d)*Shi(2*b*c/d+2*b*x)/d^2+b*Chi(2*b*c/d+2*b*x)*sinh(2*a-2
*b*c/d)/d^2-sinh(b*x+a)^2/d/(d*x+c)
```

3.13.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx = \frac{b\,\mathrm{Chi}\left(\frac{2b(c+dx)}{d}\right)\sinh\left(2a-\frac{2bc}{d}\right)-\frac{d\sinh^2(a+bx)}{c+dx}+b\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2b(c+dx)}{d}\right)}{d^2}$$

input
```
Integrate[Sinh[a + b*x]^2/(c + d*x)^2,x]
```

output
```
(b*CoshIntegral[(2*b*(c + d*x))/d]*Sinh[2*a - (2*b*c)/d] - (d*Sinh[a + b*x
]^2)/(c + d*x) + b*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/
d^2
```

3.13.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 25, 3794, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\sin(ia+ibx)^2}{(c+dx)^2}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\sin(ia+ibx)^2}{(c+dx)^2}dx$$

$$\Big\downarrow \; 3794$$

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{2ib\int \frac{i\sinh(2a+2bx)}{2(c+dx)}dx}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{b\int \frac{\sinh(2a+2bx)}{c+dx}dx}{d}-\frac{\sinh^2(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{b\int -\frac{i\sin(2ia+2ibx)}{c+dx}dx}{d}$$

$$\Big\downarrow \; 26$$

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{ib\int \frac{\sin(2ia+2ibx)}{c+dx}dx}{d}$$

$$\Big\downarrow \; 3784$$

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int \frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+\cosh\left(2a-\frac{2bc}{d}\right)\int \frac{i\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}$$

$$\Big\downarrow \; 26$$

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+i\cosh\left(2a-\frac{2bc}{d}\right)\int\frac{\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(2a-\frac{2bc}{d}\right)\int-\frac{i\sin\left(\frac{2ibc}{d}+2ibx\right)}{c+dx}dx\right)}{d}$$

$$\Big\downarrow \; 26$$

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx\right)}{c+dx}dx\right)}{d}$$

$$\Big\downarrow \; 3779$$

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}$$

$$\Big\downarrow \; 3782$$

$$-\frac{\sinh^2(a+bx)}{d(c+dx)}-\frac{ib\left(\frac{i\sinh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{i\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}$$

input
```
Int[Sinh[a + b*x]^2/(c + d*x)^2,x]
```

output
```
-(Sinh[a + b*x]^2/(d*(c + d*x))) - (I*b*((I*CoshIntegral[(2*b*c)/d + 2*b*x
]*Sinh[2*a - (2*b*c)/d])/d + (I*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)
/d + 2*b*x])/d))/d
```

3.13. $\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx$

3.13.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

3.13.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.88

method	result
risch	$\frac{1}{2(dx+c)d}-\frac{b\,\mathrm{e}^{-2bx-2a}}{4d(bdx+bc)}+\frac{b\,\mathrm{e}^{-\frac{2(ad-bc)}{d}}\,\mathrm{Ei}_1\left(2bx+2a-\frac{2(ad-bc)}{d}\right)}{2d^2}-\frac{b\,\mathrm{e}^{2bx+2a}}{4d^2\left(\frac{bc}{d}+bx\right)}-\frac{b\,\mathrm{e}^{\frac{2ad-2bc}{d}}\,\mathrm{Ei}_1\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)}{2d^2}$

input
```
int(sinh(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(d*x+c)/d-1/4*b*exp(-2*b*x-2*a)/d/(b*d*x+b*c)+1/2*b/d^2*exp(-2*(a*d-b*
c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/4*b/d^2*exp(2*b*x+2*a)/(b*c/d+b*x)-1
/2*b/d^2*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(81) = 162.

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.05

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{d\cosh\left(bx+a\right)^2+d\sinh\left(bx+a\right)^2-\left((bdx+bc)\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)-(bdx+bc)\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{2}{}\right.}{2\,(d^3x+cd^2)}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(d*cosh(b*x + a)^2 + d*sinh(b*x + a)^2 - ((b*d*x + b*c)*Ei(2*(b*d*x +
 b*c)/d) - (b*d*x + b*c)*Ei(-2*(b*d*x + b*c)/d))*cosh(-2*(b*c - a*d)/d) -
((b*d*x + b*c)*Ei(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-2*(b*d*x + b*c)/d
))*sinh(-2*(b*c - a*d)/d) - d)/(d^3*x + c*d^2)
```

3.13.6 Sympy [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sinh^2{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(sinh(a + b*x)**2/(c + d*x)**2, x)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx = -\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_2\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)d} - \frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_2\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)d} + \frac{1}{2\,(d^2x+cd)}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(2, 2*(d*x + c)*b/d)/((d*x + c)*d) -
 1/4*e^(2*a - 2*b*c/d)*exp_integral_e(2, -2*(d*x + c)*b/d)/((d*x + c)*d) +
 1/2/(d^2*x + c*d)
```

3.13.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 574 vs. 2(81) = 162.

Time = 0.32 (sec) , antiderivative size = 574, normalized size of antiderivative = 7.09

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx =$$

$$-\frac{\left(2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(-\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(\frac{2\,(bc-ad)}{d}\right)}+2\,b^3c\mathrm{Ei}\left(-\frac{2\left((dx+c)\left(b-\right.\right.}{}\right.\right.}{}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/4*(2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(-2*((d*x + c)
*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(2*(b*c - a*d)/d) +
 2*b^3*c*Ei(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)
/d)*e^(2*(b*c - a*d)/d) - 2*a*b^2*d*Ei(-2*((d*x + c)*(b - b*c/(d*x + c) +
a*d/(d*x + c)) + b*c - a*d)/d)*e^(2*(b*c - a*d)/d) - 2*(d*x + c)*(b - b*c/
(d*x + c) + a*d/(d*x + c))*b^2*Ei(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d
*x + c)) + b*c - a*d)/d)*e^(-2*(b*c - a*d)/d) - 2*b^3*c*Ei(2*((d*x + c)*(b
 - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-2*(b*c - a*d)/d) + 2
*a*b^2*d*Ei(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/
d)*e^(-2*(b*c - a*d)/d) + b^2*d*e^(2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d
*x + c))/d) + b^2*d*e^(-2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d)
 - 2*b^2*d)*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^4 + b*c*
d^4 - a*d^5)*b)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sinh(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x)^2,x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x)^2, x)
```

3.14 $\int \frac{\sinh^2(a+bx)}{(c+dx)^3} dx$

3.14.1 Optimal result

Integrand size = 16, antiderivative size = 112

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3} dx = \frac{b^2 \cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b \cosh(a+bx) \sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{b^2 \sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{d^3}$$

output
```
b^2*Chi(2*b*c/d+2*b*x)*cosh(2*a-2*b*c/d)/d^3+b^2*Shi(2*b*c/d+2*b*x)*sinh(2
*a-2*b*c/d)/d^3-b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)-1/2*sinh(b*x+a)^2/d/
(d*x+c)^2
```

3.14.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.91

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3} dx = \frac{2b^2 \cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2b(c+dx)}{d}\right) - \frac{d(d \sinh^2(a+bx) + b(c+dx) \sinh(2(a+bx)))}{(c+dx)^2} + 2b^2 \sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2b(c+dx)}{d}\right)}{2d^3}$$

input
```
Integrate[Sinh[a + b*x]^2/(c + d*x)^3,x]
```

output
```
(2*b^2*Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*(c + d*x))/d] - (d*(d*Sinh[
a + b*x]^2 + b*(c + d*x)*Sinh[2*(a + b*x)]))/(c + d*x)^2 + 2*b^2*Sinh[2*a
- (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/(2*d^3)
```

3.14.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {3042, 25, 3795, 16, 25, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ia+ibx)^2}{(c+dx)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ia+ibx)^2}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3795}$$

$$-\frac{2b^2\int -\frac{\sinh^2(a+bx)}{c+dx}dx}{d^2}+\frac{b^2\int \frac{1}{c+dx}dx}{d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\sinh^2(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{16}$$

$$-\frac{2b^2\int -\frac{\sinh^2(a+bx)}{c+dx}dx}{d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\sinh^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b^2\int \frac{\sinh^2(a+bx)}{c+dx}dx}{d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\sinh^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b^2\int -\frac{\sin(ia+ibx)^2}{c+dx}dx}{d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\sinh^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2b^2 \int \frac{\sin(ia+ibx)^2}{c+dx}dx}{d^2} - \frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{2b^2 \int \left(\frac{1}{2(c+dx)} - \frac{\cosh(2a+2bx)}{2(c+dx)}\right)dx}{d^2} - \frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2b^2\left(-\frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d} - \frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d} + \frac{\log(c+dx)}{2d}\right)}{d^2} - \frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\log(c+dx)}{d^3}$$

input
```
Int[Sinh[a + b*x]^2/(c + d*x)^3,x]
```

output
```
(b^2*Log[c + d*x])/d^3 - (b*Cosh[a + b*x]*Sinh[a + b*x])/(d^2*(c + d*x)) -
 Sinh[a + b*x]^2/(2*d*(c + d*x)^2) - (2*b^2*(-1/2*(Cosh[2*a - (2*b*c)/d]*C
oshIntegral[(2*b*c)/d + 2*b*x])/d + Log[c + d*x]/(2*d) - (Sinh[2*a - (2*b*
c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(2*d)))/d^2
```

3.14.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

3.14.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(110) = 220.

Time = 3.22 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.67

method	result
risch	$\frac{1}{4(dx+c)^2 d}+\frac{b^3 \mathrm{e}^{-2bx-2a} x}{4d(b^2d^2x^2+2b^2cdx+b^2c^2)}+\frac{b^3 \mathrm{e}^{-2bx-2a} c}{4d^2(b^2d^2x^2+2b^2cdx+b^2c^2)}-\frac{b^2 \mathrm{e}^{-2bx-2a}}{8d(b^2d^2x^2+2b^2cdx+b^2c^2)}-\frac{b^2 \mathrm{e}^{-\frac{2(ad-bc)}{d}} \mathrm{Ei}_1\left(2bx\right.}{2d^3}$

input
```
int(sinh(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/(d*x+c)^2/d+1/4*b^3*exp(-2*b*x-2*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2
)*x+1/4*b^3*exp(-2*b*x-2*a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-1/8*b^
2*exp(-2*b*x-2*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-1/2*b^2/d^3*exp(-2*(
a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/8*b^2/d^3*exp(2*b*x+2*a)/(b*c/
d+b*x)^2-1/4*b^2/d^3*exp(2*b*x+2*a)/(b*c/d+b*x)-1/2*b^2/d^3*exp(2*(a*d-b*c
)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

3.14.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(110) = 220$.

Time = 0.25 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.50

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{d^2\cosh\left(bx+a\right)^2+d^2\sinh\left(bx+a\right)^2+4\left(bd^2x+bcd\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)-d^2-2\left(\left(b^2d^2x^2+2\right.\right.}{}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(d^2*cosh(b*x + a)^2 + d^2*sinh(b*x + a)^2 + 4*(b*d^2*x + b*c*d)*cosh
(b*x + a)*sinh(b*x + a) - d^2 - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*E
i(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-2*(b*d*x
+ b*c)/d))*cosh(-2*(b*c - a*d)/d) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*Ei(2*(b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-2*(b*
d*x + b*c)/d))*sinh(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.14.6 Sympy [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sinh^2\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c)**3,x)
```

output
```
Integral(sinh(a + b*x)**2/(c + d*x)**3, x)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx = \frac{1}{4\left(d^3x^2+2\,cd^2x+c^2d\right)} - \frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_3\left(\frac{2\,(dx+c)b}{d}\right)}{4\left(dx+c\right)^2d} - \frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_3\left(-\frac{2\,(dx+c)b}{d}\right)}{4\left(dx+c\right)^2d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4/(d^3*x^2 + 2*c*d^2*x + c^2*d) - 1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(
3, 2*(d*x + c)*b/d)/((d*x + c)^2*d) - 1/4*e^(2*a - 2*b*c/d)*exp_integral_e
(3, -2*(d*x + c)*b/d)/((d*x + c)^2*d)
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(110) = 220$.

Time = 0.28 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.95

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{4\,b^2d^2x^2\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+4\,b^2d^2x^2\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}+8\,b^2cdx\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+}{}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*(4*b^2*d^2*x^2*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + 4*b^2*d^2*x^2
*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) + 8*b^2*c*d*x*Ei(2*(b*d*x + b*c
)/d)*e^(2*a - 2*b*c/d) + 8*b^2*c*d*x*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*
c/d) + 4*b^2*c^2*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + 4*b^2*c^2*Ei(-2
*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) - 2*b*d^2*x*e^(2*b*x + 2*a) + 2*b*d^2
*x*e^(-2*b*x - 2*a) - 2*b*c*d*e^(2*b*x + 2*a) + 2*b*c*d*e^(-2*b*x - 2*a) -
 d^2*e^(2*b*x + 2*a) - d^2*e^(-2*b*x - 2*a) + 2*d^2)/(d^5*x^2 + 2*c*d^4*x
+ c^2*d^3)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{{\mathrm{sinh}\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x)^3,x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x)^3, x)
```

3.15 $\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx$

3.15.1 Optimal result

Integrand size = 16, antiderivative size = 162

$$\begin{aligned}\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx = & -\frac{b^2}{3d^3(c+dx)}+\frac{2b^3\text{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{3d^4} \\ & -\frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3} \\ & -\frac{2b^2\sinh^2(a+bx)}{3d^3(c+dx)}+\frac{2b^3\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{3d^4}\end{aligned}$$

output
```
-1/3*b^2/d^3/(d*x+c)+2/3*b^3*cosh(2*a-2*b*c/d)*Shi(2*b*c/d+2*b*x)/d^4+2/3*
b^3*Chi(2*b*c/d+2*b*x)*sinh(2*a-2*b*c/d)/d^4-1/3*b*cosh(b*x+a)*sinh(b*x+a)
/d^2/(d*x+c)^2-1/3*sinh(b*x+a)^2/d/(d*x+c)^3-2/3*b^2*sinh(b*x+a)^2/d^3/(d*
x+c)
```

3.15.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.76

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx = \frac{4b^3\text{Chi}\left(\frac{2b(c+dx)}{d}\right)\sinh\left(2a-\frac{2bc}{d}\right)-\frac{d\left(\left(d^2+2b^2(c+dx)^2\right)\cosh(2(a+bx))+d(-d+b(c+dx)\sinh(2(a+bx)))\right)}{(c+dx)^3}+4b^3\cosh\left(2a-2\right.}{6d^4}$$

input
```
Integrate[Sinh[a + b*x]^2/(c + d*x)^4,x]
```

output
```
(4*b^3*CoshIntegral[(2*b*(c + d*x))/d]*Sinh[2*a - (2*b*c)/d] - (d*((d^2 +
2*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)] + d*(-d + b*(c + d*x)*Sinh[2*(a + b*x
)])))/(c + d*x)^3 + 4*b^3*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x
))/d])/(6*d^4)
```

3.15.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.062, Rules used = {3042, 25, 3795, 17, 25, 3042, 25, 3794, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ia+ibx)^2}{(c+dx)^4}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ia+ibx)^2}{(c+dx)^4}dx$$

$$\Big\downarrow \text{3795}$$

$$-\frac{2b^2\int -\frac{\sinh^2(a+bx)}{(c+dx)^2}dx}{3d^2}+\frac{b^2\int \frac{1}{(c+dx)^2}dx}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{17}$$

$$-\frac{2b^2\int -\frac{\sinh^2(a+bx)}{(c+dx)^2}dx}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b^2\int \frac{\sinh^2(a+bx)}{(c+dx)^2}dx}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b^2 \int -\frac{\sin(ia+ibx)^2}{(c+dx)^2}dx}{3d^2} - \frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3} - \frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2b^2 \int \frac{\sin(ia+ibx)^2}{(c+dx)^2}dx}{3d^2} - \frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3} - \frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3794}$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)} + \frac{2ib\int \frac{i\sinh(2a+2bx)}{2(c+dx)}dx}{d}\right)}{3d^2} - \frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3} - \frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)} - \frac{b\int \frac{\sinh(2a+2bx)}{c+dx}dx}{d}\right)}{3d^2} - \frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3} - \frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)} - \frac{b\int -\frac{i\sin(2ia+2ibx)}{c+dx}dx}{d}\right)}{3d^2} - \frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3} - \frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)} + \frac{ib\int \frac{\sin(2ia+2ibx)}{c+dx}dx}{d}\right)}{3d^2} - \frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3} - \frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)} + \frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int \frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx + \cosh\left(2a-\frac{2bc}{d}\right)\int \frac{i\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}\right)}{3d^2} - \frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3} - \frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{26}$$

3.15. $\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+i\cosh\left(2a-\frac{2bc}{d}\right)\int\frac{\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(2a-\frac{2bc}{d}\right)\int-\frac{i\sin\left(\frac{2ibc}{d}+2ibx\right)}{c+dx}dx\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx\right)}{c+dx}dx\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3779}$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{2b^2\left(\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{ib\left(\frac{i\sinh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{i\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

input
```
Int[Sinh[a + b*x]^2/(c + d*x)^4,x]
```

3.15. $\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx$

output
```
-1/3*b^2/(d^3*(c + d*x)) - (b*Cosh[a + b*x]*Sinh[a + b*x])/(3*d^2*(c + d*x
)^2) - Sinh[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*(Sinh[a + b*x]^2/(d*(c +
 d*x)) + (I*b*((I*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[2*a - (2*b*c)/d])/d
 + (I*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/d))/d))/(3*d^
2)
```

3.15.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

3.15.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(150) = 300$.

Time = 3.11 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.43

method	result
risch	$\frac{1}{6(dx+c)^3 d} - \frac{b^5 e^{-2bx-2a} x^2}{6d(d^3x^3b^3+3b^3c\,d^2x^2+3b^3c^2dx+b^3c^3)} - \frac{b^5 e^{-2bx-2a} cx}{3d^2(d^3x^3b^3+3b^3c\,d^2x^2+3b^3c^2dx+b^3c^3)} - \frac{b^5 e^{-2bx-2a} c^2}{6d^3(d^3x^3b^3+3b^3c\,d^2x^2+3b^3}$

input
```
int(sinh(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6/(d*x+c)^3/d-1/6*b^5*exp(-2*b*x-2*a)/d/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b
^3*c^2*d*x+b^3*c^3)*x^2-1/3*b^5*exp(-2*b*x-2*a)/d^2/(b^3*d^3*x^3+3*b^3*c*d
^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c*x-1/6*b^5*exp(-2*b*x-2*a)/d^3/(b^3*d^3*x^3
+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c^2+1/12*b^4*exp(-2*b*x-2*a)/d/(b^
3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*x+1/12*b^4*exp(-2*b*x-2*a
)/d^2/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c-1/12*b^3*exp(-
2*b*x-2*a)/d/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)+1/3*b^3/d
^4*exp(-2*(a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/12*b^3/d^4*exp(2*b*
x+2*a)/(b*c/d+b*x)^3-1/12*b^3/d^4*exp(2*b*x+2*a)/(b*c/d+b*x)^2-1/6*b^3/d^4
*exp(2*b*x+2*a)/(b*c/d+b*x)-1/3*b^3/d^4*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a
-2*(-a*d+b*c)/d)
```

3.15.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(150) = 300$.

Time = 0.27 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.54

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{d^3 - (2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d + d^3)\cosh{(bx+a)}^2 - 2\,(bd^3x + bcd^2)\cosh{(bx+a)}\sinh{(bx+a)} - (2}{}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*(d^3 - (2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*cosh(b*x +
a)^2 - 2*(b*d^3*x + b*c*d^2)*cosh(b*x + a)*sinh(b*x + a) - (2*b^2*d^3*x^2
+ 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*sinh(b*x + a)^2 + 2*((b^3*d^3*x^3 + 3
*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*Ei(2*(b*d*x + b*c)/d) - (b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*Ei(-2*(b*d*x + b*c)/d))*
cosh(-2*(b*c - a*d)/d) + 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
 + b^3*c^3)*Ei(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3
*c^2*d*x + b^3*c^3)*Ei(-2*(b*d*x + b*c)/d))*sinh(-2*(b*c - a*d)/d))/(d^7*x
^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

3.15.6 Sympy [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sinh^2{(a+bx)}}{(c+dx)^4}\,dx$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c)**4,x)
```

output
```
Integral(sinh(a + b*x)**2/(c + d*x)**4, x)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.68

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx = \frac{1}{6\,(d^4x^3+3\,cd^3x^2+3\,c^2d^2x+c^3d)} - \frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_4\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^3d} - \frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_4\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^3d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/6/(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x + c^3*d) - 1/4*e^(-2*a + 2*b*c/d)
*exp_integral_e(4, 2*(d*x + c)*b/d)/((d*x + c)^3*d) - 1/4*e^(2*a - 2*b*c/d
)*exp_integral_e(4, -2*(d*x + c)*b/d)/((d*x + c)^3*d)
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(150) = 300$.

Time = 0.27 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.31

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx = \frac{4\,b^3d^3x^3\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} - 4\,b^3d^3x^3\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)} + 12\,b^3cd^2x^2\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

output
```
1/12*(4*b^3*d^3*x^3*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) - 4*b^3*d^3*x^
3*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) + 12*b^3*c*d^2*x^2*Ei(2*(b*d*x
 + b*c)/d)*e^(2*a - 2*b*c/d) - 12*b^3*c*d^2*x^2*Ei(-2*(b*d*x + b*c)/d)*e^(
-2*a + 2*b*c/d) + 12*b^3*c^2*d*x*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) -
 12*b^3*c^2*d*x*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) - 2*b^2*d^3*x^2*
e^(2*b*x + 2*a) - 2*b^2*d^3*x^2*e^(-2*b*x - 2*a) + 4*b^3*c^3*Ei(2*(b*d*x +
 b*c)/d)*e^(2*a - 2*b*c/d) - 4*b^3*c^3*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*
b*c/d) - 4*b^2*c*d^2*x*e^(2*b*x + 2*a) - 4*b^2*c*d^2*x*e^(-2*b*x - 2*a) -
2*b^2*c^2*d*e^(2*b*x + 2*a) - b*d^3*x*e^(2*b*x + 2*a) - 2*b^2*c^2*d*e^(-2*
b*x - 2*a) + b*d^3*x*e^(-2*b*x - 2*a) - b*c*d^2*e^(2*b*x + 2*a) + b*c*d^2*
e^(-2*b*x - 2*a) - d^3*e^(2*b*x + 2*a) - d^3*e^(-2*b*x - 2*a) + 2*d^3)/(d^
7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sinh(a+b\,x)^2}{(c+d\,x)^4}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x)^4,x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x)^4, x)
```

3.16 $\int (c+dx)^4 \sinh^3(a+bx)\,dx$

3.16.1 Optimal result

Integrand size = 16, antiderivative size = 225

$$\begin{aligned}\int (c+dx)^4 \sinh^3(a+bx)\,dx = &-\frac{488d^4\cosh(a+bx)}{27b^5}-\frac{80d^2(c+dx)^2\cosh(a+bx)}{9b^3}\\&-\frac{2(c+dx)^4\cosh(a+bx)}{3b}+\frac{8d^4\cosh^3(a+bx)}{81b^5}\\&+\frac{160d^3(c+dx)\sinh(a+bx)}{9b^4}+\frac{8d(c+dx)^3\sinh(a+bx)}{3b^2}\\&+\frac{4d^2(c+dx)^2\cosh(a+bx)\sinh^2(a+bx)}{9b^3}\\&+\frac{(c+dx)^4\cosh(a+bx)\sinh^2(a+bx)}{3b}\\&-\frac{8d^3(c+dx)\sinh^3(a+bx)}{27b^4}-\frac{4d(c+dx)^3\sinh^3(a+bx)}{9b^2}\end{aligned}$$

output
```
-488/27*d^4*cosh(b*x+a)/b^5-80/9*d^2*(d*x+c)^2*cosh(b*x+a)/b^3-2/3*(d*x+c)
^4*cosh(b*x+a)/b+8/81*d^4*cosh(b*x+a)^3/b^5+160/9*d^3*(d*x+c)*sinh(b*x+a)/
b^4+8/3*d*(d*x+c)^3*sinh(b*x+a)/b^2+4/9*d^2*(d*x+c)^2*cosh(b*x+a)*sinh(b*x
+a)^2/b^3+1/3*(d*x+c)^4*cosh(b*x+a)*sinh(b*x+a)^2/b-8/27*d^3*(d*x+c)*sinh(
b*x+a)^3/b^4-4/9*d*(d*x+c)^3*sinh(b*x+a)^3/b^2
```

3.16.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

$$\int (c+dx)^4 \sinh^3(a+bx)\,dx$$
$$= \frac{-243(24d^4+12b^2d^2(c+dx)^2+b^4(c+dx)^4)\cosh(a+bx)+(8d^4+36b^2d^2(c+dx)^2+27b^4(c+dx)^4)\cosh}{32}$$

input
```
Integrate[(c + d*x)^4*Sinh[a + b*x]^3,x]
```

output
```
(-243*(24*d^4 + 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cosh[a + b*x] +
(8*d^4 + 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cosh[3*(a + b*x)] -
24*b*d*(c + d*x)*(-242*d^2 - 39*b^2*(c + d*x)^2 + (2*d^2 + 3*b^2*(c + d*x)
^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(324*b^5)
```

3.16.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sinh^3(a+bx)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int i(c+dx)^4 \sin(ia+ibx)^3 dx$$
$$\Big\downarrow \; 26$$
$$i\int (c+dx)^4 \sin(ia+ibx)^3 dx$$
$$\Big\downarrow \; 3792$$
$$i\left(\frac{4d^2\int -i(c+dx)^2\sinh^3(a+bx)dx}{3b^2}+\frac{2}{3}\int i(c+dx)^4\sinh(a+bx)dx+\frac{4id(c+dx)^3\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^4}{}\right.$$
$$\Big\downarrow \; 26$$
$$i\left(-\frac{4id^2\int (c+dx)^2\sinh^3(a+bx)dx}{3b^2}+\frac{2}{3}i\int (c+dx)^4\sinh(a+bx)dx+\frac{4id(c+dx)^3\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^4}{}\right.$$

$$\Big\downarrow 3042$$

$$i\left(-\frac{4id^2\int i(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}i\int -i(c+dx)^4\sin(ia+ibx)dx+\frac{4id(c+dx)^3\sinh^3(a+bx)}{9b^2}-\frac{i(c+\cdots}{}\right.$$

$$\Big\downarrow 26$$

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\int(c+dx)^4\sin(ia+ibx)dx+\frac{4id(c+dx)^3\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^4\sin\cdots}{}\right.$$

$$\Big\downarrow 3777$$

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\int(c+dx)^3\cosh(a+bx)dx}{b}\right)+\frac{4id(c+dx\cdots}{}\right.$$

$$\Big\downarrow 3042$$

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\int(c+dx)^3\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)+\frac{4id(\cdots}{}\right.$$

$$\Big\downarrow 3777$$

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}-\frac{3id\int -i(c+dx)^2\sinh(a+bx\cdots}{b}\right.}{b}\right.\right.$$

$$\Big\downarrow 26$$

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}-\frac{3d\int(c+dx)^2\sinh(a+bx)dx}{b}\right.}{b}\right.\right.$$

$$\Big\downarrow 3042$$

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}-\frac{3d\int -i(c+dx)^2\sin(ia+ibx\cdots}{b}\right.}{b}\right.\right.$$

$$\Big\downarrow 26$$

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\int(c+dx)^2\sin(ia+ibx)dx}{b}\right.}{b}\right.\right.$$

3.16. $\int(c+dx)^4\sinh^3(a+bx)\,dx$

$$\Big\downarrow \text{3777}$$

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\right.}{b}\right.}{b}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\right.}{}\right.}{b}\right.\right.$$

$$\Big\downarrow \text{3777}$$

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\right.}{}\right.}{b}\right.\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\right.}{}\right.}{b}\right.\right.$$

$$\Big\downarrow \text{3042}$$

3.16. $\int(c+dx)^4\sinh^3(a+bx)\,dx$

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\right.}{}\right.}{b}\right.\right.$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\right.}{}\right.}{b}\right.\right.$$

$$\Big\downarrow \; 3118$$

$$i\left(\frac{4d^2\int(c+dx)^2\sin(ia+ibx)^3dx}{3b^2}+\frac{4id(c+dx)^3\sinh^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{i(c+dx)^4\cosh(a+bx)}{b}-\frac{4id\left(\frac{(c+dx)^3\sinh}{b}\right.}{}\right.\right.$$

$$\Big\downarrow \; 3792$$

$$i\left(\frac{4d^2\left(\frac{2d^2\int-i\sinh^3(a+bx)dx}{9b^2}+\frac{2}{3}\int i(c+dx)^2\sinh(a+bx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}\right.$$

3.16. $\int(c+dx)^4\sinh^3(a+bx)\,dx$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{4d^2\left(-\frac{2id^2\int\sinh^3(a+bx)dx}{9b^2}+\frac{2}{3}i\int(c+dx)^2\sinh(a+bx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{4d^2\left(-\frac{2id^2\int i\sin(ia+ibx)^3dx}{9b^2}+\frac{2}{3}i\int-i(c+dx)^2\sin(ia+ibx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh^2(a+bx)\cosh(a+bx)}{3b}\right.}{3b^2}\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{4d^2\left(\frac{2d^2\int\sin(ia+ibx)^3dx}{9b^2}+\frac{2}{3}\int(c+dx)^2\sin(ia+ibx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}+\right.$$

$$\Big\downarrow \text{3113}$$

3.16. $\int(c+dx)^4\sinh^3(a+bx)\,dx$

$$i\left(\frac{4d^2\left(\frac{2id^2\int(1-\cosh^2(a+bx))\,d\cosh(a+bx)}{9b^3}+\frac{2}{3}\int(c+dx)^2\sin(ia+ibx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh^2(a+bx)\cos}{3b}\right.}{3b^2}\right.$$

$\downarrow$ 2009

$$i\left(\frac{4d^2\left(\frac{2}{3}\int(c+dx)^2\sin(ia+ibx)dx+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh^2(a+bx)\cosh(}{3b}\right.}{3b^2}\right.$$

$\downarrow$ 3777

$$i\left(\frac{4d^2\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int(c+dx)\cosh(a+bx)dx}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx}{}\right.}{3b^2}\right.$$

$\downarrow$ 3042

$$i\left(\frac{4d^2\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(}{}\right.}{3b^2}\right.$$

3.16. $\int(c+dx)^4\sinh^3(a+bx)\,dx$

$$\Big\downarrow \text{3777}$$

$$i\left(\frac{4d^2\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{id\int -i\sinh(a+bx)dx}{b}\right)}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c+dx)\sinh}{9b^2}\right.}{3b^2}\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{4d^2\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int \sinh(a+bx)dx}{b}\right)}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c+dx)\sinh^3(a}{9b^2}\right.}{3b^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{4d^2\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int -i\sin(ia+ibx)dx}{b}\right)}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c+dx)\sinh}{9b^2}\right.}{3b^2}\right.$$

input
```
Int[(c + d*x)^4*Sinh[a + b*x]^3,x]
```

output
```
$Aborted
```

3.16. $\int (c+dx)^4\sinh^3(a+bx)\,dx$

3.16.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.16.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{\left(27(dx+c)^4b^4+36d^2(dx+c)^2b^2+8d^4\right)\cosh(3bx+3a)-36d\left((dx+c)^2b^2+\frac{2d^2}{3}\right)b(dx+c)\sinh(3bx+3a)+\left(-243(dx+c)^4b^4\right.}{324}$
risch	$\frac{(27d^4x^4b^4+108b^4c\,d^3x^3+162b^4c^2d^2x^2-36b^3d^4x^3+108b^4c^3dx-108b^3c\,d^3x^2+27b^4c^4-108b^3c^2d^2x+36b^2d^4x^2-36b^3c^3d}{648b^5}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^4*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/324*((27*(d*x+c)^4*b^4+36*d^2*(d*x+c)^2*b^2+8*d^4)*cosh(3*b*x+3*a)-36*d*
((d*x+c)^2*b^2+2/3*d^2)*b*(d*x+c)*sinh(3*b*x+3*a)+(-243*(d*x+c)^4*b^4-2916
*d^2*(d*x+c)^2*b^2-5832*d^4)*cosh(b*x+a)+972*d*((d*x+c)^2*b^2+6*d^2)*b*(d*
x+c)*sinh(b*x+a)-216*b^4*c^4-2880*b^2*c^2*d^2-5824*d^4)/b^5
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(205) = 410$.

Time = 0.25 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.35

$$\int (c+dx)^4\sinh^3(a+bx)\,dx$$
$$=\frac{(27\,b^4d^4x^4+108\,b^4cd^3x^3+27\,b^4c^4+36\,b^2c^2d^2+8\,d^4+18\,(9\,b^4c^2d^2+2\,b^2d^4)x^2+36\,(3\,b^4c^3d+2\,b^2cd^3)x)}{}$$

input
```
integrate((d*x+c)^4*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/324*((27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 + 36*b^2*c^2*d^2 +
 8*d^4 + 18*(9*b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d + 2*b^2*c*d^
3)*x)*cosh(b*x + a)^3 + 3*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4
 + 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 36*(3*b^4
*c^3*d + 2*b^2*c*d^3)*x)*cosh(b*x + a)*sinh(b*x + a)^2 - 12*(3*b^3*d^4*x^3
 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d + 2*b*c*d^3 + (9*b^3*c^2*d^2 + 2*b*d^4)*x
)*sinh(b*x + a)^3 - 243*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 + 12*b^2*
c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d + 6*b^2*
c*d^3)*x)*cosh(b*x + a) + 36*(27*b^3*d^4*x^3 + 81*b^3*c*d^3*x^2 + 27*b^3*c
^3*d + 162*b*c*d^3 - (3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d + 2*b*
c*d^3 + (9*b^3*c^2*d^2 + 2*b*d^4)*x)*cosh(b*x + a)^2 + 81*(b^3*c^2*d^2 + 2
*b*d^4)*x)*sinh(b*x + a))/b^5
```

3.16.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(226) = 452$.

Time = 0.67 (sec) , antiderivative size = 772, normalized size of antiderivative = 3.43

$$\int (c+dx)^4\sinh^3(a+bx)\,dx$$
$$= \begin{cases} \frac{c^4\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2c^4\cosh^3(a+bx)}{3b} + \frac{4c^3dx\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{8c^3dx\cosh^3(a+bx)}{3b} + \frac{6c^2d^2x^2\sinh^2(a+bx)\cosh}{b} \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sinh^3(a) \end{cases}$$

input
```
integrate((d*x+c)**4*sinh(b*x+a)**3,x)
```

output
```
Piecewise((c**4*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c**4*cosh(a + b*x)**3
/(3*b) + 4*c**3*d*x*sinh(a + b*x)**2*cosh(a + b*x)/b - 8*c**3*d*x*cosh(a +
 b*x)**3/(3*b) + 6*c**2*d**2*x**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 4*c**
2*d**2*x**2*cosh(a + b*x)**3/b + 4*c*d**3*x**3*sinh(a + b*x)**2*cosh(a + b
*x)/b - 8*c*d**3*x**3*cosh(a + b*x)**3/(3*b) + d**4*x**4*sinh(a + b*x)**2*
cosh(a + b*x)/b - 2*d**4*x**4*cosh(a + b*x)**3/(3*b) - 28*c**3*d*sinh(a +
b*x)**3/(9*b**2) + 8*c**3*d*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2) - 28*c
**2*d**2*x*sinh(a + b*x)**3/(3*b**2) + 8*c**2*d**2*x*sinh(a + b*x)*cosh(a
+ b*x)**2/b**2 - 28*c*d**3*x**2*sinh(a + b*x)**3/(3*b**2) + 8*c*d**3*x**2*
sinh(a + b*x)*cosh(a + b*x)**2/b**2 - 28*d**4*x**3*sinh(a + b*x)**3/(9*b**
2) + 8*d**4*x**3*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2) + 28*c**2*d**2*si
nh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 80*c**2*d**2*cosh(a + b*x)**3/(9*b
**3) + 56*c*d**3*x*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 160*c*d**3*x*
cosh(a + b*x)**3/(9*b**3) + 28*d**4*x**2*sinh(a + b*x)**2*cosh(a + b*x)/(3
*b**3) - 80*d**4*x**2*cosh(a + b*x)**3/(9*b**3) - 488*c*d**3*sinh(a + b*x)
**3/(27*b**4) + 160*c*d**3*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4) - 488*d
**4*x*sinh(a + b*x)**3/(27*b**4) + 160*d**4*x*sinh(a + b*x)*cosh(a + b*x)*
*2/(9*b**4) + 488*d**4*sinh(a + b*x)**2*cosh(a + b*x)/(27*b**5) - 1456*d**
4*cosh(a + b*x)**3/(81*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2
*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sinh(a)**3, True))
```

3.16.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(205) = 410$.

Time $= 0.22$ (sec) , antiderivative size $= 639$, normalized size of antiderivative $= 2.84$

$$\int (c + dx)^4 \sinh^3(a + bx)\, dx$$
$$= \frac{1}{18}\, c^3 d \left(\frac{\left(3\, bxe^{(3\,a)} - e^{(3\,a)}\right)e^{(3\,bx)}}{b^2} - \frac{27\,(bxe^a - e^a)e^{(bx)}}{b^2} - \frac{27\,(bx+1)e^{(-bx-a)}}{b^2} + \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^2} \right)$$
$$+ \frac{1}{24}\, c^4 \left(\frac{e^{(3\,bx+3\,a)}}{b} - \frac{9\,e^{(bx+a)}}{b} - \frac{9\,e^{(-bx-a)}}{b} + \frac{e^{(-3\,bx-3\,a)}}{b} \right)$$
$$+ \frac{1}{36}\, c^2 d^2 \left(\frac{\left(9\, b^2x^2e^{(3\,a)} - 6\, bxe^{(3\,a)} + 2\, e^{(3\,a)}\right)e^{(3\,bx)}}{b^3} - \frac{81\,(b^2x^2e^a - 2\, bxe^a + 2\, e^a)e^{(bx)}}{b^3} - \frac{81\,(b^2x^2 + 2\, bx +}{b^3}\right.$$
$$+ \frac{1}{54}\, cd^3 \left(\frac{\left(9\, b^3x^3e^{(3\,a)} - 9\, b^2x^2e^{(3\,a)} + 6\, bxe^{(3\,a)} - 2\, e^{(3\,a)}\right)e^{(3\,bx)}}{b^4} - \frac{81\,(b^3x^3e^a - 3\, b^2x^2e^a + 6\, bxe^a - 6\, e^a)e^{(}}{b^4}\right.$$
$$+ \frac{1}{648}\, d^4 \left(\frac{\left(27\, b^4x^4e^{(3\,a)} - 36\, b^3x^3e^{(3\,a)} + 36\, b^2x^2e^{(3\,a)} - 24\, bxe^{(3\,a)} + 8\, e^{(3\,a)}\right)e^{(3\,bx)}}{b^5} - \frac{243\,(b^4x^4e^a - 4\, b^3x^3e}{}\right.$$

input
```
integrate((d*x+c)^4*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/18*c^3*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 27*(b*x*e^a - e^a)*e
^(b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 + (3*b*x + 1)*e^(-3*b*x - 3*a)/
b^2) + 1/24*c^4*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b +
e^(-3*b*x - 3*a)/b) + 1/36*c^2*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2
*e^(3*a))*e^(3*b*x)/b^3 - 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3
 - 81*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 + (9*b^2*x^2 + 6*b*x + 2)*e^(
-3*b*x - 3*a)/b^3) + 1/54*c*d^3*((9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) +
6*b*x*e^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 - 81*(b^3*x^3*e^a - 3*b^2*x^2*e^a
 + 6*b*x*e^a - 6*e^a)*e^(b*x)/b^4 - 81*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e
^(-b*x - a)/b^4 + (9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
) + 1/648*d^4*((27*b^4*x^4*e^(3*a) - 36*b^3*x^3*e^(3*a) + 36*b^2*x^2*e^(3*
a) - 24*b*x*e^(3*a) + 8*e^(3*a))*e^(3*b*x)/b^5 - 243*(b^4*x^4*e^a - 4*b^3*
x^3*e^a + 12*b^2*x^2*e^a - 24*b*x*e^a + 24*e^a)*e^(b*x)/b^5 - 243*(b^4*x^4
 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*e^(-b*x - a)/b^5 + (27*b^4*x^4 +
36*b^3*x^3 + 36*b^2*x^2 + 24*b*x + 8)*e^(-3*b*x - 3*a)/b^5)
```

3.16.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(205) = 410$.

Time = 0.27 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.91

$$\int (c+dx)^4 \sinh^3(a+bx)\,dx$$
$$= \frac{(27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 162\,b^4c^2d^2x^2 - 36\,b^3d^4x^3 + 108\,b^4c^3dx - 108\,b^3cd^3x^2 + 27\,b^4c^4 - 108\,b^3c^2d^2x +}{648\,b^5}$$
$$- \frac{3\,(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 - 4\,b^3d^4x^3 + 4\,b^4c^3dx - 12\,b^3cd^3x^2 + b^4c^4 - 12\,b^3c^2d^2x + 12\,b^2d^4x^2 -}{8\,b^5}$$
$$- \frac{3\,(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 + 4\,b^3d^4x^3 + 4\,b^4c^3dx + 12\,b^3cd^3x^2 + b^4c^4 + 12\,b^3c^2d^2x + 12\,b^2d^4x^2 +}{8\,b^5}$$
$$+ \frac{(27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 162\,b^4c^2d^2x^2 + 36\,b^3d^4x^3 + 108\,b^4c^3dx + 108\,b^3cd^3x^2 + 27\,b^4c^4 + 108\,b^3c^2d^2x}{648\,b^5}$$

input
```
integrate((d*x+c)^4*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 - 36*b^3*d
^4*x^3 + 108*b^4*c^3*d*x - 108*b^3*c*d^3*x^2 + 27*b^4*c^4 - 108*b^3*c^2*d^
2*x + 36*b^2*d^4*x^2 - 36*b^3*c^3*d + 72*b^2*c*d^3*x + 36*b^2*c^2*d^2 - 24
*b*d^4*x - 24*b*c*d^3 + 8*d^4)*e^(3*b*x + 3*a)/b^5 - 3/8*(b^4*d^4*x^4 + 4*
b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x - 12*b^3
*c*d^3*x^2 + b^4*c^4 - 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^2 - 4*b^3*c^3*d + 2
4*b^2*c*d^3*x + 12*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*d^3 + 24*d^4)*e^(b*x
+ a)/b^5 - 3/8*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^3*
d^4*x^3 + 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4 + 12*b^3*c^2*d^2*x +
12*b^2*d^4*x^2 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 + 24*b*d^4*
x + 24*b*c*d^3 + 24*d^4)*e^(-b*x - a)/b^5 + 1/648*(27*b^4*d^4*x^4 + 108*b^
4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 36*b^3*d^4*x^3 + 108*b^4*c^3*d*x + 108
*b^3*c*d^3*x^2 + 27*b^4*c^4 + 108*b^3*c^2*d^2*x + 36*b^2*d^4*x^2 + 36*b^3*
c^3*d + 72*b^2*c*d^3*x + 36*b^2*c^2*d^2 + 24*b*d^4*x + 24*b*c*d^3 + 8*d^4)
*e^(-3*b*x - 3*a)/b^5
```

3.16. $\int (c+dx)^4 \sinh^3(a+bx)\,dx$

3.16.9 Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.36

$$\int (c+dx)^4\sinh^3(a+bx)\,dx = \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,(27\,b^4\,c^4+252\,b^2\,c^2\,d^2+488\,d^4)}{27\,b^5} - \frac{2\cosh(a+b\,x)^3\,(27\,b^4\,c^4+360\,b^2\,c^2\,d^2+728\,d^4)}{81\,b^5} - \frac{4\sinh(a+b\,x)^3\,(21\,b^2\,c^3\,d+122\,c\,d^3)}{27\,b^4} + \frac{8\cosh(a+b\,x)^2\sinh(a+b\,x)\,(3\,b^2\,c^3\,d+20\,c\,d^3)}{9\,b^4} - \frac{2\,d^4\,x^4\cosh(a+b\,x)^3}{3\,b} - \frac{8\,x\cosh(a+b\,x)^3\,(3\,b^2\,c^3\,d+20\,c\,d^3)}{9\,b^3} - \frac{28\,d^4\,x^3\sinh(a+b\,x)^3}{9\,b^2} - \frac{4\,x\sinh(a+b\,x)^3\,(63\,b^2\,c^2\,d^2+122\,d^4)}{27\,b^4} - \frac{4\,x^2\cosh(a+b\,x)^3\,(9\,b^2\,c^2\,d^2+20\,d^4)}{9\,b^3} + \frac{2\,x^2\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,(9\,b^2\,c^2\,d^2+14\,d^4)}{3\,b^3} - \frac{8\,c\,d^3\,x^3\cosh(a+b\,x)^3}{3\,b} + \frac{d^4\,x^4\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b} + \frac{8\,d^4\,x^3\cosh(a+b\,x)^2\sinh(a+b\,x)}{3\,b^2} - \frac{28\,c\,d^3\,x^2\sinh(a+b\,x)^3}{3\,b^2} + \frac{8\,x\cosh(a+b\,x)^2\sinh(a+b\,x)\,(9\,b^2\,c^2\,d^2+20\,d^4)}{9\,b^4} + \frac{4\,x\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,(3\,b^2\,c^3\,d+14\,c\,d^3)}{3\,b^3} + \frac{4\,c\,d^3\,x^3\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b} + \frac{8\,c\,d^3\,x^2\cosh(a+b\,x)^2\sinh(a+b\,x)}{b^2}$$

input
```
int(sinh(a + b*x)^3*(c + d*x)^4,x)
```

output
```
(cosh(a + b*x)*sinh(a + b*x)^2*(488*d^4 + 27*b^4*c^4 + 252*b^2*c^2*d^2))/(
27*b^5) - (2*cosh(a + b*x)^3*(728*d^4 + 27*b^4*c^4 + 360*b^2*c^2*d^2))/(81
*b^5) - (4*sinh(a + b*x)^3*(122*c*d^3 + 21*b^2*c^3*d))/(27*b^4) + (8*cosh(
a + b*x)^2*sinh(a + b*x)*(20*c*d^3 + 3*b^2*c^3*d))/(9*b^4) - (2*d^4*x^4*co
sh(a + b*x)^3)/(3*b) - (8*x*cosh(a + b*x)^3*(20*c*d^3 + 3*b^2*c^3*d))/(9*b
^3) - (28*d^4*x^3*sinh(a + b*x)^3)/(9*b^2) - (4*x*sinh(a + b*x)^3*(122*d^4
 + 63*b^2*c^2*d^2))/(27*b^4) - (4*x^2*cosh(a + b*x)^3*(20*d^4 + 9*b^2*c^2*
d^2))/(9*b^3) + (2*x^2*cosh(a + b*x)*sinh(a + b*x)^2*(14*d^4 + 9*b^2*c^2*d
^2))/(3*b^3) - (8*c*d^3*x^3*cosh(a + b*x)^3)/(3*b) + (d^4*x^4*cosh(a + b*x
)*sinh(a + b*x)^2)/b + (8*d^4*x^3*cosh(a + b*x)^2*sinh(a + b*x))/(3*b^2) -
 (28*c*d^3*x^2*sinh(a + b*x)^3)/(3*b^2) + (8*x*cosh(a + b*x)^2*sinh(a + b*
x)*(20*d^4 + 9*b^2*c^2*d^2))/(9*b^4) + (4*x*cosh(a + b*x)*sinh(a + b*x)^2*
(14*c*d^3 + 3*b^2*c^3*d))/(3*b^3) + (4*c*d^3*x^3*cosh(a + b*x)*sinh(a + b*
x)^2)/b + (8*c*d^3*x^2*cosh(a + b*x)^2*sinh(a + b*x))/b^2
```

3.16. $\int (c+dx)^4 \sinh^3(a+bx)\,dx$

3.17 $\int (c+dx)^3 \sinh^3(a+bx)\,dx$

3.17.1 Optimal result

Integrand size = 16, antiderivative size = 175

$$\begin{aligned}\int (c+dx)^3 \sinh^3(a+bx)\,dx = & -\frac{40d^2(c+dx)\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^3\cosh(a+bx)}{3b} \\ & + \frac{40d^3\sinh(a+bx)}{9b^4} + \frac{2d(c+dx)^2\sinh(a+bx)}{b^2} \\ & + \frac{2d^2(c+dx)\cosh(a+bx)\sinh^2(a+bx)}{9b^3} \\ & + \frac{(c+dx)^3\cosh(a+bx)\sinh^2(a+bx)}{3b} \\ & - \frac{2d^3\sinh^3(a+bx)}{27b^4} - \frac{d(c+dx)^2\sinh^3(a+bx)}{3b^2}\end{aligned}$$

output
```
-40/9*d^2*(d*x+c)*cosh(b*x+a)/b^3-2/3*(d*x+c)^3*cosh(b*x+a)/b+40/9*d^3*sin
h(b*x+a)/b^4+2*d*(d*x+c)^2*sinh(b*x+a)/b^2+2/9*d^2*(d*x+c)*cosh(b*x+a)*sin
h(b*x+a)^2/b^3+1/3*(d*x+c)^3*cosh(b*x+a)*sinh(b*x+a)^2/b-2/27*d^3*sinh(b*x
+a)^3/b^4-1/3*d*(d*x+c)^2*sinh(b*x+a)^3/b^2
```

3.17.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.73

$$\int (c+dx)^3 \sinh^3(a+bx)\,dx$$
$$= \frac{-162b(c+dx)\left(6d^2+b^2(c+dx)^2\right)\cosh(a+bx)+6b(c+dx)\left(2d^2+3b^2(c+dx)^2\right)\cosh(3(a+bx))-4d(\cdots}{216b^4}$$

input
```
Integrate[(c + d*x)^3*Sinh[a + b*x]^3,x]
```

output
```
(-162*b*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] + 6*b*(c + d*x)*
(2*d^2 + 3*b^2*(c + d*x)^2)*Cosh[3*(a + b*x)] - 4*d*(-242*d^2 - 117*b^2*(c
 + d*x)^2 + (2*d^2 + 9*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/
(216*b^4)
```

3.17.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.41, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.375, Rules used = {3042, 26, 3792, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 3791, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sinh^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int i(c+dx)^3 \sin(ia+ibx)^3 dx$$
$$\Big\downarrow \text{26}$$
$$i\int (c+dx)^3 \sin(ia+ibx)^3 dx$$
$$\Big\downarrow \text{3792}$$
$$i\left(\frac{2d^2\int -i(c+dx)\sinh^3(a+bx)dx}{3b^2}+\frac{2}{3}\int i(c+dx)^3\sinh(a+bx)dx+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}-\frac{i(c+dx)^3\sin\cdots}{\cdots}\right.$$

$$\Big\downarrow \; 26$$

$$i\left(-\frac{2id^2\int (c+dx)\sinh^3(a+bx)dx}{3b^2}+\frac{2}{3}i\int (c+dx)^3\sinh(a+bx)dx+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}-\frac{i(c+dx)^3\sin\right.$$

$$\Big\downarrow \; 3042$$

$$i\left(-\frac{2id^2\int i(c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}i\int -i(c+dx)^3\sin(ia+ibx)dx+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}-\frac{i(c+dx\right.$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{2d^2\int (c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\int (c+dx)^3\sin(ia+ibx)dx+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}-\frac{i(c+dx)^3\sinh^2\right.$$

$$\Big\downarrow \; 3777$$

$$i\left(\frac{2d^2\int (c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\int (c+dx)^2\cosh(a+bx)dx}{b}\right)+\frac{id(c+dx)^2}{3}\right.$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{2d^2\int (c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\int (c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)+\frac{id(c+\right.$$

$$\Big\downarrow \; 3777$$

$$i\left(\frac{2d^2\int (c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2id\int -i(c+dx)\sinh(a+bx)dx}{b}\right.}{b}\right.\right.$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{2d^2\int (c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int (c+dx)\sinh(a+bx)dx}{b}\right)}{b}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{2d^2\int (c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int -i(c+dx)\sin(ia+ibx)dx}{b}\right.}{b}\right.\right.$$

$$\Big\downarrow \; 26$$

3.17. $\int (c+dx)^3\sinh^3(a+bx)\,dx$

$$i\left(\frac{2d^2\int(c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\int(c+dx)\sin(ia+ibx)dx}{b}\right)}{b}\right.\right.$$

$$\Big\downarrow \text{3777}$$

$$i\left(\frac{2d^2\int(c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int}{}\right.}{b}\right.}{b}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{2d^2\int(c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id}{}\right.}{b}\right.}{b}\right.\right.$$

$$\Big\downarrow \text{3117}$$

$$i\left(\frac{2d^2\int(c+dx)\sin(ia+ibx)^3dx}{3b^2}+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a}{b}\right.}{}\right.\right.$$

$$\Big\downarrow \text{3791}$$

$$i\left(\frac{2d^2\left(\frac{2}{3}\int i(c+dx)\sinh(a+bx)dx+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}+\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{2d^2\left(\frac{2}{3}i\int(c+dx)\sinh(a+bx)dx+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}+\right.$$

3.17. $\int(c+dx)^3\sinh^3(a+bx)\,dx$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{2d^2\left(\frac{2}{3}i\int -i(c+dx)\sin(ia+ibx)dx+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{2d^2\left(\frac{2}{3}\int (c+dx)\sin(ia+ibx)dx+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}+\frac{id(c+dx)^2\sinh^3(a+bx)}{3b^2}+\frac{2}{3}\right.$$

$$\Big\downarrow \text{3777}$$

$$i\left(\frac{2d^2\left(\frac{2}{3}\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\cosh(a+bx)dx}{b}\right)+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}+\frac{id(c+dx)^2\sinh^3(a}{3b^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{2d^2\left(\frac{2}{3}\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}+\frac{id(c+dx)^2\sinh}{3b^2}\right.$$

$$\Big\downarrow \text{3117}$$

$$i\left(\frac{2d^2\left(\frac{2}{3}\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)}{3b^2}+\frac{id(c+dx)^2\sinh^3(a+}{3b^2}\right.$$

input
```
Int[(c + d*x)^3*Sinh[a + b*x]^3,x]
```

3.17. $\int (c+dx)^3\sinh^3(a+bx)\,dx$

output
```
I*(((-1/3*I)*(c + d*x)^3*Cosh[a + b*x]*Sinh[a + b*x]^2)/b + ((I/3)*d*(c +
d*x)^2*Sinh[a + b*x]^3)/b^2 + (2*d^2*(((-1/3*I)*(c + d*x)*Cosh[a + b*x]*Si
nh[a + b*x]^2)/b + ((I/9)*d*Sinh[a + b*x]^3)/b^2 + (2*((I*(c + d*x)*Cosh[a
 + b*x])/b - (I*d*Sinh[a + b*x])/b^2))/3))/(3*b^2) + (2*((I*(c + d*x)^3*Co
sh[a + b*x])/b - ((3*I)*d*(((c + d*x)^2*Sinh[a + b*x])/b + ((2*I)*d*((I*(c
 + d*x)*Cosh[a + b*x])/b - (I*d*Sinh[a + b*x])/b^2))/b))/b))/3)
```

3.17.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.17.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{3\left((dx+c)^2b^2+\frac{2d^2}{3}\right)b(dx+c)\cosh(3bx+3a)-3d\left((dx+c)^2b^2+\frac{2d^2}{9}\right)\sinh(3bx+3a)-27\left((dx+c)^2b^2+6d^2\right)b(dx+c)\cosh(b}{36b^4}$
risch	$\frac{(9d^3x^3b^3+27b^3c\,d^2x^2+27b^3c^2dx-9b^2d^3x^2+9b^3c^3-18b^2c\,d^2x-9b^2c^2d+6b\,d^3x+6bc\,d^2-2d^3)\,\mathrm{e}^{3bx+3a}}{216b^4}-\frac{3(d^3x^3b^3+3b}{}$
derivativedivides	$\frac{\frac{d^3\left(-\frac{2(bx+a)^3\cosh(bx+a)}{3}+\frac{(bx+a)^3\cosh(bx+a)\sinh(bx+a)^2}{3}+2(bx+a)^2\sinh(bx+a)-\frac{40(bx+a)\cosh(bx+a)}{9}+\frac{40\sinh(bx+a)}{9}-\frac{(bx-}{}\right.}{b^3}}{}$
default	$\frac{\frac{d^3\left(-\frac{2(bx+a)^3\cosh(bx+a)}{3}+\frac{(bx+a)^3\cosh(bx+a)\sinh(bx+a)^2}{3}+2(bx+a)^2\sinh(bx+a)-\frac{40(bx+a)\cosh(bx+a)}{9}+\frac{40\sinh(bx+a)}{9}-\frac{(bx-}{}\right.}{b^3}}{}$

input
```
int((d*x+c)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/36*(3*((d*x+c)^2*b^2+2/3*d^2)*b*(d*x+c)*cosh(3*b*x+3*a)-3*d*((d*x+c)^2*b
^2+2/9*d^2)*sinh(3*b*x+3*a)-27*((d*x+c)^2*b^2+6*d^2)*b*(d*x+c)*cosh(b*x+a)
+81*((d*x+c)^2*b^2+2*d^2)*d*sinh(b*x+a)-24*b^3*c^3-160*b*c*d^2)/b^4
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(161) = 322$.

Time = 0.24 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.97

$$\int (c+dx)^3\sinh^3(a+bx)\,dx$$
$$=\frac{3\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\,b^3c^3+2\,bcd^2+(9\,b^3c^2d+2\,bd^3)x\right)\cosh\left(bx+a\right)^3+9\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\right.}{}$$

input
```
integrate((d*x+c)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/108*(3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 + 2*b*c*d^2 + (9*b^3
*c^2*d + 2*b*d^3)*x)*cosh(b*x + a)^3 + 9*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2
+ 3*b^3*c^3 + 2*b*c*d^2 + (9*b^3*c^2*d + 2*b*d^3)*x)*cosh(b*x + a)*sinh(b*
x + a)^2 - (9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d + 2*d^3)*sinh(b*x
 + a)^3 - 81*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 6*b*c*d^2 + 3*(b^3
*c^2*d + 2*b*d^3)*x)*cosh(b*x + a) + 3*(81*b^2*d^3*x^2 + 162*b^2*c*d^2*x +
 81*b^2*c^2*d + 162*d^3 - (9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d +
2*d^3)*cosh(b*x + a)^2)*sinh(b*x + a))/b^4
```

3.17.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(173) = 346$.

Time = 0.50 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.83

$$\int (c+dx)^3 \sinh^3(a+bx)\,dx$$
$$= \begin{cases} \frac{c^3\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2c^3\cosh^3(a+bx)}{3b} + \frac{3c^2dx\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2c^2dx\cosh^3(a+bx)}{b} + \frac{3cd^2x^2\sinh^2(a+bx)\cosh(}{b} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sinh^3(a) \end{cases}$$

input
```
integrate((d*x+c)**3*sinh(b*x+a)**3,x)
```

output
```
Piecewise((c**3*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c**3*cosh(a + b*x)**3
/(3*b) + 3*c**2*d*x*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c**2*d*x*cosh(a +
 b*x)**3/b + 3*c*d**2*x**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c*d**2*x**
2*cosh(a + b*x)**3/b + d**3*x**3*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*d**3
*x**3*cosh(a + b*x)**3/(3*b) - 7*c**2*d*sinh(a + b*x)**3/(3*b**2) + 2*c**2
*d*sinh(a + b*x)*cosh(a + b*x)**2/b**2 - 14*c*d**2*x*sinh(a + b*x)**3/(3*b
**2) + 4*c*d**2*x*sinh(a + b*x)*cosh(a + b*x)**2/b**2 - 7*d**3*x**2*sinh(a
 + b*x)**3/(3*b**2) + 2*d**3*x**2*sinh(a + b*x)*cosh(a + b*x)**2/b**2 + 14
*c*d**2*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 40*c*d**2*cosh(a + b*x)*
*3/(9*b**3) + 14*d**3*x*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 40*d**3*
x*cosh(a + b*x)**3/(9*b**3) - 122*d**3*sinh(a + b*x)**3/(27*b**4) + 40*d**
3*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*
x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sinh(a)**3, True))
```

3.17.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(161) = 322$.

Time = 0.21 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.49

$$\int (c+dx)^3 \sinh^3(a+bx)\,dx$$
$$= \frac{1}{24}c^2d\left(\frac{(3\,bxe^{(3\,a)}-e^{(3\,a)})e^{(3\,bx)}}{b^2}-\frac{27\,(bxe^a-e^a)e^{(bx)}}{b^2}-\frac{27\,(bx+1)e^{(-bx-a)}}{b^2}+\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^2}\right)$$
$$+\frac{1}{24}c^3\left(\frac{e^{(3\,bx+3\,a)}}{b}-\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}+\frac{e^{(-3\,bx-3\,a)}}{b}\right)$$
$$+\frac{1}{72}cd^2\left(\frac{(9\,b^2x^2e^{(3\,a)}-6\,bxe^{(3\,a)}+2\,e^{(3\,a)})e^{(3\,bx)}}{b^3}-\frac{81\,(b^2x^2e^a-2\,bxe^a+2\,e^a)e^{(bx)}}{b^3}-\frac{81\,(b^2x^2+2\,bx+2}{b^3}\right.$$
$$+\frac{1}{216}d^3\left(\frac{(9\,b^3x^3e^{(3\,a)}-9\,b^2x^2e^{(3\,a)}+6\,bxe^{(3\,a)}-2\,e^{(3\,a)})e^{(3\,bx)}}{b^4}-\frac{81\,(b^3x^3e^a-3\,b^2x^2e^a+6\,bxe^a-6\,e^a)e^{(}}{b^4}\right.$$

input
```
integrate((d*x+c)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/24*c^2*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 27*(b*x*e^a - e^a)*e
^(b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 + (3*b*x + 1)*e^(-3*b*x - 3*a)/
b^2) + 1/24*c^3*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b +
e^(-3*b*x - 3*a)/b) + 1/72*c*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e
^(3*a))*e^(3*b*x)/b^3 - 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 -
 81*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 + (9*b^2*x^2 + 6*b*x + 2)*e^(-3
*b*x - 3*a)/b^3) + 1/216*d^3*((9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b
*x*e^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 - 81*(b^3*x^3*e^a - 3*b^2*x^2*e^a +
6*b*x*e^a - 6*e^a)*e^(b*x)/b^4 - 81*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-
b*x - a)/b^4 + (9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4)
```

3.17.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(161) = 322$.

Time = 0.30 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.37

$$
\begin{aligned}
&\int (c+dx)^3 \sinh^3(a+bx)\,dx \\
&= \frac{(9\,b^3d^3x^3 + 27\,b^3cd^2x^2 + 27\,b^3c^2dx - 9\,b^2d^3x^2 + 9\,b^3c^3 - 18\,b^2cd^2x - 9\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 - 2\,d^3)e^{(3\,bx}}{216\,b^4} \\
&\quad - \frac{3\,(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx - 3\,b^2d^3x^2 + b^3c^3 - 6\,b^2cd^2x - 3\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 - 6\,d^3)e^{(bx+a)}}{8\,b^4} \\
&\quad - \frac{3\,(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + 3\,b^2d^3x^2 + b^3c^3 + 6\,b^2cd^2x + 3\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 + 6\,d^3)e^{(-bx-a}}{8\,b^4} \\
&\quad + \frac{(9\,b^3d^3x^3 + 27\,b^3cd^2x^2 + 27\,b^3c^2dx + 9\,b^2d^3x^2 + 9\,b^3c^3 + 18\,b^2cd^2x + 9\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 + 2\,d^3)e^{(}}{216\,b^4}
\end{aligned}
$$

input
```
integrate((d*x+c)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 27*b^3*c^2*d*x - 9*b^2*d^3*x^2 +
 9*b^3*c^3 - 18*b^2*c*d^2*x - 9*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 2*d^3)
*e^(3*b*x + 3*a)/b^4 - 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
- 3*b^2*d^3*x^2 + b^3*c^3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*
c*d^2 - 6*d^3)*e^(b*x + a)/b^4 - 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + 3*b^2*d^3*x^2 + b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^
3*x + 6*b*c*d^2 + 6*d^3)*e^(-b*x - a)/b^4 + 1/216*(9*b^3*d^3*x^3 + 27*b^3*
c*d^2*x^2 + 27*b^3*c^2*d*x + 9*b^2*d^3*x^2 + 9*b^3*c^3 + 18*b^2*c*d^2*x +
9*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 2*d^3)*e^(-3*b*x - 3*a)/b^4
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.08

$$\int (c+dx)^3 \sinh^3(a+bx)\,dx = \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,(3\,b^2\,c^3+14\,c\,d^2)}{3\,b^3} - \frac{\sinh(a+b\,x)^3\,(63\,b^2\,c^2\,d+122\,d^3)}{27\,b^4} - \frac{2\cosh(a+b\,x)^3\,(3\,b^2\,c^3+20\,c\,d^2)}{9\,b^3} + \frac{2\cosh(a+b\,x)^2\sinh(a+b\,x)\,(9\,b^2\,c^2\,d+20\,d^3)}{9\,b^4} - \frac{2\,x\cosh(a+b\,x)^3\,(9\,b^2\,c^2\,d+20\,d^3)}{9\,b^3} - \frac{2\,d^3\,x^3\cosh(a+b\,x)^3}{3\,b} - \frac{7\,d^3\,x^2\sinh(a+b\,x)^3}{3\,b^2} - \frac{14\,c\,d^2\,x\sinh(a+b\,x)^3}{3\,b^2} + \frac{x\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,(9\,b^2\,c^2\,d+14\,d^3)}{3\,b^3} - \frac{2\,c\,d^2\,x^2\cosh(a+b\,x)^3}{b} + \frac{d^3\,x^3\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b} + \frac{2\,d^3\,x^2\cosh(a+b\,x)^2\sinh(a+b\,x)}{b^2} + \frac{3\,c\,d^2\,x^2\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b} + \frac{4\,c\,d^2\,x\cosh(a+b\,x)^2\sinh(a+b\,x)}{b^2}$$

input
```
int(sinh(a + b*x)^3*(c + d*x)^3,x)
```

output
```
(cosh(a + b*x)*sinh(a + b*x)^2*(14*c*d^2 + 3*b^2*c^3))/(3*b^3) - (sinh(a +
 b*x)^3*(122*d^3 + 63*b^2*c^2*d))/(27*b^4) - (2*cosh(a + b*x)^3*(20*c*d^2
+ 3*b^2*c^3))/(9*b^3) + (2*cosh(a + b*x)^2*sinh(a + b*x)*(20*d^3 + 9*b^2*c
^2*d))/(9*b^4) - (2*x*cosh(a + b*x)^3*(20*d^3 + 9*b^2*c^2*d))/(9*b^3) - (2
*d^3*x^3*cosh(a + b*x)^3)/(3*b) - (7*d^3*x^2*sinh(a + b*x)^3)/(3*b^2) - (1
4*c*d^2*x*sinh(a + b*x)^3)/(3*b^2) + (x*cosh(a + b*x)*sinh(a + b*x)^2*(14*
d^3 + 9*b^2*c^2*d))/(3*b^3) - (2*c*d^2*x^2*cosh(a + b*x)^3)/b + (d^3*x^3*c
osh(a + b*x)*sinh(a + b*x)^2)/b + (2*d^3*x^2*cosh(a + b*x)^2*sinh(a + b*x)
)/b^2 + (3*c*d^2*x^2*cosh(a + b*x)*sinh(a + b*x)^2)/b + (4*c*d^2*x*cosh(a
+ b*x)^2*sinh(a + b*x))/b^2
```

3.18 $\int (c+dx)^2 \sinh^3(a+bx)\,dx$

3.18.1 Optimal result

Integrand size = 16, antiderivative size = 123

$$\begin{aligned}\int (c+dx)^2 \sinh^3(a+bx)\,dx = &-\frac{14d^2\cosh(a+bx)}{9b^3}-\frac{2(c+dx)^2\cosh(a+bx)}{3b}\\&+\frac{2d^2\cosh^3(a+bx)}{27b^3}+\frac{4d(c+dx)\sinh(a+bx)}{3b^2}\\&+\frac{(c+dx)^2\cosh(a+bx)\sinh^2(a+bx)}{3b}\\&-\frac{2d(c+dx)\sinh^3(a+bx)}{9b^2}\end{aligned}$$

output
```
-14/9*d^2*cosh(b*x+a)/b^3-2/3*(d*x+c)^2*cosh(b*x+a)/b+2/27*d^2*cosh(b*x+a)
^3/b^3+4/3*d*(d*x+c)*sinh(b*x+a)/b^2+1/3*(d*x+c)^2*cosh(b*x+a)*sinh(b*x+a)
^2/b-2/9*d*(d*x+c)*sinh(b*x+a)^3/b^2
```

3.18.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

$$\int (c+dx)^2 \sinh^3(a+bx)\,dx$$
$$=\frac{-81(2d^2+b^2(c+dx)^2)\cosh(a+bx)+(2d^2+9b^2(c+dx)^2)\cosh(3(a+bx))-6bd(c+dx)(-27\sinh(a+}{108b^3}$$

input
```
Integrate[(c + d*x)^2*Sinh[a + b*x]^3,x]
```

output
```
(-81*(2*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] + (2*d^2 + 9*b^2*(c + d*x)^2)
*Cosh[3*(a + b*x)] - 6*b*d*(c + d*x)*(-27*Sinh[a + b*x] + Sinh[3*(a + b*x)
]))/(108*b^3)
```

3.18.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.22, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {3042, 26, 3792, 26, 3042, 26, 3113, 2009, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sinh^3(a+bx) \, dx$$

$$\Big\downarrow \ 3042$$

$$\int i(c+dx)^2 \sin(ia+ibx)^3 dx$$

$$\Big\downarrow \ 26$$

$$i \int (c+dx)^2 \sin(ia+ibx)^3 dx$$

$$\Big\downarrow \ 3792$$

$$i\left(\frac{2d^2 \int -i \sinh^3(a+bx)dx}{9b^2} + \frac{2}{3}\int i(c+dx)^2 \sinh(a+bx)dx + \frac{2id(c+dx)\sinh^3(a+bx)}{9b^2} - \frac{i(c+dx)^2 \sinh^2(a+\right.$$

$$\Big\downarrow \ 26$$

$$i\left(-\frac{2id^2 \int \sinh^3(a+bx)dx}{9b^2} + \frac{2}{3}i\int (c+dx)^2 \sinh(a+bx)dx + \frac{2id(c+dx)\sinh^3(a+bx)}{9b^2} - \frac{i(c+dx)^2 \sinh^2(a+\right.$$

$$\Big\downarrow \ 3042$$

$$i\left(-\frac{2id^2 \int i \sin(ia+ibx)^3 dx}{9b^2} + \frac{2}{3}i\int -i(c+dx)^2 \sin(ia+ibx)dx + \frac{2id(c+dx)\sinh^3(a+bx)}{9b^2} - \frac{i(c+dx)^2 \sinh^2(\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{2d^2\int\sin(ia+ibx)^3dx}{9b^2}+\frac{2}{3}\int(c+dx)^2\sin(ia+ibx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)^2\sinh^2(a+bx)}{3b}\right.$$

$$\Big\downarrow \text{3113}$$

$$i\left(\frac{2id^2\int\left(1-\cosh^2(a+bx)\right)d\cosh(a+bx)}{9b^3}+\frac{2}{3}\int(c+dx)^2\sin(ia+ibx)dx+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+\right.$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{2}{3}\int(c+dx)^2\sin(ia+ibx)dx+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)}{}\right.$$

$$\Big\downarrow \text{3777}$$

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int(c+dx)\cosh(a+bx)dx}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\right.$$

$$\Big\downarrow \text{3777}$$

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{id\int-i\sinh(a+bx)dx}{b}\right)}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+\right.}{9b^3}\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int\sinh(a+bx)dx}{b}\right)}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right.}{9b^3}\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int-i\sin(ia+ibx)dx}{b}\right)}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+\right.}{9b^3}\right.$$

3.18. $\int(c+dx)^2\sinh^3(a+bx)\,dx$

$$\Big\downarrow \; 26$$

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}+\frac{id\int\sin(ia+ibx)dx}{b}\right)}{b}\right)+\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx\right.}{9b^3}\right.$$

$$\Big\downarrow \; 3118$$

$$i\left(\frac{2id^2\left(\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)\right)}{9b^3}+\frac{2id(c+dx)\sinh^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)}{}\right.}{}\right.\right.$$

input
```
Int[(c + d*x)^2*Sinh[a + b*x]^3,x]
```

output
```
I*((((2*I)/9)*d^2*(Cosh[a + b*x] - Cosh[a + b*x]^3/3))/b^3 - ((I/3)*(c + d
*x)^2*Cosh[a + b*x]*Sinh[a + b*x]^2)/b + (((2*I)/9)*d*(c + d*x)*Sinh[a + b
*x]^3)/b^2 + (2*((I*(c + d*x)^2*Cosh[a + b*x])/b - ((2*I)*d*(-((d*Cosh[a +
 b*x])/b^2) + ((c + d*x)*Sinh[a + b*x])/b))/b))/3)
```

3.18.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.18.4 Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.86

method	result
parallelrisch	$\frac{\left(9(dx+c)^2b^2+2d^2\right)\cosh(3bx+3a)-6bd(dx+c)\sinh(3bx+3a)+\left(-81(dx+c)^2b^2-162d^2\right)\cosh(bx+a)+162bd(dx+c)\sinh}{108b^3}$
risch	$\frac{(9b^2d^2x^2+18b^2cdx+9b^2c^2-6b\,d^2x-6bcd+2d^2)\mathrm{e}^{3bx+3a}}{216b^3}-\frac{3(b^2d^2x^2+2b^2cdx+b^2c^2-2b\,d^2x-2bcd+2d^2)\mathrm{e}^{bx+a}}{8b^3}-\frac{3(b}{}$
derivativedivides	$\frac{d^2\left(-\frac{2(bx+a)^2\cosh(bx+a)}{3}+\frac{(bx+a)^2\cosh(bx+a)\sinh(bx+a)^2}{3}+\frac{4(bx+a)\sinh(bx+a)}{3}-\frac{40\cosh(bx+a)}{27}-\frac{2(bx+a)\sinh(bx+a)^3}{9}+\frac{2\cos}{}\right.}{b^2}$
default	$\frac{d^2\left(-\frac{2(bx+a)^2\cosh(bx+a)}{3}+\frac{(bx+a)^2\cosh(bx+a)\sinh(bx+a)^2}{3}+\frac{4(bx+a)\sinh(bx+a)}{3}-\frac{40\cosh(bx+a)}{27}-\frac{2(bx+a)\sinh(bx+a)^3}{9}+\frac{2\cos}{}\right.}{b^2}$

input
```
int((d*x+c)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/108*((9*(d*x+c)^2*b^2+2*d^2)*cosh(3*b*x+3*a)-6*b*d*(d*x+c)*sinh(3*b*x+3*
a)+(-81*(d*x+c)^2*b^2-162*d^2)*cosh(b*x+a)+162*b*d*(d*x+c)*sinh(b*x+a)-72*
b^2*c^2-160*d^2)/b^3
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.62

$$\int (c+dx)^2 \sinh^3(a+bx)\,dx$$
$$= \frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2+2\,d^2)\cosh{(bx+a)}^3+3\,(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2+2\,d^2)\cosh{(bx+a)}}{}$$

input
```
integrate((d*x+c)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/108*((9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 2*d^2)*cosh(b*x + a)^3
+ 3*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 2*d^2)*cosh(b*x + a)*sinh(
b*x + a)^2 - 6*(b*d^2*x + b*c*d)*sinh(b*x + a)^3 - 81*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2 + 2*d^2)*cosh(b*x + a) + 18*(9*b*d^2*x + 9*b*c*d - (b*d^2
*x + b*c*d)*cosh(b*x + a)^2)*sinh(b*x + a))/b^3
```

3.18.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(121) = 242$.

Time = 0.36 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.31

$$\int (c+dx)^2 \sinh^3(a+bx)\,dx$$
$$= \begin{cases} \frac{c^2\sinh^2{(a+bx)}\cosh{(a+bx)}}{b} - \frac{2c^2\cosh^3{(a+bx)}}{3b} + \frac{2cdx\sinh^2{(a+bx)}\cosh{(a+bx)}}{b} - \frac{4cdx\cosh^3{(a+bx)}}{3b} + \frac{d^2x^2\sinh^2{(a+bx)}\cosh{(a+bx)}}{b} \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sinh^3{(a)} \end{cases}$$

input
```
integrate((d*x+c)**2*sinh(b*x+a)**3,x)
```

output
```
Piecewise((c**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c**2*cosh(a + b*x)**3
/(3*b) + 2*c*d*x*sinh(a + b*x)**2*cosh(a + b*x)/b - 4*c*d*x*cosh(a + b*x)*
*3/(3*b) + d**2*x**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*d**2*x**2*cosh(a
 + b*x)**3/(3*b) - 14*c*d*sinh(a + b*x)**3/(9*b**2) + 4*c*d*sinh(a + b*x)*
cosh(a + b*x)**2/(3*b**2) - 14*d**2*x*sinh(a + b*x)**3/(9*b**2) + 4*d**2*x
*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2) + 14*d**2*sinh(a + b*x)**2*cosh(a
 + b*x)/(9*b**3) - 40*d**2*cosh(a + b*x)**3/(27*b**3), Ne(b, 0)), ((c**2*x
 + c*d*x**2 + d**2*x**3/3)*sinh(a)**3, True))
```

3.18.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(111) = 222$.

Time = 0.20 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.19

$$\int (c+dx)^2 \sinh^3(a+bx)\,dx$$
$$= \frac{1}{36}cd\left(\frac{(3\,bxe^{(3\,a)} - e^{(3\,a)})e^{(3\,bx)}}{b^2} - \frac{27\,(bxe^a - e^a)e^{(bx)}}{b^2} - \frac{27\,(bx+1)e^{(-bx-a)}}{b^2} + \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^2}\right)$$
$$+ \frac{1}{24}c^2\left(\frac{e^{(3\,bx+3\,a)}}{b} - \frac{9\,e^{(bx+a)}}{b} - \frac{9\,e^{(-bx-a)}}{b} + \frac{e^{(-3\,bx-3\,a)}}{b}\right)$$
$$+ \frac{1}{216}d^2\left(\frac{(9\,b^2x^2e^{(3\,a)} - 6\,bxe^{(3\,a)} + 2\,e^{(3\,a)})e^{(3\,bx)}}{b^3} - \frac{81\,(b^2x^2e^a - 2\,bxe^a + 2\,e^a)e^{(bx)}}{b^3} - \frac{81\,(b^2x^2 + 2\,bx + 2}{b^3}\right.$$

input
```
integrate((d*x+c)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/36*c*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 27*(b*x*e^a - e^a)*e^(
b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 + (3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2) + 1/24*c^2*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^
(-3*b*x - 3*a)/b) + 1/216*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3
*a))*e^(3*b*x)/b^3 - 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 81
*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 + (9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*
x - 3*a)/b^3)
```

3.18.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(111) = 222$.

Time = 0.29 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.87

$$\int (c+dx)^2 \sinh^3(a+bx)\,dx$$
$$= \frac{(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 - 6\,bd^2x - 6\,bcd + 2\,d^2)e^{(3\,bx+3\,a)}}{216\,b^3}$$
$$- \frac{3\,(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - 2\,bd^2x - 2\,bcd + 2\,d^2)e^{(bx+a)}}{8\,b^3}$$
$$- \frac{3\,(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + 2\,bd^2x + 2\,bcd + 2\,d^2)e^{(-bx-a)}}{8\,b^3}$$
$$+ \frac{(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 + 6\,bd^2x + 6\,bcd + 2\,d^2)e^{(-3\,bx-3\,a)}}{216\,b^3}$$

input
```
integrate((d*x+c)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/216*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 6*b*d^2*x - 6*b*c*d + 2*
d^2)*e^(3*b*x + 3*a)/b^3 - 3/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b*
d^2*x - 2*b*c*d + 2*d^2)*e^(b*x + a)/b^3 - 3/8*(b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2 + 2*b*d^2*x + 2*b*c*d + 2*d^2)*e^(-b*x - a)/b^3 + 1/216*(9*b^2*d
^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 6*b*d^2*x + 6*b*c*d + 2*d^2)*e^(-3*b*x
 - 3*a)/b^3
```

3.18.9 Mupad [B] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.50

$$\int (c+dx)^2 \sinh^3(a+bx)\,dx =$$
$$-\frac{\frac{3\,d^2\cosh(a+b\,x)}{2}-\frac{d^2\cosh(3\,a+3\,b\,x)}{54}+\frac{3\,b^2\,c^2\cosh(a+b\,x)}{4}-\frac{b^2\,c^2\cosh(3\,a+3\,b\,x)}{12}+\frac{3\,b^2\,d^2\,x^2\cosh(a+b\,x)}{4}+\frac{b\,c\,d\sinh(3\,a+3\,b\,x)}{18}-$$

input
```
int(sinh(a + b*x)^3*(c + d*x)^2,x)
```

output
```
-((3*d^2*cosh(a + b*x))/2 - (d^2*cosh(3*a + 3*b*x))/54 + (3*b^2*c^2*cosh(a
 + b*x))/4 - (b^2*c^2*cosh(3*a + 3*b*x))/12 + (3*b^2*d^2*x^2*cosh(a + b*x)
)/4 + (b*c*d*sinh(3*a + 3*b*x))/18 - (3*b*d^2*x*sinh(a + b*x))/2 - (b^2*d^
2*x^2*cosh(3*a + 3*b*x))/12 + (b*d^2*x*sinh(3*a + 3*b*x))/18 - (3*b*c*d*si
nh(a + b*x))/2 - (b^2*c*d*x*cosh(3*a + 3*b*x))/6 + (3*b^2*c*d*x*cosh(a + b
*x))/2)/b^3
```

3.19 $\int (c+dx)\sinh^3(a+bx)\,dx$

3.19.1 Optimal result

Integrand size = 14, antiderivative size = 75

$$\int (c+dx)\sinh^3(a+bx)\,dx = -\frac{2(c+dx)\cosh(a+bx)}{3b}+\frac{2d\sinh(a+bx)}{3b^2}+\frac{(c+dx)\cosh(a+bx)\sinh^2(a+bx)}{3b}-\frac{d\sinh^3(a+bx)}{9b^2}$$

output
```
-2/3*(d*x+c)*cosh(b*x+a)/b+2/3*d*sinh(b*x+a)/b^2+1/3*(d*x+c)*cosh(b*x+a)*s
inh(b*x+a)^2/b-1/9*d*sinh(b*x+a)^3/b^2
```

3.19.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int (c+dx)\sinh^3(a+bx)\,dx = \frac{-27b(c+dx)\cosh(a+bx)+3b(c+dx)\cosh(3(a+bx))+d(27\sinh(a+bx)-\sinh(3(a+bx)))}{36b^2}$$

input
```
Integrate[(c + d*x)*Sinh[a + b*x]^3,x]
```

output
```
(-27*b*(c + d*x)*Cosh[a + b*x] + 3*b*(c + d*x)*Cosh[3*(a + b*x)] + d*(27*S
inh[a + b*x] - Sinh[3*(a + b*x)]))/(36*b^2)
```

3.19.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3042, 26, 3791, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sinh^3(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int i(c+dx)\sin(ia+ibx)^3dx$$

$$\Big\downarrow \ 26$$

$$i\int (c+dx)\sin(ia+ibx)^3dx$$

$$\Big\downarrow \ 3791$$

$$i\left(\frac{2}{3}\int i(c+dx)\sinh(a+bx)dx+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)$$

$$\Big\downarrow \ 26$$

$$i\left(\frac{2}{3}i\int (c+dx)\sinh(a+bx)dx+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)$$

$$\Big\downarrow \ 3042$$

$$i\left(\frac{2}{3}i\int -i(c+dx)\sin(ia+ibx)dx+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)$$

$$\Big\downarrow \ 26$$

$$i\left(\frac{2}{3}\int (c+dx)\sin(ia+ibx)dx+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)$$

$$\Big\downarrow \ 3777$$

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\cosh(a+bx)dx}{b}\right)+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)$$

$$\Big\downarrow \ 3042$$

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int \sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+$$

$$\Big\downarrow \text{3117}$$

$$i\left(\frac{2}{3}\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)+\frac{id\sinh^3(a+bx)}{9b^2}-\frac{i(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}\right)$$

input
```
Int[(c + d*x)*Sinh[a + b*x]^3,x]
```

output
```
I*(((-1/3*I)*(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2)/b + ((I/9)*d*Sinh[a
+ b*x]^3)/b^2 + (2*((I*(c + d*x)*Cosh[a + b*x])/b - (I*d*Sinh[a + b*x])/b^
2))/3)
```

3.19.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.19.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

method	result
parallelrisch	$\frac{3b(dx+c)\cosh(3bx+3a)-d\sinh(3bx+3a)-27b(dx+c)\cosh(bx+a)-24bc+27d\sinh(bx+a)}{36b^2}$
risch	$\frac{(3bdx+3bc-d)e^{3bx+3a}}{72b^2}-\frac{3(bdx+bc-d)e^{bx+a}}{8b^2}-\frac{3(bdx+bc+d)e^{-bx-a}}{8b^2}+\frac{(3bdx+3bc+d)e^{-3bx-3a}}{72b^2}$
derivativedivides	$\frac{\frac{d\left(-\frac{2(bx+a)\cosh(bx+a)}{3}+\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)^2}{3}+\frac{2\sinh(bx+a)}{3}-\frac{\sinh(bx+a)^3}{9}\right)}{b}-\frac{da\left(-\frac{2}{3}+\frac{\sinh(bx+a)^2}{3}\right)\cosh(bx+a)}{b}+}{b}$
default	$\frac{\frac{d\left(-\frac{2(bx+a)\cosh(bx+a)}{3}+\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)^2}{3}+\frac{2\sinh(bx+a)}{3}-\frac{\sinh(bx+a)^3}{9}\right)}{b}-\frac{da\left(-\frac{2}{3}+\frac{\sinh(bx+a)^2}{3}\right)\cosh(bx+a)}{b}+}{b}$

input
```
int((d*x+c)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/36*(3*b*(d*x+c)*cosh(3*b*x+3*a)-d*sinh(3*b*x+3*a)-27*b*(d*x+c)*cosh(b*x+
a)-24*b*c+27*d*sinh(b*x+a))/b^2
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int (c+dx)\sinh^3(a+bx)\,dx$$
$$=\frac{3\,(bdx+bc)\cosh\,(bx+a)^3+9\,(bdx+bc)\cosh\,(bx+a)\sinh\,(bx+a)^2-d\sinh\,(bx+a)^3-27\,(bdx+bc)\,\mathrm{c}}{36\,b^2}$$

input
```
integrate((d*x+c)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/36*(3*(b*d*x + b*c)*cosh(b*x + a)^3 + 9*(b*d*x + b*c)*cosh(b*x + a)*sinh
(b*x + a)^2 - d*sinh(b*x + a)^3 - 27*(b*d*x + b*c)*cosh(b*x + a) - 3*(d*co
sh(b*x + a)^2 - 9*d)*sinh(b*x + a))/b^2
```

3.19.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.68

$$\int (c+dx)\sinh^3(a+bx)\,dx$$
$$=\begin{cases} \frac{c\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2c\cosh^3(a+bx)}{3b} + \frac{dx\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2dx\cosh^3(a+bx)}{3b} - \frac{7d\sinh^3(a+bx)}{9b^2} + \frac{2d\sinh(a+bx)}{3} \\ \left(cx+\frac{dx^2}{2}\right)\sinh^3(a) \end{cases}$$

input
```
integrate((d*x+c)*sinh(b*x+a)**3,x)
```

output
```
Piecewise((c*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c*cosh(a + b*x)**3/(3*b)
 + d*x*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*d*x*cosh(a + b*x)**3/(3*b) - 7
*d*sinh(a + b*x)**3/(9*b**2) + 2*d*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2)
, Ne(b, 0)), ((c*x + d*x**2/2)*sinh(a)**3, True))
```

3.19.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(67) = 134.

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.88

$$\int (c+dx)\sinh^3(a+bx)\,dx$$
$$=\frac{1}{72}d\left(\frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{b^2}-\frac{27\,(bxe^a-e^a)e^{(bx)}}{b^2}-\frac{27\,(bx+1)e^{(-bx-a)}}{b^2}+\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^2}\right)$$
$$+\frac{1}{24}c\left(\frac{e^{(3\,bx+3\,a)}}{b}-\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}+\frac{e^{(-3\,bx-3\,a)}}{b}\right)$$

input
```
integrate((d*x+c)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/72*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 27*(b*x*e^a - e^a)*e^(b*
x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 + (3*b*x + 1)*e^(-3*b*x - 3*a)/b^2)
 + 1/24*c*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(-3*
b*x - 3*a)/b)
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.31

$$\int (c+dx)\sinh^3(a+bx)\,dx = \frac{(3\,bdx+3\,bc-d)e^{(3\,bx+3\,a)}}{72\,b^2} - \frac{3\,(bdx+bc-d)e^{(bx+a)}}{8\,b^2} - \frac{3\,(bdx+bc+d)e^{(-bx-a)}}{8\,b^2} + \frac{(3\,bdx+3\,bc+d)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

input
```
integrate((d*x+c)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/72*(3*b*d*x + 3*b*c - d)*e^(3*b*x + 3*a)/b^2 - 3/8*(b*d*x + b*c - d)*e^(
b*x + a)/b^2 - 3/8*(b*d*x + b*c + d)*e^(-b*x - a)/b^2 + 1/72*(3*b*d*x + 3*
b*c + d)*e^(-3*b*x - 3*a)/b^2
```

3.19.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int (c+dx)\sinh^3(a+bx)\,dx = \frac{7\,d\sinh(a+b\,x)}{9\,b^2} - \frac{c\cosh(a+b\,x) - \frac{c\cosh(a+b\,x)^3}{3} + d\,x\cosh(a+b\,x) - \frac{d\,x\cosh(a+b\,x)^3}{3}}{b} - \frac{d\cosh(a+b\,x)^2\sinh(a+b\,x)}{9\,b^2}$$

input
```
int(sinh(a + b*x)^3*(c + d*x),x)
```

output
```
(7*d*sinh(a + b*x))/(9*b^2) - (c*cosh(a + b*x) - (c*cosh(a + b*x)^3)/3 + d
*x*cosh(a + b*x) - (d*x*cosh(a + b*x)^3)/3)/b - (d*cosh(a + b*x)^2*sinh(a
+ b*x))/(9*b^2)
```

3.20 $\int \frac{\sinh^3(a+bx)}{c+dx}\,dx$

3.20.1 Optimal result

Integrand size = 16, antiderivative size = 121

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx = \frac{\mathrm{Chi}\left(\frac{3bc}{d}+3bx\right)\sinh\left(3a-\frac{3bc}{d}\right)}{4d} - \frac{3\mathrm{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{4d} - \frac{3\cosh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{bc}{d}+bx\right)}{4d} + \frac{\cosh\left(3a-\frac{3bc}{d}\right)\mathrm{Shi}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

output
```
-3/4*cosh(a-b*c/d)*Shi(b*c/d+b*x)/d+1/4*cosh(3*a-3*b*c/d)*Shi(3*b*c/d+3*b*
x)/d+1/4*Chi(3*b*c/d+3*b*x)*sinh(3*a-3*b*c/d)/d-3/4*Chi(b*c/d+b*x)*sinh(a-
b*c/d)/d
```

3.20.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx = \frac{\mathrm{Chi}\left(\frac{3b(c+dx)}{d}\right)\sinh\left(3a-\frac{3bc}{d}\right) - 3\mathrm{Chi}\left(b\left(\frac{c}{d}+x\right)\right)\sinh\left(a-\frac{bc}{d}\right) - 3\cosh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(b\left(\frac{c}{d}+x\right)\right) + \cosh\left(3a\right.}{4d}$$

input
```
Integrate[Sinh[a + b*x]^3/(c + d*x),x]
```

output
```
(CoshIntegral[(3*b*(c + d*x))/d]*Sinh[3*a - (3*b*c)/d] - 3*CoshIntegral[b*
(c/d + x)]*Sinh[a - (b*c)/d] - 3*Cosh[a - (b*c)/d]*SinhIntegral[b*(c/d + x
)] + Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*(c + d*x))/d])/(4*d)
```

3.20.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ia+ibx)^3}{c+dx}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ia+ibx)^3}{c+dx}dx$$

$$\Big\downarrow \text{3793}$$

$$i\int \left(\frac{3i\sinh(a+bx)}{4(c+dx)}-\frac{i\sinh(3a+3bx)}{4(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{i\sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{3i\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{3i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{i\cosh\left(3a\right.}{}\right.$$

input
```
Int[Sinh[a + b*x]^3/(c + d*x),x]
```

output
```
I*(((-1/4*I)*CoshIntegral[(3*b*c)/d + 3*b*x]*Sinh[3*a - (3*b*c)/d])/d + ((
(3*I)/4)*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/d + (((3*I)/4)*Cos
h[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d - ((I/4)*Cosh[3*a - (3*b*c)/
d]*SinhIntegral[(3*b*c)/d + 3*b*x])/d)
```

3.20.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.20.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.37

method	result
risch	$\frac{e^{-\frac{3(ad-bc)}{d}} \operatorname{Ei}_1\left(3bx+3a-\frac{3(ad-bc)}{d}\right)}{8d}-\frac{3\,e^{-\frac{ad-bc}{d}} \operatorname{Ei}_1\left(bx+a-\frac{ad-bc}{d}\right)}{8d}+\frac{3\,e^{\frac{ad-bc}{d}} \operatorname{Ei}_1\left(-bx-a-\frac{-ad+bc}{d}\right)}{8d}-\frac{e^{\frac{3ad-3bc}{d}} \operatorname{Ei}_1\left(-3\right.}{}$

input
```
int(sinh(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*exp(-3*(a*d-b*c)/d)*Ei(1,3*b*x+3*a-3*(a*d-b*c)/d)-3/8/d*exp(-(a*d-b*
c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+3/8/d*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c
)/d)-1/8/d*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a*d+b*c)/d)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.55

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx = -\frac{3\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)-\left(\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)-\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{3\,(bc-ad)}{d}\right)+3}{8\,d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
-1/8*(3*(Ei((b*d*x + b*c)/d) - Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d)
- (Ei(3*(b*d*x + b*c)/d) - Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)/d)
+ 3*(Ei((b*d*x + b*c)/d) + Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) - (E
i(3*(b*d*x + b*c)/d) + Ei(-3*(b*d*x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/d
```

3.20.6 Sympy [F]

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx = \int \frac{\sinh^3(a+bx)}{c+dx}\,dx$$

input
```
integrate(sinh(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(sinh(a + b*x)**3/(c + d*x), x)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{\sinh^3(a+bx)}{c+dx}\,dx &= \frac{e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_1\left(\frac{3\,(dx+c)b}{d}\right)}{8\,d} - \frac{3\,e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{8\,d}\\ &\quad + \frac{3\,e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{8\,d} - \frac{e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_1\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,d}\end{aligned}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(1, 3*(d*x + c)*b/d)/d - 3/8*e^(-a +
b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d + 3/8*e^(a - b*c/d)*exp_integral
_e(1, -(d*x + c)*b/d)/d - 1/8*e^(3*a - 3*b*c/d)*exp_integral_e(1, -3*(d*x
+ c)*b/d)/d
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx$$
$$= \frac{\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}-3\,\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+3\,\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}-\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}}{8\,d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
1/8*(Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 3*Ei((b*d*x + b*c)/d)*e^(a
- b*c/d) + 3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - Ei(-3*(b*d*x + b*c)/d)*
e^(-3*a + 3*b*c/d))/d
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx = \int \frac{\sinh(a+b\,x)^3}{c+d\,x}\,dx$$

input
```
int(sinh(a + b*x)^3/(c + d*x),x)
```

output
```
int(sinh(a + b*x)^3/(c + d*x), x)
```

3.21 $\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx$

3.21.1 Optimal result

Integrand size = 16, antiderivative size = 145

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx = -\frac{3b\cosh\left(a-\frac{bc}{d}\right)\operatorname{Chi}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cosh\left(3a-\frac{3bc}{d}\right)\operatorname{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\sinh^3(a+bx)}{d(c+dx)}-\frac{3b\sinh\left(a-\frac{bc}{d}\right)\operatorname{Shi}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\sinh\left(3a-\frac{3bc}{d}\right)\operatorname{Shi}\left(\frac{3bc}{d}+3bx\right)}{4d^2}$$

output
```
3/4*b*Chi(3*b*c/d+3*b*x)*cosh(3*a-3*b*c/d)/d^2-3/4*b*Chi(b*c/d+b*x)*cosh(a
-b*c/d)/d^2+3/4*b*Shi(3*b*c/d+3*b*x)*sinh(3*a-3*b*c/d)/d^2-3/4*b*Shi(b*c/d
+b*x)*sinh(a-b*c/d)/d^2-sinh(b*x+a)^3/d/(d*x+c)
```

3.21.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{6d\cosh(bx)\sinh(a)-2d\cosh(3bx)\sinh(3a)+6d\cosh(a)\sinh(bx)-2d\cosh(3a)\sinh(3bx)+6b(c+dx)\,(}{}$$

input
```
Integrate[Sinh[a + b*x]^3/(c + d*x)^2,x]
```

output
```
(6*d*Cosh[b*x]*Sinh[a] - 2*d*Cosh[3*b*x]*Sinh[3*a] + 6*d*Cosh[a]*Sinh[b*x]
 - 2*d*Cosh[3*a]*Sinh[3*b*x] + 6*b*(c + d*x)*(-(Cosh[a - (b*c)/d]*CoshInte
gral[b*(c/d + x)]) + Cosh[3*a - (3*b*c)/d]*CoshIntegral[(3*b*(c + d*x))/d]
 - Sinh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)] + Sinh[3*a - (3*b*c)/d]*Sin
hIntegral[(3*b*(c + d*x))/d]))/(8*d^2*(c + d*x))
```

3.21.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 26, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ia+ibx)^3}{(c+dx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ia+ibx)^3}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3794}$$

$$i\left(\frac{3ib\int \left(\frac{\cosh(a+bx)}{4(c+dx)}-\frac{\cosh(3a+3bx)}{4(c+dx)}\right)dx}{d}+\frac{i\sinh^3(a+bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{3ib\left(\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\sinh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3bc}{d}+3bx\right)}{4d}\right)}{d}+\right.$$

input
```
Int[Sinh[a + b*x]^3/(c + d*x)^2,x]
```

output
```
I*((I*Sinh[a + b*x]^3)/(d*(c + d*x)) + ((3*I)*b*((Cosh[a - (b*c)/d]*CoshIn
tegral[(b*c)/d + b*x])/(4*d) - (Cosh[3*a - (3*b*c)/d]*CoshIntegral[(3*b*c)
/d + 3*b*x])/(4*d) + (Sinh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(4*d)
 - (Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/(4*d)))/d)
```

3.21.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

3.21.4 Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.87

method	result
risch	$\frac{b\,\mathrm{e}^{-3bx-3a}}{8d(bdx+bc)}-\frac{3b\,\mathrm{e}^{-\frac{3(ad-bc)}{d}}\operatorname{Ei}_{1}\left(3bx+3a-\frac{3(ad-bc)}{d}\right)}{8d^{2}}-\frac{3b\,\mathrm{e}^{-bx-a}}{8d(bdx+bc)}+\frac{3b\,\mathrm{e}^{-\frac{ad-bc}{d}}\operatorname{Ei}_{1}\left(bx+a-\frac{ad-bc}{d}\right)}{8d^{2}}+\frac{3b\,\mathrm{e}^{bx+a}}{8d^{2}\left(\frac{bc}{d}+bx\right)}+$

input
```
int(sinh(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*b*exp(-3*b*x-3*a)/d/(b*d*x+b*c)-3/8*b/d^2*exp(-3*(a*d-b*c)/d)*Ei(1,3*b
*x+3*a-3*(a*d-b*c)/d)-3/8*b*exp(-b*x-a)/d/(b*d*x+b*c)+3/8*b/d^2*exp(-(a*d-
b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+3/8*b/d^2*exp(b*x+a)/(b*c/d+b*x)+3/8*b/d^2
*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/d)-1/8*b/d^2*exp(3*b*x+3*a)/(b*c/
d+b*x)-3/8*b/d^2*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a*d+b*c)/d)
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(137) = 274$.

Time = 0.25 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.08

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{2\,d\sinh{(bx+a)}^3 + 3\left((bdx+bc)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right) + (bdx+bc)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right) - 3\left((bdx+bc)\mathrm{Ei}\right.}{}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*d*sinh(b*x + a)^3 + 3*((b*d*x + b*c)*Ei((b*d*x + b*c)/d) + (b*d*x
+ b*c)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) - 3*((b*d*x + b*c)*Ei(3*
(b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*
d)/d) + 6*(d*cosh(b*x + a)^2 - d)*sinh(b*x + a) + 3*((b*d*x + b*c)*Ei((b*d
*x + b*c)/d) - (b*d*x + b*c)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) -
3*((b*d*x + b*c)*Ei(3*(b*d*x + b*c)/d) - (b*d*x + b*c)*Ei(-3*(b*d*x + b*c)
/d))*sinh(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

3.21.6 Sympy [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sinh^3{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(sinh(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(sinh(a + b*x)**3/(c + d*x)**2, x)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx = \frac{e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_2\left(\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)d} - \frac{3\,e^{\left(-a+\frac{bc}{d}\right)}E_2\left(\frac{(dx+c)b}{d}\right)}{8\,(dx+c)d} + \frac{3\,e^{\left(a-\frac{bc}{d}\right)}E_2\left(-\frac{(dx+c)b}{d}\right)}{8\,(dx+c)d} - \frac{e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_2\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(2, 3*(d*x + c)*b/d)/((d*x + c)*d) -
3/8*e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d) + 3/8*e^
(a - b*c/d)*exp_integral_e(2, -(d*x + c)*b/d)/((d*x + c)*d) - 1/8*e^(3*a -
 3*b*c/d)*exp_integral_e(2, -3*(d*x + c)*b/d)/((d*x + c)*d)
```

3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. 2(137) = 274.

Time = 0.33 (sec) , antiderivative size = 1076, normalized size of antiderivative = 7.42

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/8*(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(-3*((d*x + c)*
(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(3*(b*c - a*d)/d) +
3*b^3*c*Ei(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/
d)*e^(3*(b*c - a*d)/d) - 3*a*b^2*d*Ei(-3*((d*x + c)*(b - b*c/(d*x + c) + a
*d/(d*x + c)) + b*c - a*d)/d)*e^(3*(b*c - a*d)/d) - 3*(d*x + c)*(b - b*c/(
d*x + c) + a*d/(d*x + c))*b^2*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x
 + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) - 3*b^3*c*Ei(-((d*x + c)*(b - b*c
/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) + 3*a*b^2*d*
Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c
 - a*d)/d) - 3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(((d*x
+ c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-(b*c - a*d)/d
) - 3*b^3*c*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)
/d)*e^(-(b*c - a*d)/d) + 3*a*b^2*d*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/
(d*x + c)) + b*c - a*d)/d)*e^(-(b*c - a*d)/d) + 3*(d*x + c)*(b - b*c/(d*x
+ c) + a*d/(d*x + c))*b^2*Ei(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x +
c)) + b*c - a*d)/d)*e^(-3*(b*c - a*d)/d) + 3*b^3*c*Ei(3*((d*x + c)*(b - b*
c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-3*(b*c - a*d)/d) - 3*a*b^
2*d*Ei(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^
(-3*(b*c - a*d)/d) - b^2*d*e^(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x +
c))/d) + 3*b^2*d*e^((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) - ...
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sinh(a+b\,x)^3}{(c+d\,x)^2}\,dx$$

input
```
int(sinh(a + b*x)^3/(c + d*x)^2,x)
```

output
```
int(sinh(a + b*x)^3/(c + d*x)^2, x)
```

3.22 $\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx$

3.22.1 Optimal result

Integrand size = 16, antiderivative size = 184

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx = \frac{9b^2\text{Chi}\left(\frac{3bc}{d}+3bx\right)\sinh\left(3a-\frac{3bc}{d}\right)}{8d^3} - \frac{3b^2\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{8d^3} - \frac{3b\cosh(a+bx)\sinh^2(a+bx)}{2d^2(c+dx)} - \frac{\sinh^3(a+bx)}{2d(c+dx)^2} - \frac{3b^2\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{8d^3} + \frac{9b^2\cosh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3bc}{d}+3bx\right)}{8d^3}$$

output
```
-3/8*b^2*cosh(a-b*c/d)*Shi(b*c/d+b*x)/d^3+9/8*b^2*cosh(3*a-3*b*c/d)*Shi(3*
b*c/d+3*b*x)/d^3+9/8*b^2*Chi(3*b*c/d+3*b*x)*sinh(3*a-3*b*c/d)/d^3-3/8*b^2*
Chi(b*c/d+b*x)*sinh(a-b*c/d)/d^3-3/2*b*cosh(b*x+a)*sinh(b*x+a)^2/d^2/(d*x+
c)-1/2*sinh(b*x+a)^3/d/(d*x+c)^2
```

3.22.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.20

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx$$

$$= \frac{6d\cosh(bx)(b(c+dx)\cosh(a)+d\sinh(a)) - 2d\cosh(3bx)(3b(c+dx)\cosh(3a)+d\sinh(3a)) + 6d(d\cosh}{}$$

input
```
Integrate[Sinh[a + b*x]^3/(c + d*x)^3,x]
```

output
```
(6*d*Cosh[b*x]*(b*(c + d*x)*Cosh[a] + d*Sinh[a]) - 2*d*Cosh[3*b*x]*(3*b*(c
 + d*x)*Cosh[3*a] + d*Sinh[3*a]) + 6*d*(d*Cosh[a] + b*(c + d*x)*Sinh[a])*S
inh[b*x] - 2*d*(d*Cosh[3*a] + 3*b*(c + d*x)*Sinh[3*a])*Sinh[3*b*x] + 6*b^2
*(c + d*x)^2*(3*CoshIntegral[(3*b*(c + d*x))/d]*Sinh[3*a - (3*b*c)/d] - Co
shIntegral[b*(c/d + x)]*Sinh[a - (b*c)/d] - Cosh[a - (b*c)/d]*SinhIntegral
[b*(c/d + x)] + 3*Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*(c + d*x))/d]))/
(16*d^3*(c + d*x)^2)
```

3.22.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.43, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 26, 3795, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ia+ibx)^3}{(c+dx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ia+ibx)^3}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3795}$$

$$i\left(\frac{9b^2\int -\frac{i\sinh^3(a+bx)}{c+dx}dx}{2d^2}-\frac{3b^2\int \frac{i\sinh(a+bx)}{c+dx}dx}{d^2}+\frac{3ib\sinh^2(a+bx)\cosh(a+bx)}{2d^2(c+dx)}+\frac{i\sinh^3(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{9ib^2\int \frac{\sinh^3(a+bx)}{c+dx}dx}{2d^2}-\frac{3ib^2\int \frac{\sinh(a+bx)}{c+dx}dx}{d^2}+\frac{3ib\sinh^2(a+bx)\cosh(a+bx)}{2d^2(c+dx)}+\frac{i\sinh^3(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{3ib^2\int -\frac{i\sin(ia+ibx)}{c+dx}dx}{d^2}-\frac{9ib^2\int \frac{i\sin(ia+ibx)^3}{c+dx}dx}{2d^2}+\frac{3ib\sinh^2(a+bx)\cosh(a+bx)}{2d^2(c+dx)}+\frac{i\sinh^3(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{3b^2\int \frac{\sin(ia+ibx)}{c+dx}dx}{d^2}+\frac{9b^2\int \frac{\sin(ia+ibx)^3}{c+dx}dx}{2d^2}+\frac{3ib\sinh^2(a+bx)\cosh(a+bx)}{2d^2(c+dx)}+\frac{i\sinh^3(a+bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3784}$$

$$i\left(\frac{9b^2\int \frac{\sin(ia+ibx)^3}{c+dx}dx}{2d^2}-\frac{3b^2\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int \frac{i\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d^2}+\frac{3ib\sinh^2(a+b\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{9b^2\int \frac{\sin(ia+ibx)^3}{c+dx}dx}{2d^2}-\frac{3b^2\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+i\cosh\left(a-\frac{bc}{d}\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d^2}+\frac{3ib\sinh^2(a+b\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{9b^2\int \frac{\sin(ia+ibx)^3}{c+dx}dx}{2d^2}-\frac{3b^2\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(a-\frac{bc}{d}\right)\int -\frac{i\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)}{d^2}+\frac{3ib\sinh^2\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{9b^2\int \frac{\sin(ia+ibx)^3}{c+dx}dx}{2d^2}-\frac{3b^2\left(i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)}{d^2}+\frac{3ib\sinh^2(a+\right.$$

$$\Big\downarrow \text{3779}$$

3.22. $\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx$

$$i\left(-\frac{3b^2\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}+\frac{9b^2\int\frac{\sin(ia+ibx)^3}{c+dx}dx}{2d^2}+\frac{3ib\sinh^2(a+bx)\cosh(a+bx)}{2d^2(c+dx)}\right.$$

$$\Big\downarrow \text{3782}$$

$$i\left(\frac{9b^2\int\frac{\sin(ia+ibx)^3}{c+dx}dx}{2d^2}-\frac{3b^2\left(\frac{i\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}+\frac{3ib\sinh^2(a+bx)\cosh(a+bx)}{2d^2(c+dx)}\right.$$

$$\Big\downarrow \text{3793}$$

$$i\left(\frac{9b^2\int\left(\frac{3i\sinh(a+bx)}{4(c+dx)}-\frac{i\sinh(3a+3bx)}{4(c+dx)}\right)dx}{2d^2}-\frac{3b^2\left(\frac{i\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}+\frac{3ib\sinh^2(a+bx)}{2d^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{3b^2\left(\frac{i\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}+\frac{9b^2\left(-\frac{i\sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{3i\sinh\left(a-\frac{bc}{d}\right)\text{Chi}}{4d}\right.}{}\right.$$

input
```
Int[Sinh[a + b*x]^3/(c + d*x)^3,x]
```

output
```
I*((((3*I)/2)*b*Cosh[a + b*x]*Sinh[a + b*x]^2)/(d^2*(c + d*x)) + ((I/2)*Si
nh[a + b*x]^3)/(d*(c + d*x)^2) - (3*b^2*((I*CoshIntegral[(b*c)/d + b*x]*Si
nh[a - (b*c)/d])/d + (I*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d))
/d^2 + (9*b^2*(((-1/4*I)*CoshIntegral[(3*b*c)/d + 3*b*x]*Sinh[3*a - (3*b*c
)/d])/d + (((3*I)/4)*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/d + ((
(3*I)/4)*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d - ((I/4)*Cosh[3*
a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/d))/(2*d^2))
```

3.22. $\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx$

3.22.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

3.22.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(172) = 344$.

Time = 2.38 (sec) , antiderivative size = 562, normalized size of antiderivative = 3.05

method	result
risch	$-\frac{3b^3 e^{-3bx-3a} x}{16d(b^2d^2x^2+2b^2cdx+b^2c^2)} - \frac{3b^3 e^{-3bx-3a} c}{16d^2(b^2d^2x^2+2b^2cdx+b^2c^2)} + \frac{b^2 e^{-3bx-3a}}{16d(b^2d^2x^2+2b^2cdx+b^2c^2)} + \frac{9b^2 e^{-\frac{3(ad-bc)}{d}} \operatorname{Ei}_1\left(3bx+3a-\frac{3(\right.}{}}{16d^3}$

input
```
int(sinh(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/16*b^3*exp(-3*b*x-3*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*x-3/16*b^3*e
xp(-3*b*x-3*a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c+1/16*b^2*exp(-3*b*x
-3*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)+9/16*b^2/d^3*exp(-3*(a*d-b*c)/d)
*Ei(1,3*b*x+3*a-3*(a*d-b*c)/d)+3/16*b^3*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c
*d*x+b^2*c^2)*x+3/16*b^3*exp(-b*x-a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)
*c-3/16*b^2*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-3/16*b^2/d^3*e
xp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+3/16*b^2/d^3*exp(b*x+a)/(b*c/d+b*
x)^2+3/16*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)+3/16*b^2/d^3*exp((a*d-b*c)/d)*Ei(
1,-b*x-a-(-a*d+b*c)/d)-1/16*b^2/d^3*exp(3*b*x+3*a)/(b*c/d+b*x)^2-3/16*b^2/
d^3*exp(3*b*x+3*a)/(b*c/d+b*x)-9/16*b^2/d^3*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x
-3*a-3*(-a*d+b*c)/d)
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(172) = 344$.

Time = 0.25 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.88

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{2\,d^2\sinh(bx+a)^3 + 6\,(bd^2x+bcd)\cosh(bx+a)^3 + 18\,(bd^2x+bcd)\cosh(bx+a)\sinh(bx+a)^2 - 6\,(b}{}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/16*(2*d^2*sinh(b*x + a)^3 + 6*(b*d^2*x + b*c*d)*cosh(b*x + a)^3 + 18*(b
*d^2*x + b*c*d)*cosh(b*x + a)*sinh(b*x + a)^2 - 6*(b*d^2*x + b*c*d)*cosh(b
*x + a) + 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) - (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*
d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(3*(b*d*x + b*c)/d) - (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c
- a*d)/d) + 6*(d^2*cosh(b*x + a)^2 - d^2)*sinh(b*x + a) + 3*((b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*Ei(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*Ei(-3*(b*d*x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/(d^5*x^2 + 2*c*
d^4*x + c^2*d^3)
```

3.22.6 Sympy [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sinh^3{(a+bx)}}{(c+dx)^3}\,dx$$

input
```
integrate(sinh(b*x+a)**3/(d*x+c)**3,x)
```

output
```
Integral(sinh(a + b*x)**3/(c + d*x)**3, x)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.79

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx = \frac{e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_3\left(\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)^2d} - \frac{3\,e^{\left(-a+\frac{bc}{d}\right)}E_3\left(\frac{(dx+c)b}{d}\right)}{8\,(dx+c)^2d} + \frac{3\,e^{\left(a-\frac{bc}{d}\right)}E_3\left(-\frac{(dx+c)b}{d}\right)}{8\,(dx+c)^2d} - \frac{e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_3\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)^2d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(3, 3*(d*x + c)*b/d)/((d*x + c)^2*d)
- 3/8*e^(-a + b*c/d)*exp_integral_e(3, (d*x + c)*b/d)/((d*x + c)^2*d) + 3/
8*e^(a - b*c/d)*exp_integral_e(3, -(d*x + c)*b/d)/((d*x + c)^2*d) - 1/8*e^
(3*a - 3*b*c/d)*exp_integral_e(3, -3*(d*x + c)*b/d)/((d*x + c)^2*d)
```

3.22.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(172) = 344$.

Time = 0.28 (sec) , antiderivative size = 601, normalized size of antiderivative = 3.27

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx = \frac{9\,b^2d^2x^2\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)} - 3\,b^2d^2x^2\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} + 3\,b^2d^2x^2\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)} - 9\,b^2d^2x^2\mathrm{E}}{}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/16*(9*b^2*d^2*x^2*Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 3*b^2*d^2*x^
2*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) + 3*b^2*d^2*x^2*Ei(-(b*d*x + b*c)/d)*e
^(-a + b*c/d) - 9*b^2*d^2*x^2*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) +
18*b^2*c*d*x*Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 6*b^2*c*d*x*Ei((b*d
*x + b*c)/d)*e^(a - b*c/d) + 6*b^2*c*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/
d) - 18*b^2*c*d*x*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) + 9*b^2*c^2*Ei
(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 3*b^2*c^2*Ei((b*d*x + b*c)/d)*e^(a
 - b*c/d) + 3*b^2*c^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 9*b^2*c^2*Ei(-
3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) - 3*b*d^2*x*e^(3*b*x + 3*a) + 3*b*d^
2*x*e^(b*x + a) + 3*b*d^2*x*e^(-b*x - a) - 3*b*d^2*x*e^(-3*b*x - 3*a) - 3*
b*c*d*e^(3*b*x + 3*a) + 3*b*c*d*e^(b*x + a) + 3*b*c*d*e^(-b*x - a) - 3*b*c
*d*e^(-3*b*x - 3*a) - d^2*e^(3*b*x + 3*a) + 3*d^2*e^(b*x + a) - 3*d^2*e^(-
b*x - a) + d^2*e^(-3*b*x - 3*a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sinh(a+b\,x)^3}{(c+d\,x)^3}\,dx$$

input
```
int(sinh(a + b*x)^3/(c + d*x)^3,x)
```

output
```
int(sinh(a + b*x)^3/(c + d*x)^3, x)
```

3.23 $\int (c+dx)^3 \mathbf{csch}(a+bx)\,dx$

3.23.1 Optimal result

Integrand size = 14, antiderivative size = 149

$$\begin{aligned}\int (c+dx)^3 \operatorname{csch}(a+bx)\,dx = & -\frac{2(c+dx)^3 \operatorname{arctanh}\left(e^{a+bx}\right)}{b} \\ & -\frac{3d(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} \\ & +\frac{3d(c+dx)^2 \operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^2} \\ & +\frac{6d^2(c+dx) \operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} \\ & -\frac{6d^2(c+dx) \operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^3} \\ & -\frac{6d^3 \operatorname{PolyLog}\left(4,-e^{a+bx}\right)}{b^4}+\frac{6d^3 \operatorname{PolyLog}\left(4,e^{a+bx}\right)}{b^4}\end{aligned}$$

output
```
-2*(d*x+c)^3*arctanh(exp(b*x+a))/b-3*d*(d*x+c)^2*polylog(2,-exp(b*x+a))/b^
2+3*d*(d*x+c)^2*polylog(2,exp(b*x+a))/b^2+6*d^2*(d*x+c)*polylog(3,-exp(b*x
+a))/b^3-6*d^2*(d*x+c)*polylog(3,exp(b*x+a))/b^3-6*d^3*polylog(4,-exp(b*x+
a))/b^4+6*d^3*polylog(4,exp(b*x+a))/b^4
```

3.23.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.13

$$\int (c+dx)^3\text{csch}(a+bx)\,dx$$
$$=\frac{(c+dx)^3\log\left(1-e^{a+bx}\right)-(c+dx)^3\log\left(1+e^{a+bx}\right)-\frac{3d(b^2(c+dx)^2\operatorname{PolyLog}(2,-e^{a+bx})-2bd(c+dx)\operatorname{PolyLog}(3,-e^{a+bx})-}{b^3}}{b}$$

input
```
Integrate[(c + d*x)^3*Csch[a + b*x],x]
```

output
```
((c + d*x)^3*Log[1 - E^(a + b*x)] - (c + d*x)^3*Log[1 + E^(a + b*x)] - (3*
d*(b^2*(c + d*x)^2*PolyLog[2, -E^(a + b*x)] - 2*b*d*(c + d*x)*PolyLog[3, -
E^(a + b*x)] + 2*d^2*PolyLog[4, -E^(a + b*x)]))/b^3 + (3*d*(b^2*(c + d*x)^
2*PolyLog[2, E^(a + b*x)] - 2*b*d*(c + d*x)*PolyLog[3, E^(a + b*x)] + 2*d^
2*PolyLog[4, E^(a + b*x)]))/b^3)/b
```

3.23.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 26, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\text{csch}(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int i(c+dx)^3\csc(ia+ibx)dx$$
$$\Big\downarrow \text{26}$$
$$i\int (c+dx)^3\csc(ia+ibx)dx$$
$$\Big\downarrow \text{4670}$$

$$i\left(\frac{3id\int (c+dx)^2\log\left(1-e^{a+bx}\right)dx}{b}-\frac{3id\int (c+dx)^2\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2i(c+dx)^3\text{arctanh}\left(e^{a+bx}\right)}{b}\right)$$

$$\Big\downarrow \; 3011$$

$$i\left(-\frac{3id\left(\frac{2d\int (c+dx)\,\text{PolyLog}\left(2,-e^{a+bx}\right)dx}{b}-\frac{(c+dx)^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{3id\left(\frac{2d\int (c+dx)\,\text{PolyLog}\left(2,e^{a+bx}\right)dx}{b}-\frac{(c+dx)^2\,\text{PolyLo}}{b}\right.}{b}\right.$$

$$\Big\downarrow \; 7163$$

$$i\left(-\frac{3id\left(\frac{2d\left(\frac{(c+dx)\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{d\int \text{PolyLog}\left(3,-e^{a+bx}\right)dx}{b}\right)}{b}-\frac{(c+dx)^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{3id\left(\frac{2d\left(\frac{(c+dx)\,\text{PolyLog}\left(3,e^{a+bx}\right.}{b}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \; 2720$$

$$i\left(-\frac{3id\left(\frac{2d\left(\frac{(c+dx)\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{d\int e^{-a-bx}\,\text{PolyLog}\left(3,-e^{a+bx}\right)de^{a+bx}}{b^2}\right)}{b}-\frac{(c+dx)^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{3id\left(\frac{2d\left(\frac{(c+dx)\,\text{Pol}}{}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \; 7143$$

$$i\left(\frac{2i(c+dx)^3\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{3id\left(\frac{2d\left(\frac{(c+dx)\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{d\,\text{PolyLog}\left(4,-e^{a+bx}\right)}{b^2}\right)}{b}-\frac{(c+dx)^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\right.$$

input
```
Int[(c + d*x)^3*Csch[a + b*x],x]
```

3.23. $\int (c+dx)^3\text{csch}(a+bx)\,dx$

output
```
I*(((2*I)*(c + d*x)^3*ArcTanh[E^(a + b*x)])/b - ((3*I)*d*(-(((c + d*x)^2*P
olyLog[2, -E^(a + b*x)])/b) + (2*d*(((c + d*x)*PolyLog[3, -E^(a + b*x)])/b
 - (d*PolyLog[4, -E^(a + b*x)])/b^2))/b))/b + ((3*I)*d*(-(((c + d*x)^2*Pol
yLog[2, E^(a + b*x)])/b) + (2*d*(((c + d*x)*PolyLog[3, E^(a + b*x)])/b - (
d*PolyLog[4, E^(a + b*x)])/b^2))/b))/b)
```

3.23.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.23.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(142) = 284$.

Time = 1.86 (sec) , antiderivative size = 541, normalized size of antiderivative = 3.63

method	result
risch	$\frac{3d^3 \operatorname{polylog}(2,e^{bx+a})x^2}{b^2} - \frac{6d^3 \operatorname{polylog}(3,e^{bx+a})x}{b^3} - \frac{d^3 \ln(e^{bx+a}+1)x^3}{b} - \frac{d^3 \ln(e^{bx+a}+1)a^3}{b^4} - \frac{3d^3 \operatorname{polylog}(2,-e^{bx+a})x^2}{b^2} +$

input
```
int((d*x+c)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
3/b^2*d^3*polylog(2,exp(b*x+a))*x^2-6/b^3*d^3*polylog(3,exp(b*x+a))*x-1/b*
d^3*ln(exp(b*x+a)+1)*x^3-1/b^4*d^3*ln(exp(b*x+a)+1)*a^3-3/b^2*d^3*polylog(
2,-exp(b*x+a))*x^2+6/b^3*d^3*polylog(3,-exp(b*x+a))*x-6/b^3*c*d^2*polylog(
3,exp(b*x+a))+6/b^3*c*d^2*polylog(3,-exp(b*x+a))+3/b^2*c^2*d*polylog(2,exp
(b*x+a))-3/b^2*c^2*d*polylog(2,-exp(b*x+a))+2/b^4*d^3*a^3*arctanh(exp(b*x+
a))+1/b*d^3*ln(1-exp(b*x+a))*x^3+1/b^4*d^3*ln(1-exp(b*x+a))*a^3-3/b^2*c^2*
d*ln(exp(b*x+a)+1)*a-6/b^3*d^2*a^2*c*arctanh(exp(b*x+a))+6/b^2*d*a*c^2*arc
tanh(exp(b*x+a))-6/b^2*c*d^2*polylog(2,-exp(b*x+a))*x+3/b*c*d^2*ln(1-exp(b
*x+a))*x^2-3/b^3*c*d^2*ln(1-exp(b*x+a))*a^2+6/b^2*c*d^2*polylog(2,exp(b*x+
a))*x-3/b*c*d^2*ln(exp(b*x+a)+1)*x^2+3/b^3*c*d^2*ln(exp(b*x+a)+1)*a^2+3/b*
c^2*d*ln(1-exp(b*x+a))*x-2/b*c^3*arctanh(exp(b*x+a))+6*d^3*polylog(4,exp(b
*x+a))/b^4-6*d^3*polylog(4,-exp(b*x+a))/b^4+3/b^2*c^2*d*ln(1-exp(b*x+a))*a
-3/b*c^2*d*ln(exp(b*x+a)+1)*x
```

3.23.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(140) = 280.

Time = 0.25 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.66

$$\int (c+dx)^3\text{csch}(a+bx)\,dx$$
$$= \frac{6\,d^3\text{polylog}(4,\cosh{(bx+a)}+\sinh{(bx+a)})-6\,d^3\text{polylog}(4,-\cosh{(bx+a)}-\sinh{(bx+a)})+3\,(b^2d^3x}{}$$

input
```
integrate((d*x+c)^3*csch(b*x+a),x, algorithm="fricas")
```

output
```
(6*d^3*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 6*d^3*polylog(4, -cosh(
b*x + a) - sinh(b*x + a)) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*di
log(cosh(b*x + a) + sinh(b*x + a)) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*
c^2*d)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*
x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cosh(b*x + a) + sinh
(b*x + a) - 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) -
 6*(b*d^3*x + b*c*d^2)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*(b*d^
3*x + b*c*d^2)*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^4
```

3.23.6 Sympy [F]

$$\int (c+dx)^3\text{csch}(a+bx)\,dx = \int (c+dx)^3\,\text{csch}\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*csch(b*x+a),x)
```

output
```
Integral((c + d*x)**3*csch(a + b*x), x)
```

3.23.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(140) = 280$.

Time = 0.26 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.23

$$
\begin{aligned}
&\int (c+dx)^3\operatorname{csch}(a+bx)\,dx \\
&= -c^3\left(\frac{\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{b}\right) \\
&\quad - \frac{3\left(bx\log\left(e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)c^2d}{b^2} \\
&\quad + \frac{3\left(bx\log\left(-e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(e^{(bx+a)}\right)\right)c^2d}{b^2} \\
&\quad - \frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 2\,\mathrm{Li}_3\left(-e^{(bx+a)}\right)\right)cd^2}{b^3} \\
&\quad + \frac{3\left(b^2x^2\log\left(-e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right) - 2\,\mathrm{Li}_3\left(e^{(bx+a)}\right)\right)cd^2}{b^3} \\
&\quad - \frac{\left(b^3x^3\log\left(e^{(bx+a)}+1\right) + 3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 6\,bx\mathrm{Li}_3\left(-e^{(bx+a)}\right) + 6\,\mathrm{Li}_4\left(-e^{(bx+a)}\right)\right)d^3}{b^4} \\
&\quad + \frac{\left(b^3x^3\log\left(-e^{(bx+a)}+1\right) + 3\,b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right) - 6\,bx\mathrm{Li}_3\left(e^{(bx+a)}\right) + 6\,\mathrm{Li}_4\left(e^{(bx+a)}\right)\right)d^3}{b^4}
\end{aligned}
$$

input
```
integrate((d*x+c)^3*csch(b*x+a),x, algorithm="maxima")
```

output
```
-c^3*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b) - 3*(b*x*log(e^(b
*x + a) + 1) + dilog(-e^(b*x + a)))*c^2*d/b^2 + 3*(b*x*log(-e^(b*x + a) +
1) + dilog(e^(b*x + a)))*c^2*d/b^2 - 3*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b
*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))*c*d^2/b^3 + 3*(b^2*x^
2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x +
 a)))*c*d^2/b^3 - (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x
+ a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))*d^3/b
^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x
*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))*d^3/b^4
```

3.23.8 Giac [F]

$$\int (c+dx)^3\text{csch}(a+bx)\,dx = \int (dx+c)^3\,\text{csch}\,(bx+a)\;dx$$

input
```
integrate((d*x+c)^3*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csch(b*x + a), x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\text{csch}(a+bx)\,dx = \int \frac{(c+d\,x)^3}{\sinh\,(a+b\,x)}\,dx$$

input
```
int((c + d*x)^3/sinh(a + b*x),x)
```

output
```
int((c + d*x)^3/sinh(a + b*x), x)
```

3.24 $\int (c+dx)^2 \text{csch}(a+bx)\,dx$

3.24.1 Optimal result

Integrand size = 14, antiderivative size = 99

$$\int (c+dx)^2 \text{csch}(a+bx)\,dx = -\frac{2(c+dx)^2 \text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{2d(c+dx)\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{2d(c+dx)\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} + \frac{2d^2\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} - \frac{2d^2\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}$$

output
```
-2*(d*x+c)^2*arctanh(exp(b*x+a))/b-2*d*(d*x+c)*polylog(2,-exp(b*x+a))/b^2+
2*d*(d*x+c)*polylog(2,exp(b*x+a))/b^2+2*d^2*polylog(3,-exp(b*x+a))/b^3-2*d
^2*polylog(3,exp(b*x+a))/b^3
```

3.24.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.19

$$\int (c+dx)^2 \text{csch}(a+bx)\,dx = \frac{(c+dx)^2 \log\left(1-e^{a+bx}\right) - (c+dx)^2 \log\left(1+e^{a+bx}\right) - \frac{2d(b(c+dx)\,\text{PolyLog}(2,-e^{a+bx}) - d\,\text{PolyLog}(3,-e^{a+bx}))}{b^2} + \frac{2d(b(c\cdots}{}}{b}$$

input
```
Integrate[(c + d*x)^2*Csch[a + b*x],x]
```

output
```
((c + d*x)^2*Log[1 - E^(a + b*x)] - (c + d*x)^2*Log[1 + E^(a + b*x)] - (2*
d*(b*(c + d*x)*PolyLog[2, -E^(a + b*x)] - d*PolyLog[3, -E^(a + b*x)]))/b^2
 + (2*d*(b*(c + d*x)*PolyLog[2, E^(a + b*x)] - d*PolyLog[3, E^(a + b*x)]))
/b^2)/b
```

3.24.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i(c+dx)^2\csc(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$i\int (c+dx)^2\csc(ia+ibx)dx$$

$$\Big\downarrow \text{4670}$$

$$i\left(\frac{2id\int (c+dx)\log\left(1-e^{a+bx}\right)dx}{b}-\frac{2id\int (c+dx)\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2i(c+dx)^2\text{arctanh}\left(e^{a+bx}\right)}{b}\right)$$

$$\Big\downarrow \text{3011}$$

$$i\left(-\frac{2id\left(\frac{d\int \text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{(c+dx)\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2id\left(\frac{d\int \text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{(c+dx)\,\text{PolyLog}(2,e^{a+bx})}{b}\right)}{b}+\frac{2}{}\right.$$

$$\Big\downarrow \text{2720}$$

$$i\left(-\frac{2id\left(\frac{d\int e^{-a-bx}\,\text{PolyLog}(2,-e^{a+bx})de^{a+bx}}{b^2}-\frac{(c+dx)\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2id\left(\frac{d\int e^{-a-bx}\,\text{PolyLog}(2,e^{a+bx})de^{a+bx}}{b^2}-\frac{(c+dx)\,\text{I}}{}\right.}{b}\right.$$

$$\Big\downarrow \text{7143}$$

$$i\left(\frac{2i(c+dx)^2\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{2id\left(\frac{d\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)}{b}+\frac{2id\left(\frac{d\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^2}-\frac{(c+dx}{}\right.}{b}\right.$$

input
```
Int[(c + d*x)^2*Csch[a + b*x],x]
```

output
```
I*(((2*I)*(c + d*x)^2*ArcTanh[E^(a + b*x)])/b - ((2*I)*d*(-(((c + d*x)*Pol
yLog[2, -E^(a + b*x)])/b) + (d*PolyLog[3, -E^(a + b*x)])/b^2))/b + ((2*I)*
d*(-(((c + d*x)*PolyLog[2, E^(a + b*x)])/b) + (d*PolyLog[3, E^(a + b*x)])/
b^2))/b)
```

3.24.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.24.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(94) = 188$.

Time = 1.68 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.09

method	result
risch	$-\frac{2c^2 \operatorname{arctanh}(e^{bx+a})}{b} + \frac{2cd\ln(1-e^{bx+a})a}{b^2} - \frac{2cd\ln(e^{bx+a}+1)a}{b^2} - \frac{2cd\ln(e^{bx+a}+1)x}{b} - \frac{2cd\,\mathrm{polylog}(2,-e^{bx+a})}{b^2} - 2d^2a^2$ ar

input
```
int((d*x+c)^2*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/b*c^2*arctanh(exp(b*x+a))+2/b^2*c*d*ln(1-exp(b*x+a))*a-2/b^2*c*d*ln(exp
(b*x+a)+1)*a-2/b*c*d*ln(exp(b*x+a)+1)*x-2/b^2*c*d*polylog(2,-exp(b*x+a))-2
/b^3*d^2*a^2*arctanh(exp(b*x+a))+2/b*c*d*ln(1-exp(b*x+a))*x+2/b^2*c*d*poly
log(2,exp(b*x+a))-1/b^3*d^2*ln(1-exp(b*x+a))*a^2+2/b^2*d^2*polylog(2,exp(b
*x+a))*x+4/b^2*d*a*c*arctanh(exp(b*x+a))+1/b^3*d^2*ln(exp(b*x+a)+1)*a^2-2*
d^2*polylog(3,exp(b*x+a))/b^3-1/b*d^2*ln(exp(b*x+a)+1)*x^2+2*d^2*polylog(3
,-exp(b*x+a))/b^3+1/b*d^2*ln(1-exp(b*x+a))*x^2-2/b^2*d^2*polylog(2,-exp(b*
x+a))*x
```

3.24.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(92) = 184$.

Time = 0.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.44

$$\int (c+dx)^2\text{csch}(a+bx)\,dx = -\frac{2\,d^2\text{polylog}(3,\cosh{(bx+a)}+\sinh{(bx+a)}) - 2\,d^2\text{polylog}(3,-\cosh{(bx+a)}-\sinh{(bx+a)}) - 2\,(bd^2}{}$$

input
```
integrate((d*x+c)^2*csch(b*x+a),x, algorithm="fricas")
```

output
```
-(2*d^2*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 2*d^2*polylog(3, -cosh
(b*x + a) - sinh(b*x + a)) - 2*(b*d^2*x + b*c*d)*dilog(cosh(b*x + a) + sin
h(b*x + a)) + 2*(b*d^2*x + b*c*d)*dilog(-cosh(b*x + a) - sinh(b*x + a)) +
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cosh(b*x + a) + sinh(b*x + a) +
1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1
) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cosh(b*x + a) -
 sinh(b*x + a) + 1))/b^3
```

3.24.6 Sympy [F]

$$\int (c+dx)^2\text{csch}(a+bx)\,dx = \int (c+dx)^2\,\text{csch}\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*csch(b*x+a),x)
```

output
```
Integral((c + d*x)**2*csch(a + b*x), x)
```

3.24.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(92) = 184$.

Time = 0.26 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.97

$$\int (c+dx)^2 \operatorname{csch}(a+bx)\,dx$$
$$= -c^2\left(\frac{\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{\log\left(e^{(-bx-a)}-1\right)}{b}\right)$$
$$-\frac{2\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)cd}{b^2}+\frac{2\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)\right)cd}{b^2}$$
$$-\frac{\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3\left(-e^{(bx+a)}\right)\right)d^2}{b^3}$$
$$+\frac{\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right)-2\,\mathrm{Li}_3\left(e^{(bx+a)}\right)\right)d^2}{b^3}$$

input
```
integrate((d*x+c)^2*csch(b*x+a),x, algorithm="maxima")
```

output
```
-c^2*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b) - 2*(b*x*log(e^(b
*x + a) + 1) + dilog(-e^(b*x + a)))*c*d/b^2 + 2*(b*x*log(-e^(b*x + a) + 1)
 + dilog(e^(b*x + a)))*c*d/b^2 - (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dil
og(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))*d^2/b^3 + (b^2*x^2*log(-e^(
b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))*d^2/
b^3
```

3.24.8 Giac [F]

$$\int (c+dx)^2 \operatorname{csch}(a+bx)\,dx = \int (dx+c)^2 \operatorname{csch}(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csch(b*x + a), x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\text{csch}(a+bx)\,dx=\int \frac{{\left(c+d\,x\right)}^2}{\mathrm{sinh}\left(a+b\,x\right)}\,dx$$

input
```
int((c + d*x)^2/sinh(a + b*x),x)
```

output
```
int((c + d*x)^2/sinh(a + b*x), x)
```

3.25 $\int (c+dx)\text{csch}(a+bx)\,dx$

3.25.1 Optimal result

Integrand size = 12, antiderivative size = 50

$$\int (c+dx)\text{csch}(a+bx)\,dx = -\frac{2(c+dx)\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{d\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{d\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}$$

output
```
-2*(d*x+c)*arctanh(exp(b*x+a))/b-d*polylog(2,-exp(b*x+a))/b^2+d*polylog(2,
exp(b*x+a))/b^2
```

3.25.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 116 vs. 2(50) = 100.

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.32

$$\int (c+dx)\text{csch}(a+bx)\,dx = -\frac{c\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} + \frac{c\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} + 2d\left(\frac{x\log\left(1-e^{a+bx}\right)}{2b} - \frac{x\log\left(1+e^{a+bx}\right)}{2b} - \frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} + \frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}\right)$$

input
```
Integrate[(c + d*x)*Csch[a + b*x],x]
```

output
```
-((c*Log[Cosh[a/2 + (b*x)/2]])/b) + (c*Log[Sinh[a/2 + (b*x)/2]])/b + 2*d*(
(x*Log[1 - E^(a + b*x)])/(2*b) - (x*Log[1 + E^(a + b*x)])/(2*b) - PolyLog[
2, -E^(a + b*x)]/(2*b^2) + PolyLog[2, E^(a + b*x)]/(2*b^2))
```

3.25.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i(c+dx)\csc(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$i\int (c+dx)\csc(ia+ibx)dx$$

$$\Big\downarrow \text{4670}$$

$$i\left(\frac{id\int \log\left(1-e^{a+bx}\right)dx}{b}-\frac{id\int \log\left(1+e^{a+bx}\right)dx}{b}+\frac{2i(c+dx)\text{arctanh}\left(e^{a+bx}\right)}{b}\right)$$

$$\Big\downarrow \text{2715}$$

$$i\left(\frac{id\int e^{-a-bx}\log\left(1-e^{a+bx}\right)de^{a+bx}}{b^2}-\frac{id\int e^{-a-bx}\log\left(1+e^{a+bx}\right)de^{a+bx}}{b^2}+\frac{2i(c+dx)\text{arctanh}\left(e^{a+bx}\right)}{b}\right)$$

$$\Big\downarrow \text{2838}$$

$$i\left(\frac{2i(c+dx)\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\right)$$

input
```
Int[(c + d*x)*Csch[a + b*x],x]
```

output
```
I*(((2*I)*(c + d*x)*ArcTanh[E^(a + b*x)])/b + (I*d*PolyLog[2, -E^(a + b*x)
])/b^2 - (I*d*PolyLog[2, E^(a + b*x)])/b^2)
```

3.25.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.25.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.78

method	result
derivativedivides	$\frac{\frac{d\left((bx+a)\ln\left(1-\mathrm{e}^{bx+a}\right)+\mathrm{polylog}\left(2,\mathrm{e}^{bx+a}\right)-(bx+a)\ln\left(\mathrm{e}^{bx+a}+1\right)-\mathrm{polylog}\left(2,-\mathrm{e}^{bx+a}\right)\right)}{b}+\frac{2da\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}-2c\,\mathrm{arctanh}(\mathrm{e}^{bx}}{b}$
default	$\frac{\frac{d\left((bx+a)\ln\left(1-\mathrm{e}^{bx+a}\right)+\mathrm{polylog}\left(2,\mathrm{e}^{bx+a}\right)-(bx+a)\ln\left(\mathrm{e}^{bx+a}+1\right)-\mathrm{polylog}\left(2,-\mathrm{e}^{bx+a}\right)\right)}{b}+\frac{2da\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}-2c\,\mathrm{arctanh}(\mathrm{e}^{bx}}{b}$
parts	$\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)dx}{b}+\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)c}{b}+\frac{2d\left(-\frac{\mathrm{dilog}\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}-\frac{\mathrm{dilog}\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{2}-\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{}\right.}{b^2}$
risch	$-\frac{2c\,\mathrm{arctanh}(\mathrm{e}^{bx+a})}{b}+\frac{d\ln(1-\mathrm{e}^{bx+a})x}{b}+\frac{d\ln(1-\mathrm{e}^{bx+a})a}{b^2}+\frac{d\,\mathrm{polylog}(2,\mathrm{e}^{bx+a})}{b^2}-\frac{d\ln(\mathrm{e}^{bx+a}+1)x}{b}-\frac{d\ln(\mathrm{e}^{b}}{}$

input
```
int((d*x+c)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(d/b*((b*x+a)*ln(1-exp(b*x+a))+polylog(2,exp(b*x+a))-(b*x+a)*ln(exp(b*
x+a)+1)-polylog(2,-exp(b*x+a)))+2*d/b*a*arctanh(exp(b*x+a))-2*c*arctanh(ex
p(b*x+a)))
```

3.25.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.38

$$\int (c+dx)\mathrm{csch}(a+bx)\,dx$$
$$= \frac{d\mathrm{Li}_2(\cosh(bx+a)+\sinh(bx+a))-d\mathrm{Li}_2(-\cosh(bx+a)-\sinh(bx+a))-(bdx+bc)\log(\cosh(bx+}{}$$

input
```
integrate((d*x+c)*csch(b*x+a),x, algorithm="fricas")
```

output
```
(d*dilog(cosh(b*x + a) + sinh(b*x + a)) - d*dilog(-cosh(b*x + a) - sinh(b*
x + a)) - (b*d*x + b*c)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (b*c - a*
d)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b*d*x + a*d)*log(-cosh(b*x +
a) - sinh(b*x + a) + 1))/b^2
```

3.25.6 Sympy [F]

$$\int (c+dx)\text{csch}(a+bx)\,dx=\int (c+dx)\,\text{csch}\,(a+bx)\,dx$$

input
```
integrate((d*x+c)*csch(b*x+a),x)
```

output
```
Integral((c + d*x)*csch(a + b*x), x)
```

3.25.7 Maxima [F]

$$\int (c+dx)\text{csch}(a+bx)\,dx=\int (dx+c)\,\text{csch}\,(bx+a)\,\,dx$$

input
```
integrate((d*x+c)*csch(b*x+a),x, algorithm="maxima")
```

output
```
-c*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b) + 2*d*(integrate(1/
2*x/(e^(b*x + a) + 1), x) + integrate(1/2*x/(e^(b*x + a) - 1), x))
```

3.25.8 Giac [F]

$$\int (c+dx)\text{csch}(a+bx)\,dx=\int (dx+c)\,\text{csch}\,(bx+a)\,\,dx$$

input
```
integrate((d*x+c)*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*csch(b*x + a), x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\mathrm{csch}(a+bx)\,dx=\int \frac{c+d\,x}{\sinh\left(a+b\,x\right)}\,dx$$

input
```
int((c + d*x)/sinh(a + b*x),x)
```

output
```
int((c + d*x)/sinh(a + b*x), x)
```

3.26 $\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx$

3.26.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx=\operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)}{c+dx},x\right)$$

output
```
Unintegrable(csch(b*x+a)/(d*x+c),x)
```

3.26.2 Mathematica [N/A]

Not integrable

Time = 14.74 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Csch[a + b*x]/(c + d*x),x]
```

output
```
Integrate[Csch[a + b*x]/(c + d*x), x]
```

3.26.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 26, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i \csc(ia+ibx)}{c+dx}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\csc(ia+ibx)}{c+dx}dx$$
$$\Big\downarrow \text{4680}$$
$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}dx$$

input
```
Int[Csch[a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

3.26.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

3.26.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)}{dx+c}dx$$

input
```
int(csch(b*x+a)/(d*x+c),x)
```

output
```
int(csch(b*x+a)/(d*x+c),x)
```

3.26.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}(bx+a)}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)/(d*x + c), x)
```

3.26.6 Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c),x)
```

output
```
Integral(csch(a + b*x)/(c + d*x), x)
```

3.26.7 Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(csch(b*x + a)/(d*x + c), x)
```

3.26.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)/(d*x + c), x)
```

3.26.9 Mupad [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx = \int \frac{1}{\sinh(a+b\,x)\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(sinh(a + b*x)*(c + d*x)),x)
```

output
```
int(1/(sinh(a + b*x)*(c + d*x)), x)
```

3.27 $\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} dx$

3.27.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} dx = \operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Unintegrable(csch(b*x+a)/(d*x+c)^2,x)
```

3.27.2 Mathematica [N/A]

Not integrable

Time = 15.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} dx = \int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} dx$$

input
```
Integrate[Csch[a + b*x]/(c + d*x)^2,x]
```

output
```
Integrate[Csch[a + b*x]/(c + d*x)^2, x]
```

3.27.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 26, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i \csc(ia+ibx)}{(c+dx)^2}dx$$
$$\Big\downarrow \text{26}$$
$$i \int \frac{\csc(ia+ibx)}{(c+dx)^2}dx$$
$$\Big\downarrow \text{4680}$$
$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}dx$$

input
```
Int[Csch[a + b*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

3.27.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

3.27.4 Maple [N/A] (verified)

Not integrable

Time = 0.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)}{\left(dx+c\right)^{2}}dx$$

input
```
int(csch(b*x+a)/(d*x+c)^2,x)
```

output
```
int(csch(b*x+a)/(d*x+c)^2,x)
```

3.27.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)}{\left(dx+c\right)^{2}}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.27.6 Sympy [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(csch(a + b*x)/(c + d*x)**2, x)
```

3.27.7 Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(csch(b*x + a)/(d*x + c)^2, x)
```

3.27.8 Giac [N/A]

Not integrable

Time = 2.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)/(d*x + c)^2, x)
```

3.27.9 Mupad [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\sinh\left(a+b\,x\right)\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/(sinh(a + b*x)*(c + d*x)^2),x)
```

output
```
int(1/(sinh(a + b*x)*(c + d*x)^2), x)
```

3.28 $\int (c+dx)^3 \mathbf{csch}^2(a+bx)\,dx$

3.28.1 Optimal result

Integrand size = 16, antiderivative size = 103

$$\int (c+dx)^3\operatorname{csch}^2(a+bx)\,dx = -\frac{(c+dx)^3}{b}-\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3d^2(c+dx)\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}-\frac{3d^3\operatorname{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^4}$$

output
```
-(d*x+c)^3/b-(d*x+c)^3*coth(b*x+a)/b+3*d*(d*x+c)^2*ln(1-exp(2*b*x+2*a))/b^
2+3*d^2*(d*x+c)*polylog(2,exp(2*b*x+2*a))/b^3-3/2*d^3*polylog(3,exp(2*b*x+
2*a))/b^4
```

3.28.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.80

$$\int (c+dx)^3\operatorname{csch}^2(a+bx)\,dx$$
$$= \frac{-\frac{2(c+dx)^3}{-1+e^{2a}}+\frac{3d(c+dx)^2\log(1-e^{-a-bx})}{b}+\frac{3d(c+dx)^2\log(1+e^{-a-bx})}{b}-\frac{6d^2(b(c+dx)\operatorname{PolyLog}(2,-e^{-a-bx})+d\operatorname{PolyLog}(3,-e^{-a-bx}))}{b^3}-}{b}$$

input
```
Integrate[(c + d*x)^3*Csch[a + b*x]^2,x]
```

output
```
((-2*(c + d*x)^3)/(-1 + E^(2*a)) + (3*d*(c + d*x)^2*Log[1 - E^(-a - b*x)])
/b + (3*d*(c + d*x)^2*Log[1 + E^(-a - b*x)])/b - (6*d^2*(b*(c + d*x)*PolyL
og[2, -E^(-a - b*x)] + d*PolyLog[3, -E^(-a - b*x)]))/b^3 - (6*d^2*(b*(c +
d*x)*PolyLog[2, E^(-a - b*x)] + d*PolyLog[3, E^(-a - b*x)]))/b^3 + (c + d*
x)^3*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b
```

3.28.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.42, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 25, 4672, 26, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^3\csc(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^3\csc(ia+ibx)^2dx$$

$$\Big\downarrow \text{4672}$$

$$-\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{3id\int -i(c+dx)^2\coth(a+bx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{3d\int (c+dx)^2\coth(a+bx)dx}{b}-\frac{(c+dx)^3\coth(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{3d\int -i(c+dx)^2\tan\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{(c+dx)^3\coth(a+bx)}{b}-\frac{3id\int(c+dx)^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx}{b}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{(c+dx)^3\coth(a+bx)}{b}-\frac{3id\left(2i\int\frac{e^{2a+2bx-i\pi}(c+dx)^2}{1+e^{2a+2bx-i\pi}}dx-\frac{i(c+dx)^3}{3d}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{(c+dx)^3\coth(a+bx)}{b}-\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{d\int(c+dx)\log\left(1+e^{2a+2bx-i\pi}\right)dx}{b}\right)-\frac{i(c+dx)^3}{3d}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{(c+dx)^3\coth(a+bx)}{b}-\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{d\left(\frac{d\int\operatorname{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{2b}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{b}\right)-\frac{i(c+dx)^3}{3d}\right)}{b}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{(c+dx)^3\coth(a+bx)}{b}-\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{d\left(\frac{d\int e^{-2a-2bx+i\pi}\operatorname{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{b}\right)-i(\ldots}{b}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{(c+dx)^3\coth(a+bx)}{b}-\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{d\left(\frac{d\operatorname{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{4b^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{b}\right)-\frac{i(c+dx)^3}{3d}\right)}{b}$$

input

```
Int[(c + d*x)^3*Csch[a + b*x]^2,x]
```

3.28. $\int (c+dx)^3\operatorname{csch}^2(a+bx)\,dx$

output
```
-(((c + d*x)^3*Coth[a + b*x])/b) - ((3*I)*d*(((-1/3*I)*(c + d*x)^3)/d + (2
*I)*(((c + d*x)^2*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) - (d*(-1/2*((c +
d*x)*PolyLog[2, -E^(2*a - I*Pi + 2*b*x)])/b + (d*PolyLog[3, -E^(2*a - I*Pi
 + 2*b*x)])/(4*b^2)))/b)))/b
```

3.28.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.28.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(101) = 202$.

Time = 2.08 (sec) , antiderivative size = 473, normalized size of antiderivative = 4.59

method	result
risch	$-\frac{2(d^3x^3+3c\,d^2x^2+3dx\,c^2+c^3)}{(\mathrm{e}^{2bx+2a}-1)b}-\frac{2d^3x^3}{b}+\frac{4d^3a^3}{b^4}-\frac{6d^3\,\mathrm{polylog}(3,\mathrm{e}^{bx+a})}{b^4}-\frac{6d^3\,\mathrm{polylog}(3,-\mathrm{e}^{bx+a})}{b^4}-\frac{6d\,c^2\ln(\mathrm{e}^{bx+a})}{b^2}+$

input
```
int((d*x+c)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/(exp(2*b*x+2*a)-1)/b-2*d^3/b*x^3+4*
d^3/b^4*a^3-6*d^3/b^4*polylog(3,exp(b*x+a))-6*d^3/b^4*polylog(3,-exp(b*x+a
))-6*d/b^2*c^2*ln(exp(b*x+a))+3*d/b^2*c^2*ln(exp(b*x+a)-1)+6*d^2/b^3*c*pol
ylog(2,exp(b*x+a))-6*d^2/b^3*c*a^2-12*d^2/b^2*a*c*x+12*d^2/b^3*c*a*ln(exp(
b*x+a))-6*d^2/b^3*c*a*ln(exp(b*x+a)-1)+6*d^2/b^2*c*ln(1-exp(b*x+a))*x+6*d^
2/b^3*c*ln(1-exp(b*x+a))*a+6*d^2/b^2*c*ln(exp(b*x+a)+1)*x+6*d^3/b^3*a^2*x-
6*d^2/b*c*x^2+6*d^3/b^3*polylog(2,-exp(b*x+a))*x+3*d^3/b^2*ln(exp(b*x+a)+1
)*x^2+3*d^3/b^2*ln(1-exp(b*x+a))*x^2-3*d^3/b^4*ln(1-exp(b*x+a))*a^2+6*d^3/
b^3*polylog(2,exp(b*x+a))*x+6*d^2/b^3*c*polylog(2,-exp(b*x+a))+3*d/b^2*c^2
*ln(exp(b*x+a)+1)-6*d^3/b^4*a^2*ln(exp(b*x+a))+3*d^3/b^4*a^2*ln(exp(b*x+a)
-1)
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1159 vs. $2(100) = 200$.

Time = 0.28 (sec) , antiderivative size = 1159, normalized size of antiderivative = 11.25

$$\int (c+dx)^3 \operatorname{csch}^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 2*a^3*d^3 + 2*(b^3*d^3*x^3 +
 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3
)*cosh(b*x + a)^2 + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cosh(b*x + a)*sinh(b*x + a) + 2*(b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
 + a^3*d^3)*sinh(b*x + a)^2 + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*c
osh(b*x + a)^2 - 2*(b*d^3*x + b*c*d^2)*cosh(b*x + a)*sinh(b*x + a) - (b*d^
3*x + b*c*d^2)*sinh(b*x + a)^2)*dilog(cosh(b*x + a) + sinh(b*x + a)) + 6*(
b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cosh(b*x + a)^2 - 2*(b*d^3*x + b*c
*d^2)*cosh(b*x + a)*sinh(b*x + a) - (b*d^3*x + b*c*d^2)*sinh(b*x + a)^2)*d
ilog(-cosh(b*x + a) - sinh(b*x + a)) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^
2*c^2*d - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cosh(b*x + a)^2 - 2*(b
^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cosh(b*x + a)*sinh(b*x + a) - (b^2
*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sinh(b*x + a)^2)*log(cosh(b*x + a) +
 sinh(b*x + a) + 1) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3 - (b^2*c^2*d -
2*a*b*c*d^2 + a^2*d^3)*cosh(b*x + a)^2 - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*cosh(b*x + a)*sinh(b*x + a) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sin
h(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 3*(b^2*d^3*x^2 + 2*
b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3 - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b
*c*d^2 - a^2*d^3)*cosh(b*x + a)^2 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*...
```

3.28.6 Sympy [F]

$$\int (c+dx)^3 \operatorname{csch}^2(a+bx)\,dx = \int (c+dx)^3 \operatorname{csch}^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*csch(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*csch(a + b*x)**2, x)
```

3.28.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(100) = 200$.

Time = 0.34 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.11

$$
\begin{aligned}
&\int (c+dx)^3 \operatorname{csch}^2(a+bx)\,dx \\
&= -3\,c^2 d\left(\frac{2\,xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}-b} - \frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2} - \frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}\right) \\
&\quad + \frac{6\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)cd^2}{b^3} \\
&\quad + \frac{6\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)\right)cd^2}{b^3} + \frac{2\,c^3}{b(e^{(-2\,bx-2\,a)}-1)} - \frac{2\,(d^3x^3+3\,cd^2x^2)}{be^{(2\,bx+2\,a)}-b} \\
&\quad + \frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3(-e^{(bx+a)})\right)d^3}{b^4} \\
&\quad + \frac{3\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right)-2\,\mathrm{Li}_3(e^{(bx+a)})\right)d^3}{b^4} \\
&\quad - \frac{2\,(b^3d^3x^3+3\,b^3cd^2x^2)}{b^4}
\end{aligned}
$$

input
```
integrate((d*x+c)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-3*c^2*d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) +
 1)*e^(-a))/b^2 - log((e^(b*x + a) - 1)*e^(-a))/b^2) + 6*(b*x*log(e^(b*x +
 a) + 1) + dilog(-e^(b*x + a)))*c*d^2/b^3 + 6*(b*x*log(-e^(b*x + a) + 1) +
 dilog(e^(b*x + a)))*c*d^2/b^3 + 2*c^3/(b*(e^(-2*b*x - 2*a) - 1)) - 2*(d^3
*x^3 + 3*c*d^2*x^2)/(b*e^(2*b*x + 2*a) - b) + 3*(b^2*x^2*log(e^(b*x + a) +
 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))*d^3/b^4 + 3*
(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e
^(b*x + a)))*d^3/b^4 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2)/b^4
```

3.28.8 Giac [F]

$$\int (c+dx)^3\text{csch}^2(a+bx)\,dx = \int (dx+c)^3\,\text{csch}\,(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^3*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csch(b*x + a)^2, x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\text{csch}^2(a+bx)\,dx = \int \frac{(c+d\,x)^3}{\sinh\,(a+b\,x)^2}\,dx$$

input
```
int((c + d*x)^3/sinh(a + b*x)^2,x)
```

output
```
int((c + d*x)^3/sinh(a + b*x)^2, x)
```

3.29 $\int (c+dx)^2 \mathrm{csch}^2(a+bx)\,dx$

3.29.1 Optimal result

Integrand size = 16, antiderivative size = 74

$$\int (c+dx)^2 \mathrm{csch}^2(a+bx)\,dx = -\frac{(c+dx)^2}{b} - \frac{(c+dx)^2 \coth(a+bx)}{b} + \frac{2d(c+dx)\log\left(1-e^{2(a+bx)}\right)}{b^2} + \frac{d^2 \operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}$$

output
```
-(d*x+c)^2/b-(d*x+c)^2*coth(b*x+a)/b+2*d*(d*x+c)*ln(1-exp(2*b*x+2*a))/b^2+
d^2*polylog(2,exp(2*b*x+2*a))/b^3
```

3.29.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.85

$$\int (c+dx)^2 \mathrm{csch}^2(a+bx)\,dx$$

$$= \frac{-\frac{2b(c+dx)\left(b(c+dx)-d(-1+e^{2a})\log(1-e^{-a-bx})-d(-1+e^{2a})\log(1+e^{-a-bx})\right)}{-1+e^{2a}} - 2d^2 \operatorname{PolyLog}\left(2,-e^{-a-bx}\right) - 2d^2 \operatorname{PolyLog}\left(\right.}{b^3}$$

input
```
Integrate[(c + d*x)^2*Csch[a + b*x]^2,x]
```

output
```
((-2*b*(c + d*x)*(b*(c + d*x) - d*(-1 + E^(2*a))*Log[1 - E^(-a - b*x)] - d
*(-1 + E^(2*a))*Log[1 + E^(-a - b*x)]))/(-1 + E^(2*a)) - 2*d^2*PolyLog[2,
-E^(-a - b*x)] - 2*d^2*PolyLog[2, E^(-a - b*x)] + b^2*(c + d*x)^2*Csch[a]*
Csch[a + b*x]*Sinh[b*x])/b^3
```

3.29.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.43, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 25, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^2\csc(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^2\csc(ia+ibx)^2dx$$

$$\Big\downarrow \text{4672}$$

$$-\frac{(c+dx)^2\coth(a+bx)}{b}+\frac{2id\int -i(c+dx)\coth(a+bx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{2d\int (c+dx)\coth(a+bx)dx}{b}-\frac{(c+dx)^2\coth(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(c+dx)^2\coth(a+bx)}{b}+\frac{2d\int -i(c+dx)\tan\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{(c+dx)^2\coth(a+bx)}{b}-\frac{2id\int (c+dx)\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx}{b}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{(c+dx)^2\coth(a+bx)}{b}-\frac{2id\left(2i\int \frac{e^{2a+2bx-i\pi}(c+dx)}{1+e^{2a+2bx-i\pi}}dx-\frac{i(c+dx)^2}{2d}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{(c+dx)^2\coth(a+bx)}{b}-\frac{2id\left(2i\left(\frac{(c+dx)\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{d\int \log\left(1+e^{2a+2bx-i\pi}\right)dx}{2b}\right)-\frac{i(c+dx)^2}{2d}\right)}{b}$$

$$\Big\downarrow \color{blue}{2715}$$

$$-\frac{(c+dx)^2\coth(a+bx)}{b}-\frac{2id\left(2i\left(\frac{(c+dx)\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{d\int e^{-2a-2bx+i\pi}\log\left(1+e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{b}$$

$$\Big\downarrow \color{blue}{2838}$$

$$-\frac{(c+dx)^2\coth(a+bx)}{b}-\frac{2id\left(2i\left(\frac{d\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{4b^2}+\frac{(c+dx)\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}\right)-\frac{i(c+dx)^2}{2d}\right)}{b}$$

input
```
Int[(c + d*x)^2*Csch[a + b*x]^2,x]
```

output
```
-(((c + d*x)^2*Coth[a + b*x])/b) - ((2*I)*d*(((-1/2*I)*(c + d*x)^2)/d + (2
*I)*(((c + d*x)*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) + (d*PolyLog[2, -E^
(2*a - I*Pi + 2*b*x)])/(4*b^2))))/b
```

3.29.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.29.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(74) = 148$.

Time = 1.76 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.24

method	result
risch	$-\frac{2(d^2x^2+2cdx+c^2)}{(e^{2bx+2a}-1)b}+\frac{2dc\ln(e^{bx+a}+1)}{b^2}-\frac{4dc\ln(e^{bx+a})}{b^2}+\frac{2dc\ln(e^{bx+a}-1)}{b^2}-\frac{2d^2x^2}{b}-\frac{4d^2ax}{b^2}-\frac{2d^2a^2}{b^3}+\frac{2d^2\ln(1-e^{b}}{b^2}$

input
```
int((d*x+c)^2*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(d^2*x^2+2*c*d*x+c^2)/(exp(2*b*x+2*a)-1)/b+2*d/b^2*c*ln(exp(b*x+a)+1)-4
*d/b^2*c*ln(exp(b*x+a))+2*d/b^2*c*ln(exp(b*x+a)-1)-2*d^2/b*x^2-4*d^2/b^2*a
*x-2*d^2/b^3*a^2+2*d^2/b^2*ln(1-exp(b*x+a))*x+2*d^2/b^3*ln(1-exp(b*x+a))*a
+2*d^2/b^3*polylog(2,exp(b*x+a))+2*d^2/b^2*ln(exp(b*x+a)+1)*x+2*d^2/b^3*po
lylog(2,-exp(b*x+a))+4*d^2/b^3*a*ln(exp(b*x+a))-2*d^2/b^3*a*ln(exp(b*x+a)-
1)
```

3.29.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(73) = 146$.

Time = 0.24 (sec) , antiderivative size = 623, normalized size of antiderivative = 8.42

$$\int (c+dx)^2 \operatorname{csch}^2(a+bx)\,dx = -\frac{2\left(b^2c^2 - 2\,abcd + a^2d^2 + \left(b^2d^2x^2 + 2\,b^2cdx + 2\,abcd - a^2d^2\right)\cosh\left(bx+a\right)^2 + 2\left(b^2d^2x^2 + 2\,b^2cdx + 2\,a\right.}{}$$

input
```
integrate((d*x+c)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
 - a^2*d^2)*cosh(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a
^2*d^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c
*d - a^2*d^2)*sinh(b*x + a)^2 - (d^2*cosh(b*x + a)^2 + 2*d^2*cosh(b*x + a)
*sinh(b*x + a) + d^2*sinh(b*x + a)^2 - d^2)*dilog(cosh(b*x + a) + sinh(b*x
 + a)) - (d^2*cosh(b*x + a)^2 + 2*d^2*cosh(b*x + a)*sinh(b*x + a) + d^2*si
nh(b*x + a)^2 - d^2)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*d^2*x + b*
c*d - (b*d^2*x + b*c*d)*cosh(b*x + a)^2 - 2*(b*d^2*x + b*c*d)*cosh(b*x + a
)*sinh(b*x + a) - (b*d^2*x + b*c*d)*sinh(b*x + a)^2)*log(cosh(b*x + a) + s
inh(b*x + a) + 1) + (b*c*d - a*d^2 - (b*c*d - a*d^2)*cosh(b*x + a)^2 - 2*(
b*c*d - a*d^2)*cosh(b*x + a)*sinh(b*x + a) - (b*c*d - a*d^2)*sinh(b*x + a)
^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b*d^2*x + a*d^2 - (b*d^2*x +
 a*d^2)*cosh(b*x + a)^2 - 2*(b*d^2*x + a*d^2)*cosh(b*x + a)*sinh(b*x + a)
- (b*d^2*x + a*d^2)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) +
1))/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*
x + a)^2 - b^3)
```

3.29.6 Sympy [F]

$$\int (c+dx)^2 \operatorname{csch}^2(a+bx)\,dx = \int (c+dx)^2 \operatorname{csch}^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*csch(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*csch(a + b*x)**2, x)
```

3.29.7 Maxima [F]

$$\int (c+dx)^2\text{csch}^2(a+bx)\,dx = \int (dx+c)^2\,\text{csch}\,(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-2*d^2*(x^2/(b*e^(2*b*x + 2*a) - b) + 2*integrate(1/2*x/(b*e^(b*x + a) + b
), x) - 2*integrate(1/2*x/(b*e^(b*x + a) - b), x)) - 2*c*d*(2*x*e^(2*b*x +
 2*a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) + 1)*e^(-a))/b^2 - log((e
^(b*x + a) - 1)*e^(-a))/b^2) + 2*c^2/(b*(e^(-2*b*x - 2*a) - 1))
```

3.29.8 Giac [F]

$$\int (c+dx)^2\text{csch}^2(a+bx)\,dx = \int (dx+c)^2\,\text{csch}\,(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^2*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csch(b*x + a)^2, x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\text{csch}^2(a+bx)\,dx = \int \frac{(c+d\,x)^2}{\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int((c + d*x)^2/sinh(a + b*x)^2,x)
```

output
```
int((c + d*x)^2/sinh(a + b*x)^2, x)
```

3.30 $\int (c+dx)\text{csch}^2(a+bx)\,dx$

3.30.1 Optimal result

Integrand size = 14, antiderivative size = 29

$$\int (c+dx)\text{csch}^2(a+bx)\,dx = -\frac{(c+dx)\coth(a+bx)}{b}+\frac{d\log(\sinh(a+bx))}{b^2}$$

output
```
-(d*x+c)*coth(b*x+a)/b+d*ln(sinh(b*x+a))/b^2
```

3.30.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.79

$$\int (c+dx)\text{csch}^2(a+bx)\,dx = -\frac{dx\coth(a)}{b}-\frac{c\coth(a+bx)}{b}+\frac{d\log(\sinh(a+bx))}{b^2}+\frac{dx\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b}$$

input
```
Integrate[(c + d*x)*Csch[a + b*x]^2,x]
```

output
```
-((d*x*Coth[a])/b) - (c*Coth[a + b*x])/b + (d*Log[Sinh[a + b*x]])/b^2 + (d
*x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b
```

3.30.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 25, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\left((c+dx)\csc(ia+ibx)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)\csc(ia+ibx)^2dx$$

$$\Big\downarrow \text{4672}$$

$$-\frac{(c+dx)\coth(a+bx)}{b}+\frac{id\int -i\coth(a+bx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{d\int \coth(a+bx)dx}{b}-\frac{(c+dx)\coth(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(c+dx)\coth(a+bx)}{b}+\frac{d\int -i\tan\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{(c+dx)\coth(a+bx)}{b}-\frac{id\int \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx}{b}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{(c+dx)\coth(a+bx)}{b}+\frac{d\log(-i\sinh(a+bx))}{b^2}$$

input
```
Int[(c + d*x)*Csch[a + b*x]^2,x]
```

output
```
-(((c + d*x)*Coth[a + b*x])/b) + (d*Log[(-I)*Sinh[a + b*x]])/b^2
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.30.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.93

method	result	size
risch	$-\frac{2dx}{b}-\frac{2da}{b^{2}}-\frac{2(dx+c)}{\left(\mathrm{e}^{2bx+2a}-1\right)b}+\frac{d\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b^{2}}$	56
parallelrisch	$\frac{-4\ln\left(1-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+2\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-b\left(\coth\left(\frac{bx}{2}+\frac{a}{2}\right)(dx+c)+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)(dx+c)+2dx\right)}{2b^{2}}$	75

input
```
int((d*x+c)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*d/b*x-2*d/b^2*a-2*(d*x+c)/(exp(2*b*x+2*a)-1)/b+d/b^2*ln(exp(2*b*x+2*a)-
1)
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(29) = 58$.

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 5.72

$$\int (c+dx)\text{csch}^2(a+bx)\,dx =$$

$$-\frac{2\,bdx\cosh\left(bx+a\right)^2+4\,bdx\cosh\left(bx+a\right)\sinh\left(bx+a\right)+2\,bdx\sinh\left(bx+a\right)^2+2\,bc-\left(d\cosh\left(bx+a\right)\right.}{b^2\cosh\left(bx+a\right)^2+2\,b^2\cosh\left(bx+a\right)\sinh\left(bx+\right.}$$

input
```
integrate((d*x+c)*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*b*d*x*cosh(b*x + a)^2 + 4*b*d*x*cosh(b*x + a)*sinh(b*x + a) + 2*b*d*x*
sinh(b*x + a)^2 + 2*b*c - (d*cosh(b*x + a)^2 + 2*d*cosh(b*x + a)*sinh(b*x
+ a) + d*sinh(b*x + a)^2 - d)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*
x + a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*s
inh(b*x + a)^2 - b^2)
```

3.30.6 Sympy [F]

$$\int (c+dx)\text{csch}^2(a+bx)\,dx = \int (c+dx)\,\text{csch}^2\left(a+bx\right)dx$$

input
```
integrate((d*x+c)*csch(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*csch(a + b*x)**2, x)
```

3.30.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(29) = 58$.

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.14

$$\begin{aligned}&\int (c+dx)\text{csch}^2(a+bx)\,dx\\ &= -d\left(\frac{2\,xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}-b}-\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2}-\frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}\right)\\ &\quad+\frac{2\,c}{b(e^{(-2\,bx-2\,a)}-1)}\end{aligned}$$

input
```
integrate((d*x+c)*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) + 1)*e^
(-a))/b^2 - log((e^(b*x + a) - 1)*e^(-a))/b^2) + 2*c/(b*(e^(-2*b*x - 2*a)
- 1))
```

3.30.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(29) = 58$.

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.76

$$\int (c+dx)\text{csch}^2(a+bx)\,dx = -\frac{2\,bdxe^{(2\,bx+2\,a)} - de^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)}-1\right) + 2\,bc + d\log\left(e^{(2\,bx+2\,a)}-1\right)}{b^2e^{(2\,bx+2\,a)} - b^2}$$

input
```
integrate((d*x+c)*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-(2*b*d*x*e^(2*b*x + 2*a) - d*e^(2*b*x + 2*a)*log(e^(2*b*x + 2*a) - 1) + 2
*b*c + d*log(e^(2*b*x + 2*a) - 1))/(b^2*e^(2*b*x + 2*a) - b^2)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int (c+dx)\text{csch}^2(a+bx)\,dx = \frac{d\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b^2} - \frac{2\,(c+d\,x)}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} - \frac{2\,d\,x}{b}$$

input
```
int((c + d*x)/sinh(a + b*x)^2,x)
```

output
```
(d*log(exp(2*a)*exp(2*b*x) - 1))/b^2 - (2*(c + d*x))/(b*(exp(2*a + 2*b*x)
- 1)) - (2*d*x)/b
```

3.31 $\int \frac{\operatorname{csch}^2(a+bx)}{c+dx} \, dx$

3.31.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx} \, dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(a+bx)}{c+dx}, x\right)$$

output
```
Unintegrable(csch(b*x+a)^2/(d*x+c),x)
```

3.31.2 Mathematica [N/A]

Not integrable

Time = 22.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx} \, dx = \int \frac{\operatorname{csch}^2(a+bx)}{c+dx} \, dx$$

input
```
Integrate[Csch[a + b*x]^2/(c + d*x),x]
```

output
```
Integrate[Csch[a + b*x]^2/(c + d*x), x]
```

3.31.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 25, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\csc(ia+ibx)^2}{c+dx}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{\csc(ia+ibx)^2}{c+dx}dx$$
$$\Big\downarrow \text{4680}$$
$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}dx$$

input
```
Int[Csch[a + b*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

3.31.4 Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{2}}{dx+c}dx$$

input
```
int(csch(b*x+a)^2/(d*x+c),x)
```

output
```
int(csch(b*x+a)^2/(d*x+c),x)
```

3.31.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^{2}}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^2/(d*x + c), x)
```

3.31.6 Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(csch(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(csch(a + b*x)**2/(c + d*x), x)
```

3.31.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 157, normalized size of antiderivative = 9.81

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
4*d*integrate(1/4/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2*e^a + 2*b*c*
d*x*e^a + b*c^2*e^a)*e^(b*x)), x) - 4*d*integrate(-1/4/(b*d^2*x^2 + 2*b*c*
d*x + b*c^2 - (b*d^2*x^2*e^a + 2*b*c*d*x*e^a + b*c^2*e^a)*e^(b*x)), x) + 2
/(b*d*x + b*c - (b*d*x*e^(2*a) + b*c*e^(2*a))*e^(2*b*x))
```

3.31.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^2/(d*x + c), x)
```

3.31.9 Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx=\int \frac{1}{{\mathrm{sinh}\left(a+b\,x\right)}^2\,\left(c+d\,x\right)} \,d x$$

input
```
int(1/(sinh(a + b*x)^2*(c + d*x)),x)
```

output
```
int(1/(sinh(a + b*x)^2*(c + d*x)), x)
```

3.32 $\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2} \, dx$

3.32.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2} \, dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Unintegrable(csch(b*x+a)^2/(d*x+c)^2,x)
```

3.32.2 Mathematica [N/A]

Not integrable

Time = 22.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2} \, dx = \int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2} \, dx$$

input
```
Integrate[Csch[a + b*x]^2/(c + d*x)^2,x]
```

output
```
Integrate[Csch[a + b*x]^2/(c + d*x)^2, x]
```

3.32.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 25, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\csc(ia+ibx)^2}{(c+dx)^2}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{\csc(ia+ibx)^2}{(c+dx)^2}dx$$
$$\Big\downarrow \text{4680}$$
$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}dx$$

input
```
Int[Csch[a + b*x]^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

3.32.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

3.32.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}dx$$

input
```
int(csch(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(csch(b*x+a)^2/(d*x+c)^2,x)
```

3.32.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\,dx$$

input
```
integrate(csch(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.32.6 Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(csch(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(csch(a + b*x)**2/(c + d*x)**2, x)
```

3.32.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 237, normalized size of antiderivative = 14.81

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csch(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
4*d*integrate(1/2/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^
3*x^3*e^a + 3*b*c*d^2*x^2*e^a + 3*b*c^2*d*x*e^a + b*c^3*e^a)*e^(b*x)), x)
- 4*d*integrate(-1/2/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 - (b
*d^3*x^3*e^a + 3*b*c*d^2*x^2*e^a + 3*b*c^2*d*x*e^a + b*c^3*e^a)*e^(b*x)),
x) + 2/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 - (b*d^2*x^2*e^(2*a) + 2*b*c*d*x*e^(
2*a) + b*c^2*e^(2*a))*e^(2*b*x))
```

3.32.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csch(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^2/(d*x + c)^2, x)
```

3.32.9 Mupad [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\sinh{(a+b\,x)}^2\,{(c+d\,x)}^2}\,dx$$

input
```
int(1/(sinh(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(1/(sinh(a + b*x)^2*(c + d*x)^2), x)
```

3.33 $\int (c+dx)^3 \mathbf{csch}^3(a+bx)\,dx$

3.33.1 Optimal result

Integrand size = 16, antiderivative size = 256

$$
\begin{aligned}
\int (c+dx)^3 \operatorname{csch}^3(a+bx)\,dx = &-\frac{6d^2(c+dx)\operatorname{arctanh}\left(e^{a+bx}\right)}{b^3} \\
&+\frac{(c+dx)^3\operatorname{arctanh}\left(e^{a+bx}\right)}{b}-\frac{3d(c+dx)^2\operatorname{csch}(a+bx)}{2b^2} \\
&-\frac{(c+dx)^3\coth(a+bx)\operatorname{csch}(a+bx)}{2b} \\
&-\frac{3d^3\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^4} \\
&+\frac{3d(c+dx)^2\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} \\
&+\frac{3d^3\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^4}-\frac{3d(c+dx)^2\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{2b^2} \\
&-\frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} \\
&+\frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^3} \\
&+\frac{3d^3\operatorname{PolyLog}\left(4,-e^{a+bx}\right)}{b^4}-\frac{3d^3\operatorname{PolyLog}\left(4,e^{a+bx}\right)}{b^4}
\end{aligned}
$$

output
```
-6*d^2*(d*x+c)*arctanh(exp(b*x+a))/b^3+(d*x+c)^3*arctanh(exp(b*x+a))/b-3/2
*d*(d*x+c)^2*csch(b*x+a)/b^2-1/2*(d*x+c)^3*coth(b*x+a)*csch(b*x+a)/b-3*d^3
*polylog(2,-exp(b*x+a))/b^4+3/2*d*(d*x+c)^2*polylog(2,-exp(b*x+a))/b^2+3*d
^3*polylog(2,exp(b*x+a))/b^4-3/2*d*(d*x+c)^2*polylog(2,exp(b*x+a))/b^2-3*d
^2*(d*x+c)*polylog(3,-exp(b*x+a))/b^3+3*d^2*(d*x+c)*polylog(3,exp(b*x+a))/
b^3+3*d^3*polylog(4,-exp(b*x+a))/b^4-3*d^3*polylog(4,exp(b*x+a))/b^4
```

3.33.2 Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.72

$$\int (c+dx)^3 \operatorname{csch}^3(a+bx)\,dx =$$

$$-\frac{b^2(c+dx)^2(3d+b(c+dx)\coth(a+bx))\operatorname{csch}(a+bx)+b^3c^3\log\left(1-e^{a+bx}\right)-6bcd^2\log\left(1-e^{a+bx}\right)+3$$

input
```
Integrate[(c + d*x)^3*Csch[a + b*x]^3,x]
```

output
```
-1/2*(b^2*(c + d*x)^2*(3*d + b*(c + d*x)*Coth[a + b*x])*Csch[a + b*x] + b^
3*c^3*Log[1 - E^(a + b*x)] - 6*b*c*d^2*Log[1 - E^(a + b*x)] + 3*b^3*c^2*d*
x*Log[1 - E^(a + b*x)] - 6*b*d^3*x*Log[1 - E^(a + b*x)] + 3*b^3*c*d^2*x^2*
Log[1 - E^(a + b*x)] + b^3*d^3*x^3*Log[1 - E^(a + b*x)] - b^3*c^3*Log[1 +
E^(a + b*x)] + 6*b*c*d^2*Log[1 + E^(a + b*x)] - 3*b^3*c^2*d*x*Log[1 + E^(a
 + b*x)] + 6*b*d^3*x*Log[1 + E^(a + b*x)] - 3*b^3*c*d^2*x^2*Log[1 + E^(a +
 b*x)] - b^3*d^3*x^3*Log[1 + E^(a + b*x)] - 3*d*(-2*d^2 + b^2*(c + d*x)^2)
*PolyLog[2, -E^(a + b*x)] + 3*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(a
 + b*x)] + 6*b*c*d^2*PolyLog[3, -E^(a + b*x)] + 6*b*d^3*x*PolyLog[3, -E^(a
 + b*x)] - 6*b*c*d^2*PolyLog[3, E^(a + b*x)] - 6*b*d^3*x*PolyLog[3, E^(a +
 b*x)] - 6*d^3*PolyLog[4, -E^(a + b*x)] + 6*d^3*PolyLog[4, E^(a + b*x)])/b
^4
```

3.33.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {3042, 26, 4674, 26, 3042, 26, 4670, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\text{csch}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(c+dx)^3\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)^3\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{4674}$$

$$-i\left(-\frac{3d^2\int -i(c+dx)\text{csch}(a+bx)dx}{b^2}+\frac{1}{2}\int -i(c+dx)^3\text{csch}(a+bx)dx-\frac{3id(c+dx)^2\text{csch}(a+bx)}{2b^2}-\frac{i(c+dx)\right.$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{3id^2\int (c+dx)\text{csch}(a+bx)dx}{b^2}-\frac{1}{2}i\int (c+dx)^3\text{csch}(a+bx)dx-\frac{3id(c+dx)^2\text{csch}(a+bx)}{2b^2}-\frac{i(c+dx)^3\coth\right.$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{3id^2\int i(c+dx)\csc(ia+ibx)dx}{b^2}-\frac{1}{2}i\int i(c+dx)^3\csc(ia+ibx)dx-\frac{3id(c+dx)^2\text{csch}(a+bx)}{2b^2}-\frac{i(c+dx)^3\text{c}\right.$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{3d^2\int (c+dx)\csc(ia+ibx)dx}{b^2}+\frac{1}{2}\int (c+dx)^3\csc(ia+ibx)dx-\frac{3id(c+dx)^2\text{csch}(a+bx)}{2b^2}-\frac{i(c+dx)^3\text{cot}\right.$$

$$\Big\downarrow \text{4670}$$

$$-i\left(-\frac{3d^2\left(\frac{id\int\log(1-e^{a+bx})dx}{b}-\frac{id\int\log(1+e^{a+bx})dx}{b}+\frac{2i(c+dx)\text{arctanh}(e^{a+bx})}{b}\right)}{b^2}+\frac{1}{2}\left(\frac{3id\int(c+dx)^2\log\left(1-e^{a+bx}\right)d}{b}\right.\right.$$

$$\Big\downarrow \text{2715}$$

$$-i\left(-\frac{3d^2\left(\frac{id\int e^{-a-bx}\log(1-e^{a+bx})de^{a+bx}}{b^2}-\frac{id\int e^{-a-bx}\log(1+e^{a+bx})de^{a+bx}}{b^2}+\frac{2i(c+dx)\text{arctanh}(e^{a+bx})}{b}\right)}{b^2}+\frac{1}{2}\left(\frac{3id\int(c+dx}{}\right.\right.$$

$$\Big\downarrow \text{2838}$$

$$-i\left(\frac{1}{2}\left(\frac{3id\int(c+dx)^2\log\left(1-e^{a+bx}\right)dx}{b}-\frac{3id\int(c+dx)^2\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2i(c+dx)^3\text{arctanh}(e^{a+bx})}{b}\right)-\right.$$

$$\Big\downarrow \text{3011}$$

$$-i\left(\frac{1}{2}\left(-\frac{3id\left(\frac{2d\int(c+dx)\operatorname{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3id\left(\frac{2d\int(c+dx)\operatorname{PolyLog}(2,e^{a+bx})dx}{b}-\frac{(c+dx)^2}{}\right.}{b}\right.\right.$$

$$\Big\downarrow \text{7163}$$

$$-i\left(\frac{1}{2}\left(-\frac{3id\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{d\int\operatorname{PolyLog}\left(3,-e^{a+bx}\right)dx}{b}\right)}{b}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3id\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}(}{b}\right.}{}\right.}{}\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$-i\left(\frac{1}{2}\left(-\frac{3id\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b}-\frac{d\int e^{-a-bx}\operatorname{PolyLog}\left(3,-e^{a+bx}\right)de^{a+bx}}{b^2}\right)}{b}-\frac{(c+dx)^2\operatorname{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{3id\left(\frac{2d\left(\frac{(c}{}\right.}{}\right.}{}\right.\right.$$

3.33. $\int(c+dx)^3\text{csch}^3(a+bx)\,dx$

$$\Big\downarrow \text{7143}$$

$$-i\left(-\frac{3d^2\left(\frac{2i(c+dx)\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\right)}{b^2}+\frac{1}{2}\left(\frac{2i(c+dx)^3\text{arctanh}\left(e^{a+bx}\right)}{b}-\right.\right.$$

input
```
Int[(c + d*x)^3*Csch[a + b*x]^3,x]
```

output
```
(-I)*((((-3*I)/2)*d*(c + d*x)^2*Csch[a + b*x])/b^2 - ((I/2)*(c + d*x)^3*Co
th[a + b*x]*Csch[a + b*x])/b - (3*d^2*(((2*I)*(c + d*x)*ArcTanh[E^(a + b*x
)])/b + (I*d*PolyLog[2, -E^(a + b*x)])/b^2 - (I*d*PolyLog[2, E^(a + b*x)])
/b^2))/b^2 + (((2*I)*(c + d*x)^3*ArcTanh[E^(a + b*x)])/b - ((3*I)*d*(-(((c
 + d*x)^2*PolyLog[2, -E^(a + b*x)])/b) + (2*d*(((c + d*x)*PolyLog[3, -E^(a
 + b*x)])/b - (d*PolyLog[4, -E^(a + b*x)])/b^2))/b))/b + ((3*I)*d*(-(((c +
 d*x)^2*PolyLog[2, E^(a + b*x)])/b) + (2*d*(((c + d*x)*PolyLog[3, E^(a + b
*x)])/b - (d*PolyLog[4, E^(a + b*x)])/b^2))/b))/b)/2)
```

3.33.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.33. $\int (c+dx)^3 \operatorname{csch}^3(a+bx)\,dx$

3.33.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(238) = 476$.

Time = 1.95 (sec) , antiderivative size = 876, normalized size of antiderivative = 3.42

method	result	size
risch	Expression too large to display	876

input
```
int((d*x+c)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/2/b^2*d^3*polylog(2,exp(b*x+a))*x^2+3/b^3*d^3*polylog(3,exp(b*x+a))*x+1
/2/b*d^3*ln(exp(b*x+a)+1)*x^3+1/2/b^4*d^3*ln(exp(b*x+a)+1)*a^3+3/2/b^2*d^3
*polylog(2,-exp(b*x+a))*x^2-3/b^3*d^3*polylog(3,-exp(b*x+a))*x+3/b^3*c*d^2
*polylog(3,exp(b*x+a))-3/b^3*c*d^2*polylog(3,-exp(b*x+a))-3/2/b^2*c^2*d*po
lylog(2,exp(b*x+a))+3/2/b^2*c^2*d*polylog(2,-exp(b*x+a))-1/b^4*d^3*a^3*arc
tanh(exp(b*x+a))-1/2/b*d^3*ln(1-exp(b*x+a))*x^3-1/2/b^4*d^3*ln(1-exp(b*x+a
))*a^3-exp(b*x+a)*(exp(2*b*x+2*a)*b*d^3*x^3+3*exp(2*b*x+2*a)*b*c*d^2*x^2+3
*exp(2*b*x+2*a)*b*c^2*d*x+b*d^3*x^3+3*exp(2*b*x+2*a)*d^3*x^2+exp(2*b*x+2*a
)*b*c^3+3*b*c*d^2*x^2+6*exp(2*b*x+2*a)*c*d^2*x+3*b*c^2*d*x+3*exp(2*b*x+2*a
)*c^2*d-3*d^3*x^2+b*c^3-6*c*d^2*x-3*c^2*d)/b^2/(exp(2*b*x+2*a)-1)^2+3/2/b^
2*c^2*d*ln(exp(b*x+a)+1)*a+3/b^3*d^2*a^2*c*arctanh(exp(b*x+a))-3/b^2*d*a*c
^2*arctanh(exp(b*x+a))+3/b^2*c*d^2*polylog(2,-exp(b*x+a))*x-3/2/b*c*d^2*ln
(1-exp(b*x+a))*x^2+3/2/b^3*c*d^2*ln(1-exp(b*x+a))*a^2-3/b^2*c*d^2*polylog(
2,exp(b*x+a))*x+3/2/b*c*d^2*ln(exp(b*x+a)+1)*x^2-3/2/b^3*c*d^2*ln(exp(b*x+
a)+1)*a^2-3/2/b*c^2*d*ln(1-exp(b*x+a))*x+1/b*c^3*arctanh(exp(b*x+a))-3/b^4
*d^3*ln(exp(b*x+a)+1)*a+6/b^4*d^3*a*arctanh(exp(b*x+a))-6/b^3*c*d^2*arctan
h(exp(b*x+a))-3*d^3*polylog(4,exp(b*x+a))/b^4-3*d^3*polylog(2,-exp(b*x+a))
/b^4+3*d^3*polylog(2,exp(b*x+a))/b^4+3*d^3*polylog(4,-exp(b*x+a))/b^4-3/2/
b^2*c^2*d*ln(1-exp(b*x+a))*a+3/2/b*c^2*d*ln(exp(b*x+a)+1)*x+3/b^3*d^3*ln(1
-exp(b*x+a))*x+3/b^4*d^3*ln(1-exp(b*x+a))*a-3/b^3*d^3*ln(exp(b*x+a)+1)*...
```

3.33.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4008 vs. $2(234) = 468$.

Time = 0.32 (sec) , antiderivative size = 4008, normalized size of antiderivative = 15.66

$$\int (c+dx)^3 \operatorname{csch}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(b^3*d^3*x^3 + b^3*c^3 + 3*b^2*c^2*d + 3*(b^3*c*d^2 + b^2*d^3)*x^2
 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*x)*cosh(b*x + a)^3 + 6*(b^3*d^3*x^3 + b^3*c
^3 + 3*b^2*c^2*d + 3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^
2)*x)*cosh(b*x + a)*sinh(b*x + a)^2 + 2*(b^3*d^3*x^3 + b^3*c^3 + 3*b^2*c^2
*d + 3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*x)*sinh(b*x
 + a)^3 + 2*(b^3*d^3*x^3 + b^3*c^3 - 3*b^2*c^2*d + 3*(b^3*c*d^2 - b^2*d^3)
*x^2 + 3*(b^3*c^2*d - 2*b^2*c*d^2)*x)*cosh(b*x + a) + 3*(b^2*d^3*x^2 + 2*b
^2*c*d^2*x + b^2*c^2*d + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)
*cosh(b*x + a)^4 + 4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos
h(b*x + a)*sinh(b*x + a)^3 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*
d^3)*sinh(b*x + a)^4 - 2*d^3 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d
- 2*d^3)*cosh(b*x + a)^2 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*
d^3 - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a)^2)
*sinh(b*x + a)^2 + 4*((b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*co
sh(b*x + a)^3 - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x
 + a))*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 3*(b^2*d^3*x^
2 + 2*b^2*c*d^2*x + b^2*c^2*d + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d -
 2*d^3)*cosh(b*x + a)^4 + 4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d
^3)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2
*d - 2*d^3)*sinh(b*x + a)^4 - 2*d^3 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + ...
```

3.33.6 Sympy [F]

$$\int (c+dx)^3\text{csch}^3(a+bx)\,dx = \int (c+dx)^3\,\text{csch}^3\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*csch(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**3*csch(a + b*x)**3, x)
```

3.33.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(234) = 468$.

Time = 0.33 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.36

$$
\begin{aligned}
&\int (c+dx)^3 \operatorname{csch}^3(a+bx)\,dx \\
&= \frac{1}{2}\,c^3\left(\frac{\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{2\left(e^{(-bx-a)}+e^{(-3\,bx-3\,a)}\right)}{b(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1)}\right) \\
&+ \frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3(-e^{(bx+a)})\right)cd^2}{2\,b^3} \\
&- \frac{3\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right)-2\,\mathrm{Li}_3(e^{(bx+a)})\right)cd^2}{2\,b^3} \\
&- \frac{3\,cd^2\log\left(e^{(bx+a)}+1\right)}{b^3} + \frac{3\,cd^2\log\left(e^{(bx+a)}-1\right)}{b^3} \\
&- \frac{\left(bd^3x^3e^{(3\,a)}+3\,c^2de^{(3\,a)}+3\,(bcd^2+d^3)x^2e^{(3\,a)}+3\,(bc^2d+2\,cd^2)xe^{(3\,a)}\right)e^{(3\,bx)}+\left(bd^3x^3e^{a}-3\,c^2de^{a}+3\,(\right.}{b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2} \\
&+ \frac{\left(b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(-e^{(bx+a)})+6\,\mathrm{Li}_4(-e^{(bx+a)})\right)d^3}{2\,b^4} \\
&- \frac{\left(b^3x^3\log\left(-e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(e^{(bx+a)})+6\,\mathrm{Li}_4(e^{(bx+a)})\right)d^3}{2\,b^4} \\
&+ \frac{3\left(b^2c^2d-2\,d^3\right)\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{2\,b^4} \\
&- \frac{3\left(b^2c^2d-2\,d^3\right)\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{2\,b^4}
\end{aligned}
$$

input
```
integrate((d*x+c)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*c^3*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + 2*(e^(-b*x -
a) + e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))) +
3/2*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(
3, -e^(b*x + a)))*c*d^2/b^3 - 3/2*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*d
ilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))*c*d^2/b^3 - 3*c*d^2*log(e^(
b*x + a) + 1)/b^3 + 3*c*d^2*log(e^(b*x + a) - 1)/b^3 - ((b*d^3*x^3*e^(3*a)
 + 3*c^2*d*e^(3*a) + 3*(b*c*d^2 + d^3)*x^2*e^(3*a) + 3*(b*c^2*d + 2*c*d^2)
*x*e^(3*a))*e^(3*b*x) + (b*d^3*x^3*e^a - 3*c^2*d*e^a + 3*(b*c*d^2 - d^3)*x
^2*e^a + 3*(b*c^2*d - 2*c*d^2)*x*e^a)*e^(b*x))/(b^2*e^(4*b*x + 4*a) - 2*b^
2*e^(2*b*x + 2*a) + b^2) + 1/2*(b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*d
ilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x
 + a)))*d^3/b^4 - 1/2*(b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(
b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))*d^3
/b^4 + 3/2*(b^2*c^2*d - 2*d^3)*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x +
 a)))/b^4 - 3/2*(b^2*c^2*d - 2*d^3)*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(
b*x + a)))/b^4
```

3.33.8 Giac [F]

$$\int (c+dx)^3\text{csch}^3(a+bx)\,dx=\int (dx+c)^3\,\text{csch}\,(bx+a)^3\,\,dx$$

input
```
integrate((d*x+c)^3*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csch(b*x + a)^3, x)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\text{csch}^3(a+bx)\,dx=\int \frac{(c+d\,x)^3}{\sinh\,(a+b\,x)^3}\,dx$$

input
```
int((c + d*x)^3/sinh(a + b*x)^3,x)
```

output
```
int((c + d*x)^3/sinh(a + b*x)^3, x)
```

3.34 $\int (c+dx)^2 \mathrm{csch}^3(a+bx)\,dx$

3.34.1 Optimal result

Integrand size = 16, antiderivative size = 154

$$\begin{aligned}\int (c+dx)^2\mathrm{csch}^3(a+bx)\,dx = {} & \frac{(c+dx)^2\operatorname{arctanh}\left(e^{a+bx}\right)}{b}-\frac{d^2\operatorname{arctanh}(\cosh(a+bx))}{b^3}\\ & -\frac{d(c+dx)\mathrm{csch}(a+bx)}{b^2}\\ & -\frac{(c+dx)^2\coth(a+bx)\mathrm{csch}(a+bx)}{2b}\\ & +\frac{d(c+dx)\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}\\ & -\frac{d(c+dx)\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\\ & -\frac{d^2\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}+\frac{d^2\operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^3}\end{aligned}$$

output
```
(d*x+c)^2*arctanh(exp(b*x+a))/b-d^2*arctanh(cosh(b*x+a))/b^3-d*(d*x+c)*csc
h(b*x+a)/b^2-1/2*(d*x+c)^2*coth(b*x+a)*csch(b*x+a)/b+d*(d*x+c)*polylog(2,-
exp(b*x+a))/b^2-d*(d*x+c)*polylog(2,exp(b*x+a))/b^2-d^2*polylog(3,-exp(b*x
+a))/b^3+d^2*polylog(3,exp(b*x+a))/b^3
```

3.34.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 420 vs. $2(154) = 308$.

Time = 6.72 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.73

$$\int (c+dx)^2 \operatorname{csch}^3(a+bx)\,dx = -\frac{d(c+dx)\operatorname{csch}(a)}{b^2} + \frac{(-c^2-2cdx-d^2x^2)\operatorname{csch}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}$$
$$+ \frac{-b^2c^2\log\left(1-e^{a+bx}\right)+2d^2\log\left(1-e^{a+bx}\right)-2b^2cdx\log\left(1-e^{a+bx}\right)-b^2d^2x^2\log\left(1-e^{a+bx}\right)+b^2c^2\log}{}$$
$$+ \frac{(-c^2-2cdx-d^2x^2)\operatorname{sech}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}$$
$$+ \frac{\operatorname{csch}\left(\frac{a}{2}\right)\operatorname{csch}\left(\frac{a}{2}+\frac{bx}{2}\right)\left(cd\sinh\left(\frac{bx}{2}\right)+d^2x\sinh\left(\frac{bx}{2}\right)\right)}{2b^2}$$
$$+ \frac{\operatorname{sech}\left(\frac{a}{2}\right)\operatorname{sech}\left(\frac{a}{2}+\frac{bx}{2}\right)\left(cd\sinh\left(\frac{bx}{2}\right)+d^2x\sinh\left(\frac{bx}{2}\right)\right)}{2b^2}$$

input
```
Integrate[(c + d*x)^2*Csch[a + b*x]^3,x]
```

output
```
-((d*(c + d*x)*Csch[a])/b^2) + ((-c^2 - 2*c*d*x - d^2*x^2)*Csch[a/2 + (b*x
)/2]^2)/(8*b) + (-(b^2*c^2*Log[1 - E^(a + b*x)]) + 2*d^2*Log[1 - E^(a + b*
x)] - 2*b^2*c*d*x*Log[1 - E^(a + b*x)] - b^2*d^2*x^2*Log[1 - E^(a + b*x)]
+ b^2*c^2*Log[1 + E^(a + b*x)] - 2*d^2*Log[1 + E^(a + b*x)] + 2*b^2*c*d*x*
Log[1 + E^(a + b*x)] + b^2*d^2*x^2*Log[1 + E^(a + b*x)] + 2*b*d*(c + d*x)*
PolyLog[2, -E^(a + b*x)] - 2*b*d*(c + d*x)*PolyLog[2, E^(a + b*x)] - 2*d^2
*PolyLog[3, -E^(a + b*x)] + 2*d^2*PolyLog[3, E^(a + b*x)])/(2*b^3) + ((-c^
2 - 2*c*d*x - d^2*x^2)*Sech[a/2 + (b*x)/2]^2)/(8*b) + (Csch[a/2]*Csch[a/2
+ (b*x)/2]*(c*d*Sinh[(b*x)/2] + d^2*x*Sinh[(b*x)/2]))/(2*b^2) + (Sech[a/2]
*Sech[a/2 + (b*x)/2]*(c*d*Sinh[(b*x)/2] + d^2*x*Sinh[(b*x)/2]))/(2*b^2)
```

3.34.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 26, 4674, 26, 3042, 26, 4257, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\text{csch}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(c+dx)^2\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)^2\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{4674}$$

$$-i\left(-\frac{d^2\int -i\text{csch}(a+bx)dx}{b^2}+\frac{1}{2}\int -i(c+dx)^2\text{csch}(a+bx)dx-\frac{id(c+dx)\text{csch}(a+bx)}{b^2}-\frac{i(c+dx)^2\coth(a+}{2b}\right.$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{id^2\int \text{csch}(a+bx)dx}{b^2}-\frac{1}{2}i\int (c+dx)^2\text{csch}(a+bx)dx-\frac{id(c+dx)\text{csch}(a+bx)}{b^2}-\frac{i(c+dx)^2\coth(a+bx)\text{csc}}{2b}\right.$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{id^2\int i\csc(ia+ibx)dx}{b^2}-\frac{1}{2}i\int i(c+dx)^2\csc(ia+ibx)dx-\frac{id(c+dx)\text{csch}(a+bx)}{b^2}-\frac{i(c+dx)^2\coth(a+bx)}{2b}\right.$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{d^2\int \csc(ia+ibx)dx}{b^2}+\frac{1}{2}\int (c+dx)^2\csc(ia+ibx)dx-\frac{id(c+dx)\text{csch}(a+bx)}{b^2}-\frac{i(c+dx)^2\coth(a+bx)\text{cs}}{2b}\right.$$

$$\Big\downarrow \text{4257}$$

$$-i\left(\frac{1}{2}\int (c+dx)^2\csc(ia+ibx)dx-\frac{id^2\text{arctanh}(\cosh(a+bx))}{b^3}-\frac{id(c+dx)\text{csch}(a+bx)}{b^2}-\frac{i(c+dx)^2\coth(a+b}{2b}\right.$$

$$\Big\downarrow \text{4670}$$

$$-i\left(\frac{1}{2}\left(\frac{2id\int (c+dx)\log\left(1-e^{a+bx}\right)dx}{b}-\frac{2id\int (c+dx)\log\left(1+e^{a+bx}\right)dx}{b}+\frac{2i(c+dx)^2\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-\frac{id^2}{}\right.$$

$$\Big\downarrow \text{3011}$$

3.34. $\int (c+dx)^2\text{csch}^3(a+bx)\,dx$

$$-i\left(\frac{1}{2}\left(-\frac{2id\left(\frac{d\int \text{PolyLog}(2,-e^{a+bx})dx}{b}-\frac{(c+dx)\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2id\left(\frac{d\int \text{PolyLog}(2,e^{a+bx})dx}{b}-\frac{(c+dx)\,\text{PolyLog}(2,e^{a+bx})}{b}\right.}{b}\right.\right.$$

$$\Big\downarrow \textbf{2720}$$

$$-i\left(\frac{1}{2}\left(-\frac{2id\left(\frac{d\int e^{-a-bx}\,\text{PolyLog}(2,-e^{a+bx})de^{a+bx}}{b^2}-\frac{(c+dx)\,\text{PolyLog}(2,-e^{a+bx})}{b}\right)}{b}+\frac{2id\left(\frac{d\int e^{-a-bx}\,\text{PolyLog}(2,e^{a+bx})de^{a+bx}}{b^2}-\right.}{b}\right.\right.$$

$$\Big\downarrow \textbf{7143}$$

$$-i\left(-\frac{id^2\text{arctanh}(\cosh(a+bx))}{b^3}+\frac{1}{2}\left(\frac{2i(c+dx)^2\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{2id\left(\frac{d\,\text{PolyLog}(3,-e^{a+bx})}{b^2}-\frac{(c+dx)\,\text{PolyLog}(2,-e^{a+}}{b}\right.}{b}\right.\right.$$

input
```
Int[(c + d*x)^2*Csch[a + b*x]^3,x]
```

output
```
(-I)*(((-I)*d^2*ArcTanh[Cosh[a + b*x]])/b^3 - (I*d*(c + d*x)*Csch[a + b*x]
)/b^2 - ((I/2)*(c + d*x)^2*Coth[a + b*x]*Csch[a + b*x])/b + (((2*I)*(c + d
*x)^2*ArcTanh[E^(a + b*x)])/b - ((2*I)*d*(-(((c + d*x)*PolyLog[2, -E^(a +
b*x)])/b) + (d*PolyLog[3, -E^(a + b*x)])/b^2))/b + ((2*I)*d*(-(((c + d*x)*
PolyLog[2, E^(a + b*x)])/b) + (d*PolyLog[3, E^(a + b*x)])/b^2))/b)/2)
```

3.34.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.34. $\int (c+dx)^2 \text{csch}^3(a+bx)\,dx$

3.34.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(147) = 294$.

Time = 1.76 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.88

method	result
risch	$-\frac{e^{bx+a}\left(e^{2bx+2a} b\, d^2 x^2+2\, e^{2bx+2a} bcdx+e^{2bx+2a} b\, c^2+b\, d^2 x^2+2\, e^{2bx+2a} d^2 x+2bcdx+2\, e^{2bx+2a} cd+b\, c^2-2d^2 x-2cd\right)}{b^2\left(e^{2bx+2a}-1\right)^2}+\frac{d^2\, \mathrm{polylog}(}{b^3}$

input
```
int((d*x+c)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-exp(b*x+a)*(exp(2*b*x+2*a)*b*d^2*x^2+2*exp(2*b*x+2*a)*b*c*d*x+exp(2*b*x+2
*a)*b*c^2+b*d^2*x^2+2*exp(2*b*x+2*a)*d^2*x+2*b*c*d*x+2*exp(2*b*x+2*a)*c*d+
b*c^2-2*d^2*x-2*c*d)/b^2/(exp(2*b*x+2*a)-1)^2+d^2*polylog(3,exp(b*x+a))/b^
3-d^2*polylog(3,-exp(b*x+a))/b^3-1/2/b*d^2*ln(1-exp(b*x+a))*x^2-1/b^2*d^2*
polylog(2,exp(b*x+a))*x+1/2/b*d^2*ln(exp(b*x+a)+1)*x^2+1/b^2*d^2*polylog(2
,-exp(b*x+a))*x-1/b^2*c*d*polylog(2,exp(b*x+a))+1/b^2*c*d*polylog(2,-exp(b
*x+a))+1/b^3*a^2*d^2*arctanh(exp(b*x+a))+1/b*c^2*arctanh(exp(b*x+a))+1/2/b
^3*d^2*ln(1-exp(b*x+a))*a^2-1/2/b^3*d^2*ln(exp(b*x+a)+1)*a^2-2/b^2*a*c*d*a
rctanh(exp(b*x+a))-1/b*c*d*ln(1-exp(b*x+a))*x-1/b^2*c*d*ln(1-exp(b*x+a))*a
+1/b*c*d*ln(exp(b*x+a)+1)*x+1/b^2*c*d*ln(exp(b*x+a)+1)*a-2/b^3*d^2*arctanh
(exp(b*x+a))
```

3.34.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2218 vs. $2(145) = 290$.

Time = 0.26 (sec) , antiderivative size = 2218, normalized size of antiderivative = 14.40

$$\int (c+dx)^2 \operatorname{csch}^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*cosh(b*x
 + a)^3 + 6*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*cosh
(b*x + a)*sinh(b*x + a)^2 + 2*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*
d + b*d^2)*x)*sinh(b*x + a)^3 + 2*(b^2*d^2*x^2 + b^2*c^2 - 2*b*c*d + 2*(b^
2*c*d - b*d^2)*x)*cosh(b*x + a) + 2*((b*d^2*x + b*c*d)*cosh(b*x + a)^4 + 4
*(b*d^2*x + b*c*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*d^2*x + b*c*d)*sinh(
b*x + a)^4 + b*d^2*x + b*c*d - 2*(b*d^2*x + b*c*d)*cosh(b*x + a)^2 - 2*(b*
d^2*x + b*c*d - 3*(b*d^2*x + b*c*d)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*(
(b*d^2*x + b*c*d)*cosh(b*x + a)^3 - (b*d^2*x + b*c*d)*cosh(b*x + a))*sinh(
b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 2*((b*d^2*x + b*c*d)*cosh
(b*x + a)^4 + 4*(b*d^2*x + b*c*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*d^2*x
 + b*c*d)*sinh(b*x + a)^4 + b*d^2*x + b*c*d - 2*(b*d^2*x + b*c*d)*cosh(b*x
 + a)^2 - 2*(b*d^2*x + b*c*d - 3*(b*d^2*x + b*c*d)*cosh(b*x + a)^2)*sinh(b
*x + a)^2 + 4*((b*d^2*x + b*c*d)*cosh(b*x + a)^3 - (b*d^2*x + b*c*d)*cosh(
b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x + a)) - (b^2*d^2*
x^2 + 2*b^2*c*d*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cosh(b*x
 + a)^4 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cosh(b*x + a)*si
nh(b*x + a)^3 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sinh(b*x + a
)^4 + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cosh(b*x +
 a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 3*(b^2*d^2*x^2 + 2*b^2...
```

3.34.6 Sympy [F]

$$\int (c+dx)^2\text{csch}^3(a+bx)\,dx = \int (c+dx)^2\,\text{csch}^3\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*csch(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**2*csch(a + b*x)**3, x)
```

3.34.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(145) = 290$.

Time = 0.34 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.55

$$\int (c+dx)^2 \operatorname{csch}^3(a+bx)\,dx$$
$$= \frac{1}{2}c^2\left(\frac{\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{2\left(e^{(-bx-a)}+e^{(-3bx-3a)}\right)}{b(2e^{(-2bx-2a)}-e^{(-4bx-4a)}-1)}\right)$$
$$+ \frac{\left(bx\log\left(e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(-e^{(bx+a)}\right)\right)cd}{b^2} - \frac{\left(bx\log\left(-e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(e^{(bx+a)}\right)\right)cd}{b^2}$$
$$- \frac{\left(bd^2x^2e^{(3a)}+2\,cde^{(3a)}+2\,(bcd+d^2)xe^{(3a)}\right)e^{(3bx)}+\left(bd^2x^2e^a-2\,cde^a+2\,(bcd-d^2)xe^a\right)e^{(bx)}}{b^2e^{(4bx+4a)}-2\,b^2e^{(2bx+2a)}+b^2}$$
$$+ \frac{\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\operatorname{Li}_2\left(-e^{(bx+a)}\right)-2\operatorname{Li}_3(-e^{(bx+a)})\right)d^2}{2\,b^3}$$
$$- \frac{\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\operatorname{Li}_2\left(e^{(bx+a)}\right)-2\operatorname{Li}_3(e^{(bx+a)})\right)d^2}{2\,b^3}$$
$$- \frac{d^2\log\left(e^{(bx+a)}+1\right)}{b^3} + \frac{d^2\log\left(e^{(bx+a)}-1\right)}{b^3}$$

input
```
integrate((d*x+c)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*c^2*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + 2*(e^(-b*x -
a) + e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))) +
(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))*c*d/b^2 - (b*x*log(-e^(b*
x + a) + 1) + dilog(e^(b*x + a)))*c*d/b^2 - ((b*d^2*x^2*e^(3*a) + 2*c*d*e^
(3*a) + 2*(b*c*d + d^2)*x*e^(3*a))*e^(3*b*x) + (b*d^2*x^2*e^a - 2*c*d*e^a
+ 2*(b*c*d - d^2)*x*e^a)*e^(b*x))/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x +
2*a) + b^2) + 1/2*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)
) - 2*polylog(3, -e^(b*x + a)))*d^2/b^3 - 1/2*(b^2*x^2*log(-e^(b*x + a) +
1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))*d^2/b^3 - d^2*l
og(e^(b*x + a) + 1)/b^3 + d^2*log(e^(b*x + a) - 1)/b^3
```

3.34.8 Giac [F]

$$\int (c+dx)^2\text{csch}^3(a+bx)\,dx = \int (dx+c)^2\,\text{csch}\,(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^2*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csch(b*x + a)^3, x)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\text{csch}^3(a+bx)\,dx = \int \frac{(c+d\,x)^2}{\sinh\,(a+b\,x)^3}\,dx$$

input
```
int((c + d*x)^2/sinh(a + b*x)^3,x)
```

output
```
int((c + d*x)^2/sinh(a + b*x)^3, x)
```

3.35 $\int (c+dx)\text{csch}^3(a+bx)\,dx$

3.35.1 Optimal result

Integrand size = 14, antiderivative size = 92

$$\begin{aligned}\int (c+dx)\text{csch}^3(a+bx)\,dx=&\frac{(c+dx)\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{d\text{csch}(a+bx)}{2b^2}\\&-\frac{(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b}\\&+\frac{d\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{d\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}\end{aligned}$$

output
```
(d*x+c)*arctanh(exp(b*x+a))/b-1/2*d*csch(b*x+a)/b^2-1/2*(d*x+c)*coth(b*x+a
)*csch(b*x+a)/b+1/2*d*polylog(2,-exp(b*x+a))/b^2-1/2*d*polylog(2,exp(b*x+a
))/b^2
```

3.35.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 271 vs. $2(92) = 184$.

Time = 0.22 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.95

$$\int (c+dx)\text{csch}^3(a+bx)\,dx = -\frac{dx\text{csch}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} - \frac{c\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{dx\log\left(1-e^{a+bx}\right)}{2b} + \frac{dx\log\left(1+e^{a+bx}\right)}{2b} + \frac{c\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{c\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{d\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} - \frac{d\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2} - \frac{dx\text{sech}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} - \frac{c\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{d\text{csch}\left(\frac{a}{2}\right)\text{csch}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{4b^2} + \frac{d\text{sech}\left(\frac{a}{2}\right)\text{sech}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{4b^2}$$

input
```
Integrate[(c + d*x)*Csch[a + b*x]^3,x]
```

output
```
-1/8*(d*x*Csch[a/2 + (b*x)/2]^2)/b - (c*Csch[(a + b*x)/2]^2)/(8*b) - (d*x*
Log[1 - E^(a + b*x)])/(2*b) + (d*x*Log[1 + E^(a + b*x)])/(2*b) + (c*Log[Co
sh[(a + b*x)/2]])/(2*b) - (c*Log[Sinh[(a + b*x)/2]])/(2*b) + (d*PolyLog[2,
 -E^(a + b*x)])/(2*b^2) - (d*PolyLog[2, E^(a + b*x)])/(2*b^2) - (d*x*Sech[
a/2 + (b*x)/2]^2)/(8*b) - (c*Sech[(a + b*x)/2]^2)/(8*b) + (d*Csch[a/2]*Csc
h[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(4*b^2) + (d*Sech[a/2]*Sech[a/2 + (b*x)/2]
*Sinh[(b*x)/2])/(4*b^2)
```

3.35.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 26, 4673, 26, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\text{csch}^3(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -i(c+dx)\csc(ia+ibx)^3dx$$

$$\Big\downarrow \ 26$$

$$-i\int (c+dx)\csc(ia+ibx)^3dx$$

$$\Big\downarrow \ 4673$$

$$-i\left(\frac{1}{2}\int -i(c+dx)\text{csch}(a+bx)dx-\frac{id\text{csch}(a+bx)}{2b^2}-\frac{i(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \ 26$$

$$-i\left(-\frac{1}{2}i\int (c+dx)\text{csch}(a+bx)dx-\frac{id\text{csch}(a+bx)}{2b^2}-\frac{i(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \ 3042$$

$$-i\left(-\frac{1}{2}i\int i(c+dx)\csc(ia+ibx)dx-\frac{id\text{csch}(a+bx)}{2b^2}-\frac{i(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \ 26$$

$$-i\left(\frac{1}{2}\int (c+dx)\csc(ia+ibx)dx-\frac{id\text{csch}(a+bx)}{2b^2}-\frac{i(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \ 4670$$

$$-i\left(\frac{1}{2}\left(\frac{id\int \log\left(1-e^{a+bx}\right)dx}{b}-\frac{id\int \log\left(1+e^{a+bx}\right)dx}{b}+\frac{2i(c+dx)\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-\frac{id\text{csch}(a+bx)}{2b^2}-\frac{i(c+$$

$$\Big\downarrow \ 2715$$

$$-i\left(\frac{1}{2}\left(\frac{id\int e^{-a-bx}\log\left(1-e^{a+bx}\right)de^{a+bx}}{b^2}-\frac{id\int e^{-a-bx}\log\left(1+e^{a+bx}\right)de^{a+bx}}{b^2}+\frac{2i(c+dx)\text{arctanh}\left(e^{a+bx}\right)}{b}\right)-\right.$$

$$\Big\downarrow \ 2838$$

$$-i\left(\frac{1}{2}\left(\frac{2i(c+dx)\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}\right)-\frac{id\text{csch}(a+bx)}{2b^2}-\frac{i(c+$$

input
```
Int[(c + d*x)*Csch[a + b*x]^3,x]
```

3.35. $\int (c+dx)\text{csch}^3(a+bx)\,dx$

output
```
(-I)*(((-1/2*I)*d*Csch[a + b*x])/b^2 - ((I/2)*(c + d*x)*Coth[a + b*x]*Csch
[a + b*x])/b + (((2*I)*(c + d*x)*ArcTanh[E^(a + b*x)])/b + (I*d*PolyLog[2,
 -E^(a + b*x)])/b^2 - (I*d*PolyLog[2, E^(a + b*x)])/b^2)/2)
```

3.35.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

3.35.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(81) = 162$.

Time = 1.36 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.14

method	result
risch	$-\frac{e^{bx+a}\left(bdx\,e^{2bx+2a}+b\,e^{2bx+2a}c+bdx+e^{2bx+2a}d+bc-d\right)}{b^{2}\left(e^{2bx+2a}-1\right)^{2}}+\frac{c\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}-\frac{d\ln\left(1-e^{bx+a}\right)x}{2b}-\frac{d\ln\left(1-e^{bx+a}\right)a}{2b^{2}}-\frac{d\,\mathrm{poly}}{}$

input
```
int((d*x+c)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-exp(b*x+a)*(b*d*x*exp(2*b*x+2*a)+b*exp(2*b*x+2*a)*c+b*d*x+exp(2*b*x+2*a)*
d+b*c-d)/b^2/(exp(2*b*x+2*a)-1)^2+1/b*c*arctanh(exp(b*x+a))-1/2/b*d*ln(1-e
xp(b*x+a))*x-1/2/b^2*d*ln(1-exp(b*x+a))*a-1/2*d*polylog(2,exp(b*x+a))/b^2+
1/2/b*d*ln(exp(b*x+a)+1)*x+1/2/b^2*d*ln(exp(b*x+a)+1)*a+1/2*d*polylog(2,-e
xp(b*x+a))/b^2-1/b^2*d*a*arctanh(exp(b*x+a))
```

3.35.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. $2(79) = 158$.

Time = 0.26 (sec) , antiderivative size = 1026, normalized size of antiderivative = 11.15

$$\int (c+dx)\operatorname{csch}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(b*d*x + b*c + d)*cosh(b*x + a)^3 + 6*(b*d*x + b*c + d)*cosh(b*x +
 a)*sinh(b*x + a)^2 + 2*(b*d*x + b*c + d)*sinh(b*x + a)^3 + 2*(b*d*x + b*c
 - d)*cosh(b*x + a) + (d*cosh(b*x + a)^4 + 4*d*cosh(b*x + a)*sinh(b*x + a)
^3 + d*sinh(b*x + a)^4 - 2*d*cosh(b*x + a)^2 + 2*(3*d*cosh(b*x + a)^2 - d)
*sinh(b*x + a)^2 + 4*(d*cosh(b*x + a)^3 - d*cosh(b*x + a))*sinh(b*x + a) +
 d)*dilog(cosh(b*x + a) + sinh(b*x + a)) - (d*cosh(b*x + a)^4 + 4*d*cosh(b
*x + a)*sinh(b*x + a)^3 + d*sinh(b*x + a)^4 - 2*d*cosh(b*x + a)^2 + 2*(3*d
*cosh(b*x + a)^2 - d)*sinh(b*x + a)^2 + 4*(d*cosh(b*x + a)^3 - d*cosh(b*x
+ a))*sinh(b*x + a) + d)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - ((b*d*x +
 b*c)*cosh(b*x + a)^4 + 4*(b*d*x + b*c)*cosh(b*x + a)*sinh(b*x + a)^3 + (b
*d*x + b*c)*sinh(b*x + a)^4 + b*d*x - 2*(b*d*x + b*c)*cosh(b*x + a)^2 - 2*
(b*d*x - 3*(b*d*x + b*c)*cosh(b*x + a)^2 + b*c)*sinh(b*x + a)^2 + b*c + 4*
((b*d*x + b*c)*cosh(b*x + a)^3 - (b*d*x + b*c)*cosh(b*x + a))*sinh(b*x + a
))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + ((b*c - a*d)*cosh(b*x + a)^4 +
 4*(b*c - a*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*c - a*d)*sinh(b*x + a)^4
 - 2*(b*c - a*d)*cosh(b*x + a)^2 + 2*(3*(b*c - a*d)*cosh(b*x + a)^2 - b*c
+ a*d)*sinh(b*x + a)^2 + b*c - a*d + 4*((b*c - a*d)*cosh(b*x + a)^3 - (b*c
 - a*d)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) -
1) + ((b*d*x + a*d)*cosh(b*x + a)^4 + 4*(b*d*x + a*d)*cosh(b*x + a)*sinh(b
*x + a)^3 + (b*d*x + a*d)*sinh(b*x + a)^4 + b*d*x - 2*(b*d*x + a*d)*cos...
```

3.35.6 Sympy [F]

$$\int (c+dx)\text{csch}^3(a+bx)\,dx = \int (c+dx)\,\text{csch}^3\,(a+bx)\,dx$$

input
```
integrate((d*x+c)*csch(b*x+a)**3,x)
```

output
```
Integral((c + d*x)*csch(a + b*x)**3, x)
```

3.35.7 Maxima [F]

$$\int (c+dx)\text{csch}^3(a+bx)\,dx = \int (dx+c)\,\text{csch}\,(bx+a)^3\,\,dx$$

input
```
integrate((d*x+c)*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
-d*(((b*x*e^(3*a) + e^(3*a))*e^(3*b*x) + (b*x*e^a - e^a)*e^(b*x))/(b^2*e^(
4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) + 8*integrate(1/16*x/(e^(b*x +
 a) + 1), x) + 8*integrate(1/16*x/(e^(b*x + a) - 1), x)) + 1/2*c*(log(e^(-
b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + 2*(e^(-b*x - a) + e^(-3*b*x -
3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1)))
```

3.35.8 Giac [F]

$$\int (c+dx)\text{csch}^3(a+bx)\,dx = \int (dx+c)\,\text{csch}\,(bx+a)^3\,\,dx$$

input
```
integrate((d*x+c)*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*csch(b*x + a)^3, x)
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\text{csch}^3(a+bx)\,dx = \int \frac{c+d\,x}{\sinh\left(a+b\,x\right)^3}\,dx$$

input
```
int((c + d*x)/sinh(a + b*x)^3,x)
```

output
```
int((c + d*x)/sinh(a + b*x)^3, x)
```

3.36 $\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx$

3.36.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^3(a+bx)}{c+dx},x\right)$$

output
```
Unintegrable(csch(b*x+a)^3/(d*x+c),x)
```

3.36.2 Mathematica [N/A]

Not integrable

Time = 65.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Csch[a + b*x]^3/(c + d*x),x]
```

output
```
Integrate[Csch[a + b*x]^3/(c + d*x), x]
```

3.36.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i \csc(ia+ibx)^3}{c+dx}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i \int \frac{\csc(ia+ibx)^3}{c+dx}dx$$

$$\Big\downarrow \text{ 4680}$$

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}dx$$

input
```
Int[Csch[a + b*x]^3/(c + d*x),x]
```

output
```
$Aborted
```

3.36.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

3.36.4 Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{3}}{dx+c}dx$$

input
```
int(csch(b*x+a)^3/(d*x+c),x)
```

output
```
int(csch(b*x+a)^3/(d*x+c),x)
```

3.36.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^{3}}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^3/(d*x + c), x)
```

3.36.6 Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx$$

input
```
integrate(csch(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(csch(a + b*x)**3/(c + d*x), x)
```

3.36.7 Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 433, normalized size of antiderivative = 27.06

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
-((b*d*x*e^(3*a) + (b*c - d)*e^(3*a))*e^(3*b*x) + (b*d*x*e^a + (b*c + d)*e
^a)*e^(b*x))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2*e^(4*a) +
 2*b^2*c*d*x*e^(4*a) + b^2*c^2*e^(4*a))*e^(4*b*x) - 2*(b^2*d^2*x^2*e^(2*a)
 + 2*b^2*c*d*x*e^(2*a) + b^2*c^2*e^(2*a))*e^(2*b*x)) - 8*integrate(1/16*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3*e^a + 3*b^2*c*d^2*x^2*e^a + 3*b^
2*c^2*d*x*e^a + b^2*c^3*e^a)*e^(b*x)), x) - 8*integrate(-1/16*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3 - (b^2*d^3*x^3*e^a + 3*b^2*c*d^2*x^2*e^a + 3*b^2*c^2*d*x*
e^a + b^2*c^3*e^a)*e^(b*x)), x)
```

3.36.8 Giac [N/A]

Not integrable

Time = 2.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^3}{dx+c}\,dx$$

input
```
integrate(csch(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^3/(d*x + c), x)
```

3.36.9 Mupad [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx = \int \frac{1}{\sinh\left(a+b\,x\right)^3\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(sinh(a + b*x)^3*(c + d*x)),x)
```

output
```
int(1/(sinh(a + b*x)^3*(c + d*x)), x)
```

3.37 $\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} \, dx$

3.37.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} \, dx = \operatorname{Int}\left(\frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Unintegrable(csch(b*x+a)^3/(d*x+c)^2,x)
```

3.37.2 Mathematica [N/A]

Not integrable

Time = 69.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} \, dx = \int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} \, dx$$

input
```
Integrate[Csch[a + b*x]^3/(c + d*x)^2,x]
```

output
```
Integrate[Csch[a + b*x]^3/(c + d*x)^2, x]
```

3.37.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} \, dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{i \csc(ia+ibx)^3}{(c+dx)^2}dx$$
$$\Big\downarrow \; 26$$
$$-i\int \frac{\csc(ia+ibx)^3}{(c+dx)^2}dx$$
$$\Big\downarrow \; 4680$$
$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}dx$$

input
```
Int[Csch[a + b*x]^3/(c + d*x)^2,x]
```

output
```
$Aborted
```

3.37.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

3.37.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{3}}{\left(dx+c\right)^{2}}dx$$

input
```
int(csch(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(csch(b*x+a)^3/(d*x+c)^2,x)
```

3.37.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^{3}}{\left(dx+c\right)^{2}}\,dx$$

input
```
integrate(csch(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.37.6 Sympy [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(csch(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(csch(a + b*x)**3/(c + d*x)**2, x)
```

3.37.7 Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 545, normalized size of antiderivative = 34.06

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}(bx+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate(csch(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-((b*d*x*e^(3*a) + (b*c - 2*d)*e^(3*a))*e^(3*b*x) + (b*d*x*e^a + (b*c + 2*
d)*e^a)*e^(b*x))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
+ (b^2*d^3*x^3*e^(4*a) + 3*b^2*c*d^2*x^2*e^(4*a) + 3*b^2*c^2*d*x*e^(4*a) +
 b^2*c^3*e^(4*a))*e^(4*b*x) - 2*(b^2*d^3*x^3*e^(2*a) + 3*b^2*c*d^2*x^2*e^(
2*a) + 3*b^2*c^2*d*x*e^(2*a) + b^2*c^3*e^(2*a))*e^(2*b*x)) - 8*integrate(1
/16*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)/(b^2*d^4*x^4 + 4*b^2*c*d
^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4*e^a +
4*b^2*c*d^3*x^3*e^a + 6*b^2*c^2*d^2*x^2*e^a + 4*b^2*c^3*d*x*e^a + b^2*c^4*
e^a)*e^(b*x)), x) - 8*integrate(-1/16*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
 - 6*d^2)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d
*x + b^2*c^4 - (b^2*d^4*x^4*e^a + 4*b^2*c*d^3*x^3*e^a + 6*b^2*c^2*d^2*x^2*
e^a + 4*b^2*c^3*d*x*e^a + b^2*c^4*e^a)*e^(b*x)), x)
```

3.37.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(csch(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.37.9 Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\sinh\left(a+b\,x\right)^3\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/(sinh(a + b*x)^3*(c + d*x)^2),x)
```

output
```
int(1/(sinh(a + b*x)^3*(c + d*x)^2), x)
```

3.38 $\int (c+dx)^{5/2}\sinh(a+bx)\,dx$

3.38.1 Optimal result

Integrand size = 16, antiderivative size = 171

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx = \frac{15d^2\sqrt{c+dx}\cosh(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\cosh(a+bx)}{b} - \frac{15d^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} - \frac{15d^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} - \frac{5d(c+dx)^{3/2}\sinh(a+bx)}{2b^2}$$

output
```
(d*x+c)^(5/2)*cosh(b*x+a)/b-5/2*d*(d*x+c)^(3/2)*sinh(b*x+a)/b^2-15/16*d^(5
/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(7/2)-15/1
6*d^(5/2)*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(7/2
)+15/4*d^2*cosh(b*x+a)*(d*x+c)^(1/2)/b^3
```

3.38.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.63

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx = \frac{d^3e^{-a-\frac{bc}{d}}\left(-e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{b(c+dx)}{d}\right)\right)}{2b^4\sqrt{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/2)*Sinh[a + b*x],x]
```

output
```
(d^3*E^(-a - (b*c)/d)*(-(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[7/2, -((b*
(c + d*x))/d)]) + E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[7/2, (b*(c + d
*x))/d]))/(2*b^4*Sqrt[c + d*x])
```

3.38.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.19, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(c+dx)^{5/2}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)^{5/2}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\int (c+dx)^{3/2}\cosh(a+bx)dx}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\int (c+dx)^{3/2}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{2b}\right)$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3id\int -i\sqrt{c+dx}\sinh(a+bx)dx}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3d\int\sqrt{c+dx}\sinh(a+bx)dx}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3d\int -i\sqrt{c+dx}\sin(ia+ibx)dx}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \textbf{26}$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\int\sqrt{c+dx}\sin(ia+ibx)dx}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \textbf{3777}$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\int\frac{\cosh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \textbf{3788}$$

3.38. $\int (c+dx)^{5/2}\sinh(a+bx)\,dx$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{1}{2}i\int-\frac{ie^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{ie^{-a-bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}\right)}{2b}\right.$$

$$\Big\downarrow \ 26$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{1}{2}\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx+\frac{1}{2}\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \ 2611$$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\int e^{a+\frac{b(c+dx)}{d}}}{d}\right.}{2b}\right.}{2b}\right.}{2b}\right.$$

$$\Big\downarrow \ 2633$$

3.38. $\int(c+dx)^{5/2}\sinh(a+bx)\,dx$

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{er}}{2}\right.}{2b}\right.}{2b}\right.}{2b}\right.$$

$\downarrow$ 2634

$$-i\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erf}}{2\sqrt{}}\right.}{2b}\right.}{2b}\right.}{2b}\right.$$

input
```
Int[(c + d*x)^(5/2)*Sinh[a + b*x],x]
```

3.38. $\int (c+dx)^{5/2}\sinh(a+bx)\,dx$

output
```
(-I)*((I*(c + d*x)^(5/2)*Cosh[a + b*x])/b - (((5*I)/2)*d*((((3*I)/2)*d*((I
*Sqrt[c + d*x]*Cosh[a + b*x])/b - ((I/2)*d*((E^(-a + (b*c)/d)*Sqrt[Pi]*Erf
[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) + (E^(a - (b*c)/d)*
Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d])))/b))/
b + ((c + d*x)^(3/2)*Sinh[a + b*x])/b))/b)
```

3.38.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.38.4 Maple [F]

$$\int (dx+c)^{\frac{5}{2}} \sinh (bx+a)\, dx$$

input
```
int((d*x+c)^(5/2)*sinh(b*x+a),x)
```

output
```
int((d*x+c)^(5/2)*sinh(b*x+a),x)
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(131) = 262$.

Time = 0.26 (sec) , antiderivative size = 521, normalized size of antiderivative = 3.05

$$\int (c+dx)^{5/2} \sinh(a+bx)\, dx =$$

$$-\frac{15\sqrt{\pi}\left(d^3\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right) - d^3\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right) + \left(d^3\cosh\left(-\frac{bc-ad}{d}\right) - d^3\sinh\left(-\frac{bc}{}\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-1/16*(15*sqrt(pi)*(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^3*cosh(b*x
+ a)*sinh(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) - d^3*sinh(-(b*c - a
*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 15*sqrt(pi
)*(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-(b*c -
 a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) + d^3*sinh(-(b*c - a*d)/d))*sinh(b*x
+ a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - 2*(4*b^3*d^2*x^2 + 4*b^3*
c^2 + 10*b^2*c*d + 15*b*d^2 + (4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15
*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^2 + 2*(4*b^3*d^2*x^2 +
 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x
 + a)*sinh(b*x + a) + (4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 +
 2*(4*b^3*c*d - 5*b^2*d^2)*x)*sinh(b*x + a)^2 + 2*(4*b^3*c*d + 5*b^2*d^2)*
x)*sqrt(d*x + c))/(b^4*cosh(b*x + a) + b^4*sinh(b*x + a))
```

3.38.6 Sympy [F]

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx = \int (c+dx)^{\frac{5}{2}}\sinh\left(a+bx\right)dx$$

input
```
integrate((d*x+c)**(5/2)*sinh(b*x+a),x)
```

output
```
Integral((c + d*x)**(5/2)*sinh(a + b*x), x)
```

3.38.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(131) = 262$.

Time = 0.22 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.80

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx = \frac{32\,(dx+c)^{\frac{7}{2}}\sinh\left(bx+a\right) - \frac{\left(\frac{105\,\sqrt{\pi}d^4\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^4\sqrt{-\frac{b}{d}}} + \frac{105\,\sqrt{\pi}d^4\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^4\sqrt{\frac{b}{d}}} - \frac{2\left(8\,(dx+c)^{\frac{7}{2}}b^3de^{\left(\cdots\right.}\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
integrate((d*x+c)^(5/2)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/112*(32*(d*x + c)^(7/2)*sinh(b*x + a) - (105*sqrt(pi)*d^4*erf(sqrt(d*x +
 c)*sqrt(-b/d))*e^(a - b*c/d)/(b^4*sqrt(-b/d)) + 105*sqrt(pi)*d^4*erf(sqrt
(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^4*sqrt(b/d)) - 2*(8*(d*x + c)^(7/2)
*b^3*d*e^(b*c/d) + 28*(d*x + c)^(5/2)*b^2*d^2*e^(b*c/d) + 70*(d*x + c)^(3/
2)*b*d^3*e^(b*c/d) + 105*sqrt(d*x + c)*d^4*e^(b*c/d))*e^(-a - (d*x + c)*b/
d)/b^4 + 2*(8*(d*x + c)^(7/2)*b^3*d*e^a - 28*(d*x + c)^(5/2)*b^2*d^2*e^a +
 70*(d*x + c)^(3/2)*b*d^3*e^a - 105*sqrt(d*x + c)*d^4*e^a)*e^((d*x + c)*b/
d - b*c/d)/b^4)*b/d)/d
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.36

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx = \frac{\frac{15\sqrt{\pi}d^4\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}b^3} + \frac{15\sqrt{\pi}d^4\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}b^3} + \frac{2\left(4(dx+c)^{\frac{5}{2}}b^2d-10(dx+c)^{\frac{3}{2}}bd^2+15\sqrt{dx+c}d\right.}{b^3}}{16\,d}$$

input
```
integrate((d*x+c)^(5/2)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/16*(15*sqrt(pi)*d^4*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(s
qrt(b*d)*b^3) + 15*sqrt(pi)*d^4*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c
- a*d)/d)/(sqrt(-b*d)*b^3) + 2*(4*(d*x + c)^(5/2)*b^2*d - 10*(d*x + c)^(3/
2)*b*d^2 + 15*sqrt(d*x + c)*d^3)*e^(((d*x + c)*b - b*c + a*d)/d)/b^3 + 2*(
4*(d*x + c)^(5/2)*b^2*d + 10*(d*x + c)^(3/2)*b*d^2 + 15*sqrt(d*x + c)*d^3)
*e^(-((d*x + c)*b - b*c + a*d)/d)/b^3)/d
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx = \int \sinh(a+b\,x)\,\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(sinh(a + b*x)*(c + d*x)^(5/2),x)
```

output
```
int(sinh(a + b*x)*(c + d*x)^(5/2), x)
```

3.39 $\int (c+dx)^{3/2}\sinh(a+bx)\,dx$

3.39.1 Optimal result

Integrand size = 16, antiderivative size = 146

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx=\frac{(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{3d^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3d\sqrt{c+dx}\sinh(a+bx)}{2b^2}$$

output
```
(d*x+c)^(3/2)*cosh(b*x+a)/b-3/8*d^(3/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*Pi^(1/2)/b^(5/2)+3/8*d^(3/2)*exp(a-b*c/d)*erfi(b^(1/2)*(d*x
+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(5/2)-3/2*d*sinh(b*x+a)*(d*x+c)^(1/2)/b^2
```

3.39.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.73

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx=\frac{de^{-a-\frac{bc}{d}}\sqrt{c+dx}\left(-\frac{e^{2a}\Gamma\left(\frac{5}{2},-\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}}+\frac{e^{\frac{2bc}{d}}\Gamma\left(\frac{5}{2},\frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}}\right)}{2b^2}$$

input
```
Integrate[(c + d*x)^(3/2)*Sinh[a + b*x],x]
```

output
```
(d*E^(-a - (b*c)/d)*Sqrt[c + d*x]*(-((E^(2*a)*Gamma[5/2, -((b*(c + d*x))/d
)])/Sqrt[-((b*(c + d*x))/d)]) + (E^((2*b*c)/d)*Gamma[5/2, (b*(c + d*x))/d]
)/Sqrt[(b*(c + d*x))/d]))/(2*b^2)
```

3.39.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(c+dx)^{3/2}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)^{3/2}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\int \sqrt{c+dx}\cosh(a+bx)dx}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\int \sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{2b}\right)$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{id\int -\frac{i\sinh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{d\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{d\int -\frac{i\sin(ia+ibx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\int \frac{\sin(ia+ibx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \; 3789$$

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{1}{2}i\int \frac{e^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{e^{-a-bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \; 2611$$

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \; 2633$$

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{2b}\right)}{2b}\right)$$

$$\Big\downarrow \; 2634$$

3.39. $\int (c+dx)^{3/2}\sinh(a+bx)\,dx$

$$-i\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{2b}\right)}{2b}\right)$$

input
```
Int[(c + d*x)^(3/2)*Sinh[a + b*x],x]
```

output
```
(-I)*((I*(c + d*x)^(3/2)*Cosh[a + b*x])/b - (((3*I)/2)*d*(((I/2)*d*(((-1/2
*I)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[
b]*Sqrt[d]) + ((I/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])
/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))/b + (Sqrt[c + d*x]*Sinh[a + b*x])/b))/b)
```

3.39.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.39.4 Maple [F]

$$\int (dx+c)^{\frac{3}{2}}\sinh(bx+a)\,dx$$

input
```
int((d*x+c)^(3/2)*sinh(b*x+a),x)
```

output
```
int((d*x+c)^(3/2)*sinh(b*x+a),x)
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(110) = 220$.

Time = 0.24 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.64

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx =$$

$$-\frac{3\sqrt{\pi}\left(d^2\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right)-d^2\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right)+\left(d^2\cosh\left(-\frac{bc-ad}{d}\right)-d^2\sinh\left(-\frac{bc-}{}\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-1/8*(3*sqrt(pi)*(d^2*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x +
a)*sinh(-(b*c - a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) - d^2*sinh(-(b*c - a*d
)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(pi)*(
d^2*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-(b*c - a*
d)/d) + (d^2*cosh(-(b*c - a*d)/d) + d^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a
))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - 2*(2*b^2*d*x + 2*b^2*c + (2*
b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x + a)^2 + 2*(2*b^2*d*x + 2*b^2*c - 3*b*
d)*cosh(b*x + a)*sinh(b*x + a) + (2*b^2*d*x + 2*b^2*c - 3*b*d)*sinh(b*x +
a)^2 + 3*b*d)*sqrt(d*x + c))/(b^3*cosh(b*x + a) + b^3*sinh(b*x + a))
```

3.39.6 Sympy [F]

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sinh(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*sinh(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*sinh(a + b*x), x)
```

3.39.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(110) = 220$.

Time = 0.22 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.84

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx = \frac{16\,(dx+c)^{\frac{5}{2}}\sinh(bx+a) + \frac{\left(\frac{15\,\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} - \frac{15\,\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} + \frac{2\left(4\,(dx+c)^{\frac{5}{2}}b^2de^{\left(\frac{bc}{d}\right.}\right.}{}}{}}{40}$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/40*(16*(d*x + c)^(5/2)*sinh(b*x + a) + (15*sqrt(pi)*d^3*erf(sqrt(d*x + c
)*sqrt(-b/d))*e^(a - b*c/d)/(b^3*sqrt(-b/d)) - 15*sqrt(pi)*d^3*erf(sqrt(d*
x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^3*sqrt(b/d)) + 2*(4*(d*x + c)^(5/2)*b^
2*d*e^(b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(b*c/d) + 15*sqrt(d*x + c)*d^3*
e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^3 - 2*(4*(d*x + c)^(5/2)*b^2*d*e^a - 1
0*(d*x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x + c)*b/d -
 b*c/d)/b^3)*b/d)/d
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.38

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx = \frac{\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}b^2} - \frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}b^2} + \frac{2\left(2(dx+c)^{\frac{3}{2}}bd-3\sqrt{dx+c}d^2\right)e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b^2}}{8d}$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/8*(3*sqrt(pi)*d^3*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sqr
t(b*d)*b^2) - 3*sqrt(pi)*d^3*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - a
*d)/d)/(sqrt(-b*d)*b^2) + 2*(2*(d*x + c)^(3/2)*b*d - 3*sqrt(d*x + c)*d^2)*
e^(((d*x + c)*b - b*c + a*d)/d)/b^2 + 2*(2*(d*x + c)^(3/2)*b*d + 3*sqrt(d*
x + c)*d^2)*e^(-((d*x + c)*b - b*c + a*d)/d)/b^2)/d
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx = \int \sinh(a+b\,x)\,\left(c+d\,x\right)^{3/2}\,dx$$

input
```
int(sinh(a + b*x)*(c + d*x)^(3/2),x)
```

output
```
int(sinh(a + b*x)*(c + d*x)^(3/2), x)
```

3.40 $\int \sqrt{c+dx}\sinh(a+bx)\,dx$

3.40.1 Optimal result

Integrand size = 16, antiderivative size = 123

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx = \frac{\sqrt{c+dx}\cosh(a+bx)}{b} - \frac{\sqrt{d}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{d}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}$$

output
```
-1/4*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/b^(
3/2)-1/4*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)
/b^(3/2)+cosh(b*x+a)*(d*x+c)^(1/2)/b
```

3.40.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx = \frac{e^{-a-\frac{bc}{d}}\sqrt{c+dx}\left(\frac{e^{2a}\Gamma\left(\frac{3}{2},-\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}} + \frac{e^{\frac{2bc}{d}}\Gamma\left(\frac{3}{2},\frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}}\right)}{2b}$$

input
```
Integrate[Sqrt[c + d*x]*Sinh[a + b*x],x]
```

output
```
(E^(-a - (b*c)/d)*Sqrt[c + d*x]*((E^(2*a)*Gamma[3/2, -((b*(c + d*x))/d)])/
Sqrt[-((b*(c + d*x))/d)] + (E^((2*b*c)/d)*Gamma[3/2, (b*(c + d*x))/d])/Sqr
t[(b*(c + d*x))/d]))/(2*b)
```

3.40.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 26, 3777, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sqrt{c+dx}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sqrt{c+dx}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{2b}\right)$$

$$\Big\downarrow \text{3788}$$

$$-i\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{1}{2}i\int -\frac{ie^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{ie^{-a-bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{1}{2}\int \frac{e^{-a-bx}}{\sqrt{c+dx}}dx+\frac{1}{2}\int \frac{e^{a+bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)$$

$$\Big\downarrow \text{2611}$$

$$-i\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{2b}\right)$$

$$\Big\downarrow \text{2633}$$

$$-i\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{2b}\right)$$

$$\Big\downarrow \text{2634}$$

$$-i\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{2b}\right)$$

input
```
Int[Sqrt[c + d*x]*Sinh[a + b*x],x]
```

output
```
(-I)*((I*Sqrt[c + d*x]*Cosh[a + b*x])/b - ((I/2)*d*((E^(-a + (b*c)/d)*Sqrt
[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) + (E^(a - (
b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]
)))/b)
```

3.40.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.40.4 Maple [F]

$$\int \sinh (bx + a) \sqrt{dx + c}dx$$

input
```
int(sinh(b*x+a)*(d*x+c)^(1/2),x)
```

output
```
int(sinh(b*x+a)*(d*x+c)^(1/2),x)
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(91) = 182$.

Time = 0.27 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.45

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx =$$

$$-\frac{\sqrt{\pi}\left(d\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)-d\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(d\cosh\left(-\frac{bc-ad}{d}\right)-d\sinh\left(-\frac{bc-ad}{d}\right)\right.\right.}{}$$

input
```
integrate(sinh(b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*(d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)*sin
h(-(b*c - a*d)/d) + (d*cosh(-(b*c - a*d)/d) - d*sinh(-(b*c - a*d)/d))*sinh
(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - sqrt(pi)*(d*cosh(b*x +
 a)*cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)*sinh(-(b*c - a*d)/d) + (d*cosh(
-(b*c - a*d)/d) + d*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sq
rt(d*x + c)*sqrt(-b/d)) - 2*(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*
x + a) + b*sinh(b*x + a)^2 + b)*sqrt(d*x + c))/(b^2*cosh(b*x + a) + b^2*si
nh(b*x + a))
```

3.40.6 Sympy [F]

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx = \int \sqrt{c+dx}\sinh\left(a+bx\right)dx$$

input
```
integrate(sinh(b*x+a)*(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*sinh(a + b*x), x)
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(91) = 182$.

Time = 0.19 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.87

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx$$

$$= \frac{8\left(dx+c\right)^{\frac{3}{2}}\sinh\left(bx+a\right) - \frac{\left(\frac{3\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} + \frac{3\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} - \frac{2\left(2\left(dx+c\right)^{\frac{3}{2}}bde^{\left(\frac{bc}{d}\right)} + 3\sqrt{dx+c}d^2e^{(\cdots}\right.}{b^2}\right.}{d}}{12\,d}$$

input
```
integrate(sinh(b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/12*(8*(d*x + c)^(3/2)*sinh(b*x + a) - (3*sqrt(pi)*d^2*erf(sqrt(d*x + c)*
sqrt(-b/d))*e^(a - b*c/d)/(b^2*sqrt(-b/d)) + 3*sqrt(pi)*d^2*erf(sqrt(d*x +
 c)*sqrt(b/d))*e^(-a + b*c/d)/(b^2*sqrt(b/d)) - 2*(2*(d*x + c)^(3/2)*b*d*e
^(b*c/d) + 3*sqrt(d*x + c)*d^2*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^2 + 2*(
2*(d*x + c)^(3/2)*b*d*e^a - 3*sqrt(d*x + c)*d^2*e^a)*e^((d*x + c)*b/d - b*
c/d)/b^2)*b/d)/d
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.37

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx$$
$$= \frac{\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}b} + \frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}b} + \frac{2\sqrt{dx+c}de^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b} + \frac{2\sqrt{dx+c}de^{\left(-\frac{(dx+c)b-bc+ad}{d}\right)}}{b}}{4\,d}$$

input
```
integrate(sinh(b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sqrt(
b*d)*b) + sqrt(pi)*d^2*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - a*d)/d)
/(sqrt(-b*d)*b) + 2*sqrt(d*x + c)*d*e^(((d*x + c)*b - b*c + a*d)/d)/b + 2*
sqrt(d*x + c)*d*e^(-((d*x + c)*b - b*c + a*d)/d)/b)/d
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx = \int \sinh(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

input
```
int(sinh(a + b*x)*(c + d*x)^(1/2),x)
```

output
```
int(sinh(a + b*x)*(c + d*x)^(1/2), x)
```

3.41 $\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx$

3.41.1 Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} + \frac{e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

output
```
-1/2*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(1/2)/d^(
1/2)+1/2*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(1/2)
/d^(1/2)
```

3.41.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx = \frac{e^{-a-\frac{bc}{d}}\left(e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)\right)}{2b\sqrt{c+dx}}$$

input
```
Integrate[Sinh[a + b*x]/Sqrt[c + d*x],x]
```

output
```
(E^(-a - (b*c)/d)*(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d
*x))/d)] + E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]
))/(2*b*Sqrt[c + d*x])
```

3.41.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}dx$$
$$\Big\downarrow \textbf{3042}$$
$$\int -\frac{i\sin(ia+ibx)}{\sqrt{c+dx}}dx$$
$$\Big\downarrow \textbf{26}$$
$$-i\int \frac{\sin(ia+ibx)}{\sqrt{c+dx}}dx$$
$$\Big\downarrow \textbf{3789}$$
$$-i\left(\frac{1}{2}i\int \frac{e^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{e^{-a-bx}}{\sqrt{c+dx}}dx\right)$$
$$\Big\downarrow \textbf{2611}$$
$$-i\left(\frac{i\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)$$
$$\Big\downarrow \textbf{2633}$$
$$-i\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)$$
$$\Big\downarrow \textbf{2634}$$
$$-i\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)$$

input
```
Int[Sinh[a + b*x]/Sqrt[c + d*x],x]
```

output
```
(-I)*(((-1/2*I)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]])/(Sqrt[b]*Sqrt[d]) + ((I/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sq
rt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]))
```

3.41.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.41.4 Maple [F]

$$\int \frac{\sinh(bx+a)}{\sqrt{dx+c}}dx$$

input
```
int(sinh(b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(sinh(b*x+a)/(d*x+c)^(1/2),x)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\sqrt{\pi}\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)-\sinh\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)+\sqrt{\pi}\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)+\sinh\left(-\frac{bc-ad}{d}\right)\right)}{2\,b}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(pi)*sqrt(b/d)*(cosh(-(b*c - a*d)/d) - sinh(-(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(b/d)) + sqrt(pi)*sqrt(-b/d)*(cosh(-(b*c - a*d)/d) + si
nh(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-b/d)))/b
```

3.41.6 Sympy [F]

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x+c)**(1/2),x)
```

output
```
Integral(sinh(a + b*x)/sqrt(c + d*x), x)
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(74) = 148.

Time = 0.22 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.74

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx$$
$$= \frac{4\sqrt{dx+c}\sinh(bx+a) + \frac{\left(\frac{\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}} - \frac{\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b\sqrt{\frac{b}{d}}} - \frac{2\sqrt{dx+c}de^{\left(a+\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b} + 2\sqrt{dx+c}de^{\left(\right.}}{d}}{2d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(4*sqrt(d*x + c)*sinh(b*x + a) + (sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(-b
/d))*e^(a - b*c/d)/(b*sqrt(-b/d)) - sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(b/d)
)*e^(-a + b*c/d)/(b*sqrt(b/d)) - 2*sqrt(d*x + c)*d*e^(a + (d*x + c)*b/d -
b*c/d)/b + 2*sqrt(d*x + c)*d*e^(-a - (d*x + c)*b/d + b*c/d)/b)*b/d)/d
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\left(\frac{\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc}{d}\right)}}{\sqrt{bd}} - \frac{\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-2ad}{d}\right)}}{\sqrt{-bd}}\right)e^{(-a)}}{2d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^(b*c/d)/sqrt(b*d) - sqrt
(pi)*d*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - 2*a*d)/d)/sqrt(-b*d))*e
^(-a)/d
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a + bx)}{\sqrt{c + dx}}\,dx = \int \frac{\sinh(a + b\,x)}{\sqrt{c + d\,x}}\,dx$$

input
```
int(sinh(a + b*x)/(c + d*x)^(1/2),x)
```

output
```
int(sinh(a + b*x)/(c + d*x)^(1/2), x)
```

3.42 $\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx$

3.42.1 Optimal result

Integrand size = 16, antiderivative size = 118

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx=\frac{\sqrt{b}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{b}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh(a+bx)}{d\sqrt{c+dx}}$$

output
```
exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*Pi^(1/2)/d^(3/2)+
exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*Pi^(1/2)/d^(3/2)-
2*sinh(b*x+a)/d/(d*x+c)^(1/2)
```

3.42.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.02

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx=\frac{e^{-a-\frac{bc}{d}}\left(e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)-e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)-2e^{a+\frac{bc}{d}}\sinh(a+b}{d\sqrt{c+dx}}$$

input
```
Integrate[Sinh[a + b*x]/(c + d*x)^(3/2),x]
```

output
```
(E^(-a - (b*c)/d)*(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d
*x))/d)] - E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]
 - 2*E^(a + (b*c)/d)*Sinh[a + b*x]))/(d*Sqrt[c + d*x])
```

3.42.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 26, 3778, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ia+ibx)}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ia+ibx)}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{3778}$$

$$-i\left(\frac{2ib\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{2ib\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)$$

$$\Big\downarrow \text{3788}$$

$$-i\left(\frac{2ib\left(\frac{1}{2}i\int -\frac{ie^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{ie^{-a-bx}}{\sqrt{c+dx}}dx\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{2ib\left(\frac{1}{2}\int \frac{e^{-a-bx}}{\sqrt{c+dx}}dx+\frac{1}{2}\int \frac{e^{a+bx}}{\sqrt{c+dx}}dx\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)$$

$$\Big\downarrow \text{2611}$$

$$-i\left(\frac{2ib\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)$$

$$\Big\downarrow \text{2633}$$

$$-i\left(\frac{2ib\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)$$

$$\Big\downarrow \text{2634}$$

$$-i\left(\frac{2ib\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)$$

input
```
Int[Sinh[a + b*x]/(c + d*x)^(3/2),x]
```

output
```
(-I)*(((2*I)*b*((E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqr
t[d]])/(2*Sqrt[b]*Sqrt[d]) + (E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[
c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d])))/d - ((2*I)*Sinh[a + b*x])/(d*Sqr
t[c + d*x]))
```

3.42.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.42.4 Maple [F]

$$\int \frac{\sinh(bx+a)}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int(sinh(b*x+a)/(d*x+c)^(3/2),x)
```

output
```
int(sinh(b*x+a)/(d*x+c)^(3/2),x)
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(90) = 180$.

Time = 0.25 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.87

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{\sqrt{\pi}\left((dx+c)\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right) - (dx+c)\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right) + ((\ldots}{\ldots}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
(sqrt(pi)*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(b
*x + a)*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b*c - a*d)/d) - (d*x + c)
*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d
)) - sqrt(pi)*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + (d*x + c)*co
sh(b*x + a)*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b*c - a*d)/d) + (d*x
+ c)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqr
t(-b/d)) - sqrt(d*x + c)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
+ sinh(b*x + a)^2 - 1))/((d^2*x + c*d)*cosh(b*x + a) + (d^2*x + c*d)*sinh(
b*x + a))
```

3.42.6 Sympy [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh{(a+bx)}}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x+c)**(3/2),x)
```

output
```
Integral(sinh(a + b*x)/(c + d*x)**(3/2), x)
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{\frac{\left(\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{\sqrt{\frac{b}{d}}}\right)b}{d}-\frac{2\sinh(bx+a)}{\sqrt{dx+c}}}{d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
((sqrt(pi)*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/sqrt(-b/d) + sqrt(p
i)*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/sqrt(b/d))*b/d - 2*sinh(b*x
 + a)/sqrt(d*x + c))/d
```

3.42.8 Giac [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh(bx+a)}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)/(d*x + c)^(3/2), x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh(a+b\,x)}{(c+d\,x)^{3/2}}\,dx$$

input
```
int(sinh(a + b*x)/(c + d*x)^(3/2),x)
```

output
```
int(sinh(a + b*x)/(c + d*x)^(3/2), x)
```

3.43 $\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx$

3.43.1 Optimal result

Integrand size = 16, antiderivative size = 149

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{4b\cosh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}}$$

output
```
-2/3*sinh(b*x+a)/d/(d*x+c)^(3/2)-2/3*b^(3/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(5/2)+2/3*b^(3/2)*exp(a-b*c/d)*erfi(b^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(5/2)-4/3*b*cosh(b*x+a)/d^2/(d*x+c)^(1/
2)
```

3.43.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.08

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{2b\left(\frac{e^a\left(-e^{bx}+e^{-\frac{bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)\right)}{d\sqrt{c+dx}} + \frac{e^{-a-bx}\left(-1+e^{b\left(\frac{c}{d}+x\right)}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)\right)}{d\sqrt{c+dx}}\right)}{3d} - \frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}}$$

input
```
Integrate[Sinh[a + b*x]/(c + d*x)^(5/2),x]
```

output
```
(2*b*((E^a*(-E^(b*x) + (Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x)
)/d)])/E^((b*c)/d)))/(d*Sqrt[c + d*x]) + (E^(-a - b*x)*(-1 + E^(b*(c/d + x
))*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]))/(d*Sqrt[c + d*x])))
/(3*d) - (2*Sinh[a + b*x])/(3*d*(c + d*x)^(3/2))
```

3.43.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 26, 3778, 3042, 3778, 26, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\sin(ia+ibx)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\sin(ia+ibx)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \; 3778$$

$$-i\left(\frac{2ib\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}dx}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$-i\left(\frac{2ib\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{(c+dx)^{3/2}}dx}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow \; 3778$$

$$-i \left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}+\frac{2ib\int -\frac{i\sinh(a+bx)}{\sqrt{c+dx}}dx}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow 26$$

$$-i \left(\frac{2ib\left(\frac{2b\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow 3042$$

$$-i \left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}+\frac{2b\int -\frac{i\sin(ia+ibx)}{\sqrt{c+dx}}dx}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow 26$$

$$-i \left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\int \frac{\sin(ia+ibx)}{\sqrt{c+dx}}dx}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow 3789$$

$$-i \left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{1}{2}i\int \frac{e^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{e^{-a-bx}}{\sqrt{c+dx}}dx\right)}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow 2611$$

$$-i \left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow 2633$$

3.43. $\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow \text{2634}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)$$

input
```
Int[Sinh[a + b*x]/(c + d*x)^(5/2),x]
```

output
```
(-I)*((((2*I)/3)*b*((-2*Cosh[a + b*x])/(d*Sqrt[c + d*x]) - ((2*I)*b*(((-1/
2*I)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt
[b]*Sqrt[d]) + ((I/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x]
)/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))/d))/d - (((2*I)/3)*Sinh[a + b*x])/(d*(c +
d*x)^(3/2)))
```

3.43.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.43.4 Maple [F]

$$\int \frac{\sinh (bx + a)}{(dx + c)^{\frac{5}{2}}} dx$$

input
```
int(sinh(b*x+a)/(d*x+c)^(5/2),x)
```

output
```
int(sinh(b*x+a)/(d*x+c)^(5/2),x)
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(111) = 222$.

Time = 0.27 (sec) , antiderivative size = 532, normalized size of antiderivative = 3.57

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx =$$

$$-\frac{2\sqrt{\pi}\left(\left(bd^2x^2+2\,bcdx+bc^2\right)\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)-\left(bd^2x^2+2\,bcdx+bc^2\right)\cosh\left(bx+a\right)\sinh\left(-\frac{bc}{}\right.\right.}{}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-(b*c
 - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-(b*c - a*
d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d) - (b*d^2*x^2
 + 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(s
qrt(d*x + c)*sqrt(b/d)) + 2*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh
(b*x + a)*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x
+ a)*sinh(-(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c -
a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x +
 a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) + (2*b*d*x + (2*b*d*x + 2*b*
c + d)*cosh(b*x + a)^2 + 2*(2*b*d*x + 2*b*c + d)*cosh(b*x + a)*sinh(b*x +
a) + (2*b*d*x + 2*b*c + d)*sinh(b*x + a)^2 + 2*b*c - d)*sqrt(d*x + c))/((d
^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a) + (d^4*x^2 + 2*c*d^3*x + c^2*d
^2)*sinh(b*x + a))
```

3.43.6 Sympy [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh\left(a+bx\right)}{\left(c+dx\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x+c)**(5/2),x)
```

output
```
Integral(sinh(a + b*x)/(c + d*x)**(5/2), x)
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{\frac{\left(\frac{\sqrt{\frac{(dx+c)b}{d}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{\sqrt{-\frac{(dx+c)b}{d}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}\right)b}{d}+\frac{2\sinh(bx+a)}{(dx+c)^{\frac{3}{2}}}}{3\,d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*((sqrt((d*x + c)*b/d)*e^(-a + b*c/d)*gamma(-1/2, (d*x + c)*b/d)/sqrt(
d*x + c) + sqrt(-(d*x + c)*b/d)*e^(a - b*c/d)*gamma(-1/2, -(d*x + c)*b/d)/
sqrt(d*x + c))*b/d + 2*sinh(b*x + a)/(d*x + c)^(3/2))/d
```

3.43.8 Giac [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh(bx+a)}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)/(d*x + c)^(5/2), x)
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh(a+b\,x)}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(sinh(a + b*x)/(c + d*x)^(5/2),x)
```

output
```
int(sinh(a + b*x)/(c + d*x)^(5/2), x)
```

3.44 $\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx$

3.44.1 Optimal result

Integrand size = 16, antiderivative size = 174

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{4b\cosh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{4b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{4b^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\sinh(a+bx)}{15d^3\sqrt{c+dx}}$$

output
```
-4/15*b*cosh(b*x+a)/d^2/(d*x+c)^(3/2)-2/5*sinh(b*x+a)/d/(d*x+c)^(5/2)+4/15
*b^(5/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(7/2)
+4/15*b^(5/2)*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^
(7/2)-8/15*b^2*sinh(b*x+a)/d^3/(d*x+c)^(1/2)
```

3.44.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.97

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{2\left(-b(c+dx)\left(e^{a-\frac{bc}{d}}\left(e^{b\left(\frac{c}{d}+x\right)}(d+2b(c+dx))+2d\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)\right)+\right.}{15d^3(c}$$

input
```
Integrate[Sinh[a + b*x]/(c + d*x)^(7/2),x]
```

output
```
(2*(-(b*(c + d*x)*(E^(a - (b*c)/d)*(E^(b*(c/d + x))*(d + 2*b*(c + d*x)) +
2*d*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/d)]) + E^(-a - b
*x)*(d - 2*b*(c + d*x) + 2*d*E^(b*(c/d + x))*((b*(c + d*x))/d)^(3/2)*Gamma
[1/2, (b*(c + d*x))/d]))) - 3*d^2*Sinh[a + b*x]))/(15*d^3*(c + d*x)^(5/2))
```

3.44.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.19, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {3042, 26, 3778, 3042, 3778, 26, 3042, 26, 3778, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i \sin(ia+ibx)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i \int \frac{\sin(ia+ibx)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{3778}$$

$$-i \left(\frac{2ib \int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}dx}{5d} - \frac{2i \sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i \left(\frac{2ib \int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{(c+dx)^{5/2}}dx}{5d} - \frac{2i \sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \text{3778}$$

$$-i \left(\frac{2ib \left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}} + \frac{2ib \int -\frac{i \sinh(a+bx)}{(c+dx)^{3/2}}dx}{3d}\right)}{5d} - \frac{2i \sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{2ib\left(\frac{2b\int\frac{\sinh(a+bx)}{(c+dx)^{3/2}}dx}{3d}-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}+\frac{2b\int-\frac{i\sin(ia+ibx)}{(c+dx)^{3/2}}dx}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\int\frac{\sin(ia+ibx)}{(c+dx)^{3/2}}dx}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \text{3778}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\int\frac{\cosh(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

3.44. $\int\frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx$

$$\Big\downarrow \text{3788}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\left(\frac{1}{2}i\int -\frac{ie^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{ie^{-a-bx}}{\sqrt{c+dx}}dx\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\left(\frac{1}{2}\int \frac{e^{-a-bx}}{\sqrt{c+dx}}dx+\frac{1}{2}\int \frac{e^{a+bx}}{\sqrt{c+dx}}dx\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \text{2611}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \text{2633}$$

3.44. $\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \textbf{2634}$$

$$-i\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}\right)}{5d}-\frac{2i\sinh(a+bx)}{5d(c+dx)^{5/2}}\right)$$

input
```
Int[Sinh[a + b*x]/(c + d*x)^(7/2),x]
```

3.44. $\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx$

output
```
(-I)*((((-2*I)/5)*Sinh[a + b*x])/(d*(c + d*x)^(5/2)) + (((2*I)/5)*b*((-2*C
osh[a + b*x])/(3*d*(c + d*x)^(3/2)) - (((2*I)/3)*b*(((2*I)*b*((E^(-a + (b*
c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) +
 (E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[
b]*Sqrt[d])))/d - ((2*I)*Sinh[a + b*x])/(d*Sqrt[c + d*x])))/d))/d)
```

3.44.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.44.4 Maple [F]

$$\int \frac{\sinh(bx+a)}{(dx+c)^{\frac{7}{2}}}dx$$

input
```
int(sinh(b*x+a)/(d*x+c)^(7/2),x)
```

output
```
int(sinh(b*x+a)/(d*x+c)^(7/2),x)
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(132) = 264$.

Time = 0.27 (sec) , antiderivative size = 855, normalized size of antiderivative = 4.91

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{4\sqrt{\pi}\left((b^2d^3x^3+3\,b^2cd^2x^2+3\,b^2c^2dx+b^2c^3)\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right)-(b^2d^3x^3+3$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(4*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-(b*c - a*d)/d) - (b^2*d
^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*
sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 4*sqrt(pi)*((b^2*d
^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-(b
*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*c
osh(b*x + a)*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d
)*erf(sqrt(d*x + c)*sqrt(-b/d)) + (4*b^2*d^2*x^2 + 4*b^2*c^2 - 2*b*c*d - (
4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cos
h(b*x + a)^2 - 2*(4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c
*d + b*d^2)*x)*cosh(b*x + a)*sinh(b*x + a) - (4*b^2*d^2*x^2 + 4*b^2*c^2 +
2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*sinh(b*x + a)^2 + 3*d^2 + 2*(4*
b^2*c*d - b*d^2)*x)*sqrt(d*x + c))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x +
 c^3*d^3)*cosh(b*x + a) + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*
sinh(b*x + a))
```

3.44.6 Sympy [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh{(a+bx)}}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x+c)**(7/2),x)
```

output
```
Integral(sinh(a + b*x)/(c + d*x)**(7/2), x)
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.66

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{\frac{\left(\frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}\right)b}{d}+\frac{2\sinh(bx+a)}{(dx+c)^{\frac{5}{2}}}}{5\,d}$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-1/5*((((d*x + c)*b/d)^(3/2)*e^(-a + b*c/d)*gamma(-3/2, (d*x + c)*b/d)/(d*
x + c)^(3/2) + (-(d*x + c)*b/d)^(3/2)*e^(a - b*c/d)*gamma(-3/2, -(d*x + c)
*b/d)/(d*x + c)^(3/2))*b/d + 2*sinh(b*x + a)/(d*x + c)^(5/2))/d
```

3.44.8 Giac [F]

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh{(bx+a)}}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)/(d*x + c)^(7/2), x)
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh(a+b\,x)}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(sinh(a + b*x)/(c + d*x)^(7/2),x)
```

output
```
int(sinh(a + b*x)/(c + d*x)^(7/2), x)
```

3.45 $\int (c+dx)^{5/2} \sinh^2(a+bx)\,dx$

3.45.1 Optimal result

Integrand size = 18, antiderivative size = 239

$$\begin{aligned}\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx = {} & -\frac{5d(c+dx)^{3/2}}{16b^2} \\ & -\frac{(c+dx)^{7/2}}{7d}+\frac{15d^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}} \\ & -\frac{15d^{5/2}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}}+\frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b} \\ & -\frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2}+\frac{15d^2\sqrt{c+dx}\sinh(2a+2bx)}{64b^3}\end{aligned}$$

output
```
-5/16*d*(d*x+c)^(3/2)/b^2-1/7*(d*x+c)^(7/2)/d+1/2*(d*x+c)^(5/2)*cosh(b*x+a
)*sinh(b*x+a)/b-5/8*d*(d*x+c)^(3/2)*sinh(b*x+a)^2/b^2+15/512*d^(5/2)*exp(-
2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b
^(7/2)-15/512*d^(5/2)*exp(2*a-2*b*c/d)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^2*sinh(2*b*x+2*a)*(d*x+c)^(1/2)/
b^3
```

3.45.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.57

$$\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx = \frac{-\frac{64(c+dx)^4}{d}-\frac{7\sqrt{2}d^3e^{2a-\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{2b(c+dx)}{d}\right)}{b^4}-\frac{7\sqrt{2}d^3e^{-2a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{2b(c+dx)}{d}\right)}{b^4}}{448\sqrt{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/2)*Sinh[a + b*x]^2,x]
```

output
```
((-64*(c + d*x)^4)/d - (7*Sqrt[2]*d^3*E^(2*a - (2*b*c)/d)*Sqrt[-((b*(c + d
*x))/d)]*Gamma[7/2, (-2*b*(c + d*x))/d])/b^4 - (7*Sqrt[2]*d^3*E^(-2*a + (2
*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[7/2, (2*b*(c + d*x))/d])/b^4)/(448*Sq
rt[c + d*x])
```

3.45.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 25, 3792, 17, 25, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -(c+dx)^{5/2}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int (c+dx)^{5/2}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 3792}$$

$$-\frac{15d^2\int -\sqrt{c+dx}\sinh^2(a+bx)dx}{16b^2}-\frac{1}{2}\int (c+dx)^{5/2}dx-\frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}$$

$$\Big\downarrow \; 17$$

$$-\frac{15d^2\int -\sqrt{c+dx}\sinh^2(a+bx)dx}{16b^2}-\frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow \; 25$$

$$\frac{15d^2\int \sqrt{c+dx}\sinh^2(a+bx)dx}{16b^2}-\frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{15d^2\int -\sqrt{c+dx}\sin(ia+ibx)^2dx}{16b^2}-\frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow \; 25$$

$$-\frac{15d^2\int \sqrt{c+dx}\sin(ia+ibx)^2dx}{16b^2}-\frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow \; 3793$$

$$-\frac{15d^2\int \left(\frac{1}{2}\sqrt{c+dx}-\frac{1}{2}\sqrt{c+dx}\cosh(2a+2bx)\right)dx}{16b^2}-\frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2}-\frac{15d^2\left(-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}+\frac{(c+dx)^{3/2}}{3d}\right)}{16b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{7/2}}{7d}$$

input
```
Int[(c + d*x)^(5/2)*Sinh[a + b*x]^2,x]
```

3.45. $\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx$

output
```
-1/7*(c + d*x)^(7/2)/d + ((c + d*x)^(5/2)*Cosh[a + b*x]*Sinh[a + b*x])/(2*
b) - (5*d*(c + d*x)^(3/2)*Sinh[a + b*x]^2)/(8*b^2) - (15*d^2*((c + d*x)^(3
/2)/(3*d) - (Sqrt[d]*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*
Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) + (Sqrt[d]*E^(2*a - (2*b*c)/d)*Sqrt[
Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[
c + d*x]*Sinh[2*a + 2*b*x])/(4*b)))/(16*b^2)
```

3.45.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.45.4 Maple [F]

$$\int (dx+c)^{\frac{5}{2}} \sinh (bx+a)^2 \, dx$$

input
```
int((d*x+c)^(5/2)*sinh(b*x+a)^2,x)
```

output
```
int((d*x+c)^(5/2)*sinh(b*x+a)^2,x)
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. 2(183) = 366.

Time = 0.26 (sec) , antiderivative size = 1001, normalized size of antiderivative = 4.19

$$\int (c+dx)^{5/2} \sinh^2(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3584*(105*sqrt(2)*sqrt(pi)*(d^4*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) -
 d^4*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^4*cosh(-2*(b*c - a*d)/d)
- d^4*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^4*cosh(b*x + a)*cosh(
-2*(b*c - a*d)/d) - d^4*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a
))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 105*sqrt(2)*sqrt(pi)*(
d^4*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^4*cosh(b*x + a)^2*sinh(-2*(
b*c - a*d)/d) + (d^4*cosh(-2*(b*c - a*d)/d) + d^4*sinh(-2*(b*c - a*d)/d))*
sinh(b*x + a)^2 + 2*(d^4*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^4*cosh(b
*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt
(d*x + c)*sqrt(-b/d)) - 4*(112*b^3*d^3*x^2 + 112*b^3*c^2*d + 140*b^2*c*d^2
 - 7*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c
*d^2 - 5*b^2*d^3)*x)*cosh(b*x + a)^4 - 28*(16*b^3*d^3*x^2 + 16*b^3*c^2*d -
 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x)*cosh(b*x + a)*si
nh(b*x + a)^3 - 7*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3
 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x)*sinh(b*x + a)^4 + 105*b*d^3 + 128*(b^4*d
^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*cosh(b*x + a)^2 + 2*(6
4*b^4*d^3*x^3 + 192*b^4*c*d^2*x^2 + 192*b^4*c^2*d*x + 64*b^4*c^3 - 21*(16*
b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*
b^2*d^3)*x)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 28*(8*b^3*c*d^2 + 5*b^2*d^3
)*x - 4*(7*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4...
```

3.45.6 Sympy [F]

$$\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx = \int (c+dx)^{\frac{5}{2}}\sinh^2{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(5/2)*sinh(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(5/2)*sinh(a + b*x)**2, x)
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.18

$$\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx =$$

$$-\frac{512\,(dx+c)^{\frac{7}{2}} + \frac{105\sqrt{2}\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2a-\frac{2bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} - \frac{105\sqrt{2}\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2a+\frac{2bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} + \frac{28\left(16\,(dx+c)^{\frac{5}{2}}b^2\right.}{}}{}$$

input
```
integrate((d*x+c)^(5/2)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3584*(512*(d*x + c)^(7/2) + 105*sqrt(2)*sqrt(pi)*d^3*erf(sqrt(2)*sqrt(d
*x + c)*sqrt(-b/d))*e^(2*a - 2*b*c/d)/(b^3*sqrt(-b/d)) - 105*sqrt(2)*sqrt(
pi)*d^3*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b^3*sqrt(
b/d)) + 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*b*c/d) + 20*(d*x + c)^(3/2)*b*d^
2*e^(2*b*c/d) + 15*sqrt(d*x + c)*d^3*e^(2*b*c/d))*e^(-2*a - 2*(d*x + c)*b/
d)/b^3 - 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*a) - 20*(d*x + c)^(3/2)*b*d^2*e
^(2*a) + 15*sqrt(d*x + c)*d^3*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^3)/
d
```

3.45.8 Giac [F]

$$\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx = \int (dx+c)^{\frac{5}{2}}\sinh(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^(5/2)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(5/2)*sinh(b*x + a)^2, x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx = \int \sinh(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

input
```
int(sinh(a + b*x)^2*(c + d*x)^(5/2),x)
```

output
```
int(sinh(a + b*x)^2*(c + d*x)^(5/2), x)
```

3.46 $\int (c+dx)^{3/2} \sinh^2(a+bx)\,dx$

3.46.1 Optimal result

Integrand size = 18, antiderivative size = 211

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx = -\frac{3d\sqrt{c+dx}}{16b^2}-\frac{(c+dx)^{5/2}}{5d} + \frac{3d^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}} + \frac{3d^{3/2}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}} + \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a+bx)}{2b}-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}$$

output
```
-1/5*(d*x+c)^(5/2)/d+1/2*(d*x+c)^(3/2)*cosh(b*x+a)*sinh(b*x+a)/b+3/128*d^(
3/2)*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*
Pi^(1/2)/b^(5/2)+3/128*d^(3/2)*exp(2*a-2*b*c/d)*erfi(2^(1/2)*b^(1/2)*(d*x+
c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d*(d*x+c)^(1/2)/b^2-3/8*d*
sinh(b*x+a)^2*(d*x+c)^(1/2)/b^2
```

3.46.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.65

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx = \frac{\sqrt{c+dx}\left(-32(c+dx)^2-\frac{5\sqrt{2}d^2e^{2a-\frac{2bc}{d}}\Gamma\left(\frac{5}{2},-\frac{2b(c+dx)}{d}\right)}{b^2\sqrt{-\frac{b(c+dx)}{d}}}-\frac{5\sqrt{2}d^2e^{-2a+\frac{2bc}{d}}\Gamma\left(\frac{5}{2},\frac{2b(c+dx)}{d}\right)}{b^2\sqrt{\frac{b(c+dx)}{d}}}\right)}{160d}$$

input
```
Integrate[(c + d*x)^(3/2)*Sinh[a + b*x]^2,x]
```

output
```
(Sqrt[c + d*x]*(-32*(c + d*x)^2 - (5*Sqrt[2]*d^2*E^(2*a - (2*b*c)/d)*Gamma
[5/2, (-2*b*(c + d*x))/d])/(b^2*Sqrt[-((b*(c + d*x))/d)]) - (5*Sqrt[2]*d^2
*E^(-2*a + (2*b*c)/d)*Gamma[5/2, (2*b*(c + d*x))/d])/(b^2*Sqrt[(b*(c + d*x
))/d])))/(160*d)
```

3.46.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 25, 3792, 17, 25, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^{3/2}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^{3/2}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{3d^2\int -\frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{16b^2}-\frac{1}{2}\int (c+dx)^{3/2}dx-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{3d^2\int -\frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{16b^2}-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{3d^2\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{16b^2}-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d^2\int -\frac{\sin(ia+ibx)^2}{\sqrt{c+dx}}dx}{16b^2}-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3d^2\int \frac{\sin(ia+ibx)^2}{\sqrt{c+dx}}dx}{16b^2}-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{3d^2\int \left(\frac{1}{2\sqrt{c+dx}}-\frac{\cosh(2a+2bx)}{2\sqrt{c+dx}}\right)dx}{16b^2}-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3d^2\left(-\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}\right)}{16b^2}-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{(c+dx)^{5/2}}{5d}$$

input
```
Int[(c + d*x)^(3/2)*Sinh[a + b*x]^2,x]
```

output
```
-1/5*(c + d*x)^(5/2)/d - (3*d^2*(Sqrt[c + d*x]/d - (E^(-2*a + (2*b*c)/d)*S
qrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d]
) - (E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/S
qrt[d]])/(4*Sqrt[b]*Sqrt[d])))/(16*b^2) + ((c + d*x)^(3/2)*Cosh[a + b*x]*S
inh[a + b*x])/(2*b) - (3*d*Sqrt[c + d*x]*Sinh[a + b*x]^2)/(8*b^2)
```

3.46. $\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx$

3.46.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.46.4 Maple [F]

$$\int (dx+c)^{\frac{3}{2}} \sinh (bx+a)^2 \, dx$$

input
```
int((d*x+c)^(3/2)*sinh(b*x+a)^2,x)
```

output
```
int((d*x+c)^(3/2)*sinh(b*x+a)^2,x)
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(159) = 318$.

Time = 0.27 (sec) , antiderivative size = 755, normalized size of antiderivative = 3.58

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx = \frac{15\sqrt{2}\sqrt{\pi}\left(d^3\cosh(bx+a)^2\cosh\left(-\frac{2(bc-ad)}{d}\right) - d^3\cosh(bx+a)^2\sinh\left(-\frac{2(bc-ad)}{d}\right) + \left(d^3\cosh\left(-\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/640*(15*sqrt(2)*sqrt(pi)*(d^3*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - d
^3*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^3*cosh(-2*(b*c - a*d)/d) -
d^3*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^3*cosh(b*x + a)*cosh(-2
*(b*c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))
*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 15*sqrt(2)*sqrt(pi)*(d^3
*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^3*cosh(b*x + a)^2*sinh(-2*(b*c
 - a*d)/d) + (d^3*cosh(-2*(b*c - a*d)/d) + d^3*sinh(-2*(b*c - a*d)/d))*sin
h(b*x + a)^2 + 2*(d^3*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^3*cosh(b*x
+ a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*
x + c)*sqrt(-b/d)) - 4*(20*b^2*d^2*x - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^
2)*cosh(b*x + a)^4 - 20*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)*
sinh(b*x + a)^3 - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*sinh(b*x + a)^4 +
20*b^2*c*d + 15*b*d^2 + 32*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*cosh(b*x
+ a)^2 + 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 - 15*(4*b^2*d^2*x +
 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 4*(5*(4*b^2*d^2*x
 + 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)^3 - 16*(b^3*d^2*x^2 + 2*b^3*c*d*x +
b^3*c^2)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^3*d*cosh(b*x + a)
^2 + 2*b^3*d*cosh(b*x + a)*sinh(b*x + a) + b^3*d*sinh(b*x + a)^2)
```

3.46.6 Sympy [F]

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sinh^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*sinh(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*sinh(a + b*x)**2, x)
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.13

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx =$$

$$-\frac{128\,(dx+c)^{\frac{5}{2}} - \frac{15\,\sqrt{2}\sqrt{\pi}d^2\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} - \frac{15\,\sqrt{2}\sqrt{\pi}d^2\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} + \frac{20\left(4\,(dx+c)^{\frac{3}{2}}\,bde^{(2}\right.}{}}{640\,d}$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/640*(128*(d*x + c)^(5/2) - 15*sqrt(2)*sqrt(pi)*d^2*erf(sqrt(2)*sqrt(d*x
 + c)*sqrt(-b/d))*e^(2*a - 2*b*c/d)/(b^2*sqrt(-b/d)) - 15*sqrt(2)*sqrt(pi)
*d^2*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b^2*sqrt(b/d
)) + 20*(4*(d*x + c)^(3/2)*b*d*e^(2*b*c/d) + 3*sqrt(d*x + c)*d^2*e^(2*b*c/
d))*e^(-2*a - 2*(d*x + c)*b/d)/b^2 - 20*(4*(d*x + c)^(3/2)*b*d*e^(2*a) - 3
*sqrt(d*x + c)*d^2*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^2)/d
```

3.46.8 Giac [F]

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx = \int (dx+c)^{\frac{3}{2}}\sinh\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)*sinh(b*x + a)^2, x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx = \int \sinh(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

input
```
int(sinh(a + b*x)^2*(c + d*x)^(3/2),x)
```

output
```
int(sinh(a + b*x)^2*(c + d*x)^(3/2), x)
```

3.47 $\int \sqrt{c+dx}\sinh^2(a+bx)\,dx$

3.47.1 Optimal result

Integrand size = 18, antiderivative size = 166

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx = -\frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{d}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{d}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}$$

output
```
-1/3*(d*x+c)^(3/2)/d+1/32*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/32*exp(2*a-2*b*c/d)*erfi(2
^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/4
*sinh(2*b*x+2*a)*(d*x+c)^(1/2)/b
```

3.47.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.78

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx = \frac{1}{48}\sqrt{c+dx}\left(-\frac{16(c+dx)}{d}+\frac{3\sqrt{2}e^{2a-\frac{2bc}{d}}\Gamma\left(\frac{3}{2},-\frac{2b(c+dx)}{d}\right)}{b\sqrt{-\frac{b(c+dx)}{d}}} - \frac{3\sqrt{2}e^{-2a+\frac{2bc}{d}}\Gamma\left(\frac{3}{2},\frac{2b(c+dx)}{d}\right)}{b\sqrt{\frac{b(c+dx)}{d}}}\right)$$

input
```
Integrate[Sqrt[c + d*x]*Sinh[a + b*x]^2,x]
```

output
```
(Sqrt[c + d*x]*((-16*(c + d*x))/d + (3*Sqrt[2]*E^(2*a - (2*b*c)/d)*Gamma[3
/2, (-2*b*(c + d*x))/d])/(b*Sqrt[-((b*(c + d*x))/d)]) - (3*Sqrt[2]*E^(-2*a
 + (2*b*c)/d)*Gamma[3/2, (2*b*(c + d*x))/d])/(b*Sqrt[(b*(c + d*x))/d])))/4
8
```

3.47.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\sqrt{c+dx}\sin(ia+ibx)^2dx$$
$$\Big\downarrow \text{25}$$
$$-\int \sqrt{c+dx}\sin(ia+ibx)^2dx$$
$$\Big\downarrow \text{3793}$$
$$-\int \left(\frac{1}{2}\sqrt{c+dx}-\frac{1}{2}\sqrt{c+dx}\cosh(2a+2bx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}-\frac{(c+dx)^{3/2}}{3d}$$

input
```
Int[Sqrt[c + d*x]*Sinh[a + b*x]^2,x]
```

output
```
-1/3*(c + d*x)^(3/2)/d + (Sqrt[d]*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqr
t[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(2*a - (2*
b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3
/2)) + (Sqrt[c + d*x]*Sinh[2*a + 2*b*x])/(4*b)
```

3.47.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.47.4 Maple [F]

$$\int \sinh{(bx + a)}^2 \sqrt{dx + c}dx$$

input
```
int(sinh(b*x+a)^2*(d*x+c)^(1/2),x)
```

output
```
int(sinh(b*x+a)^2*(d*x+c)^(1/2),x)
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(122) = 244$.

Time = 0.26 (sec) , antiderivative size = 590, normalized size of antiderivative = 3.55

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx$$
$$= \frac{3\sqrt{2}\sqrt{\pi}\left(d^2\cosh\left(bx+a\right)^2\cosh\left(-\frac{2\,(bc-ad)}{d}\right) - d^2\cosh\left(bx+a\right)^2\sinh\left(-\frac{2\,(bc-ad)}{d}\right) + \left(d^2\cosh\left(-\frac{2\,(bc-ad)}{d}\right)\right.\right.}{}$$

input
```
integrate(sinh(b*x+a)^2*(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/96*(3*sqrt(2)*sqrt(pi)*(d^2*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - d^2
*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^2*cosh(-2*(b*c - a*d)/d) - d^
2*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^2*cosh(b*x + a)*cosh(-2*(
b*c - a*d)/d) - d^2*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*s
qrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(2)*sqrt(pi)*(d^2*co
sh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^2*cosh(b*x + a)^2*sinh(-2*(b*c -
a*d)/d) + (d^2*cosh(-2*(b*c - a*d)/d) + d^2*sinh(-2*(b*c - a*d)/d))*sinh(b
*x + a)^2 + 2*(d^2*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^2*cosh(b*x + a
)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x +
 c)*sqrt(-b/d)) + 4*(3*b*d*cosh(b*x + a)^4 + 12*b*d*cosh(b*x + a)*sinh(b*x
 + a)^3 + 3*b*d*sinh(b*x + a)^4 - 8*(b^2*d*x + b^2*c)*cosh(b*x + a)^2 - 2*
(4*b^2*d*x - 9*b*d*cosh(b*x + a)^2 + 4*b^2*c)*sinh(b*x + a)^2 - 3*b*d + 4*
(3*b*d*cosh(b*x + a)^3 - 4*(b^2*d*x + b^2*c)*cosh(b*x + a))*sinh(b*x + a))
*sqrt(d*x + c))/(b^2*d*cosh(b*x + a)^2 + 2*b^2*d*cosh(b*x + a)*sinh(b*x +
a) + b^2*d*sinh(b*x + a)^2)
```

3.47.6 Sympy [F]

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx = \int \sqrt{c+dx}\sinh^2\left(a+bx\right)dx$$

input
```
integrate(sinh(b*x+a)**2*(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*sinh(a + b*x)**2, x)
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.14

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx =$$

$$-\frac{\frac{3\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2a-\frac{2bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}-\frac{3\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2a+\frac{2bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}+32\,(dx+c)^{\frac{3}{2}}-\frac{12\sqrt{dx+c}de^{\left(2a+\frac{2(dx}{d}\right.}}{b}}{96\,d}$$

input
```
integrate(sinh(b*x+a)^2*(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/96*(3*sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a -
 2*b*c/d)/(b*sqrt(-b/d)) - 3*sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*
sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b*sqrt(b/d)) + 32*(d*x + c)^(3/2) - 12*sqrt
(d*x + c)*d*e^(2*a + 2*(d*x + c)*b/d - 2*b*c/d)/b + 12*sqrt(d*x + c)*d*e^(
-2*a - 2*(d*x + c)*b/d + 2*b*c/d)/b)/d
```

3.47.8 Giac [F]

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx = \int \sqrt{dx+c}\sinh\left(bx+a\right)^2\,dx$$

input
```
integrate(sinh(b*x+a)^2*(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*sinh(b*x + a)^2, x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx = \int \sinh(a+b\,x)^2\,\sqrt{c+d\,x}\,dx$$

input
```
int(sinh(a + b*x)^2*(c + d*x)^(1/2),x)
```

output
```
int(sinh(a + b*x)^2*(c + d*x)^(1/2), x)
```

3.48 $\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx$

3.48.1 Optimal result

Integrand size = 18, antiderivative size = 139

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{d} + \frac{e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}} + \frac{e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}$$

output
```
1/8*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*P
i^(1/2)/b^(1/2)/d^(1/2)+1/8*exp(2*a-2*b*c/d)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)-(d*x+c)^(1/2)/d
```

3.48.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{d} + \frac{e^{2a-\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{c+dx}} - \frac{e^{-2a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{c+dx}}$$

input
```
Integrate[Sinh[a + b*x]^2/Sqrt[c + d*x],x]
```

output
```
-(Sqrt[c + d*x]/d) + (E^(2*a - (2*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1
/2, (-2*b*(c + d*x))/d])/(4*Sqrt[2]*b*Sqrt[c + d*x]) - (E^(-2*a + (2*b*c)/
d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*x))/d])/(4*Sqrt[2]*b*Sqrt[
c + d*x])
```

3.48.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\sin(ia+ibx)^2}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sin(ia+ibx)^2}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 3793}$$

$$-\int \left(\frac{1}{2\sqrt{c+dx}}-\frac{\cosh(2a+2bx)}{2\sqrt{c+dx}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}-\frac{\sqrt{c+dx}}{d}$$

input
```
Int[Sinh[a + b*x]^2/Sqrt[c + d*x],x]
```

output
```
-(Sqrt[c + d*x]/d) + (E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]
*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d]) + (E^(2*a - (2*b*c)/d)*Sqrt[
Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d])
```

3.48.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.48.4 Maple [F]

$$\int \frac{\sinh\left(bx+a\right)^{2}}{\sqrt{dx+c}}dx$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^(1/2),x)
```

output
```
int(sinh(b*x+a)^2/(d*x+c)^(1/2),x)
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{2}\sqrt{\pi}\left(d\cosh\left(-\frac{2\,(bc-ad)}{d}\right)-d\sinh\left(-\frac{2\,(bc-ad)}{d}\right)\right)\sqrt{\frac{b}{d}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)-\sqrt{2}\sqrt{\pi}\left(d\cosh\left(-\frac{2\,(bc-ad)}{d}\right)\right.}{8\,bd}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(sqrt(2)*sqrt(pi)*(d*cosh(-2*(b*c - a*d)/d) - d*sinh(-2*(b*c - a*d)/d)
)*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(2)*sqrt(pi)*(d*cos
h(-2*(b*c - a*d)/d) + d*sinh(-2*(b*c - a*d)/d))*sqrt(-b/d)*erf(sqrt(2)*sqr
t(d*x + c)*sqrt(-b/d)) - 8*sqrt(d*x + c)*b)/(b*d)
```

3.48.6 Sympy [F]

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sinh^2{(a+bx)}}{\sqrt{c+dx}}\,dx$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c)**(1/2),x)
```

output
```
Integral(sinh(a + b*x)**2/sqrt(c + d*x), x)
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.77

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{\sqrt{\frac{b}{d}}}-8\sqrt{dx+c}}{8\,d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/8*(sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a - 2*b*c
/d)/sqrt(-b/d) + sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(
-2*a + 2*b*c/d)/sqrt(b/d) - 8*sqrt(d*x + c))/d
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx =$$
$$-\frac{\left(\frac{\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{2\,bc}{d}\right)}}{\sqrt{bd}}+\frac{\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{2\,(bc-2\,ad)}{d}\right)}}{\sqrt{-bd}}+8\sqrt{dx+c}e^{(2\,a)}\right)e^{(-2\,a)}}{8\,d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)/d)*e^(2*b*c/
d)/sqrt(b*d) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(-b*d)*sqrt(d*x + c)/d)
*e^(-2*(b*c - 2*a*d)/d)/sqrt(-b*d) + 8*sqrt(d*x + c)*e^(2*a))*e^(-2*a)/d
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sinh(a+b\,x)^2}{\sqrt{c+d\,x}}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x)^(1/2),x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x)^(1/2), x)
```

3.49 $\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx$

3.49.1 Optimal result

Integrand size = 18, antiderivative size = 142

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{\sqrt{b}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{b}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}$$

output
```
-1/2*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*
2^(1/2)*Pi^(1/2)/d^(3/2)+1/2*exp(2*a-2*b*c/d)*erfi(2^(1/2)*b^(1/2)*(d*x+c)
^(1/2)/d^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-2*sinh(b*x+a)^2/d/(d*x+c)
^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{e^{-2\left(a+b\left(\frac{c}{d}+x\right)\right)}\left(\sqrt{2}e^{4a+2bx}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\left(-\left(-1+e^{2(a+bx)}\right)^2+\sqrt{2}e\right.\right.}{2d\sqrt{c+dx}}$$

input
```
Integrate[Sinh[a + b*x]^2/(c + d*x)^(3/2),x]
```

output
```
(Sqrt[2]*E^(4*a + 2*b*x)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-2*b*(c + d*
x))/d] + E^((2*b*c)/d)*(-(-1 + E^(2*(a + b*x)))^2 + Sqrt[2]*E^((2*b*(c + d
*x))/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*x))/d]))/(2*d*E^(2*(a
 + b*(c/d + x)))*Sqrt[c + d*x])
```

3.49.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 25, 3794, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ia+ibx)^2}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ia+ibx)^2}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{3794}$$

$$-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{4ib\int \frac{i\sinh(2a+2bx)}{2\sqrt{c+dx}}dx}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\int \frac{\sinh(2a+2bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}+\frac{2b\int -\frac{i\sin(2ia+2ibx)}{\sqrt{c+dx}}dx}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\int\frac{\sin(2ia+2ibx)}{\sqrt{c+dx}}dx}{d}$$

$$\Big\downarrow \text{3789}$$

$$-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{1}{2}i\int\frac{e^{2(a+bx)}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{e^{-2(a+bx)}}{\sqrt{c+dx}}dx\right)}{d}$$

$$\Big\downarrow \text{2611}$$

$$-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\int e^{2\left(a-\frac{bc}{d}\right)+\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-2\left(a-\frac{bc}{d}\right)-\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}\right)}{d}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-2\left(a-\frac{bc}{d}\right)-\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}\right)}{d}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}$$

input
```
Int[Sinh[a + b*x]^2/(c + d*x)^(3/2),x]
```

output
```
((-2*I)*b*(((-1/2*I)*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*
Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + ((I/2)*E^(2*a - (2*b*c)/d)*Sq
rt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]))
)/d - (2*Sinh[a + b*x]^2)/(d*Sqrt[c + d*x])
```

3.49.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

3.49. $\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx$

3.49.4 Maple [F]

$$\int \frac{\sinh (bx+a)^2}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^(3/2),x)
```

output
```
int(sinh(b*x+a)^2/(d*x+c)^(3/2),x)
```

3.49.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(109) = 218$.

Time = 0.27 (sec) , antiderivative size = 571, normalized size of antiderivative = 4.02

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx =$$
$$-\frac{\sqrt{2}\sqrt{\pi}\left((dx+c)\cosh(bx+a)^2\cosh\left(-\frac{2(bc-ad)}{d}\right)-(dx+c)\cosh(bx+a)^2\sinh\left(-\frac{2(bc-ad)}{d}\right)+\left((dx+c)\right.\right.}{}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) -
 (d*x + c)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((d*x + c)*cosh(-2*(b*
c - a*d)/d) - (d*x + c)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((d*x
+ c)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-
2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(
b/d)) + sqrt(2)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d)
 + (d*x + c)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((d*x + c)*cosh(-2*(
b*c - a*d)/d) + (d*x + c)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((d*
x + c)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)*sinh
(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sq
rt(-b/d)) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x
+ a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4
*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*sqrt(d*x + c))/((d^2
*x + c*d)*cosh(b*x + a)^2 + 2*(d^2*x + c*d)*cosh(b*x + a)*sinh(b*x + a) +
(d^2*x + c*d)*sinh(b*x + a)^2)
```

3.49.6 Sympy [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh^2(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c)**(3/2),x)
```

output
```
Integral(sinh(a + b*x)**2/(c + d*x)**(3/2), x)
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{\frac{\sqrt{2}\sqrt{\frac{(dx+c)b}{d}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{2(dx+c)b}{d}\right)}{\sqrt{dx+c}} + \frac{\sqrt{2}\sqrt{-\frac{(dx+c)b}{d}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{2(dx+c)b}{d}\right)}{\sqrt{dx+c}} - \frac{4}{\sqrt{dx+c}}}{4\,d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(sqrt(2)*sqrt((d*x + c)*b/d)*e^(2*(b*c - a*d)/d)*gamma(-1/2, 2*(d*x +
 c)*b/d)/sqrt(d*x + c) + sqrt(2)*sqrt(-(d*x + c)*b/d)*e^(-2*(b*c - a*d)/d)
*gamma(-1/2, -2*(d*x + c)*b/d)/sqrt(d*x + c) - 4/sqrt(d*x + c))/d
```

3.49.8 Giac [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh(bx+a)^2}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^2/(d*x + c)^(3/2), x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh(a+b\,x)^2}{(c+d\,x)^{3/2}}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x)^(3/2),x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x)^(3/2), x)
```

3.50 $\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx$

3.50.1 Optimal result

Integrand size = 18, antiderivative size = 174

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{2b^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2b^{3/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}$$

output
```
-2/3*sinh(b*x+a)^2/d/(d*x+c)^(3/2)+2/3*b^(3/2)*exp(-2*a+2*b*c/d)*erf(2^(1/
2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(5/2)+2/3*b^(3/2)*exp
(2*a-2*b*c/d)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)
/d^(5/2)-8/3*b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)^(1/2)
```

3.50.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2e^{-2\left(a+\frac{bc}{d}\right)}\left(\sqrt{2}de^{4a}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)+\sqrt{2}de^{\frac{4bc}{d}}\left(\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)+e^{2\left(a+\frac{bc}{d}\right)}(d\sinh}{3d^2(c+dx)^{3/2}}$$

input
```
Integrate[Sinh[a + b*x]^2/(c + d*x)^(5/2),x]
```

output
```
(-2*(Sqrt[2]*d*E^(4*a)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-2*b*(c + d*
x))/d] + Sqrt[2]*d*E^((4*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (2*b*(
c + d*x))/d] + E^(2*(a + (b*c)/d))*(d*Sinh[a + b*x]^2 + 2*b*(c + d*x)*Sinh
[2*(a + b*x)])))/(3*d^2*E^(2*(a + (b*c)/d))*(c + d*x)^(3/2))
```

3.50.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 25, 3795, 17, 25, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ia+ibx)^2}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ia+ibx)^2}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{3795}$$

$$-\frac{16b^2\int -\frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b^2\int \frac{1}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{17}$$

$$-\frac{16b^2\int -\frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{16b^2\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{16b^2\int -\frac{\sin(ia+ibx)^2}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow 25$$

$$-\frac{16b^2\int \frac{\sin(ia+ibx)^2}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow 3793$$

$$-\frac{16b^2\int \left(\frac{1}{2\sqrt{c+dx}}-\frac{\cosh(2a+2bx)}{2\sqrt{c+dx}}\right)dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow 2009$$

$$-\frac{16b^2\left(-\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}\right)}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

input
```
Int[Sinh[a + b*x]^2/(c + d*x)^(5/2),x]
```

output
```
(16*b^2*Sqrt[c + d*x])/(3*d^3) - (16*b^2*(Sqrt[c + d*x]/d - (E^(-2*a + (2*
b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b
]*Sqrt[d]) - (E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c
+ d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d])))/(3*d^2) - (8*b*Cosh[a + b*x]*Sinh[
a + b*x])/(3*d^2*Sqrt[c + d*x]) - (2*Sinh[a + b*x]^2)/(3*d*(c + d*x)^(3/2)
)
```

3.50.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

3.50.4 Maple [F]

$$\int \frac{\sinh\left(bx+a\right)^{2}}{\left(dx+c\right)^{\frac{5}{2}}}dx$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^(5/2),x)
```

output
```
int(sinh(b*x+a)^2/(d*x+c)^(5/2),x)
```

3.50.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 864 vs. $2(134) = 268$.

Time = 0.26 (sec) , antiderivative size = 864, normalized size of antiderivative = 4.97

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(4*sqrt(2)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*c
osh(-2*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*si
nh(-2*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-2*(b*c - a*d
)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-2*(b*c - a*d)/d))*sinh(b*x +
a)^2 + 2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-2*(b*c - a*d
)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d
))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 4*sqrt(
2)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-2*(b*c
- a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-2*(b*c -
 a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-2*(b*c - a*d)/d) + (b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b
*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x
+ a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) - ((4*b*d*x + 4*b*c
 + d)*cosh(b*x + a)^4 + 4*(4*b*d*x + 4*b*c + d)*cosh(b*x + a)*sinh(b*x + a
)^3 + (4*b*d*x + 4*b*c + d)*sinh(b*x + a)^4 - 4*b*d*x - 2*d*cosh(b*x + a)^
2 + 2*(3*(4*b*d*x + 4*b*c + d)*cosh(b*x + a)^2 - d)*sinh(b*x + a)^2 - 4*b*
c + 4*((4*b*d*x + 4*b*c + d)*cosh(b*x + a)^3 - d*cosh(b*x + a))*sinh(b*x +
 a) + d)*sqrt(d*x + c))/((d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)^2 +
 2*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)*sinh(b*x + a) + (d^4*x...
```

3.50.6 Sympy [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh^2(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c)**(5/2),x)
```

output
```
Integral(sinh(a + b*x)**2/(c + d*x)**(5/2), x)
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.68

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx =$$
$$-\frac{\frac{3\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{3\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}-\frac{2}{(dx+c)^{\frac{3}{2}}}}{6\,d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*(3*sqrt(2)*((d*x + c)*b/d)^(3/2)*e^(2*(b*c - a*d)/d)*gamma(-3/2, 2*(d
*x + c)*b/d)/(d*x + c)^(3/2) + 3*sqrt(2)*(-(d*x + c)*b/d)^(3/2)*e^(-2*(b*c
 - a*d)/d)*gamma(-3/2, -2*(d*x + c)*b/d)/(d*x + c)^(3/2) - 2/(d*x + c)^(3/
2))/d
```

3.50.8 Giac [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh(bx+a)^2}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^2/(d*x + c)^(5/2), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh(a+b\,x)^2}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x)^(5/2),x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x)^(5/2), x)
```

3.51 $\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx$

3.51.1 Optimal result

Integrand size = 18, antiderivative size = 220

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8b^{5/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\sinh^2(a+bx)}{15d^3\sqrt{c+dx}}$$

output
```
-8/15*b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)^(3/2)-2/5*sinh(b*x+a)^2/d/(d*x
+c)^(5/2)-8/15*b^(5/2)*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(7/2)+8/15*b^(5/2)*exp(2*a-2*b*c/d)*erfi(2^(1
/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(7/2)-16/15*b^2/d^3/
(d*x+c)^(1/2)-32/15*b^2*sinh(b*x+a)^2/d^3/(d*x+c)^(1/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.93

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{6d^2+e^{2a}\left(-3d^2e^{2bx}-4be^{-\frac{2bc}{d}}(c+dx)\left(e^{\frac{2b(c+dx)}{d}}(d+4b(c+dx))+4\sqrt{2}d\left(-\frac{b(c+dx)}{d}\right)^{3}\right.\right.}{}$$

input
```
Integrate[Sinh[a + b*x]^2/(c + d*x)^(7/2),x]
```

output
```
(6*d^2 + E^(2*a)*(-3*d^2*E^(2*b*x) - (4*b*(c + d*x)*(E^((2*b*(c + d*x))/d)
*(d + 4*b*(c + d*x)) + 4*Sqrt[2]*d*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (
-2*b*(c + d*x))/d]))/E^((2*b*c)/d)) + (-3*d^2 + 4*b*(c + d*x)*(d - 4*b*(c
+ d*x) + 4*Sqrt[2]*d*E^((2*b*(c + d*x))/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1
/2, (2*b*(c + d*x))/d]))/E^(2*(a + b*x)))/(30*d^3*(c + d*x)^(5/2))
```

3.51.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 25, 3795, 17, 25, 3042, 25, 3794, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ia+ibx)^2}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ia+ibx)^2}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{3795}$$

$$-\frac{16b^2\int -\frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}dx}{15d^2}+\frac{8b^2\int \frac{1}{(c+dx)^{3/2}}dx}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \ 17$$

$$-\frac{16b^2\int -\frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}dx}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \ 25$$

$$\frac{16b^2\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}dx}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \ 3042$$

$$\frac{16b^2\int -\frac{\sin(ia+ibx)^2}{(c+dx)^{3/2}}dx}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \ 25$$

$$-\frac{16b^2\int \frac{\sin(ia+ibx)^2}{(c+dx)^{3/2}}dx}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \ 3794$$

$$-\frac{16b^2\left(\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}+\frac{4ib\int \frac{i\sinh(2a+2bx)}{2\sqrt{c+dx}}dx}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \ 27$$

$$-\frac{16b^2\left(\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2b\int \frac{\sinh(2a+2bx)}{\sqrt{c+dx}}dx}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{16b^2\left(\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2b\int -\frac{i\sin(2ia+2ibx)}{\sqrt{c+dx}}dx}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \ 26$$

3.51. $\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx$

$$-\frac{16b^2\left(\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}+\frac{2ib\int\frac{\sin(2ia+2ibx)}{\sqrt{c+dx}}dx}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{3789}$$

$$-\frac{16b^2\left(\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}+\frac{2ib\left(\frac{1}{2}i\int\frac{e^{2(a+bx)}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{e^{-2(a+bx)}}{\sqrt{c+dx}}dx\right)}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{2611}$$

$$-\frac{16b^2\left(\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}+\frac{2ib\left(\frac{i\int e^{2\left(a-\frac{bc}{d}\right)+\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-2\left(a-\frac{bc}{d}\right)-\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}\right)}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{16b^2\left(\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}+\frac{2ib\left(\frac{i\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-2\left(a-\frac{bc}{d}\right)-\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}\right)}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{16b^2\left(\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}+\frac{2ib\left(\frac{i\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

3.51. $\int\frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}dx$

input
```
Int[Sinh[a + b*x]^2/(c + d*x)^(7/2),x]
```

output
```
(-16*b^2)/(15*d^3*Sqrt[c + d*x]) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(15*d
^2*(c + d*x)^(3/2)) - (2*Sinh[a + b*x]^2)/(5*d*(c + d*x)^(5/2)) - (16*b^2*
(((2*I)*b*(((-1/2*I)*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*
Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + ((I/2)*E^(2*a - (2*b*c)/d)*Sq
rt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]))
)/d + (2*Sinh[a + b*x]^2)/(d*Sqrt[c + d*x])))/(15*d^2)
```

3.51.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

3.51.4 Maple [F]

$$\int \frac{\sinh\left(bx+a\right)^{2}}{\left(dx+c\right)^{\frac{7}{2}}}dx$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^(7/2),x)
```

output
```
int(sinh(b*x+a)^2/(d*x+c)^(7/2),x)
```

3.51.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1352 vs. $2(172) = 344$.

Time = 0.28 (sec) , antiderivative size = 1352, normalized size of antiderivative = 6.15

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/30*(16*sqrt(2)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d
) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-2*(b*
c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*si
nh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - (b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-2*(b
*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)
) + 16*sqrt(2)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d
^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) +
 ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-2*(b*c -
 a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(
-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b^2*d^3*x^
3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-2*(b*c
- a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))
 + (16*b^2*d^2*x^2 + (16*b^2*d^2*x^2 + 16*b^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8
*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^4 + 4*(16*b^2*d^2*x^2 + 16*b^2*c^2 +...
```

3.51.6 Sympy [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh^2(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c)**(7/2),x)
```

output
```
Integral(sinh(a + b*x)**2/(c + d*x)**(7/2), x)
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.54

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{\frac{5\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}+\frac{5\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}-\frac{1}{(dx+c)^{\frac{5}{2}}}}{5\,d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-1/5*(5*sqrt(2)*((d*x + c)*b/d)^(5/2)*e^(2*(b*c - a*d)/d)*gamma(-5/2, 2*(d
*x + c)*b/d)/(d*x + c)^(5/2) + 5*sqrt(2)*(-(d*x + c)*b/d)^(5/2)*e^(-2*(b*c
 - a*d)/d)*gamma(-5/2, -2*(d*x + c)*b/d)/(d*x + c)^(5/2) - 1/(d*x + c)^(5/
2))/d
```

3.51.8 Giac [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh(bx+a)^2}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^2/(d*x + c)^(7/2), x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh(a+b\,x)^2}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x)^(7/2),x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x)^(7/2), x)
```

3.52 $\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx$

3.52.1 Optimal result

Integrand size = 18, antiderivative size = 251

$$\begin{aligned}\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx &= -\frac{16b^2}{105d^3(c+dx)^{3/2}} + \frac{32b^{7/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} \\ &+ \frac{32b^{7/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} \\ &- \frac{128b^3\cosh(a+bx)\sinh(a+bx)}{105d^4\sqrt{c+dx}} - \frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2\sinh^2(a+bx)}{105d^3(c+dx)^{3/2}}\end{aligned}$$

output
```
-16/105*b^2/d^3/(d*x+c)^(3/2)-8/35*b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)^(
5/2)-2/7*sinh(b*x+a)^2/d/(d*x+c)^(7/2)-32/105*b^2*sinh(b*x+a)^2/d^3/(d*x+c
)^(3/2)+32/105*b^(7/2)*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(9/2)+32/105*b^(7/2)*exp(2*a-2*b*c/d)*erfi(2^
(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(9/2)-128/105*b^3*
cosh(b*x+a)*sinh(b*x+a)/d^4/(d*x+c)^(1/2)
```

3.52.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.88

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \frac{2\left(-8b^2d(c+dx)^2+16\sqrt{2}b^3e^{2a-\frac{2bc}{d}}(c+dx)^3\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)-16\sqrt{2}b^3e^{-2}\right.}{}$$

input
```
Integrate[Sinh[a + b*x]^2/(c + d*x)^(9/2),x]
```

output
```
(2*(-8*b^2*d*(c + d*x)^2 + 16*Sqrt[2]*b^3*E^(2*a - (2*b*c)/d)*(c + d*x)^3*
Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-2*b*(c + d*x))/d] - 16*Sqrt[2]*b^3*E
^(-2*a + (2*b*c)/d)*(c + d*x)^3*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c +
 d*x))/d] - 15*d^3*Sinh[a + b*x]^2 - 16*b^2*d*(c + d*x)^2*Sinh[a + b*x]^2
- 6*b*d^2*(c + d*x)*Sinh[2*(a + b*x)] - 32*b^3*(c + d*x)^3*Sinh[2*(a + b*x
)]))/(105*d^4*(c + d*x)^(7/2))
```

3.52.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {3042, 25, 3795, 17, 25, 3042, 25, 3795, 17, 25, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\sin(ia+ibx)^2}{(c+dx)^{9/2}}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\sin(ia+ibx)^2}{(c+dx)^{9/2}}dx$$

$$\Big\downarrow \; 3795$$

$$-\frac{16b^2\int -\frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}dx}{35d^2}+\frac{8b^2\int \frac{1}{(c+dx)^{5/2}}dx}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}$$

$$\Big\downarrow \text{17}$$

$$-\frac{16b^2\int -\frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}dx}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{16b^2\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}dx}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{16b^2\int -\frac{\sin(ia+ibx)^2}{(c+dx)^{5/2}}dx}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{16b^2\int \frac{\sin(ia+ibx)^2}{(c+dx)^{5/2}}dx}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3795}$$

$$-\frac{16b^2\left(\frac{16b^2\int -\frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b^2\int \frac{1}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}\right)}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{17}$$

$$-\frac{16b^2\left(\frac{16b^2\int -\frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{16b^2\left(-\frac{16b^2\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.52. $\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx$

$$-\frac{16b^2\left(-\frac{16b^2\int -\frac{\sin(ia+ibx)^2}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{16b^2\left(\frac{16b^2\int \frac{\sin(ia+ibx)^2}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \; 3793$$

$$-\frac{16b^2\left(\frac{16b^2\int \left(\frac{1}{2\sqrt{c+dx}}-\frac{\cosh(2a+2bx)}{2\sqrt{c+dx}}\right)dx}{3d^2}+\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{16b^2\left(\frac{16b^2\left(-\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}\right)}{3d^2}+\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}\right.}{35d^2}$$

$$\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

input
```
Int[Sinh[a + b*x]^2/(c + d*x)^(9/2),x]
```

output
```
(-16*b^2)/(105*d^3*(c + d*x)^(3/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(3
5*d^2*(c + d*x)^(5/2)) - (2*Sinh[a + b*x]^2)/(7*d*(c + d*x)^(7/2)) - (16*b
^2*((-16*b^2*Sqrt[c + d*x])/(3*d^3) + (16*b^2*(Sqrt[c + d*x]/d - (E^(-2*a
+ (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*S
qrt[b]*Sqrt[d]) - (E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sq
rt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d])))/(3*d^2) + (8*b*Cosh[a + b*x]*
Sinh[a + b*x])/(3*d^2*Sqrt[c + d*x]) + (2*Sinh[a + b*x]^2)/(3*d*(c + d*x)^
(3/2))))/(35*d^2)
```

3.52. $\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx$

3.52.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

3.52.4 Maple [F]

$$\int \frac{\sinh\left(bx+a\right)^{2}}{\left(dx+c\right)^{\frac{9}{2}}}dx$$

input
```
int(sinh(b*x+a)^2/(d*x+c)^(9/2),x)
```

output
```
int(sinh(b*x+a)^2/(d*x+c)^(9/2),x)
```

3.52.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1827 vs. $2(199) = 398$.

Time = 0.28 (sec) , antiderivative size = 1827, normalized size of antiderivative = 7.28

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
1/210*(64*sqrt(2)*sqrt(pi)*((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2
*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - (
b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^
4)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((b^3*d^4*x^4 + 4*b^3*c*d^3*x^
3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(-2*(b*c - a*d)/d) -
(b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c
^4)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^3*d^4*x^4 + 4*b^3*c*d^
3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*cosh(-2
*(b*c - a*d)/d) - (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b
^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))
*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 64*sqrt(2)*sqrt(pi)*((b^
3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)
*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 +
 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)^2*sinh(-2*(b*c
 - a*d)/d) + ((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c
^3*d*x + b^3*c^4)*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3
+ 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*sinh(-2*(b*c - a*d)/d))*sin
h(b*x + a)^2 + 2*((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b
^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4
+ 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b...
```

3.52.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(sinh(b*x+a)**2/(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.47

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx = -\frac{\frac{14\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{7}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}}+\frac{14\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{7}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}}-\frac{1}{(dx+c)^{\frac{7}{2}}}}{7d}$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
-1/7*(14*sqrt(2)*((d*x + c)*b/d)^(7/2)*e^(2*(b*c - a*d)/d)*gamma(-7/2, 2*(
d*x + c)*b/d)/(d*x + c)^(7/2) + 14*sqrt(2)*(-(d*x + c)*b/d)^(7/2)*e^(-2*(b
*c - a*d)/d)*gamma(-7/2, -2*(d*x + c)*b/d)/(d*x + c)^(7/2) - 1/(d*x + c)^(
7/2))/d
```

3.52.8 Giac [F]

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \int \frac{\sinh(bx+a)^2}{(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^2/(d*x + c)^(9/2), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \int \frac{\sinh(a+b\,x)^2}{(c+d\,x)^{9/2}}\,dx$$

input
```
int(sinh(a + b*x)^2/(c + d*x)^(9/2),x)
```

output
```
int(sinh(a + b*x)^2/(c + d*x)^(9/2), x)
```

3.53 $\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx$

3.53.1 Optimal result

Integrand size = 18, antiderivative size = 381

$$\begin{aligned}\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx &= -\frac{45d^2\sqrt{c+dx}\cosh(a+bx)}{16b^3}\\ &\quad-\frac{2(c+dx)^{5/2}\cosh(a+bx)}{3b}+\frac{5d^2\sqrt{c+dx}\cosh(3a+3bx)}{144b^3}\\ &\quad+\frac{45d^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}\\ &\quad-\frac{5d^{5/2}e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{45d^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}\\ &\quad-\frac{5d^{5/2}e^{3a-\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{5d(c+dx)^{3/2}\sinh(a+bx)}{3b^2}\\ &\quad+\frac{(c+dx)^{5/2}\cosh(a+bx)\sinh^2(a+bx)}{3b}-\frac{5d(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}\end{aligned}$$

output
```
-2/3*(d*x+c)^(5/2)*cosh(b*x+a)/b+5/3*d*(d*x+c)^(3/2)*sinh(b*x+a)/b^2+1/3*(
d*x+c)^(5/2)*cosh(b*x+a)*sinh(b*x+a)^2/b-5/18*d*(d*x+c)^(3/2)*sinh(b*x+a)^
3/b^2-5/1728*d^(5/2)*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)-5/1728*d^(5/2)*exp(3*a-3*b*c/d)*erfi(3^(1
/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)+45/64*d^(5/2)*
exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(7/2)+45/64*d^
(5/2)*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(7/2)-45
/16*d^2*cosh(b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*cosh(3*b*x+3*a)*(d*x+c)^(1
/2)/b^3
```

3.53.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.51

$$\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx = \frac{e^{-3\left(a+\frac{bc}{d}\right)}(c+dx)^{3/2}\left(\sqrt{3}be^{6a}(c+dx)\Gamma\left(\frac{7}{2},-\frac{3b(c+dx)}{d}\right)-243be^{4a+\frac{2bc}{d}}(c+dx)\Gamma\left(\frac{7}{2},-\frac{b(c+dx)}{d}\right)+de\right.}{648b^2\left(-\frac{b(c+dx)}{d}\right)^{5/2}}$$

input
```
Integrate[(c + d*x)^(5/2)*Sinh[a + b*x]^3,x]
```

output
```
((c + d*x)^(3/2)*(Sqrt[3]*b*E^(6*a)*(c + d*x)*Gamma[7/2, (-3*b*(c + d*x))/
d] - 243*b*E^(4*a + (2*b*c)/d)*(c + d*x)*Gamma[7/2, -((b*(c + d*x))/d)] +
d*E^((4*b*c)/d)*Sqrt[-((b^2*(c + d*x)^2)/d^2)]*(-243*E^(2*a)*Gamma[7/2, (b
*(c + d*x))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[7/2, (3*b*(c + d*x))/d])))/(6
48*b^2*E^(3*(a + (b*c)/d))*(-((b*(c + d*x))/d))^(5/2))
```

3.53.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.10 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.48, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {3042, 26, 3792, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3788, 26, 2611, 2633, 2634, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i(c+dx)^{5/2}\sin(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int (c+dx)^{5/2}\sin(ia+ibx)^3dx$$

$$\Big\downarrow \text{3792}$$

$$i\left(\frac{5d^2\int -i\sqrt{c+dx}\sinh^3(a+bx)dx}{12b^2}+\frac{2}{3}\int i(c+dx)^{5/2}\sinh(a+bx)dx+\frac{5id(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}-\frac{i(c+d\ldots}{}\right.$$

$$\Big\downarrow \ 26$$

$$i\left(-\frac{5id^2\int \sqrt{c+dx}\sinh^3(a+bx)dx}{12b^2}+\frac{2}{3}i\int (c+dx)^{5/2}\sinh(a+bx)dx+\frac{5id(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}-\frac{i(c+d\ldots}{}\right.$$

$$\Big\downarrow \ 3042$$

$$i\left(-\frac{5id^2\int i\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}i\int -i(c+dx)^{5/2}\sin(ia+ibx)dx+\frac{5id(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}-\frac{i(c\ldots}{}\right.$$

$$\Big\downarrow \ 26$$

$$i\left(\frac{5d^2\int \sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\int (c+dx)^{5/2}\sin(ia+ibx)dx+\frac{5id(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}-\frac{i(c+dx)^{5\ldots}}{}\right.$$

$$\Big\downarrow \ 3777$$

$$i\left(\frac{5d^2\int \sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\int (c+dx)^{3/2}\cosh(a+bx)dx}{2b}\right)+\frac{5id(c+\ldots}{}\right.$$

$$\Big\downarrow \ 3042$$

$$i\left(\frac{5d^2\int \sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\int (c+dx)^{3/2}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{2b}\right)+\frac{5\ldots}{}\right.$$

$$\Big\downarrow \ 3777$$

$$i\left(\frac{5d^2\int \sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3id\int -i\sqrt{c+dx}\sinh(a+\ldots}{2b}\right.}{2b}\right.\right.$$

$$\Big\downarrow \ 26$$

$$i\left(\frac{5d^2\int \sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3d\int \sqrt{c+dx}\sinh(a+bx)\ldots}{2b}\right.}{2b}\right.\right.$$

$$\Big\downarrow \ 3042$$

3.53. $\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx$

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3d\int-i\sqrt{c+dx}\sin(ia+i}{2b}\right.}{2b}\right.\right.$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\int\sqrt{c+dx}\sin(ia+ibx}{2b}\right.}{2b}\right.\right.$$

$$\Big\downarrow \; 3777$$

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}\right.}{2}\right.}{2b}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}\right.}{}\right.}{2b}\right.\right.$$

$$\Big\downarrow \; 3788$$

3.53. $\int(c+dx)^{5/2}\sinh^3(a+bx)\,dx$

$$i\Bigg(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\Bigg(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\bigg(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\Big(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}}{}}{2b}$$

$\Big\downarrow$ 26

$$i\Bigg(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\Bigg(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\bigg(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\Big(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}}{}}{2b}$$

$\Big\downarrow$ 2611

$$i\Bigg(\frac{5d^2\int\sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\Bigg(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\bigg(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\Big(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}}{}}{}$$

$\Big\downarrow$ 2633

3.53. $\int(c+dx)^{5/2}\sinh^3(a+bx)\,dx$

$$i\left(\frac{5d^2\int \sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}\cdots\right.}{\cdots}\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \text{2634}$$

$$i\left(\frac{5d^2\int \sqrt{c+dx}\sin(ia+ibx)^3dx}{12b^2}+\frac{5id(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{5id\left(\frac{(c+dx)^{3/2}\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \text{3793}$$

3.53. $\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx$

$$i\left(\frac{5d^2\int\left(\frac{3}{4}i\sqrt{c+dx}\sinh(a+bx)-\frac{1}{4}i\sqrt{c+dx}\sinh(3a+3bx)\right)dx}{12b^2}+\frac{5id(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}+\frac{2}{3}\left(\frac{i(c+dx)\cdots}{\cdots}\right.\right.$$

$\big\downarrow$ 2009

$$i\left(\frac{5id(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}+\frac{5d^2\left(-\frac{3i\sqrt{\pi}\sqrt{d}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{i\sqrt{\frac{\pi}{3}}\sqrt{d}e^{\frac{3bc}{d}-3a}\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{3i\sqrt{\pi}\sqrt{d}e^{a-\frac{bc}{d}}\text{er}\cdots}{16b^{3}\cdots}\right.}{\cdots}\right.$$

input
```
Int[(c + d*x)^(5/2)*Sinh[a + b*x]^3,x]
```

3.53. $\int(c+dx)^{5/2}\sinh^3(a+bx)\,dx$

output
```
I*((5*d^2*((((3*I)/4)*Sqrt[c + d*x]*Cosh[a + b*x])/b - ((I/12)*Sqrt[c + d*
x]*Cosh[3*a + 3*b*x])/b - (((3*I)/16)*Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Er
f[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/b^(3/2) + ((I/48)*Sqrt[d]*E^(-3*a + (3
*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/b^(3/2)
- (((3*I)/16)*Sqrt[d]*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x]
)/Sqrt[d]])/b^(3/2) + ((I/48)*Sqrt[d]*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[
(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/b^(3/2)))/(12*b^2) - ((I/3)*(c +
 d*x)^(5/2)*Cosh[a + b*x]*Sinh[a + b*x]^2)/b + (((5*I)/18)*d*(c + d*x)^(3/
2)*Sinh[a + b*x]^3)/b^2 + (2*((I*(c + d*x)^(5/2)*Cosh[a + b*x])/b - (((5*I
)/2)*d*((((3*I)/2)*d*((I*Sqrt[c + d*x]*Cosh[a + b*x])/b - ((I/2)*d*((E^(-a
 + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt
[d]) + (E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2
*Sqrt[b]*Sqrt[d])))/b))/b + ((c + d*x)^(3/2)*Sinh[a + b*x])/b))/b))/3)
```

3.53.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.53.4 Maple [F]

$$\int (dx+c)^{\frac{5}{2}} \sinh(bx+a)^3 \, dx$$

input
```
int((d*x+c)^(5/2)*sinh(b*x+a)^3,x)
```

output
```
int((d*x+c)^(5/2)*sinh(b*x+a)^3,x)
```

3.53.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2090 vs. $2(291) = 582$.

Time = 0.27 (sec) , antiderivative size = 2090, normalized size of antiderivative = 5.49

$$\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/1728*(5*sqrt(3)*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) -
d^3*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^3*cosh(-3*(b*c - a*d)/d) -
 d^3*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-
3*(b*c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)
^2 + 3*(d^3*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d^3*cosh(b*x + a)^2*s
inh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*
sqrt(b/d)) - 5*sqrt(3)*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d
) + d^3*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^3*cosh(-3*(b*c - a*d)/
d) + d^3*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*co
sh(-3*(b*c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x
+ a)^2 + 3*(d^3*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + d^3*cosh(b*x + a)
^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x
+ c)*sqrt(-b/d)) - 1215*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d)
 - d^3*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) -
d^3*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-(b*
c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*
(d^3*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - d^3*cosh(b*x + a)^2*sinh(-(b*c
 - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 1215*s
qrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)^3*si
nh(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) + d^3*sinh(-(b*c - a*d)/...
```

3.53.6 Sympy [F]

$$\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx = \int (c+dx)^{\frac{5}{2}}\sinh^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(5/2)*sinh(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**(5/2)*sinh(a + b*x)**3, x)
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx =$$

$$-\frac{\frac{5\sqrt{3}\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}}+\frac{5\sqrt{3}\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}}-\frac{1215\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}}-\frac{1}{}}{}$$

input
```
integrate((d*x+c)^(5/2)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/1728*(5*sqrt(3)*sqrt(pi)*d^3*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3
*a - 3*b*c/d)/(b^3*sqrt(-b/d)) + 5*sqrt(3)*sqrt(pi)*d^3*erf(sqrt(3)*sqrt(d
*x + c)*sqrt(b/d))*e^(-3*a + 3*b*c/d)/(b^3*sqrt(b/d)) - 1215*sqrt(pi)*d^3*
erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^3*sqrt(-b/d)) - 1215*sqrt(p
i)*d^3*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^3*sqrt(b/d)) + 162*(
4*(d*x + c)^(5/2)*b^2*d*e^(b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(b*c/d) + 1
5*sqrt(d*x + c)*d^3*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^3 - 6*(12*(d*x + c
)^(5/2)*b^2*d*e^(3*b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(3*b*c/d) + 5*sqrt(
d*x + c)*d^3*e^(3*b*c/d))*e^(-3*a - 3*(d*x + c)*b/d)/b^3 - 6*(12*(d*x + c)
^(5/2)*b^2*d*e^(3*a) - 10*(d*x + c)^(3/2)*b*d^2*e^(3*a) + 5*sqrt(d*x + c)*
d^3*e^(3*a))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^3 + 162*(4*(d*x + c)^(5/2)*b^
2*d*e^a - 10*(d*x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x
 + c)*b/d - b*c/d)/b^3)/d
```

3.53.8 Giac [F]

$$\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx = \int (dx+c)^{\frac{5}{2}}\sinh(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^(5/2)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(5/2)*sinh(b*x + a)^3, x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx=\int \sinh(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

input
```
int(sinh(a + b*x)^3*(c + d*x)^(5/2),x)
```

output
```
int(sinh(a + b*x)^3*(c + d*x)^(5/2), x)
```

3.54 $\int (c+dx)^{3/2} \sinh^3(a+bx)\,dx$

3.54.1 Optimal result

Integrand size = 18, antiderivative size = 325

$$\begin{aligned}\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx =\\ &-\frac{2(c+dx)^{3/2}\cosh(a+bx)}{3b}+\frac{9d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}\\ &-\frac{d^{3/2}e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{9d^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}\\ &+\frac{d^{3/2}e^{3a-\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{d\sqrt{c+dx}\sinh(a+bx)}{b^2}\\ &+\frac{(c+dx)^{3/2}\cosh(a+bx)\sinh^2(a+bx)}{3b}-\frac{d\sqrt{c+dx}\sinh^3(a+bx)}{6b^2}\end{aligned}$$

output
```
-2/3*(d*x+c)^(3/2)*cosh(b*x+a)/b+1/3*(d*x+c)^(3/2)*cosh(b*x+a)*sinh(b*x+a)
^2/b-1/288*d^(3/2)*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)+1/288*d^(3/2)*exp(3*a-3*b*c/d)*erfi(3^(1/2)
*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)+9/32*d^(3/2)*exp(
-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(5/2)-9/32*d^(3/2)
*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(5/2)+d*sinh(
b*x+a)*(d*x+c)^(1/2)/b^2-1/6*d*sinh(b*x+a)^3*(d*x+c)^(1/2)/b^2
```

3.54.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.65

$$\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx = \frac{de^{-3\left(a+\frac{bc}{d}\right)}\sqrt{c+dx}\left(-\sqrt{3}e^{6a}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{5}{2},-\frac{3b(c+dx)}{d}\right)+81e^{4a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{5}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{4bc}{d}}\right.}{216b^2\sqrt{-\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[(c + d*x)^(3/2)*Sinh[a + b*x]^3,x]
```

output
```
(d*Sqrt[c + d*x]*(-(Sqrt[3]*E^(6*a)*Sqrt[(b*(c + d*x))/d]*Gamma[5/2, (-3*b
*(c + d*x))/d]) + 81*E^(4*a + (2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[5/2,
-((b*(c + d*x))/d)] + E^((4*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*(-81*E^(2*a)*
Gamma[5/2, (b*(c + d*x))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[5/2, (3*b*(c + d
*x))/d])))/(216*b^2*E^(3*(a + (b*c)/d))*Sqrt[-((b^2*(c + d*x)^2)/d^2)])
```

3.54.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.49, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 26, 3792, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3789, 2611, 2633, 2634, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i(c+dx)^{3/2}\sin(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int (c+dx)^{3/2}\sin(ia+ibx)^3dx$$

$$\Big\downarrow \text{3792}$$

$$i\left(\frac{d^2\int -\frac{i\sinh^3(a+bx)}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\int i(c+dx)^{3/2}\sinh(a+bx)dx+\frac{id\sqrt{c+dx}\sinh^3(a+bx)}{6b^2}-\frac{i(c+dx)^{3/2}\sinh^2(a+bx)}{3b}\right.$$

$$\Big\downarrow \; 26$$

$$i\left(-\frac{id^2\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}i\int (c+dx)^{3/2}\sinh(a+bx)dx+\frac{id\sqrt{c+dx}\sinh^3(a+bx)}{6b^2}-\frac{i(c+dx)^{3/2}\sinh^2(a+bx)}{3b}\right.$$

$$\Big\downarrow \; 3042$$

$$i\left(-\frac{id^2\int \frac{i\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}i\int -i(c+dx)^{3/2}\sin(ia+ibx)dx+\frac{id\sqrt{c+dx}\sinh^3(a+bx)}{6b^2}-\frac{i(c+dx)^{3/2}\sinh^2(a}{3b}\right.$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{d^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\int (c+dx)^{3/2}\sin(ia+ibx)dx+\frac{id\sqrt{c+dx}\sinh^3(a+bx)}{6b^2}-\frac{i(c+dx)^{3/2}\sinh^2(a+bx)c}{3b}\right.$$

$$\Big\downarrow \; 3777$$

$$i\left(\frac{d^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\int \sqrt{c+dx}\cosh(a+bx)dx}{2b}\right)+\frac{id\sqrt{c+dx}\sinh^3(a+b}{6b^2}\right.$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{d^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\int \sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{2b}\right)+\frac{id\sqrt{c+dx}\sinh^3(}{6b^2}\right.$$

$$\Big\downarrow \; 3777$$

$$i\left(\frac{d^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{id\int -\frac{i\sinh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)+\frac{id\sqrt{c+dx}}{}\right.$$

$$\Big\downarrow \; 26$$

3.54. $\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx$

$$i\left(\frac{d^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{d\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)+\frac{id\sqrt{c+dx}\sin}{6b^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{d^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{d\int -\frac{i\sin(ia+ibx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)+\frac{id\sqrt{c+dx}}{}\right.$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{d^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\int \frac{\sin(ia+ibx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)+\frac{id\sqrt{c+dx}\sin}{6b}\right.$$

$$\Big\downarrow \text{3789}$$

$$i\left(\frac{d^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{1}{2}i\int \frac{e^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{e^{-a-bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}\right)\right.$$

$$\Big\downarrow \text{2611}$$

$$i\left(\frac{d^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-a-\frac{b(}{}}}{}\right.}{2b}\right.}{2b}\right.\right.$$

$$\Big\downarrow \text{2633}$$

3.54. $\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx$

$$i\Bigg(\frac{d^2\int\frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\Bigg(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\Bigg(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\Big(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-a-b}}{}}{2b}}{2b}$$

$\downarrow$ 2634

$$i\Bigg(\frac{d^2\int\frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{id\sqrt{c+dx}\sinh^3(a+bx)}{6b^2}+\frac{2}{3}\Bigg(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\Bigg(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\Big(\frac{i\sqrt{}}{}}{}}{}$$

$\downarrow$ 3793

$$i\Bigg(\frac{d^2\int\left(\frac{3i\sinh(a+bx)}{4\sqrt{c+dx}}-\frac{i\sinh(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{12b^2}+\frac{id\sqrt{c+dx}\sinh^3(a+bx)}{6b^2}+\frac{2}{3}\Bigg(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\Bigg(\frac{\sqrt{c+dx}}{}}{}$$

$\downarrow$ 2009

3.54. $\int(c+dx)^{3/2}\sinh^3(a+bx)\,dx$

$$i\left(\frac{d^2\left(-\frac{3i\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{i\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{3i\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}\right.}{12b^2}\right.$$

input
```
Int[(c + d*x)^(3/2)*Sinh[a + b*x]^3,x]
```

output
```
I*((d^2*((((-3*I)/8)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])
/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + ((I/8)*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[
(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + (((3*I)/8)*E
^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sq
rt[d]) - ((I/8)*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[
c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))/(12*b^2) - ((I/3)*(c + d*x)^(3/2)*
Cosh[a + b*x]*Sinh[a + b*x]^2)/b + ((I/6)*d*Sqrt[c + d*x]*Sinh[a + b*x]^3)
/b^2 + (2*((I*(c + d*x)^(3/2)*Cosh[a + b*x])/b - (((3*I)/2)*d*(((I/2)*d*((
(-1/2*I)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(
Sqrt[b]*Sqrt[d]) + ((I/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c +
d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))/b + (Sqrt[c + d*x]*Sinh[a + b*x])/b))/
b))/3)
```

3.54.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.54. $\int (c+dx)^{3/2} \sinh^3(a+bx)\,dx$

3.54.4 Maple [F]

$$\int (dx+c)^{\frac{3}{2}} \sinh (bx+a)^3 \, dx$$

input
```
int((d*x+c)^(3/2)*sinh(b*x+a)^3,x)
```

output
```
int((d*x+c)^(3/2)*sinh(b*x+a)^3,x)
```

3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1543 vs. $2(245) = 490$.

Time = 0.27 (sec) , antiderivative size = 1543, normalized size of antiderivative = 4.75

$$\int (c+dx)^{3/2} \sinh^3(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/288*(sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d^2
*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) - d^
2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3*(
b*c - a*d)/d) - d^2*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2
+ 3*(d^2*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d^2*cosh(b*x + a)^2*sinh
(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqr
t(b/d)) + sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d
^2*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) +
d^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3
*(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^
2 + 3*(d^2*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + d^2*cosh(b*x + a)^2*si
nh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*
sqrt(-b/d)) - 81*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) - d^2*
cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) - d^2*sin
h(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-(b*c - a*d
)/d) - d^2*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^2*co
sh(b*x + a)^2*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x + a)^2*sinh(-(b*c - a*d)
/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 81*sqrt(pi)*(
d^2*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)^3*sinh(-(b*c
- a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) + d^2*sinh(-(b*c - a*d)/d))*sinh(...
```

3.54.6 Sympy [F]

$$\int (c+dx)^{3/2} \sinh^3(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}} \sinh^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*sinh(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**(3/2)*sinh(a + b*x)**3, x)
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.32

$$\int (c+dx)^{3/2} \sinh^3(a+bx)\,dx = \frac{\frac{\sqrt{3}\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} - \frac{\sqrt{3}\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} - \frac{81\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}}}{}$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/288*(sqrt(3)*sqrt(pi)*d^2*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a -
 3*b*c/d)/(b^2*sqrt(-b/d)) - sqrt(3)*sqrt(pi)*d^2*erf(sqrt(3)*sqrt(d*x + c
)*sqrt(b/d))*e^(-3*a + 3*b*c/d)/(b^2*sqrt(b/d)) - 81*sqrt(pi)*d^2*erf(sqrt
(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^2*sqrt(-b/d)) + 81*sqrt(pi)*d^2*erf
(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^2*sqrt(b/d)) - 54*(2*(d*x + c)
^(3/2)*b*d*e^(b*c/d) + 3*sqrt(d*x + c)*d^2*e^(b*c/d))*e^(-a - (d*x + c)*b/
d)/b^2 + 6*(2*(d*x + c)^(3/2)*b*d*e^(3*b*c/d) + sqrt(d*x + c)*d^2*e^(3*b*c
/d))*e^(-3*a - 3*(d*x + c)*b/d)/b^2 + 6*(2*(d*x + c)^(3/2)*b*d*e^(3*a) - s
qrt(d*x + c)*d^2*e^(3*a))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^2 - 54*(2*(d*x +
 c)^(3/2)*b*d*e^a - 3*sqrt(d*x + c)*d^2*e^a)*e^((d*x + c)*b/d - b*c/d)/b^2
)/d
```

3.54.8 Giac [F]

$$\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx = \int (dx+c)^{\frac{3}{2}}\sinh{(bx+a)}^3\,dx$$

input
```
integrate((d*x+c)^(3/2)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)*sinh(b*x + a)^3, x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx = \int \sinh(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

input
```
int(sinh(a + b*x)^3*(c + d*x)^(3/2),x)
```

output
```
int(sinh(a + b*x)^3*(c + d*x)^(3/2), x)
```

3.55 $\int \sqrt{c+dx}\sinh^3(a+bx)\,dx$

3.55.1 Optimal result

Integrand size = 18, antiderivative size = 275

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx = -\frac{3\sqrt{c+dx}\cosh(a+bx)}{4b}+\frac{\sqrt{c+dx}\cosh(3a+3bx)}{12b}+\frac{3\sqrt{d}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{d}e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}+\frac{3\sqrt{d}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{d}e^{3a-\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}$$

output
```
-1/144*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2
)*3^(1/2)*Pi^(1/2)/b^(3/2)-1/144*exp(3*a-3*b*c/d)*erfi(3^(1/2)*b^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*d^(1/2)*3^(1/2)*Pi^(1/2)/b^(3/2)+3/16*exp(-a+b*c/d)*er
f(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)+3/16*exp(a-b*c/d
)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-3/4*cosh(b*
x+a)*(d*x+c)^(1/2)/b+1/12*cosh(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

3.55.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.76

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx$$
$$= \frac{e^{-3\left(a+\frac{bc}{d}\right)}\sqrt{c+dx}\left(\sqrt{3}e^{6a}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{3b(c+dx)}{d}\right)-27e^{4a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{4bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\left(\right.\right.}{72b\sqrt{-\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Sinh[a + b*x]^3,x]
```

output
```
(Sqrt[c + d*x]*(Sqrt[3]*E^(6*a)*Sqrt[(b*(c + d*x))/d]*Gamma[3/2, (-3*b*(c
+ d*x))/d] - 27*E^(4*a + (2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[3/2, -((b*
(c + d*x))/d)] + E^((4*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*(-27*E^(2*a)*Gamma
[3/2, (b*(c + d*x))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[3/2, (3*b*(c + d*x))/
d])))/(72*b*E^(3*(a + (b*c)/d))*Sqrt[-((b^2*(c + d*x)^2)/d^2)])
```

3.55.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int i\sqrt{c+dx}\sin(ia+ibx)^3dx$$
$$\Big\downarrow \; 26$$
$$i\int \sqrt{c+dx}\sin(ia+ibx)^3dx$$
$$\Big\downarrow \; 3793$$

$$i\int\left(\frac{3}{4}i\sqrt{c+dx}\sinh(a+bx)-\frac{1}{4}i\sqrt{c+dx}\sinh(3a+3bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{3i\sqrt{\pi}\sqrt{d}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{i\sqrt{\frac{\pi}{3}}\sqrt{d}e^{\frac{3bc}{d}-3a}\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{3i\sqrt{\pi}\sqrt{d}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{i\sqrt{\frac{\pi}{3}}\sqrt{d}e^{3}\right.$$

input
```
Int[Sqrt[c + d*x]*Sinh[a + b*x]^3,x]
```

output
```
I*((((3*I)/4)*Sqrt[c + d*x]*Cosh[a + b*x])/b - ((I/12)*Sqrt[c + d*x]*Cosh[
3*a + 3*b*x])/b - (((3*I)/16)*Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[
b]*Sqrt[c + d*x])/Sqrt[d]])/b^(3/2) + ((I/48)*Sqrt[d]*E^(-3*a + (3*b*c)/d)
*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/b^(3/2) - (((3*I
)/16)*Sqrt[d]*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d
]])/b^(3/2) + ((I/48)*Sqrt[d]*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]
*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/b^(3/2))
```

3.55.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.55.4 Maple [F]

$$\int \sinh (bx+a)^3 \sqrt{dx+c}dx$$

input
```
int(sinh(b*x+a)^3*(d*x+c)^(1/2),x)
```

output
```
int(sinh(b*x+a)^3*(d*x+c)^(1/2),x)
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. 2(201) = 402.

Time = 0.26 (sec) , antiderivative size = 1216, normalized size of antiderivative = 4.42

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/144*(sqrt(3)*sqrt(pi)*(d*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d*cos
h(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d*cosh(-3*(b*c - a*d)/d) - d*sinh(-
3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-3*(b*c - a*d)
/d) - d*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(
b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/
d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(3
)*sqrt(pi)*(d*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d*cosh(b*x + a)^3*s
inh(-3*(b*c - a*d)/d) + (d*cosh(-3*(b*c - a*d)/d) + d*sinh(-3*(b*c - a*d)/
d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) + d*cosh(b
*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cos
h(-3*(b*c - a*d)/d) + d*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x +
 a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d)) - 27*sqrt(pi)*(d*cos
h(b*x + a)^3*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d)
 + (d*cosh(-(b*c - a*d)/d) - d*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(
d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)*sinh(-(b*c - a*d)/d
))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - d*cosh(b*
x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*
sqrt(b/d)) + 27*sqrt(pi)*(d*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + d*cosh(
b*x + a)^3*sinh(-(b*c - a*d)/d) + (d*cosh(-(b*c - a*d)/d) + d*sinh(-(b*c -
 a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d...
```

3.55.6 Sympy [F]

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx = \int \sqrt{c+dx}\sinh^3{(a+bx)}\,dx$$

input
```
integrate(sinh(b*x+a)**3*(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*sinh(a + b*x)**3, x)
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.21

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx =$$
$$-\frac{\frac{\sqrt{3}\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}+\frac{\sqrt{3}\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}-\frac{27\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}-\frac{27\sqrt{\pi}}{}}{}$$

input
```
integrate(sinh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/144*(sqrt(3)*sqrt(pi)*d*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a -
3*b*c/d)/(b*sqrt(-b/d)) + sqrt(3)*sqrt(pi)*d*erf(sqrt(3)*sqrt(d*x + c)*sqr
t(b/d))*e^(-3*a + 3*b*c/d)/(b*sqrt(b/d)) - 27*sqrt(pi)*d*erf(sqrt(d*x + c)
*sqrt(-b/d))*e^(a - b*c/d)/(b*sqrt(-b/d)) - 27*sqrt(pi)*d*erf(sqrt(d*x + c
)*sqrt(b/d))*e^(-a + b*c/d)/(b*sqrt(b/d)) - 6*sqrt(d*x + c)*d*e^(3*a + 3*(
d*x + c)*b/d - 3*b*c/d)/b + 54*sqrt(d*x + c)*d*e^(a + (d*x + c)*b/d - b*c/
d)/b + 54*sqrt(d*x + c)*d*e^(-a - (d*x + c)*b/d + b*c/d)/b - 6*sqrt(d*x +
c)*d*e^(-3*a - 3*(d*x + c)*b/d + 3*b*c/d)/b)/d
```

3.55.8 Giac [F]

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx = \int \sqrt{dx+c}\sinh\left(bx+a\right)^3\,dx$$

input
```
integrate(sinh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*sinh(b*x + a)^3, x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx = \int \sinh(a+b\,x)^3\,\sqrt{c+d\,x}\,dx$$

input
```
int(sinh(a + b*x)^3*(c + d*x)^(1/2),x)
```

output
```
int(sinh(a + b*x)^3*(c + d*x)^(1/2), x)
```

3.56 $\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx$

3.56.1 Optimal result

Integrand size = 18, antiderivative size = 228

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{3e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} - \frac{e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} - \frac{3e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} + \frac{e^{3a-\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}$$

output
```
-1/24*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)
*Pi^(1/2)/b^(1/2)/d^(1/2)+1/24*exp(3*a-3*b*c/d)*erfi(3^(1/2)*b^(1/2)*(d*x+
c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)+3/8*exp(-a+b*c/d)*erf(b
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(1/2)/d^(1/2)-3/8*exp(a-b*c/d)*er
fi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(1/2)/d^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.84

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{e^{-3\left(a+\frac{bc}{d}\right)}\left(\sqrt{3}e^{6a}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right) - 9e^{4a+\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right) + e^{\frac{4bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\left(-9e^{2a}\Gamma \cdots\right.\right.}{24b\sqrt{c+dx}}$$

input
```
Integrate[Sinh[a + b*x]^3/Sqrt[c + d*x],x]
```

output
```
(Sqrt[3]*E^(6*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-3*b*(c + d*x))/d] -
 9*E^(4*a + (2*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x))
/d)] + E^((4*b*c)/d)*Sqrt[(b*(c + d*x))/d]*(-9*E^(2*a)*Gamma[1/2, (b*(c +
d*x))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[1/2, (3*b*(c + d*x))/d]))/(24*b*E^(
3*(a + (b*c)/d))*Sqrt[c + d*x])
```

3.56.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\sin(ia+ibx)^3}{\sqrt{c+dx}}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx$$
$$\Big\downarrow \text{3793}$$
$$i\int \left(\frac{3i\sinh(a+bx)}{4\sqrt{c+dx}}-\frac{i\sinh(3a+3bx)}{4\sqrt{c+dx}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$i\left(-\frac{3i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{i\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{3i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\text{erfi}\left(\sqrt{3}\right.}{8\sqrt{b}\sqrt{d}}\right.$$

input
```
Int[Sinh[a + b*x]^3/Sqrt[c + d*x],x]
```

output
```
I*((((-3*I)/8)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[
d]])/(Sqrt[b]*Sqrt[d]) + ((I/8)*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[
3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + (((3*I)/8)*E^(a -
(b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d])
 - ((I/8)*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*
x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]))
```

3.56.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.56.4 Maple [F]

$$\int \frac{\sinh(bx+a)^3}{\sqrt{dx+c}}dx$$

input
```
int(sinh(b*x+a)^3/(d*x+c)^(1/2),x)
```

output
```
int(sinh(b*x+a)^3/(d*x+c)^(1/2),x)
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.11

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\sqrt{3}\sqrt{\pi}\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{3(bc-ad)}{d}\right)-\sinh\left(-\frac{3(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)+\sqrt{3}\sqrt{\pi}\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{3(bc-\cdots}{d}\cdots\right.\right.}{\cdots}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(sqrt(3)*sqrt(pi)*sqrt(b/d)*(cosh(-3*(b*c - a*d)/d) - sinh(-3*(b*c -
 a*d)/d))*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) + sqrt(3)*sqrt(pi)*sqrt(-b/
d)*(cosh(-3*(b*c - a*d)/d) + sinh(-3*(b*c - a*d)/d))*erf(sqrt(3)*sqrt(d*x
+ c)*sqrt(-b/d)) - 9*sqrt(pi)*sqrt(b/d)*(cosh(-(b*c - a*d)/d) - sinh(-(b*c
 - a*d)/d))*erf(sqrt(d*x + c)*sqrt(b/d)) - 9*sqrt(pi)*sqrt(-b/d)*(cosh(-(b
*c - a*d)/d) + sinh(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-b/d)))/b
```

3.56.6 Sympy [F]

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(sinh(b*x+a)**3/(d*x+c)**(1/2),x)
```

output
```
Integral(sinh(a + b*x)**3/sqrt(c + d*x), x)
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.78

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}-\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{\sqrt{\frac{b}{d}}}-\frac{9\sqrt{\pi}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{9\sqrt{\pi}\operatorname{erf}\left(\sqrt{dx\cdots}\right.}{\cdots}}{24\,d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/24*(sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a - 3*b*
c/d)/sqrt(-b/d) - sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d))*e^
(-3*a + 3*b*c/d)/sqrt(b/d) - 9*sqrt(pi)*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a
 - b*c/d)/sqrt(-b/d) + 9*sqrt(pi)*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c
/d)/sqrt(b/d))/d
```

3.56.8 Giac [F]

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sinh(bx+a)^3}{\sqrt{dx+c}}\,dx$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^3/sqrt(d*x + c), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\sinh(a+b\,x)^3}{\sqrt{c+d\,x}}\,dx$$

input
```
int(sinh(a + b*x)^3/(c + d*x)^(1/2),x)
```

output
```
int(sinh(a + b*x)^3/(c + d*x)^(1/2), x)
```

3.57 $\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx$

3.57.1 Optimal result

Integrand size = 18, antiderivative size = 246

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{3\sqrt{b}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}+\frac{\sqrt{b}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{3\sqrt{b}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}+\frac{\sqrt{b}e^{3a-\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}-\frac{2\sinh^3(a+bx)}{d\sqrt{c+dx}}$$

output
```
-3/4*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*Pi^(1/2)/d^(
3/2)-3/4*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*Pi^(1/2)
/d^(3/2)+1/4*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*
b^(1/2)*3^(1/2)*Pi^(1/2)/d^(3/2)+1/4*exp(3*a-3*b*c/d)*erfi(3^(1/2)*b^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/d^(3/2)-2*sinh(b*x+a)^3/d
/(d*x+c)^(1/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{e^{-3\left(a+b\left(\frac{c}{d}+x\right)\right)}\left(\sqrt{3}e^{6a+3bx}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right)-3e^{4a+\frac{2bc}{d}+3bx}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\right.\right.}{}$$

input
```
Integrate[Sinh[a + b*x]^3/(c + d*x)^(3/2),x]
```

output
```
(Sqrt[3]*E^(6*a + 3*b*x)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-3*b*(c + d*
x))/d] - 3*E^(4*a + (2*b*c)/d + 3*b*x)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2,
 -((b*(c + d*x))/d)] - E^((3*b*c)/d)*((-1 + E^(2*(a + b*x)))^3 - 3*E^(2*a
+ (b*c)/d + 3*b*x)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, b*(c/d + x)] + Sqrt[3]
*E^((3*b*(c + d*x))/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (3*b*(c + d*x))/d]
))/(4*d*E^(3*(a + b*(c/d + x)))*Sqrt[c + d*x])
```

3.57.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 26, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i \sin(ia+ibx)^3}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{3794}$$

$$i\left(\frac{6ib\int \left(\frac{\cosh(a+bx)}{4\sqrt{c+dx}}-\frac{\cosh(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d}+\frac{2i\sinh^3(a+bx)}{d\sqrt{c+dx}}\right)$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{6ib\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}\right)}{d}+\right.$$

input
```
Int[Sinh[a + b*x]^3/(c + d*x)^(3/2),x]
```

output
```
I*(((6*I)*b*((E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d
]])/(8*Sqrt[b]*Sqrt[d]) - (E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sq
rt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (E^(a - (b*c)/d)*Sqrt
[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) - (E^(3*a
- (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*
Sqrt[b]*Sqrt[d])))/d + ((2*I)*Sinh[a + b*x]^3)/(d*Sqrt[c + d*x]))
```

3.57.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

3.57.4 Maple [F]

$$\int \frac{\sinh\left(bx+a\right)^{3}}{\left(dx+c\right)^{\frac{3}{2}}}dx$$

input
```
int(sinh(b*x+a)^3/(d*x+c)^(3/2),x)
```

output
```
int(sinh(b*x+a)^3/(d*x+c)^(3/2),x)
```

3.57.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1346 vs. $2(182) = 364$.

Time = 0.26 (sec) , antiderivative size = 1346, normalized size of antiderivative = 5.47

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(3)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) -
(d*x + c)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + ((d*x + c)*cosh(-3*(b*c
 - a*d)/d) - (d*x + c)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x +
 c)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-3
*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cosh(-3*(b
*c - a*d)/d) - (d*x + c)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x
+ a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(3)*sqrt(pi)*((
d*x + c)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)^
3*sinh(-3*(b*c - a*d)/d) + ((d*x + c)*cosh(-3*(b*c - a*d)/d) + (d*x + c)*s
inh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x + c)*cosh(b*x + a)*cosh(-
3*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*
x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + (d*x + c)
*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqr
t(3)*sqrt(d*x + c)*sqrt(-b/d)) - 3*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*cos
h(-(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((d*x
 + c)*cosh(-(b*c - a*d)/d) - (d*x + c)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)
^3 + 3*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(b*x
+ a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*
cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sin
h(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(pi)*((d*x +...
```

3.57.6 Sympy [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh^3{(a+bx)}}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)**3/(d*x+c)**(3/2),x)
```

output
```
Integral(sinh(a + b*x)**3/(c + d*x)**(3/2), x)
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.80

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{\frac{\sqrt{3}\sqrt{\frac{(dx+c)b}{d}}e^{\left(\frac{3\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{3\,(dx+c)b}{d}\right)}{\sqrt{dx+c}} - \frac{\sqrt{3}\sqrt{-\frac{(dx+c)b}{d}}e^{\left(-\frac{3\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{3\,(dx+c)b}{d}\right)}{\sqrt{dx+c}} - \frac{3\sqrt{\frac{(dx+c)b}{d}}e}{}}{8\,d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(sqrt(3)*sqrt((d*x + c)*b/d)*e^(3*(b*c - a*d)/d)*gamma(-1/2, 3*(d*x +
c)*b/d)/sqrt(d*x + c) - sqrt(3)*sqrt(-(d*x + c)*b/d)*e^(-3*(b*c - a*d)/d)*
gamma(-1/2, -3*(d*x + c)*b/d)/sqrt(d*x + c) - 3*sqrt((d*x + c)*b/d)*e^(-a
+ b*c/d)*gamma(-1/2, (d*x + c)*b/d)/sqrt(d*x + c) + 3*sqrt(-(d*x + c)*b/d)
*e^(a - b*c/d)*gamma(-1/2, -(d*x + c)*b/d)/sqrt(d*x + c))/d
```

3.57.8 Giac [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh{(bx+a)}^3}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^3/(d*x + c)^(3/2), x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\sinh(a+b\,x)^3}{(c+d\,x)^{3/2}}\,dx$$

input
```
int(sinh(a + b*x)^3/(c + d*x)^(3/2),x)
```

output
```
int(sinh(a + b*x)^3/(c + d*x)^(3/2), x)
```

3.58 $\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx$

3.58.1 Optimal result

Integrand size = 18, antiderivative size = 277

$$\begin{aligned}\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx =&\ \frac{b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} \\ &-\frac{b^{3/2}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}-\frac{b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} \\ &+\frac{b^{3/2}e^{3a-\frac{3bc}{d}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}-\frac{4b\cosh(a+bx)\sinh^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\sinh^3(a+bx)}{3d(c+dx)^{3/2}}\end{aligned}$$

output

```
-2/3*sinh(b*x+a)^3/d/(d*x+c)^(3/2)+1/2*b^(3/2)*exp(-a+b*c/d)*erf(b^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(5/2)-1/2*b^(3/2)*exp(a-b*c/d)*erfi(b^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(5/2)-1/2*b^(3/2)*exp(-3*a+3*b*c/d)*e
rf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/d^(5/2)+1/2*b^(
3/2)*exp(3*a-3*b*c/d)*erfi(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*
Pi^(1/2)/d^(5/2)-4*b*cosh(b*x+a)*sinh(b*x+a)^2/d^2/(d*x+c)^(1/2)
```

3.58.2 Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.91

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{e^{-3\left(a+\frac{bc}{d}\right)}\left(-3\sqrt{3}de^{6a}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right)+3de^{4a+\frac{2bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\right.\right.}{}$$

input
```
Integrate[Sinh[a + b*x]^3/(c + d*x)^(5/2),x]
```

output
```
(-3*Sqrt[3]*d*E^(6*a)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-3*b*(c + d*x
))/d] + 3*d*E^(4*a + (2*b*c)/d)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b
*(c + d*x))/d)] - 3*d*E^(2*a + (4*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/
2, (b*(c + d*x))/d] + 3*Sqrt[3]*d*E^((6*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Ga
mma[1/2, (3*b*(c + d*x))/d] - 4*E^(3*(a + (b*c)/d))*Sinh[a + b*x]^2*(6*b*(
c + d*x)*Cosh[a + b*x] + d*Sinh[a + b*x]))/(6*d^2*E^(3*(a + (b*c)/d))*(c +
 d*x)^(3/2))
```

3.58.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.52, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 26, 3795, 26, 3042, 26, 3789, 2611, 2633, 2634, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ia+ibx)^3}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ia+ibx)^3}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{3795}$$

$$i\left(\frac{12b^2\int -\frac{i\sinh^3(a+bx)}{\sqrt{c+dx}}dx}{d^2}-\frac{8b^2\int \frac{i\sinh(a+bx)}{\sqrt{c+dx}}dx}{d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{d^2\sqrt{c+dx}}+\frac{2i\sinh^3(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow \ 26$$

$$i\left(-\frac{12ib^2\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}dx}{d^2}-\frac{8ib^2\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}dx}{d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{d^2\sqrt{c+dx}}+\frac{2i\sinh^3(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$i\left(-\frac{8ib^2\int -\frac{i\sin(ia+ibx)}{\sqrt{c+dx}}dx}{d^2}-\frac{12ib^2\int \frac{i\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{d^2\sqrt{c+dx}}+\frac{2i\sinh^3(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow \ 26$$

$$i\left(-\frac{8b^2\int \frac{\sin(ia+ibx)}{\sqrt{c+dx}}dx}{d^2}+\frac{12b^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{d^2\sqrt{c+dx}}+\frac{2i\sinh^3(a+bx)}{3d(c+dx)^{3/2}}\right)$$

$$\Big\downarrow \ 3789$$

$$i\left(-\frac{8b^2\left(\frac{1}{2}i\int \frac{e^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{e^{-a-bx}}{\sqrt{c+dx}}dx\right)}{d^2}+\frac{12b^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{d^2\sqrt{c+dx}}+\frac{2i\sinh^3(a+\right.$$

$$\Big\downarrow \ 2611$$

$$i\left(-\frac{8b^2\left(\frac{i\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d^2}+\frac{12b^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+\right.$$

$$\Big\downarrow \ 2633$$

$$i\left(-\frac{8b^2\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d^2}+\frac{12b^2\int \frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+\right.$$

$$\Big\downarrow \ 2634$$

3.58. $\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx$

$$i\left(\frac{12b^2\int\frac{\sin(ia+ibx)^3}{\sqrt{c+dx}}dx}{d^2}-\frac{8b^2\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{d^2\sqrt{c+dx}}\right.$$

$$\Big\downarrow \textbf{3793}$$

$$i\left(\frac{12b^2\int\left(\frac{3i\sinh(a+bx)}{4\sqrt{c+dx}}-\frac{i\sinh(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d^2}-\frac{8b^2\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d^2}+\frac{4ib\sinh^2(a+}{d^2}\right.$$

$$\Big\downarrow \textbf{2009}$$

$$i\left(-\frac{8b^2\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d^2}+\frac{12b^2\left(-\frac{3i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{i\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\text{erf}\left(\frac{\sqrt{3}\sqrt{b}}{}\right.}{8\sqrt{b}\sqrt{d}}\right.}{}\right.$$

input
```
Int[Sinh[a + b*x]^3/(c + d*x)^(5/2),x]
```

output
```
I*((-8*b^2*(((-1/2*I)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x]
)/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + ((I/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt
[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))/d^2 + (12*b^2*((((-3*I)/8
)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]
*Sqrt[d]) + ((I/8)*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sq
rt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + (((3*I)/8)*E^(a - (b*c)/d)*Sqrt
[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) - ((I/8)*E^(
3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])
/(Sqrt[b]*Sqrt[d])))/d^2 + ((4*I)*b*Cosh[a + b*x]*Sinh[a + b*x]^2)/(d^2*Sq
rt[c + d*x]) + (((2*I)/3)*Sinh[a + b*x]^3)/(d*(c + d*x)^(3/2)))
```

3.58. $\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx$

3.58.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.58. $\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx$

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

3.58.4 Maple [F]

$$\int \frac{\sinh\left(bx+a\right)^{3}}{\left(dx+c\right)^{\frac{5}{2}}}dx$$

input
```
int(sinh(b*x+a)^3/(d*x+c)^(5/2),x)
```

output
```
int(sinh(b*x+a)^3/(d*x+c)^(5/2),x)
```

3.58.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2059 vs. 2(209) = 418.

Time = 0.30 (sec) , antiderivative size = 2059, normalized size of antiderivative = 7.43

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(6*sqrt(3)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3
*cosh(-3*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*
sinh(-3*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-3*(b*c - a
*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-3*(b*c - a*d)/d))*sinh(b*x
+ a)^3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-3*(b*c - a
*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-3*(b*c - a*d)
/d))*sinh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*
cosh(-3*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*s
inh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*
sqrt(b/d)) + 6*sqrt(3)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x
+ a)^3*cosh(-3*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x +
 a)^3*sinh(-3*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-3*(b
*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-3*(b*c - a*d)/d))*sin
h(b*x + a)^3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-3*(b
*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-3*(b*c
- a*d)/d))*sinh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x +
 a)^2*cosh(-3*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x +
a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*
x + c)*sqrt(-b/d)) - 6*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x
+ a)^3*cosh(-(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x ...
```

3.58.6 Sympy [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh^3(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)**3/(d*x+c)**(5/2),x)
```

output
```
Integral(sinh(a + b*x)**3/(c + d*x)**(5/2), x)
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.71

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{3\left(\frac{\sqrt{3}\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}} - \frac{\sqrt{3}\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}} - \cdots\right.}{8\,d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
3/8*(sqrt(3)*((d*x + c)*b/d)^(3/2)*e^(3*(b*c - a*d)/d)*gamma(-3/2, 3*(d*x
+ c)*b/d)/(d*x + c)^(3/2) - sqrt(3)*(-(d*x + c)*b/d)^(3/2)*e^(-3*(b*c - a*
d)/d)*gamma(-3/2, -3*(d*x + c)*b/d)/(d*x + c)^(3/2) - ((d*x + c)*b/d)^(3/2
)*e^(-a + b*c/d)*gamma(-3/2, (d*x + c)*b/d)/(d*x + c)^(3/2) + (-(d*x + c)*
b/d)^(3/2)*e^(a - b*c/d)*gamma(-3/2, -(d*x + c)*b/d)/(d*x + c)^(3/2))/d
```

3.58.8 Giac [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh(bx+a)^3}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^3/(d*x + c)^(5/2), x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\sinh(a+b\,x)^3}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(sinh(a + b*x)^3/(c + d*x)^(5/2),x)
```

output
```
int(sinh(a + b*x)^3/(c + d*x)^(5/2), x)
```

3.59 $\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx$

3.59.1 Optimal result

Integrand size = 18, antiderivative size = 331

$$\begin{aligned}\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx &= -\frac{b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} \\ &+ \frac{3b^{5/2}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{b^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} \\ &+ \frac{3b^{5/2}e^{3a-\frac{3bc}{d}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{16b^2\sinh(a+bx)}{5d^3\sqrt{c+dx}} \\ &- \frac{4b\cosh(a+bx)\sinh^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sinh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\sinh^3(a+bx)}{5d^3\sqrt{c+dx}}\end{aligned}$$

output
```
-4/5*b*cosh(b*x+a)*sinh(b*x+a)^2/d^2/(d*x+c)^(3/2)-2/5*sinh(b*x+a)^3/d/(d*
x+c)^(5/2)-1/5*b^(5/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi
^(1/2)/d^(7/2)-1/5*b^(5/2)*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*Pi^(1/2)/d^(7/2)+3/5*b^(5/2)*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+
c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/d^(7/2)+3/5*b^(5/2)*exp(3*a-3*b*c/d)*er
fi(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/d^(7/2)-16/5*b^
2*sinh(b*x+a)/d^3/(d*x+c)^(1/2)-24/5*b^2*sinh(b*x+a)^3/d^3/(d*x+c)^(1/2)
```

3.59.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.14

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{e^{-3a}\left(2e^{6a}\left(-d^2e^{3bx}-2be^{-\frac{3bc}{d}}(c+dx)\left(e^{\frac{3b(c+dx)}{d}}(d+6b(c+dx))+6\sqrt{3}d\left(-\frac{b(c+dx)}{d}\right)^{3}\right.\right.\right.}{}$$

input
```
Integrate[Sinh[a + b*x]^3/(c + d*x)^(7/2),x]
```

output
```
(2*E^(6*a)*(-(d^2*E^(3*b*x)) - (2*b*(c + d*x)*(E^((3*b*(c + d*x))/d)*(d +
6*b*(c + d*x)) + 6*Sqrt[3]*d*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-3*b*(
c + d*x))/d]))/E^((3*b*c)/d)) + 2*E^(4*a)*(3*d^2*E^(b*x) + (2*b*(c + d*x)*
(E^(b*(c/d + x))*(d + 2*b*(c + d*x)) + 2*d*(-((b*(c + d*x))/d))^(3/2)*Gamm
a[1/2, -((b*(c + d*x))/d)]))/E^((b*c)/d)) + E^(2*a - b*x)*(-6*d^2 + 4*b*d*
(c + d*x) - 8*b^2*(c + d*x)^2 + 8*d^2*E^(b*(c/d + x))*((b*(c + d*x))/d)^(5
/2)*Gamma[1/2, (b*(c + d*x))/d]) - (2*(-d^2 + 2*b*(c + d*x)*(d - 6*b*(c +
d*x) + 6*Sqrt[3]*d*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2
, (3*b*(c + d*x))/d])))/E^(3*b*x))/(40*d^3*E^(3*a)*(c + d*x)^(5/2))
```

3.59.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.45, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 26, 3795, 26, 3042, 26, 3778, 3042, 3788, 26, 2611, 2633, 2634, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\sin(ia+ibx)^3}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\sin(ia+ibx)^3}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \; 3795$$

$$i\left(\frac{12b^2\int -\frac{i\sinh^3(a+bx)}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\int \frac{i\sinh(a+bx)}{(c+dx)^{3/2}}dx}{5d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}}+\frac{2i\sinh^3(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(-\frac{12ib^2\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8ib^2\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}dx}{5d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}}+\frac{2i\sinh^3(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(-\frac{8ib^2\int -\frac{i\sin(ia+ibx)}{(c+dx)^{3/2}}dx}{5d^2}-\frac{12ib^2\int \frac{i\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}}+\frac{2i\sinh^3(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(-\frac{8b^2\int \frac{\sin(ia+ibx)}{(c+dx)^{3/2}}dx}{5d^2}+\frac{12b^2\int \frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}}+\frac{2i\sinh^3(a+bx)}{5d(c+dx)^{5/2}}\right)$$

$$\Big\downarrow \; 3778$$

$$i\left(\frac{12b^2\int \frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(\frac{2ib\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}}+\frac{2i\sinh^3(a+bx}{5d(c+dx)^{5/2}}\right.$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{12b^2\int \frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(\frac{2ib\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}}+\frac{2i\sinh^3(a}{5d(c+d}\right.$$

$$\Big\downarrow \; 3788$$

3.59. $\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx$

$$i\left(\frac{12b^2\int\frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(\frac{2ib\left(\frac{1}{2}i\int-\frac{ie^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{ie^{-a-bx}}{\sqrt{c+dx}}dx\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}}\right.$$

$$\Big\downarrow 26$$

$$i\left(\frac{12b^2\int\frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(\frac{2ib\left(\frac{1}{2}\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx+\frac{1}{2}\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4ib\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}}+\frac{2}{}\right.$$

$$\Big\downarrow 2611$$

$$i\left(\frac{12b^2\int\frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(\frac{2ib\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4ib\sinh^2(a+b}{5d^2(c+}\right.$$

$$\Big\downarrow 2633$$

$$i\left(-\frac{8b^2\left(\frac{2ib\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{12b^2\int\frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{4ib\sinh^2(a}{5d^2(}\right.$$

$$\Big\downarrow 2634$$

3.59. $\int\frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx$

$$i\left(\frac{12b^2\int\frac{\sin(ia+ibx)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(\frac{2ib\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4ib\sinh^2(a+b}{5d^2(c-}\right.$$

$$\Big\downarrow \text{3794}$$

$$i\left(\frac{12b^2\left(\frac{6ib\int\left(\frac{\cosh(a+bx)}{4\sqrt{c+dx}}-\frac{\cosh(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d}+\frac{2i\sinh^3(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}-\frac{8b^2\left(\frac{2ib\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}-\right.}{5d^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{12b^2\left(\frac{6ib\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}\right)}{d}+\frac{2}{}\right.}{5d^2}\right.$$

input
```
Int[Sinh[a + b*x]^3/(c + d*x)^(7/2),x]
```

3.59. $\int\frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx$

output
```
I*((((4*I)/5)*b*Cosh[a + b*x]*Sinh[a + b*x]^2)/(d^2*(c + d*x)^(3/2)) + (((
2*I)/5)*Sinh[a + b*x]^3)/(d*(c + d*x)^(5/2)) - (8*b^2*(((2*I)*b*((E^(-a +
(b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]
) + (E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sq
rt[b]*Sqrt[d])))/d - ((2*I)*Sinh[a + b*x])/(d*Sqrt[c + d*x])))/(5*d^2) + (
12*b^2*(((6*I)*b*((E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/S
qrt[d]])/(8*Sqrt[b]*Sqrt[d]) - (E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[
3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (E^(a - (b*c)/d)
*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) - (E^
(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]
)/(8*Sqrt[b]*Sqrt[d])))/d + ((2*I)*Sinh[a + b*x]^3)/(d*Sqrt[c + d*x])))/(5
*d^2))
```

3.59.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

3.59.4 Maple [F]

$$\int \frac{\sinh \left(bx+a\right)^{3}}{\left(dx+c\right)^{\frac{7}{2}}}dx$$

input
```
int(sinh(b*x+a)^3/(d*x+c)^(7/2),x)
```

output
```
int(sinh(b*x+a)^3/(d*x+c)^(7/2),x)
```

3.59.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3286 vs. $2(253) = 506$.

Time = 0.31 (sec) , antiderivative size = 3286, normalized size of antiderivative = 9.93

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/20*(12*sqrt(3)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c
*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d)
 + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-3*(b*c
 - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin
h(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) - (b^2*d^3*
x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-3*(b*
c - a*d)/d))*sinh(b*x + a)^2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-3*(b*c -
a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) - 1
2*sqrt(3)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c
^3)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^
2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + ((b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-3*(b*c - a*d)
/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-3*(b
*c - a*d)/d))*sinh(b*x + a)^3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*
c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3
*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-3*(b*c - ...
```

3.59.6 Sympy [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh^3(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)**3/(d*x+c)**(7/2),x)
```

output
```
Integral(sinh(a + b*x)**3/(c + d*x)**(7/2), x)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.60

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{3\left(\frac{3\sqrt{3}\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}} - \frac{3\sqrt{3}\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},-\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}} - \cdots\right.}{8\,d}$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
3/8*(3*sqrt(3)*((d*x + c)*b/d)^(5/2)*e^(3*(b*c - a*d)/d)*gamma(-5/2, 3*(d*
x + c)*b/d)/(d*x + c)^(5/2) - 3*sqrt(3)*(-(d*x + c)*b/d)^(5/2)*e^(-3*(b*c
- a*d)/d)*gamma(-5/2, -3*(d*x + c)*b/d)/(d*x + c)^(5/2) - ((d*x + c)*b/d)^
(5/2)*e^(-a + b*c/d)*gamma(-5/2, (d*x + c)*b/d)/(d*x + c)^(5/2) + (-(d*x +
 c)*b/d)^(5/2)*e^(a - b*c/d)*gamma(-5/2, -(d*x + c)*b/d)/(d*x + c)^(5/2))/
d
```

3.59.8 Giac [F]

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh\left(bx+a\right)^3}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^3/(d*x + c)^(7/2), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\sinh(a+b\,x)^3}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(sinh(a + b*x)^3/(c + d*x)^(7/2),x)
```

output
```
int(sinh(a + b*x)^3/(c + d*x)^(7/2), x)
```

3.60 $\int (dx)^{3/2} \sinh(fx)\,dx$

3.60.1 Optimal result

Integrand size = 12, antiderivative size = 111

$$\int (dx)^{3/2}\sinh(fx)\,dx=\frac{(dx)^{3/2}\cosh(fx)}{f}-\frac{3d^{3/2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}+\frac{3d^{3/2}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}-\frac{3d\sqrt{dx}\sinh(fx)}{2f^2}$$

output
```
(d*x)^(3/2)*cosh(f*x)/f-3/8*d^(3/2)*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))*Pi^(1
/2)/f^(5/2)+3/8*d^(3/2)*erfi(f^(1/2)*(d*x)^(1/2)/d^(1/2))*Pi^(1/2)/f^(5/2)
-3/2*d*sinh(f*x)*(d*x)^(1/2)/f^2
```

3.60.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.45

$$\int (dx)^{3/2}\sinh(fx)\,dx=\frac{d^2\left(\sqrt{-fx}\,\Gamma\left(\frac{5}{2},-fx\right)+\sqrt{fx}\,\Gamma\left(\frac{5}{2},fx\right)\right)}{2f^3\sqrt{dx}}$$

input
```
Integrate[(d*x)^(3/2)*Sinh[f*x],x]
```

output
```
(d^2*(Sqrt[-(f*x)]*Gamma[5/2, -(f*x)] + Sqrt[f*x]*Gamma[5/2, f*x]))/(2*f^3
*Sqrt[d*x])
```

3.60.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.26, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2}\sinh(fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i(dx)^{3/2}\sin(ifx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (dx)^{3/2}\sin(ifx)dx$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\int \sqrt{dx}\cosh(fx)dx}{2f}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\int \sqrt{dx}\sin\left(ifx+\frac{\pi}{2}\right)dx}{2f}\right)$$

$$\Big\downarrow \text{3777}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\left(\frac{\sqrt{dx}\sinh(fx)}{f}-\frac{id\int -\frac{i\sinh(fx)}{\sqrt{dx}}dx}{2f}\right)}{2f}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\left(\frac{\sqrt{dx}\sinh(fx)}{f}-\frac{d\int \frac{\sinh(fx)}{\sqrt{dx}}dx}{2f}\right)}{2f}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\left(\frac{\sqrt{dx}\sinh(fx)}{f}-\frac{d\int -\frac{i\sin(ifx)}{\sqrt{dx}}dx}{2f}\right)}{2f}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\left(\frac{\sqrt{dx}\sinh(fx)}{f}+\frac{id\int \frac{\sin(ifx)}{\sqrt{dx}}dx}{2f}\right)}{2f}\right)$$

$$\Big\downarrow \text{3789}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\left(\frac{\sqrt{dx}\sinh(fx)}{f}+\frac{id\left(\frac{1}{2}i\int \frac{e^{fx}}{\sqrt{dx}}dx-\frac{1}{2}i\int \frac{e^{-fx}}{\sqrt{dx}}dx\right)}{2f}\right)}{2f}\right)$$

$$\Big\downarrow \text{2611}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\left(\frac{\sqrt{dx}\sinh(fx)}{f}+\frac{id\left(\frac{i\int e^{fx}d\sqrt{dx}}{d}-\frac{i\int e^{-fx}d\sqrt{dx}}{d}\right)}{2f}\right)}{2f}\right)$$

$$\Big\downarrow \text{2633}$$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\left(\frac{\sqrt{dx}\sinh(fx)}{f}+\frac{id\left(\frac{i\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{i\int e^{-fx}d\sqrt{dx}}{d}\right)}{2f}\right)}{2f}\right)$$

$$\Big\downarrow \text{2634}$$

3.60. $\int (dx)^{3/2}\sinh(fx)\,dx$

$$-i\left(\frac{i(dx)^{3/2}\cosh(fx)}{f}-\frac{3id\left(\frac{\sqrt{dx}\sinh(fx)}{f}+\frac{id\left(\frac{i\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{i\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{2f}\right)}{2f}\right)$$

input
```
Int[(d*x)^(3/2)*Sinh[f*x],x]
```

output
```
(-I)*((I*(d*x)^(3/2)*Cosh[f*x])/f - (((3*I)/2)*d*(((I/2)*d*(((-1/2*I)*Sqrt
[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt[f]) + ((I/2)*Sqrt[Pi]
*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt[f])))/f + (Sqrt[d*x]*Sin
h[f*x])/f))/f)
```

3.60.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.60.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.19

method	result	size
meijerg	$-\frac{2(dx)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\left(-\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{7}{2}}(-14fx+21)\mathrm{e}^{fx}}{112\sqrt{\pi}\,f^{3}}+\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{7}{2}}(14fx+21)\mathrm{e}^{-fx}}{112\sqrt{\pi}\,f^{3}}-\frac{3(if)^{\frac{7}{2}}\sqrt{2}\,\operatorname{erf}(\sqrt{x}\,\sqrt{f})}{32f^{\frac{7}{2}}}+\frac{3(if)^{\frac{7}{2}}\sqrt{2}\,\operatorname{erfi}(\sqrt{x}\,\sqrt{f})}{32f^{\frac{7}{2}}}\right)}{x^{\frac{3}{2}}(if)^{\frac{3}{2}}f}$	132

input
```
int((d*x)^(3/2)*sinh(f*x),x,method=_RETURNVERBOSE)
```

output
```
-2*(d*x)^(3/2)/x^(3/2)*2^(1/2)/(I*f)^(3/2)*Pi^(1/2)/f*(-1/112/Pi^(1/2)*x^(
1/2)*2^(1/2)*(I*f)^(7/2)*(-14*f*x+21)/f^3*exp(f*x)+1/112/Pi^(1/2)*x^(1/2)*
2^(1/2)*(I*f)^(7/2)*(14*f*x+21)/f^3*exp(-f*x)-3/32*(I*f)^(7/2)*2^(1/2)/f^(
7/2)*erf(x^(1/2)*f^(1/2))+3/32*(I*f)^(7/2)*2^(1/2)/f^(7/2)*erfi(x^(1/2)*f^
(1/2)))
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(77) = 154.

Time = 0.25 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.70

$$\int (dx)^{3/2}\sinh(fx)\,dx =$$

$$-\frac{3\sqrt{\pi}(d^2\cosh(fx)+d^2\sinh(fx))\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)+3\sqrt{\pi}(d^2\cosh(fx)+d^2\sinh(fx))\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\right.}{8\,(f}$$

input
```
integrate((d*x)^(3/2)*sinh(f*x),x, algorithm="fricas")
```

output
```
-1/8*(3*sqrt(pi)*(d^2*cosh(f*x) + d^2*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*s
qrt(f/d)) + 3*sqrt(pi)*(d^2*cosh(f*x) + d^2*sinh(f*x))*sqrt(-f/d)*erf(sqrt
(d*x)*sqrt(-f/d)) - 2*(2*d*f^2*x + (2*d*f^2*x - 3*d*f)*cosh(f*x)^2 + 2*(2*
d*f^2*x - 3*d*f)*cosh(f*x)*sinh(f*x) + (2*d*f^2*x - 3*d*f)*sinh(f*x)^2 + 3
*d*f)*sqrt(d*x))/(f^3*cosh(f*x) + f^3*sinh(f*x))
```

3.60.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.63 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.20

$$\int (dx)^{3/2}\sinh(fx)\,dx = \frac{7d^{\frac{3}{2}}x^{\frac{3}{2}}\cosh{(fx)}\Gamma\left(\frac{7}{4}\right)}{4f\Gamma\left(\frac{11}{4}\right)} - \frac{21d^{\frac{3}{2}}\sqrt{x}\sinh{(fx)}\Gamma\left(\frac{7}{4}\right)}{8f^2\Gamma\left(\frac{11}{4}\right)} + \frac{21\sqrt{2}\sqrt{\pi}d^{\frac{3}{2}}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{7}{4}\right)}{16f^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((d*x)**(3/2)*sinh(f*x),x)
```

output
```
7*d**(3/2)*x**(3/2)*cosh(f*x)*gamma(7/4)/(4*f*gamma(11/4)) - 21*d**(3/2)*s
qrt(x)*sinh(f*x)*gamma(7/4)/(8*f**2*gamma(11/4)) + 21*sqrt(2)*sqrt(pi)*d**
(3/2)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/sqrt(pi)
)*gamma(7/4)/(16*f**(5/2)*gamma(11/4))
```

3.60.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(77) = 154$.

Time = 0.19 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.58

$$\int (dx)^{3/2}\sinh(fx)\,dx = \frac{16\,(dx)^{\frac{5}{2}}\sinh{(fx)} - \frac{f\left(\frac{15\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f^3\sqrt{\frac{f}{d}}} - \frac{15\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f^3\sqrt{-\frac{f}{d}}} + \frac{2\left(4\,(dx)^{\frac{5}{2}}df^2 - 10\,(dx)^{\frac{3}{2}}d^2f + 15\sqrt{d}\right.}{f^3}\right.}{d}}{40\,d}$$

input
```
integrate((d*x)^(3/2)*sinh(f*x),x, algorithm="maxima")
```

output
```
1/40*(16*(d*x)^(5/2)*sinh(f*x) - f*(15*sqrt(pi)*d^3*erf(sqrt(d*x)*sqrt(f/d
))/(f^3*sqrt(f/d)) - 15*sqrt(pi)*d^3*erf(sqrt(d*x)*sqrt(-f/d))/(f^3*sqrt(-
f/d)) + 2*(4*(d*x)^(5/2)*d*f^2 - 10*(d*x)^(3/2)*d^2*f + 15*sqrt(d*x)*d^3)*
e^(f*x)/f^3 - 2*(4*(d*x)^(5/2)*d*f^2 + 10*(d*x)^(3/2)*d^2*f + 15*sqrt(d*x)
*d^3)*e^(-f*x)/f^3)/d)/d
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.32

$$\int (dx)^{3/2}\sinh(fx)\,dx = \frac{1}{8}\,d\left(\frac{\frac{3\,\sqrt{\pi}d^3\,\mathrm{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}\,f^2}+\frac{2\left(2\,\sqrt{dx}d^2\,fx+3\,\sqrt{dx}d^2\right)e^{(-fx)}}{f^2}}{d^2}-\frac{\frac{3\,\sqrt{\pi}d^3\,\mathrm{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}\,f^2}-\frac{2\left(2\,\sqrt{dx}d^2\right.}{}}{d^2}\right.$$

input
```
integrate((d*x)^(3/2)*sinh(f*x),x, algorithm="giac")
```

output
```
1/8*d*((3*sqrt(pi)*d^3*erf(-sqrt(d*f)*sqrt(d*x)/d)/(sqrt(d*f)*f^2) + 2*(2*
sqrt(d*x)*d^2*f*x + 3*sqrt(d*x)*d^2)*e^(-f*x)/f^2)/d^2 - (3*sqrt(pi)*d^3*e
rf(-sqrt(-d*f)*sqrt(d*x)/d)/(sqrt(-d*f)*f^2) - 2*(2*sqrt(d*x)*d^2*f*x - 3*
sqrt(d*x)*d^2)*e^(f*x)/f^2)/d^2)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2}\sinh(fx)\,dx = \int \sinh(f\,x)\,\left(d\,x\right)^{3/2}\,dx$$

input
```
int(sinh(f*x)*(d*x)^(3/2),x)
```

output
```
int(sinh(f*x)*(d*x)^(3/2), x)
```

3.61 $\int \sqrt{dx} \sinh(fx) \, dx$

3.61.1 Optimal result

Integrand size = 12, antiderivative size = 92

$$\int \sqrt{dx} \sinh(fx) \, dx = \frac{\sqrt{dx} \cosh(fx)}{f} - \frac{\sqrt{d}\sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} - \frac{\sqrt{d}\sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}$$

output
```
-1/4*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/f^(3/2)-1/4*erfi(f^
(1/2)*(d*x)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/f^(3/2)+cosh(f*x)*(d*x)^(1/2)/
f
```

3.61.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int \sqrt{dx} \sinh(fx) \, dx = \frac{d\left(-\sqrt{-fx}\Gamma\left(\frac{3}{2}, -fx\right) + \sqrt{fx}\Gamma\left(\frac{3}{2}, fx\right)\right)}{2f^2\sqrt{dx}}$$

input
```
Integrate[Sqrt[d*x]*Sinh[f*x],x]
```

output
```
(d*(-(Sqrt[-(f*x)]*Gamma[3/2, -(f*x)]) + Sqrt[f*x]*Gamma[3/2, f*x]))/(2*f^
2*Sqrt[d*x])
```

3.61.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 26, 3777, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\sinh(fx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\sqrt{dx}\sin(ifx)dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \sqrt{dx}\sin(ifx)dx$$

$$\Big\downarrow \text{ 3777}$$

$$-i\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\int \frac{\cosh(fx)}{\sqrt{dx}}dx}{2f}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\int \frac{\sin\left(ifx+\frac{\pi}{2}\right)}{\sqrt{dx}}dx}{2f}\right)$$

$$\Big\downarrow \text{ 3788}$$

$$-i\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\left(\frac{1}{2}i\int -\frac{ie^{fx}}{\sqrt{dx}}dx-\frac{1}{2}i\int \frac{ie^{-fx}}{\sqrt{dx}}dx\right)}{2f}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\left(\frac{1}{2}\int \frac{e^{-fx}}{\sqrt{dx}}dx+\frac{1}{2}\int \frac{e^{fx}}{\sqrt{dx}}dx\right)}{2f}\right)$$

$$\Big\downarrow \text{ 2611}$$

$$-i\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\left(\frac{\int e^{-fx}d\sqrt{dx}}{d}+\frac{\int e^{fx}d\sqrt{dx}}{d}\right)}{2f}\right)$$

$$\Big\downarrow \text{2633}$$

$$-i\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\left(\frac{\int e^{-fx}d\sqrt{dx}}{d}+\frac{\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{2f}\right)$$

$$\Big\downarrow \text{2634}$$

$$-i\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\left(\frac{\sqrt{\pi}\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}+\frac{\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{2f}\right)$$

input
```
Int[Sqrt[d*x]*Sinh[f*x],x]
```

output
```
(-I)*((I*Sqrt[d*x]*Cosh[f*x])/f - ((I/2)*d*((Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*
x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f]) + (Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqr
t[d]])/(2*Sqrt[d]*Sqrt[f])))/f)
```

3.61.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.61.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.30

method	result	size
meijerg	$-\frac{\sqrt{\pi}\,\sqrt{dx}\,\sqrt{2}\left(\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{5}{2}}\,{\mathrm e}^{-fx}}{4\sqrt{\pi}\,f^{2}}+\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{5}{2}}\,{\mathrm e}^{fx}}{4\sqrt{\pi}\,f^{2}}-\frac{(if)^{\frac{5}{2}}\sqrt{2}\,\operatorname{erf}\left(\sqrt{x}\,\sqrt{f}\right)}{8f^{\frac{5}{2}}}-\frac{(if)^{\frac{5}{2}}\sqrt{2}\,\operatorname{erfi}\left(\sqrt{x}\,\sqrt{f}\right)}{8f^{\frac{5}{2}}}\right)}{\sqrt{x}\,\sqrt{if}\,f}$	120

input
```
int(sinh(f*x)*(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-Pi^(1/2)*(d*x)^(1/2)/x^(1/2)*2^(1/2)/(I*f)^(1/2)/f*(1/4/Pi^(1/2)*x^(1/2)*
2^(1/2)*(I*f)^(5/2)/f^2*exp(-f*x)+1/4/Pi^(1/2)*x^(1/2)*2^(1/2)*(I*f)^(5/2)
/f^2*exp(f*x)-1/8*(I*f)^(5/2)*2^(1/2)/f^(5/2)*erf(x^(1/2)*f^(1/2))-1/8*(I*
f)^(5/2)*2^(1/2)/f^(5/2)*erfi(x^(1/2)*f^(1/2)))
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(62) = 124$.

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.49

$$\int \sqrt{dx}\sinh(fx)\,dx =$$
$$-\frac{\sqrt{\pi}(d\cosh(fx)+d\sinh(fx))\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)-\sqrt{\pi}(d\cosh(fx)+d\sinh(fx))\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\right.}{4\left(f^2\cosh(fx)+f^2\sinh(fx)\right)}$$

input
```
integrate(sinh(f*x)*(d*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*(d*cosh(f*x) + d*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/
d)) - sqrt(pi)*(d*cosh(f*x) + d*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d*x)*sqrt(-
f/d)) - 2*(f*cosh(f*x)^2 + 2*f*cosh(f*x)*sinh(f*x) + f*sinh(f*x)^2 + f)*sq
rt(d*x))/(f^2*cosh(f*x) + f^2*sinh(f*x))
```

3.61.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int \sqrt{dx}\sinh(fx)\,dx = \frac{5\sqrt{d}\sqrt{x}\cosh(fx)\Gamma\left(\frac{5}{4}\right)}{4f\Gamma\left(\frac{9}{4}\right)} - \frac{5\sqrt{2}\sqrt{\pi}\sqrt{d}e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{8f^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(sinh(f*x)*(d*x)**(1/2),x)
```

output
```
5*sqrt(d)*sqrt(x)*cosh(f*x)*gamma(5/4)/(4*f*gamma(9/4)) - 5*sqrt(2)*sqrt(p
i)*sqrt(d)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/sqrt(
pi))*gamma(5/4)/(8*f**(3/2)*gamma(9/4))
```

3.61.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(62) = 124$.

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.62

$$\int \sqrt{dx}\sinh(fx)\,dx$$
$$= \frac{8\,(dx)^{\frac{3}{2}}\sinh(fx) - \frac{f\left(\frac{3\,\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f^2\sqrt{\frac{f}{d}}} + \frac{3\,\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f^2\sqrt{-\frac{f}{d}}} + \frac{2\left(2\,(dx)^{\frac{3}{2}}df - 3\,\sqrt{dx}d^2\right)e^{(fx)}}{f^2} - \frac{2\left(2\,(dx)^{\frac{3}{2}}df + 3\,\sqrt{dx}d^2\right)e^{(-fx)}}{f^2}\right)}{d}}{12\,d}$$

input
```
integrate(sinh(f*x)*(d*x)^(1/2),x, algorithm="maxima")
```

output
```
1/12*(8*(d*x)^(3/2)*sinh(f*x) - f*(3*sqrt(pi)*d^2*erf(sqrt(d*x)*sqrt(f/d))
/(f^2*sqrt(f/d)) + 3*sqrt(pi)*d^2*erf(sqrt(d*x)*sqrt(-f/d))/(f^2*sqrt(-f/d
)) + 2*(2*(d*x)^(3/2)*d*f - 3*sqrt(d*x)*d^2)*e^(f*x)/f^2 - 2*(2*(d*x)^(3/2
)*d*f + 3*sqrt(d*x)*d^2)*e^(-f*x)/f^2)/d)/d
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17

$$\int \sqrt{dx}\sinh(fx)\,dx = \frac{\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}f} + \frac{2\,\sqrt{dx}de^{(-fx)}}{f}}{4\,d} + \frac{\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}f} + \frac{2\,\sqrt{dx}de^{(fx)}}{f}}{4\,d}$$

input
```
integrate(sinh(f*x)*(d*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(pi)*d^2*erf(-sqrt(d*f)*sqrt(d*x)/d)/(sqrt(d*f)*f) + 2*sqrt(d*x)*
d*e^(-f*x)/f)/d + 1/4*(sqrt(pi)*d^2*erf(-sqrt(-d*f)*sqrt(d*x)/d)/(sqrt(-d*
f)*f) + 2*sqrt(d*x)*d*e^(f*x)/f)/d
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\sinh(fx)\,dx = \int \sinh(f\,x)\,\sqrt{d\,x}\,dx$$

input
```
int(sinh(f*x)*(d*x)^(1/2),x)
```

output
```
int(sinh(f*x)*(d*x)^(1/2), x)
```

3.62 $\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx$

3.62.1 Optimal result

Integrand size = 12, antiderivative size = 77

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx = -\frac{\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
-1/2*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))*Pi^(1/2)/d^(1/2)/f^(1/2)+1/2*erfi(f^
(1/2)*(d*x)^(1/2)/d^(1/2))*Pi^(1/2)/d^(1/2)/f^(1/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx = \frac{\sqrt{-fx}\,\Gamma\left(\frac{1}{2},-fx\right) + \sqrt{fx}\,\Gamma\left(\frac{1}{2},fx\right)}{2f\sqrt{dx}}$$

input
```
Integrate[Sinh[f*x]/Sqrt[d*x],x]
```

output
```
(Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] + Sqrt[f*x]*Gamma[1/2, f*x])/(2*f*Sqrt[d*
x])
```

3.62.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ifx)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ifx)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{3789}$$

$$-i\left(\frac{1}{2}i\int \frac{e^{fx}}{\sqrt{dx}}dx-\frac{1}{2}i\int \frac{e^{-fx}}{\sqrt{dx}}dx\right)$$

$$\Big\downarrow \text{2611}$$

$$-i\left(\frac{i\int e^{fx}d\sqrt{dx}}{d}-\frac{i\int e^{-fx}d\sqrt{dx}}{d}\right)$$

$$\Big\downarrow \text{2633}$$

$$-i\left(\frac{i\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{i\int e^{-fx}d\sqrt{dx}}{d}\right)$$

$$\Big\downarrow \text{2634}$$

$$-i\left(\frac{i\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{i\sqrt{\pi}\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)$$

input
```
Int[Sinh[f*x]/Sqrt[d*x],x]
```

output
```
(-I)*(((-1/2*I)*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt[f
]) + ((I/2)*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt[f]))
```

3.62.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.62.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

method	result	size
meijerg	$-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{i f}\left(-\frac{(i f)^{\frac{3}{2}}\sqrt{2}\,\operatorname{erf}\left(\sqrt{x}\,\sqrt{f}\right)}{2 f^{\frac{3}{2}}}+\frac{(i f)^{\frac{3}{2}}\sqrt{2}\,\operatorname{erfi}\left(\sqrt{x}\,\sqrt{f}\right)}{2 f^{\frac{3}{2}}}\right)}{2\sqrt{d x}\,f}$	71

input
```
int(sinh(f*x)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*Pi^(1/2)/(d*x)^(1/2)*x^(1/2)*2^(1/2)*(I*f)^(1/2)/f*(-1/2*(I*f)^(3/2)*
2^(1/2)/f^(3/2)*erf(x^(1/2)*f^(1/2))+1/2*(I*f)^(3/2)*2^(1/2)/f^(3/2)*erfi(
x^(1/2)*f^(1/2)))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx = -\frac{\sqrt{\pi}\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)+\sqrt{\pi}\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{2\,f}$$

input
```
integrate(sinh(f*x)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(pi)*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d)) + sqrt(pi)*sqrt(-f/d)*er
f(sqrt(d*x)*sqrt(-f/d)))/f
```

3.62.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx = \frac{3\sqrt{2}\sqrt{\pi}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{4\sqrt{d}\sqrt{f}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(sinh(f*x)/(d*x)**(1/2),x)
```

output
```
3*sqrt(2)*sqrt(pi)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*p
i/4)/sqrt(pi))*gamma(3/4)/(4*sqrt(d)*sqrt(f)*gamma(7/4))
```

3.62.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(49) = 98$.

Time = 0.18 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.51

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx = \frac{4\sqrt{dx}\sinh(fx) - \frac{\left(\frac{2\sqrt{dx}de^{(fx)}}{f} - \frac{2\sqrt{dx}de^{(-fx)}}{f} + \frac{\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f\sqrt{\frac{f}{d}}} - \frac{\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f\sqrt{-\frac{f}{d}}}\right)f}{d}}{2\,d}$$

input
```
integrate(sinh(f*x)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(4*sqrt(d*x)*sinh(f*x) - (2*sqrt(d*x)*d*e^(f*x)/f - 2*sqrt(d*x)*d*e^(-
f*x)/f + sqrt(pi)*d*erf(sqrt(d*x)*sqrt(f/d))/(f*sqrt(f/d)) - sqrt(pi)*d*er
f(sqrt(d*x)*sqrt(-f/d))/(f*sqrt(-f/d)))*f/d)/d
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx = \frac{\frac{\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}} - \frac{\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}}}{2\,d}$$

input
```
integrate(sinh(f*x)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(pi)*d*erf(-sqrt(d*f)*sqrt(d*x)/d)/sqrt(d*f) - sqrt(pi)*d*erf(-sq
rt(-d*f)*sqrt(d*x)/d)/sqrt(-d*f))/d
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx = \int \frac{\sinh(f\,x)}{\sqrt{d\,x}}\,dx$$

input
```
int(sinh(f*x)/(d*x)^(1/2),x)
```

output
```
int(sinh(f*x)/(d*x)^(1/2), x)
```

3.63 $\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx$

3.63.1 Optimal result

Integrand size = 12, antiderivative size = 87

$$\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx = \frac{\sqrt{f}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{f}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sinh(fx)}{d\sqrt{dx}}$$

output
```
erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))*f^(1/2)*Pi^(1/2)/d^(3/2)+erfi(f^(1/2)*(d*
x)^(1/2)/d^(1/2))*f^(1/2)*Pi^(1/2)/d^(3/2)-2*sinh(f*x)/d/(d*x)^(1/2)
```

3.63.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.56

$$\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx = \frac{x\left(\sqrt{-fx}\,\Gamma\left(\frac{1}{2},-fx\right) - \sqrt{fx}\,\Gamma\left(\frac{1}{2},fx\right) - 2\sinh(fx)\right)}{(dx)^{3/2}}$$

input
```
Integrate[Sinh[f*x]/(d*x)^(3/2),x]
```

output
```
(x*(Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] - Sqrt[f*x]*Gamma[1/2, f*x] - 2*Sinh[f
*x]))/(d*x)^(3/2)
```

3.63.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 26, 3778, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(fx)}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ifx)}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ifx)}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{3778}$$

$$-i\left(\frac{2if\int \frac{\cosh(fx)}{\sqrt{dx}}dx}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{2if\int \frac{\sin\left(ifx+\frac{\pi}{2}\right)}{\sqrt{dx}}dx}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)$$

$$\Big\downarrow \text{3788}$$

$$-i\left(\frac{2if\left(\frac{1}{2}i\int -\frac{ie^{fx}}{\sqrt{dx}}dx-\frac{1}{2}i\int \frac{ie^{-fx}}{\sqrt{dx}}dx\right)}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{2if\left(\frac{1}{2}\int \frac{e^{-fx}}{\sqrt{dx}}dx+\frac{1}{2}\int \frac{e^{fx}}{\sqrt{dx}}dx\right)}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)$$

$$\Big\downarrow \text{2611}$$

$$-i\left(\frac{2if\left(\frac{\int e^{-fx}d\sqrt{dx}}{d}+\frac{\int e^{fx}d\sqrt{dx}}{d}\right)}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)$$

$$\Big\downarrow \text{2633}$$

$$-i\left(\frac{2if\left(\frac{\int e^{-fx}d\sqrt{dx}}{d}+\frac{\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)$$

$$\Big\downarrow \text{2634}$$

$$-i\left(\frac{2if\left(\frac{\sqrt{\pi}\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}+\frac{\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)$$

input
```
Int[Sinh[f*x]/(d*x)^(3/2),x]
```

output
```
(-I)*(((2*I)*f*((Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqr
t[f]) + (Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f])))
/d - ((2*I)*Sinh[f*x])/(d*Sqrt[d*x]))
```

3.63.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.63.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.38

method	result	size
meijerg	$-\frac{\sqrt{\pi}\, x^{\frac{3}{2}} \sqrt{2}\, (if)^{\frac{3}{2}} \left(\frac{2\sqrt{2}\, \sqrt{if}\, \mathrm{e}^{-fx}}{\sqrt{\pi}\, \sqrt{x}\, f}-\frac{2\sqrt{2}\, \sqrt{if}\, \mathrm{e}^{fx}}{\sqrt{\pi}\, \sqrt{x}\, f}+\frac{2\sqrt{if}\, \sqrt{2}\, \operatorname{erf}\left(\sqrt{x}\, \sqrt{f}\right)}{\sqrt{f}}+\frac{2\sqrt{if}\, \sqrt{2}\, \operatorname{erfi}\left(\sqrt{x}\, \sqrt{f}\right)}{\sqrt{f}}\right)}{4(dx)^{\frac{3}{2}} f}$	120

input
```
int(sinh(f*x)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*Pi^(1/2)/(d*x)^(3/2)*x^(3/2)*2^(1/2)*(I*f)^(3/2)/f*(2/Pi^(1/2)/x^(1/2
)*2^(1/2)*(I*f)^(1/2)/f*exp(-f*x)-2/Pi^(1/2)/x^(1/2)*2^(1/2)*(I*f)^(1/2)/f
*exp(f*x)+2*(I*f)^(1/2)*2^(1/2)/f^(1/2)*erf(x^(1/2)*f^(1/2))+2*(I*f)^(1/2)
*2^(1/2)/f^(1/2)*erfi(x^(1/2)*f^(1/2)))
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(61) = 122$.

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

$$\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx = \frac{\sqrt{\pi}(dx\cosh(fx) + dx\sinh(fx))\sqrt{\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right) - \sqrt{\pi}(dx\cosh(fx) + dx\sinh(fx))\sqrt{}}{d^2x\cosh(fx) + d^2}$$

input
```
integrate(sinh(f*x)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
(sqrt(pi)*(d*x*cosh(f*x) + d*x*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d
)) - sqrt(pi)*(d*x*cosh(f*x) + d*x*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d*x)*sqr
t(-f/d)) - sqrt(d*x)*(cosh(f*x)^2 + 2*cosh(f*x)*sinh(f*x) + sinh(f*x)^2 -
1))/(d^2*x*cosh(f*x) + d^2*x*sinh(f*x))
```

3.63.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx = \frac{\sqrt{2}\sqrt{\pi}\sqrt{f}e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{2d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} - \frac{\sinh(fx)\Gamma\left(\frac{1}{4}\right)}{2d^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(sinh(f*x)/(d*x)**(3/2),x)
```

output
```
sqrt(2)*sqrt(pi)*sqrt(f)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*exp
(I*pi/4)/sqrt(pi))*gamma(1/4)/(2*d**(3/2)*gamma(5/4)) - sinh(f*x)*gamma(1/
4)/(2*d**(3/2)*sqrt(x)*gamma(5/4))
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx = \frac{\frac{f\left(\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{\sqrt{\frac{f}{d}}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{\sqrt{-\frac{f}{d}}}\right)}{d}-\frac{2\sinh(fx)}{\sqrt{dx}}}{d}$$

input
```
integrate(sinh(f*x)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
(f*(sqrt(pi)*erf(sqrt(d*x)*sqrt(f/d))/sqrt(f/d) + sqrt(pi)*erf(sqrt(d*x)*s
qrt(-f/d))/sqrt(-f/d))/d - 2*sinh(f*x)/sqrt(d*x))/d
```

3.63.8 Giac [F]

$$\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx = \int \frac{\sinh(fx)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(f*x)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(f*x)/(d*x)^(3/2), x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx = \int \frac{\sinh(f\,x)}{(d\,x)^{3/2}}\,dx$$

input
```
int(sinh(f*x)/(d*x)^(3/2),x)
```

output
```
int(sinh(f*x)/(d*x)^(3/2), x)
```

3.64 $\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx$

3.64.1 Optimal result

Integrand size = 12, antiderivative size = 114

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx = -\frac{4f\cosh(fx)}{3d^2\sqrt{dx}} - \frac{2f^{3/2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2f^{3/2}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{2\sinh(fx)}{3d(dx)^{3/2}}$$

output
```
-2/3*sinh(f*x)/d/(d*x)^(3/2)-2/3*f^(3/2)*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))*
Pi^(1/2)/d^(5/2)+2/3*f^(3/2)*erfi(f^(1/2)*(d*x)^(1/2)/d^(1/2))*Pi^(1/2)/d^
(5/2)-4/3*f*cosh(f*x)/d^2/(d*x)^(1/2)
```

3.64.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx = -\frac{e^{-fx}x\left(-1+e^{2fx}+2fx+2e^{2fx}fx+2e^{fx}(-fx)^{3/2}\Gamma\left(\frac{1}{2},-fx\right)-2e^{fx}(fx)^{3/2}\Gamma\left(\frac{1}{2},fx\right)\right)}{3(dx)^{5/2}}$$

input
```
Integrate[Sinh[f*x]/(d*x)^(5/2),x]
```

output
```
-1/3*(x*(-1 + E^(2*f*x) + 2*f*x + 2*E^(2*f*x)*f*x + 2*E^(f*x)*(-(f*x))^(3/
2)*Gamma[1/2, -(f*x)] - 2*E^(f*x)*(f*x)^(3/2)*Gamma[1/2, f*x]))/(E^(f*x)*(
d*x)^(5/2))
```

3.64.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 26, 3778, 3042, 3778, 26, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\sin(ifx)}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\sin(ifx)}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 3778}$$

$$-i\left(\frac{2if\int \frac{\cosh(fx)}{(dx)^{3/2}}dx}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{2if\int \frac{\sin\left(ifx+\frac{\pi}{2}\right)}{(dx)^{3/2}}dx}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$$\Big\downarrow \text{ 3778}$$

$$-i\left(\frac{2if\left(-\frac{2\cosh(fx)}{d\sqrt{dx}}+\frac{2if\int -\frac{i\sinh(fx)}{\sqrt{dx}}dx}{d}\right)}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\frac{2if\left(\frac{2f\int \frac{\sinh(fx)}{\sqrt{dx}}dx}{d}-\frac{2\cosh(fx)}{d\sqrt{dx}}\right)}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{2if\left(-\frac{2\cosh(fx)}{d\sqrt{dx}}+\frac{2f\int -\frac{i\sin(ifx)}{\sqrt{dx}}dx}{d}\right)}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{2if\left(-\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{2if\int \frac{\sin(ifx)}{\sqrt{dx}}dx}{d}\right)}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$$\Big\downarrow \text{3789}$$

$$-i\left(\frac{2if\left(-\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{2if\left(\frac{1}{2}i\int \frac{e^{fx}}{\sqrt{dx}}dx-\frac{1}{2}i\int \frac{e^{-fx}}{\sqrt{dx}}dx\right)}{d}\right)}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$$\Big\downarrow \text{2611}$$

$$-i\left(\frac{2if\left(-\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{2if\left(\frac{i\int e^{fx}d\sqrt{dx}}{d}-\frac{i\int e^{-fx}d\sqrt{dx}}{d}\right)}{d}\right)}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$$\Big\downarrow \text{2633}$$

$$-i\left(\frac{2if\left(-\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{2if\left(\frac{i\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{i\int e^{-fx}d\sqrt{dx}}{d}\right)}{d}\right)}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

$$\Big\downarrow \text{2634}$$

3.64. $\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx$

$$-i\left(\frac{2if\left(-\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{2if\left(\frac{i\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{i\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{d}\right)}{3d}-\frac{2i\sinh(fx)}{3d(dx)^{3/2}}\right)$$

input
```
Int[Sinh[f*x]/(d*x)^(5/2),x]
```

output
```
(-I)*((((2*I)/3)*f*((-2*Cosh[f*x])/(d*Sqrt[d*x]) - ((2*I)*f*(((-1/2*I)*Sqr
t[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt[f]) + ((I/2)*Sqrt[Pi
]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt[f])))/d))/d - (((2*I)/3
)*Sinh[f*x])/(d*(d*x)^(3/2)))
```

3.64.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.64.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.16

method	result	size
meijerg	$-\frac{\sqrt{\pi}\, x^{\frac{5}{2}} \sqrt{2}\, (i f)^{\frac{5}{2}} \left(-\frac{4 \sqrt{2}\, (2 f x+1) {\mathrm e}^{f x}}{3 \sqrt{\pi}\, x^{\frac{3}{2}} \sqrt{i f}\, f}+\frac{4 \sqrt{2}\, (-2 f x+1) {\mathrm e}^{-f x}}{3 \sqrt{\pi}\, x^{\frac{3}{2}} \sqrt{i f}\, f}-\frac{8 \sqrt{2}\, \sqrt{f}\, \operatorname{erf}\left(\sqrt{x}\, \sqrt{f}\right)}{3 \sqrt{i f}}+\frac{8 \sqrt{2}\, \sqrt{f}\, \operatorname{erfi}\left(\sqrt{x}\, \sqrt{f}\right)}{3 \sqrt{i f}}\right)}{8 (d x)^{\frac{5}{2}} f}$	132

input
```
int(sinh(f*x)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*Pi^(1/2)/(d*x)^(5/2)*x^(5/2)*2^(1/2)*(I*f)^(5/2)/f*(-4/3/Pi^(1/2)/x^(
3/2)*2^(1/2)/(I*f)^(1/2)*(2*f*x+1)/f*exp(f*x)+4/3/Pi^(1/2)/x^(3/2)*2^(1/2)
/(I*f)^(1/2)*(-2*f*x+1)/f*exp(-f*x)-8/3/(I*f)^(1/2)*2^(1/2)*f^(1/2)*erf(x^
(1/2)*f^(1/2))+8/3/(I*f)^(1/2)*2^(1/2)*f^(1/2)*erfi(x^(1/2)*f^(1/2)))
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(78) = 156.

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.56

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}\, dx =$$

$$-\frac{2\sqrt{\pi}(dfx^2\cosh(fx)+dfx^2\sinh(fx))\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)+2\sqrt{\pi}(dfx^2\cosh(fx)+dfx^2\sinh(fx))\sqrt{-\frac{f}{d}}}{3\,(d^3x^2\cosh}$$

input
```
integrate(sinh(f*x)/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(pi)*(d*f*x^2*cosh(f*x) + d*f*x^2*sinh(f*x))*sqrt(f/d)*erf(sqr
t(d*x)*sqrt(f/d)) + 2*sqrt(pi)*(d*f*x^2*cosh(f*x) + d*f*x^2*sinh(f*x))*sqr
t(-f/d)*erf(sqrt(d*x)*sqrt(-f/d)) + ((2*f*x + 1)*cosh(f*x)^2 + 2*(2*f*x +
1)*cosh(f*x)*sinh(f*x) + (2*f*x + 1)*sinh(f*x)^2 + 2*f*x - 1)*sqrt(d*x))/(
d^3*x^2*cosh(f*x) + d^3*x^2*sinh(f*x))
```

3.64.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx = -\frac{\sqrt{2}\sqrt{\pi}f^{\frac{3}{2}}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{1}{4}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{3}{4}\right)} + \frac{f\cosh{(fx)}\Gamma\left(-\frac{1}{4}\right)}{3d^{\frac{5}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{\sinh{(fx)}\Gamma\left(-\frac{1}{4}\right)}{6d^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(sinh(f*x)/(d*x)**(5/2),x)
```

output
```
-sqrt(2)*sqrt(pi)*f**(3/2)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)
*exp(I*pi/4)/sqrt(pi))*gamma(-1/4)/(3*d**(5/2)*gamma(3/4)) + f*cosh(f*x)*g
amma(-1/4)/(3*d**(5/2)*sqrt(x)*gamma(3/4)) + sinh(f*x)*gamma(-1/4)/(6*d**(
5/2)*x**(3/2)*gamma(3/4))
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx = -\frac{\frac{f\left(\frac{\sqrt{fx}\Gamma\left(-\frac{1}{2},fx\right)}{\sqrt{dx}}+\frac{\sqrt{-fx}\Gamma\left(-\frac{1}{2},-fx\right)}{\sqrt{dx}}\right)}{d}+\frac{2\sinh(fx)}{(dx)^{\frac{3}{2}}}}{3\,d}$$

input
```
integrate(sinh(f*x)/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(f*(sqrt(f*x)*gamma(-1/2, f*x)/sqrt(d*x) + sqrt(-f*x)*gamma(-1/2, -f*
x)/sqrt(d*x))/d + 2*sinh(f*x)/(d*x)^(3/2))/d
```

3.64.8 Giac [F]

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx = \int \frac{\sinh(fx)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(f*x)/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sinh(f*x)/(d*x)^(5/2), x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx = \int \frac{\sinh(f\,x)}{(d\,x)^{5/2}}\,dx$$

input
```
int(sinh(f*x)/(d*x)^(5/2),x)
```

output
```
int(sinh(f*x)/(d*x)^(5/2), x)
```

3.65 $\int \sqrt{c+dx}\,\mathrm{csch}(a+bx)\,dx$

3.65.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \sqrt{c+dx}\,\mathrm{csch}(a+bx)\,dx = \mathrm{Int}\left(\sqrt{c+dx}\,\mathrm{csch}(a+bx),x\right)$$

output
```
Unintegrable(csch(b*x+a)*(d*x+c)^(1/2),x)
```

3.65.2 Mathematica [N/A]

Not integrable

Time = 28.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \sqrt{c+dx}\,\mathrm{csch}(a+bx)\,dx = \int \sqrt{c+dx}\,\mathrm{csch}(a+bx)\,dx$$

input
```
Integrate[Sqrt[c + d*x]*Csch[a + b*x],x]
```

output
```
Integrate[Sqrt[c + d*x]*Csch[a + b*x], x]
```

3.65.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\text{csch}(a+bx)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int i\sqrt{c+dx}\csc(ia+ibx)dx$$
$$\Big\downarrow \text{ 26}$$
$$i\int \sqrt{c+dx}\csc(ia+ibx)dx$$
$$\Big\downarrow \text{ 4680}$$
$$\int \sqrt{c+dx}\text{csch}(a+bx)dx$$

input
```
Int[Sqrt[c + d*x]*Csch[a + b*x],x]
```

output
```
$Aborted
```

3.65.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

3.65.4 Maple [N/A] (verified)

Not integrable

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \operatorname{csch}\left(bx+a\right)\sqrt{dx+c}dx$$

input
```
int(csch(b*x+a)*(d*x+c)^(1/2),x)
```

output
```
int(csch(b*x+a)*(d*x+c)^(1/2),x)
```

3.65.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\operatorname{csch}(a+bx)\,dx=\int \sqrt{dx+c}\operatorname{csch}\left(bx+a\right)\,dx$$

input
```
integrate(csch(b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*csch(b*x + a), x)
```

3.65.6 Sympy [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{c+dx}\operatorname{csch}(a+bx)\,dx = \int \sqrt{c+dx}\operatorname{csch}(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)*(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*csch(a + b*x), x)
```

3.65.7 Maxima [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\operatorname{csch}(a+bx)\,dx = \int \sqrt{dx+c}\operatorname{csch}(bx+a)\,dx$$

input
```
integrate(csch(b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*csch(b*x + a), x)
```

3.65.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\operatorname{csch}(a+bx)\,dx = \int \sqrt{dx+c}\operatorname{csch}(bx+a)\,dx$$

input
```
integrate(csch(b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*csch(b*x + a), x)
```

3.65.9 Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \sqrt{c+dx}\operatorname{csch}(a+bx)\,dx = \int \frac{\sqrt{c+d\,x}}{\sinh\left(a+b\,x\right)}\,dx$$

input
```
int((c + d*x)^(1/2)/sinh(a + b*x),x)
```

output
```
int((c + d*x)^(1/2)/sinh(a + b*x), x)
```

3.66 $\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx$

3.66.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx=\operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}},x\right)$$

output
```
Unintegrable(csch(b*x+a)/(d*x+c)^(1/2),x)
```

3.66.2 Mathematica [N/A]

Not integrable

Time = 36.87 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx$$

input
```
Integrate[Csch[a + b*x]/Sqrt[c + d*x],x]
```

output
```
Integrate[Csch[a + b*x]/Sqrt[c + d*x], x]
```

3.66.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(a+bx)}{\sqrt{c+dx}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\csc(ia+ibx)}{\sqrt{c+dx}}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\csc(ia+ibx)}{\sqrt{c+dx}}dx$$
$$\Big\downarrow \text{4680}$$
$$\int \frac{\text{csch}(a+bx)}{\sqrt{c+dx}}dx$$

input
```
Int[Csch[a + b*x]/Sqrt[c + d*x],x]
```

output
```
$Aborted
```

3.66.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

3.66.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{csch}(bx+a)}{\sqrt{dx+c}}dx$$

input
```
int(csch(b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(csch(b*x+a)/(d*x+c)^(1/2),x)
```

3.66.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\operatorname{csch}(bx+a)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)/sqrt(d*x + c), x)
```

3.66.6 Sympy [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c)**(1/2),x)
```

output
```
Integral(csch(a + b*x)/sqrt(c + d*x), x)
```

3.66.7 Maxima [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\operatorname{csch}(bx+a)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csch(b*x + a)/sqrt(d*x + c), x)
```

3.66.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\operatorname{csch}(bx+a)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(csch(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)/sqrt(d*x + c), x)
```

3.66.9 Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{1}{\sinh(a+b\,x)\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/(sinh(a + b*x)*(c + d*x)^(1/2)),x)
```

output
```
int(1/(sinh(a + b*x)*(c + d*x)^(1/2)), x)
```

3.67 $\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx$

3.67.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx = -\frac{3\cosh(x)\sqrt{\sinh(x)}}{4x} - \frac{\sinh^{\frac{3}{2}}(x)}{2x^2} + \frac{3}{8}\text{Int}\left(\frac{1}{x\sqrt{\sinh(x)}},x\right) + \frac{9}{8}\text{Int}\left(\frac{\sinh^{\frac{3}{2}}(x)}{x},x\right)$$

output
```
-1/2*sinh(x)^(3/2)/x^2-3/4*cosh(x)*sinh(x)^(1/2)/x+9/8*Unintegrable(sinh(x
)^(3/2)/x,x)+3/8*Unintegrable(1/x/sinh(x)^(1/2),x)
```

3.67.2 Mathematica [N/A]

Not integrable

Time = 4.86 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx$$

input
```
Integrate[Sinh[x]^(3/2)/x^3,x]
```

output
```
Integrate[Sinh[x]^(3/2)/x^3, x]
```

3.67.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3795, 3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(-i\sin(ix))^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{3795}$$

$$\frac{9}{8}\int \frac{\sinh^{\frac{3}{2}}(x)}{x}dx + \frac{3}{8}\int \frac{1}{x\sqrt{\sinh(x)}}dx - \frac{\sinh^{\frac{3}{2}}(x)}{2x^2} - \frac{3\sqrt{\sinh(x)}\cosh(x)}{4x}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{8}\int \frac{1}{x\sqrt{-i\sin(ix)}}dx + \frac{9}{8}\int \frac{(-i\sin(ix))^{3/2}}{x}dx - \frac{\sinh^{\frac{3}{2}}(x)}{2x^2} - \frac{3\sqrt{\sinh(x)}\cosh(x)}{4x}$$

$$\Big\downarrow \text{3807}$$

$$\frac{9}{8}\int \frac{\sinh^{\frac{3}{2}}(x)}{x}dx + \frac{3}{8}\int \frac{1}{x\sqrt{\sinh(x)}}dx - \frac{\sinh^{\frac{3}{2}}(x)}{2x^2} - \frac{3\sqrt{\sinh(x)}\cosh(x)}{4x}$$

input
```
Int[Sinh[x]^(3/2)/x^3,x]
```

output
```
$Aborted
```

3.67.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.67.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sinh(x)^{\frac{3}{2}}}{x^3}dx$$

input
```
int(sinh(x)^(3/2)/x^3,x)
```

output
```
int(sinh(x)^(3/2)/x^3,x)
```

3.67.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sinh(x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

3.67.6 Sympy [N/A]

Not integrable

Time = 4.45 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx$$

input
```
integrate(sinh(x)**(3/2)/x**3,x)
```

output
```
Integral(sinh(x)**(3/2)/x**3, x)
```

3.67.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\sinh(x)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(sinh(x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sinh(x)^(3/2)/x^3, x)
```

3.67.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\sinh(x)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(sinh(x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sinh(x)^(3/2)/x^3, x)
```

3.67.9 Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\sinh(x)^{3/2}}{x^3}\,dx$$

input
```
int(sinh(x)^(3/2)/x^3,x)
```

output
```
int(sinh(x)^(3/2)/x^3, x)
```

3.68 $\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)}-x\sqrt{\sinh(x)}\right)\,dx$

3.68.1 Optimal result

Integrand size = 18, antiderivative size = 20

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)}-x\sqrt{\sinh(x)}\right)\,dx=-\frac{2x\cosh(x)}{\sqrt{\sinh(x)}}+4\sqrt{\sinh(x)}$$

output
```
-2*x*cosh(x)/sinh(x)^(1/2)+4*sinh(x)^(1/2)
```

3.68.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)}-x\sqrt{\sinh(x)}\right)\,dx=\frac{-2x\cosh(x)+4\sinh(x)}{\sqrt{\sinh(x)}}$$

input
```
Integrate[x/Sinh[x]^(3/2) - x*Sqrt[Sinh[x]],x]
```

output
```
(-2*x*Cosh[x] + 4*Sinh[x])/Sqrt[Sinh[x]]
```

3.68.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)}-x\sqrt{\sinh(x)}\right) dx$$
$$\Big\downarrow \; 2009$$
$$4\sqrt{\sinh(x)}-\frac{2x\cosh(x)}{\sqrt{\sinh(x)}}$$

input
```
Int[x/Sinh[x]^(3/2) - x*Sqrt[Sinh[x]],x]
```

output
```
(-2*x*Cosh[x])/Sqrt[Sinh[x]] + 4*Sqrt[Sinh[x]]
```

3.68.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.68.4 Maple [F]

$$\int \left(\frac{x}{\sinh \left(x\right)^{\frac{3}{2}}}-x\sqrt{\sinh \left(x\right)}\right) dx$$

input
```
int(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x)
```

output
```
int(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x)
```

3.68.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)} - x\sqrt{\sinh(x)}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.68.6 Sympy [F]

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)} - x\sqrt{\sinh(x)}\right) dx = -\int \left(-\frac{x}{\sinh^{\frac{3}{2}}(x)}\right) dx - \int x\sqrt{\sinh(x)}\, dx$$

input
```
integrate(x/sinh(x)**(3/2)-x*sinh(x)**(1/2),x)
```

output
```
-Integral(-x/sinh(x)**(3/2), x) - Integral(x*sqrt(sinh(x)), x)
```

3.68.7 Maxima [F]

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)} - x\sqrt{\sinh(x)}\right) dx = \int -x\sqrt{\sinh(x)} + \frac{x}{\sinh(x)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-x*sqrt(sinh(x)) + x/sinh(x)^(3/2), x)
```

3.68.8 Giac [F]

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)} - x\sqrt{\sinh(x)}\right) dx = \int -x\sqrt{\sinh(x)} + \frac{x}{\sinh(x)^{\frac{3}{2}}} dx$$

input
```
integrate(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x*sqrt(sinh(x)) + x/sinh(x)^(3/2), x)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)} - x\sqrt{\sinh(x)}\right) dx = -\frac{2\sqrt{\frac{e^x}{2} - \frac{e^{-x}}{2}}\,(x - 2\,e^{2\,x} + x\,e^{2\,x} + 2)}{e^{2\,x} - 1}$$

input
```
int(x/sinh(x)^(3/2) - x*sinh(x)^(1/2),x)
```

output
```
-(2*(exp(x)/2 - exp(-x)/2)^(1/2)*(x - 2*exp(2*x) + x*exp(2*x) + 2))/(exp(2
*x) - 1)
```

3.69 $\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)\,dx$

3.69.1 Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)\,dx=-\frac{2x\cosh(x)}{3\sinh^{\frac{3}{2}}(x)}-\frac{4}{3\sqrt{\sinh(x)}}$$

output
```
-2/3*x*cosh(x)/sinh(x)^(3/2)-4/3/sinh(x)^(1/2)
```

3.69.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)\,dx=\frac{1}{6}(-8\text{csch}(x)-4x\coth(x)\text{csch}(x))\sqrt{\sinh(x)}$$

input
```
Integrate[x/Sinh[x]^(5/2) + x/(3*Sqrt[Sinh[x]]),x]
```

output
```
((-8*Csch[x] - 4*x*Coth[x]*Csch[x])*Sqrt[Sinh[x]])/6
```

3.69.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{4}{3\sqrt{\sinh(x)}}-\frac{2x\cosh(x)}{3\sinh^{\frac{3}{2}}(x)}$$

input
```
Int[x/Sinh[x]^(5/2) + x/(3*Sqrt[Sinh[x]]),x]
```

output
```
(-2*x*Cosh[x])/(3*Sinh[x]^(3/2)) - 4/(3*Sqrt[Sinh[x]])
```

3.69.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.69.4 Maple [F]

$$\int \left(\frac{x}{\sinh\left(x\right)^{\frac{5}{2}}}+\frac{x}{3\sqrt{\sinh\left(x\right)}}\right)dx$$

input
```
int(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x)
```

output
```
int(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x)
```

3.69.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(16) = 32$.

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.50

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)\,dx =$$
$$-\frac{4\left((x+2)\cosh(x)^3+3\,(x+2)\cosh(x)\sinh(x)^2+(x+2)\sinh(x)^3+(x-2)\cosh(x)+\left(3\,(x+2)\cos\right.\right.}{3\left(\cosh(x)^4+4\,\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\,\cosh(x)^2-1\right)\sinh(x)^2-2\,\cosh(x)^2+4\left(\cos\right.\right.}$$

input
```
integrate(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x, algorithm="fricas")
```

output
```
-4/3*((x + 2)*cosh(x)^3 + 3*(x + 2)*cosh(x)*sinh(x)^2 + (x + 2)*sinh(x)^3
+ (x - 2)*cosh(x) + (3*(x + 2)*cosh(x)^2 + x - 2)*sinh(x))*sqrt(sinh(x))/(
cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^
2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

3.69.6 Sympy [F]

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)\,dx = \frac{\int \frac{3x}{\sinh^{\frac{5}{2}}(x)}\,dx+\int \frac{x}{\sqrt{\sinh(x)}}\,dx}{3}$$

input
```
integrate(x/sinh(x)**(5/2)+1/3*x/sinh(x)**(1/2),x)
```

output
```
(Integral(3*x/sinh(x)**(5/2), x) + Integral(x/sqrt(sinh(x)), x))/3
```

3.69.7 Maxima [F]

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right)\,dx = \int \frac{x}{3\sqrt{\sinh(x)}}+\frac{x}{\sinh(x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/3*x/sqrt(sinh(x)) + x/sinh(x)^(5/2), x)
```

3.69.8 Giac [F]

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right) dx=\int \frac{x}{3\sqrt{\sinh(x)}}+\frac{x}{\sinh(x)^{\frac{5}{2}}}\, dx$$

input
```
integrate(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/3*x/sqrt(sinh(x)) + x/sinh(x)^(5/2), x)
```

3.69.9 Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right) dx=-\frac{4\,e^{x}\,\sqrt{\frac{e^{x}}{2}-\frac{e^{-x}}{2}}\,\left(x+2\,e^{2\,x}+x\,e^{2\,x}-2\right)}{3\,\left(e^{2\,x}-1\right)^2}$$

input
```
int(x/(3*sinh(x)^(1/2)) + x/sinh(x)^(5/2),x)
```

output
```
-(4*exp(x)*(exp(x)/2 - exp(-x)/2)^(1/2)*(x + 2*exp(2*x) + x*exp(2*x) - 2))
/(3*(exp(2*x) - 1)^2)
```

3.70 $\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right)\,dx$

3.70.1 Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right)\,dx=-\frac{2x\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}-\frac{4}{15\sinh^{\frac{3}{2}}(x)}+\frac{6x\cosh(x)}{5\sqrt{\sinh(x)}}-\frac{12\sqrt{\sinh(x)}}{5}$$

output
```
-2/5*x*cosh(x)/sinh(x)^(5/2)-4/15/sinh(x)^(3/2)+6/5*x*cosh(x)/sinh(x)^(1/2
)-12/5*sinh(x)^(1/2)
```

3.70.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right)\,dx=\frac{-21x\cosh(x)+9x\cosh(3x)+46\sinh(x)-18\sinh(3x)}{30\sinh^{\frac{5}{2}}(x)}$$

input
```
Integrate[x/Sinh[x]^(7/2) + (3*x*Sqrt[Sinh[x]])/5,x]
```

output
```
(-21*x*Cosh[x] + 9*x*Cosh[3*x] + 46*Sinh[x] - 18*Sinh[3*x])/(30*Sinh[x]^(5
/2))
```

3.70.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4}{15\sinh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\sinh(x)}}{5}-\frac{2x\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}+\frac{6x\cosh(x)}{5\sqrt{\sinh(x)}}$$

input
```
Int[x/Sinh[x]^(7/2) + (3*x*Sqrt[Sinh[x]])/5,x]
```

output
```
(-2*x*Cosh[x])/(5*Sinh[x]^(5/2)) - 4/(15*Sinh[x]^(3/2)) + (6*x*Cosh[x])/(5
*Sqrt[Sinh[x]]) - (12*Sqrt[Sinh[x]])/5
```

3.70.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.70.4 Maple [F]

$$\int \left(\frac{x}{\sinh\left(x\right)^{\frac{7}{2}}}+\frac{3x\sqrt{\sinh\left(x\right)}}{5}\right) dx$$

input
```
int(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x)
```

output
```
int(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x)
```

3.70.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)} + \frac{3}{5}x\sqrt{\sinh(x)}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.70.6 Sympy [F]

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)} + \frac{3}{5}x\sqrt{\sinh(x)}\right) dx = \frac{\int \frac{5x}{\sinh^{\frac{7}{2}}(x)}\, dx + \int 3x\sqrt{\sinh(x)}\, dx}{5}$$

input
```
integrate(x/sinh(x)**(7/2)+3/5*x*sinh(x)**(1/2),x)
```

output
```
(Integral(5*x/sinh(x)**(7/2), x) + Integral(3*x*sqrt(sinh(x)), x))/5
```

3.70.7 Maxima [F]

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)} + \frac{3}{5}x\sqrt{\sinh(x)}\right) dx = \int \frac{3}{5}\, x\sqrt{\sinh(x)} + \frac{x}{\sinh(x)^{\frac{7}{2}}}\, dx$$

input
```
integrate(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(3/5*x*sqrt(sinh(x)) + x/sinh(x)^(7/2), x)
```

3.70.8 Giac [F]

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)} + \frac{3}{5}x\sqrt{\sinh(x)}\right) dx = \int \frac{3}{5}\,x\sqrt{\sinh(x)} + \frac{x}{\sinh(x)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(3/5*x*sqrt(sinh(x)) + x/sinh(x)^(7/2), x)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.36

$$\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)} + \frac{3}{5}x\sqrt{\sinh(x)}\right) dx = \frac{12\,x\,e^{2\,x}\,\sqrt{\frac{e^x}{2}-\frac{e^{-x}}{2}}}{5\,\left(e^{2\,x}-1\right)} - \frac{e^{2\,x}\,\left(\frac{8\,x}{5}+\frac{16}{15}\right)\,\sqrt{\frac{e^x}{2}-\frac{e^{-x}}{2}}}{\left(e^{2\,x}-1\right)^2} - \left(\frac{6\,x}{5}+\frac{12}{5}\right)\,\sqrt{\frac{e^x}{2}-\frac{e^{-x}}{2}} - \frac{16\,x\,e^{2\,x}\,\sqrt{\frac{e^x}{2}-\frac{e^{-x}}{2}}}{5\,\left(e^{2\,x}-1\right)^3}$$

input
```
int((3*x*sinh(x)^(1/2))/5 + x/sinh(x)^(7/2),x)
```

output
```
(12*x*exp(2*x)*(exp(x)/2 - exp(-x)/2)^(1/2))/(5*(exp(2*x) - 1)) - (exp(2*x
)*((8*x)/5 + 16/15)*(exp(x)/2 - exp(-x)/2)^(1/2))/(exp(2*x) - 1)^2 - ((6*x
)/5 + 12/5)*(exp(x)/2 - exp(-x)/2)^(1/2) - (16*x*exp(2*x)*(exp(x)/2 - exp(
-x)/2)^(1/2))/(5*(exp(2*x) - 1)^3)
```

3.71 $\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)}-x^2\sqrt{\sinh(x)}\right)\,dx$

3.71.1 Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)}-x^2\sqrt{\sinh(x)}\right)\,dx=-\frac{2x^2\cosh(x)}{\sqrt{\sinh(x)}}+8x\sqrt{\sinh(x)}-\frac{16iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{\sinh(x)}}{\sqrt{i\sinh(x)}}$$

output
```
-2*x^2*cosh(x)/sinh(x)^(1/2)+8*x*sinh(x)^(1/2)-16*I*(sin(1/4*Pi+1/2*I*x)^2
)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)
^(1/2)/(I*sinh(x))^(1/2)
```

3.71.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)}-x^2\sqrt{\sinh(x)}\right)\,dx=$$
$$-\frac{2\left(x^2\cosh(x)-4(-2+x)\sinh(x)-8\sqrt{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\cosh(2x)+\sinh(2x)\right)(-\cosh(\right.}{\sqrt{\sinh(x)}}$$

input
```
Integrate[x^2/Sinh[x]^(3/2) - x^2*Sqrt[Sinh[x]],x]
```

output
```
(-2*(x^2*Cosh[x] - 4*(-2 + x)*Sinh[x] - 8*Sqrt[2]*Hypergeometric2F1[-1/4,
1/2, 3/4, Cosh[2*x] + Sinh[2*x]]*(-Cosh[x] + Sinh[x])*Sqrt[-(Sinh[x]*(Cosh
[x] + Sinh[x]))]))/Sqrt[Sinh[x]]
```

3.71.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)}-x^2\sqrt{\sinh(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2x^2\cosh(x)}{\sqrt{\sinh(x)}}+8x\sqrt{\sinh(x)}-\frac{16i\sqrt{\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{\sqrt{i\sinh(x)}}$$

input
```
Int[x^2/Sinh[x]^(3/2) - x^2*Sqrt[Sinh[x]],x]
```

output
```
(-2*x^2*Cosh[x])/Sqrt[Sinh[x]] + 8*x*Sqrt[Sinh[x]] - ((16*I)*EllipticE[Pi/
4 - (I/2)*x, 2]*Sqrt[Sinh[x]])/Sqrt[I*Sinh[x]]
```

3.71.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.71.4 Maple [F]

$$\int \left(\frac{x^2}{\sinh(x)^{\frac{3}{2}}}-x^2\sqrt{\sinh(x)}\right)dx$$

input
```
int(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x)
```

output
```
int(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x)
```

3.71.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)} - x^2\sqrt{\sinh(x)}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.71.6 Sympy [F]

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)} - x^2\sqrt{\sinh(x)}\right) dx = -\int \left(-\frac{x^2}{\sinh^{\frac{3}{2}}(x)}\right) dx - \int x^2\sqrt{\sinh(x)}\, dx$$

input
```
integrate(x**2/sinh(x)**(3/2)-x**2*sinh(x)**(1/2),x)
```

output
```
-Integral(-x**2/sinh(x)**(3/2), x) - Integral(x**2*sqrt(sinh(x)), x)
```

3.71.7 Maxima [F]

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)} - x^2\sqrt{\sinh(x)}\right) dx = \int -x^2\sqrt{\sinh(x)} + \frac{x^2}{\sinh(x)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-x^2*sqrt(sinh(x)) + x^2/sinh(x)^(3/2), x)
```

3.71.8 Giac [F]

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)} - x^2\sqrt{\sinh(x)}\right) dx = \int -x^2\sqrt{\sinh(x)} + \frac{x^2}{\sinh(x)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x^2*sqrt(sinh(x)) + x^2/sinh(x)^(3/2), x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)} - x^2\sqrt{\sinh(x)}\right) dx = -\int x^2 \sqrt{\sinh(x)} - \frac{x^2}{\sinh(x)^{3/2}} \, dx$$

input
```
int(x^2/sinh(x)^(3/2) - x^2*sinh(x)^(1/2),x)
```

output
```
-int(x^2*sinh(x)^(1/2) - x^2/sinh(x)^(3/2), x)
```

3.72 $\int (c+dx)^m (b\sinh(e+fx))^n \, dx$

3.72.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (c+dx)^m (b\sinh(e+fx))^n \, dx = \mathrm{Int}((c+dx)^m (b\sinh(e+fx))^n, x)$$

output
```
Unintegrable((d*x+c)^m*(b*sinh(f*x+e))^n,x)
```

3.72.2 Mathematica [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\sinh(e+fx))^n \, dx = \int (c+dx)^m (b\sinh(e+fx))^n \, dx$$

input
```
Integrate[(c + d*x)^m*(b*Sinh[e + f*x])^n,x]
```

output
```
Integrate[(c + d*x)^m*(b*Sinh[e + f*x])^n, x]
```

3.72.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m (b\sinh (e+fx))^n \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^m (-ib\sin (ie+ifx))^ndx$$
$$\Big\downarrow \text{3807}$$
$$\int (c+dx)^m (b\sinh (e+fx))^ndx$$

input
```
Int[(c + d*x)^m*(b*Sinh[e + f*x])^n,x]
```

output
```
$Aborted
```

3.72.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.72.4 Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx+c)^m (b\sinh(fx+e))^n \, dx$$

input
```
int((d*x+c)^m*(b*sinh(f*x+e))^n,x)
```

output
```
int((d*x+c)^m*(b*sinh(f*x+e))^n,x)
```

3.72.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\sinh(e+fx))^n \, dx = \int (dx+c)^m (b\sinh(fx+e))^n \, dx$$

input
```
integrate((d*x+c)^m*(b*sinh(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*(b*sinh(f*x + e))^n, x)
```

3.72.6 Sympy [N/A]

Not integrable

Time = 12.87 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (c+dx)^m (b\sinh(e+fx))^n \, dx = \int (b\sinh(e+fx))^n (c+dx)^m \, dx$$

input
```
integrate((d*x+c)**m*(b*sinh(f*x+e))**n,x)
```

output
```
Integral((b*sinh(e + f*x))**n*(c + d*x)**m, x)
```

3.72.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\sinh(e+fx))^n\,dx = \int (dx+c)^m (b\sinh(fx+e))^n\,dx$$

input
```
integrate((d*x+c)^m*(b*sinh(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*(b*sinh(f*x + e))^n, x)
```

3.72.8 Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\sinh(e+fx))^n\,dx = \int (dx+c)^m (b\sinh(fx+e))^n\,dx$$

input
```
integrate((d*x+c)^m*(b*sinh(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*(b*sinh(f*x + e))^n, x)
```

3.72.9 Mupad [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\sinh(e+fx))^n\,dx = \int (b\sinh(e+f\,x))^n\,(c+d\,x)^m\,dx$$

input
```
int((b*sinh(e + f*x))^n*(c + d*x)^m,x)
```

output
```
int((b*sinh(e + f*x))^n*(c + d*x)^m, x)
```

3.73 $\int (c+dx)^m \sinh^3(a+bx)\,dx$

3.73.1 Optimal result

Integrand size = 16, antiderivative size = 237

$$\int (c+dx)^m \sinh^3(a+bx)\,dx = \frac{3^{-1-m}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3b(c+dx)}{d}\right)}{8b} - \frac{3e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)}{8b} - \frac{3e^{-a+\frac{bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{b(c+dx)}{d}\right)}{8b} + \frac{3^{-1-m}e^{-3a+\frac{3bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3b(c+dx)}{d}\right)}{8b}$$

output
```
1/8*3^(-1-m)*exp(3*a-3*b*c/d)*(d*x+c)^m*GAMMA(1+m,-3*b*(d*x+c)/d)/b/((-b*(
d*x+c)/d)^m)-3/8*exp(a-b*c/d)*(d*x+c)^m*GAMMA(1+m,-b*(d*x+c)/d)/b/((-b*(d*
x+c)/d)^m)-3/8*exp(-a+b*c/d)*(d*x+c)^m*GAMMA(1+m,b*(d*x+c)/d)/b/((b*(d*x+c
)/d)^m)+1/8*3^(-1-m)*exp(-3*a+3*b*c/d)*(d*x+c)^m*GAMMA(1+m,3*b*(d*x+c)/d)/
b/((b*(d*x+c)/d)^m)
```

3.73.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.87

$$\int (c+dx)^m \sinh^3(a+bx)\,dx$$
$$=\frac{3^{-1-m}e^{-3\left(a+\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{6a}\left(b\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(1+m,-\frac{3b(c+dx)}{d}\right)-3^{2+m}e^{4a+\frac{2bc}{d}}\left(b\left(\frac{c}{d}+x\right)\right.\right.}{8b}$$

input
```
Integrate[(c + d*x)^m*Sinh[a + b*x]^3,x]
```

output
```
(3^(-1 - m)*(c + d*x)^m*(E^(6*a)*(b*(c/d + x))^m*Gamma[1 + m, (-3*b*(c + d
*x))/d] - 3^(2 + m)*E^(4*a + (2*b*c)/d)*(b*(c/d + x))^m*Gamma[1 + m, -((b*
(c + d*x))/d)] + E^((4*b*c)/d)*(-((b*(c + d*x))/d))^m*(-(3^(2 + m)*E^(2*a)
*Gamma[1 + m, (b*(c + d*x))/d]) + E^((2*b*c)/d)*Gamma[1 + m, (3*b*(c + d*x
))/d])))/(8*b*E^(3*(a + (b*c)/d))*(-((b^2*(c + d*x)^2)/d^2))^m)
```

3.73.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)(c+dx)^m\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int i\sin(ia+ibx)^3(c+dx)^m dx$$
$$\Big\downarrow \text{26}$$
$$i\int (c+dx)^m\sin(ia+ibx)^3dx$$
$$\Big\downarrow \text{3793}$$
$$i\int \left(\frac{3}{4}i(c+dx)^m\sinh(a+bx)-\frac{1}{4}i(c+dx)^m\sinh(3a+3bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{i3^{-m-1}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3b(c+dx)}{d}\right)}{8b}+\frac{3ie^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\right.}{8b}\right.$$

input
```
Int[(c + d*x)^m*Sinh[a + b*x]^3,x]
```

output
```
I*(((-1/8*I)*3^(-1 - m)*E^(3*a - (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (-3*b
*(c + d*x))/d])/(b*(-((b*(c + d*x))/d))^m) + (((3*I)/8)*E^(a - (b*c)/d)*(c
 + d*x)^m*Gamma[1 + m, -((b*(c + d*x))/d)])/(b*(-((b*(c + d*x))/d))^m) + (
((3*I)/8)*E^(-a + (b*c)/d)*(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))/d])/(b*(
(b*(c + d*x))/d)^m) - ((I/8)*3^(-1 - m)*E^(-3*a + (3*b*c)/d)*(c + d*x)^m*G
amma[1 + m, (3*b*(c + d*x))/d])/(b*((b*(c + d*x))/d)^m))
```

3.73.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.73.4 Maple [F]

$$\int (dx+c)^m \sinh(bx+a)^3\, dx$$

input
```
int((d*x+c)^m*sinh(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*sinh(b*x+a)^3,x)
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.43

$$\int (c+dx)^m \sinh^3(a+bx)\, dx$$
$$= \frac{\cosh\left(\frac{dm\log\left(\frac{3b}{d}\right)-3bc+3ad}{d}\right)\Gamma\left(m+1,\frac{3(bdx+bc)}{d}\right)-9\cosh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)\Gamma\left(m+1,\frac{bdx+bc}{d}\right)-9\cosh\left(\frac{dm}{}\right.}{}$$

input
```
integrate((d*x+c)^m*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/24*(cosh((d*m*log(3*b/d) - 3*b*c + 3*a*d)/d)*gamma(m + 1, 3*(b*d*x + b*c
)/d) - 9*cosh((d*m*log(b/d) - b*c + a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d)
- 9*cosh((d*m*log(-b/d) + b*c - a*d)/d)*gamma(m + 1, -(b*d*x + b*c)/d) + c
osh((d*m*log(-3*b/d) + 3*b*c - 3*a*d)/d)*gamma(m + 1, -3*(b*d*x + b*c)/d)
- gamma(m + 1, 3*(b*d*x + b*c)/d)*sinh((d*m*log(3*b/d) - 3*b*c + 3*a*d)/d)
 + 9*gamma(m + 1, (b*d*x + b*c)/d)*sinh((d*m*log(b/d) - b*c + a*d)/d) + 9*
gamma(m + 1, -(b*d*x + b*c)/d)*sinh((d*m*log(-b/d) + b*c - a*d)/d) - gamma
(m + 1, -3*(b*d*x + b*c)/d)*sinh((d*m*log(-3*b/d) + 3*b*c - 3*a*d)/d))/b
```

3.73.6 Sympy [F]

$$\int (c+dx)^m \sinh^3(a+bx)\, dx = \int (c+dx)^m \sinh^3(a+bx)\, dx$$

input
```
integrate((d*x+c)**m*sinh(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**m*sinh(a + b*x)**3, x)
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.68

$$\int (c+dx)^m \sinh^3(a+bx)\,dx = \frac{(dx+c)^{m+1}e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_{-m}\left(\frac{3\,(dx+c)b}{d}\right)}{8\,d} - \frac{3\,(dx+c)^{m+1}e^{\left(-a+\frac{bc}{d}\right)}E_{-m}\left(\frac{(dx+c)b}{d}\right)}{8\,d} + \frac{3\,(dx+c)^{m+1}e^{\left(a-\frac{bc}{d}\right)}E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{8\,d} - \frac{(dx+c)^{m+1}e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_{-m}\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,d}$$

input
```
integrate((d*x+c)^m*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*(d*x + c)^(m + 1)*e^(-3*a + 3*b*c/d)*exp_integral_e(-m, 3*(d*x + c)*b/
d)/d - 3/8*(d*x + c)^(m + 1)*e^(-a + b*c/d)*exp_integral_e(-m, (d*x + c)*b
/d)/d + 3/8*(d*x + c)^(m + 1)*e^(a - b*c/d)*exp_integral_e(-m, -(d*x + c)*
b/d)/d - 1/8*(d*x + c)^(m + 1)*e^(3*a - 3*b*c/d)*exp_integral_e(-m, -3*(d*
x + c)*b/d)/d
```

3.73.8 Giac [F]

$$\int (c+dx)^m \sinh^3(a+bx)\,dx = \int (dx+c)^m \sinh\left(bx+a\right)^3\,dx$$

input
```
integrate((d*x+c)^m*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sinh(b*x + a)^3, x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \sinh^3(a+bx)\,dx = \int \sinh(a+b\,x)^3\,(c+d\,x)^m\,dx$$

input
```
int(sinh(a + b*x)^3*(c + d*x)^m,x)
```

output
```
int(sinh(a + b*x)^3*(c + d*x)^m, x)
```

3.74 $\int (c+dx)^m \sinh^2(a+bx)\,dx$

3.74.1 Optimal result

Integrand size = 16, antiderivative size = 144

$$\int (c+dx)^m \sinh^2(a+bx)\,dx = -\frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-3-m}e^{-2a+\frac{2bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2b(c+dx)}{d}\right)}{b}$$

output
```
-1/2*(d*x+c)^(1+m)/d/(1+m)+2^(-3-m)*exp(2*a-2*b*c/d)*(d*x+c)^m*GAMMA(1+m,-
2*b*(d*x+c)/d)/b/((-b*(d*x+c)/d)^m)-2^(-3-m)*exp(-2*a+2*b*c/d)*(d*x+c)^m*G
AMMA(1+m,2*b*(d*x+c)/d)/b/((b*(d*x+c)/d)^m)
```

3.74.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91

$$\int (c+dx)^m \sinh^2(a+bx)\,dx = \frac{1}{8}(c+dx)^m \left(-\frac{4(c+dx)}{d(1+m)} + \frac{2^{-m}e^{2a-\frac{2bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-m}e^{-2a+\frac{2bc}{d}}\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2b(c+dx)}{d}\right)}{b}\right)$$

input
```
Integrate[(c + d*x)^m*Sinh[a + b*x]^2,x]
```

output
```
((c + d*x)^m*((-4*(c + d*x))/(d*(1 + m)) + (E^(2*a - (2*b*c)/d)*Gamma[1 +
m, (-2*b*(c + d*x))/d])/(2^m*b*(-((b*(c + d*x))/d))^m) - (E^(-2*a + (2*b*c
)/d)*Gamma[1 + m, (2*b*(c + d*x))/d])/(2^m*b*((b*(c + d*x))/d)^m)))/8
```

3.74.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^2\left(-(c+dx)^m\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^m \sin(ia+ibx)^2 dx$$

$$\Big\downarrow \text{3793}$$

$$-\int \left(\frac{1}{2}(c+dx)^m-\frac{1}{2}(c+dx)^m\cosh(2a+2bx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2^{-m-3}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2b(c+dx)}{d}\right)}{b}-\frac{2^{-m-3}e^{\frac{2bc}{d}-2a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2b(c+dx)}{d}\right)}{b}-\frac{(c+dx)^{m+1}}{2d(m+1)}$$

input
```
Int[(c + d*x)^m*Sinh[a + b*x]^2,x]
```

output
```
-1/2*(c + d*x)^(1 + m)/(d*(1 + m)) + (2^(-3 - m)*E^(2*a - (2*b*c)/d)*(c +
d*x)^m*Gamma[1 + m, (-2*b*(c + d*x))/d])/(b*(-((b*(c + d*x))/d))^m) - (2^(
-3 - m)*E^(-2*a + (2*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (2*b*(c + d*x))/d])/
(b*((b*(c + d*x))/d)^m)
```

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.74.4 Maple [F]

$$\int (dx+c)^m \sinh (bx+a)^2 \, dx$$

input
```
int((d*x+c)^m*sinh(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*sinh(b*x+a)^2,x)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.67

$$\int (c+dx)^m \sinh^2(a+bx) \, dx =$$
$$-\frac{(dm+d)\cosh\left(\frac{dm\log\left(\frac{2b}{d}\right)-2bc+2ad}{d}\right)\Gamma\left(m+1,\frac{2(bdx+bc)}{d}\right)-(dm+d)\cosh\left(\frac{dm\log\left(-\frac{2b}{d}\right)+2bc-2ad}{d}\right)\Gamma\left(m\right.}{}$$

input
```
integrate((d*x+c)^m*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*((d*m + d)*cosh((d*m*log(2*b/d) - 2*b*c + 2*a*d)/d)*gamma(m + 1, 2*(b
*d*x + b*c)/d) - (d*m + d)*cosh((d*m*log(-2*b/d) + 2*b*c - 2*a*d)/d)*gamma
(m + 1, -2*(b*d*x + b*c)/d) - (d*m + d)*gamma(m + 1, 2*(b*d*x + b*c)/d)*si
nh((d*m*log(2*b/d) - 2*b*c + 2*a*d)/d) + (d*m + d)*gamma(m + 1, -2*(b*d*x
+ b*c)/d)*sinh((d*m*log(-2*b/d) + 2*b*c - 2*a*d)/d) + 4*(b*d*x + b*c)*cosh
(m*log(d*x + c)) + 4*(b*d*x + b*c)*sinh(m*log(d*x + c)))/(b*d*m + b*d)
```

3.74.6 Sympy [F]

$$\int (c+dx)^m \sinh^2(a+bx) \, dx = \int (c+dx)^m \sinh^2 (a+bx) \, dx$$

input
```
integrate((d*x+c)**m*sinh(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*sinh(a + b*x)**2, x)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.71

$$\int (c+dx)^m \sinh^2(a+bx)\,dx = -\frac{(dx+c)^{m+1}e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_{-m}\left(\frac{2\,(dx+c)b}{d}\right)}{4\,d} - \frac{(dx+c)^{m+1}e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_{-m}\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,d} - \frac{(dx+c)^{m+1}}{2\,d(m+1)}$$

input
```
integrate((d*x+c)^m*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(d*x + c)^(m + 1)*e^(-2*a + 2*b*c/d)*exp_integral_e(-m, 2*(d*x + c)*b
/d)/d - 1/4*(d*x + c)^(m + 1)*e^(2*a - 2*b*c/d)*exp_integral_e(-m, -2*(d*x
 + c)*b/d)/d - 1/2*(d*x + c)^(m + 1)/(d*(m + 1))
```

3.74.8 Giac [F]

$$\int (c+dx)^m \sinh^2(a+bx)\,dx = \int (dx+c)^m \sinh(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sinh(b*x + a)^2, x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \sinh^2(a+bx)\,dx = \int \sinh(a+b\,x)^2\,(c+d\,x)^m\,dx$$

input
```
int(sinh(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(sinh(a + b*x)^2*(c + d*x)^m, x)
```

3.75 $\int (c+dx)^m \sinh(a+bx)\,dx$

3.75.1 Optimal result

Integrand size = 14, antiderivative size = 110

$$\int (c+dx)^m \sinh(a+bx)\,dx = \frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)}{2b} + \frac{e^{-a+\frac{bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{b(c+dx)}{d}\right)}{2b}$$

output
```
1/2*exp(a-b*c/d)*(d*x+c)^m*GAMMA(1+m,-b*(d*x+c)/d)/b/((-b*(d*x+c)/d)^m)+1/
2*exp(-a+b*c/d)*(d*x+c)^m*GAMMA(1+m,b*(d*x+c)/d)/b/((b*(d*x+c)/d)^m)
```

3.75.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.92

$$\int (c+dx)^m \sinh(a+bx)\,dx = \frac{e^{-a-\frac{bc}{d}}(c+dx)^m\left(e^{2a}\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\left(b\left(\frac{c}{d}+x\right)\right)^{-m}\Gamma\left(1+m,\frac{b(c+dx)}{d}\right)\right)}{2b}$$

input
```
Integrate[(c + d*x)^m*Sinh[a + b*x],x]
```

output
```
(E^(-a - (b*c)/d)*(c + d*x)^m*((E^(2*a)*Gamma[1 + m, -((b*(c + d*x))/d)])/
(-((b*(c + d*x))/d))^m + (E^((2*b*c)/d)*Gamma[1 + m, (b*(c + d*x))/d])/(b*
(c/d + x))^m))/(2*b)
```

3.75.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ia+ibx)(c+dx)^m dx$$

$$\Big\downarrow \text{26}$$

$$-i\int (c+dx)^m\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3789}$$

$$-i\left(\frac{1}{2}i\int e^{a+bx}(c+dx)^m dx-\frac{1}{2}i\int e^{-a-bx}(c+dx)^m dx\right)$$

$$\Big\downarrow \text{2612}$$

$$-i\left(\frac{ie^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{b(c+dx)}{d}\right)}{2b}+\frac{ie^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{b(c+dx)}{d}\right)}{2b}\right)$$

input
```
Int[(c + d*x)^m*Sinh[a + b*x],x]
```

output
```
(-I)*(((I/2)*E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 + m, -((b*(c + d*x))/d)])
/(b*(-((b*(c + d*x))/d))^m) + ((I/2)*E^(-a + (b*c)/d)*(c + d*x)^m*Gamma[1
+ m, (b*(c + d*x))/d])/(b*((b*(c + d*x))/d)^m))
```

3.75.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.75.4 Maple [F]

$$\int (dx+c)^m \sinh (bx+a)\, dx$$

input
```
int((d*x+c)^m*sinh(b*x+a),x)
```

output
```
int((d*x+c)^m*sinh(b*x+a),x)
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.53

$$\int (c+dx)^m \sinh(a+bx)\,dx$$
$$= \frac{\cosh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)\Gamma\left(m+1,\frac{bdx+bc}{d}\right)+\cosh\left(\frac{dm\log\left(-\frac{b}{d}\right)+bc-ad}{d}\right)\Gamma\left(m+1,-\frac{bdx+bc}{d}\right)-\Gamma\left(m+1,\frac{bdx+bc}{d}\right.}{2b}$$

input
```
integrate((d*x+c)^m*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh((d*m*log(b/d) - b*c + a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d) + co
sh((d*m*log(-b/d) + b*c - a*d)/d)*gamma(m + 1, -(b*d*x + b*c)/d) - gamma(m
 + 1, (b*d*x + b*c)/d)*sinh((d*m*log(b/d) - b*c + a*d)/d) - gamma(m + 1, -
(b*d*x + b*c)/d)*sinh((d*m*log(-b/d) + b*c - a*d)/d))/b
```

3.75.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m \sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int (c+dx)^m \sinh(a+bx)\,dx = \frac{(dx+c)^{m+1}e^{\left(-a+\frac{bc}{d}\right)}E_{-m}\left(\frac{(dx+c)b}{d}\right)}{2\,d} - \frac{(dx+c)^{m+1}e^{\left(a-\frac{bc}{d}\right)}E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{2\,d}$$

input
```
integrate((d*x+c)^m*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(d*x + c)^(m + 1)*e^(-a + b*c/d)*exp_integral_e(-m, (d*x + c)*b/d)/d -
 1/2*(d*x + c)^(m + 1)*e^(a - b*c/d)*exp_integral_e(-m, -(d*x + c)*b/d)/d
```

3.75.8 Giac [F]

$$\int (c+dx)^m \sinh(a+bx)\,dx = \int (dx+c)^m \sinh(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sinh(b*x + a), x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \sinh(a+bx)\,dx = \int \sinh(a+b\,x)\,(c+d\,x)^m\,dx$$

input
```
int(sinh(a + b*x)*(c + d*x)^m,x)
```

output
```
int(sinh(a + b*x)*(c + d*x)^m, x)
```

3.76 $\int (c+dx)^m \text{csch}(a+bx)\,dx$

3.76.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (c+dx)^m \text{csch}(a+bx)\,dx = \text{Int}((c+dx)^m \text{csch}(a+bx),x)$$

output
```
Unintegrable((d*x+c)^m*csch(b*x+a),x)
```

3.76.2 Mathematica [N/A]

Not integrable

Time = 11.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \text{csch}(a+bx)\,dx = \int (c+dx)^m \text{csch}(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csch[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Csch[a + b*x], x]
```

3.76.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 26, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)(c+dx)^m dx$$

$$\Big\downarrow \text{26}$$

$$i\int (c+dx)^m\csc(ia+ibx)dx$$

$$\Big\downarrow \text{4680}$$

$$\int \operatorname{csch}(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csch[a + b*x],x]
```

output
```
$Aborted
```

3.76.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

3.76.4 Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \operatorname{csch}(bx+a)\,dx$$

input
```
int((d*x+c)^m*csch(b*x+a),x)
```

output
```
int((d*x+c)^m*csch(b*x+a),x)
```

3.76.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \operatorname{csch}(a+bx)\,dx = \int (dx+c)^m \operatorname{csch}(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csch(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csch(b*x + a), x)
```

3.76.6 Sympy [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \operatorname{csch}(a+bx)\,dx=\int (c+dx)^m \operatorname{csch}(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*csch(b*x+a),x)
```

output
```
Integral((c + d*x)**m*csch(a + b*x), x)
```

3.76.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \operatorname{csch}(a+bx)\,dx=\int (dx+c)^m \operatorname{csch}(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csch(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csch(b*x + a), x)
```

3.76.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \operatorname{csch}(a+bx)\,dx=\int (dx+c)^m \operatorname{csch}(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csch(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csch(b*x + a), x)
```

3.76.9 Mupad [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int (c+dx)^m \operatorname{csch}(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\sinh(a+b\,x)}\,dx$$

input
```
int((c + d*x)^m/sinh(a + b*x),x)
```

output
```
int((c + d*x)^m/sinh(a + b*x), x)
```

3.77 $\int (c+dx)^m \mathbf{csch}^2(a+bx)\,dx$

3.77.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx = \operatorname{Int}\left((c+dx)^m \operatorname{csch}^2(a+bx),x\right)$$

output
```
Unintegrable((d*x+c)^m*csch(b*x+a)^2,x)
```

3.77.2 Mathematica [N/A]

Not integrable

Time = 3.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx = \int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csch[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Csch[a + b*x]^2, x]
```

3.77.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 25, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(ia+ibx)^2\left(-(c+dx)^m\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^m\csc(ia+ibx)^2dx$$

$$\Big\downarrow \text{4680}$$

$$\int \text{csch}^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csch[a + b*x]^2,x]
```

output
```
$Aborted
```

3.77.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

3.77.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \operatorname{csch}(bx+a)^2\,dx$$

input
```
int((d*x+c)^m*csch(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*csch(b*x+a)^2,x)
```

3.77.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx = \int (dx+c)^m \operatorname{csch}(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csch(b*x + a)^2, x)
```

3.77.6 Sympy [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx = \int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*csch(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*csch(a + b*x)**2, x)
```

3.77.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx = \int (dx+c)^m \operatorname{csch}(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csch(b*x + a)^2, x)
```

3.77.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx = \int (dx+c)^m \operatorname{csch}(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csch(b*x + a)^2, x)
```

3.77.9 Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\sinh\left(a+b\,x\right)^2}\,dx$$

input
```
int((c + d*x)^m/sinh(a + b*x)^2,x)
```

output
```
int((c + d*x)^m/sinh(a + b*x)^2, x)
```

3.78 $\int x^{3+m} \sinh(a+bx)\, dx$

3.78.1 Optimal result

Integrand size = 12, antiderivative size = 59

$$\int x^{3+m}\sinh(a+bx)\,dx=-\frac{e^a x^m(-bx)^{-m}\Gamma(4+m,-bx)}{2b^4}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(4+m,bx)}{2b^4}$$

output
```
-1/2*exp(a)*x^m*GAMMA(4+m,-b*x)/b^4/((-b*x)^m)+1/2*x^m*GAMMA(4+m,b*x)/b^4/
exp(a)/((b*x)^m)
```

3.78.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^{3+m}\sinh(a+bx)\,dx=\frac{e^{-a}x^m\left(-e^{2a}(-bx)^{-m}\Gamma(4+m,-bx)+(bx)^{-m}\Gamma(4+m,bx)\right)}{2b^4}$$

input
```
Integrate[x^(3 + m)*Sinh[a + b*x],x]
```

output
```
(x^m*(-((E^(2*a)*Gamma[4 + m, -(b*x)])/(-(b*x))^m) + Gamma[4 + m, b*x]/(b*
x)^m))/(2*b^4*E^a)
```

3.78.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+3}\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -ix^{m+3}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int x^{m+3}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{ 3789}$$

$$-i\left(\frac{1}{2}i\int e^{a+bx}x^{m+3}dx-\frac{1}{2}i\int e^{-a-bx}x^{m+3}dx\right)$$

$$\Big\downarrow \text{ 2612}$$

$$-i\left(\frac{ie^{-a}x^m(bx)^{-m}\Gamma(m+4,bx)}{2b^4}-\frac{ie^ax^m(-bx)^{-m}\Gamma(m+4,-bx)}{2b^4}\right)$$

input
```
Int[x^(3 + m)*Sinh[a + b*x],x]
```

output
```
(-I)*(((-1/2*I)*E^a*x^m*Gamma[4 + m, -(b*x)])/(b^4*(-(b*x))^m) + ((I/2)*x^
m*Gamma[4 + m, b*x])/(b^4*E^a*(b*x)^m))
```

3.78.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.78.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
meijerg	$\frac{x^{4+m} \operatorname{hypergeom}\left(\left[2+\frac{m}{2}\right],\left[\frac{1}{2},\frac{m}{2}+3\right],\frac{b^{2} x^{2}}{4}\right) \sinh (a)}{4+m}+\frac{b\, x^{m+5} \operatorname{hypergeom}\left(\left[\frac{m}{2}+\frac{5}{2}\right],\left[\frac{3}{2},\frac{7}{2}+\frac{m}{2}\right],\frac{b^{2} x^{2}}{4}\right) \cosh (a)}{m+5}$	73

input
```
int(x^(3+m)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(4+m)*x^(4+m)*hypergeom([2+1/2*m],[1/2,1/2*m+3],1/4*b^2*x^2)*sinh(a)+b/(
m+5)*x^(m+5)*hypergeom([1/2*m+5/2],[3/2,7/2+1/2*m],1/4*b^2*x^2)*cosh(a)
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int x^{3+m}\sinh(a+bx)\,dx$$
$$= \frac{\cosh\left((m+3)\log\left(b\right)+a\right)\Gamma(m+4,bx)+\cosh\left((m+3)\log\left(-b\right)-a\right)\Gamma(m+4,-bx)-\Gamma(m+4,-bx)\sin}{2\,b}$$

input
```
integrate(x^(3+m)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh((m + 3)*log(b) + a)*gamma(m + 4, b*x) + cosh((m + 3)*log(-b) - a
)*gamma(m + 4, -b*x) - gamma(m + 4, -b*x)*sinh((m + 3)*log(-b) - a) - gamm
a(m + 4, b*x)*sinh((m + 3)*log(b) + a))/b
```

3.78.6 Sympy [F(-2)]

Exception generated.

$$\int x^{3+m}\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**(3+m)*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^{3+m}\sinh(a+bx)\,dx = \frac{1}{2}\left(bx\right)^{-m-4}x^{m+4}e^{(-a)}\Gamma(m+4,bx) - \frac{1}{2}\left(-bx\right)^{-m-4}x^{m+4}e^{a}\Gamma(m+4,-bx)$$

input
```
integrate(x^(3+m)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(b*x)^(-m - 4)*x^(m + 4)*e^(-a)*gamma(m + 4, b*x) - 1/2*(-b*x)^(-m - 4
)*x^(m + 4)*e^a*gamma(m + 4, -b*x)
```

3.78.8 Giac [F]

$$\int x^{3+m}\sinh(a+bx)\,dx = \int x^{m+3}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^(3+m)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m + 3)*sinh(b*x + a), x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int x^{3+m}\sinh(a+bx)\,dx = \int x^{m+3}\sinh(a+b\,x)\,dx$$

input
```
int(x^(m + 3)*sinh(a + b*x),x)
```

output
```
int(x^(m + 3)*sinh(a + b*x), x)
```

3.79 $\int x^{2+m} \sinh(a+bx)\,dx$

3.79.1 Optimal result

Integrand size = 12, antiderivative size = 59

$$\int x^{2+m} \sinh(a+bx)\,dx = \frac{e^a x^m (-bx)^{-m} \Gamma(3+m,-bx)}{2b^3} + \frac{e^{-a} x^m (bx)^{-m} \Gamma(3+m,bx)}{2b^3}$$

output
```
1/2*exp(a)*x^m*GAMMA(3+m,-b*x)/b^3/((-b*x)^m)+1/2*x^m*GAMMA(3+m,b*x)/b^3/e
xp(a)/((b*x)^m)
```

3.79.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x^{2+m} \sinh(a+bx)\,dx = \frac{e^{-a} x^m \left(e^{2a} (-bx)^{-m} \Gamma(3+m,-bx) + (bx)^{-m} \Gamma(3+m,bx)\right)}{2b^3}$$

input
```
Integrate[x^(2 + m)*Sinh[a + b*x],x]
```

output
```
(x^m*((E^(2*a)*Gamma[3 + m, -(b*x)])/(-(b*x))^m + Gamma[3 + m, b*x]/(b*x)^
m))/(2*b^3*E^a)
```

3.79.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+2}\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix^{m+2}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int x^{m+2}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3789}$$

$$-i\left(\frac{1}{2}i\int e^{a+bx}x^{m+2}dx-\frac{1}{2}i\int e^{-a-bx}x^{m+2}dx\right)$$

$$\Big\downarrow \text{2612}$$

$$-i\left(\frac{ie^ax^m(-bx)^{-m}\Gamma(m+3,-bx)}{2b^3}+\frac{ie^{-a}x^m(bx)^{-m}\Gamma(m+3,bx)}{2b^3}\right)$$

input
```
Int[x^(2 + m)*Sinh[a + b*x],x]
```

output
```
(-I)*(((I/2)*E^a*x^m*Gamma[3 + m, -(b*x)])/(b^3*(-(b*x))^m) + ((I/2)*x^m*G
amma[3 + m, b*x])/(b^3*E^a*(b*x)^m))
```

3.79.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.79.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
meijerg	$\frac{x^{3+m} \operatorname{hypergeom}\left(\left[\frac{3}{2}+\frac{m}{2}\right],\left[\frac{1}{2},\frac{m}{2}+\frac{5}{2}\right],\frac{b^{2} x^{2}}{4}\right) \sinh \left(a \right)}{3+m}+\frac{b \,x^{4+m} \operatorname{hypergeom}\left(\left[2+\frac{m}{2}\right],\left[\frac{3}{2},\frac{m}{2}+3\right],\frac{b^{2} x^{2}}{4}\right) \cosh \left(a \right)}{4+m}$	73

input
```
int(x^(2+m)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(3+m)*x^(3+m)*hypergeom([3/2+1/2*m],[1/2,1/2*m+5/2],1/4*b^2*x^2)*sinh(a)
+b/(4+m)*x^(4+m)*hypergeom([2+1/2*m],[3/2,1/2*m+3],1/4*b^2*x^2)*cosh(a)
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int x^{2+m}\sinh(a+bx)\,dx$$
$$=\frac{\cosh\left((m+2)\log\left(b\right)+a\right)\Gamma(m+3,bx)+\cosh\left((m+2)\log\left(-b\right)-a\right)\Gamma(m+3,-bx)-\Gamma(m+3,-bx)\sin}{2\,b}$$

input
```
integrate(x^(2+m)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh((m + 2)*log(b) + a)*gamma(m + 3, b*x) + cosh((m + 2)*log(-b) - a
)*gamma(m + 3, -b*x) - gamma(m + 3, -b*x)*sinh((m + 2)*log(-b) - a) - gamm
a(m + 3, b*x)*sinh((m + 2)*log(b) + a))/b
```

3.79.6 Sympy [F(-2)]

Exception generated.

$$\int x^{2+m}\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**(2+m)*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^{2+m}\sinh(a+bx)\,dx = \frac{1}{2}\left(bx\right)^{-m-3}x^{m+3}e^{(-a)}\Gamma(m+3,bx) - \frac{1}{2}\left(-bx\right)^{-m-3}x^{m+3}e^{a}\Gamma(m+3,-bx)$$

input
```
integrate(x^(2+m)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(b*x)^(-m - 3)*x^(m + 3)*e^(-a)*gamma(m + 3, b*x) - 1/2*(-b*x)^(-m - 3
)*x^(m + 3)*e^a*gamma(m + 3, -b*x)
```

3.79.8 Giac [F]

$$\int x^{2+m}\sinh(a+bx)\,dx=\int x^{m+2}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^(2+m)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m + 2)*sinh(b*x + a), x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int x^{2+m}\sinh(a+bx)\,dx=\int x^{m+2}\sinh(a+b\,x)\,dx$$

input
```
int(x^(m + 2)*sinh(a + b*x),x)
```

output
```
int(x^(m + 2)*sinh(a + b*x), x)
```

3.80 $\int x^{1+m}\sinh(a+bx)\,dx$

3.80.1 Optimal result

Integrand size = 12, antiderivative size = 59

$$\int x^{1+m}\sinh(a+bx)\,dx=-\frac{e^a x^m(-bx)^{-m}\Gamma(2+m,-bx)}{2b^2}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(2+m,bx)}{2b^2}$$

output
```
-1/2*exp(a)*x^m*GAMMA(2+m,-b*x)/b^2/((-b*x)^m)+1/2*x^m*GAMMA(2+m,b*x)/b^2/
exp(a)/((b*x)^m)
```

3.80.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^{1+m}\sinh(a+bx)\,dx=\frac{e^{-a}x^m\left(-e^{2a}(-bx)^{-m}\Gamma(2+m,-bx)+(bx)^{-m}\Gamma(2+m,bx)\right)}{2b^2}$$

input
```
Integrate[x^(1 + m)*Sinh[a + b*x],x]
```

output
```
(x^m*(-((E^(2*a)*Gamma[2 + m, -(b*x)])/(-(b*x))^m) + Gamma[2 + m, b*x]/(b*
x)^m))/(2*b^2*E^a)
```

3.80.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+1} \sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix^{m+1}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int x^{m+1}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3789}$$

$$-i\left(\frac{1}{2}i\int e^{a+bx}x^{m+1}dx-\frac{1}{2}i\int e^{-a-bx}x^{m+1}dx\right)$$

$$\Big\downarrow \text{2612}$$

$$-i\left(\frac{ie^{-a}x^m(bx)^{-m}\Gamma(m+2,bx)}{2b^2}-\frac{ie^{a}x^m(-bx)^{-m}\Gamma(m+2,-bx)}{2b^2}\right)$$

input
```
Int[x^(1 + m)*Sinh[a + b*x],x]
```

output
```
(-I)*(((-1/2*I)*E^a*x^m*Gamma[2 + m, -(b*x)])/(b^2*(-(b*x))^m) + ((I/2)*x^
m*Gamma[2 + m, b*x])/(b^2*E^a*(b*x)^m))
```

3.80.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.80.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
meijerg	$\frac{x^{2+m}\,\text{hypergeom}\left(\left[1+\frac{m}{2}\right],\left[\frac{1}{2},2+\frac{m}{2}\right],\frac{b^{2}x^{2}}{4}\right)\sinh\left(a\right)}{2+m}+\frac{b\,x^{3+m}\,\text{hypergeom}\left(\left[\frac{3}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{m}{2}+\frac{5}{2}\right],\frac{b^{2}x^{2}}{4}\right)\cosh\left(a\right)}{3+m}$	73

input
```
int(x^(1+m)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(2+m)*x^(2+m)*hypergeom([1+1/2*m],[1/2,2+1/2*m],1/4*b^2*x^2)*sinh(a)+b/(
3+m)*x^(3+m)*hypergeom([3/2+1/2*m],[3/2,1/2*m+5/2],1/4*b^2*x^2)*cosh(a)
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int x^{1+m}\sinh(a+bx)\,dx$$
$$=\frac{\cosh\left((m+1)\log\left(b\right)+a\right)\Gamma(m+2,bx)+\cosh\left((m+1)\log\left(-b\right)-a\right)\Gamma(m+2,-bx)-\Gamma(m+2,-bx)\sin}{2\,b}$$

input
```
integrate(x^(1+m)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh((m + 1)*log(b) + a)*gamma(m + 2, b*x) + cosh((m + 1)*log(-b) - a
)*gamma(m + 2, -b*x) - gamma(m + 2, -b*x)*sinh((m + 1)*log(-b) - a) - gamm
a(m + 2, b*x)*sinh((m + 1)*log(b) + a))/b
```

3.80.6 Sympy [F(-2)]

Exception generated.

$$\int x^{1+m}\sinh(a+bx)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x**(1+m)*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^{1+m}\sinh(a+bx)\,dx=\frac{1}{2}\,(bx)^{-m-2}\,x^{m+2}e^{(-a)}\Gamma(m+2,bx)\\-\frac{1}{2}\,(-bx)^{-m-2}\,x^{m+2}e^{a}\Gamma(m+2,-bx)$$

input
```
integrate(x^(1+m)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(b*x)^(-m - 2)*x^(m + 2)*e^(-a)*gamma(m + 2, b*x) - 1/2*(-b*x)^(-m - 2
)*x^(m + 2)*e^a*gamma(m + 2, -b*x)
```

3.80.8 Giac [F]

$$\int x^{1+m}\sinh(a+bx)\,dx = \int x^{m+1}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^(1+m)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m + 1)*sinh(b*x + a), x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int x^{1+m}\sinh(a+bx)\,dx = \int x^{m+1}\sinh(a+b\,x)\,dx$$

input
```
int(x^(m + 1)*sinh(a + b*x),x)
```

output
```
int(x^(m + 1)*sinh(a + b*x), x)
```

3.81 $\int x^m \sinh(a+bx)\,dx$

3.81.1 Optimal result

Integrand size = 10, antiderivative size = 59

$$\int x^m \sinh(a+bx)\,dx=\frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b}$$

output
```
1/2*exp(a)*x^m*GAMMA(1+m,-b*x)/b/((-b*x)^m)+1/2*x^m*GAMMA(1+m,b*x)/b/exp(a
)/((b*x)^m)
```

3.81.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x^m \sinh(a+bx)\,dx=\frac{e^{-a}x^m\left(e^{2a}(-bx)^{-m}\Gamma(1+m,-bx)+(bx)^{-m}\Gamma(1+m,bx)\right)}{2b}$$

input
```
Integrate[x^m*Sinh[a + b*x],x]
```

output
```
(x^m*((E^(2*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + Gamma[1 + m, b*x]/(b*x)^
m))/(2*b*E^a)
```

3.81.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix^m \sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int x^m \sin(ia+ibx)dx$$

$$\Big\downarrow \text{3789}$$

$$-i\left(\frac{1}{2}i\int e^{a+bx}x^m dx-\frac{1}{2}i\int e^{-a-bx}x^m dx\right)$$

$$\Big\downarrow \text{2612}$$

$$-i\left(\frac{ie^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b}+\frac{ie^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{2b}\right)$$

input
```
Int[x^m*Sinh[a + b*x],x]
```

output
```
(-I)*(((I/2)*E^a*x^m*Gamma[1 + m, -(b*x)])/(b*(-(b*x))^m) + ((I/2)*x^m*Gam
ma[1 + m, b*x])/(b*E^a*(b*x)^m))
```

3.81.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.81.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
meijerg	$\frac{x^{1+m}\,\text{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2}\right],\left[\frac{1}{2},\frac{3}{2}+\frac{m}{2}\right],\frac{b^{2}x^{2}}{4}\right)\sinh(a)}{1+m}+\frac{b\,x^{2+m}\,\text{hypergeom}\left(\left[1+\frac{m}{2}\right],\left[\frac{3}{2},2+\frac{m}{2}\right],\frac{b^{2}x^{2}}{4}\right)\cosh(a)}{2+m}$	73

input
```
int(x^m*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(1+m)*x^(1+m)*hypergeom([1/2+1/2*m],[1/2,3/2+1/2*m],1/4*b^2*x^2)*sinh(a)
+b/(2+m)*x^(2+m)*hypergeom([1+1/2*m],[3/2,2+1/2*m],1/4*b^2*x^2)*cosh(a)
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.32

$$\int x^m \sinh(a+bx)\,dx$$
$$= \frac{\cosh\left(m\log\left(b\right)+a\right)\Gamma(m+1,bx)+\cosh\left(m\log\left(-b\right)-a\right)\Gamma(m+1,-bx)-\Gamma(m+1,-bx)\sinh\left(m\log\left(-\right.\right.}{2\,b}$$

input
```
integrate(x^m*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh(m*log(b) + a)*gamma(m + 1, b*x) + cosh(m*log(-b) - a)*gamma(m +
1, -b*x) - gamma(m + 1, -b*x)*sinh(m*log(-b) - a) - gamma(m + 1, b*x)*sinh
(m*log(b) + a))/b
```

3.81.6 Sympy [F(-2)]

Exception generated.

$$\int x^m \sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**m*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\begin{aligned}\int x^m \sinh(a+bx)\,dx =& \frac{1}{2}\,(bx)^{-m-1}\,x^{m+1}e^{(-a)}\Gamma(m+1,bx)\\ &-\frac{1}{2}\,(-bx)^{-m-1}\,x^{m+1}e^{a}\Gamma(m+1,-bx)\end{aligned}$$

input
```
integrate(x^m*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(b*x)^(-m - 1)*x^(m + 1)*e^(-a)*gamma(m + 1, b*x) - 1/2*(-b*x)^(-m - 1
)*x^(m + 1)*e^a*gamma(m + 1, -b*x)
```

3.81.8 Giac [F]

$$\int x^m \sinh(a+bx)\,dx = \int x^m \sinh(bx+a)\,dx$$

input
```
integrate(x^m*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*sinh(b*x + a), x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int x^m \sinh(a+bx)\,dx = \int x^m \sinh(a+b\,x)\,dx$$

input
```
int(x^m*sinh(a + b*x),x)
```

output
```
int(x^m*sinh(a + b*x), x)
```

3.82 $\int x^{-1+m} \sinh(a+bx)\,dx$

3.82.1 Optimal result

Integrand size = 12, antiderivative size = 49

$$\int x^{-1+m} \sinh(a+bx)\,dx = -\frac{1}{2}e^a x^m (-bx)^{-m}\Gamma(m,-bx) + \frac{1}{2}e^{-a}x^m (bx)^{-m}\Gamma(m,bx)$$

output
```
-1/2*exp(a)*x^m*GAMMA(m,-b*x)/((-b*x)^m)+1/2*x^m*GAMMA(m,b*x)/exp(a)/((b*x
)^m)
```

3.82.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int x^{-1+m} \sinh(a+bx)\,dx = -\frac{1}{2}e^a x^m (-bx)^{-m}\Gamma(m,-bx) + \frac{1}{2}e^{-a}x^m (bx)^{-m}\Gamma(m,bx)$$

input
```
Integrate[x^(-1 + m)*Sinh[a + b*x],x]
```

output
```
-1/2*(E^a*x^m*Gamma[m, -(b*x)])/(-(b*x))^m + (x^m*Gamma[m, b*x])/(2*E^a*(b
*x)^m)
```

3.82.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-1}\sinh(a+bx)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -ix^{m-1}\sin(ia+ibx)dx$$
$$\Big\downarrow \text{ 26}$$
$$-i\int x^{m-1}\sin(ia+ibx)dx$$
$$\Big\downarrow \text{ 3789}$$
$$-i\left(\frac{1}{2}i\int e^{a+bx}x^{m-1}dx-\frac{1}{2}i\int e^{-a-bx}x^{m-1}dx\right)$$
$$\Big\downarrow \text{ 2612}$$
$$-i\left(\frac{1}{2}ie^{-a}x^m(bx)^{-m}\Gamma(m,bx)-\frac{1}{2}ie^a x^m(-bx)^{-m}\Gamma(m,-bx)\right)$$

input
```
Int[x^(-1 + m)*Sinh[a + b*x],x]
```

output
```
(-I)*(((-1/2*I)*E^a*x^m*Gamma[m, -(b*x)])/(-(b*x))^m + ((I/2)*x^m*Gamma[m,
 b*x])/(E^a*(b*x)^m))
```

3.82.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.82.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

method	result	size
meijerg	$\frac{x^{m} \operatorname{hypergeom}\left(\left[\frac{m}{2}\right],\left[\frac{1}{2},1+\frac{m}{2}\right],\frac{b^{2} x^{2}}{4}\right) \sinh \left(a \right)}{m}+\frac{b \,x^{1+m} \operatorname{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{m}{2}\right],\frac{b^{2} x^{2}}{4}\right) \cosh \left(a \right)}{1+m}$	67

input
```
int(x^(m-1)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/m*x^m*hypergeom([1/2*m],[1/2,1+1/2*m],1/4*b^2*x^2)*sinh(a)+b/(1+m)*x^(1+
m)*hypergeom([1/2+1/2*m],[3/2,3/2+1/2*m],1/4*b^2*x^2)*cosh(a)
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int x^{-1+m}\sinh(a+bx)\,dx$$
$$=\frac{\cosh\left((m-1)\log\left(b\right)+a\right)\Gamma(m,bx)+\cosh\left((m-1)\log\left(-b\right)-a\right)\Gamma(m,-bx)-\Gamma(m,-bx)\sinh\left((m-1)\mathrm{l}\right.}{2\,b}$$

input
```
integrate(x^(-1+m)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh((m - 1)*log(b) + a)*gamma(m, b*x) + cosh((m - 1)*log(-b) - a)*ga
mma(m, -b*x) - gamma(m, -b*x)*sinh((m - 1)*log(-b) - a) - gamma(m, b*x)*si
nh((m - 1)*log(b) + a))/b
```

3.82.6 Sympy [F(-2)]

Exception generated.

$$\int x^{-1+m}\sinh(a+bx)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x**(-1+m)*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int x^{-1+m}\sinh(a+bx)\,dx=\frac{x^{m}e^{(-a)}\Gamma(m,bx)}{2\,(bx)^{m}}-\frac{x^{m}e^{a}\Gamma(m,-bx)}{2\,(-bx)^{m}}$$

input
```
integrate(x^(-1+m)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x^m*e^(-a)*gamma(m, b*x)/(b*x)^m - 1/2*x^m*e^a*gamma(m, -b*x)/(-b*x)^m
```

3.82.8 Giac [F]

$$\int x^{-1+m}\sinh(a+bx)\,dx=\int x^{m-1}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^(-1+m)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m - 1)*sinh(b*x + a), x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1+m}\sinh(a+bx)\,dx=\int x^{m-1}\sinh(a+b\,x)\,dx$$

input
```
int(x^(m - 1)*sinh(a + b*x),x)
```

output
```
int(x^(m - 1)*sinh(a + b*x), x)
```

3.83 $\int x^{-2+m}\sinh(a+bx)\,dx$

3.83.1 Optimal result

Integrand size = 12, antiderivative size = 55

$$\int x^{-2+m}\sinh(a+bx)\,dx=\frac{1}{2}be^{a}x^{m}(-bx)^{-m}\Gamma(-1+m,-bx)+\frac{1}{2}be^{-a}x^{m}(bx)^{-m}\Gamma(-1+m,bx)$$

output
```
1/2*b*exp(a)*x^m*GAMMA(-1+m,-b*x)/((-b*x)^m)+1/2*b*x^m*GAMMA(-1+m,b*x)/exp
(a)/((b*x)^m)
```

3.83.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^{-2+m}\sinh(a+bx)\,dx=\frac{1}{2}be^{-a}x^{m}\left(e^{2a}(-bx)^{-m}\Gamma(-1+m,-bx)+(bx)^{-m}\Gamma(-1+m,bx)\right)$$

input
```
Integrate[x^(-2 + m)*Sinh[a + b*x],x]
```

output
```
(b*x^m*((E^(2*a)*Gamma[-1 + m, -(b*x)])/(-(b*x))^m + Gamma[-1 + m, b*x]/(b
*x)^m))/(2*E^a)
```

3.83.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-2}\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix^{m-2}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int x^{m-2}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3789}$$

$$-i\left(\frac{1}{2}i\int e^{a+bx}x^{m-2}dx-\frac{1}{2}i\int e^{-a-bx}x^{m-2}dx\right)$$

$$\Big\downarrow \text{2612}$$

$$-i\left(\frac{1}{2}ie^{a}bx^{m}(-bx)^{-m}\Gamma(m-1,-bx)+\frac{1}{2}ie^{-a}bx^{m}(bx)^{-m}\Gamma(m-1,bx)\right)$$

input
```
Int[x^(-2 + m)*Sinh[a + b*x],x]
```

output
```
(-I)*(((I/2)*b*E^a*x^m*Gamma[-1 + m, -(b*x)])/(-(b*x))^m + ((I/2)*b*x^m*Ga
mma[-1 + m, b*x])/(E^a*(b*x)^m))
```

3.83.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.83.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

method	result	size
meijerg	$\frac{x^{m-1}\,\mathrm{hypergeom}\left(\left[-\frac{1}{2}+\frac{m}{2}\right],\left[\frac{1}{2},\frac{1}{2}+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\sinh(a)}{m-1}+\frac{b\,x^m\,\mathrm{hypergeom}\left(\left[\frac{m}{2}\right],\left[\frac{3}{2},1+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\cosh(a)}{m}$	67

input
```
int(x^(m-2)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(m-1)*x^(m-1)*hypergeom([-1/2+1/2*m],[1/2,1/2+1/2*m],1/4*b^2*x^2)*sinh(a
)+b/m*x^m*hypergeom([1/2*m],[3/2,1+1/2*m],1/4*b^2*x^2)*cosh(a)
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.56

$$\int x^{-2+m}\sinh(a+bx)\,dx = \frac{\cosh\left((m-2)\log\left(b\right)+a\right)\Gamma(m-1,bx)+\cosh\left((m-2)\log\left(-b\right)-a\right)\Gamma(m-1,-bx)-\Gamma(m-1,-bx)\sin}{2\,b}$$

input
```
integrate(x^(-2+m)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh((m - 2)*log(b) + a)*gamma(m - 1, b*x) + cosh((m - 2)*log(-b) - a
)*gamma(m - 1, -b*x) - gamma(m - 1, -b*x)*sinh((m - 2)*log(-b) - a) - gamm
a(m - 1, b*x)*sinh((m - 2)*log(b) + a))/b
```

3.83.6 Sympy [F(-2)]

Exception generated.

$$\int x^{-2+m}\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**(-2+m)*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^{-2+m}\sinh(a+bx)\,dx = \frac{1}{2}\left(bx\right)^{-m+1}x^{m-1}e^{(-a)}\Gamma(m-1,bx) - \frac{1}{2}\left(-bx\right)^{-m+1}x^{m-1}e^{a}\Gamma(m-1,-bx)$$

input
```
integrate(x^(-2+m)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(b*x)^(-m + 1)*x^(m - 1)*e^(-a)*gamma(m - 1, b*x) - 1/2*(-b*x)^(-m + 1
)*x^(m - 1)*e^a*gamma(m - 1, -b*x)
```

3.83.8 Giac [F]

$$\int x^{-2+m}\sinh(a+bx)\,dx = \int x^{m-2}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^(-2+m)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m - 2)*sinh(b*x + a), x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int x^{-2+m}\sinh(a+bx)\,dx = \int x^{m-2}\sinh(a+b\,x)\,dx$$

input
```
int(x^(m - 2)*sinh(a + b*x),x)
```

output
```
int(x^(m - 2)*sinh(a + b*x), x)
```

3.84 $\int x^{-3+m}\sinh(a+bx)\,dx$

3.84.1 Optimal result

Integrand size = 12, antiderivative size = 59

$$\int x^{-3+m}\sinh(a+bx)\,dx = -\frac{1}{2}b^2e^ax^m(-bx)^{-m}\Gamma(-2+m,-bx) + \frac{1}{2}b^2e^{-a}x^m(bx)^{-m}\Gamma(-2+m,bx)$$

output
```
-1/2*b^2*exp(a)*x^m*GAMMA(-2+m,-b*x)/((-b*x)^m)+1/2*b^2*x^m*GAMMA(-2+m,b*x
)/exp(a)/((b*x)^m)
```

3.84.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^{-3+m}\sinh(a+bx)\,dx = \frac{1}{2}b^2e^{-a}x^m\left(-e^{2a}(-bx)^{-m}\Gamma(-2+m,-bx)+(bx)^{-m}\Gamma(-2+m,bx)\right)$$

input
```
Integrate[x^(-3 + m)*Sinh[a + b*x],x]
```

output
```
(b^2*x^m*(-((E^(2*a)*Gamma[-2 + m, -(b*x)])/(-(b*x))^m) + Gamma[-2 + m, b*
x]/(b*x)^m))/(2*E^a)
```

3.84.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-3}\sinh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix^{m-3}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int x^{m-3}\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3789}$$

$$-i\left(\frac{1}{2}i\int e^{a+bx}x^{m-3}dx-\frac{1}{2}i\int e^{-a-bx}x^{m-3}dx\right)$$

$$\Big\downarrow \text{2612}$$

$$-i\left(\frac{1}{2}ie^{-a}b^2x^m(bx)^{-m}\Gamma(m-2,bx)-\frac{1}{2}ie^ab^2x^m(-bx)^{-m}\Gamma(m-2,-bx)\right)$$

input
```
Int[x^(-3 + m)*Sinh[a + b*x],x]
```

output
```
(-I)*(((-1/2*I)*b^2*E^a*x^m*Gamma[-2 + m, -(b*x)])/(-(b*x))^m + ((I/2)*b^2
*x^m*Gamma[-2 + m, b*x])/(E^a*(b*x)^m))
```

3.84.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.84.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

method	result	size
meijerg	$\frac{x^{m-2}\,\mathrm{hypergeom}\left(\left[\frac{m}{2}-1\right],\left[\frac{1}{2},\frac{m}{2}\right],\frac{b^{2}x^{2}}{4}\right)\sinh\left(a\right)}{m-2}+\frac{b\,x^{m-1}\,\mathrm{hypergeom}\left(\left[-\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{1}{2}+\frac{m}{2}\right],\frac{b^{2}x^{2}}{4}\right)\cosh\left(a\right)}{m-1}$	71

input
```
int(x^(m-3)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(m-2)*x^(m-2)*hypergeom([1/2*m-1],[1/2,1/2*m],1/4*b^2*x^2)*sinh(a)+b/(m-
1)*x^(m-1)*hypergeom([-1/2+1/2*m],[3/2,1/2+1/2*m],1/4*b^2*x^2)*cosh(a)
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int x^{-3+m}\sinh(a+bx)\,dx$$
$$=\frac{\cosh\left((m-3)\log\left(b\right)+a\right)\Gamma(m-2,bx)+\cosh\left((m-3)\log\left(-b\right)-a\right)\Gamma(m-2,-bx)-\Gamma(m-2,-bx)\sin}{2\,b}$$

input
```
integrate(x^(-3+m)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh((m - 3)*log(b) + a)*gamma(m - 2, b*x) + cosh((m - 3)*log(-b) - a
)*gamma(m - 2, -b*x) - gamma(m - 2, -b*x)*sinh((m - 3)*log(-b) - a) - gamm
a(m - 2, b*x)*sinh((m - 3)*log(b) + a))/b
```

3.84.6 Sympy [F(-2)]

Exception generated.

$$\int x^{-3+m}\sinh(a+bx)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x**(-3+m)*sinh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^{-3+m}\sinh(a+bx)\,dx=\frac{1}{2}\left(bx\right)^{-m+2}x^{m-2}e^{(-a)}\Gamma(m-2,bx)-\frac{1}{2}\left(-bx\right)^{-m+2}x^{m-2}e^{a}\Gamma(m-2,-bx)$$

input
```
integrate(x^(-3+m)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(b*x)^(-m + 2)*x^(m - 2)*e^(-a)*gamma(m - 2, b*x) - 1/2*(-b*x)^(-m + 2
)*x^(m - 2)*e^a*gamma(m - 2, -b*x)
```

3.84.8 Giac [F]

$$\int x^{-3+m}\sinh(a+bx)\,dx=\int x^{m-3}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(x^(-3+m)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m - 3)*sinh(b*x + a), x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int x^{-3+m}\sinh(a+bx)\,dx=\int x^{m-3}\sinh(a+b\,x)\,dx$$

input
```
int(x^(m - 3)*sinh(a + b*x),x)
```

output
```
int(x^(m - 3)*sinh(a + b*x), x)
```

3.85 $\int x^{3+m} \sinh^2(a+bx)\,dx$

3.85.1 Optimal result

Integrand size = 14, antiderivative size = 86

$$\int x^{3+m} \sinh^2(a+bx)\,dx = -\frac{x^{4+m}}{2(4+m)} - \frac{2^{-6-m}e^{2a}x^m(-bx)^{-m}\Gamma(4+m,-2bx)}{b^4} - \frac{2^{-6-m}e^{-2a}x^m(bx)^{-m}\Gamma(4+m,2bx)}{b^4}$$

output
```
-1/2*x^(4+m)/(4+m)-2^(-6-m)*exp(2*a)*x^m*GAMMA(4+m,-2*b*x)/b^4/((-b*x)^m)-
2^(-6-m)*x^m*GAMMA(4+m,2*b*x)/b^4/exp(2*a)/((b*x)^m)
```

3.85.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int x^{3+m} \sinh^2(a+bx)\,dx = \frac{1}{64}x^m\left(-\frac{32x^4}{4+m} - \frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(4+m,-2bx)}{b^4} - \frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(4+m,2bx)}{b^4}\right)$$

input
```
Integrate[x^(3 + m)*Sinh[a + b*x]^2,x]
```

output
```
(x^m*((-32*x^4)/(4 + m) - (E^(2*a)*Gamma[4 + m, -2*b*x])/(2^m*b^4*(-(b*x))
^m) - Gamma[4 + m, 2*b*x]/(2^m*b^4*E^(2*a)*(b*x)^m)))/64
```

3.85.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+3}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^{m+3}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^{m+3}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$-\int \left(\frac{x^{m+3}}{2}-\frac{1}{2}x^{m+3}\cosh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}2^{-m-6}x^m(-bx)^{-m}\Gamma(m+4,-2bx)}{b^4}-\frac{e^{-2a}2^{-m-6}x^m(bx)^{-m}\Gamma(m+4,2bx)}{b^4}-\frac{x^{m+4}}{2(m+4)}$$

input
```
Int[x^(3 + m)*Sinh[a + b*x]^2,x]
```

output
```
-1/2*x^(4 + m)/(4 + m) - (2^(-6 - m)*E^(2*a)*x^m*Gamma[4 + m, -2*b*x])/(b^
4*(-(b*x))^m) - (2^(-6 - m)*x^m*Gamma[4 + m, 2*b*x])/(b^4*E^(2*a)*(b*x)^m)
```

3.85.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.85.4 Maple [F]

$$\int x^{3+m} \sinh (bx+a)^2 \, dx$$

input
```
int(x^(3+m)*sinh(b*x+a)^2,x)
```

output
```
int(x^(3+m)*sinh(b*x+a)^2,x)
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.58

$$\int x^{3+m} \sinh^2(a+bx) \, dx = -\frac{4\,bx \cosh\left((m+3)\log\left(x\right)\right) + (m+4)\cosh\left((m+3)\log\left(2\,b\right)+2\,a\right)\Gamma(m+4,2\,bx) - (m+4)\cosh\left((m+\right.}{}$$

input
```
integrate(x^(3+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*cosh((m + 3)*log(x)) + (m + 4)*cosh((m + 3)*log(2*b) + 2*a)*ga
mma(m + 4, 2*b*x) - (m + 4)*cosh((m + 3)*log(-2*b) - 2*a)*gamma(m + 4, -2*
b*x) - (m + 4)*gamma(m + 4, 2*b*x)*sinh((m + 3)*log(2*b) + 2*a) + (m + 4)*
gamma(m + 4, -2*b*x)*sinh((m + 3)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 3)*lo
g(x)))/(b*m + 4*b)
```

3.85.6 Sympy [F]

$$\int x^{3+m}\sinh^2(a+bx)\,dx=\int x^{m+3}\sinh^2{(a+bx)}\,dx$$

input
```
integrate(x**(3+m)*sinh(b*x+a)**2,x)
```

output
```
Integral(x**(m + 3)*sinh(a + b*x)**2, x)
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int x^{3+m}\sinh^2(a+bx)\,dx=-\frac{1}{4}\,(2\,bx)^{-m-4}\,x^{m+4}e^{(-2\,a)}\Gamma(m+4,2\,bx)-\frac{1}{4}\,(-2\,bx)^{-m-4}\,x^{m+4}e^{(2\,a)}\Gamma(m+4,-2\,bx)-\frac{x^{m+4}}{2\,(m+4)}$$

input
```
integrate(x^(3+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x)^(-m - 4)*x^(m + 4)*e^(-2*a)*gamma(m + 4, 2*b*x) - 1/4*(-2*b*x
)^(-m - 4)*x^(m + 4)*e^(2*a)*gamma(m + 4, -2*b*x) - 1/2*x^(m + 4)/(m + 4)
```

3.85.8 Giac [F]

$$\int x^{3+m}\sinh^2(a+bx)\,dx=\int x^{m+3}\sinh{(bx+a)}^2\,dx$$

input
```
integrate(x^(3+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m + 3)*sinh(b*x + a)^2, x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int x^{3+m}\sinh^2(a+bx)\,dx = \int x^{m+3}\sinh(a+b\,x)^2\,dx$$

input
```
int(x^(m + 3)*sinh(a + b*x)^2,x)
```

output
```
int(x^(m + 3)*sinh(a + b*x)^2, x)
```

3.86 $\int x^{2+m} \sinh^2(a+bx)\,dx$

3.86.1 Optimal result

Integrand size = 14, antiderivative size = 85

$$\int x^{2+m} \sinh^2(a+bx)\,dx = -\frac{x^{3+m}}{2(3+m)} + \frac{2^{-5-m}e^{2a}x^m(-bx)^{-m}\Gamma(3+m,-2bx)}{b^3} - \frac{2^{-5-m}e^{-2a}x^m(bx)^{-m}\Gamma(3+m,2bx)}{b^3}$$

output
```
-1/2*x^(3+m)/(3+m)+2^(-5-m)*exp(2*a)*x^m*GAMMA(3+m,-2*b*x)/b^3/((-b*x)^m)-
2^(-5-m)*x^m*GAMMA(3+m,2*b*x)/b^3/exp(2*a)/((b*x)^m)
```

3.86.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{2+m} \sinh^2(a+bx)\,dx = \frac{1}{32}x^m\left(-\frac{16x^3}{3+m} + \frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(3+m,-2bx)}{b^3} - \frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(3+m,2bx)}{b^3}\right)$$

input
```
Integrate[x^(2 + m)*Sinh[a + b*x]^2,x]
```

output
```
(x^m*((-16*x^3)/(3 + m) + (E^(2*a)*Gamma[3 + m, -2*b*x])/(2^m*b^3*(-(b*x))
^m) - Gamma[3 + m, 2*b*x]/(2^m*b^3*E^(2*a)*(b*x)^m)))/32
```

3.86.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+2}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -x^{m+2}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int x^{m+2}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 3793}$$

$$-\int \left(\frac{x^{m+2}}{2}-\frac{1}{2}x^{m+2}\cosh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^{2a}2^{-m-5}x^m(-bx)^{-m}\Gamma(m+3,-2bx)}{b^3}-\frac{e^{-2a}2^{-m-5}x^m(bx)^{-m}\Gamma(m+3,2bx)}{b^3}-\frac{x^{m+3}}{2(m+3)}$$

input
```
Int[x^(2 + m)*Sinh[a + b*x]^2,x]
```

output
```
-1/2*x^(3 + m)/(3 + m) + (2^(-5 - m)*E^(2*a)*x^m*Gamma[3 + m, -2*b*x])/(b^
3*(-(b*x))^m) - (2^(-5 - m)*x^m*Gamma[3 + m, 2*b*x])/(b^3*E^(2*a)*(b*x)^m)
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.86.4 Maple [F]

$$\int x^{2+m} \sinh\left(bx+a\right)^{2} dx$$

input
```
int(x^(2+m)*sinh(b*x+a)^2,x)
```

output
```
int(x^(2+m)*sinh(b*x+a)^2,x)
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.60

$$\int x^{2+m} \sinh^2(a+bx)\, dx = -\frac{4\,bx\cosh\left((m+2)\log\left(x\right)\right)+(m+3)\cosh\left((m+2)\log\left(2\,b\right)+2\,a\right)\Gamma(m+3,2\,bx)-(m+3)\cosh\left((m+\right.}{}$$

input
```
integrate(x^(2+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*cosh((m + 2)*log(x)) + (m + 3)*cosh((m + 2)*log(2*b) + 2*a)*ga
mma(m + 3, 2*b*x) - (m + 3)*cosh((m + 2)*log(-2*b) - 2*a)*gamma(m + 3, -2*
b*x) - (m + 3)*gamma(m + 3, 2*b*x)*sinh((m + 2)*log(2*b) + 2*a) + (m + 3)*
gamma(m + 3, -2*b*x)*sinh((m + 2)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 2)*lo
g(x)))/(b*m + 3*b)
```

3.86.6 Sympy [F]

$$\int x^{2+m}\sinh^2(a+bx)\,dx = \int x^{m+2}\sinh^2{(a+bx)}\,dx$$

input
```
integrate(x**(2+m)*sinh(b*x+a)**2,x)
```

output
```
Integral(x**(m + 2)*sinh(a + b*x)**2, x)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\begin{aligned}\int x^{2+m}\sinh^2(a+bx)\,dx = &-\frac{1}{4}\,(2\,bx)^{-m-3}\,x^{m+3}e^{(-2\,a)}\Gamma(m+3,2\,bx)\\ &-\frac{1}{4}\,(-2\,bx)^{-m-3}\,x^{m+3}e^{(2\,a)}\Gamma(m+3,-2\,bx)-\frac{x^{m+3}}{2\,(m+3)}\end{aligned}$$

input
```
integrate(x^(2+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x)^(-m - 3)*x^(m + 3)*e^(-2*a)*gamma(m + 3, 2*b*x) - 1/4*(-2*b*x
)^(-m - 3)*x^(m + 3)*e^(2*a)*gamma(m + 3, -2*b*x) - 1/2*x^(m + 3)/(m + 3)
```

3.86.8 Giac [F]

$$\int x^{2+m}\sinh^2(a+bx)\,dx = \int x^{m+2}\sinh{(bx+a)}^2\,dx$$

input
```
integrate(x^(2+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m + 2)*sinh(b*x + a)^2, x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int x^{2+m} \sinh^2(a+bx)\,dx = \int x^{m+2} \sinh(a+b\,x)^2\,dx$$

input
```
int(x^(m + 2)*sinh(a + b*x)^2,x)
```

output
```
int(x^(m + 2)*sinh(a + b*x)^2, x)
```

3.87 $\int x^{1+m} \sinh^2(a+bx)\,dx$

3.87.1 Optimal result

Integrand size = 14, antiderivative size = 86

$$\int x^{1+m} \sinh^2(a+bx)\,dx = -\frac{x^{2+m}}{2(2+m)} - \frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(2+m,-2bx)}{b^2} - \frac{2^{-4-m}e^{-2a}x^m(bx)^{-m}\Gamma(2+m,2bx)}{b^2}$$

output
```
-1/2*x^(2+m)/(2+m)-2^(-4-m)*exp(2*a)*x^m*GAMMA(2+m,-2*b*x)/b^2/((-b*x)^m)-
2^(-4-m)*x^m*GAMMA(2+m,2*b*x)/b^2/exp(2*a)/((b*x)^m)
```

3.87.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int x^{1+m} \sinh^2(a+bx)\,dx = \frac{1}{16}x^m\left(-\frac{8x^2}{2+m} - \frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(2+m,-2bx)}{b^2} - \frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(2+m,2bx)}{b^2}\right)$$

input
```
Integrate[x^(1 + m)*Sinh[a + b*x]^2,x]
```

output
```
(x^m*((-8*x^2)/(2 + m) - (E^(2*a)*Gamma[2 + m, -2*b*x])/(2^m*b^2*(-(b*x))^
m) - Gamma[2 + m, 2*b*x]/(2^m*b^2*E^(2*a)*(b*x)^m)))/16
```

3.87.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+1}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^{m+1}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^{m+1}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$-\int \left(\frac{x^{m+1}}{2}-\frac{1}{2}x^{m+1}\cosh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\Gamma(m+2,-2bx)}{b^2}-\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\Gamma(m+2,2bx)}{b^2}-\frac{x^{m+2}}{2(m+2)}$$

input
```
Int[x^(1 + m)*Sinh[a + b*x]^2,x]
```

output
```
-1/2*x^(2 + m)/(2 + m) - (2^(-4 - m)*E^(2*a)*x^m*Gamma[2 + m, -2*b*x])/(b^
2*(-(b*x))^m) - (2^(-4 - m)*x^m*Gamma[2 + m, 2*b*x])/(b^2*E^(2*a)*(b*x)^m)
```

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.87.4 Maple [F]

$$\int x^{1+m} \sinh (bx+a)^2 \, dx$$

input
```
int(x^(1+m)*sinh(b*x+a)^2,x)
```

output
```
int(x^(1+m)*sinh(b*x+a)^2,x)
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.58

$$\int x^{1+m} \sinh^2(a+bx) \, dx =$$
$$-\frac{4\,bx\cosh\left((m+1)\log\left(x\right)\right)+(m+2)\cosh\left((m+1)\log\left(2\,b\right)+2\,a\right)\Gamma(m+2,2\,bx)-(m+2)\cosh\left((m+\right.}{}$$

input
```
integrate(x^(1+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*cosh((m + 1)*log(x)) + (m + 2)*cosh((m + 1)*log(2*b) + 2*a)*ga
mma(m + 2, 2*b*x) - (m + 2)*cosh((m + 1)*log(-2*b) - 2*a)*gamma(m + 2, -2*
b*x) - (m + 2)*gamma(m + 2, 2*b*x)*sinh((m + 1)*log(2*b) + 2*a) + (m + 2)*
gamma(m + 2, -2*b*x)*sinh((m + 1)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 1)*lo
g(x)))/(b*m + 2*b)
```

3.87.6 Sympy [F]

$$\int x^{1+m}\sinh^2(a+bx)\,dx=\int x^{m+1}\sinh^2{(a+bx)}\,dx$$

input
```
integrate(x**(1+m)*sinh(b*x+a)**2,x)
```

output
```
Integral(x**(m + 1)*sinh(a + b*x)**2, x)
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\begin{aligned}\int x^{1+m}\sinh^2(a+bx)\,dx=&-\frac{1}{4}\,(2\,bx)^{-m-2}\,x^{m+2}e^{(-2\,a)}\Gamma(m+2,2\,bx)\\&-\frac{1}{4}\,(-2\,bx)^{-m-2}\,x^{m+2}e^{(2\,a)}\Gamma(m+2,-2\,bx)-\frac{x^{m+2}}{2\,(m+2)}\end{aligned}$$

input
```
integrate(x^(1+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x)^(-m - 2)*x^(m + 2)*e^(-2*a)*gamma(m + 2, 2*b*x) - 1/4*(-2*b*x
)^(-m - 2)*x^(m + 2)*e^(2*a)*gamma(m + 2, -2*b*x) - 1/2*x^(m + 2)/(m + 2)
```

3.87.8 Giac [F]

$$\int x^{1+m}\sinh^2(a+bx)\,dx=\int x^{m+1}\sinh{(bx+a)}^2\,dx$$

input
```
integrate(x^(1+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m + 1)*sinh(b*x + a)^2, x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int x^{1+m}\sinh^2(a+bx)\,dx=\int x^{m+1}\sinh(a+b\,x)^2\,dx$$

input
```
int(x^(m + 1)*sinh(a + b*x)^2,x)
```

output
```
int(x^(m + 1)*sinh(a + b*x)^2, x)
```

3.88 $\int x^m \sinh^2(a+bx)\,dx$

3.88.1 Optimal result

Integrand size = 12, antiderivative size = 85

$$\int x^m \sinh^2(a+bx)\,dx = -\frac{x^{1+m}}{2(1+m)} + \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} - \frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}$$

output
```
-1/2*x^(1+m)/(1+m)+2^(-3-m)*exp(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)-2^
(-3-m)*x^m*GAMMA(1+m,2*b*x)/b/exp(2*a)/((b*x)^m)
```

3.88.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int x^m \sinh^2(a+bx)\,dx = \frac{1}{8}x^m\left(-\frac{4x}{1+m} + \frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(1+m,-2bx)}{b} - \frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(1+m,2bx)}{b}\right)$$

input
```
Integrate[x^m*Sinh[a + b*x]^2,x]
```

output
```
(x^m*((-4*x)/(1 + m) + (E^(2*a)*Gamma[1 + m, -2*b*x])/(2^m*b*(-(b*x))^m) -
 Gamma[1 + m, 2*b*x]/(2^m*b*E^(2*a)*(b*x)^m)))/8
```

3.88.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh^2(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -x^m \sin(ia+ibx)^2dx$$
$$\Big\downarrow \text{25}$$
$$-\int x^m \sin(ia+ibx)^2dx$$
$$\Big\downarrow \text{3793}$$
$$-\int \left(\frac{x^m}{2}-\frac{1}{2}x^m\cosh(2a+2bx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b}-\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b}-\frac{x^{m+1}}{2(m+1)}$$

input
```
Int[x^m*Sinh[a + b*x]^2,x]
```

output
```
-1/2*x^(1 + m)/(1 + m) + (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*
(-(b*x))^m) - (2^(-3 - m)*x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m)
```

3.88.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.88.4 Maple [F]

$$\int x^{m} \sinh\left(bx+a\right)^{2} dx$$

input
```
int(x^m*sinh(b*x+a)^2,x)
```

output
```
int(x^m*sinh(b*x+a)^2,x)
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44

$$\int x^m \sinh^2(a+bx)\,dx = -\frac{4\,bx\cosh\left(m\log\left(x\right)\right) + (m+1)\cosh\left(m\log\left(2\,b\right)+2\,a\right)\Gamma(m+1,2\,bx) - (m+1)\cosh\left(m\log\left(-2\,b\right)-2\right.}{}$$

input
```
integrate(x^m*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*cosh(m*log(x)) + (m + 1)*cosh(m*log(2*b) + 2*a)*gamma(m + 1, 2
*b*x) - (m + 1)*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) - (m + 1)*gam
ma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) + (m + 1)*gamma(m + 1, -2*b*x)*sin
h(m*log(-2*b) - 2*a) + 4*b*x*sinh(m*log(x)))/(b*m + b)
```

3.88.6 Sympy [F]

$$\int x^m \sinh^2(a+bx)\,dx = \int x^m \sinh^2{(a+bx)}\,dx$$

input
```
integrate(x**m*sinh(b*x+a)**2,x)
```

output
```
Integral(x**m*sinh(a + b*x)**2, x)
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int x^m \sinh^2(a+bx)\,dx = -\frac{1}{4}\,(2\,bx)^{-m-1}\,x^{m+1}e^{(-2\,a)}\Gamma(m+1,2\,bx) - \frac{1}{4}\,(-2\,bx)^{-m-1}\,x^{m+1}e^{(2\,a)}\Gamma(m+1,-2\,bx) - \frac{x^{m+1}}{2\,(m+1)}$$

input
```
integrate(x^m*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x)^(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) - 1/4*(-2*b*x
)^(-m - 1)*x^(m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) - 1/2*x^(m + 1)/(m + 1)
```

3.88.8 Giac [F]

$$\int x^m \sinh^2(a+bx)\,dx = \int x^m \sinh{(bx+a)}^2\,dx$$

input
```
integrate(x^m*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*sinh(b*x + a)^2, x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int x^m \sinh^2(a+bx)\,dx = \int x^m \sinh(a+b\,x)^2\,dx$$

input
```
int(x^m*sinh(a + b*x)^2,x)
```

output
```
int(x^m*sinh(a + b*x)^2, x)
```

3.89 $\int x^{-1+m} \sinh^2(a+bx)\,dx$

3.89.1 Optimal result

Integrand size = 14, antiderivative size = 72

$$\int x^{-1+m} \sinh^2(a+bx)\,dx = -\frac{x^m}{2m} - 2^{-2-m}e^{2a}x^m(-bx)^{-m}\Gamma(m,-2bx) - 2^{-2-m}e^{-2a}x^m(bx)^{-m}\Gamma(m,2bx)$$

output
```
-1/2*x^m/m-2^(-2-m)*exp(2*a)*x^m*GAMMA(m,-2*b*x)/((-b*x)^m)-2^(-2-m)*x^m*G
AMMA(m,2*b*x)/exp(2*a)/((b*x)^m)
```

3.89.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int x^{-1+m} \sinh^2(a+bx)\,dx = -\frac{x^m(2+2^{-m}e^{2a}m(-bx)^{-m}\Gamma(m,-2bx)+2^{-m}e^{-2a}m(bx)^{-m}\Gamma(m,2bx))}{4m}$$

input
```
Integrate[x^(-1 + m)*Sinh[a + b*x]^2,x]
```

output
```
-1/4*(x^m*(2 + (E^(2*a)*m*Gamma[m, -2*b*x])/(2^m*(-(b*x))^m) + (m*Gamma[m,
 2*b*x])/(2^m*E^(2*a)*(b*x)^m)))/m
```

3.89.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-1}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^{m-1}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^{m-1}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$-\int \left(\frac{x^{m-1}}{2}-\frac{1}{2}x^{m-1}\cosh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$e^{2a}\left(-2^{-m-2}\right)x^m(-bx)^{-m}\Gamma(m,-2bx)-e^{-2a}2^{-m-2}x^m(bx)^{-m}\Gamma(m,2bx)-\frac{x^m}{2m}$$

input
```
Int[x^(-1 + m)*Sinh[a + b*x]^2,x]
```

output
```
-1/2*x^m/m - (2^(-2 - m)*E^(2*a)*x^m*Gamma[m, -2*b*x])/(-(b*x))^m - (2^(-2
 - m)*x^m*Gamma[m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

3.89.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.89.4 Maple [F]

$$\int x^{m-1} \sinh (bx+a)^2 \, dx$$

input
```
int(x^(m-1)*sinh(b*x+a)^2,x)
```

output
```
int(x^(m-1)*sinh(b*x+a)^2,x)
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.62

$$\int x^{-1+m} \sinh^2(a+bx) \, dx = -\frac{4\,bx\cosh\left((m-1)\log\left(x\right)\right) + m\cosh\left((m-1)\log\left(2\,b\right)+2\,a\right)\Gamma(m,2\,bx) - m\cosh\left((m-1)\log\left(-2\,b\right) -$$

input
```
integrate(x^(-1+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*cosh((m - 1)*log(x)) + m*cosh((m - 1)*log(2*b) + 2*a)*gamma(m,
 2*b*x) - m*cosh((m - 1)*log(-2*b) - 2*a)*gamma(m, -2*b*x) - m*gamma(m, 2*
b*x)*sinh((m - 1)*log(2*b) + 2*a) + m*gamma(m, -2*b*x)*sinh((m - 1)*log(-2
*b) - 2*a) + 4*b*x*sinh((m - 1)*log(x)))/(b*m)
```

3.89.6 Sympy [F]

$$\int x^{-1+m}\sinh^2(a+bx)\,dx=\int x^{m-1}\sinh^2{(a+bx)}\,dx$$

input
```
integrate(x**(-1+m)*sinh(b*x+a)**2,x)
```

output
```
Integral(x**(m - 1)*sinh(a + b*x)**2, x)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int x^{-1+m}\sinh^2(a+bx)\,dx=-\frac{x^m e^{(-2\,a)}\Gamma(m,2\,bx)}{4\,(2\,bx)^m}-\frac{x^m e^{(2\,a)}\Gamma(m,-2\,bx)}{4\,(-2\,bx)^m}-\frac{x^m}{2\,m}$$

input
```
integrate(x^(-1+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*x^m*e^(-2*a)*gamma(m, 2*b*x)/(2*b*x)^m - 1/4*x^m*e^(2*a)*gamma(m, -2*
b*x)/(-2*b*x)^m - 1/2*x^m/m
```

3.89.8 Giac [F]

$$\int x^{-1+m}\sinh^2(a+bx)\,dx=\int x^{m-1}\sinh{(bx+a)}^2\,dx$$

input
```
integrate(x^(-1+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m - 1)*sinh(b*x + a)^2, x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1+m}\sinh^2(a+bx)\,dx = \int x^{m-1}\sinh(a+b\,x)^2\,dx$$

input
```
int(x^(m - 1)*sinh(a + b*x)^2,x)
```

output
```
int(x^(m - 1)*sinh(a + b*x)^2, x)
```

3.90 $\int x^{-2+m}\sinh^2(a+bx)\,dx$

3.90.1 Optimal result

Integrand size = 14, antiderivative size = 83

$$\int x^{-2+m}\sinh^2(a+bx)\,dx=\frac{x^{-1+m}}{2(1-m)}+2^{-1-m}be^{2a}x^m(-bx)^{-m}\Gamma(-1+m,-2bx)-2^{-1-m}be^{-2a}x^m(bx)^{-m}\Gamma(-1+m,2bx)$$

output
```
1/2*x^(-1+m)/(1-m)+2^(-1-m)*b*exp(2*a)*x^m*GAMMA(-1+m,-2*b*x)/((-b*x)^m)-2
^(-1-m)*b*x^m*GAMMA(-1+m,2*b*x)/exp(2*a)/((b*x)^m)
```

3.90.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int x^{-2+m}\sinh^2(a+bx)\,dx=\frac{1}{2}x^m\left(\frac{1}{x-mx}+2^{-m}be^{2a}(-bx)^{-m}\Gamma(-1+m,-2bx)-2^{-m}be^{-2a}(bx)^{-m}\Gamma(-1+m,2bx)\right)$$

input
```
Integrate[x^(-2 + m)*Sinh[a + b*x]^2,x]
```

output
```
(x^m*((x - m*x)^(-1) + (b*E^(2*a)*Gamma[-1 + m, -2*b*x])/(2^m*(-(b*x))^m)
- (b*Gamma[-1 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)))/2
```

3.90.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-2}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^{m-2}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^{m-2}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3793}$$

$$-\int\left(\frac{x^{m-2}}{2}-\frac{1}{2}x^{m-2}\cosh(2a+2bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$e^{2a}b2^{-m-1}x^m(-bx)^{-m}\Gamma(m-1,-2bx)-e^{-2a}b2^{-m-1}x^m(bx)^{-m}\Gamma(m-1,2bx)+\frac{x^{m-1}}{2(1-m)}$$

input
```
Int[x^(-2 + m)*Sinh[a + b*x]^2,x]
```

output
```
x^(-1 + m)/(2*(1 - m)) + (2^(-1 - m)*b*E^(2*a)*x^m*Gamma[-1 + m, -2*b*x])/
(-(b*x))^m - (2^(-1 - m)*b*x^m*Gamma[-1 + m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

3.90.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.90.4 Maple [F]

$$\int x^{m-2} \sinh(bx+a)^2\, dx$$

input
```
int(x^(m-2)*sinh(b*x+a)^2,x)
```

output
```
int(x^(m-2)*sinh(b*x+a)^2,x)
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.64

$$\int x^{-2+m} \sinh^2(a+bx)\, dx = -\frac{4\,bx\cosh\left((m-2)\log(x)\right) + (m-1)\cosh\left((m-2)\log(2\,b)+2\,a\right)\Gamma(m-1,2\,bx) - (m-1)\cosh\left((m-\right.}{}$$

input
```
integrate(x^(-2+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*cosh((m - 2)*log(x)) + (m - 1)*cosh((m - 2)*log(2*b) + 2*a)*ga
mma(m - 1, 2*b*x) - (m - 1)*cosh((m - 2)*log(-2*b) - 2*a)*gamma(m - 1, -2*
b*x) - (m - 1)*gamma(m - 1, 2*b*x)*sinh((m - 2)*log(2*b) + 2*a) + (m - 1)*
gamma(m - 1, -2*b*x)*sinh((m - 2)*log(-2*b) - 2*a) + 4*b*x*sinh((m - 2)*lo
g(x)))/(b*m - b)
```

3.90.6 Sympy [F]

$$\int x^{-2+m}\sinh^2(a+bx)\,dx = \int x^{m-2}\sinh^2{(a+bx)}\,dx$$

input
```
integrate(x**(-2+m)*sinh(b*x+a)**2,x)
```

output
```
Integral(x**(m - 2)*sinh(a + b*x)**2, x)
```

3.90.7 Maxima [F(-2)]

Exception generated.

$$\int x^{-2+m}\sinh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^(-2+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(m-2>0)', see `assume?` for more
details)Is
```

3.90.8 Giac [F]

$$\int x^{-2+m}\sinh^2(a+bx)\,dx = \int x^{m-2}\sinh{(bx+a)}^2\,dx$$

input
```
integrate(x^(-2+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m - 2)*sinh(b*x + a)^2, x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int x^{-2+m}\sinh^2(a+bx)\,dx=\int x^{m-2}\,\sinh(a+b\,x)^2\,dx$$

input
```
int(x^(m - 2)*sinh(a + b*x)^2,x)
```

output
```
int(x^(m - 2)*sinh(a + b*x)^2, x)
```

3.91 $\int x^{-3+m}\sinh^2(a+bx)\,dx$

3.91.1 Optimal result

Integrand size = 14, antiderivative size = 84

$$\int x^{-3+m}\sinh^2(a+bx)\,dx=\frac{x^{-2+m}}{2(2-m)}-2^{-m}b^2e^{2a}x^m(-bx)^{-m}\Gamma(-2+m,-2bx)-2^{-m}b^2e^{-2a}x^m(bx)^{-m}\Gamma(-2+m,2bx)$$

output
```
1/2*x^(-2+m)/(2-m)-b^2*exp(2*a)*x^m*GAMMA(-2+m,-2*b*x)/(2^m)/((-b*x)^m)-b^
2*x^m*GAMMA(-2+m,2*b*x)/(2^m)/exp(2*a)/((b*x)^m)
```

3.91.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int x^{-3+m}\sinh^2(a+bx)\,dx=x^m\left(\frac{1}{(4-2m)x^2}-2^{-m}b^2e^{2a}(-bx)^{-m}\Gamma(-2+m,-2bx)-2^{-m}b^2e^{-2a}(bx)^{-m}\Gamma(-2+m,2bx)\right)$$

input
```
Integrate[x^(-3 + m)*Sinh[a + b*x]^2,x]
```

output
```
x^m*(1/((4 - 2*m)*x^2) - (b^2*E^(2*a)*Gamma[-2 + m, -2*b*x])/(2^m*(-(b*x))
^m) - (b^2*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m))
```

3.91.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-3}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -x^{m-3}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \; 25$$

$$-\int x^{m-3}\sin(ia+ibx)^2dx$$

$$\Big\downarrow \; 3793$$

$$-\int \left(\frac{x^{m-3}}{2}-\frac{1}{2}x^{m-3}\cosh(2a+2bx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$-e^{2a}b^2 2^{-m}x^m(-bx)^{-m}\Gamma(m-2,-2bx)-e^{-2a}b^2 2^{-m}x^m(bx)^{-m}\Gamma(m-2,2bx)+\frac{x^{m-2}}{2(2-m)}$$

input
```
Int[x^(-3 + m)*Sinh[a + b*x]^2,x]
```

output
```
x^(-2 + m)/(2*(2 - m)) - (b^2*E^(2*a)*x^m*Gamma[-2 + m, -2*b*x])/(2^m*(-(b
*x))^m) - (b^2*x^m*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)
```

3.91.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.91.4 Maple [F]

$$\int x^{m-3} \sinh(bx+a)^2 \, dx$$

input
```
int(x^(m-3)*sinh(b*x+a)^2,x)
```

output
```
int(x^(m-3)*sinh(b*x+a)^2,x)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.62

$$\int x^{-3+m}\sinh^2(a+bx)\,dx = -\frac{4\,bx\cosh\left((m-3)\log\left(x\right)\right)+(m-2)\cosh\left((m-3)\log\left(2\,b\right)+2\,a\right)\Gamma(m-2,2\,bx)-(m-2)\cosh\left((m-\right.}{}$$

input
```
integrate(x^(-3+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*b*x*cosh((m - 3)*log(x)) + (m - 2)*cosh((m - 3)*log(2*b) + 2*a)*ga
mma(m - 2, 2*b*x) - (m - 2)*cosh((m - 3)*log(-2*b) - 2*a)*gamma(m - 2, -2*
b*x) - (m - 2)*gamma(m - 2, 2*b*x)*sinh((m - 3)*log(2*b) + 2*a) + (m - 2)*
gamma(m - 2, -2*b*x)*sinh((m - 3)*log(-2*b) - 2*a) + 4*b*x*sinh((m - 3)*lo
g(x)))/(b*m - 2*b)
```

3.91.6 Sympy [F]

$$\int x^{-3+m}\sinh^2(a+bx)\,dx = \int x^{m-3}\sinh^2(a+bx)\,dx$$

input
```
integrate(x**(-3+m)*sinh(b*x+a)**2,x)
```

output
```
Integral(x**(m - 3)*sinh(a + b*x)**2, x)
```

3.91.7 Maxima [F(-2)]

Exception generated.

$$\int x^{-3+m}\sinh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^(-3+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(m-3>0)', see `assume?` for more
details)Is
```

3.91.8 Giac [F]

$$\int x^{-3+m}\sinh^2(a+bx)\,dx = \int x^{m-3}\sinh(bx+a)^2\,dx$$

input
```
integrate(x^(-3+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m - 3)*sinh(b*x + a)^2, x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int x^{-3+m} \sinh^2(a+bx)\, dx = \int x^{m-3} \sinh(a+b\,x)^2\, dx$$

input
```
int(x^(m - 3)*sinh(a + b*x)^2,x)
```

output
```
int(x^(m - 3)*sinh(a + b*x)^2, x)
```

3.92 $\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3} x \sqrt{\operatorname{csch}(x)}\right) dx$

3.92.1 Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3} x \sqrt{\operatorname{csch}(x)}\right) dx=-\frac{4}{9 \operatorname{csch}^{\frac{3}{2}}(x)}+\frac{2 x \cosh(x)}{3 \sqrt{\operatorname{csch}(x)}}$$

output
```
-4/9/csch(x)^(3/2)+2/3*x*cosh(x)/csch(x)^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3} x \sqrt{\operatorname{csch}(x)}\right) dx=\frac{2(-2+3 x \coth(x))}{9 \operatorname{csch}^{\frac{3}{2}}(x)}$$

input
```
Integrate[x/Csch[x]^(3/2) + (x*Sqrt[Csch[x]])/3,x]
```

output
```
(2*(-2 + 3*x*Coth[x]))/(9*Csch[x]^(3/2))
```

3.92.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\operatorname{csch}(x)}\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{4}{9\operatorname{csch}^{\frac{3}{2}}(x)}$$

input
```
Int[x/Csch[x]^(3/2) + (x*Sqrt[Csch[x]])/3,x]
```

output
```
-4/(9*Csch[x]^(3/2)) + (2*x*Cosh[x])/(3*Sqrt[Csch[x]])
```

3.92.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.92.4 Maple [F]

$$\int \left(\frac{x}{\operatorname{csch}\left(x\right)^{\frac{3}{2}}}+\frac{x\sqrt{\operatorname{csch}\left(x\right)}}{3}\right)dx$$

input
```
int(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x)
```

output
```
int(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x)
```

3.92.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\operatorname{csch}(x)}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.92.6 Sympy [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\operatorname{csch}(x)}\right)\,dx=\frac{\int \frac{3x}{\operatorname{csch}^{\frac{3}{2}}(x)}\,dx+\int x\sqrt{\operatorname{csch}(x)}\,dx}{3}$$

input
```
integrate(x/csch(x)**(3/2)+1/3*x*csch(x)**(1/2),x)
```

output
```
(Integral(3*x/csch(x)**(3/2), x) + Integral(x*sqrt(csch(x)), x))/3
```

3.92.7 Maxima [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\operatorname{csch}(x)}\right)\,dx=\int \frac{1}{3}\,x\sqrt{\operatorname{csch}(x)}+\frac{x}{\operatorname{csch}(x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/3*x*sqrt(csch(x)) + x/csch(x)^(3/2), x)
```

3.92.8 Giac [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\operatorname{csch}(x)}\right)\,dx=\int \frac{1}{3}\,x\sqrt{\operatorname{csch}\left(x\right)}+\frac{x}{\operatorname{csch}\left(x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/3*x*sqrt(csch(x)) + x/csch(x)^(3/2), x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\operatorname{csch}(x)}\right)\,dx=\int \frac{x\,\sqrt{\frac{1}{\sinh(x)}}}{3}+\frac{x}{{\left(\frac{1}{\sinh(x)}\right)}^{3/2}}\,dx$$

input
```
int((x*(1/sinh(x))^(1/2))/3 + x/(1/sinh(x))^(3/2),x)
```

output
```
int((x*(1/sinh(x))^(1/2))/3 + x/(1/sinh(x))^(3/2), x)
```

3.93 $\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)dx$

3.93.1 Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)dx=-\frac{4}{25\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{2x\cosh(x)}{5\operatorname{csch}^{\frac{3}{2}}(x)}$$

output
```
-4/25/csch(x)^(5/2)+2/5*x*cosh(x)/csch(x)^(3/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)dx=\frac{2(-2+5x\coth(x))}{25\operatorname{csch}^{\frac{5}{2}}(x)}$$

input
```
Integrate[x/Csch[x]^(5/2) + (3*x)/(5*Sqrt[Csch[x]]),x]
```

output
```
(2*(-2 + 5*x*Coth[x]))/(25*Csch[x]^(5/2))
```

3.93.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{2x\cosh(x)}{5\operatorname{csch}^{\frac{3}{2}}(x)}-\frac{4}{25\operatorname{csch}^{\frac{5}{2}}(x)}$$

input
```
Int[x/Csch[x]^(5/2) + (3*x)/(5*Sqrt[Csch[x]]),x]
```

output
```
-4/(25*Csch[x]^(5/2)) + (2*x*Cosh[x])/(5*Csch[x]^(3/2))
```

3.93.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.93.4 Maple [F]

$$\int \left(\frac{x}{\operatorname{csch}\left(x\right)^{\frac{5}{2}}}+\frac{3x}{5\sqrt{\operatorname{csch}\left(x\right)}}\right)dx$$

input
```
int(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x)
```

output
```
int(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x)
```

3.93.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.93.6 Sympy [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)\,dx=\frac{\int \frac{5x}{\operatorname{csch}^{\frac{5}{2}}(x)}\,dx+\int \frac{3x}{\sqrt{\operatorname{csch}(x)}}\,dx}{5}$$

input
```
integrate(x/csch(x)**(5/2)+3/5*x/csch(x)**(1/2),x)
```

output
```
(Integral(5*x/csch(x)**(5/2), x) + Integral(3*x/sqrt(csch(x)), x))/5
```

3.93.7 Maxima [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)\,dx=\int \frac{3\,x}{5\,\sqrt{\operatorname{csch}\left(x\right)}}+\frac{x}{\operatorname{csch}\left(x\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(3/5*x/sqrt(csch(x)) + x/csch(x)^(5/2), x)
```

3.93.8 Giac [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)\,dx=\int \frac{3\,x}{5\,\sqrt{\operatorname{csch}\left(x\right)}}+\frac{x}{\operatorname{csch}\left(x\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(3/5*x/sqrt(csch(x)) + x/csch(x)^(5/2), x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)\,dx=\int \frac{3\,x}{5\,\sqrt{\frac{1}{\sinh(x)}}}+\frac{x}{{\left(\frac{1}{\sinh(x)}\right)}^{5/2}}\,dx$$

input
```
int((3*x)/(5*(1/sinh(x))^(1/2)) + x/(1/sinh(x))^(5/2),x)
```

output
```
int((3*x)/(5*(1/sinh(x))^(1/2)) + x/(1/sinh(x))^(5/2), x)
```

3.94 $\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}-\frac{5}{21} x \sqrt{\operatorname{csch}(x)}\right) dx$

3.94.1 Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}-\frac{5}{21} x \sqrt{\operatorname{csch}(x)}\right) dx=-\frac{4}{49\operatorname{csch}^{\frac{7}{2}}(x)}+\frac{2x\cosh(x)}{7\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{20}{63\operatorname{csch}^{\frac{3}{2}}(x)}-\frac{10x\cosh(x)}{21\sqrt{\operatorname{csch}(x)}}$$

output
```
-4/49/csch(x)^(7/2)+2/7*x*cosh(x)/csch(x)^(5/2)+20/63/csch(x)^(3/2)-10/21*
x*cosh(x)/csch(x)^(1/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}-\frac{5}{21} x \sqrt{\operatorname{csch}(x)}\right) dx=\sqrt{\operatorname{csch}(x)}\left(-\frac{167}{882}+\frac{88}{441}\cosh(2x)-\frac{1}{98}\cosh(4x)-\frac{13}{42}x\sinh(2x)+\frac{1}{28}x\sinh(4x)\right)$$

input
```
Integrate[x/Csch[x]^(7/2) - (5*x*Sqrt[Csch[x]])/21,x]
```

output
```
Sqrt[Csch[x]]*(-167/882 + (88*Cosh[2*x])/441 - Cosh[4*x]/98 - (13*x*Sinh[2
*x])/42 + (x*Sinh[4*x])/28)
```

3.94.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}-\frac{5}{21} x \sqrt{\operatorname{csch}(x)}\right) \, dx$$
$$\Big\downarrow \ 2009$$
$$\frac{20}{63 \operatorname{csch}^{\frac{3}{2}}(x)}-\frac{4}{49 \operatorname{csch}^{\frac{7}{2}}(x)}+\frac{2 x \cosh(x)}{7 \operatorname{csch}^{\frac{5}{2}}(x)}-\frac{10 x \cosh(x)}{21 \sqrt{\operatorname{csch}(x)}}$$

input
```
Int[x/Csch[x]^(7/2) - (5*x*Sqrt[Csch[x]])/21,x]
```

output
```
-4/(49*Csch[x]^(7/2)) + (2*x*Cosh[x])/(7*Csch[x]^(5/2)) + 20/(63*Csch[x]^(
3/2)) - (10*x*Cosh[x])/(21*Sqrt[Csch[x]])
```

3.94.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.94.4 Maple [F]

$$\int \left(\frac{x}{\operatorname{csch}(x)^{\frac{7}{2}}}-\frac{5 x \sqrt{\operatorname{csch}(x)}}{21}\right) dx$$

input
```
int(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x)
```

output
```
int(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x)
```

3.94.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)} - \frac{5}{21}x\sqrt{\operatorname{csch}(x)}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.94.6 Sympy [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)} - \frac{5}{21}x\sqrt{\operatorname{csch}(x)}\right) dx = -\frac{\int \left(-\frac{21x}{\operatorname{csch}^{\frac{7}{2}}(x)}\right) dx + \int 5x\sqrt{\operatorname{csch}(x)}\, dx}{21}$$

input
```
integrate(x/csch(x)**(7/2)-5/21*x*csch(x)**(1/2),x)
```

output
```
-(Integral(-21*x/csch(x)**(7/2), x) + Integral(5*x*sqrt(csch(x)), x))/21
```

3.94.7 Maxima [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)} - \frac{5}{21}x\sqrt{\operatorname{csch}(x)}\right) dx = \int -\frac{5}{21}x\sqrt{\operatorname{csch}(x)} + \frac{x}{\operatorname{csch}(x)^{\frac{7}{2}}}\, dx$$

input
```
integrate(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-5/21*x*sqrt(csch(x)) + x/csch(x)^(7/2), x)
```

3.94.8 Giac [F]

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}-\frac{5}{21}x\sqrt{\operatorname{csch}(x)}\right)\,dx=\int -\frac{5}{21}\,x\sqrt{\operatorname{csch}\left(x\right)}+\frac{x}{\operatorname{csch}\left(x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-5/21*x*sqrt(csch(x)) + x/csch(x)^(7/2), x)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}-\frac{5}{21}x\sqrt{\operatorname{csch}(x)}\right)\,dx=-\int \frac{5\,x\,\sqrt{\frac{1}{\sinh(x)}}}{21}-\frac{x}{{\left(\frac{1}{\sinh(x)}\right)}^{7/2}}\,dx$$

input
```
int(x/(1/sinh(x))^(7/2) - (5*x*(1/sinh(x))^(1/2))/21,x)
```

output
```
-int((5*x*(1/sinh(x))^(1/2))/21 - x/(1/sinh(x))^(7/2), x)
```

3.95 $\int \left(\frac{x^2}{\text{csch}^{\frac{3}{2}}(x)}+\frac{1}{3} x^2 \sqrt{\text{csch}(x)}\right) \, dx$

3.95.1 Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \left(\frac{x^2}{\text{csch}^{\frac{3}{2}}(x)}+\frac{1}{3} x^2 \sqrt{\text{csch}(x)}\right) \, dx=-\frac{8x}{9\text{csch}^{\frac{3}{2}}(x)}+\frac{16\cosh(x)}{27\sqrt{\text{csch}(x)}}+\frac{2x^2\cosh(x)}{3\sqrt{\text{csch}(x)}}-\frac{16}{27} i \sqrt{\text{csch}(x)}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}$$

output
```
-8/9*x/csch(x)^(3/2)+16/27*cosh(x)/csch(x)^(1/2)+2/3*x^2*cosh(x)/csch(x)^(
1/2)-16/27*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(c
os(1/4*Pi+1/2*I*x),2^(1/2))*csch(x)^(1/2)*(I*sinh(x))^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \left(\frac{x^2}{\text{csch}^{\frac{3}{2}}(x)}+\frac{1}{3} x^2 \sqrt{\text{csch}(x)}\right) \, dx=\frac{4(3+\text{csch}^2(x))\left(-12x+(8+9x^2)\coth(x)+\frac{8\text{csch}(x)\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),2\right)}{\sqrt{i\sinh(x)}}\right)}{27(-1+3\cosh(2x))\text{csch}^{\frac{7}{2}}(x)}$$

input
```
Integrate[x^2/Csch[x]^(3/2) + (x^2*Sqrt[Csch[x]])/3,x]
```

output
```
(4*(3 + Csch[x]^2)*(-12*x + (8 + 9*x^2)*Coth[x] + (8*Csch[x]*EllipticF[(Pi
 - (2*I)*x)/4, 2])/Sqrt[I*Sinh[x]]))/(27*(-1 + 3*Cosh[2*x])*Csch[x]^(7/2))
```

3.95.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)dx$$

$$\Big\downarrow \; \color{blue}{2009}$$

$$\frac{2x^2\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{16\cosh(x)}{27\sqrt{\operatorname{csch}(x)}}-\frac{16}{27}i\sqrt{i\sinh(x)}\sqrt{\operatorname{csch}(x)}\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)$$

input
```
Int[x^2/Csch[x]^(3/2) + (x^2*Sqrt[Csch[x]])/3,x]
```

output
```
(-8*x)/(9*Csch[x]^(3/2)) + (16*Cosh[x])/(27*Sqrt[Csch[x]]) + (2*x^2*Cosh[x
])/(3*Sqrt[Csch[x]]) - ((16*I)/27)*Sqrt[Csch[x]]*EllipticF[Pi/4 - (I/2)*x,
 2]*Sqrt[I*Sinh[x]]
```

3.95.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.95.4 Maple [F]

$$\int \left(\frac{x^2}{\operatorname{csch}(x)^{\frac{3}{2}}}+\frac{x^2\sqrt{\operatorname{csch}(x)}}{3}\right)dx$$

input
```
int(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x)
```

output
```
int(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x)
```

3.95.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.95.6 Sympy [F]

$$\int \left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)dx = \frac{\int \frac{3x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}\,dx+\int x^2\sqrt{\operatorname{csch}(x)}\,dx}{3}$$

input
```
integrate(x**2/csch(x)**(3/2)+1/3*x**2*csch(x)**(1/2),x)
```

output
```
(Integral(3*x**2/csch(x)**(3/2), x) + Integral(x**2*sqrt(csch(x)), x))/3
```

3.95.7 Maxima [F]

$$\int \left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)\,dx=\int \frac{1}{3}\,x^2\sqrt{\operatorname{csch}\left(x\right)}+\frac{x^2}{\operatorname{csch}\left(x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/3*x^2*sqrt(csch(x)) + x^2/csch(x)^(3/2), x)
```

3.95.8 Giac [F]

$$\int \left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)\,dx=\int \frac{1}{3}\,x^2\sqrt{\operatorname{csch}\left(x\right)}+\frac{x^2}{\operatorname{csch}\left(x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/3*x^2*sqrt(csch(x)) + x^2/csch(x)^(3/2), x)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)\,dx=\int \frac{x^2\,\sqrt{\frac{1}{\operatorname{sinh}(x)}}}{3}+\frac{x^2}{{\left(\frac{1}{\operatorname{sinh}(x)}\right)}^{3/2}}\,dx$$

input
```
int((x^2*(1/sinh(x))^(1/2))/3 + x^2/(1/sinh(x))^(3/2),x)
```

output
```
int((x^2*(1/sinh(x))^(1/2))/3 + x^2/(1/sinh(x))^(3/2), x)
```

3.96 $\int (c+dx)^3(a+ia\sinh(e+fx))\,dx$

3.96.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (c+dx)^3(a+ia\sinh(e+fx))\,dx = \frac{a(c+dx)^4}{4d}+\frac{6iad^2(c+dx)\cosh(e+fx)}{f^3} + \frac{ia(c+dx)^3\cosh(e+fx)}{f}-\frac{6iad^3\sinh(e+fx)}{f^4} - \frac{3iad(c+dx)^2\sinh(e+fx)}{f^2}$$

output
```
1/4*a*(d*x+c)^4/d+6*I*a*d^2*(d*x+c)*cosh(f*x+e)/f^3+I*a*(d*x+c)^3*cosh(f*x
+e)/f-6*I*a*d^3*sinh(f*x+e)/f^4-3*I*a*d*(d*x+c)^2*sinh(f*x+e)/f^2
```

3.96.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.31

$$\int (c+dx)^3(a+ia\sinh(e+fx))\,dx = \frac{a(f^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+4if(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\cosh(e+fx)-12id(}{4f^4}$$

input
```
Integrate[(c + d*x)^3*(a + I*a*Sinh[e + f*x]),x]
```

output
```
(a*(f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + (4*I)*f*(c + d*x)*
(c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f*x] - (12*I)*d*(c^2*
f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x]))/(4*f^4)
```

3.96.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+ia\sinh(e+fx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)^3(a+a\sin(ie+ifx))dx$$

$$\Big\downarrow \; 3798$$

$$\int \left(a(c+dx)^3+ia(c+dx)^3\sinh(e+fx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{6iad^2(c+dx)\cosh(e+fx)}{f^3}-\frac{3iad(c+dx)^2\sinh(e+fx)}{f^2}+\frac{ia(c+dx)^3\cosh(e+fx)}{f}+\frac{a(c+dx)^4}{4d}-\frac{6iad^3\sinh(e+fx)}{f^4}$$

input
```
Int[(c + d*x)^3*(a + I*a*Sinh[e + f*x]),x]
```

output
```
(a*(c + d*x)^4)/(4*d) + ((6*I)*a*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (I*a*(
c + d*x)^3*Cosh[e + f*x])/f - ((6*I)*a*d^3*Sinh[e + f*x])/f^4 - ((3*I)*a*d
*(c + d*x)^2*Sinh[e + f*x])/f^2
```

3.96.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.96.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{\left(if(dx+c)\left((dx+c)^2f^2+6d^2\right)\cosh(fx+e)-3id\left((dx+c)^2f^2+2d^2\right)\sinh(fx+e)+f\left(x\left(\frac{1}{2}d^2x^2+cdx+c^2\right)\left(\frac{dx}{2}+c\right)f^3+ic^3 f\right.\right.}{f^4}$
risch	$\frac{a\,d^3x^4}{4}+a\,d^2c\,x^3+\frac{3ad\,c^2x^2}{2}+a\,c^3x+\frac{a\,c^4}{4d}+\frac{ia\left(d^3x^3f^3+3c\,d^2f^3x^2+3c^2d\,f^3x-3d^3f^2x^2+c^3f^3-6c\,d^2f^2x\right.}{2f^4}$
parts	$\frac{a(dx+c)^4}{4d}+\frac{ia\left(\frac{d^3\left((fx+e)^3\cosh(fx+e)-3(fx+e)^2\sinh(fx+e)+6(fx+e)\cosh(fx+e)-6\sinh(fx+e)\right)}{f^3}-\frac{3d^3e\left((fx+e)^2\cosh(fx+\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{d^3a(fx+e)^4}{4f^3}+ic^3a\cosh(fx+e)-\frac{d^3ea(fx+e)^3}{f^3}+\frac{3id^3e^2a((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f^3}+\frac{d^2ca(fx+e)^3}{f^2}+\frac{3id^2e^2ca\cosh(fx+e}{f^2}}{}$
default	$\frac{\frac{d^3a(fx+e)^4}{4f^3}+ic^3a\cosh(fx+e)-\frac{d^3ea(fx+e)^3}{f^3}+\frac{3id^3e^2a((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f^3}+\frac{d^2ca(fx+e)^3}{f^2}+\frac{3id^2e^2ca\cosh(fx+e}{f^2}}{}$

input
```
int((d*x+c)^3*(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
(I*f*(d*x+c)*((d*x+c)^2*f^2+6*d^2)*cosh(f*x+e)-3*I*d*((d*x+c)^2*f^2+2*d^2)
*sinh(f*x+e)+f*(x*(1/2*d^2*x^2+c*d*x+c^2)*(1/2*d*x+c)*f^3+I*c^3*f^2+6*I*c*
d^2))*a/f^4
```

3.96.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(88) = 176$.

Time = 0.25 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.90

$$\int (c+dx)^3(a+ia\sinh(e+fx))\,dx$$
$$= \frac{(2i\,ad^3f^3x^3 + 2i\,ac^3f^3 + 6i\,ac^2df^2 + 12i\,acd^2f + 12i\,ad^3 - 6\,(-i\,acd^2f^3 - i\,ad^3f^2)x^2 - 6\,(-i\,ac^2df^3 - 2i}{}$$

input
```
integrate((d*x+c)^3*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*(2*I*a*d^3*f^3*x^3 + 2*I*a*c^3*f^3 + 6*I*a*c^2*d*f^2 + 12*I*a*c*d^2*f
+ 12*I*a*d^3 - 6*(-I*a*c*d^2*f^3 - I*a*d^3*f^2)*x^2 - 6*(-I*a*c^2*d*f^3 -
2*I*a*c*d^2*f^2 - 2*I*a*d^3*f)*x - 2*(-I*a*d^3*f^3*x^3 - I*a*c^3*f^3 + 3*I
*a*c^2*d*f^2 - 6*I*a*c*d^2*f + 6*I*a*d^3 + 3*(-I*a*c*d^2*f^3 + I*a*d^3*f^2
)*x^2 + 3*(-I*a*c^2*d*f^3 + 2*I*a*c*d^2*f^2 - 2*I*a*d^3*f)*x)*e^(2*f*x + 2
*e) + (a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4
*x)*e^(f*x + e))*e^(-f*x - e)/f^4
```

3.96.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 517, normalized size of antiderivative = 5.28

$$\int (c+dx)^3(a+ia\sinh(e+fx))\,dx = ac^3x + \frac{3ac^2dx^2}{2} + acd^2x^3 + \frac{ad^3x^4}{4}$$
$$+ \begin{cases} \frac{((2iac^3f^7+6iac^2df^7x+6iac^2df^6+6iacd^2f^7x^2+12iacd^2f^6x+12iacd^2f^5+2iad^3f^7x^3+6iad^3f^6x^2+12iad^3f^5x+12iad^3f^4)e^{-fx}+(2iac^3f^7e^{}}{} \\ \frac{x^4(iad^3e^{2e}-iad^3)e^{-e}}{8} + \frac{x^3(iacd^2e^{2e}-iacd^2)e^{-e}}{2} + \frac{x^2\cdot(3iac^2de^{2e}-3iac^2d)e^{-e}}{4} + \frac{x(iac^3e^{2e}-iac^3)e^{-e}}{2} \end{cases}$$

input
```
integrate((d*x+c)**3*(a+I*a*sinh(f*x+e)),x)
```

output
```
a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 + Piecewise((
((2*I*a*c**3*f**7 + 6*I*a*c**2*d*f**7*x + 6*I*a*c**2*d*f**6 + 6*I*a*c*d**2
*f**7*x**2 + 12*I*a*c*d**2*f**6*x + 12*I*a*c*d**2*f**5 + 2*I*a*d**3*f**7*x
**3 + 6*I*a*d**3*f**6*x**2 + 12*I*a*d**3*f**5*x + 12*I*a*d**3*f**4)*exp(-f
*x) + (2*I*a*c**3*f**7*exp(2*e) + 6*I*a*c**2*d*f**7*x*exp(2*e) - 6*I*a*c**
2*d*f**6*exp(2*e) + 6*I*a*c*d**2*f**7*x**2*exp(2*e) - 12*I*a*c*d**2*f**6*x
*exp(2*e) + 12*I*a*c*d**2*f**5*exp(2*e) + 2*I*a*d**3*f**7*x**3*exp(2*e) -
6*I*a*d**3*f**6*x**2*exp(2*e) + 12*I*a*d**3*f**5*x*exp(2*e) - 12*I*a*d**3*
f**4*exp(2*e))*exp(f*x))*exp(-e)/(4*f**8), Ne(f**8*exp(e), 0)), (x**4*(I*a
*d**3*exp(2*e) - I*a*d**3)*exp(-e)/8 + x**3*(I*a*c*d**2*exp(2*e) - I*a*c*d
**2)*exp(-e)/2 + x**2*(3*I*a*c**2*d*exp(2*e) - 3*I*a*c**2*d)*exp(-e)/4 + x
*(I*a*c**3*exp(2*e) - I*a*c**3)*exp(-e)/2, True))
```

3.96.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(88) = 176.

Time = 0.20 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.40

$$
\begin{aligned}
&\int (c+dx)^3(a+ia\sinh(e+fx))\,dx \\
&= \frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{3}{2}i\,ac^2d\left(\frac{(fxe^e-e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) \\
&\quad + \frac{3}{2}i\,acd^2\left(\frac{(f^2x^2e^e-2\,fxe^e+2\,e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2+2\,fx+2)e^{(-fx-e)}}{f^3}\right) \\
&\quad + \frac{1}{2}i\,ad^3\left(\frac{(f^3x^3e^e-3\,f^2x^2e^e+6\,fxe^e-6\,e^e)e^{(fx)}}{f^4} + \frac{(f^3x^3+3\,f^2x^2+6\,fx+6)e^{(-fx-e)}}{f^4}\right) \\
&\quad + \frac{i\,ac^3\cosh(fx+e)}{f}
\end{aligned}
$$

input
```
integrate((d*x+c)^3*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3/2*I*a*c^2*d*((
f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + 3/2*I*a*c*d^2*(
(f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-
f*x - e)/f^3) + 1/2*I*a*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6*f*x*e^e - 6*
e^e)*e^(f*x)/f^4 + (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x - e)/f^4) + I
*a*c^3*cosh(f*x + e)/f
```

3.96.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(88) = 176$.

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.67

$$\int (c+dx)^3(a+ia\sinh(e+fx))\,dx = \frac{1}{4}ad^3x^4 + acd^2x^3 + \frac{3}{2}ac^2dx^2 + ac^3x - \frac{(-i\,ad^3f^3x^3 - 3i\,acd^2f^3x^2 - 3i\,ac^2df^3x + 3i\,ad^3f^2x^2 - i\,ac^3f^3 + 6i\,acd^2f^2x + 3i\,ac^2df^2 - 6i\,ad^3fx - \cdots}{2\,f^4} - \frac{(-i\,ad^3f^3x^3 - 3i\,acd^2f^3x^2 - 3i\,ac^2df^3x - 3i\,ad^3f^2x^2 - i\,ac^3f^3 - 6i\,acd^2f^2x - 3i\,ac^2df^2 - 6i\,ad^3fx - \cdots}{2\,f^4}$$

input
```
integrate((d*x+c)^3*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x - 1/2*(-I*a*d^3*f^
3*x^3 - 3*I*a*c*d^2*f^3*x^2 - 3*I*a*c^2*d*f^3*x + 3*I*a*d^3*f^2*x^2 - I*a*
c^3*f^3 + 6*I*a*c*d^2*f^2*x + 3*I*a*c^2*d*f^2 - 6*I*a*d^3*f*x - 6*I*a*c*d^
2*f + 6*I*a*d^3)*e^(f*x + e)/f^4 - 1/2*(-I*a*d^3*f^3*x^3 - 3*I*a*c*d^2*f^3
*x^2 - 3*I*a*c^2*d*f^3*x - 3*I*a*d^3*f^2*x^2 - I*a*c^3*f^3 - 6*I*a*c*d^2*f
^2*x - 3*I*a*c^2*d*f^2 - 6*I*a*d^3*f*x - 6*I*a*c*d^2*f - 6*I*a*d^3)*e^(-f*
x - e)/f^4
```

3.96.9 Mupad [B] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.00

$$\int (c+dx)^3(a+ia\sinh(e+fx))\,dx = \frac{\cosh(e+f\,x)\,\left(a\,c^3\,f^2+6\,a\,c\,d^2\right)\,1\mathrm{i}}{f^3} - \frac{\sinh(e+f\,x)\,\left(a\,c^2\,d\,f^2+2\,a\,d^3\right)\,3\mathrm{i}}{f^4} + \frac{a\,d^3\,x^4}{4} + a\,c^3\,x + \frac{x\cosh(e+f\,x)\,\left(a\,c^2\,d\,f^2+2\,a\,d^3\right)\,3\mathrm{i}}{f^3} + \frac{3\,a\,c^2\,d\,x^2}{2} + a\,c\,d^2\,x^3 + \frac{a\,d^3\,x^3\cosh(e+f\,x)\,1\mathrm{i}}{f} - \frac{a\,d^3\,x^2\sinh(e+f\,x)\,3\mathrm{i}}{f^2} - \frac{a\,c\,d^2\,x\sinh(e+f\,x)\,6\mathrm{i}}{f^2} + \frac{a\,c\,d^2\,x^2\cosh(e+f\,x)\,3\mathrm{i}}{f}$$

input
```
int((a + a*sinh(e + f*x)*1i)*(c + d*x)^3,x)
```

output
```
(cosh(e + f*x)*(a*c^3*f^2 + 6*a*c*d^2)*1i)/f^3 - (sinh(e + f*x)*(2*a*d^3 +
 a*c^2*d*f^2)*3i)/f^4 + (a*d^3*x^4)/4 + a*c^3*x + (x*cosh(e + f*x)*(2*a*d^
3 + a*c^2*d*f^2)*3i)/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 + (a*d^3*x^3*co
sh(e + f*x)*1i)/f - (a*d^3*x^2*sinh(e + f*x)*3i)/f^2 - (a*c*d^2*x*sinh(e +
 f*x)*6i)/f^2 + (a*c*d^2*x^2*cosh(e + f*x)*3i)/f
```

3.96. $\int (c+dx)^3(a+ia\sinh(e+fx))\,dx$

3.97 $\int (c+dx)^2(a+ia\sinh(e+fx))\,dx$

3.97.1 Optimal result

Integrand size = 21, antiderivative size = 74

$$\int (c+dx)^2(a+ia\sinh(e+fx))\,dx = \frac{a(c+dx)^3}{3d}+\frac{2iad^2\cosh(e+fx)}{f^3}+\frac{ia(c+dx)^2\cosh(e+fx)}{f}-\frac{2iad(c+dx)\sinh(e+fx)}{f^2}$$

output
```
1/3*a*(d*x+c)^3/d+2*I*a*d^2*cosh(f*x+e)/f^3+I*a*(d*x+c)^2*cosh(f*x+e)/f-2*
I*a*d*(d*x+c)*sinh(f*x+e)/f^2
```

3.97.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int (c+dx)^2(a+ia\sinh(e+fx))\,dx = \frac{a(f^3x(3c^2+3cdx+d^2x^2)+3i(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)-6idf(c+dx)\sinh(e+fx))}{3f^3}$$

input
```
Integrate[(c + d*x)^2*(a + I*a*Sinh[e + f*x]),x]
```

output
```
(a*(f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + (3*I)*(c^2*f^2 + 2*c*d*f^2*x + d^2
*(2 + f^2*x^2))*Cosh[e + f*x] - (6*I)*d*f*(c + d*x)*Sinh[e + f*x]))/(3*f^3
)
```

3.97.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+ia\sinh(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2(a+a\sin(ie+ifx))dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a(c+dx)^2+ia(c+dx)^2\sinh(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2iad(c+dx)\sinh(e+fx)}{f^2}+\frac{ia(c+dx)^2\cosh(e+fx)}{f}+\frac{a(c+dx)^3}{3d}+\frac{2iad^2\cosh(e+fx)}{f^3}$$

input
```
Int[(c + d*x)^2*(a + I*a*Sinh[e + f*x]),x]
```

output
```
(a*(c + d*x)^3)/(3*d) + ((2*I)*a*d^2*Cosh[e + f*x])/f^3 + (I*a*(c + d*x)^2
*Cosh[e + f*x])/f - ((2*I)*a*d*(c + d*x)*Sinh[e + f*x])/f^2
```

3.97.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.97.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

method	result
parallelrisch	$\frac{\left(i\left((dx+c)^2 f^2+2d^2\right)\cosh(fx+e)-2if(dx+c)d\sinh(fx+e)+x\left(\frac{1}{3}d^2x^2+cdx+c^2\right)f^3+ic^2f^2+2id^2\right)a}{f^3}$
risch	$\frac{a\,d^2x^3}{3}+adc\,x^2+ax\,c^2+\frac{a\,c^3}{3d}+\frac{ia\left(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2fx-2cdf+2d^2\right)e^{fx+e}}{2f^3}+\frac{ia\left(d^2x^2f^2+2cd\,f^2\right.}{}$
parts	$\frac{a(dx+c)^3}{3d}+\frac{ia\left(\frac{d^2\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{2d^2e((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f^2}+\frac{2dc((}{}\right.}{f}$
derivativedivides	$\frac{\frac{d^2a(fx+e)^3}{3f^2}+\frac{id^2a\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{d^2ea(fx+e)^2}{f^2}-\frac{2id^2ea((fx+e)\cosh(fx+e)-\sinh(}{f^2}}{}$
default	$\frac{\frac{d^2a(fx+e)^3}{3f^2}+\frac{id^2a\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{d^2ea(fx+e)^2}{f^2}-\frac{2id^2ea((fx+e)\cosh(fx+e)-\sinh(}{f^2}}{}$

input
```
int((d*x+c)^2*(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
(I*((d*x+c)^2*f^2+2*d^2)*cosh(f*x+e)-2*I*f*(d*x+c)*d*sinh(f*x+e)+x*(1/3*d^
2*x^2+c*d*x+c^2)*f^3+I*c^2*f^2+2*I*d^2)*a/f^3
```

3.97.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(66) = 132$.

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.32

$$\int (c+dx)^2(a+ia\sinh(e+fx))\,dx$$
$$=\frac{(3i\,ad^2f^2x^2+3i\,ac^2f^2+6i\,acdf+6i\,ad^2-6\,(-i\,acdf^2-i\,ad^2f)x-3\,(-i\,ad^2f^2x^2-i\,ac^2f^2+2i\,acdf-}{6\,f^3}$$

input
```
integrate((d*x+c)^2*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
1/6*(3*I*a*d^2*f^2*x^2 + 3*I*a*c^2*f^2 + 6*I*a*c*d*f + 6*I*a*d^2 - 6*(-I*a
*c*d*f^2 - I*a*d^2*f)*x - 3*(-I*a*d^2*f^2*x^2 - I*a*c^2*f^2 + 2*I*a*c*d*f
- 2*I*a*d^2 + 2*(-I*a*c*d*f^2 + I*a*d^2*f)*x)*e^(2*f*x + 2*e) + 2*(a*d^2*f
^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x)*e^(f*x + e))*e^(-f*x - e)/f^3
```

3.97.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 314, normalized size of antiderivative = 4.24

$$\int (c+dx)^2(a+ia\sinh(e+fx))\,dx=ac^2x+acdx^2+\frac{ad^2x^3}{3}$$
$$+\begin{cases}\frac{((2iac^2f^5+4iacdf^5x+4iacdf^4+2iad^2f^5x^2+4iad^2f^4x+4iad^2f^3)e^{-fx}+(2iac^2f^5e^{2e}+4iacdf^5xe^{2e}-4iacdf^4e^{2e}+2iad^2f^5x^2e^{2e}-4iad^2f^4xe}{4f^6} \\ \frac{x^3(iad^2e^{2e}-iad^2)e^{-e}}{6}+\frac{x^2(iacde^{2e}-iacd)e^{-e}}{2}+\frac{x(iac^2e^{2e}-iac^2)e^{-e}}{2}\end{cases}$$

input
```
integrate((d*x+c)**2*(a+I*a*sinh(f*x+e)),x)
```

output
```
a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 + Piecewise((((2*I*a*c**2*f**5 + 4*I
*a*c*d*f**5*x + 4*I*a*c*d*f**4 + 2*I*a*d**2*f**5*x**2 + 4*I*a*d**2*f**4*x
+ 4*I*a*d**2*f**3)*exp(-f*x) + (2*I*a*c**2*f**5*exp(2*e) + 4*I*a*c*d*f**5*
x*exp(2*e) - 4*I*a*c*d*f**4*exp(2*e) + 2*I*a*d**2*f**5*x**2*exp(2*e) - 4*I
*a*d**2*f**4*x*exp(2*e) + 4*I*a*d**2*f**3*exp(2*e))*exp(f*x))*exp(-e)/(4*f
**6), Ne(f**6*exp(e), 0)), (x**3*(I*a*d**2*exp(2*e) - I*a*d**2)*exp(-e)/6
+ x**2*(I*a*c*d*exp(2*e) - I*a*c*d)*exp(-e)/2 + x*(I*a*c**2*exp(2*e) - I*a
*c**2)*exp(-e)/2, True))
```

3.97.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(66) = 132.

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.91

$$\int (c+dx)^2(a+ia\sinh(e+fx))\,dx$$
$$=\frac{1}{3}\,ad^2x^3+acdx^2+ac^2x+i\,acd\left(\frac{(fxe^e-e^e)e^{(fx)}}{f^2}+\frac{(fx+1)e^{(-fx-e)}}{f^2}\right)$$
$$+\frac{1}{2}i\,ad^2\left(\frac{(f^2x^2e^e-2\,fxe^e+2\,e^e)e^{(fx)}}{f^3}+\frac{(f^2x^2+2\,fx+2)e^{(-fx-e)}}{f^3}\right)$$
$$+\frac{i\,ac^2\cosh{(fx+e)}}{f}$$

input
```
integrate((d*x+c)^2*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + I*a*c*d*((f*x*e^e - e^e)*e^(f*x)/f^2
 + (f*x + 1)*e^(-f*x - e)/f^2) + 1/2*I*a*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2
*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + I*a*c^2*cosh
(f*x + e)/f
```

3.97.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(66) = 132.

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.03

$$\int (c+dx)^2(a+ia\sinh(e+fx))\,dx$$
$$=\frac{1}{3}\,ad^2x^3+acdx^2+ac^2x$$
$$-\frac{(-i\,ad^2f^2x^2-2i\,acdf^2x-i\,ac^2f^2+2i\,ad^2fx+2i\,acdf-2i\,ad^2)e^{(fx+e)}}{2\,f^3}$$
$$+\frac{(i\,ad^2f^2x^2+2i\,acdf^2x+i\,ac^2f^2+2i\,ad^2fx+2i\,acdf+2i\,ad^2)e^{(-fx-e)}}{2\,f^3}$$

input
```
integrate((d*x+c)^2*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x - 1/2*(-I*a*d^2*f^2*x^2 - 2*I*a*c*d*f^
2*x - I*a*c^2*f^2 + 2*I*a*d^2*f*x + 2*I*a*c*d*f - 2*I*a*d^2)*e^(f*x + e)/f
^3 + 1/2*(I*a*d^2*f^2*x^2 + 2*I*a*c*d*f^2*x + I*a*c^2*f^2 + 2*I*a*d^2*f*x
+ 2*I*a*c*d*f + 2*I*a*d^2)*e^(-f*x - e)/f^3
```

3.97.9 Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.59

$$\int (c+dx)^2(a+ia\sinh(e+fx))\,dx$$
$$= \frac{-\frac{a\,f\,\left(6\mathrm{i}\,x\sinh(e+f\,x)\,d^2+6\mathrm{i}\,c\sinh(e+f\,x)\,d\right)}{3}+\frac{a\,f^2\,\left(c^2\cosh(e+f\,x)\,3\mathrm{i}+d^2\,x^2\cosh(e+f\,x)\,3\mathrm{i}+c\,d\,x\cosh(e+f\,x)\,6\mathrm{i}\right)}{3}+a\,d^2\cosh(e+f\,x}{f^3}$$
$$+\frac{a\,(3\,c^2\,x+3\,c\,d\,x^2+d^2\,x^3)}{3}$$

input
```
int((a + a*sinh(e + f*x)*1i)*(c + d*x)^2,x)
```

output
```
((a*f^2*(c^2*cosh(e + f*x)*3i + d^2*x^2*cosh(e + f*x)*3i + c*d*x*cosh(e +
f*x)*6i))/3 - (a*f*(d^2*x*sinh(e + f*x)*6i + c*d*sinh(e + f*x)*6i))/3 + a*
d^2*cosh(e + f*x)*2i)/f^3 + (a*(3*c^2*x + d^2*x^3 + 3*c*d*x^2))/3
```

3.98 $\int (c+dx)(a+ia\sinh(e+fx))\,dx$

3.98.1 Optimal result

Integrand size = 19, antiderivative size = 50

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx=\frac{a(c+dx)^2}{2d}+\frac{ia(c+dx)\cosh(e+fx)}{f}-\frac{iad\sinh(e+fx)}{f^2}$$

output
```
1/2*a*(d*x+c)^2/d+I*a*(d*x+c)*cosh(f*x+e)/f-I*a*d*sinh(f*x+e)/f^2
```

3.98.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx =\frac{a(f^2x(2c+dx)+2if(c+dx)\cosh(e+fx)-2id\sinh(e+fx))}{2f^2}$$

input
```
Integrate[(c + d*x)*(a + I*a*Sinh[e + f*x]),x]
```

output
```
(a*(f^2*x*(2*c + d*x) + (2*I)*f*(c + d*x)*Cosh[e + f*x] - (2*I)*d*Sinh[e +
 f*x]))/(2*f^2)
```

3.98.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int (c+dx)(a+a\sin(ie+ifx))dx$$
$$\Big\downarrow \text{ 3798}$$
$$\int (a(c+dx)+ia(c+dx)\sinh(e+fx))dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{ia(c+dx)\cosh(e+fx)}{f}+\frac{a(c+dx)^2}{2d}-\frac{iad\sinh(e+fx)}{f^2}$$

input
```
Int[(c + d*x)*(a + I*a*Sinh[e + f*x]),x]
```

output
```
(a*(c + d*x)^2)/(2*d) + (I*a*(c + d*x)*Cosh[e + f*x])/f - (I*a*d*Sinh[e +
f*x])/f^2
```

3.98.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.98.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

method	result	size
parallelrisch	$\frac{a\left(if(dx+c)\cosh(fx+e)-i\sinh(fx+e)d+f\left(x\left(\frac{dx}{2}+c\right)f+ic\right)\right)}{f^2}$	48
risch	$\frac{ad\,x^2}{2}+acx+\frac{ia(dfx+cf-d)\mathrm{e}^{fx+e}}{2f^2}+\frac{ia(dfx+cf+d)\mathrm{e}^{-fx-e}}{2f^2}$	62
parts	$a\left(\frac{1}{2}d\,x^2+cx\right)+\frac{ia\left(\frac{d((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{de\cosh(fx+e)}{f}+c\cosh(fx+e)\right)}{f}$	69
derivativedivides	$\frac{\frac{da(fx+e)^2}{2f}+\frac{ida((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{idea\cosh(fx+e)}{f}+ac(fx+e)+iac\cosh(fx+e)}{f}$	96
default	$\frac{\frac{da(fx+e)^2}{2f}+\frac{ida((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{idea\cosh(fx+e)}{f}+ac(fx+e)+iac\cosh(fx+e)}{f}$	96

input
```
int((d*x+c)*(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
a*(I*f*(d*x+c)*cosh(f*x+e)-I*sinh(f*x+e)*d+f*(x*(1/2*d*x+c)*f+I*c))/f^2
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.62

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx$$
$$=\frac{\left(i\,adfx+i\,acf+i\,ad+(i\,adfx+i\,acf-i\,ad)e^{(2\,fx+2\,e)}+(adf^2x^2+2\,acf^2x)e^{(fx+e)}\right)e^{(-fx-e)}}{2\,f^2}$$

input
```
integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(I*a*d*f*x + I*a*c*f + I*a*d + (I*a*d*f*x + I*a*c*f - I*a*d)*e^(2*f*x
+ 2*e) + (a*d*f^2*x^2 + 2*a*c*f^2*x)*e^(f*x + e))*e^(-f*x - e)/f^2
```

3.98.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.24

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx$$
$$= acx + \frac{adx^2}{2} + \begin{cases} \frac{((2iacf^3+2iadf^3x+2iadf^2)e^{-fx}+(2iacf^3e^{2e}+2iadf^3xe^{2e}-2iadf^2e^{2e})e^{fx})e^{-e}}{4f^4} & \text{for } f^4e^e \neq 0 \\ \frac{x^2(iade^{2e}-iad)e^{-e}}{4} + \frac{x(iace^{2e}-iac)e^{-e}}{2} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x)
```

output
```
a*c*x + a*d*x**2/2 + Piecewise((((2*I*a*c*f**3 + 2*I*a*d*f**3*x + 2*I*a*d*
f**2)*exp(-f*x) + (2*I*a*c*f**3*exp(2*e) + 2*I*a*d*f**3*x*exp(2*e) - 2*I*a
*d*f**2*exp(2*e))*exp(f*x))*exp(-e)/(4*f**4), Ne(f**4*exp(e), 0)), (x**2*(
I*a*d*exp(2*e) - I*a*d)*exp(-e)/4 + x*(I*a*c*exp(2*e) - I*a*c)*exp(-e)/2,
True))
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx = \frac{1}{2}adx^2 + acx + \frac{1}{2}i\,ad\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{i\,ac\cosh(fx+e)}{f}$$

input
```
integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + a*c*x + 1/2*I*a*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e
^(-f*x - e)/f^2) + I*a*c*cosh(f*x + e)/f
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx = \frac{1}{2}\,adx^2 + acx - \frac{(-i\,adfx - i\,acf + i\,ad)e^{(fx+e)}}{2\,f^2} - \frac{(-i\,adfx - i\,acf - i\,ad)e^{(-fx-e)}}{2\,f^2}$$

input
```
integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
1/2*a*d*x^2 + a*c*x - 1/2*(-I*a*d*f*x - I*a*c*f + I*a*d)*e^(f*x + e)/f^2 -
 1/2*(-I*a*d*f*x - I*a*c*f - I*a*d)*e^(-f*x - e)/f^2
```

3.98.9 Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int (c+dx)(a+ia\sinh(e+fx))\,dx = \frac{\frac{a\,f\,(c\cosh(e+f\,x)\,2\mathrm{i}+d\,x\cosh(e+f\,x)\,2\mathrm{i})}{2} - a\,d\sinh(e+f\,x)\,1\mathrm{i}}{f^2} + \frac{a\,(d\,x^2+2\,c\,x)}{2}$$

input
```
int((a + a*sinh(e + f*x)*1i)*(c + d*x),x)
```

output
```
((a*f*(c*cosh(e + f*x)*2i + d*x*cosh(e + f*x)*2i))/2 - a*d*sinh(e + f*x)*1
i)/f^2 + (a*(2*c*x + d*x^2))/2
```

3.99 $\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx$

3.99.1 Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx=\frac{a\log(c+dx)}{d}+\frac{ia\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{ia\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d}$$

output
```
a*ln(d*x+c)/d+I*a*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d-I*a*Chi(c*f/d+f*x)*sinh(
-e+c*f/d)/d
```

3.99.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx=\frac{a\left(\log(c+dx)+i\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+i\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{d}$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])/(c + d*x),x]
```

output
```
(a*(Log[c + d*x] + I*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + I*Cosh[
e - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/d
```

3.99.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+a\sin(ie+ifx)}{c+dx}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a}{c+dx}+\frac{ia\sinh(e+fx)}{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ia\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{ia\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

input
```
Int[(a + I*a*Sinh[e + f*x])/(c + d*x),x]
```

output
```
(a*Log[c + d*x])/d + (I*a*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d
 + (I*a*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

3.99.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.99.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.37

method	result	size
risch	$\frac{a\ln(dx+c)}{d}+\frac{ia\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{Ei}_1\left(fx+e+\frac{cf-de}{d}\right)}{2d}-\frac{ia\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{Ei}_1\left(-fx-e-\frac{cf-de}{d}\right)}{2d}$	96

input
```
int((a+I*a*sinh(f*x+e))/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*ln(d*x+c)/d+1/2*I*a/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*I*a/d
*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx=\frac{i\,a\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}-i\,a\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}+2\,a\log\left(\frac{dx+c}{d}\right)}{2\,d}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(I*a*Ei((d*f*x + c*f)/d)*e^((d*e - c*f)/d) - I*a*Ei(-(d*f*x + c*f)/d)*
e^(-(d*e - c*f)/d) + 2*a*log((d*x + c)/d))/d
```

3.99.6 Sympy [F]

$$\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx = ia\left(\int \left(-\frac{i}{c+dx}\right)dx + \int \frac{\sinh(e+fx)}{c+dx}\,dx\right)$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c),x)
```

output
```
I*a*(Integral(-I/(c + d*x), x) + Integral(sinh(e + f*x)/(c + d*x), x))
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx = \frac{1}{2}i\,a\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right) + \frac{a\log(dx+c)}{d}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*I*a*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(e - c*f/d)
*exp_integral_e(1, -(d*x + c)*f/d)/d) + a*log(d*x + c)/d
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

$$\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx = -\frac{-i\,a\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} + i\,a\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} - 2\,a\log(dx+c)}{2\,d}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c),x, algorithm="giac")
```

output
```
-1/2*(-I*a*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + I*a*Ei(-(d*f*x + c*f)/d)*e^
(-e + c*f/d) - 2*a*log(d*x + c))/d
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx=\int \frac{a+a\sinh(e+f\,x)\;1\mathrm{i}}{c+d\,x}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)/(c + d*x),x)
```

output
```
int((a + a*sinh(e + f*x)*1i)/(c + d*x), x)
```

3.100 $\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx$

3.100.1 Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx=-\frac{a}{d(c+dx)}+\frac{iaf\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{ia\sinh(e+fx)}{d(c+dx)}+\frac{iaf\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(\frac{cf}{d}+fx\right)}{d^2}$$

output
```
-a/d/(d*x+c)+I*a*f*Chi(c*f/d+f*x)*cosh(-e+c*f/d)/d^2-I*a*f*Shi(c*f/d+f*x)*
sinh(-e+c*f/d)/d^2-I*a*sinh(f*x+e)/d/(d*x+c)
```

3.100.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx=\frac{ia\left(f(c+dx)\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(f\left(\frac{c}{d}+x\right)\right)-d(-i+\sinh(e+fx))+f(c+dx)\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(f\left(\frac{c}{d}+x\right.\right.\right.}{d^2(c+dx)}$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])/(c + d*x)^2,x]
```

output
```
(I*a*(f*(c + d*x)*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] - d*(-I + Si
nh[e + f*x]) + f*(c + d*x)*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/(
d^2*(c + d*x))
```

3.100.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{a+a\sin(ie+ifx)}{(c+dx)^2}dx$$
$$\Big\downarrow \ 3798$$
$$\int \left(\frac{a}{(c+dx)^2}+\frac{ia\sinh(e+fx)}{(c+dx)^2}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{iaf\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{iaf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{ia\sinh(e+fx)}{d(c+dx)}-\frac{a}{d(c+dx)}$$

input
```
Int[(a + I*a*Sinh[e + f*x])/(c + d*x)^2,x]
```

output
```
-(a/(d*(c + d*x))) + (I*a*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])
/d^2 - (I*a*Sinh[e + f*x])/(d*(c + d*x)) + (I*a*f*Sinh[e - (c*f)/d]*SinhIn
tegral[(c*f)/d + f*x])/d^2
```

3.100.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.100.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.61

method	result	size
risch	$-\frac{a}{d(dx+c)}+\frac{iaf\,\mathrm{e}^{-fx-e}}{2d(dfx+cf)}-\frac{iaf\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{Ei}_1\left(fx+e+\frac{cf-de}{d}\right)}{2d^2}-\frac{iaf\,\mathrm{e}^{fx+e}}{2d^2\left(\frac{cf}{d}+fx\right)}-\frac{iaf\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{Ei}_1\left(-fx-e-\frac{cf-de}{d}\right)}{2d^2}$	153

input
```
int((a+I*a*sinh(f*x+e))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/(d*x+c)+1/2*I*a*f*exp(-f*x-e)/d/(d*f*x+c*f)-1/2*I*a*f/d^2*exp((c*f-d*
e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*I*a*f/d^2*exp(f*x+e)/(c*f/d+f*x)-1/2*I*a
*f/d^2*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.41

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx$$
$$=\frac{\left(-i\,ade^{(2\,fx+2\,e)}+i\,ad+\left((i\,adfx+i\,acf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}+(i\,adfx+i\,acf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}-\right.\right.}{2\,(d^3x+cd^2)}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(-I*a*d*e^(2*f*x + 2*e) + I*a*d + ((I*a*d*f*x + I*a*c*f)*Ei((d*f*x + c
*f)/d)*e^((d*e - c*f)/d) + (I*a*d*f*x + I*a*c*f)*Ei(-(d*f*x + c*f)/d)*e^(-
(d*e - c*f)/d) - 2*a*d)*e^(f*x + e))*e^(-f*x - e)/(d^3*x + c*d^2)
```

3.100.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c)**2,x)
```

output
```
Timed out
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx = \frac{1}{2}ia\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right) - \frac{a}{d^2x+cd}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*I*a*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/((d*x + c)*d) - e
^(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d)) - a/(d^2*x +
 c*d)
```

3.100.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(91) = 182$.

Time = 0.28 (sec) , antiderivative size = 630, normalized size of antiderivative = 6.63

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx$$
$$= \frac{1}{2}ia\left(\frac{\left((dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^2\text{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}-def^2\text{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+\right.}{d}\right.\right.}{\left((dx+c)d^4\left(\frac{de}{dx+c}-\frac{cf}{dx+c}\right.\right.}\right.$$
$$-\frac{a}{(dx+c)d}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*I*a*(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*
(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - d*
e*f^2*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^
((d*e - c*f)/d) + c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)
- d*e + c*f)/d)*e^((d*e - c*f)/d) - d*f^2*e^((d*x + c)*(d*e/(d*x + c) - c*
f/(d*x + c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x + c) - c*f/(d*x + c) +
f) - d^5*e + c*d^4*f)*f) + ((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*
f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(
-(d*e - c*f)/d) - d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) +
f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + c*f^3*Ei(-((d*x + c)*(d*e/(d*x + c
) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + d*f^2*e^(-(d*x
 + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*
x + c) - c*f/(d*x + c) + f) - d^5*e + c*d^4*f)*f)) - a/((d*x + c)*d)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx = \int \frac{a+a\sinh(e+f\,x)\,1\mathrm{i}}{(c+d\,x)^2}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)/(c + d*x)^2,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)/(c + d*x)^2, x)
```

3.101 $\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3}\,dx$

3.101.1 Optimal result

Integrand size = 21, antiderivative size = 131

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3}\,dx = -\frac{a}{2d(c+dx)^2}-\frac{iaf\cosh(e+fx)}{2d^2(c+dx)} + \frac{iaf^2\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{ia\sinh(e+fx)}{2d(c+dx)^2} + \frac{iaf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{2d^3}$$

output
```
-1/2*a/d/(d*x+c)^2-1/2*I*a*f*cosh(f*x+e)/d^2/(d*x+c)+1/2*I*a*f^2*cosh(-e+c
*f/d)*Shi(c*f/d+f*x)/d^3-1/2*I*a*f^2*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d^3-1/2
*I*a*sinh(f*x+e)/d/(d*x+c)^2
```

3.101.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3}\,dx = \frac{ia\left(f^2(c+dx)^2\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)-d(f(c+dx)\cosh(e+fx)+d(-i+\sinh(e+fx)))+f^2(c\right.}{2d^3(c+dx)^2}$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])/(c + d*x)^3,x]
```

output
```
((I/2)*a*(f^2*(c + d*x)^2*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] - d*
(f*(c + d*x)*Cosh[e + f*x] + d*(-I + Sinh[e + f*x])) + f^2*(c + d*x)^2*Cos
h[e - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/(d^3*(c + d*x)^2)
```

3.101.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{a+a\sin(ie+ifx)}{(c+dx)^3}dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(\frac{a}{(c+dx)^3}+\frac{ia\sinh(e+fx)}{(c+dx)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{iaf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{iaf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{iaf\cosh(e+fx)}{2d^2(c+dx)}-\frac{ia\sinh(e+fx)}{2d(c+dx)^2}-\frac{a}{2d(c+dx)^2}$$

input
```
Int[(a + I*a*Sinh[e + f*x])/(c + d*x)^3,x]
```

output
```
-1/2*a/(d*(c + d*x)^2) - ((I/2)*a*f*Cosh[e + f*x])/(d^2*(c + d*x)) + ((I/2
)*a*f^2*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - ((I/2)*a*Sinh
[e + f*x])/(d*(c + d*x)^2) + ((I/2)*a*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(
c*f)/d + f*x])/d^3
```

3.101.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.101.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(119) = 238$.

Time = 1.07 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.31

method	result
risch	$-\frac{a}{2d(dx+c)^2}-\frac{ia\,f^3\mathrm{e}^{-fx-e}x}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{ia\,f^3\mathrm{e}^{-fx-e}c}{4d^2(d^2x^2f^2+2cd\,f^2x+c^2f^2)}+\frac{ia\,f^2\mathrm{e}^{-fx-e}}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)}+\frac{ia\,f^2\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{Ei}}{4}$

input
```
int((a+I*a*sinh(f*x+e))/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/(d*x+c)^2-1/4*I*a*f^3*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*
f^2)*x-1/4*I*a*f^3*exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/4
*I*a*f^2*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/4*I*a*f^2/d^3*e
xp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/4*I*a*f^2/d^3*exp(f*x+e)/(c*f/d+
f*x)^2-1/4*I*a*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)-1/4*I*a*f^2/d^3*exp(-(c*f-d*
e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.69

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3}\,dx$$
$$= \frac{\left(-i\,ad^2fx - i\,acdf + i\,ad^2 + (-i\,ad^2fx - i\,acdf - i\,ad^2)e^{(2\,fx+2\,e)} - \left(2\,ad^2 - (i\,ad^2f^2x^2 + 2i\,acdf^2x + i\right.\right.}{4\,(d^5x^2 + 2\,c}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*(-I*a*d^2*f*x - I*a*c*d*f + I*a*d^2 + (-I*a*d^2*f*x - I*a*c*d*f - I*a*
d^2)*e^(2*f*x + 2*e) - (2*a*d^2 - (I*a*d^2*f^2*x^2 + 2*I*a*c*d*f^2*x + I*a
*c^2*f^2)*Ei((d*f*x + c*f)/d)*e^((d*e - c*f)/d) - (-I*a*d^2*f^2*x^2 - 2*I*
a*c*d*f^2*x - I*a*c^2*f^2)*Ei(-(d*f*x + c*f)/d)*e^(-(d*e - c*f)/d))*e^(f*x
 + e))*e^(-f*x - e)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.101.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c)**3,x)
```

output
```
Timed out
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3}\,dx = \frac{1}{2}i\,a\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right) - \frac{a}{2\,(d^3x^2 + 2\,cd^2x + c^2d)}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/2*I*a*(e^(-e + c*f/d)*exp_integral_e(3, (d*x + c)*f/d)/((d*x + c)^2*d) -
 e^(e - c*f/d)*exp_integral_e(3, -(d*x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a/
(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

3.101.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(115) = 230$.

Time = 0.27 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.46

$$\int \frac{a + ia\sinh(e + fx)}{(c + dx)^3}\,dx$$
$$= \frac{i\,ad^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} - i\,ad^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} + 2i\,acdf^2x\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} - 2i\,acdf^2x\mathrm{E}}{}$$

input
```
integrate((a+I*a*sinh(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(I*a*d^2*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - I*a*d^2*f^2*x^2*E
i(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + 2*I*a*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e
^(e - c*f/d) - 2*I*a*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + I*a*c
^2*f^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - I*a*c^2*f^2*Ei(-(d*f*x + c*f)/d
)*e^(-e + c*f/d) - I*a*d^2*f*x*e^(f*x + e) - I*a*d^2*f*x*e^(-f*x - e) - I*
a*c*d*f*e^(f*x + e) - I*a*c*d*f*e^(-f*x - e) - I*a*d^2*e^(f*x + e) + I*a*d
^2*e^(-f*x - e) - 2*a*d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + ia\sinh(e + fx)}{(c + dx)^3}\,dx = \int \frac{a + a\sinh(e + f\,x)\,1\mathrm{i}}{(c + d\,x)^3}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)/(c + d*x)^3,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)/(c + d*x)^3, x)
```

3.102 $\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx$

3.102.1 Optimal result

Integrand size = 23, antiderivative size = 245

$$\begin{aligned}\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx = {} & \frac{3a^2cd^2x}{4f^2}+\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d} \\ & +\frac{12ia^2d^2(c+dx)\cosh(e+fx)}{f^3} \\ & +\frac{2ia^2(c+dx)^3\cosh(e+fx)}{f} \\ & -\frac{12ia^2d^3\sinh(e+fx)}{f^4} \\ & -\frac{6ia^2d(c+dx)^2\sinh(e+fx)}{f^2} \\ & -\frac{3a^2d^2(c+dx)\cosh(e+fx)\sinh(e+fx)}{4f^3} \\ & -\frac{a^2(c+dx)^3\cosh(e+fx)\sinh(e+fx)}{2f} \\ & +\frac{3a^2d^3\sinh^2(e+fx)}{8f^4} \\ & +\frac{3a^2d(c+dx)^2\sinh^2(e+fx)}{4f^2}\end{aligned}$$

output
```
3/4*a^2*c*d^2*x/f^2+3/8*a^2*d^3*x^2/f^2+3/8*a^2*(d*x+c)^4/d+12*I*a^2*d^2*(
d*x+c)*cosh(f*x+e)/f^3+2*I*a^2*(d*x+c)^3*cosh(f*x+e)/f-12*I*a^2*d^3*sinh(f
*x+e)/f^4-6*I*a^2*d*(d*x+c)^2*sinh(f*x+e)/f^2-3/4*a^2*d^2*(d*x+c)*cosh(f*x
+e)*sinh(f*x+e)/f^3-1/2*a^2*(d*x+c)^3*cosh(f*x+e)*sinh(f*x+e)/f+3/8*a^2*d^
3*sinh(f*x+e)^2/f^4+3/4*a^2*d*(d*x+c)^2*sinh(f*x+e)^2/f^2
```

3.102.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.90

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx$$
$$= \frac{a^2(6f^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+32if(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\cosh(e+fx)+3d}{}$$

input
```
Integrate[(c + d*x)^3*(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(a^2*(6*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + (32*I)*f*(c +
d*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f*x] + 3*d*(2*c^
2*f^2 + 4*c*d*f^2*x + d^2*(1 + 2*f^2*x^2))*Cosh[2*(e + f*x)] - (96*I)*d*(c
^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x] - 2*f*(c + d*x)*(2
*c^2*f^2 + 4*c*d*f^2*x + d^2*(3 + 2*f^2*x^2))*Sinh[2*(e + f*x)]))/(16*f^4)
```

3.102.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx$$
$$\Big\downarrow \ 3042$$
$$\int (c+dx)^3(a+a\sin(ie+ifx))^2dx$$
$$\Big\downarrow \ 3798$$

$$\int \left(-a^2(c+dx)^3\sinh^2(e+fx)+2ia^2(c+dx)^3\sinh(e+fx)+a^2(c+dx)^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{12ia^2d^2(c+dx)\cosh(e+fx)}{f^3}-\frac{3a^2d^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{4f^3}+\frac{3a^2d(c+dx)^2\sinh^2(e+fx)}{4f^2}-\frac{6ia^2d(c+dx)^2\sinh(e+fx)}{f^2}+\frac{2ia^2(c+dx)^3\cosh(e+fx)}{f}-\frac{a^2(c+dx)^3\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{3a^2d(c+dx)^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}+\frac{3a^2d^3\sinh^2(e+fx)}{8f^4}-\frac{12ia^2d^3\sinh(e+fx)}{f^4}$$

input
```
Int[(c + d*x)^3*(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(3*a^2*d*(c + d*x)^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8*d) + ((12*I)*a^2*d^
2*(c + d*x)*Cosh[e + f*x])/f^3 + ((2*I)*a^2*(c + d*x)^3*Cosh[e + f*x])/f -
 ((12*I)*a^2*d^3*Sinh[e + f*x])/f^4 - ((6*I)*a^2*d*(c + d*x)^2*Sinh[e + f*
x])/f^2 - (3*a^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(4*f^3) - (a^2
*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) + (3*a^2*d^3*Sinh[e + f*x]
^2)/(8*f^4) + (3*a^2*d*(c + d*x)^2*Sinh[e + f*x]^2)/(4*f^2)
```

3.102.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.102.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.78

method	result
parallelrisch	$\frac{2\left(-\frac{f(dx+c)\left((dx+c)^2f^2+\frac{3d^2}{2}\right)\sinh(2fx+2e)}{8}+\frac{3\left((dx+c)^2f^2+\frac{d^2}{2}\right)d\cosh(2fx+2e)}{16}+if(dx+c)\left((dx+c)^2f^2+6d^2\right)\cosh(fx+\right.}{f^4}$
risch	$\frac{3a^2d^3x^4}{8}+\frac{3a^2c\,d^2x^3}{2}+\frac{9a^2c^2d\,x^2}{4}+\frac{3c^3a^2x}{2}+\frac{3a^2c^4}{8d}-\frac{a^2\left(4d^3x^3f^3+12c\,d^2f^3x^2+12c^2d\,f^3x-6d^3f^2x^2+4c^3f^3\right.}{32f}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/8*f*(d*x+c)*((d*x+c)^2*f^2+3/2*d^2)*sinh(2*f*x+2*e)+3/16*((d*x+c)^2*
f^2+1/2*d^2)*d*cosh(2*f*x+2*e)+I*f*(d*x+c)*((d*x+c)^2*f^2+6*d^2)*cosh(f*x+
e)-3*I*d*((d*x+c)^2*f^2+2*d^2)*sinh(f*x+e)+3/4*x*(1/2*d^2*x^2+c*d*x+c^2)*(
1/2*d*x+c)*f^4+I*c^3*f^3-3/16*c^2*d*f^2+6*I*c*d^2*f-3/32*d^3)*a^2/f^4
```

3.102.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 596 vs. 2(223) = 446.

Time = 0.26 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.43

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx$$
$$=\frac{(4\,a^2d^3f^3x^3+4\,a^2c^3f^3+6\,a^2c^2df^2+6\,a^2cd^2f+3\,a^2d^3+6\,(2\,a^2cd^2f^3+a^2d^3f^2)x^2+6\,(2\,a^2c^2df^3+2\,a^2cd}{}$$

input
```
integrate((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/32*(4*a^2*d^3*f^3*x^3 + 4*a^2*c^3*f^3 + 6*a^2*c^2*d*f^2 + 6*a^2*c*d^2*f
+ 3*a^2*d^3 + 6*(2*a^2*c*d^2*f^3 + a^2*d^3*f^2)*x^2 + 6*(2*a^2*c^2*d*f^3 +
 2*a^2*c*d^2*f^2 + a^2*d^3*f)*x - (4*a^2*d^3*f^3*x^3 + 4*a^2*c^3*f^3 - 6*a
^2*c^2*d*f^2 + 6*a^2*c*d^2*f - 3*a^2*d^3 + 6*(2*a^2*c*d^2*f^3 - a^2*d^3*f^
2)*x^2 + 6*(2*a^2*c^2*d*f^3 - 2*a^2*c*d^2*f^2 + a^2*d^3*f)*x)*e^(4*f*x + 4
*e) - 32*(-I*a^2*d^3*f^3*x^3 - I*a^2*c^3*f^3 + 3*I*a^2*c^2*d*f^2 - 6*I*a^2
*c*d^2*f + 6*I*a^2*d^3 + 3*(-I*a^2*c*d^2*f^3 + I*a^2*d^3*f^2)*x^2 + 3*(-I*
a^2*c^2*d*f^3 + 2*I*a^2*c*d^2*f^2 - 2*I*a^2*d^3*f)*x)*e^(3*f*x + 3*e) + 12
*(a^2*d^3*f^4*x^4 + 4*a^2*c*d^2*f^4*x^3 + 6*a^2*c^2*d*f^4*x^2 + 4*a^2*c^3*
f^4*x)*e^(2*f*x + 2*e) - 32*(-I*a^2*d^3*f^3*x^3 - I*a^2*c^3*f^3 - 3*I*a^2*
c^2*d*f^2 - 6*I*a^2*c*d^2*f - 6*I*a^2*d^3 + 3*(-I*a^2*c*d^2*f^3 - I*a^2*d^
3*f^2)*x^2 + 3*(-I*a^2*c^2*d*f^3 - 2*I*a^2*c*d^2*f^2 - 2*I*a^2*d^3*f)*x)*e
^(f*x + e))*e^(-2*f*x - 2*e)/f^4
```

3.102.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 1134, normalized size of antiderivative = 4.63

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx = \frac{3a^2c^3x}{2} + \frac{9a^2c^2dx^2}{4} + \frac{3a^2cd^2x^3}{2} + \frac{3a^2d^3x^4}{8}$$
$$+ \begin{cases} \frac{((128a^2c^3f^{15}e^e+384a^2c^2df^{15}xe^e+192a^2c^2df^{14}e^e+384a^2cd^2f^{15}x^2e^e+384a^2cd^2f^{14}xe^e+192a^2cd^2f^{13}e^e+128a^2d^3f^{15}x^3e^e+192a^2d^3f^{14}x^2 \cdots}{\cdots} \\ \frac{x^4(-a^2d^3e^{4e}+4ia^2d^3e^{3e}-4ia^2d^3e^e-a^2d^3)e^{-2e}}{16} + \frac{x^3(-a^2cd^2e^{4e}+4ia^2cd^2e^{3e}-4ia^2cd^2e^e-a^2cd^2)e^{-2e}}{4} + \frac{x^2(-3a^2c^2de^{4e}+12ia^2c^2de^{3e} \cdots}{8 \cdots} \end{cases}$$

input
```
integrate((d*x+c)**3*(a+I*a*sinh(f*x+e))**2,x)
```

output
```
3*a**2*c**3*x/2 + 9*a**2*c**2*d*x**2/4 + 3*a**2*c*d**2*x**3/2 + 3*a**2*d**
3*x**4/8 + Piecewise((((128*a**2*c**3*f**15*exp(e) + 384*a**2*c**2*d*f**15
*x*exp(e) + 192*a**2*c**2*d*f**14*exp(e) + 384*a**2*c*d**2*f**15*x**2*exp(
e) + 384*a**2*c*d**2*f**14*x*exp(e) + 192*a**2*c*d**2*f**13*exp(e) + 128*a
**2*d**3*f**15*x**3*exp(e) + 192*a**2*d**3*f**14*x**2*exp(e) + 192*a**2*d*
*3*f**13*x*exp(e) + 96*a**2*d**3*f**12*exp(e))*exp(-2*f*x) + (-128*a**2*c*
*3*f**15*exp(5*e) - 384*a**2*c**2*d*f**15*x*exp(5*e) + 192*a**2*c**2*d*f**
14*exp(5*e) - 384*a**2*c*d**2*f**15*x**2*exp(5*e) + 384*a**2*c*d**2*f**14*
x*exp(5*e) - 192*a**2*c*d**2*f**13*exp(5*e) - 128*a**2*d**3*f**15*x**3*exp
(5*e) + 192*a**2*d**3*f**14*x**2*exp(5*e) - 192*a**2*d**3*f**13*x*exp(5*e)
 + 96*a**2*d**3*f**12*exp(5*e))*exp(2*f*x) + (1024*I*a**2*c**3*f**15*exp(2
*e) + 3072*I*a**2*c**2*d*f**15*x*exp(2*e) + 3072*I*a**2*c**2*d*f**14*exp(2
*e) + 3072*I*a**2*c*d**2*f**15*x**2*exp(2*e) + 6144*I*a**2*c*d**2*f**14*x*
exp(2*e) + 6144*I*a**2*c*d**2*f**13*exp(2*e) + 1024*I*a**2*d**3*f**15*x**3
*exp(2*e) + 3072*I*a**2*d**3*f**14*x**2*exp(2*e) + 6144*I*a**2*d**3*f**13*
x*exp(2*e) + 6144*I*a**2*d**3*f**12*exp(2*e))*exp(-f*x) + (1024*I*a**2*c**
3*f**15*exp(4*e) + 3072*I*a**2*c**2*d*f**15*x*exp(4*e) - 3072*I*a**2*c**2*
d*f**14*exp(4*e) + 3072*I*a**2*c*d**2*f**15*x**2*exp(4*e) - 6144*I*a**2*c*
d**2*f**14*x*exp(4*e) + 6144*I*a**2*c*d**2*f**13*exp(4*e) + 1024*I*a**2*d*
*3*f**15*x**3*exp(4*e) - 3072*I*a**2*d**3*f**14*x**2*exp(4*e) + 6144*I*...
```

3.102.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(223) = 446$.

Time = 0.23 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.14

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx = \frac{1}{4}\,a^2d^3x^4 + a^2cd^2x^3 + \frac{3}{2}\,a^2c^2dx^2$$
$$+\frac{3}{16}\left(4\,x^2 - \frac{\left(2\,fxe^{(2\,e)} - e^{(2\,e)}\right)e^{(2\,fx)}}{f^2} + \frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)a^2c^2d$$
$$+\frac{1}{16}\left(8\,x^3 - \frac{3\left(2\,f^2x^2e^{(2\,e)} - 2\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(2\,fx)}}{f^3} + \frac{3\,(2\,f^2x^2+2\,fx+1)e^{(-2\,fx-2\,e)}}{f^3}\right)a^2cd^2$$
$$+\frac{1}{32}\left(4\,x^4 - \frac{\left(4\,f^3x^3e^{(2\,e)} - 6\,f^2x^2e^{(2\,e)} + 6\,fxe^{(2\,e)} - 3\,e^{(2\,e)}\right)e^{(2\,fx)}}{f^4} + \frac{(4\,f^3x^3+6\,f^2x^2+6\,fx+3)e^{(-2\,fx}}{f^4}\right.$$
$$+\frac{1}{8}\,a^2c^3\left(4\,x - \frac{e^{(2\,fx+2\,e)}}{f} + \frac{e^{(-2\,fx-2\,e)}}{f}\right) + a^2c^3x$$
$$+3i\,a^2c^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right)$$
$$+3i\,a^2cd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2+2\,fx+2)e^{(-fx-e)}}{f^3}\right)$$
$$+i\,a^2d^3\left(\frac{(f^3x^3e^e - 3\,f^2x^2e^e + 6\,fxe^e - 6\,e^e)e^{(fx)}}{f^4} + \frac{(f^3x^3+3\,f^2x^2+6\,fx+6)e^{(-fx-e)}}{f^4}\right)$$
$$+\frac{2i\,a^2c^3\cosh{(fx+e)}}{f}$$

input
```
integrate((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + 3/16*(4*x^2 - (2*f*x
*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*
c^2*d + 1/16*(8*x^3 - 3*(2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2
*f*x)/f^3 + 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*a^2*c*d^2 + 1/
32*(4*x^4 - (4*f^3*x^3*e^(2*e) - 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) - 3*e^(
2*e))*e^(2*f*x)/f^4 + (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x - 2*e)
/f^4)*a^2*d^3 + 1/8*a^2*c^3*(4*x - e^(2*f*x + 2*e)/f + e^(-2*f*x - 2*e)/f)
 + a^2*c^3*x + 3*I*a^2*c^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-
f*x - e)/f^2) + 3*I*a^2*c*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f
^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + I*a^2*d^3*((f^3*x^3*e^e - 3
*f^2*x^2*e^e + 6*f*x*e^e - 6*e^e)*e^(f*x)/f^4 + (f^3*x^3 + 3*f^2*x^2 + 6*f
*x + 6)*e^(-f*x - e)/f^4) + 2*I*a^2*c^3*cosh(f*x + e)/f
```

3.102. $\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx$

3.102.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(223) = 446$.

Time = 0.29 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.37

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx = \frac{3}{8}a^2d^3x^4 + \frac{3}{2}a^2cd^2x^3 + \frac{9}{4}a^2c^2dx^2 + \frac{3}{2}a^2c^3x$$
$$- \frac{(4a^2d^3f^3x^3 + 12a^2cd^2f^3x^2 + 12a^2c^2df^3x - 6a^2d^3f^2x^2 + 4a^2c^3f^3 - 12a^2cd^2f^2x - 6a^2c^2df^2 + 6a^2d^3f}{32f^4}$$
$$+ \frac{(ia^2d^3f^3x^3 + 3ia^2cd^2f^3x^2 + 3ia^2c^2df^3x - 3ia^2d^3f^2x^2 + ia^2c^3f^3 - 6ia^2cd^2f^2x - 3ia^2c^2df^2 + 6ia^2d^3}{f^4}$$
$$+ \frac{(ia^2d^3f^3x^3 + 3ia^2cd^2f^3x^2 + 3ia^2c^2df^3x + 3ia^2d^3f^2x^2 + ia^2c^3f^3 + 6ia^2cd^2f^2x + 3ia^2c^2df^2 + 6ia^2d^3}{f^4}$$
$$+ \frac{(4a^2d^3f^3x^3 + 12a^2cd^2f^3x^2 + 12a^2c^2df^3x + 6a^2d^3f^2x^2 + 4a^2c^3f^3 + 12a^2cd^2f^2x + 6a^2c^2df^2 + 6a^2d^3f}{32f^4}$$

input
```
integrate((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
3/8*a^2*d^3*x^4 + 3/2*a^2*c*d^2*x^3 + 9/4*a^2*c^2*d*x^2 + 3/2*a^2*c^3*x -
1/32*(4*a^2*d^3*f^3*x^3 + 12*a^2*c*d^2*f^3*x^2 + 12*a^2*c^2*d*f^3*x - 6*a^
2*d^3*f^2*x^2 + 4*a^2*c^3*f^3 - 12*a^2*c*d^2*f^2*x - 6*a^2*c^2*d*f^2 + 6*a
^2*d^3*f*x + 6*a^2*c*d^2*f - 3*a^2*d^3)*e^(2*f*x + 2*e)/f^4 + (I*a^2*d^3*f
^3*x^3 + 3*I*a^2*c*d^2*f^3*x^2 + 3*I*a^2*c^2*d*f^3*x - 3*I*a^2*d^3*f^2*x^2
 + I*a^2*c^3*f^3 - 6*I*a^2*c*d^2*f^2*x - 3*I*a^2*c^2*d*f^2 + 6*I*a^2*d^3*f
*x + 6*I*a^2*c*d^2*f - 6*I*a^2*d^3)*e^(f*x + e)/f^4 + (I*a^2*d^3*f^3*x^3 +
 3*I*a^2*c*d^2*f^3*x^2 + 3*I*a^2*c^2*d*f^3*x + 3*I*a^2*d^3*f^2*x^2 + I*a^2
*c^3*f^3 + 6*I*a^2*c*d^2*f^2*x + 3*I*a^2*c^2*d*f^2 + 6*I*a^2*d^3*f*x + 6*I
*a^2*c*d^2*f + 6*I*a^2*d^3)*e^(-f*x - e)/f^4 + 1/32*(4*a^2*d^3*f^3*x^3 + 1
2*a^2*c*d^2*f^3*x^2 + 12*a^2*c^2*d*f^3*x + 6*a^2*d^3*f^2*x^2 + 4*a^2*c^3*f
^3 + 12*a^2*c*d^2*f^2*x + 6*a^2*c^2*d*f^2 + 6*a^2*d^3*f*x + 6*a^2*c*d^2*f
+ 3*a^2*d^3)*e^(-2*f*x - 2*e)/f^4
```

3.102.9 Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.60

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx$$
$$=\frac{a^2\,(3\,d^3\cosh(2\,e+2\,f\,x)+24\,c^3\,f^4\,x-4\,c^3\,f^3\sinh(2\,e+2\,f\,x)+6\,d^3\,f^4\,x^4+6\,c^2\,d\,f^2\cosh(2\,e+2\,f\,x)+}{}$$

input
```
int((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^3,x)
```

output
```
(a^2*(3*d^3*cosh(2*e + 2*f*x) - d^3*sinh(e + f*x)*192i + c^3*f^3*cosh(e +
f*x)*32i + 24*c^3*f^4*x - 4*c^3*f^3*sinh(2*e + 2*f*x) + 6*d^3*f^4*x^4 + 6*
c^2*d*f^2*cosh(2*e + 2*f*x) + 36*c^2*d*f^4*x^2 + 24*c*d^2*f^4*x^3 + d^3*f^
3*x^3*cosh(e + f*x)*32i - d^3*f^2*x^2*sinh(e + f*x)*96i + c*d^2*f*cosh(e +
 f*x)*192i + d^3*f*x*cosh(e + f*x)*192i + 6*d^3*f^2*x^2*cosh(2*e + 2*f*x)
- 4*d^3*f^3*x^3*sinh(2*e + 2*f*x) - 6*c*d^2*f*sinh(2*e + 2*f*x) - c^2*d*f^
2*sinh(e + f*x)*96i - 6*d^3*f*x*sinh(2*e + 2*f*x) + c^2*d*f^3*x*cosh(e + f
*x)*96i - c*d^2*f^2*x*sinh(e + f*x)*192i + 12*c*d^2*f^2*x*cosh(2*e + 2*f*x
) + c*d^2*f^3*x^2*cosh(e + f*x)*96i - 12*c^2*d*f^3*x*sinh(2*e + 2*f*x) - 1
2*c*d^2*f^3*x^2*sinh(2*e + 2*f*x)))/(16*f^4)
```

3.103 $\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx$

3.103.1 Optimal result

Integrand size = 23, antiderivative size = 174

$$\begin{aligned}\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx &= \frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}+\frac{4ia^2d^2\cosh(e+fx)}{f^3}\\ &+\frac{2ia^2(c+dx)^2\cosh(e+fx)}{f}\\ &-\frac{4ia^2d(c+dx)\sinh(e+fx)}{f^2}\\ &-\frac{a^2d^2\cosh(e+fx)\sinh(e+fx)}{4f^3}\\ &-\frac{a^2(c+dx)^2\cosh(e+fx)\sinh(e+fx)}{2f}\\ &+\frac{a^2d(c+dx)\sinh^2(e+fx)}{2f^2}\end{aligned}$$

output
```
1/4*a^2*d^2*x/f^2+1/2*a^2*(d*x+c)^3/d+4*I*a^2*d^2*cosh(f*x+e)/f^3+2*I*a^2*
(d*x+c)^2*cosh(f*x+e)/f-4*I*a^2*d*(d*x+c)*sinh(f*x+e)/f^2-1/4*a^2*d^2*cosh
(f*x+e)*sinh(f*x+e)/f^3-1/2*a^2*(d*x+c)^2*cosh(f*x+e)*sinh(f*x+e)/f+1/2*a^
2*d*(d*x+c)*sinh(f*x+e)^2/f^2
```

3.103.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.09

$$\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx$$
$$= \frac{a^2(12c^2f^3x+12cdf^3x^2+4d^2f^3x^3+16i(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)+2df(c+dx)\cosh(}{}$$

input
```
Integrate[(c + d*x)^2*(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(a^2*(12*c^2*f^3*x + 12*c*d*f^3*x^2 + 4*d^2*f^3*x^3 + (16*I)*(c^2*f^2 + 2*
c*d*f^2*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x] + 2*d*f*(c + d*x)*Cosh[2*(e +
 f*x)] - (32*I)*c*d*f*Sinh[e + f*x] - (32*I)*d^2*f*x*Sinh[e + f*x] - d^2*S
inh[2*(e + f*x)] - 2*c^2*f^2*Sinh[2*(e + f*x)] - 4*c*d*f^2*x*Sinh[2*(e + f
*x)] - 2*d^2*f^2*x^2*Sinh[2*(e + f*x)]))/(8*f^3)
```

3.103.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^2(a+a\sin(ie+ifx))^2dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(-a^2(c+dx)^2\sinh^2(e+fx)+2ia^2(c+dx)^2\sinh(e+fx)+a^2(c+dx)^2\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^2d(c+dx)\sinh^2(e+fx)}{2f^2}-\frac{4ia^2d(c+dx)\sinh(e+fx)}{f^2}+\frac{2ia^2(c+dx)^2\cosh(e+fx)}{f}-\frac{a^2(c+dx)^2\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{a^2(c+dx)^3}{2d}+\frac{4ia^2d^2\cosh(e+fx)}{f^3}-\frac{a^2d^2\sinh(e+fx)\cosh(e+fx)}{4f^3}+\frac{a^2d^2x}{4f^2}$$

input
```
Int[(c + d*x)^2*(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(a^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(2*d) + ((4*I)*a^2*d^2*Cosh[e + f*
x])/f^3 + ((2*I)*a^2*(c + d*x)^2*Cosh[e + f*x])/f - ((4*I)*a^2*d*(c + d*x)
*Sinh[e + f*x])/f^2 - (a^2*d^2*Cosh[e + f*x]*Sinh[e + f*x])/(4*f^3) - (a^2
*(c + d*x)^2*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) + (a^2*d*(c + d*x)*Sinh[e
+ f*x]^2)/(2*f^2)
```

3.103.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.103.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{2\left(\frac{\left(-(dx+c)^2f^2-\frac{d^2}{2}\right)\sinh(2fx+2e)}{8}+\frac{df(dx+c)\cosh(2fx+2e)}{8}+i\left((dx+c)^2f^2+2d^2\right)\cosh(fx+e)-2if(dx+c)d\sinh(fx+e)\right.}{f^3}$
risch	$\frac{a^2d^2x^3}{2}+\frac{3a^2dc\,x^2}{2}+\frac{3a^2x\,c^2}{2}+\frac{a^2c^3}{2d}-\frac{a^2\left(2d^2x^2f^2+4cd\,f^2x+2c^2f^2-2d^2fx-2cdf+d^2\right)e^{2fx+2e}}{16f^3}+\frac{ia^2\left(d^2x^2f\right.}{}$
parts	$\frac{a^2(dx+c)^3}{3d}+\frac{2ia^2\left(\frac{d^2\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{2d^2e((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f^2}+\right.}{f}$
derivativedivides	$\frac{\frac{d^2a^2(fx+e)^3}{3f^2}-\frac{4id^2e\,a^2((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f^2}-\frac{d^2a^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^3}{6}-\frac{(fx+e)\cosh(fx+e}{2}\right.}{f^2}}{}$
default	$\frac{\frac{d^2a^2(fx+e)^3}{3f^2}-\frac{4id^2e\,a^2((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f^2}-\frac{d^2a^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^3}{6}-\frac{(fx+e)\cosh(fx+e}{2}\right.}{f^2}}{}$

input
```
int((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(1/8*(-(d*x+c)^2*f^2-1/2*d^2)*sinh(2*f*x+2*e)+1/8*d*f*(d*x+c)*cosh(2*f*x
+2*e)+I*((d*x+c)^2*f^2+2*d^2)*cosh(f*x+e)-2*I*f*(d*x+c)*d*sinh(f*x+e)+3/4*
x*(1/3*d^2*x^2+c*d*x+c^2)*f^3+I*c^2*f^2-1/8*c*d*f+2*I*d^2)*a^2/f^3
```

3.103.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(158) = 316.

Time = 0.26 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.02

$$\int(c+dx)^2(a+ia\sinh(e+fx))^2\,dx$$
$$=\frac{\left(2\,a^2d^2f^2x^2+2\,a^2c^2f^2+2\,a^2cdf+a^2d^2+2\,(2\,a^2cdf^2+a^2d^2f)x-(2\,a^2d^2f^2x^2+2\,a^2c^2f^2-2\,a^2cdf+a^2\right.}{}$$

input
```
integrate((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/16*(2*a^2*d^2*f^2*x^2 + 2*a^2*c^2*f^2 + 2*a^2*c*d*f + a^2*d^2 + 2*(2*a^2
*c*d*f^2 + a^2*d^2*f)*x - (2*a^2*d^2*f^2*x^2 + 2*a^2*c^2*f^2 - 2*a^2*c*d*f
 + a^2*d^2 + 2*(2*a^2*c*d*f^2 - a^2*d^2*f)*x)*e^(4*f*x + 4*e) - 16*(-I*a^2
*d^2*f^2*x^2 - I*a^2*c^2*f^2 + 2*I*a^2*c*d*f - 2*I*a^2*d^2 + 2*(-I*a^2*c*d
*f^2 + I*a^2*d^2*f)*x)*e^(3*f*x + 3*e) + 8*(a^2*d^2*f^3*x^3 + 3*a^2*c*d*f^
3*x^2 + 3*a^2*c^2*f^3*x)*e^(2*f*x + 2*e) - 16*(-I*a^2*d^2*f^2*x^2 - I*a^2*
c^2*f^2 - 2*I*a^2*c*d*f - 2*I*a^2*d^2 + 2*(-I*a^2*c*d*f^2 - I*a^2*d^2*f)*x
)*e^(f*x + e))*e^(-2*f*x - 2*e)/f^3
```

3.103.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 694, normalized size of antiderivative = 3.99

$$\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx = \frac{3a^2c^2x}{2} + \frac{3a^2cdx^2}{2} + \frac{a^2d^2x^3}{2} + \begin{cases} \frac{((32a^2c^2f^{11}e^e+64a^2cdf^{11}xe^e+32a^2cdf^{10}e^e+32a^2d^2f^{11}x^2e^e+32a^2d^2f^{10}xe^e+16a^2d^2f^9e^e)e^{-2fx}+(-32a^2c^2f^{11}e^{5e}-64a^2cdf^{11}xe^{5e}+32a \cdots}{} \\ \frac{x^3(-a^2d^2e^{4e}+4ia^2d^2e^{3e}-4ia^2d^2e^e-a^2d^2)e^{-2e}}{12} + \frac{x^2(-a^2cde^{4e}+4ia^2cde^{3e}-4ia^2cde^e-a^2cd)e^{-2e}}{4} + \frac{x(-a^2c^2e^{4e}+4ia^2c^2e^{3e}-4ia^2c^2e \cdots}{4} \end{cases}$$

input
```
integrate((d*x+c)**2*(a+I*a*sinh(f*x+e))**2,x)
```

output
```
3*a**2*c**2*x/2 + 3*a**2*c*d*x**2/2 + a**2*d**2*x**3/2 + Piecewise((((32*a
**2*c**2*f**11*exp(e) + 64*a**2*c*d*f**11*x*exp(e) + 32*a**2*c*d*f**10*exp
(e) + 32*a**2*d**2*f**11*x**2*exp(e) + 32*a**2*d**2*f**10*x*exp(e) + 16*a*
*2*d**2*f**9*exp(e))*exp(-2*f*x) + (-32*a**2*c**2*f**11*exp(5*e) - 64*a**2
*c*d*f**11*x*exp(5*e) + 32*a**2*c*d*f**10*exp(5*e) - 32*a**2*d**2*f**11*x*
*2*exp(5*e) + 32*a**2*d**2*f**10*x*exp(5*e) - 16*a**2*d**2*f**9*exp(5*e))*
exp(2*f*x) + (256*I*a**2*c**2*f**11*exp(2*e) + 512*I*a**2*c*d*f**11*x*exp(
2*e) + 512*I*a**2*c*d*f**10*exp(2*e) + 256*I*a**2*d**2*f**11*x**2*exp(2*e)
 + 512*I*a**2*d**2*f**10*x*exp(2*e) + 512*I*a**2*d**2*f**9*exp(2*e))*exp(-
f*x) + (256*I*a**2*c**2*f**11*exp(4*e) + 512*I*a**2*c*d*f**11*x*exp(4*e) -
 512*I*a**2*c*d*f**10*exp(4*e) + 256*I*a**2*d**2*f**11*x**2*exp(4*e) - 512
*I*a**2*d**2*f**10*x*exp(4*e) + 512*I*a**2*d**2*f**9*exp(4*e))*exp(f*x))*e
xp(-3*e)/(256*f**12), Ne(f**12*exp(3*e), 0)), (x**3*(-a**2*d**2*exp(4*e) +
 4*I*a**2*d**2*exp(3*e) - 4*I*a**2*d**2*exp(e) - a**2*d**2)*exp(-2*e)/12 +
 x**2*(-a**2*c*d*exp(4*e) + 4*I*a**2*c*d*exp(3*e) - 4*I*a**2*c*d*exp(e) -
a**2*c*d)*exp(-2*e)/4 + x*(-a**2*c**2*exp(4*e) + 4*I*a**2*c**2*exp(3*e) -
4*I*a**2*c**2*exp(e) - a**2*c**2)*exp(-2*e)/4, True))
```

3.103.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(158) = 316.

Time = 0.19 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.87

$$
\begin{aligned}
&\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx \\
&= \frac{1}{3}\,a^2d^2x^3 + a^2cdx^2 + \frac{1}{8}\left(4\,x^2 - \frac{\left(2\,fxe^{(2\,e)} - e^{(2\,e)}\right)e^{(2\,fx)}}{f^2} + \frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)a^2cd \\
&\quad + \frac{1}{48}\left(8\,x^3 - \frac{3\left(2\,f^2x^2e^{(2\,e)} - 2\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(2\,fx)}}{f^3} + \frac{3\left(2\,f^2x^2 + 2\,fx + 1\right)e^{(-2\,fx-2\,e)}}{f^3}\right)a^2d^2 \\
&\quad + \frac{1}{8}\,a^2c^2\left(4\,x - \frac{e^{(2\,fx+2\,e)}}{f} + \frac{e^{(-2\,fx-2\,e)}}{f}\right) + a^2c^2x \\
&\quad + 2i\,a^2cd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) \\
&\quad + i\,a^2d^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right) \\
&\quad + \frac{2i\,a^2c^2\cosh\left(fx+e\right)}{f}
\end{aligned}
$$

input
```
integrate((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + 1/8*(4*x^2 - (2*f*x*e^(2*e) - e^(2*e))*e^(
2*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*c*d + 1/48*(8*x^3 - 3*(
2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x)/f^3 + 3*(2*f^2*x^2
+ 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*a^2*d^2 + 1/8*a^2*c^2*(4*x - e^(2*f*x +
 2*e)/f + e^(-2*f*x - 2*e)/f) + a^2*c^2*x + 2*I*a^2*c*d*((f*x*e^e - e^e)*e
^(f*x)/f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + I*a^2*d^2*((f^2*x^2*e^e - 2*f*x
*e^e + 2*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + 2*I*
a^2*c^2*cosh(f*x + e)/f
```

3.103.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(158) = 316$.

Time = 0.27 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.91

$$
\begin{aligned}
&\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx \\
&= \frac{1}{2}\,a^2d^2x^3 + \frac{3}{2}\,a^2cdx^2 + \frac{3}{2}\,a^2c^2x \\
&\quad - \frac{(2\,a^2d^2f^2x^2 + 4\,a^2cdf^2x + 2\,a^2c^2f^2 - 2\,a^2d^2fx - 2\,a^2cdf + a^2d^2)e^{(2\,fx+2\,e)}}{16\,f^3} \\
&\quad + \frac{(i\,a^2d^2f^2x^2 + 2i\,a^2cdf^2x + i\,a^2c^2f^2 - 2i\,a^2d^2fx - 2i\,a^2cdf + 2i\,a^2d^2)e^{(fx+e)}}{f^3} \\
&\quad - \frac{(-i\,a^2d^2f^2x^2 - 2i\,a^2cdf^2x - i\,a^2c^2f^2 - 2i\,a^2d^2fx - 2i\,a^2cdf - 2i\,a^2d^2)e^{(-fx-e)}}{f^3} \\
&\quad + \frac{(2\,a^2d^2f^2x^2 + 4\,a^2cdf^2x + 2\,a^2c^2f^2 + 2\,a^2d^2fx + 2\,a^2cdf + a^2d^2)e^{(-2\,fx-2\,e)}}{16\,f^3}
\end{aligned}
$$

input
```
integrate((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/2*a^2*d^2*x^3 + 3/2*a^2*c*d*x^2 + 3/2*a^2*c^2*x - 1/16*(2*a^2*d^2*f^2*x^
2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - 2*a^2*d^2*f*x - 2*a^2*c*d*f + a^2*d^
2)*e^(2*f*x + 2*e)/f^3 + (I*a^2*d^2*f^2*x^2 + 2*I*a^2*c*d*f^2*x + I*a^2*c^
2*f^2 - 2*I*a^2*d^2*f*x - 2*I*a^2*c*d*f + 2*I*a^2*d^2)*e^(f*x + e)/f^3 - (
-I*a^2*d^2*f^2*x^2 - 2*I*a^2*c*d*f^2*x - I*a^2*c^2*f^2 - 2*I*a^2*d^2*f*x -
 2*I*a^2*c*d*f - 2*I*a^2*d^2)*e^(-f*x - e)/f^3 + 1/16*(2*a^2*d^2*f^2*x^2 +
 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 + 2*a^2*d^2*f*x + 2*a^2*c*d*f + a^2*d^2)*
e^(-2*f*x - 2*e)/f^3
```

3.103.9 Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.25

$$
\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx = \frac{a^2\,\left(12\,c^2\,x+12\,c\,d\,x^2+4\,d^2\,x^3\right)}{8}
$$
$$
+\frac{\frac{a^2\,\left(-d^2\,\sinh(2\,e+2\,f\,x)+d^2\,\cosh(e+f\,x)\,32\mathrm{i}\right)}{8}+\frac{a^2\,f^2\,\left(-2\,c^2\,\sinh(2\,e+2\,f\,x)-2\,d^2\,x^2\,\sinh(2\,e+2\,f\,x)-4\,c\,d\,x\,\sinh(2\,e+2\,f\,x)+c^2\,\cosh(e+f\,x)\right.}{8}}{}
$$

input
```
int((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^2,x)
```

output
```
(a^2*(12*c^2*x + 4*d^2*x^3 + 12*c*d*x^2))/8 + ((a^2*(d^2*cosh(e + f*x)*32i
 - d^2*sinh(2*e + 2*f*x)))/8 + (a^2*f^2*(c^2*cosh(e + f*x)*16i - 2*c^2*sin
h(2*e + 2*f*x) + d^2*x^2*cosh(e + f*x)*16i - 2*d^2*x^2*sinh(2*e + 2*f*x) +
 c*d*x*cosh(e + f*x)*32i - 4*c*d*x*sinh(2*e + 2*f*x)))/8 - (a^2*f*(d^2*x*s
inh(e + f*x)*32i - 2*d^2*x*cosh(2*e + 2*f*x) + c*d*sinh(e + f*x)*32i - 2*c
*d*cosh(2*e + 2*f*x)))/8)/f^3
```

3.103. $\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx$

3.104 $\int (c+dx)(a+ia\sinh(e+fx))^2\,dx$

3.104.1 Optimal result

Integrand size = 21, antiderivative size = 122

$$
\begin{aligned}
\int (c+dx)(a+ia\sinh(e+fx))^2\,dx = {} & \frac{1}{2}a^2cx+\frac{1}{4}a^2dx^2+\frac{a^2(c+dx)^2}{2d} \\
& +\frac{2ia^2(c+dx)\cosh(e+fx)}{f}-\frac{2ia^2d\sinh(e+fx)}{f^2} \\
& -\frac{a^2(c+dx)\cosh(e+fx)\sinh(e+fx)}{2f} \\
& +\frac{a^2d\sinh^2(e+fx)}{4f^2}
\end{aligned}
$$

output
```
1/2*a^2*c*x+1/4*a^2*d*x^2+1/2*a^2*(d*x+c)^2/d+2*I*a^2*(d*x+c)*cosh(f*x+e)/
f-2*I*a^2*d*sinh(f*x+e)/f^2-1/2*a^2*(d*x+c)*cosh(f*x+e)*sinh(f*x+e)/f+1/4*
a^2*d*sinh(f*x+e)^2/f^2
```

3.104.2 Mathematica [A] (verified)

Time = 12.52 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

$$
\begin{aligned}
& \int (c+dx)(a+ia\sinh(e+fx))^2\,dx \\
& = \frac{a^2(16if(c+dx)\cosh(e+fx)+d\cosh(2(e+fx))-2(3(e+fx)(de-2cf-dfx)+8id\sinh(e+fx)+}{8f^2}
\end{aligned}
$$

input
```
Integrate[(c + d*x)*(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(a^2*((16*I)*f*(c + d*x)*Cosh[e + f*x] + d*Cosh[2*(e + f*x)] - 2*(3*(e + f
*x)*(d*e - 2*c*f - d*f*x) + (8*I)*d*Sinh[e + f*x] + f*(c + d*x)*Sinh[2*(e
+ f*x)])))/(8*f^2)
```

3.104.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+ia\sinh(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)(a+a\sin(ie+ifx))^2dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(-\left(a^2(c+dx)\sinh^2(e+fx)\right)+2ia^2(c+dx)\sinh(e+fx)+a^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2ia^2(c+dx)\cosh(e+fx)}{f}-\frac{a^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{3a^2(c+dx)^2}{4d}+\frac{a^2d\sinh^2(e+fx)}{4f^2}-\frac{2ia^2d\sinh(e+fx)}{f^2}$$

input
```
Int[(c + d*x)*(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(3*a^2*(c + d*x)^2)/(4*d) + ((2*I)*a^2*(c + d*x)*Cosh[e + f*x])/f - ((2*I)
*a^2*d*Sinh[e + f*x])/f^2 - (a^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(2
*f) + (a^2*d*Sinh[e + f*x]^2)/(4*f^2)
```

3.104.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.104.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{2\left(-\frac{(dx+c)f\sinh(2fx+2e)}{8}+\frac{d\cosh(2fx+2e)}{16}+if(dx+c)\cosh(fx+e)-i\sinh(fx+e)d+\frac{3x\left(\frac{dx}{2}+c\right)f^2}{4}+icf-\frac{d}{16}\right)a^2}{f^2}$
risch	$\frac{3a^2dx^2}{4}+\frac{3a^2cx}{2}-\frac{a^2(2dfx+2cf-d)e^{2fx+2e}}{16f^2}+\frac{ia^2(dfx+cf-d)e^{fx+e}}{f^2}+\frac{ia^2(dfx+cf+d)e^{-fx-e}}{f^2}+\frac{a^2(2dfx+2\cdots}{}$
parts	$a^2\left(\frac{1}{2}dx^2+cx\right)+\frac{2ia^2\left(\frac{d((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{de\cosh(fx+e)}{f}+c\cosh(fx+e)\right)}{f}-\frac{a^2\left(\frac{d\left(\frac{(fx+e)\cosh(\cdots}{}\right.}{}\right.}{}$
derivativedivides	$\frac{da^2(fx+e)^2}{2f}+\frac{2ida^2((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{da^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)}{f}-\frac{dea^2(\cdots}{}$
default	$\frac{da^2(fx+e)^2}{2f}+\frac{2ida^2((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{da^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)}{f}-\frac{dea^2(\cdots}{}$

input
```
int((d*x+c)*(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/8*(d*x+c)*f*sinh(2*f*x+2*e)+1/16*d*cosh(2*f*x+2*e)+I*f*(d*x+c)*cosh(
f*x+e)-I*sinh(f*x+e)*d+3/4*x*(1/2*d*x+c)*f^2+I*c*f-1/16*d)*a^2/f^2
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.34

$$\int (c+dx)(a+ia\sinh(e+fx))^2\,dx$$
$$=\frac{(2\,a^2dfx+2\,a^2cf+a^2d-(2\,a^2dfx+2\,a^2cf-a^2d)e^{(4\,fx+4\,e)}-16\,(-i\,a^2dfx-i\,a^2cf+i\,a^2d)e^{(3\,fx+3\,e)}+}{16\,f^2}$$

input
```
integrate((d*x+c)*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/16*(2*a^2*d*f*x + 2*a^2*c*f + a^2*d - (2*a^2*d*f*x + 2*a^2*c*f - a^2*d)*
e^(4*f*x + 4*e) - 16*(-I*a^2*d*f*x - I*a^2*c*f + I*a^2*d)*e^(3*f*x + 3*e)
+ 12*(a^2*d*f^2*x^2 + 2*a^2*c*f^2*x)*e^(2*f*x + 2*e) - 16*(-I*a^2*d*f*x -
I*a^2*c*f - I*a^2*d)*e^(f*x + e))*e^(-2*f*x - 2*e)/f^2
```

3.104.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.93

$$\int (c+dx)(a+ia\sinh(e+fx))^2\,dx=\frac{3a^2cx}{2}+\frac{3a^2dx^2}{4}$$
$$+\begin{cases}\frac{((32a^2cf^7e^e+32a^2df^7xe^e+16a^2df^6e^e)e^{-2fx}+(-32a^2cf^7e^{5e}-32a^2df^7xe^{5e}+16a^2df^6e^{5e})e^{2fx}+(256ia^2cf^7e^{2e}+256ia^2df^7xe^{2e}+256ia^2df}{256f^8} \\ \frac{x^2(-a^2de^{4e}+4ia^2de^{3e}-4ia^2de^e-a^2d)e^{-2e}}{8}+\frac{x(-a^2ce^{4e}+4ia^2ce^{3e}-4ia^2ce^e-a^2c)e^{-2e}}{4}\end{cases}$$

input
```
integrate((d*x+c)*(a+I*a*sinh(f*x+e))**2,x)
```

output
```
3*a**2*c*x/2 + 3*a**2*d*x**2/4 + Piecewise(((((32*a**2*c*f**7*exp(e) + 32*a
**2*d*f**7*x*exp(e) + 16*a**2*d*f**6*exp(e))*exp(-2*f*x) + (-32*a**2*c*f**
7*exp(5*e) - 32*a**2*d*f**7*x*exp(5*e) + 16*a**2*d*f**6*exp(5*e))*exp(2*f*
x) + (256*I*a**2*c*f**7*exp(2*e) + 256*I*a**2*d*f**7*x*exp(2*e) + 256*I*a*
*2*d*f**6*exp(2*e))*exp(-f*x) + (256*I*a**2*c*f**7*exp(4*e) + 256*I*a**2*d
*f**7*x*exp(4*e) - 256*I*a**2*d*f**6*exp(4*e))*exp(f*x))*exp(-3*e)/(256*f*
*8), Ne(f**8*exp(3*e), 0)), (x**2*(-a**2*d*exp(4*e) + 4*I*a**2*d*exp(3*e)
- 4*I*a**2*d*exp(e) - a**2*d)*exp(-2*e)/8 + x*(-a**2*c*exp(4*e) + 4*I*a**2
*c*exp(3*e) - 4*I*a**2*c*exp(e) - a**2*c)*exp(-2*e)/4, True))
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.37

$$\begin{aligned}&\int (c+dx)(a+ia\sinh(e+fx))^2\,dx\\&=\frac{1}{2}\,a^2dx^2+\frac{1}{16}\left(4\,x^2-\frac{\left(2\,fxe^{(2\,e)}-e^{(2\,e)}\right)e^{(2\,fx)}}{f^2}+\frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)a^2d\\&\quad+\frac{1}{8}\,a^2c\left(4\,x-\frac{e^{(2\,fx+2\,e)}}{f}+\frac{e^{(-2\,fx-2\,e)}}{f}\right)+a^2cx\\&\quad+i\,a^2d\left(\frac{(fxe^e-e^e)e^{(fx)}}{f^2}+\frac{(fx+1)e^{(-fx-e)}}{f^2}\right)+\frac{2i\,a^2c\cosh\left(fx+e\right)}{f}\end{aligned}$$

input
```
integrate((d*x+c)*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*d*x^2 + 1/16*(4*x^2 - (2*f*x*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2
*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*d + 1/8*a^2*c*(4*x - e^(2*f*x + 2*e)/f
 + e^(-2*f*x - 2*e)/f) + a^2*c*x + I*a^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 +
(f*x + 1)*e^(-f*x - e)/f^2) + 2*I*a^2*c*cosh(f*x + e)/f
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.27

$$\begin{aligned}\int (c+dx)(a+ia\sinh(e+fx))^2\,dx&=\frac{3}{4}\,a^2dx^2+\frac{3}{2}\,a^2cx-\frac{(2\,a^2dfx+2\,a^2cf-a^2d)e^{(2\,fx+2\,e)}}{16\,f^2}\\&\quad+\frac{(i\,a^2dfx+i\,a^2cf-i\,a^2d)e^{(fx+e)}}{f^2}\\&\quad+\frac{(i\,a^2dfx+i\,a^2cf+i\,a^2d)e^{(-fx-e)}}{f^2}\\&\quad+\frac{(2\,a^2dfx+2\,a^2cf+a^2d)e^{(-2\,fx-2\,e)}}{16\,f^2}\end{aligned}$$

input
```
integrate((d*x+c)*(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
3/4*a^2*d*x^2 + 3/2*a^2*c*x - 1/16*(2*a^2*d*f*x + 2*a^2*c*f - a^2*d)*e^(2*
f*x + 2*e)/f^2 + (I*a^2*d*f*x + I*a^2*c*f - I*a^2*d)*e^(f*x + e)/f^2 + (I*
a^2*d*f*x + I*a^2*c*f + I*a^2*d)*e^(-f*x - e)/f^2 + 1/16*(2*a^2*d*f*x + 2*
a^2*c*f + a^2*d)*e^(-2*f*x - 2*e)/f^2
```

3.104.9 Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

$$\int (c+dx)(a+ia\sinh(e+fx))^2\,dx = \frac{a^2\,(6\,d\,x^2+12\,c\,x)}{8} - \frac{\frac{a^2\,(-d\cosh(2\,e+2\,f\,x)+d\sinh(e+f\,x)\;16\mathrm{i})}{8} - \frac{a^2\,f\,(c\cosh(e+f\,x)\;16\mathrm{i}-2\,c\sinh(2\,e+2\,f\,x)-2\,d\,x\sinh(2\,e+2\,f\,x)+d\,x\cosh(e+f\,x)\;16\mathrm{i})}{8}}{f^2}$$

input
```
int((a + a*sinh(e + f*x)*1i)^2*(c + d*x),x)
```

output
```
(a^2*(12*c*x + 6*d*x^2))/8 - ((a^2*(d*sinh(e + f*x)*16i - d*cosh(2*e + 2*f
*x)))/8 - (a^2*f*(c*cosh(e + f*x)*16i - 2*c*sinh(2*e + 2*f*x) - 2*d*x*sinh
(2*e + 2*f*x) + d*x*cosh(e + f*x)*16i))/8)/f^2
```

3.105 $\int \frac{(a+ia\sinh(e+fx))^2}{c+dx} \, dx$

3.105.1 Optimal result

Integrand size = 23, antiderivative size = 149

$$\begin{aligned}\int \frac{(a+ia\sinh(e+fx))^2}{c+dx} \, dx = &-\frac{a^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d} \\ &+\frac{3a^2\log(c+dx)}{2d}+\frac{2ia^2\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d} \\ &+\frac{2ia^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d} \\ &-\frac{a^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{2d}\end{aligned}$$

output
```
-1/2*a^2*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/d+3/2*a^2*ln(d*x+c)/d+2*I*a
^2*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d+1/2*a^2*Shi(2*c*f/d+2*f*x)*sinh(-2*e+2*
c*f/d)/d-2*I*a^2*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d
```

3.105.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.79

$$\int \frac{(a+ia\sinh(e+fx))^2}{c+dx} \, dx = -\frac{a^2\left(\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2f(c+dx)}{d}\right)-3\log(c+dx)-4i\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)-4i\cosh\left(e-\frac{cf}{d}\right)\text{S}\right.}{2d}$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])^2/(c + d*x),x]
```

output
```
-1/2*(a^2*(Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d] - 3*Log[c
 + d*x] - (4*I)*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] - (4*I)*Cosh[e
 - (c*f)/d]*SinhIntegral[f*(c/d + x)] + Sinh[2*e - (2*c*f)/d]*SinhIntegral
[(2*f*(c + d*x))/d]))/d
```

3.105.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3799, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+a\sin(ie+ifx))^2}{c+dx}dx$$

$$\Big\downarrow \text{3799}$$

$$4a^2\int \frac{\sinh^4\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$4a^2\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^4}{c+dx}dx$$

$$\Big\downarrow \text{3793}$$

$$4a^2\int \left(-\frac{\cosh(2e+2fx)}{8(c+dx)}+\frac{i\sinh(e+fx)}{2(c+dx)}+\frac{3}{8(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$4a^2\left(\frac{i\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{2d}-\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{8d}-\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{8d}+\frac{i\,\text{co}}{}\right.$$

input
```
Int[(a + I*a*Sinh[e + f*x])^2/(c + d*x),x]
```

output
```
4*a^2*(-1/8*(Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d + (3
*Log[c + d*x])/(8*d) + ((I/2)*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d
])/d + ((I/2)*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d - (Sinh[2*e
 - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*d))
```

3.105.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.105.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.30

method	result
risch	$-\frac{i a^{2} {\mathrm e}^{-\frac{c f -d e}{d}} \operatorname{Ei}_{1}\left(-f x -e -\frac{c f -d e}{d}\right)}{d}+\frac{3 a^{2} \ln \left(d x +c \right)}{2 d}+\frac{a^{2} {\mathrm e}^{-\frac{2 \left(c f -d e \right)}{d}} \operatorname{Ei}_{1}\left(-2 f x -2 e -\frac{2 \left(c f -d e \right)}{d}\right)}{4 d}+\frac{a^{2} {\mathrm e}^{\frac{2 c f -2 d e}{d}} \operatorname{Ei}_{1}\left(2 f x +2 e -\right.}{4 d}$

input
```
int((a+I*a*sinh(f*x+e))^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-I*a^2/d*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)+3/2*a^2*ln(d*x+c)/d+1/
4*a^2/d*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)+1/4*a^2/d*exp(2
*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)+I*a^2/d*exp((c*f-d*e)/d)*Ei(1,
f*x+e+(c*f-d*e)/d)
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00

$$\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx = -\frac{a^2\text{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(\frac{2\,(de-cf)}{d}\right)} - 4i\,a^2\text{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)} + 4i\,a^2\text{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)} + a^2\text{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)}{4\,d}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

output
```
-1/4*(a^2*Ei(2*(d*f*x + c*f)/d)*e^(2*(d*e - c*f)/d) - 4*I*a^2*Ei((d*f*x +
c*f)/d)*e^((d*e - c*f)/d) + 4*I*a^2*Ei(-(d*f*x + c*f)/d)*e^(-(d*e - c*f)/d
) + a^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*(d*e - c*f)/d) - 6*a^2*log((d*x + c)/
d))/d
```

3.105.6 Sympy [F]

$$\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx = -a^2\left(\int \frac{\sinh^2(e+fx)}{c+dx}\,dx + \int \left(-\frac{2i\sinh(e+fx)}{c+dx}\right)dx + \int \left(-\frac{1}{c+dx}\right)dx\right)$$

input
```
integrate((a+I*a*sinh(f*x+e))**2/(d*x+c),x)
```

output
```
-a**2*(Integral(sinh(e + f*x)**2/(c + d*x), x) + Integral(-2*I*sinh(e + f*
x)/(c + d*x), x) + Integral(-1/(c + d*x), x))
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01

$$\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx$$
$$=\frac{1}{4}a^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_1\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{e^{\left(2e-\frac{2cf}{d}\right)}E_1\left(-\frac{2(dx+c)f}{d}\right)}{d}+\frac{2\log(dx+c)}{d}\right)$$
$$+ia^2\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right)+\frac{a^2\log(dx+c)}{d}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

output
```
1/4*a^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/d + e^(2*e
- 2*c*f/d)*exp_integral_e(1, -2*(d*x + c)*f/d)/d + 2*log(d*x + c)/d) + I*a
^2*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(e - c*f/d)*exp_
integral_e(1, -(d*x + c)*f/d)/d) + a^2*log(d*x + c)/d
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx=$$
$$-\frac{a^2\mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right)e^{\left(2e-\frac{2cf}{d}\right)}-4i\,a^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+4i\,a^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+a^2\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)e}{4d}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

output
```
-1/4*(a^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) - 4*I*a^2*Ei((d*f*x + c*
f)/d)*e^(e - c*f/d) + 4*I*a^2*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + a^2*Ei
(-2*(d*f*x + c*f)/d)*e^(-2*e + 2*c*f/d) - 6*a^2*log(d*x + c))/d
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx = \int \frac{\left(a+a\sinh(e+f\,x)\ 1\mathrm{i}\right)^2}{c+d\,x}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^2/(c + d*x),x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^2/(c + d*x), x)
```

3.106 $\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx$

3.106.1 Optimal result

Integrand size = 23, antiderivative size = 170

$$\begin{aligned}\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx = &-\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)} \\ &+\frac{2ia^2f\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(\frac{cf}{d}+fx\right)}{d^2} \\ &-\frac{a^2f\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2} \\ &+\frac{2ia^2f\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(\frac{cf}{d}+fx\right)}{d^2} \\ &-\frac{a^2f\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2cf}{d}+2fx\right)}{d^2}\end{aligned}$$

output
```
2*I*a^2*f*Chi(c*f/d+f*x)*cosh(-e+c*f/d)/d^2-4*a^2*cosh(1/2*e+1/4*I*Pi+1/2*
f*x)^4/d/(d*x+c)-a^2*f*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/d^2+a^2*f*Chi
(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/d^2-2*I*a^2*f*Shi(c*f/d+f*x)*sinh(-e+c*
f/d)/d^2
```

3.106.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.26

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx$$
$$= \frac{a^2\left(-3d+d\cosh(2(e+fx))+4if(c+dx)\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)-2f(c+dx)\mathrm{Chi}\left(\frac{2f(c+dx)}{d}\right)\sinh$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])^2/(c + d*x)^2,x]
```

output
```
(a^2*(-3*d + d*Cosh[2*(e + f*x)] + (4*I)*f*(c + d*x)*Cosh[e - (c*f)/d]*Cos
hIntegral[f*(c/d + x)] - 2*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*Sin
h[2*e - (2*c*f)/d] - (4*I)*d*Sinh[e + f*x] + (4*I)*c*f*Sinh[e - (c*f)/d]*S
inhIntegral[f*(c/d + x)] + (4*I)*d*f*x*Sinh[e - (c*f)/d]*SinhIntegral[f*(c
/d + x)] - 2*c*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] - 2
*d*f*x*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d]))/(2*d^2*(c +
 d*x))
```

3.106.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3799, 3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+a\sin(ie+ifx))^2}{(c+dx)^2}dx$$
$$\Big\downarrow \text{3799}$$
$$4a^2\int \frac{\sinh^4\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)}{(c+dx)^2}dx$$
$$\Big\downarrow \text{3042}$$

$$4a^2 \int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^4}{(c+dx)^2}dx$$

$$\Big\downarrow \; 3794$$

$$4a^2\left(\frac{2if\int\left(\frac{\cosh(e+fx)}{4(c+dx)}+\frac{i\sinh(2e+2fx)}{8(c+dx)}\right)dx}{d}-\frac{\cosh^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{d(c+dx)}\right)$$

$$\Big\downarrow \; 2009$$

$$4a^2\left(\frac{2if\left(\frac{i\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{8d}+\frac{\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{4d}+\frac{\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{4d}+\frac{i\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{8d}\right.}{d}\right.$$

input
```
Int[(a + I*a*Sinh[e + f*x])^2/(c + d*x)^2,x]
```

output
```
4*a^2*(-(Cosh[e/2 + (I/4)*Pi + (f*x)/2]^4/(d*(c + d*x))) + ((2*I)*f*((Cosh
[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/(4*d) + ((I/8)*CoshIntegral[(2*
c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d + (Sinh[e - (c*f)/d]*SinhIntegral
[(c*f)/d + f*x])/(4*d) + ((I/8)*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)
/d + 2*f*x])/d))/d)
```

3.106.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.106.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.84

method	result
risch	$-\frac{i a^{2} f \,{\mathrm e}^{f x +e}}{d^{2} \left(\frac{c f}{d}+f x \right)}-\frac{i a^{2} f \,{\mathrm e}^{-\frac{c f -d e}{d}} \operatorname{Ei}_{1}\left(-f x -e -\frac{c f -d e}{d}\right)}{d^{2}}-\frac{3 a^{2}}{2 d \left(d x +c \right)}+\frac{f \,a^{2} {\mathrm e}^{-2 f x -2 e}}{4 d \left(d f x +c f \right)}-\frac{f \,a^{2} {\mathrm e}^{\frac{2 c f -2 d e}{d}} \operatorname{Ei}_{1}\left(2 f x +2 e +\frac{2 c f -2 d e}{d}\right)}{2 d^{2}}$

input
```
int((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-I*a^2*f/d^2*exp(f*x+e)/(c*f/d+f*x)-I*a^2*f/d^2*exp(-(c*f-d*e)/d)*Ei(1,-f*
x-e-(c*f-d*e)/d)-3/2*a^2/d/(d*x+c)+1/4*f*a^2*exp(-2*f*x-2*e)/d/(d*f*x+c*f)
-1/2*f*a^2/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)+1/4*f*a^2/
d^2*exp(2*f*x+2*e)/(c*f/d+f*x)+1/2*f*a^2/d^2*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f
*x-2*e-2*(c*f-d*e)/d)+I*a^2*f*exp(-f*x-e)/d/(d*f*x+c*f)-I*a^2*f/d^2*exp((c
*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.56

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx$$
$$= \frac{\left(a^2de^{(4\,fx+4\,e)} - 4i\,a^2de^{(3\,fx+3\,e)} + 4i\,a^2de^{(fx+e)} + a^2d - 2\left(3\,a^2d + (a^2dfx + a^2cf)\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(\frac{2\,(de-cf}{d}\right.}\right.\right.}{}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/4*(a^2*d*e^(4*f*x + 4*e) - 4*I*a^2*d*e^(3*f*x + 3*e) + 4*I*a^2*d*e^(f*x
+ e) + a^2*d - 2*(3*a^2*d + (a^2*d*f*x + a^2*c*f)*Ei(2*(d*f*x + c*f)/d)*e^
(2*(d*e - c*f)/d) + 2*(-I*a^2*d*f*x - I*a^2*c*f)*Ei((d*f*x + c*f)/d)*e^((d
*e - c*f)/d) + 2*(-I*a^2*d*f*x - I*a^2*c*f)*Ei(-(d*f*x + c*f)/d)*e^(-(d*e
- c*f)/d) - (a^2*d*f*x + a^2*c*f)*Ei(-2*(d*f*x + c*f)/d)*e^(-2*(d*e - c*f)
/d))*e^(2*f*x + 2*e))*e^(-2*f*x - 2*e)/(d^3*x + c*d^2)
```

3.106.6 Sympy [F]

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx = -a^2\left(\int \frac{\sinh^2(e+fx)}{c^2+2cdx+d^2x^2}\,dx + \int \left(-\frac{2i\sinh(e+fx)}{c^2+2cdx+d^2x^2}\right)dx + \int \left(-\frac{1}{c^2+2cdx+d^2x^2}\right)dx\right)$$

input
```
integrate((a+I*a*sinh(f*x+e))**2/(d*x+c)**2,x)
```

output
```
-a**2*(Integral(sinh(e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integr
al(-2*I*sinh(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(-1/(c**2
 + 2*c*d*x + d**2*x**2), x))
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.08

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx = \frac{1}{4}a^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_2\left(\frac{2(dx+c)f}{d}\right)}{(dx+c)d} + \frac{e^{\left(2e-\frac{2cf}{d}\right)}E_2\left(-\frac{2(dx+c)f}{d}\right)}{(dx+c)d} - \frac{2}{d^2x+cd}\right) + i\,a^2\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right) - \frac{a^2}{d^2x+cd}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/4*a^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)*
d) + e^(2*e - 2*c*f/d)*exp_integral_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d) -
 2/(d^2*x + c*d)) + I*a^2*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)
/((d*x + c)*d) - e^(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c
)*d)) - a^2/(d^2*x + c*d)
```

3.106.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(159) = 318$.

Time = 0.34 (sec) , antiderivative size = 1134, normalized size of antiderivative = 6.67

$$\int \frac{(a + ia \sinh(e + fx))^2}{(c + dx)^2} \, dx = \text{Too large to display}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/4*(2*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(2*((d*x +
 c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d
) - 2*a^2*d*e*f^2*Ei(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*
e + c*f)/d)*e^(2*(d*e - c*f)/d) + 2*a^2*c*f^3*Ei(2*((d*x + c)*(d*e/(d*x +
c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d) - 4*I*(d*x + c
)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d*e/(d*x + c)
 - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) + 4*I*a^2*d*e*f^2*
Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e
- c*f)/d) - 4*I*a^2*c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f
) - d*e + c*f)/d)*e^((d*e - c*f)/d) - 4*I*(d*x + c)*a^2*(d*e/(d*x + c) - c
*f/(d*x + c) + f)*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) -
 d*e + c*f)/d)*e^(-(d*e - c*f)/d) + 4*I*a^2*d*e*f^2*Ei(-((d*x + c)*(d*e/(d
*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) - 4*I*a^2*
c*f^3*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e
^(-(d*e - c*f)/d) - 2*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^
2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(
-2*(d*e - c*f)/d) + 2*a^2*d*e*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d
*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) - 2*a^2*c*f^3*Ei(-2*((d*
x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*
f)/d) - a^2*d*f^2*e^(2*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d)...
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx=\int \frac{{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2}{{\left(c+d\,x\right)}^2} \,d x$$

input
```
int((a + a*sinh(e + f*x)*1i)^2/(c + d*x)^2,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^2/(c + d*x)^2, x)
```

3.107 $\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx$

3.107.1 Optimal result

Integrand size = 23, antiderivative size = 236

$$\begin{aligned}\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx = &-\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2}\\ &-\frac{a^2f^2\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)}{d^3}\\ &+\frac{ia^2f^2\operatorname{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^3}\\ &-\frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)}\\ &+\frac{ia^2f^2\cosh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(\frac{cf}{d}+fx\right)}{d^3}\\ &-\frac{a^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{d^3}\end{aligned}$$

output
```
-a^2*f^2*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/d^3-2*a^2*cosh(1/2*e+1/4*I*
Pi+1/2*f*x)^4/d/(d*x+c)^2+I*a^2*f^2*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d^3+a^2*
f^2*Shi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/d^3-I*a^2*f^2*Chi(c*f/d+f*x)*sin
h(-e+c*f/d)/d^3-4*a^2*f*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^3*sinh(1/2*e+1/4*I*Pi
+1/2*f*x)/d^2/(d*x+c)
```

3.107.2 Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.84

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx$$
$$= \frac{a^2\left(-4f^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2f(c+dx)}{d}\right)+4if^2\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+\frac{d(-3d-4if(c+dx)\cosh(e+fx)+d\,\text{co}}{}\right.}{}$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])^2/(c + d*x)^3,x]
```

output
```
(a^2*(-4*f^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d] + (4*I)
*f^2*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + (d*(-3*d - (4*I)*f*(c +
 d*x)*Cosh[e + f*x] + d*Cosh[2*(e + f*x)] - (4*I)*d*Sinh[e + f*x] + 2*c*f*
Sinh[2*(e + f*x)] + 2*d*f*x*Sinh[2*(e + f*x)]))/(c + d*x)^2 + (4*I)*f^2*Co
sh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] - 4*f^2*Sinh[2*e - (2*c*f)/d]*Si
nhIntegral[(2*f*(c + d*x))/d]))/(4*d^3)
```

3.107.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3799, 3042, 3795, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(a+a\sin(ie+ifx))^2}{(c+dx)^3}dx$$
$$\Big\downarrow \; 3799$$
$$4a^2\int \frac{\sinh^4\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)}{(c+dx)^3}dx$$
$$\Big\downarrow \; 3042$$

$$4a^2\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^4}{(c+dx)^3}dx$$

$$\Big\downarrow \; 3795$$

$$4a^2\left(\frac{2f^2\int \frac{\cosh^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{c+dx}dx}{d^2}-\frac{3f^2\int \frac{\cosh^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{c+dx}dx}{2d^2}-\frac{f\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{d^2(c+dx)}-\frac{\cosh^4\left(\right.}{2d}\right.$$

$$\Big\downarrow \; 3042$$

$$4a^2\left(-\frac{3f^2\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2}{c+dx}dx}{2d^2}+\frac{2f^2\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^4}{c+dx}dx}{d^2}-\frac{f\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{d^2(c+dx)}-\frac{\cosh^4\left(\right.}{2d}\right.$$

$$\Big\downarrow \; 3793$$

$$4a^2\left(-\frac{3f^2\int \left(\frac{i\sinh(e+fx)}{2(c+dx)}+\frac{1}{2(c+dx)}\right)dx}{2d^2}+\frac{2f^2\int \left(-\frac{\cosh(2e+2fx)}{8(c+dx)}+\frac{i\sinh(e+fx)}{2(c+dx)}+\frac{3}{8(c+dx)}\right)dx}{d^2}-\frac{f\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right.}{d^2(}\right.$$

$$\Big\downarrow \; 2009$$

$$4a^2\left(-\frac{3f^2\left(\frac{i\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{2d}+\frac{i\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d}+\frac{\log(c+dx)}{2d}\right)}{2d^2}+\frac{2f^2\left(\frac{i\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{2d}-\frac{\text{Chi}(}{}\right.}{}\right.$$

input
```
Int[(a + I*a*Sinh[e + f*x])^2/(c + d*x)^3,x]
```

output
```
4*a^2*(-1/2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^4/(d*(c + d*x)^2) - (f*Cosh[e/2
 + (I/4)*Pi + (f*x)/2]^3*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/(d^2*(c + d*x)) -
 (3*f^2*(Log[c + d*x]/(2*d) + ((I/2)*CoshIntegral[(c*f)/d + f*x]*Sinh[e -
(c*f)/d])/d + ((I/2)*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d))/(2
*d^2) + (2*f^2*(-1/8*(Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x
])/d + (3*Log[c + d*x])/(8*d) + ((I/2)*CoshIntegral[(c*f)/d + f*x]*Sinh[e
- (c*f)/d])/d + ((I/2)*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d -
(Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*d)))/d^2)
```

3.107. $\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx$

3.107.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.107.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(214) = 428.

Time = 2.53 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.65

method	result
risch	$-\frac{i a^2 f^2 \mathrm{e}^{f x+e}}{2 d^3 \left(\frac{c f}{d}+f x\right)^2}-\frac{i a^2 f^2 \mathrm{e}^{f x+e}}{2 d^3 \left(\frac{c f}{d}+f x\right)}-\frac{i a^2 f^2 \mathrm{e}^{-\frac{c f-d e}{d}} \mathrm{Ei}_1\left(-f x-e-\frac{c f-d e}{d}\right)}{2 d^3}-\frac{3 a^2}{4 d (d x+c)^2}-\frac{f^3 a^2 \mathrm{e}^{-2 f x-2 e} x}{4 d (d^2 x^2 f^2+2 c d f^2 x+c^2 f^2)}-$

input
```
int((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*a^2*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/2*I*a^2*f^2/d^3*exp(f*x+e)/(
c*f/d+f*x)-1/2*I*a^2*f^2/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)-3/
4*a^2/d/(d*x+c)^2-1/4*f^3*a^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c
^2*f^2)*x-1/4*f^3*a^2*exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2
)*c+1/8*f^2*a^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/2*f^
2*a^2/d^3*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)+1/8*f^2*a^2/d^3
*exp(2*f*x+2*e)/(c*f/d+f*x)^2+1/4*f^2*a^2/d^3*exp(2*f*x+2*e)/(c*f/d+f*x)+1
/2*f^2*a^2/d^3*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)-1/2*I*a^
2*f^3*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x-1/2*I*a^2*f^3*exp(
-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/2*I*a^2*f^2*exp(-f*x-e)/
d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/2*I*a^2*f^2/d^3*exp((c*f-d*e)/d)*Ei(
1,f*x+e+(c*f-d*e)/d)
```

3.107.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(209) = 418$.

Time = 0.26 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.92

$$\int \frac{(a + ia\sinh(e + fx))^2}{(c + dx)^3}\,dx =$$
$$-\frac{\left(2\,a^2d^2fx + 2\,a^2cdf - a^2d^2 - (2\,a^2d^2fx + 2\,a^2cdf + a^2d^2)e^{(4\,fx+4\,e)} + 4\,(i\,a^2d^2fx + i\,a^2cdf + i\,a^2d^2)e^{(3}\right.}{}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/8*(2*a^2*d^2*f*x + 2*a^2*c*d*f - a^2*d^2 - (2*a^2*d^2*f*x + 2*a^2*c*d*f
 + a^2*d^2)*e^(4*f*x + 4*e) + 4*(I*a^2*d^2*f*x + I*a^2*c*d*f + I*a^2*d^2)*
e^(3*f*x + 3*e) + 2*(3*a^2*d^2 + 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^
2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d)*e^(2*(d*e - c*f)/d) + 2*(-I*a^2*d^2*f^2*x
^2 - 2*I*a^2*c*d*f^2*x - I*a^2*c^2*f^2)*Ei((d*f*x + c*f)/d)*e^((d*e - c*f)
/d) + 2*(I*a^2*d^2*f^2*x^2 + 2*I*a^2*c*d*f^2*x + I*a^2*c^2*f^2)*Ei(-(d*f*x
 + c*f)/d)*e^(-(d*e - c*f)/d) + 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2
*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*e^(-2*(d*e - c*f)/d))*e^(2*f*x + 2*e) + 4
*(I*a^2*d^2*f*x + I*a^2*c*d*f - I*a^2*d^2)*e^(f*x + e))*e^(-2*f*x - 2*e)/(
d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.107.6 Sympy [F]

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx = -a^2\left(\int \frac{\sinh^2(e+fx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int\left(-\frac{2i\sinh(e+fx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right)dx + \int\left(-\frac{1}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right)dx\right)$$

input
```
integrate((a+I*a*sinh(f*x+e))**2/(d*x+c)**3,x)
```

output
```
-a**2*(Integral(sinh(e + f*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3), x) + Integral(-2*I*sinh(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**
2 + d**3*x**3), x) + Integral(-1/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3), x))
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.87

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx = -\frac{1}{4}a^2\left(\frac{1}{d^3x^2+2\,cd^2x+c^2d} - \frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_3\left(\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d} - \frac{e^{\left(2\,e-\frac{2\,cf}{d}\right)}E_3\left(-\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d}\right) + i\,a^2\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right) - \frac{a^2}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*a^2*(1/(d^3*x^2 + 2*c*d^2*x + c^2*d) - e^(-2*e + 2*c*f/d)*exp_integra
l_e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*d) - e^(2*e - 2*c*f/d)*exp_integral_e
(3, -2*(d*x + c)*f/d)/((d*x + c)^2*d)) + I*a^2*(e^(-e + c*f/d)*exp_integra
l_e(3, (d*x + c)*f/d)/((d*x + c)^2*d) - e^(e - c*f/d)*exp_integral_e(3, -(
d*x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

3.107.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 682 vs. 2(209) = 418.

Time = 0.27 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.89

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx =$$
$$-\frac{4\,a^2d^2f^2x^2\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(2\,e-\frac{2\,cf}{d}\right)}-4i\,a^2d^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+4i\,a^2d^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}}{}$$

input
```
integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/8*(4*a^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) - 4*I*a^2*
d^2*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 4*I*a^2*d^2*f^2*x^2*Ei(-(d
*f*x + c*f)/d)*e^(-e + c*f/d) + 4*a^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e
^(-2*e + 2*c*f/d) + 8*a^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d
) - 8*I*a^2*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 8*I*a^2*c*d*f^2*
x*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + 8*a^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f
)/d)*e^(-2*e + 2*c*f/d) + 4*a^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c
*f/d) - 4*I*a^2*c^2*f^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 4*I*a^2*c^2*f^
2*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + 4*a^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/
d)*e^(-2*e + 2*c*f/d) - 2*a^2*d^2*f*x*e^(2*f*x + 2*e) + 4*I*a^2*d^2*f*x*e^
(f*x + e) + 4*I*a^2*d^2*f*x*e^(-f*x - e) + 2*a^2*d^2*f*x*e^(-2*f*x - 2*e)
- 2*a^2*c*d*f*e^(2*f*x + 2*e) + 4*I*a^2*c*d*f*e^(f*x + e) + 4*I*a^2*c*d*f*
e^(-f*x - e) + 2*a^2*c*d*f*e^(-2*f*x - 2*e) - a^2*d^2*e^(2*f*x + 2*e) + 4*
I*a^2*d^2*e^(f*x + e) - 4*I*a^2*d^2*e^(-f*x - e) - a^2*d^2*e^(-2*f*x - 2*e
) + 6*a^2*d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx = \int \frac{\left(a+a\sinh(e+f\,x)\,1\mathrm{i}\right)^2}{\left(c+d\,x\right)^3}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^2/(c + d*x)^3,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^2/(c + d*x)^3, x)
```

3.108 $\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx$

3.108.1 Optimal result

Integrand size = 23, antiderivative size = 132

$$\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx = \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{af^2} - \frac{12d^2(c+dx)\operatorname{PolyLog}\left(2,-ie^{e+fx}\right)}{af^3} + \frac{12d^3\operatorname{PolyLog}\left(3,-ie^{e+fx}\right)}{af^4} + \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}$$

output
```
(d*x+c)^3/a/f-6*d*(d*x+c)^2*ln(1+I*exp(f*x+e))/a/f^2-12*d^2*(d*x+c)*polylo
g(2,-I*exp(f*x+e))/a/f^3+12*d^3*polylog(3,-I*exp(f*x+e))/a/f^4+(d*x+c)^3*t
anh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f
```

3.108.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.56

$$\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx = \frac{2\left(\frac{3de^e\left(\frac{e^{-e}(c+dx)^3}{3d}+\frac{\left(i+e^{-e}\right)(c+dx)^2\log\left(1-ie^{-e-fx}\right)}{f}-\frac{2ide^{-e}\left(-i+e^e\right)\left(f(c+dx)\operatorname{PolyLog}\left(2,ie^{-e-fx}\right)+d\operatorname{PolyLog}\left(3,ie^{-e-fx}\right)\right)}{f^3}\right)}{-1-ie^e}+\frac{}{\left(\cosh\left(\frac{e}{2}\right)+\right.}\right.}{af}$$

input
```
Integrate[(c + d*x)^3/(a + I*a*Sinh[e + f*x]),x]
```

output
```
(2*((3*d*E^e*((c + d*x)^3/(3*d*E^e) + ((I + E^(-e))*(c + d*x)^2*Log[1 - I*
E^(-e - f*x)])/f - ((2*I)*d*(-I + E^e)*(f*(c + d*x)*PolyLog[2, I*E^(-e - f
*x)] + d*PolyLog[3, I*E^(-e - f*x)]))/(E^e*f^3)))/(-1 - I*E^e) + ((c + d*x
)^3*Sinh[(f*x)/2])/((Cosh[e/2] + I*Sinh[e/2])*(Cosh[(e + f*x)/2] + I*Sinh[
(e + f*x)/2]))))/(a*f)
```

3.108.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 4199, 26, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^3}{a+a\sin(ie+ifx)}dx$$
$$\Big\downarrow \text{3799}$$
$$\frac{\int -(c+dx)^3\text{csch}^2\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)dx}{2a}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\int -(c+dx)^3\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int (c+dx)^3\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int (c+dx)^3\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6id\int -i(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6d\int (c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6d\int -i(c+dx)^2\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{6id\int (c+dx)^2\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \text{4199}$$

$$\frac{\frac{6id\left(2i\int \frac{ie^{e+fx}(c+dx)^2}{1+ie^{e+fx}}dx-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{6id\left(-2\int \frac{e^{e+fx}(c+dx)^2}{1+ie^{e+fx}}dx-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{6id\left(-2\left(\frac{2id\int (c+dx)\log\left(1+ie^{e+fx}\right)dx}{f}-\frac{i(c+dx)^2\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\frac{6id\left(-2\left(\frac{2id\left(\frac{d\int \operatorname{PolyLog}\left(2,-ie^{e+fx}\right)dx}{f}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-ie^{e+fx}\right)}{f}\right)}{f}-\frac{i(c+dx)^2\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \text{2720}$$

3.108. $\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx$

$$\frac{\frac{6id\left(-2\left(\frac{2id\left(\frac{d\int e^{-e-fx}\operatorname{PolyLog}\left(2,-ie^{e+fx}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-ie^{e+fx}\right)}{f}\right)}{f}-\frac{i(c+dx)^2\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}\right.}{f}}{2a}$$

$$\Big\downarrow \ 7143$$

$$\frac{\frac{6id\left(-2\left(\frac{2id\left(\frac{d\operatorname{PolyLog}\left(3,-ie^{e+fx}\right)}{f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-ie^{e+fx}\right)}{f}\right)}{f}-\frac{i(c+dx)^2\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

input
```
Int[(c + d*x)^3/(a + I*a*Sinh[e + f*x]),x]
```

output
```
(((6*I)*d*(((-1/3*I)*(c + d*x)^3)/d - 2*(((-I)*(c + d*x)^2*Log[1 + I*E^(e
+ f*x)])/f + ((2*I)*d*(-(((c + d*x)*PolyLog[2, (-I)*E^(e + f*x)])/f) + (d*
PolyLog[3, (-I)*E^(e + f*x)])/f^2))/f)))/f + (2*(c + d*x)^3*Tanh[e/2 + (I/
4)*Pi + (f*x)/2])/f)/(2*a)
```

3.108.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.108. $\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx$

3.108.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(119) = 238$.

Time = 1.44 (sec) , antiderivative size = 435, normalized size of antiderivative = 3.30

method	result
risch	$\frac{2i(d^3x^3+3c\,d^2x^2+3dx\,c^2+c^3)}{fa(e^{fx+e}-i)}+\frac{6d^2c\,x^2}{af}+\frac{2d^3x^3}{af}+\frac{12d^2cex}{a\,f^2}+\frac{6d^2c\,e^2}{a\,f^3}+\frac{6d^3\ln(1+ie^{fx+e})e^2}{a\,f^4}-\frac{4d^3e^3}{a\,f^4}-\frac{12d^3\,\mathrm{polylog}(\cdots}{a\,f\cdots}$

input
```
int((d*x+c)^3/(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(f*x+e)-I)+6/a/f*d^2*c*x^2
+2/a/f*d^3*x^3+12/a/f^2*d^2*c*e*x+6/a/f^3*d^2*c*e^2+6/a/f^4*d^3*ln(1+I*exp
(f*x+e))*e^2-4/a/f^4*d^3*e^3-12/a/f^3*d^3*polylog(2,-I*exp(f*x+e))*x-6/a/f
^2*d^3*ln(1+I*exp(f*x+e))*x^2+12*d^3*polylog(3,-I*exp(f*x+e))/a/f^4-12/a/f
^2*d^2*c*ln(1+I*exp(f*x+e))*x-6/a/f^4*d^3*e^2*ln(exp(f*x+e)-I)+6/a/f^4*d^3
*e^2*ln(exp(f*x+e))+12/a/f^3*d^2*c*e*ln(exp(f*x+e)-I)-12/a/f^3*d^2*c*e*ln(
exp(f*x+e))-6/a/f^2*d*ln(exp(f*x+e)-I)*c^2-12/a/f^3*d^2*c*ln(1+I*exp(f*x+e
))*e-12/a/f^3*d^2*c*polylog(2,-I*exp(f*x+e))-6/a/f^3*d^3*e^2*x+6/a/f^2*d*l
n(exp(f*x+e))*c^2
```

3.108.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(114) = 228$.

Time = 0.25 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.76

$$\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx =$$
$$-\frac{2\left(i\,d^3e^3-3i\,cd^2e^2f+3i\,c^2def^2-i\,c^3f^3+6\left(-i\,d^3fx-i\,cd^2f+(d^3fx+cd^2f)e^{(fx+e)}\right)\mathrm{Li}_2\left(-i\,e^{(fx+e)}\right)\cdots}{\cdots}$$

input
```
integrate((d*x+c)^3/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
-2*(I*d^3*e^3 - 3*I*c*d^2*e^2*f + 3*I*c^2*d*e*f^2 - I*c^3*f^3 + 6*(-I*d^3*
f*x - I*c*d^2*f + (d^3*f*x + c*d^2*f)*e^(f*x + e))*dilog(-I*e^(f*x + e)) -
 (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + d^3*e^3 - 3*c*d^2*e^2*f
+ 3*c^2*d*e*f^2)*e^(f*x + e) + 3*(-I*d^3*e^2 + 2*I*c*d^2*e*f - I*c^2*d*f^2
 + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*e^(f*x + e))*log(e^(f*x + e) - I) +
 3*(-I*d^3*f^2*x^2 - 2*I*c*d^2*f^2*x + I*d^3*e^2 - 2*I*c*d^2*e*f + (d^3*f^
2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*e^(f*x + e))*log(I*e^(f*x +
 e) + 1) - 6*(d^3*e^(f*x + e) - I*d^3)*polylog(3, -I*e^(f*x + e)))/(a*f^4*
e^(f*x + e) - I*a*f^4)
```

3.108.6 Sympy [F]

$$\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx = \frac{2ic^3+6ic^2dx+6icd^2x^2+2id^3x^3}{afe^ee^{fx}-iaf} - \frac{6id\left(\int \frac{c^2}{e^ee^{fx}-i}\,dx+\int \frac{d^2x^2}{e^ee^{fx}-i}\,dx+\int \frac{2cdx}{e^ee^{fx}-i}\,dx\right)}{af}$$

input
```
integrate((d*x+c)**3/(a+I*a*sinh(f*x+e)),x)
```

output
```
(2*I*c**3 + 6*I*c**2*d*x + 6*I*c*d**2*x**2 + 2*I*d**3*x**3)/(a*f*exp(e)*ex
p(f*x) - I*a*f) - 6*I*d*(Integral(c**2/(exp(e)*exp(f*x) - I), x) + Integra
l(d**2*x**2/(exp(e)*exp(f*x) - I), x) + Integral(2*c*d*x/(exp(e)*exp(f*x)
- I), x))/(a*f)
```

3.108.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(114) = 228$.

Time = 0.31 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.80

$$\begin{aligned}&\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx\\&=6\,c^2d\left(\frac{xe^{(fx+e)}}{afe^{(fx+e)}-i\,af}-\frac{\log\left(\left(e^{(fx+e)}-i\right)e^{(-e)}\right)}{af^2}\right)-\frac{2\,c^3}{(i\,ae^{(-fx-e)}-a)f}\\&\quad-\frac{2\,(-i\,d^3x^3-3i\,cd^2x^2)}{afe^{(fx+e)}-i\,af}-\frac{12\,\left(fx\log\left(i\,e^{(fx+e)}+1\right)+\mathrm{Li}_2\left(-i\,e^{(fx+e)}\right)\right)cd^2}{af^3}\\&\quad-\frac{6\,\left(f^2x^2\log\left(i\,e^{(fx+e)}+1\right)+2\,fx\mathrm{Li}_2\left(-i\,e^{(fx+e)}\right)-2\,\mathrm{Li}_3\left(-i\,e^{(fx+e)}\right)\right)d^3}{af^4}\\&\quad+\frac{2\,(d^3f^3x^3+3\,cd^2f^3x^2)}{af^4}\end{aligned}$$

input
```
integrate((d*x+c)^3/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
6*c^2*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) - I*a*f) - log((e^(f*x + e) - I)*e
^(-e))/(a*f^2)) - 2*c^3/((I*a*e^(-f*x - e) - a)*f) - 2*(-I*d^3*x^3 - 3*I*c
*d^2*x^2)/(a*f*e^(f*x + e) - I*a*f) - 12*(f*x*log(I*e^(f*x + e) + 1) + dil
og(-I*e^(f*x + e)))*c*d^2/(a*f^3) - 6*(f^2*x^2*log(I*e^(f*x + e) + 1) + 2*
f*x*dilog(-I*e^(f*x + e)) - 2*polylog(3, -I*e^(f*x + e)))*d^3/(a*f^4) + 2*
(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2)/(a*f^4)
```

3.108.8 Giac [F]

$$\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx=\int \frac{(dx+c)^3}{i\,a\sinh\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^3/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(I*a*sinh(f*x + e) + a), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx = \int \frac{(c+d\,x)^3}{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((c + d*x)^3/(a + a*sinh(e + f*x)*1i),x)
```

output
```
int((c + d*x)^3/(a + a*sinh(e + f*x)*1i), x)
```

3.109 $\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx$

3.109.1 Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx = \frac{(c+dx)^2}{af} - \frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{af^2} - \frac{4d^2\,\text{PolyLog}\left(2,-ie^{e+fx}\right)}{af^3} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}$$

output
```
(d*x+c)^2/a/f-4*d*(d*x+c)*ln(1+I*exp(f*x+e))/a/f^2-4*d^2*polylog(2,-I*exp(
f*x+e))/a/f^3+(d*x+c)^2*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f
```

3.109.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx = \frac{2\left(\frac{\frac{if(c+dx)\left(f(c+dx)+2d(1+ie^e)\log\left(1-ie^{-e-fx}\right)\right)}{-i+e^e}+2d^2\,\text{PolyLog}\left(2,ie^{-e-fx}\right)}{f^2}+\frac{(c+dx)^2\sinh\left(\frac{fx}{2}\right)}{\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}\right)}{af}$$

input
```
Integrate[(c + d*x)^2/(a + I*a*Sinh[e + f*x]),x]
```

output
```
(2*(((I*f*(c + d*x)*(f*(c + d*x) + 2*d*(1 + I*E^e)*Log[1 - I*E^(-e - f*x)]
))/(-I + E^e) + 2*d^2*PolyLog[2, I*E^(-e - f*x)])/f^2 + ((c + d*x)^2*Sinh[
(f*x)/2])/((Cosh[e/2] + I*Sinh[e/2])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)
/2]))))/(a*f)
```

3.109.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 4199, 26, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{(c+dx)^2}{a+a\sin(ie+ifx)}dx$$
$$\Big\downarrow \ 3799$$
$$\frac{\int -(c+dx)^2\text{csch}^2\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)dx}{2a}$$
$$\Big\downarrow \ 25$$
$$-\frac{\int -(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$
$$\Big\downarrow \ 25$$
$$\frac{\int (c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$
$$\Big\downarrow \ 3042$$
$$\frac{\int (c+dx)^2\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$
$$\Big\downarrow \ 4672$$
$$\frac{\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4id\int -i(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\int(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\int-i(c+dx)\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{4id\int(c+dx)\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 4199$$

$$\frac{\frac{4id\left(2i\int\frac{ie^{e+fx}(c+dx)}{1+ie^{e+fx}}dx-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{4id\left(-2\int\frac{e^{e+fx}(c+dx)}{1+ie^{e+fx}}dx-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 2620$$

$$\frac{\frac{4id\left(-2\left(\frac{id\int\log\left(1+ie^{e+fx}\right)dx}{f}-\frac{i(c+dx)\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 2715$$

$$\frac{\frac{4id\left(-2\left(\frac{id\int e^{-e-fx}\log\left(1+ie^{e+fx}\right)de^{e+fx}}{f^2}-\frac{i(c+dx)\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \; 2838$$

$$\frac{\frac{4id\left(-2\left(-\frac{i(c+dx)\log\left(1+ie^{e+fx}\right)}{f}-\frac{id\,\mathrm{PolyLog}\left(2,-ie^{e+fx}\right)}{f^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

input
```
Int[(c + d*x)^2/(a + I*a*Sinh[e + f*x]),x]
```

3.109. $\int\frac{(c+dx)^2}{a+ia\sinh(e+fx)}dx$

output
```
(((4*I)*d*(((-1/2*I)*(c + d*x)^2)/d - 2*(((-I)*(c + d*x)*Log[1 + I*E^(e +
f*x)])/f - (I*d*PolyLog[2, (-I)*E^(e + f*x)])/f^2)))/f + (2*(c + d*x)^2*Ta
nh[e/2 + (I/4)*Pi + (f*x)/2])/f)/(2*a)
```

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.109.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(90) = 180$.

Time = 1.34 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.25

method	result
risch	$\frac{2i\left(d^{2} x^{2}+2 c d x+c^{2}\right)}{f a\left(\mathrm{e}^{f x+e}-i\right)}-\frac{4 d \ln\left(\mathrm{e}^{f x+e}-i\right) c}{a\,f^{2}}+\frac{4 d \ln\left(\mathrm{e}^{f x+e}\right) c}{a\,f^{2}}+\frac{2 d^{2} x^{2}}{a f}+\frac{4 d^{2} e x}{a\,f^{2}}+\frac{2 d^{2} e^{2}}{a\,f^{3}}-\frac{4 d^{2} \ln\left(1+i \mathrm{e}^{f x+e}\right) x}{a\,f^{2}}-\frac{4 d^{2} \ln\left(1+i\mathrm{e}\right.}{a\,f^{3}}$

input
```
int((d*x+c)^2/(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2*I*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(f*x+e)-I)-4/a/f^2*d*ln(exp(f*x+e)-I)*c+
4/a/f^2*d*ln(exp(f*x+e))*c+2/a/f*d^2*x^2+4/a/f^2*d^2*e*x+2/a/f^3*d^2*e^2-4
/a/f^2*d^2*ln(1+I*exp(f*x+e))*x-4/a/f^3*d^2*ln(1+I*exp(f*x+e))*e-4*d^2*pol
ylog(2,-I*exp(f*x+e))/a/f^3+4/a/f^3*d^2*e*ln(exp(f*x+e)-I)-4/a/f^3*d^2*e*l
n(exp(f*x+e))
```

3.109.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(86) = 172$.

Time = 0.24 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.99

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx = -\frac{2\left(-i\,d^2e^2+2i\,cdef-i\,c^2f^2+2\left(d^2e^{(fx+e)}-i\,d^2\right)\mathrm{Li}_2\left(-i\,e^{(fx+e)}\right)-\left(d^2f^2x^2+2\,cdf^2x-d^2e^2+2\,cdef\right.}{}$$

input
```
integrate((d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
-2*(-I*d^2*e^2 + 2*I*c*d*e*f - I*c^2*f^2 + 2*(d^2*e^(f*x + e) - I*d^2)*dil
og(-I*e^(f*x + e)) - (d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*e^(
f*x + e) + 2*(I*d^2*e - I*c*d*f - (d^2*e - c*d*f)*e^(f*x + e))*log(e^(f*x
+ e) - I) + 2*(-I*d^2*f*x - I*d^2*e + (d^2*f*x + d^2*e)*e^(f*x + e))*log(I
*e^(f*x + e) + 1))/(a*f^3*e^(f*x + e) - I*a*f^3)
```

3.109.6 Sympy [F]

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx = \frac{2ic^2+4icdx+2id^2x^2}{afe^e e^{fx}-iaf} - \frac{4id\left(\int \frac{c}{e^e e^{fx}-i}\,dx + \int \frac{dx}{e^e e^{fx}-i}\,dx\right)}{af}$$

input
```
integrate((d*x+c)**2/(a+I*a*sinh(f*x+e)),x)
```

output
```
(2*I*c**2 + 4*I*c*d*x + 2*I*d**2*x**2)/(a*f*exp(e)*exp(f*x) - I*a*f) - 4*I
*d*(Integral(c/(exp(e)*exp(f*x) - I), x) + Integral(d*x/(exp(e)*exp(f*x) -
 I), x))/(a*f)
```

3.109.7 Maxima [F]

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx = \int \frac{(dx+c)^2}{i\,a\sinh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
-2*d^2*(-I*x^2/(a*f*e^(f*x + e) - I*a*f) + 2*I*integrate(x/(a*f*e^(f*x + e
) - I*a*f), x)) + 4*c*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) - I*a*f) - log((e^
(f*x + e) - I)*e^(-e))/(a*f^2)) - 2*c^2/((I*a*e^(-f*x - e) - a)*f)
```

3.109.8 Giac [F]

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx = \int \frac{(dx+c)^2}{i\,a\sinh\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(I*a*sinh(f*x + e) + a), x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx = \int \frac{(c+d\,x)^2}{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((c + d*x)^2/(a + a*sinh(e + f*x)*1i),x)
```

output
```
int((c + d*x)^2/(a + a*sinh(e + f*x)*1i), x)
```

3.110 $\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx$

3.110.1 Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx=-\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{af^2}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}$$

output
```
-2*d*ln(cosh(1/2*e+1/4*I*Pi+1/2*f*x))/a/f^2+(d*x+c)*tanh(1/2*e+1/4*I*Pi+1/
2*f*x)/a/f
```

3.110.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 185 vs. 2(63) = 126.

Time = 0.38 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.94

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx$$
$$=\frac{idfx\cosh\left(e+\frac{fx}{2}\right)+\cosh\left(\frac{fx}{2}\right)\left(-2id\arctan\left(\operatorname{sech}\left(e+\frac{fx}{2}\right)\sinh\left(\frac{fx}{2}\right)\right)-d\log(\cosh(e+fx))\right)+2cf\sinh}{af^2\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)\left(\cos\right.}$$

input
```
Integrate[(c + d*x)/(a + I*a*Sinh[e + f*x]),x]
```

output
```
(I*d*f*x*Cosh[e + (f*x)/2] + Cosh[(f*x)/2]*((-2*I)*d*ArcTan[Sech[e + (f*x)
/2]*Sinh[(f*x)/2]] - d*Log[Cosh[e + f*x]]) + 2*c*f*Sinh[(f*x)/2] + d*f*x*S
inh[(f*x)/2] + 2*d*ArcTan[Sech[e + (f*x)/2]*Sinh[(f*x)/2]]*Sinh[e + (f*x)/
2] - I*d*Log[Cosh[e + f*x]]*Sinh[e + (f*x)/2])/(a*f^2*(Cosh[e/2] + I*Sinh[
e/2])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

3.110.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{c+dx}{a+a\sin(ie+ifx)}dx$$

$$\Big\downarrow \ 3799$$

$$\frac{\int -\left((c+dx)\text{csch}^2\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)\right)dx}{2a}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -\left((c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)dx}{2a}$$

$$\Big\downarrow \ 25$$

$$\frac{\int (c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int (c+dx)\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \ 4672$$

$$\frac{\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2id\int -i\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2d\int \tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2d\int -i\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\frac{2id\int \tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}}{2a}$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{f^2}}{2a}$$

input
```
Int[(c + d*x)/(a + I*a*Sinh[e + f*x]),x]
```

output
```
((-4*d*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/f^2 + (2*(c + d*x)*Tanh[e/2 +
(I/4)*Pi + (f*x)/2])/f)/(2*a)
```

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.110.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

method	result
risch	$\frac{2dx}{fa}+\frac{2de}{f^2a}+\frac{2i(dx+c)}{fa(\mathrm{e}^{fx+e}-i)}-\frac{2d\ln(\mathrm{e}^{fx+e}-i)}{f^2a}$
parallelrisch	$\frac{2\left(i-\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d\ln\left(1-\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-2\left(i-\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d\ln\left(-i+\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+f\left((-1+i)xd\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)+(-\right.}{f^2a\left(i-\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}$

input
```
int((d*x+c)/(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2*d/f/a*x+2*d/f^2/a*e+2*I*(d*x+c)/f/a/(exp(f*x+e)-I)-2*d/f^2/a*ln(exp(f*x+
e)-I)
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx=\frac{2\left(dfxe^{(fx+e)}+i\,cf-\left(de^{(fx+e)}-i\,d\right)\log\left(e^{(fx+e)}-i\right)\right)}{af^2e^{(fx+e)}-i\,af^2}$$

input
```
integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
2*(d*f*x*e^(f*x + e) + I*c*f - (d*e^(f*x + e) - I*d)*log(e^(f*x + e) - I))
/(a*f^2*e^(f*x + e) - I*a*f^2)
```

3.110.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx = \frac{2ic+2idx}{afe^{e}e^{fx}-iaf} + \frac{2dx}{af} - \frac{2d\log\left(e^{fx}-ie^{-e}\right)}{af^{2}}$$

input
```
integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

output
```
(2*I*c + 2*I*d*x)/(a*f*exp(e)*exp(f*x) - I*a*f) + 2*d*x/(a*f) - 2*d*log(ex
p(f*x) - I*exp(-e))/(a*f**2)
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx = 2\,d\left(\frac{xe^{(fx+e)}}{afe^{(fx+e)}-i\,af} - \frac{\log\left(\left(e^{(fx+e)}-i\right)e^{(-e)}\right)}{af^{2}}\right) - \frac{2\,c}{(i\,ae^{(-fx-e)}-a)f}$$

input
```
integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
2*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) - I*a*f) - log((e^(f*x + e) - I)*e^(-e
))/(a*f^2)) - 2*c/((I*a*e^(-f*x - e) - a)*f)
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx = \frac{2\left(dfxe^{(fx+e)} - de^{(fx+e)}\log\left(e^{(fx+e)}-i\right) + i\,cf + i\,d\log\left(e^{(fx+e)}-i\right)\right)}{af^2e^{(fx+e)} - i\,af^2}$$

input
```
integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
2*(d*f*x*e^(f*x + e) - d*e^(f*x + e)*log(e^(f*x + e) - I) + I*c*f + I*d*lo
g(e^(f*x + e) - I))/(a*f^2*e^(f*x + e) - I*a*f^2)
```

3.110.9 Mupad [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx = \frac{(c+d\,x)\;2\mathrm{i}}{a\,f\,\left(\mathrm{e}^{e+f\,x}-\mathrm{i}\right)} + \frac{2\,d\,x}{a\,f} - \frac{2\,d\,\ln\left(\mathrm{e}^{f\,x}\,\mathrm{e}^{e}-\mathrm{i}\right)}{a\,f^2}$$

input
```
int((c + d*x)/(a + a*sinh(e + f*x)*1i),x)
```

output
```
((c + d*x)*2i)/(a*f*(exp(e + f*x) - 1i)) + (2*d*x)/(a*f) - (2*d*log(exp(f*
x)*exp(e) - 1i))/(a*f^2)
```

3.111 $\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx$

3.111.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \mathrm{Int}\left(\frac{1}{(c+dx)(a+ia\sinh(e+fx))},x\right)$$

output
```
Unintegrable(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

3.111.2 Mathematica [N/A]

Not integrable

Time = 26.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + I*a*Sinh[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)*(a + I*a*Sinh[e + f*x])), x]
```

3.111.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)(a+a\sin(ie+ifx))}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}dx$$

input
```
Int[1/((c + d*x)*(a + I*a*Sinh[e + f*x])),x]
```

output
```
$Aborted
```

3.111.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.111.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(dx+c)\,(a+ia\sinh{(fx+e)})}dx$$

input
```
int(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

output
```
int(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

3.111.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 5.48

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)(i\,a\sinh{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
((-I*a*d*f*x - I*a*c*f + (a*d*f*x + a*c*f)*e^(f*x + e))*integral(2*I*d/(-I
*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*
c^2*f)*e^(f*x + e)), x) + 2*I)/(-I*a*d*f*x - I*a*c*f + (a*d*f*x + a*c*f)*e
^(f*x + e))
```

3.111.6 Sympy [N/A]

Not integrable

Time = 3.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 5.09

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \frac{2i}{-iacf-iadfx+(acfe^{e}+adfxe^{e})\,e^{fx}} + \frac{2id\int \frac{1}{c^2e^{e}e^{fx}-ic^2+2cdxe^{e}e^{fx}-2icdx+d^2x^2e^{e}e^{fx}-id^2x^2}\,dx}{af}$$

input
```
integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

output
```
2*I/(-I*a*c*f - I*a*d*f*x + (a*c*f*exp(e) + a*d*f*x*exp(e))*exp(f*x)) + 2*
I*d*Integral(1/(c**2*exp(e)*exp(f*x) - I*c**2 + 2*c*d*x*exp(e)*exp(f*x) -
2*I*c*d*x + d**2*x**2*exp(e)*exp(f*x) - I*d**2*x**2), x)/(a*f)
```

3.111.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.43

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)(i\,a\sinh{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
2*I*d*integrate(1/(-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f + (a*d^2*f*x
^2*e^e + 2*a*c*d*f*x*e^e + a*c^2*f*e^e)*e^(f*x)), x) + 2*I/(-I*a*d*f*x - I
*a*c*f + (a*d*f*x*e^e + a*c*f*e^e)*e^(f*x))
```

3.111.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)(i\,a\sinh{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(I*a*sinh(f*x + e) + a)), x)
```

3.111.9 Mupad [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(a+a\sinh(e+f\,x)\ 1\mathrm{i})\ (c+d\,x)}\,dx$$

input
```
int(1/((a + a*sinh(e + f*x)*1i)*(c + d*x)),x)
```

output
```
int(1/((a + a*sinh(e + f*x)*1i)*(c + d*x)), x)
```

3.112 $\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx$

3.112.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+ia\sinh(e+fx))},x\right)$$

output
```
Unintegrable(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x)
```

3.112.2 Mathematica [N/A]

Not integrable

Time = 26.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])), x]
```

3.112.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(c+dx)^2(a+a\sin(ie+ifx))}dx$$
$$\Big\downarrow \text{ 3807}$$
$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}dx$$

input
```
Int[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])),x]
```

output
```
$Aborted
```

3.112.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.112.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(dx+c)^2 (a+ia\sinh(fx+e))}dx$$

input
```
int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x)
```

output
```
int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x)
```

3.112.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 204, normalized size of antiderivative = 8.87

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(i\,a\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
((-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f + (a*d^2*f*x^2 + 2*a*c*d*f*x
+ a*c^2*f)*e^(f*x + e))*integral(4*I*d/(-I*a*d^3*f*x^3 - 3*I*a*c*d^2*f*x^2
 - 3*I*a*c^2*d*f*x - I*a*c^3*f + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*
d*f*x + a*c^3*f)*e^(f*x + e)), x) + 2*I)/(-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x -
 I*a*c^2*f + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*e^(f*x + e))
```

3.112.6 Sympy [N/A]

Not integrable

Time = 10.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 8.13

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx$$
$$= \frac{2i}{-iac^2f-2iacdfx-iad^2fx^2+(ac^2fe^e+2acdfxe^e+ad^2fx^2e^e)\,e^{fx}} + \frac{4id\int \frac{1}{c^3e^ee^{fx}-ic^3+3c^2dxe^ee^{fx}-3ic^2dx+3cd^2x^2e^ee^{fx}-3icd^2x^2+d^3x^3e^ee^{fx}-id^3x^3}\,dx}{af}$$

input
```
integrate(1/(d*x+c)**2/(a+I*a*sinh(f*x+e)),x)
```

output
```
2*I/(-I*a*c**2*f - 2*I*a*c*d*f*x - I*a*d**2*f*x**2 + (a*c**2*f*exp(e) + 2*
a*c*d*f*x*exp(e) + a*d**2*f*x**2*exp(e))*exp(f*x)) + 4*I*d*Integral(1/(c**
3*exp(e)*exp(f*x) - I*c**3 + 3*c**2*d*x*exp(e)*exp(f*x) - 3*I*c**2*d*x + 3
*c*d**2*x**2*exp(e)*exp(f*x) - 3*I*c*d**2*x**2 + d**3*x**3*exp(e)*exp(f*x)
 - I*d**3*x**3), x)/(a*f)
```

3.112.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 6.87

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(i\,a\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
4*I*d*integrate(1/(-I*a*d^3*f*x^3 - 3*I*a*c*d^2*f*x^2 - 3*I*a*c^2*d*f*x -
I*a*c^3*f + (a*d^3*f*x^3*e^e + 3*a*c*d^2*f*x^2*e^e + 3*a*c^2*d*f*x*e^e + a
*c^3*f*e^e)*e^(f*x)), x) + 2*I/(-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f
 + (a*d^2*f*x^2*e^e + 2*a*c*d*f*x*e^e + a*c^2*f*e^e)*e^(f*x))
```

3.112.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(i\,a\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(I*a*sinh(f*x + e) + a)), x)
```

3.112.9 Mupad [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(a+a\sinh\left(e+f\,x\right)\,1\mathrm{i})\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((a + a*sinh(e + f*x)*1i)*(c + d*x)^2),x)
```

output
```
int(1/((a + a*sinh(e + f*x)*1i)*(c + d*x)^2), x)
```

3.113 $\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx$

3.113.1 Optimal result

Integrand size = 23, antiderivative size = 305

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx &= \frac{(c+dx)^3}{3a^2f}-\frac{2d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{a^2f^2}\\&\quad+\frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2f^4}\\&\quad-\frac{4d^2(c+dx)\operatorname{PolyLog}\left(2,-ie^{e+fx}\right)}{a^2f^3}\\&\quad+\frac{4d^3\operatorname{PolyLog}\left(3,-ie^{e+fx}\right)}{a^2f^4}+\frac{d(c+dx)^2\operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2a^2f^2}\\&\quad-\frac{2d^2(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{a^2f^3}\\&\quad+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f}\\&\quad+\frac{(c+dx)^3\operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f}\end{aligned}$$

output
```
1/3*(d*x+c)^3/a^2/f-2*d*(d*x+c)^2*ln(1+I*exp(f*x+e))/a^2/f^2+4*d^3*ln(cosh
(1/2*e+1/4*I*Pi+1/2*f*x))/a^2/f^4-4*d^2*(d*x+c)*polylog(2,-I*exp(f*x+e))/a
^2/f^3+4*d^3*polylog(3,-I*exp(f*x+e))/a^2/f^4+1/2*d*(d*x+c)^2*sech(1/2*e+1
/4*I*Pi+1/2*f*x)^2/a^2/f^2-2*d^2*(d*x+c)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/
f^3+1/3*(d*x+c)^3*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f+1/6*(d*x+c)^3*sech(1/
2*e+1/4*I*Pi+1/2*f*x)^2*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f
```

3.113.2 Mathematica [A] (verified)

Time = 2.89 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.55

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx$$
$$= \frac{\frac{2d\left(-6d^2x+3c^2f^2x+3(1+ie^e)(2d^2-c^2f^2)x+3cdf^2x^2+d^2f^2x^3+6cd(1+ie^e)fx\log\left(1-ie^{-e-fx}\right)+3d^2(1+ie^e)fx^2\log\left(1-ie^{-e-fx}\right)+\frac{3(1+ie^e)\left(-2\right.}{}}{-1-ie^e}}{}$$

input
```
Integrate[(c + d*x)^3/(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
((2*d*(-6*d^2*x + 3*c^2*f^2*x + 3*(1 + I*E^e)*(2*d^2 - c^2*f^2)*x + 3*c*d*
f^2*x^2 + d^2*f^2*x^3 + 6*c*d*(1 + I*E^e)*f*x*Log[1 - I*E^(-e - f*x)] + 3*
d^2*(1 + I*E^e)*f*x^2*Log[1 - I*E^(-e - f*x)] + (3*(1 + I*E^e)*(-2*d^2 + c
^2*f^2)*Log[I - E^(e + f*x)])/f - 6*c*d*(1 + I*E^e)*PolyLog[2, I*E^(-e - f
*x)] - 6*d^2*(1 + I*E^e)*x*PolyLog[2, I*E^(-e - f*x)] - (6*d^2*(1 + I*E^e)
*PolyLog[3, I*E^(-e - f*x)])/f))/(-1 - I*E^e) + ((c + d*x)*(3*d*f*(c + d*x
)*Cosh[(f*x)/2] + (6*I)*d^2*Cosh[e + (f*x)/2] + I*(c^2*f^2 + 2*c*d*f^2*x +
 d^2*(-6 + f^2*x^2))*Cosh[e + (3*f*x)/2] + 3*(c^2*f^2 + 2*c*d*f^2*x + d^2*
(-4 + f^2*x^2))*Sinh[(f*x)/2] + (3*I)*d*f*(c + d*x)*Sinh[e + (f*x)/2]))/((
Cosh[e/2] + I*Sinh[e/2])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^3))/(3*
a^2*f^3)
```

3.113.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.01, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.696, Rules used = {3042, 3799, 3042, 4674, 3042, 4672, 26, 3042, 26, 3956, 4199, 26, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^3}{(a+a\sin(ie+ifx))^2}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)^3\text{csch}^4\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)^3\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^4dx}{4a^2}$$

$$\Big\downarrow \text{4674}$$

$$\frac{-\frac{4d^2\int (c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f^2}+\frac{2}{3}\int (c+dx)^3\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx+\frac{2d(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}\right.}{}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{4d^2\int (c+dx)\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2dx}{f^2}+\frac{2}{3}\int (c+dx)^3\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2dx+\frac{2d(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\right.}{}}{4a^2}$$

$$\Big\downarrow \text{4672}$$

$$\frac{-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2id\int -i\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6id\int -i(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{4a^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2d\int \tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6d\int (c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)+\frac{2d(}{}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2d\int -i\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6d\int -i(c+dx)^2\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)+}{4a^2}$$

$$\Big\downarrow \text{26}$$

3.113. $\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx$

$$\frac{-\frac{4d^2\left(\frac{2id\int \tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{6id\int (c+dx)^2\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{2d}{}}{4a^2}$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\frac{2}{3}\left(\frac{6id\int (c+dx)^2\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{f^2}\right)}{f^2}+\frac{2d(c}{}}{4a^2}$$

$$\Big\downarrow \text{ 4199}$$

$$\frac{\frac{2}{3}\left(\frac{6id\left(2i\int \frac{ie^{e+fx}(c+dx)^2}{1+ie^{e+fx}}dx-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{f^2}\right)}{f^2}+}{4a^2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\frac{2}{3}\left(\frac{6id\left(-2\int \frac{e^{e+fx}(c+dx)^2}{1+ie^{e+fx}}dx-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{f^2}\right)}{f^2}+}{4a^2}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{\frac{2}{3}\left(\frac{6id\left(-2\left(\frac{2id\int (c+dx)\log\left(1+ie^{e+fx}\right)dx}{f}-\frac{i(c+dx)^2\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{f}{2}\right.}{f}\right.}{}}{4a^2}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{2}{3}\left(\frac{6id\left(-2\left(\frac{2id\left(\frac{d\int \text{PolyLog}\left(2,-ie^{e+fx}\right)dx}{f}-\frac{(c+dx)\,\text{PolyLog}\left(2,-ie^{e+fx}\right)}{f}\right)}{f}-\frac{i(c+dx)^2\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i}{}\right.}{f}\right.$$

$$\Big\downarrow \text{ 2720}$$

3.113. $\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx$

$$\frac{\frac{2}{3}\left(\frac{6id\left(-2\left(\frac{2id\left(\frac{d\int e^{-e-fx}\operatorname{PolyLog}\left(2,-ie^{e+fx}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-ie^{e+fx}\right)}{f}\right)}{f}-\frac{i(c+dx)^2\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}+\frac{2(c+dx)^3\tan\cdots}{\cdots}\right.}{\cdots}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{f^2}\right)}{f^2}+\frac{2}{3}\left(\frac{6id\left(-2\left(\frac{2id\left(\frac{d\operatorname{PolyLog}\left(3,-ie^{e+fx}\right)}{f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-ie^{e+fx}\right)}{f}\right)}{f}-\frac{i(c+dx\cdots}{\cdots}\right.\right.}{f}\right.\cdots$$

input
```
Int[(c + d*x)^3/(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
((2*d*(c + d*x)^2*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2)/f^2 + (2*(c + d*x)^3*S
ech[e/2 + (I/4)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f) - (4
*d^2*((-4*d*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/f^2 + (2*(c + d*x)*Tanh[e
/2 + (I/4)*Pi + (f*x)/2])/f))/f^2 + (2*(((6*I)*d*(((-1/3*I)*(c + d*x)^3)/d
 - 2*(((-I)*(c + d*x)^2*Log[1 + I*E^(e + f*x)])/f + ((2*I)*d*(-(((c + d*x)
*PolyLog[2, (-I)*E^(e + f*x)])/f) + (d*PolyLog[3, (-I)*E^(e + f*x)])/f^2))
/f)))/f + (2*(c + d*x)^3*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f))/3)/(4*a^2)
```

3.113.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.113. $\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx$

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.113.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(249) = 498$.

Time = 1.94 (sec) , antiderivative size = 723, normalized size of antiderivative = 2.37

method	result
risch	$\frac{2f^2c^3\mathrm{e}^{fx+e}-4id^3x\,\mathrm{e}^{2fx+2e}-4ic\,d^2\mathrm{e}^{2fx+2e}-2if\,c^2d\,\mathrm{e}^{2fx+2e}-8d^3x\,\mathrm{e}^{fx+e}-8c\,d^2\mathrm{e}^{fx+e}-2if\,d^3x^2\mathrm{e}^{2fx+2e}-\frac{2ic^3f^2}{3}-2if^2c\,d^2x^2+4id}{}$

input
```
int((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(3*f^2*c^3*exp(f*x+e)-6*I*d^3*x*exp(2*f*x+2*e)-6*I*c*d^2*exp(2*f*x+2*e
)-3*I*f*c^2*d*exp(2*f*x+2*e)-12*d^3*x*exp(f*x+e)-12*c*d^2*exp(f*x+e)-3*I*f
*d^3*x^2*exp(2*f*x+2*e)-I*c^3*f^2-3*I*f^2*c*d^2*x^2+6*I*d^3*x+9*f^2*c*d^2*
x^2*exp(f*x+e)+9*f^2*c^2*d*x*exp(f*x+e)-6*f*c*d^2*x*exp(f*x+e)-3*I*f^2*c^2
*d*x-6*I*f*c*d^2*x*exp(2*f*x+2*e)-I*f^2*d^3*x^3+6*I*c*d^2-3*f*d^3*x^2*exp(
f*x+e)-3*f*c^2*d*exp(f*x+e)+3*f^2*d^3*x^3*exp(f*x+e))/(exp(f*x+e)-I)^3/f^3
/a^2-2/a^2/f^3*d^3*e^2*x-2/a^2/f^2*d^3*ln(1+I*exp(f*x+e))*x^2-4/a^2/f^3*d^
3*polylog(2,-I*exp(f*x+e))*x+2/a^2/f*d^2*c*x^2+2/a^2/f^3*d^2*c*e^2-4/a^2/f
^3*d^2*c*polylog(2,-I*exp(f*x+e))+4/a^2/f^4*d^3*ln(exp(f*x+e)-I)-4/a^2/f^4
*d^3*ln(exp(f*x+e))-4/3/a^2/f^4*d^3*e^3+4*d^3*polylog(3,-I*exp(f*x+e))/a^2
/f^4+4/a^2/f^2*d^2*c*e*x-4/a^2/f^2*d^2*c*ln(1+I*exp(f*x+e))*x-4/a^2/f^3*d^
2*c*ln(1+I*exp(f*x+e))*e-4/a^2/f^3*d^2*ln(exp(f*x+e))*c*e+4/a^2/f^3*d^2*ln
(exp(f*x+e)-I)*c*e+2/3/a^2/f*d^3*x^3+2/a^2/f^4*d^3*ln(1+I*exp(f*x+e))*e^2+
2/a^2/f^2*d*ln(exp(f*x+e))*c^2-2/a^2/f^2*d*ln(exp(f*x+e)-I)*c^2+2/a^2/f^4*
d^3*ln(exp(f*x+e))*e^2-2/a^2/f^4*d^3*ln(exp(f*x+e)-I)*e^2
```

3.113.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 917 vs. $2(239) = 478$.

Time = 0.26 (sec) , antiderivative size = 917, normalized size of antiderivative = 3.01

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx = -\frac{2\left(-i\,d^3e^3-3i\,c^2def^2+i\,c^3f^3+6i\,d^3e+3\left(i\,cd^2e^2-2i\,cd^2\right)f+6\left(i\,d^3fx+i\,cd^2f+(d^3fx+cd^2f)e^{(3}\right.\right.}{}$$

input
```
integrate((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-2/3*(-I*d^3*e^3 - 3*I*c^2*d*e*f^2 + I*c^3*f^3 + 6*I*d^3*e + 3*(I*c*d^2*e^
2 - 2*I*c*d^2)*f + 6*(I*d^3*f*x + I*c*d^2*f + (d^3*f*x + c*d^2*f)*e^(3*f*x
 + 3*e) + 3*(-I*d^3*f*x - I*c*d^2*f)*e^(2*f*x + 2*e) - 3*(d^3*f*x + c*d^2*
f)*e^(f*x + e))*dilog(-I*e^(f*x + e)) - (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d
^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - 6*d^3*e + 3*(c^2*d*f^3 - 2*d^3*f)
*x)*e^(3*f*x + 3*e) + 3*(I*d^3*f^3*x^3 + I*d^3*e^3 - 6*I*d^3*e + (3*I*c^2*
d*e + I*c^2*d)*f^2 + (3*I*c*d^2*f^3 + I*d^3*f^2)*x^2 + (-3*I*c*d^2*e^2 + 2
*I*c*d^2)*f + (3*I*c^2*d*f^3 + 2*I*c*d^2*f^2 - 4*I*d^3*f)*x)*e^(2*f*x + 2*
e) + 3*(d^3*f^2*x^2 + d^3*e^3 - c^3*f^3 - 6*d^3*e + (3*c^2*d*e + c^2*d)*f^
2 - (3*c*d^2*e^2 - 4*c*d^2)*f + 2*(c*d^2*f^2 - d^3*f)*x)*e^(f*x + e) + 3*(
I*d^3*e^2 - 2*I*c*d^2*e*f + I*c^2*d*f^2 - 2*I*d^3 + (d^3*e^2 - 2*c*d^2*e*f
 + c^2*d*f^2 - 2*d^3)*e^(3*f*x + 3*e) + 3*(-I*d^3*e^2 + 2*I*c*d^2*e*f - I*
c^2*d*f^2 + 2*I*d^3)*e^(2*f*x + 2*e) - 3*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^
2 - 2*d^3)*e^(f*x + e))*log(e^(f*x + e) - I) + 3*(I*d^3*f^2*x^2 + 2*I*c*d^
2*f^2*x - I*d^3*e^2 + 2*I*c*d^2*e*f + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e
^2 + 2*c*d^2*e*f)*e^(3*f*x + 3*e) + 3*(-I*d^3*f^2*x^2 - 2*I*c*d^2*f^2*x +
I*d^3*e^2 - 2*I*c*d^2*e*f)*e^(2*f*x + 2*e) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*
x - d^3*e^2 + 2*c*d^2*e*f)*e^(f*x + e))*log(I*e^(f*x + e) + 1) - 6*(d^3*e^
(3*f*x + 3*e) - 3*I*d^3*e^(2*f*x + 2*e) - 3*d^3*e^(f*x + e) + I*d^3)*polyl
og(3, -I*e^(f*x + e)))/(a^2*f^4*e^(3*f*x + 3*e) - 3*I*a^2*f^4*e^(2*f*x ...
```

3.113.6 Sympy [F]

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx$$
$$= \frac{-2ic^3f^2 - 6ic^2df^2x - 6icd^2f^2x^2 + 12icd^2 - 2id^3f^2x^3 + 12id^3x + (-6ic^2dfe^{2e} - 12icd^2fxe^{2e} - 12icd^2e^{2e}}{3a^2f^3}$$
$$- \frac{2id\left(\int\left(-\frac{2d^2}{e^ee^{fx}-i}\right)dx + \int\frac{c^2f^2}{e^ee^{fx}-i}\,dx + \int\frac{d^2f^2x^2}{e^ee^{fx}-i}\,dx + \int\frac{2cdf^2x}{e^ee^{fx}-i}\,dx\right)}{a^2f^3}$$

input
```
integrate((d*x+c)**3/(a+I*a*sinh(f*x+e))**2,x)
```

output
```
(-2*I*c**3*f**2 - 6*I*c**2*d*f**2*x - 6*I*c*d**2*f**2*x**2 + 12*I*c*d**2 -
 2*I*d**3*f**2*x**3 + 12*I*d**3*x + (-6*I*c**2*d*f*exp(2*e) - 12*I*c*d**2*
f*x*exp(2*e) - 12*I*c*d**2*exp(2*e) - 6*I*d**3*f*x**2*exp(2*e) - 12*I*d**3
*x*exp(2*e))*exp(2*f*x) + (6*c**3*f**2*exp(e) + 18*c**2*d*f**2*x*exp(e) -
6*c**2*d*f*exp(e) + 18*c*d**2*f**2*x**2*exp(e) - 12*c*d**2*f*x*exp(e) - 24
*c*d**2*exp(e) + 6*d**3*f**2*x**3*exp(e) - 6*d**3*f*x**2*exp(e) - 24*d**3*
x*exp(e))*exp(f*x))/(3*a**2*f**3*exp(3*e)*exp(3*f*x) - 9*I*a**2*f**3*exp(2
*e)*exp(2*f*x) - 9*a**2*f**3*exp(e)*exp(f*x) + 3*I*a**2*f**3) - 2*I*d*(Int
egral(-2*d**2/(exp(e)*exp(f*x) - I), x) + Integral(c**2*f**2/(exp(e)*exp(f
*x) - I), x) + Integral(d**2*f**2*x**2/(exp(e)*exp(f*x) - I), x) + Integra
l(2*c*d*f**2*x/(exp(e)*exp(f*x) - I), x))/(a**2*f**3)
```

3.113.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(239) = 478$.

Time = 0.38 (sec) , antiderivative size = 635, normalized size of antiderivative = 2.08

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx$$
$$= 2\,c^2 d\left(\frac{fxe^{(3\,fx+3\,e)} - \left(3i\,fxe^{(2\,e)} + i\,e^{(2\,e)}\right)e^{(2\,fx)} - e^{(fx+e)}}{a^2 f^2 e^{(3\,fx+3\,e)} - 3i\,a^2 f^2 e^{(2\,fx+2\,e)} - 3\,a^2 f^2 e^{(fx+e)} + i\,a^2 f^2} - \frac{\log\left(-i\left(i\,e^{(fx+e)}+1\right)e^{(-e)}\right)}{a^2 f^2}\right)$$
$$+ \frac{2}{3}\,c^3\left(\frac{3\,e^{(-fx-e)}}{\left(3\,a^2 e^{(-fx-e)} - 3i\,a^2 e^{(-2\,fx-2\,e)} - a^2 e^{(-3\,fx-3\,e)} + i\,a^2\right)f} + \frac{i}{\left(3\,a^2 e^{(-fx-e)} - 3i\,a^2 e^{(-2\,fx-2\,e)} - a^2 e^{(-3}\right.}\right.$$
$$- \frac{2\left(i\,d^3 f^2 x^3 + 3i\,cd^2 f^2 x^2 - 6i\,d^3 x - 6i\,cd^2 - 3\left(-i\,d^3 fx^2 e^{(2\,e)} - 2i\,cd^2 e^{(2\,e)} + 2\left(-i\,cd^2 fe^{(2\,e)} - i\,d^3 e^{(2\,e)}\right)\right.\right.}{3\left(a^2 f^3 e^{(3\,fx+3\,e)} - 3i\,a^2 f^3 e^{(2\,fx+2\,e)} - \right.}$$
$$- \frac{4\left(fx\log\left(i\,e^{(fx+e)}+1\right) + \mathrm{Li}_2\left(-i\,e^{(fx+e)}\right)\right)cd^2}{a^2 f^3} - \frac{4\,d^3 x}{a^2 f^3}$$
$$- \frac{2\left(f^2 x^2 \log\left(i\,e^{(fx+e)}+1\right) + 2\,fx\mathrm{Li}_2\left(-i\,e^{(fx+e)}\right) - 2\,\mathrm{Li}_3\left(-i\,e^{(fx+e)}\right)\right)d^3}{a^2 f^4}$$
$$+ \frac{4\,d^3\log\left(e^{(fx+e)} - i\right)}{a^2 f^4} + \frac{2\left(d^3 f^3 x^3 + 3\,cd^2 f^3 x^2\right)}{3\,a^2 f^4}$$

input
```
integrate((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
2*c^2*d*((f*x*e^(3*f*x + 3*e) - (3*I*f*x*e^(2*e) + I*e^(2*e))*e^(2*f*x) -
e^(f*x + e))/(a^2*f^2*e^(3*f*x + 3*e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^
2*f^2*e^(f*x + e) + I*a^2*f^2) - log(-I*(I*e^(f*x + e) + 1)*e^(-e))/(a^2*f
^2)) + 2/3*c^3*(3*e^(-f*x - e)/((3*a^2*e^(-f*x - e) - 3*I*a^2*e^(-2*f*x -
2*e) - a^2*e^(-3*f*x - 3*e) + I*a^2)*f) + I/((3*a^2*e^(-f*x - e) - 3*I*a^2
*e^(-2*f*x - 2*e) - a^2*e^(-3*f*x - 3*e) + I*a^2)*f)) - 2/3*(I*d^3*f^2*x^3
 + 3*I*c*d^2*f^2*x^2 - 6*I*d^3*x - 6*I*c*d^2 - 3*(-I*d^3*f*x^2*e^(2*e) - 2
*I*c*d^2*e^(2*e) + 2*(-I*c*d^2*f*e^(2*e) - I*d^3*e^(2*e))*x)*e^(2*f*x) - 3
*(d^3*f^2*x^3*e^e - 4*c*d^2*e^e + (3*c*d^2*f^2*e^e - d^3*f*e^e)*x^2 - 2*(c
*d^2*f*e^e + 2*d^3*e^e)*x)*e^(f*x))/(a^2*f^3*e^(3*f*x + 3*e) - 3*I*a^2*f^3
*e^(2*f*x + 2*e) - 3*a^2*f^3*e^(f*x + e) + I*a^2*f^3) - 4*(f*x*log(I*e^(f*
x + e) + 1) + dilog(-I*e^(f*x + e)))*c*d^2/(a^2*f^3) - 4*d^3*x/(a^2*f^3) -
 2*(f^2*x^2*log(I*e^(f*x + e) + 1) + 2*f*x*dilog(-I*e^(f*x + e)) - 2*polyl
og(3, -I*e^(f*x + e)))*d^3/(a^2*f^4) + 4*d^3*log(e^(f*x + e) - I)/(a^2*f^4
) + 2/3*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2)/(a^2*f^4)
```

3.113.8 Giac [F]

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^3}{(i\,a\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(I*a*sinh(f*x + e) + a)^2, x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^3}{(a+a\sinh(e+f\,x)\ 1\mathrm{i})^2}\,dx$$

input
```
int((c + d*x)^3/(a + a*sinh(e + f*x)*1i)^2,x)
```

output
```
int((c + d*x)^3/(a + a*sinh(e + f*x)*1i)^2, x)
```

3.114 $\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx$

3.114.1 Optimal result

Integrand size = 23, antiderivative size = 241

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx &= \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{3a^2f^2} \\ &\quad - \frac{4d^2\operatorname{PolyLog}\left(2,-ie^{e+fx}\right)}{3a^2f^3} + \frac{d(c+dx)\operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2} \\ &\quad - \frac{2d^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^3} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f} \\ &\quad + \frac{(c+dx)^2\operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f}\end{aligned}$$

output
```
1/3*(d*x+c)^2/a^2/f-4/3*d*(d*x+c)*ln(1+I*exp(f*x+e))/a^2/f^2-4/3*d^2*polyl
og(2,-I*exp(f*x+e))/a^2/f^3+1/3*d*(d*x+c)*sech(1/2*e+1/4*I*Pi+1/2*f*x)^2/a
^2/f^2-2/3*d^2*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f^3+1/3*(d*x+c)^2*tanh(1/2
*e+1/4*I*Pi+1/2*f*x)/a^2/f+1/6*(d*x+c)^2*sech(1/2*e+1/4*I*Pi+1/2*f*x)^2*ta
nh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f
```

3.114.2 Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx$$
$$=\frac{\frac{2if(c+dx)\left(f(c+dx)+2d(1+ie^e)\log\left(1-ie^{-e-fx}\right)\right)}{-i+e^e}+4d^2\,\text{PolyLog}\left(2,ie^{-e-fx}\right)+\frac{2df(c+dx)\cosh\left(\frac{fx}{2}\right)+2id^2\cosh\left(e+\frac{fx}{2}\right)+i(c^2f^2+}{}}{3a^2f^3}$$

input
```
Integrate[(c + d*x)^2/(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(((2*I)*f*(c + d*x)*(f*(c + d*x) + 2*d*(1 + I*E^e)*Log[1 - I*E^(-e - f*x)]
))/(-I + E^e) + 4*d^2*PolyLog[2, I*E^(-e - f*x)] + (2*d*f*(c + d*x)*Cosh[(
f*x)/2] + (2*I)*d^2*Cosh[e + (f*x)/2] + I*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-2
 + f^2*x^2))*Cosh[e + (3*f*x)/2] + (3*c^2*f^2 + 6*c*d*f^2*x + d^2*(-4 + 3*
f^2*x^2))*Sinh[(f*x)/2] + (2*I)*d*f*(c + d*x)*Sinh[e + (f*x)/2])/((Cosh[e/
2] + I*Sinh[e/2])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^3))/(3*a^2*f^3
)
```

3.114.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.696, Rules used = {3042, 3799, 3042, 4674, 3042, 4254, 24, 4672, 26, 3042, 26, 4199, 26, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{(c+dx)^2}{(a+a\sin(ie+ifx))^2}dx$$
$$\Big\downarrow \ 3799$$
$$\frac{\int (c+dx)^2\text{csch}^4\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)dx}{4a^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\int (c+dx)^2 \csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^4 dx}{4a^2}$$

$$\Big\downarrow \textbf{4674}$$

$$\frac{\frac{2}{3}\int (c+dx)^2 \operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right) dx - \frac{4d^2 \int \operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right) dx}{3f^2} + \frac{4d(c+dx)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2} + \frac{2(c+dx)^2 \tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}}{4a^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{2}{3}\int (c+dx)^2 \csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2 dx - \frac{4d^2 \int \csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2 dx}{3f^2} + \frac{4d(c+dx)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2} + \frac{2(c+dx)^2 \tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\mathrm{se}}{3f}}{4a^2}$$

$$\Big\downarrow \textbf{4254}$$

$$\frac{\frac{2}{3}\int (c+dx)^2 \csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2 dx - \frac{8id^2 \int 1d\left(-i\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{3f^3} + \frac{4d(c+dx)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2} + \frac{2(c+dx)^2 \tanh\left(\frac{e}{2}+\frac{fx}{2}+\right.}{3}}{4a^2}$$

$$\Big\downarrow \textbf{24}$$

$$\frac{\frac{2}{3}\int (c+dx)^2 \csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2 dx + \frac{4d(c+dx)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2} + \frac{2(c+dx)^2 \tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f} - \frac{8d^2 \tanh}{}}{4a^2}$$

$$\Big\downarrow \textbf{4672}$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2 \tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f} - \frac{4id\int -i(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right) dx}{f}\right) + \frac{4d(c+dx)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2} + \frac{2(c+dx)^2 \tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\mathrm{se}}{3f}}{4a^2}$$

$$\Big\downarrow \textbf{26}$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2 \tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f} - \frac{4d\int (c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right) dx}{f}\right) + \frac{4d(c+dx)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2} + \frac{2(c+dx)^2 \tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\operatorname{sech}^2}{3f}}{4a^2}$$

$$\Big\downarrow \textbf{3042}$$

3.114. $\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\, dx$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\int -i(c+dx)\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sec}}{3f}}{4a^2}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{2}{3}\left(\frac{4id\int(c+dx)\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}}{3f}}{4a^2}$$

$$\Big\downarrow \; 4199$$

$$\frac{\frac{2}{3}\left(\frac{4id\left(2i\int\frac{ie^{e+fx}(c+dx)}{1+ie^{e+fx}}dx-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{s}}{3f}}{4a^2}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{2}{3}\left(\frac{4id\left(-2\int\frac{e^{e+fx}(c+dx)}{1+ie^{e+fx}}dx-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{s}}{3f}}{4a^2}$$

$$\Big\downarrow \; 2620$$

$$\frac{\frac{2}{3}\left(\frac{4id\left(-2\left(\frac{id\int\log\left(1+ie^{e+fx}\right)dx}{f}-\frac{i(c+dx)\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}-}{4a^2}$$

$$\Big\downarrow \; 2715$$

$$\frac{\frac{2}{3}\left(\frac{4id\left(-2\left(\frac{id\int e^{-e-fx}\log\left(1+ie^{e+fx}\right)de^{e+fx}}{f^2}-\frac{i(c+dx)\log\left(1+ie^{e+fx}\right)}{f}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}\right.}{3f^2}}{4a^2}$$

$$\Big\downarrow \; 2838$$

$$\frac{\frac{2}{3}\left(\frac{4id\left(-2\left(-\frac{i(c+dx)\log\left(1+ie^{e+fx}\right)}{f}-\frac{id\,\text{PolyLog}\left(2,-ie^{e+fx}\right)}{f^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right.}{3f^2}}{4a^2}$$

3.114. $\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx$

input
```
Int[(c + d*x)^2/(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
((4*d*(c + d*x)*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2)/(3*f^2) - (8*d^2*Tanh[e/
2 + (I/4)*Pi + (f*x)/2])/(3*f^3) + (2*(c + d*x)^2*Sech[e/2 + (I/4)*Pi + (f
*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f) + (2*(((4*I)*d*(((-1/2*I)*(
c + d*x)^2)/d - 2*(((-I)*(c + d*x)*Log[1 + I*E^(e + f*x)])/f - (I*d*PolyLo
g[2, (-I)*E^(e + f*x)])/f^2)))/f + (2*(c + d*x)^2*Tanh[e/2 + (I/4)*Pi + (f
*x)/2])/f))/3)/(4*a^2)
```

3.114.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

3.114.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.55

method	result
risch	$\frac{-\frac{2ic^2f^2}{3}-\frac{4if\,d^2x\,\mathrm{e}^{2fx+2e}}{3}-\frac{4ifcd\,\mathrm{e}^{2fx+2e}}{3}-\frac{4id^2\mathrm{e}^{2fx+2e}}{3}-\frac{4if^2cdx}{3}+\frac{4id^2}{3}-\frac{4f\,d^2x\,\mathrm{e}^{fx+e}}{3}-\frac{4fcd\,\mathrm{e}^{fx+e}}{3}-\frac{2if^2d^2x^2}{3}-\frac{8d^2\mathrm{e}^{fx+e}}{3}+2f^2d^2x}{\left(\mathrm{e}^{fx+e}-i\right)^3f^3a^2}$

input
```
int((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(-I*c^2*f^2-2*I*f*d^2*x*exp(2*f*x+2*e)-2*I*f*c*d*exp(2*f*x+2*e)-2*I*d^
2*exp(2*f*x+2*e)-2*I*f^2*c*d*x+2*I*d^2-2*f*d^2*x*exp(f*x+e)-2*f*c*d*exp(f*
x+e)-I*f^2*d^2*x^2-4*d^2*exp(f*x+e)+3*f^2*d^2*x^2*exp(f*x+e)+6*f^2*c*d*x*e
xp(f*x+e)+3*f^2*c^2*exp(f*x+e))/(exp(f*x+e)-I)^3/f^3/a^2-4/3/a^2/f^2*d*ln(
exp(f*x+e)-I)*c+4/3/a^2/f^2*d*ln(exp(f*x+e))*c+2/3/a^2/f*d^2*x^2+4/3/a^2/f
^2*d^2*e*x+2/3/a^2/f^3*d^2*e^2-4/3/a^2/f^2*d^2*ln(1+I*exp(f*x+e))*x-4/3/a^
2/f^3*d^2*ln(1+I*exp(f*x+e))*e-4/3*d^2*polylog(2,-I*exp(f*x+e))/a^2/f^3+4/
3/a^2/f^3*d^2*e*ln(exp(f*x+e)-I)-4/3/a^2/f^3*d^2*e*ln(exp(f*x+e))
```

3.114.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(180) = 360$.

Time = 0.24 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.00

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx =$$
$$-\frac{2\left(i\,d^2e^2-2i\,cdef+i\,c^2f^2-2i\,d^2+2\left(d^2e^{(3\,fx+3\,e)}-3i\,d^2e^{(2\,fx+2\,e)}-3\,d^2e^{(fx+e)}+i\,d^2\right)\mathrm{Li}_2\left(-i\,e^{(fx+e)}\right)\right.}{}$$

input
```
integrate((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-2/3*(I*d^2*e^2 - 2*I*c*d*e*f + I*c^2*f^2 - 2*I*d^2 + 2*(d^2*e^(3*f*x + 3*
e) - 3*I*d^2*e^(2*f*x + 2*e) - 3*d^2*e^(f*x + e) + I*d^2)*dilog(-I*e^(f*x
+ e)) - (d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*e^(3*f*x + 3*e)
+ (3*I*d^2*f^2*x^2 - 3*I*d^2*e^2 + 2*I*d^2 + 2*(3*I*c*d*e + I*c*d)*f + 2*(
3*I*c*d*f^2 + I*d^2*f)*x)*e^(2*f*x + 2*e) - (3*d^2*e^2 + 3*c^2*f^2 - 2*d^2
*f*x - 4*d^2 - 2*(3*c*d*e + c*d)*f)*e^(f*x + e) + 2*(-I*d^2*e + I*c*d*f -
(d^2*e - c*d*f)*e^(3*f*x + 3*e) + 3*(I*d^2*e - I*c*d*f)*e^(2*f*x + 2*e) +
3*(d^2*e - c*d*f)*e^(f*x + e))*log(e^(f*x + e) - I) + 2*(I*d^2*f*x + I*d^2
*e + (d^2*f*x + d^2*e)*e^(3*f*x + 3*e) + 3*(-I*d^2*f*x - I*d^2*e)*e^(2*f*x
 + 2*e) - 3*(d^2*f*x + d^2*e)*e^(f*x + e))*log(I*e^(f*x + e) + 1))/(a^2*f^
3*e^(3*f*x + 3*e) - 3*I*a^2*f^3*e^(2*f*x + 2*e) - 3*a^2*f^3*e^(f*x + e) +
I*a^2*f^3)
```

3.114.6 Sympy [F]

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx$$
$$= \frac{-2ic^2f^2 - 4icdf^2x - 2id^2f^2x^2 + 4id^2 + (-4icdfe^{2e} - 4id^2fxe^{2e} - 4id^2e^{2e})\,e^{2fx} + (6c^2f^2e^e + 12cdf^2xe^e -}{3a^2f^3e^{3e}e^{3fx} - 9ia^2f^3e^{2e}e^{2fx} - 9a^2f^3e^ee^{fx} + 3ia^2f^3}$$
$$- \frac{4id\left(\int \frac{c}{e^ee^{fx}-i}\,dx + \int \frac{dx}{e^ee^{fx}-i}\,dx\right)}{3a^2f}$$

input
```
integrate((d*x+c)**2/(a+I*a*sinh(f*x+e))**2,x)
```

output
```
(-2*I*c**2*f**2 - 4*I*c*d*f**2*x - 2*I*d**2*f**2*x**2 + 4*I*d**2 + (-4*I*c
*d*f*exp(2*e) - 4*I*d**2*f*x*exp(2*e) - 4*I*d**2*exp(2*e))*exp(2*f*x) + (6
*c**2*f**2*exp(e) + 12*c*d*f**2*x*exp(e) - 4*c*d*f*exp(e) + 6*d**2*f**2*x*
*2*exp(e) - 4*d**2*f*x*exp(e) - 8*d**2*exp(e))*exp(f*x))/(3*a**2*f**3*exp(
3*e)*exp(3*f*x) - 9*I*a**2*f**3*exp(2*e)*exp(2*f*x) - 9*a**2*f**3*exp(e)*e
xp(f*x) + 3*I*a**2*f**3) - 4*I*d*(Integral(c/(exp(e)*exp(f*x) - I), x) + I
ntegral(d*x/(exp(e)*exp(f*x) - I), x))/(3*a**2*f)
```

3.114.7 Maxima [F]

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(i\,a\sinh\left(fx+e\right)+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-2/3*d^2*((I*f^2*x^2 - 2*(-I*f*x*e^(2*e) - I*e^(2*e))*e^(2*f*x) - (3*f^2*x
^2*e^e - 2*f*x*e^e - 4*e^e)*e^(f*x) - 2*I)/(a^2*f^3*e^(3*f*x + 3*e) - 3*I*
a^2*f^3*e^(2*f*x + 2*e) - 3*a^2*f^3*e^(f*x + e) + I*a^2*f^3) + 6*I*integra
te(1/3*x/(a^2*f*e^(f*x + e) - I*a^2*f), x)) + 4/3*c*d*((f*x*e^(3*f*x + 3*e
) - (3*I*f*x*e^(2*e) + I*e^(2*e))*e^(2*f*x) - e^(f*x + e))/(a^2*f^2*e^(3*f
*x + 3*e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*f^2*e^(f*x + e) + I*a^2*f^
2) - log(-I*(I*e^(f*x + e) + 1)*e^(-e))/(a^2*f^2)) + 2/3*c^2*(3*e^(-f*x -
e)/((3*a^2*e^(-f*x - e) - 3*I*a^2*e^(-2*f*x - 2*e) - a^2*e^(-3*f*x - 3*e)
+ I*a^2)*f) + I/((3*a^2*e^(-f*x - e) - 3*I*a^2*e^(-2*f*x - 2*e) - a^2*e^(-
3*f*x - 3*e) + I*a^2)*f))
```

3.114.8 Giac [F]

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(i\,a\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(I*a*sinh(f*x + e) + a)^2, x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^2}{(a+a\sinh(e+f\,x)\ 1\mathrm{i})^2}\,dx$$

input
```
int((c + d*x)^2/(a + a*sinh(e + f*x)*1i)^2,x)
```

output
```
int((c + d*x)^2/(a + a*sinh(e + f*x)*1i)^2, x)
```

3.115 $\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx$

3.115.1 Optimal result

Integrand size = 21, antiderivative size = 158

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx = -\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{3a^2f^2}+\frac{d\,\mathrm{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)\mathrm{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f}$$

output
```
-2/3*d*ln(cosh(1/2*e+1/4*I*Pi+1/2*f*x))/a^2/f^2+1/6*d*sech(1/2*e+1/4*I*Pi+
1/2*f*x)^2/a^2/f^2+1/3*(d*x+c)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f+1/6*(d*x
+c)*sech(1/2*e+1/4*I*Pi+1/2*f*x)^2*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f
```

3.115.2 Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.53

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx = \frac{\left(-i\cosh\left(\frac{1}{2}(e+fx)\right)+\sinh\left(\frac{1}{2}(e+fx)\right)\right)\left(d\cosh\left(\frac{1}{2}(e+fx)\right)\left(-2i+3e+3fx-6\arctan\left(\tanh\left(\frac{1}{2}(e+\right.\right.\right.\right.}{}$$

input
```
Integrate[(c + d*x)/(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(((-I)*Cosh[(e + f*x)/2] + Sinh[(e + f*x)/2])*(d*Cosh[(e + f*x)/2]*(-2*I +
 3*e + 3*f*x - 6*ArcTan[Tanh[(e + f*x)/2]] + (3*I)*Log[Cosh[e + f*x]]) + C
osh[(3*(e + f*x))/2]*(-(d*e) + 2*c*f + d*f*x + 2*d*ArcTan[Tanh[(e + f*x)/2
]] - I*d*Log[Cosh[e + f*x]]) + (2*I)*((-I)*d + 2*d*e - 3*c*f - d*f*x - 4*d
*ArcTan[Tanh[(e + f*x)/2]] + d*Cosh[e + f*x]*(e + f*x - 2*ArcTan[Tanh[(e +
 f*x)/2]] + I*Log[Cosh[e + f*x]]) + (2*I)*d*Log[Cosh[e + f*x]])*Sinh[(e +
f*x)/2]))/(6*a^2*f^2*(-I + Sinh[e + f*x])^2)
```

3.115.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 3799, 3042, 4673, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{(a+a\sin(ie+ifx))^2}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)\text{csch}^4\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^4dx}{4a^2}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\frac{2}{3}\int (c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2}{3}\int (c+dx)\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^2dx+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \; 4672$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2id\int -i\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2d\int \tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2d\int -i\tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{2}{3}\left(\frac{2id\int \tan\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \; 3956$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{f^2}\right)+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}}{4a^2}$$

input
```
Int[(c + d*x)/(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
((2*d*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2)/(3*f^2) + (2*(c + d*x)*Sech[e/2 +
(I/4)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f) + (2*((-4*d*Lo
g[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/f^2 + (2*(c + d*x)*Tanh[e/2 + (I/4)*Pi
+ (f*x)/2])/f))/3)/(4*a^2)
```

3.115. $\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx$

3.115.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

3.115.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.72

method	result
risch	$\frac{2dx}{3f\,a^{2}}+\frac{2de}{3f^{2}a^{2}}-\frac{2i\left(3ifdx\,{\mathrm e}^{fx+e}+3ifc\,{\mathrm e}^{fx+e}-id\,{\mathrm e}^{fx+e}+dfx+d\,{\mathrm e}^{2fx+2e}+cf\right)}{3\left({\mathrm e}^{fx+e}-i\right)^{3}f^{2}a^{2}}-\frac{2d\ln\left({\mathrm e}^{fx+e}-i\right)}{3f^{2}a^{2}}$
parallelrisch	$18\left(i\cosh\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{i\cosh\left(\frac{3fx}{2}+\frac{3e}{2}\right)}{3}-\frac{\sinh\left(\frac{3fx}{2}+\frac{3e}{2}\right)}{3}-\sinh\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d\ln\left(1-\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-18\left(i\cosh\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{i\cosh\left(3\right.}{3}\right.$

input
```
int((d*x+c)/(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*d/f/a^2*x+2/3*d/f^2/a^2*e-2/3*I*(3*I*f*d*x*exp(f*x+e)+3*I*f*c*exp(f*x+
e)-I*d*exp(f*x+e)+d*f*x+d*exp(2*f*x+2*e)+c*f)/(exp(f*x+e)-I)^3/f^2/a^2-2/3
*d/f^2/a^2*ln(exp(f*x+e)-I)
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx$$
$$= \frac{2\left(dfxe^{(3\,fx+3\,e)} - i\,cf - (3i\,dfx + i\,d)e^{(2\,fx+2\,e)} + (3\,cf - d)e^{(fx+e)} - \left(de^{(3\,fx+3\,e)} - 3i\,de^{(2\,fx+2\,e)} - 3\,de^{(fx}\right.\right.}{3\left(a^2f^2e^{(3\,fx+3\,e)} - 3i\,a^2f^2e^{(2\,fx+2\,e)} - 3\,a^2f^2e^{(fx+e)} + i\,a^2f^2\right)}$$

input
```
integrate((d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
2/3*(d*f*x*e^(3*f*x + 3*e) - I*c*f - (3*I*d*f*x + I*d)*e^(2*f*x + 2*e) + (
3*c*f - d)*e^(f*x + e) - (d*e^(3*f*x + 3*e) - 3*I*d*e^(2*f*x + 2*e) - 3*d*
e^(f*x + e) + I*d)*log(e^(f*x + e) - I))/(a^2*f^2*e^(3*f*x + 3*e) - 3*I*a^
2*f^2*e^(2*f*x + 2*e) - 3*a^2*f^2*e^(f*x + e) + I*a^2*f^2)
```

3.115.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx = \frac{-2icf - 2idfx - 2ide^{2e}e^{2fx} + \left(6cfe^{e} + 6dfxe^{e} - 2de^{e}\right)e^{fx}}{3a^2f^2e^{3e}e^{3fx} - 9ia^2f^2e^{2e}e^{2fx} - 9a^2f^2e^{e}e^{fx} + 3ia^2f^2} + \frac{2dx}{3a^2f} - \frac{2d\log\left(e^{fx} - ie^{-e}\right)}{3a^2f^2}$$

input
```
integrate((d*x+c)/(a+I*a*sinh(f*x+e))**2,x)
```

output
```
(-2*I*c*f - 2*I*d*f*x - 2*I*d*exp(2*e)*exp(2*f*x) + (6*c*f*exp(e) + 6*d*f*
x*exp(e) - 2*d*exp(e))*exp(f*x))/(3*a**2*f**2*exp(3*e)*exp(3*f*x) - 9*I*a*
*2*f**2*exp(2*e)*exp(2*f*x) - 9*a**2*f**2*exp(e)*exp(f*x) + 3*I*a**2*f**2)
 + 2*d*x/(3*a**2*f) - 2*d*log(exp(f*x) - I*exp(-e))/(3*a**2*f**2)
```

3.115.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(110) = 220$.

Time = 0.21 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.61

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx$$
$$= \frac{2}{3}\,d\left(\frac{fxe^{(3\,fx+3\,e)} - \left(3i\,fxe^{(2\,e)} + i\,e^{(2\,e)}\right)e^{(2\,fx)} - e^{(fx+e)}}{a^2f^2e^{(3\,fx+3\,e)} - 3i\,a^2f^2e^{(2\,fx+2\,e)} - 3\,a^2f^2e^{(fx+e)} + i\,a^2f^2} - \frac{\log\left(-i\left(i\,e^{(fx+e)}+1\right)e^{(-e)}\right)}{a^2f^2}\right)$$
$$+ \frac{2}{3}\,c\left(\frac{3\,e^{(-fx-e)}}{\left(3\,a^2e^{(-fx-e)} - 3i\,a^2e^{(-2\,fx-2\,e)} - a^2e^{(-3\,fx-3\,e)} + i\,a^2\right)f} + \frac{i}{\left(3\,a^2e^{(-fx-e)} - 3i\,a^2e^{(-2\,fx-2\,e)} - a^2e^{(-3}\right.}\right.$$

input
```
integrate((d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
2/3*d*((f*x*e^(3*f*x + 3*e) - (3*I*f*x*e^(2*e) + I*e^(2*e))*e^(2*f*x) - e^
(f*x + e))/(a^2*f^2*e^(3*f*x + 3*e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*
f^2*e^(f*x + e) + I*a^2*f^2) - log(-I*(I*e^(f*x + e) + 1)*e^(-e))/(a^2*f^2
)) + 2/3*c*(3*e^(-f*x - e)/((3*a^2*e^(-f*x - e) - 3*I*a^2*e^(-2*f*x - 2*e)
 - a^2*e^(-3*f*x - 3*e) + I*a^2)*f) + I/((3*a^2*e^(-f*x - e) - 3*I*a^2*e^(
-2*f*x - 2*e) - a^2*e^(-3*f*x - 3*e) + I*a^2)*f))
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.23

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx$$
$$= \frac{2\left(dfxe^{(3\,fx+3\,e)} - 3i\,dfxe^{(2\,fx+2\,e)} + 3\,cfe^{(fx+e)} - de^{(3\,fx+3\,e)}\log\left(e^{(fx+e)} - i\right) + 3i\,de^{(2\,fx+2\,e)}\log\left(e^{(fx+e)} -\right.\right.}{3\left(a^2f^2e^{(3\,fx+3\,e)} - 3i\,a^2f^2e^{(2\,fx+2\,e)} - 3\right.}$$

input
```
integrate((d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
2/3*(d*f*x*e^(3*f*x + 3*e) - 3*I*d*f*x*e^(2*f*x + 2*e) + 3*c*f*e^(f*x + e)
 - d*e^(3*f*x + 3*e)*log(e^(f*x + e) - I) + 3*I*d*e^(2*f*x + 2*e)*log(e^(f
*x + e) - I) + 3*d*e^(f*x + e)*log(e^(f*x + e) - I) - I*c*f - I*d*e^(2*f*x
 + 2*e) - d*e^(f*x + e) - I*d*log(e^(f*x + e) - I))/(a^2*f^2*e^(3*f*x + 3*
e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*f^2*e^(f*x + e) + I*a^2*f^2)
```

3.115.9 Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx =$$
$$-\frac{\frac{2\,d\,\ln\left(\mathrm{e}^{f\,x}\,\mathrm{e}^{e}-\mathrm{i}\right)}{3}+\mathrm{e}^{e+f\,x}\,\left(-\frac{d\,2\mathrm{i}}{3}+c\,f\,2\mathrm{i}+d\,\ln\left(\mathrm{e}^{f\,x}\,\mathrm{e}^{e}-\mathrm{i}\right)\,2\mathrm{i}\right)+\frac{2\,d\,\mathrm{e}^{2\,e+2\,f\,x}}{3}+f\,\left(\frac{2\,c}{3}+2\,d\,x\,\mathrm{e}^{2\,e+2\,f\,x}+\frac{d\,x\,\mathrm{e}^{3\,e}}{}\right.}{a^2\,f^2\,\left(1+\mathrm{e}^{e+f\,x}\,1\mathrm{i}\right)^3}$$

input
```
int((c + d*x)/(a + a*sinh(e + f*x)*1i)^2,x)
```

output
```
-((2*d*log(exp(f*x)*exp(e) - 1i))/3 + exp(e + f*x)*(c*f*2i - (d*2i)/3 + d*
log(exp(f*x)*exp(e) - 1i)*2i) + (2*d*exp(2*e + 2*f*x))/3 + f*((2*c)/3 + 2*
d*x*exp(2*e + 2*f*x) + (d*x*exp(3*e + 3*f*x)*2i)/3) - 2*d*exp(2*e + 2*f*x)
*log(exp(f*x)*exp(e) - 1i) - (d*exp(3*e + 3*f*x)*log(exp(f*x)*exp(e) - 1i)
*2i)/3)/(a^2*f^2*(exp(e + f*x)*1i + 1)^3)
```

3.116 $\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx$

3.116.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a+ia\sinh(e+fx))^2},x\right)$$

output
```
Unintegrable(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x)
```

3.116.2 Mathematica [N/A]

Not integrable

Time = 25.92 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + I*a*Sinh[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)*(a + I*a*Sinh[e + f*x])^2), x]
```

3.116.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)(a+a\sin(ie+ifx))^2}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}dx$$

input
```
Int[1/((c + d*x)*(a + I*a*Sinh[e + f*x])^2),x]
```

output
```
$Aborted
```

3.116.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.116.4 Maple [N/A] (verified)

Not integrable

Time = 0.51 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(dx+c)\,(a+ia\sinh{(fx+e)})^2}dx$$

input
```
int(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x)
```

3.116.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 793, normalized size of antiderivative = 34.48

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(i\,a\sinh{(fx+e)}+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
(-2*I*d^2*f^2*x^2 - 4*I*c*d*f^2*x - 2*I*c^2*f^2 + 4*I*d^2 - 2*(-I*d^2*f*x
- I*c*d*f + 2*I*d^2)*e^(2*f*x + 2*e) + 2*(3*d^2*f^2*x^2 + 3*c^2*f^2 + c*d*
f - 4*d^2 + (6*c*d*f^2 + d^2*f)*x)*e^(f*x + e) - 3*(-I*a^2*d^3*f^3*x^3 - 3
*I*a^2*c*d^2*f^3*x^2 - 3*I*a^2*c^2*d*f^3*x - I*a^2*c^3*f^3 - (a^2*d^3*f^3*
x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(3*f*x + 3*
e) + 3*(I*a^2*d^3*f^3*x^3 + 3*I*a^2*c*d^2*f^3*x^2 + 3*I*a^2*c^2*d*f^3*x +
I*a^2*c^3*f^3)*e^(2*f*x + 2*e) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2
+ 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(f*x + e))*integral(-2*(-I*d^3*f^2*x^
2 - 2*I*c*d^2*f^2*x - I*c^2*d*f^2 + 6*I*d^3)/(-3*I*a^2*d^4*f^3*x^4 - 12*I*
a^2*c*d^3*f^3*x^3 - 18*I*a^2*c^2*d^2*f^3*x^2 - 12*I*a^2*c^3*d*f^3*x - 3*I*
a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3
*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(f*x + e)), x))/(3*I*a^2*d^3*f^3
*x^3 + 9*I*a^2*c*d^2*f^3*x^2 + 9*I*a^2*c^2*d*f^3*x + 3*I*a^2*c^3*f^3 + 3*(
a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e
^(3*f*x + 3*e) - 9*(I*a^2*d^3*f^3*x^3 + 3*I*a^2*c*d^2*f^3*x^2 + 3*I*a^2*c^
2*d*f^3*x + I*a^2*c^3*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*
d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(f*x + e))
```

3.116.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))**2,x)
```

output
```
Timed out
```

3.116.7 Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 606, normalized size of antiderivative = 26.35

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(i\,a\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-2*(I*d^2*f^2*x^2 + 2*I*c*d*f^2*x + I*c^2*f^2 - 2*I*d^2 + (-I*d^2*f*x*e^(2
*e) - I*c*d*f*e^(2*e) + 2*I*d^2*e^(2*e))*e^(2*f*x) - (3*d^2*f^2*x^2*e^e +
3*c^2*f^2*e^e + c*d*f*e^e - 4*d^2*e^e + (6*c*d*f^2*e^e + d^2*f*e^e)*x)*e^(
f*x))/(3*I*a^2*d^3*f^3*x^3 + 9*I*a^2*c*d^2*f^3*x^2 + 9*I*a^2*c^2*d*f^3*x +
 3*I*a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^3*e^(3*e) + 3*a^2*c*d^2*f^3*x^2*e^(3*e
) + 3*a^2*c^2*d*f^3*x*e^(3*e) + a^2*c^3*f^3*e^(3*e))*e^(3*f*x) - 9*(I*a^2*
d^3*f^3*x^3*e^(2*e) + 3*I*a^2*c*d^2*f^3*x^2*e^(2*e) + 3*I*a^2*c^2*d*f^3*x*
e^(2*e) + I*a^2*c^3*f^3*e^(2*e))*e^(2*f*x) - 9*(a^2*d^3*f^3*x^3*e^e + 3*a^
2*c*d^2*f^3*x^2*e^e + 3*a^2*c^2*d*f^3*x*e^e + a^2*c^3*f^3*e^e)*e^(f*x)) -
integrate(2/3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 6*d^3)/(a^2*d^4*f
^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x +
 a^2*c^4*f^3 - (-I*a^2*d^4*f^3*x^4*e^e - 4*I*a^2*c*d^3*f^3*x^3*e^e - 6*I*a
^2*c^2*d^2*f^3*x^2*e^e - 4*I*a^2*c^3*d*f^3*x*e^e - I*a^2*c^4*f^3*e^e)*e^(f
*x)), x)
```

3.116.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(i\,a\sinh\left(fx+e\right)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(I*a*sinh(f*x + e) + a)^2), x)
```

3.116.9 Mupad [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a + a*sinh(e + f*x)*1i)^2*(c + d*x)),x)
```

output
```
int(1/((a + a*sinh(e + f*x)*1i)^2*(c + d*x)), x)
```

3.117 $\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2} dx$

3.117.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2} dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2},x\right)$$

output
```
Unintegrable(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x)
```

3.117.2 Mathematica [N/A]

Not integrable

Time = 26.61 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2} dx = \int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2} dx$$

input
```
Integrate[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])^2), x]
```

3.117.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)^2(a+a\sin(ie+ifx))^2}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}dx$$

input
```
Int[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])^2),x]
```

output
```
$Aborted
```

3.117.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.117.4 Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(dx+c)^2 (a+ia\sinh(fx+e))^2}dx$$

input
```
int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x)
```

3.117.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 965, normalized size of antiderivative = 41.96

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(i\,a\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
(-2*I*d^2*f^2*x^2 - 4*I*c*d*f^2*x - 2*I*c^2*f^2 + 12*I*d^2 - 4*(-I*d^2*f*x
 - I*c*d*f + 3*I*d^2)*e^(2*f*x + 2*e) + 2*(3*d^2*f^2*x^2 + 3*c^2*f^2 + 2*c
*d*f - 12*d^2 + 2*(3*c*d*f^2 + d^2*f)*x)*e^(f*x + e) - 3*(-I*a^2*d^4*f^3*x
^4 - 4*I*a^2*c*d^3*f^3*x^3 - 6*I*a^2*c^2*d^2*f^3*x^2 - 4*I*a^2*c^3*d*f^3*x
 - I*a^2*c^4*f^3 - (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*
f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(3*f*x + 3*e) + 3*(I*a^2*d^4*
f^3*x^4 + 4*I*a^2*c*d^3*f^3*x^3 + 6*I*a^2*c^2*d^2*f^3*x^2 + 4*I*a^2*c^3*d*
f^3*x + I*a^2*c^4*f^3)*e^(2*f*x + 2*e) + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*
f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(f*x
+ e))*integral(-4*(-I*d^3*f^2*x^2 - 2*I*c*d^2*f^2*x - I*c^2*d*f^2 + 12*I*d
^3)/(-3*I*a^2*d^5*f^3*x^5 - 15*I*a^2*c*d^4*f^3*x^4 - 30*I*a^2*c^2*d^3*f^3*
x^3 - 30*I*a^2*c^3*d^2*f^3*x^2 - 15*I*a^2*c^4*d*f^3*x - 3*I*a^2*c^5*f^3 +
3*(a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 + 10*a^2
*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)*e^(f*x + e)), x))/(3*I
*a^2*d^4*f^3*x^4 + 12*I*a^2*c*d^3*f^3*x^3 + 18*I*a^2*c^2*d^2*f^3*x^2 + 12*
I*a^2*c^3*d*f^3*x + 3*I*a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3
*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(3*f*x +
 3*e) - 9*(I*a^2*d^4*f^3*x^4 + 4*I*a^2*c*d^3*f^3*x^3 + 6*I*a^2*c^2*d^2*f^3
*x^2 + 4*I*a^2*c^3*d*f^3*x + I*a^2*c^4*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^4*f
^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*...
```

3.117.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**2/(a+I*a*sinh(f*x+e))**2,x)
```

output
```
Timed out
```

3.117.7 Maxima [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 723, normalized size of antiderivative = 31.43

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(i\,a\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-2*(I*d^2*f^2*x^2 + 2*I*c*d*f^2*x + I*c^2*f^2 - 6*I*d^2 + 2*(-I*d^2*f*x*e^
(2*e) - I*c*d*f*e^(2*e) + 3*I*d^2*e^(2*e))*e^(2*f*x) - (3*d^2*f^2*x^2*e^e
+ 3*c^2*f^2*e^e + 2*c*d*f*e^e - 12*d^2*e^e + 2*(3*c*d*f^2*e^e + d^2*f*e^e)
*x)*e^(f*x))/(3*I*a^2*d^4*f^3*x^4 + 12*I*a^2*c*d^3*f^3*x^3 + 18*I*a^2*c^2*
d^2*f^3*x^2 + 12*I*a^2*c^3*d*f^3*x + 3*I*a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4*
e^(3*e) + 4*a^2*c*d^3*f^3*x^3*e^(3*e) + 6*a^2*c^2*d^2*f^3*x^2*e^(3*e) + 4*
a^2*c^3*d*f^3*x*e^(3*e) + a^2*c^4*f^3*e^(3*e))*e^(3*f*x) - 9*(I*a^2*d^4*f^
3*x^4*e^(2*e) + 4*I*a^2*c*d^3*f^3*x^3*e^(2*e) + 6*I*a^2*c^2*d^2*f^3*x^2*e^
(2*e) + 4*I*a^2*c^3*d*f^3*x*e^(2*e) + I*a^2*c^4*f^3*e^(2*e))*e^(2*f*x) - 9
*(a^2*d^4*f^3*x^4*e^e + 4*a^2*c*d^3*f^3*x^3*e^e + 6*a^2*c^2*d^2*f^3*x^2*e^
e + 4*a^2*c^3*d*f^3*x*e^e + a^2*c^4*f^3*e^e)*e^(f*x)) - integrate(4/3*(d^3
*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 12*d^3)/(a^2*d^5*f^3*x^5 + 5*a^2*c*
d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*
d*f^3*x + a^2*c^5*f^3 - (-I*a^2*d^5*f^3*x^5*e^e - 5*I*a^2*c*d^4*f^3*x^4*e^
e - 10*I*a^2*c^2*d^3*f^3*x^3*e^e - 10*I*a^2*c^3*d^2*f^3*x^2*e^e - 5*I*a^2*
c^4*d*f^3*x*e^e - I*a^2*c^5*f^3*e^e)*e^(f*x)), x)
```

3.117.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(i\,a\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(I*a*sinh(f*x + e) + a)^2), x)
```

3.117.9 Mupad [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{\left(a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}\right)^2\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^2),x)
```

output
```
int(1/((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^2), x)
```

3.118 $\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx$

3.118.1 Optimal result

Integrand size = 21, antiderivative size = 181

$$\begin{aligned}\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx = &-\frac{384x\sqrt{a+ia\sinh(e+fx)}}{f^4}-\frac{16x^3\sqrt{a+ia\sinh(e+fx)}}{f^2}\\ &+\frac{768\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f^5}\\ &+\frac{96x^2\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f^3}\\ &+\frac{2x^4\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f}\end{aligned}$$

output
```
-384*x*(a+I*a*sinh(f*x+e))^(1/2)/f^4-16*x^3*(a+I*a*sinh(f*x+e))^(1/2)/f^2+
768*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^5+96*x^2*(a+I
*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^3+2*x^4*(a+I*a*sinh(f
*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f
```

3.118.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.78

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx$$
$$=\frac{2\left(i(384+192ifx+48f^2x^2+8if^3x^3+f^4x^4)\cosh\left(\frac{1}{2}(e+fx)\right)+(384-192ifx+48f^2x^2-8if^3x^3+f^4}{f^5\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[x^4*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
(2*(I*(384 + (192*I)*f*x + 48*f^2*x^2 + (8*I)*f^3*x^3 + f^4*x^4)*Cosh[(e +
 f*x)/2] + (384 - (192*I)*f*x + 48*f^2*x^2 - (8*I)*f^3*x^3 + f^4*x^4)*Sinh
[(e + f*x)/2])*Sqrt[a + I*a*Sinh[e + f*x]])/(f^5*(Cosh[(e + f*x)/2] + I*Si
nh[(e + f*x)/2]))
```

3.118.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.13, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.762, Rules used = {3042, 3800, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int x^4\sqrt{a+a\sin(ie+ifx)}dx$$
$$\Big\downarrow \text{3800}$$
$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^4\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$
$$\Big\downarrow \text{3042}$$
$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^4\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx$$
$$\Big\downarrow \text{3777}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^4\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{8i\int -ix^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^4\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{8\int x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^4\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{8\int -ix^3\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\int x^3\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x^4\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)$$

$$\Big\downarrow \text{3777}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\int x^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f}+\frac{2x^4\sinh\left(\frac{e}{2}+\frac{f}{2}\right.}{f}\right.$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\int x^2\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{f}\right)}{f}+\frac{2x^4\sinh\left(\frac{e}{2}+\frac{f}{2}\right.}{f}\right.$$

$$\Big\downarrow \text{3777}$$

3.118. $\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\int -ix\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f}\right)}{f}\right.$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\int x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f}\right)}{f}\right.$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\int -ix\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)}{f}\right)}{f}\right.$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\left(\frac{4i\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)}{f}\right)}{f}\right.$$

$$\Big\downarrow \text{3777}$$

3.118. $\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f}\right.}{f}\right.}{f}\right.$$

$$\Big\downarrow \; 3042$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{f}\right)}{f}\right.}{f}\right.}{f}\right.$$

$$\Big\downarrow \; 3118$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8i\left(\frac{2ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}\right)}{f}+\cdots\right.}{f}\right.}{f}\right.$$

3.118. $\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx$

input
```
Int[x^4*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
Sech[e/2 + (I/4)*Pi + (f*x)/2]*((2*x^4*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f +
 ((8*I)*(((2*I)*x^3*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/f - ((6*I)*((2*x^2*Sin
h[e/2 + (I/4)*Pi + (f*x)/2])/f + ((4*I)*(((2*I)*x*Cosh[e/2 + (I/4)*Pi + (f
*x)/2])/f - ((4*I)*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f^2))/f))/f))/f)*Sqrt[a
 + I*a*Sinh[e + f*x]]
```

3.118.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.118.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.96

method	result
risch	$\frac{i\sqrt{2}\,\sqrt{a\left(i\mathrm{e}^{2fx+2e}-i+2\,\mathrm{e}^{fx+e}\right)\mathrm{e}^{-fx-e}}\,\left(ix^4f^4+f^4x^4\mathrm{e}^{fx+e}+8ix^3f^3-8\,\mathrm{e}^{fx+e}f^3x^3+48ix^2f^2+48f^2x^2\mathrm{e}^{fx+e}+192ixf-192fx\,\mathrm{e}^{fx+e}+\right.}{\left(i\mathrm{e}^{2fx+2e}-i+2\,\mathrm{e}^{fx+e}\right)f^5}$

input
```
int(x^4*(a+I*a*sinh(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*2^(1/2)*(a*(I*exp(2*f*x+2*e)-I+2*exp(f*x+e))*exp(-f*x-e))^(1/2)/(I*exp(2
*f*x+2*e)-I+2*exp(f*x+e))*(I*x^4*f^4+f^4*x^4*exp(f*x+e)+8*I*x^3*f^3-8*exp(
f*x+e)*f^3*x^3+48*I*x^2*f^2+48*f^2*x^2*exp(f*x+e)+192*I*x*f-192*f*x*exp(f*
x+e)+384*I+384*exp(f*x+e))*(exp(f*x+e)-I)/f^5
```

3.118.5 Fricas [F(-2)]

Exception generated.

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.118.6 Sympy [F]

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx = \int x^4\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}\,dx$$

input
```
integrate(x**4*(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(x**4*sqrt(I*a*(sinh(e + f*x) - I)), x)
```

3.118.7 Maxima [F]

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx=\int\sqrt{i\,a\sinh\left(fx+e\right)+a}x^4\,dx$$

input
```
integrate(x^4*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)*x^4, x)
```

3.118.8 Giac [F]

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx=\int\sqrt{i\,a\sinh\left(fx+e\right)+a}x^4\,dx$$

input
```
integrate(x^4*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)*x^4, x)
```

3.118.9 Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.82

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx$$
$$=\frac{\sqrt{2}\,\left(\mathrm{e}^{e+f\,x}+1\mathrm{i}\right)\,\sqrt{a\,\mathrm{e}^{-e-f\,x}\,{\left(\mathrm{e}^{e+f\,x}-\mathrm{i}\right)}^2\,1\mathrm{i}}\,\left(384\,\mathrm{e}^{e+f\,x}+f\,x\,192\mathrm{i}+f^2\,x^2\,48\mathrm{i}+f^3\,x^3\,8\mathrm{i}+f^4\,x^4\,1\mathrm{i}+48\,f^2\,x\right.}{f^5\,\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)}$$

input
```
int(x^4*(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

output
```
(2^(1/2)*(exp(e + f*x) + 1i)*(a*exp(- e - f*x)*(exp(e + f*x) - 1i)^2*1i)^(
1/2)*(384*exp(e + f*x) + f*x*192i + f^2*x^2*48i + f^3*x^3*8i + f^4*x^4*1i
+ 48*f^2*x^2*exp(e + f*x) - 8*f^3*x^3*exp(e + f*x) + f^4*x^4*exp(e + f*x)
- 192*f*x*exp(e + f*x) + 384i))/(f^5*(exp(2*e + 2*f*x) + 1))
```

3.119 $\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx$

3.119.1 Optimal result

Integrand size = 21, antiderivative size = 136

$$\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx = -\frac{96\sqrt{a+ia\sinh(e+fx)}}{f^4}-\frac{12x^2\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{48x\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f^3}+\frac{2x^3\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f}$$

output
```
-96*(a+I*a*sinh(f*x+e))^(1/2)/f^4-12*x^2*(a+I*a*sinh(f*x+e))^(1/2)/f^2+48*
x*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^3+2*x^3*(a+I*a*
sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f
```

3.119.2 Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92

$$\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx$$
$$=\frac{2\left(i\left(48i+24fx+6if^2x^2+f^3x^3\right)\cosh\left(\frac{1}{2}(e+fx)\right)+\left(-48i+24fx-6if^2x^2+f^3x^3\right)\sinh\left(\frac{1}{2}(e+fx)\right)\right)}{f^4\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[x^3*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
(2*(I*(48*I + 24*f*x + (6*I)*f^2*x^2 + f^3*x^3)*Cosh[(e + f*x)/2] + (-48*I
 + 24*f*x - (6*I)*f^2*x^2 + f^3*x^3)*Sinh[(e + f*x)/2])*Sqrt[a + I*a*Sinh[
e + f*x]])/(f^4*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

3.119.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3800, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3\sqrt{a+a\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{3800}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^3\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\int -ix^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \text{26}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6\int x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \text{3042}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6\int -ix^2\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \; 26$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{6i\int x^2\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)$$

$$\Big\downarrow \; 3777$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\int x\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f}+\frac{2x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}\right.}{f}\right.$$

$$\Big\downarrow \; 3042$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{f}\right)}{f}+\frac{2x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}\right.}{f}\right.$$

$$\Big\downarrow \; 3777$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int -i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f}\right)}{f}\right.$$

$$\Big\downarrow \; 26$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2\int\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f}\right)}{f}+\right.$$

3.119. $\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2\int -i\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)}{f}\right)}{f}\right.$$

$$\Big\downarrow \text{26}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\frac{2i\int\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)}{f}\right)}{f}\right.$$

$$\Big\downarrow \text{3118}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}\right)}{f}\right)}{f}+\right.$$

input
```
Int[x^3*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
Sech[e/2 + (I/4)*Pi + (f*x)/2]*((2*x^3*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f +
 ((6*I)*(((2*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/f - ((4*I)*((-4*Cosh[e
/2 + (I/4)*Pi + (f*x)/2])/f^2 + (2*x*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f))/f
))/f)*Sqrt[a + I*a*Sinh[e + f*x]]
```

3.119. $\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx$

3.119.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.119.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11

method	result
risch	$\frac{i\sqrt{2}\,\sqrt{a\left(i\mathrm{e}^{2fx+2e}-i+2\,\mathrm{e}^{fx+e}\right)\mathrm{e}^{-fx-e}}\,\left(i x^{3} f^{3}+\mathrm{e}^{fx+e} f^{3} x^{3}+6 i x^{2} f^{2}-6 f^{2} x^{2} \mathrm{e}^{fx+e}+24 i x f+24 f x\,\mathrm{e}^{fx+e}+48 i-48\,\mathrm{e}^{fx+e}\right)\left(\mathrm{e}^{fx+e}-i\right)}{\left(i\mathrm{e}^{2fx+2e}-i+2\,\mathrm{e}^{fx+e}\right) f^{4}}$

input
```
int(x^3*(a+I*a*sinh(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*2^(1/2)*(a*(I*exp(2*f*x+2*e)-I+2*exp(f*x+e))*exp(-f*x-e))^(1/2)/(I*exp(2
*f*x+2*e)-I+2*exp(f*x+e))*(I*x^3*f^3+exp(f*x+e)*f^3*x^3+6*I*x^2*f^2-6*f^2*
x^2*exp(f*x+e)+24*I*x*f+24*f*x*exp(f*x+e)+48*I-48*exp(f*x+e))*(exp(f*x+e)-
I)/f^4
```

3.119.5 Fricas [F(-2)]

Exception generated.

$$\int x^3 \sqrt{a + ia \sinh(e + fx)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.119.6 Sympy [F]

$$\int x^3 \sqrt{a + ia \sinh(e + fx)}\, dx = \int x^3 \sqrt{ia\,(\sinh\,(e + fx) - i)}\, dx$$

input
```
integrate(x**3*(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(x**3*sqrt(I*a*(sinh(e + f*x) - I)), x)
```

3.119.7 Maxima [F]

$$\int x^3 \sqrt{a + ia \sinh(e + fx)}\, dx = \int \sqrt{i\,a \sinh\,(fx + e) + a}x^3\, dx$$

input
```
integrate(x^3*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)*x^3, x)
```

3.119.8 Giac [F]

$$\int x^3 \sqrt{a + ia \sinh(e + fx)}\, dx = \int \sqrt{i\, a \sinh\left(fx + e\right) + a} x^3\, dx$$

input
```
integrate(x^3*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)*x^3, x)
```

3.119.9 Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93

$$\int x^3 \sqrt{a + ia \sinh(e + fx)}\, dx$$
$$= \frac{\sqrt{2}\left(e^{e+f\,x} + 1\mathrm{i}\right)\, \sqrt{a\, e^{-e-f\,x}\left(e^{e+f\,x} - \mathrm{i}\right)^2 1\mathrm{i}}\left(f^3\, x^3\, e^{e+f\,x} + f\, x\, 24\mathrm{i} + f^2\, x^2\, 6\mathrm{i} + f^3\, x^3\, 1\mathrm{i} - 6\, f^2\, x^2\, e^{e+f\,x} - 48\right.}{f^4\, \left(e^{2\,e+2\,f\,x} + 1\right)}$$

input
```
int(x^3*(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

output
```
(2^(1/2)*(exp(e + f*x) + 1i)*(a*exp(- e - f*x)*(exp(e + f*x) - 1i)^2*1i)^(
1/2)*(f*x*24i - 48*exp(e + f*x) + f^2*x^2*6i + f^3*x^3*1i - 6*f^2*x^2*exp(
e + f*x) + f^3*x^3*exp(e + f*x) + 24*f*x*exp(e + f*x) + 48i))/(f^4*(exp(2*
e + 2*f*x) + 1))
```

3.120 $\int x^2 \sqrt{a+ia\sinh(e+fx)}\,dx$

3.120.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx=-\frac{8x\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{16\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f^3}+\frac{2x^2\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f}$$

output
```
-8*x*(a+I*a*sinh(f*x+e))^(1/2)/f^2+16*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e
+1/4*I*Pi+1/2*f*x)/f^3+2*x^2*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi
+1/2*f*x)/f
```

3.120.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx=\frac{2\left(i(8+4ifx+f^2x^2)\cosh\left(\frac{1}{2}(e+fx)\right)+(8-4ifx+f^2x^2)\sinh\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a+ia\sinh(e+fx)}}{f^3\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[x^2*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
(2*(I*(8 + (4*I)*f*x + f^2*x^2)*Cosh[(e + f*x)/2] + (8 - (4*I)*f*x + f^2*x
^2)*Sinh[(e + f*x)/2])*Sqrt[a + I*a*Sinh[e + f*x]])/(f^3*(Cosh[(e + f*x)/2
] + I*Sinh[(e + f*x)/2]))
```

3.120.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3800, 3042, 3777, 26, 3042, 26, 3777, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2\sqrt{a+a\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{3800}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^2\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\int -ix\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\int x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\int -ix\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{4i\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)$$

$$\Big\downarrow \text{3777}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f}+\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\right.}{f}\right.$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{f}\right)}{f}+\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\right.}{f}\right.$$

$$\Big\downarrow \text{3118}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}\right)}{f}+\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right.}{f}\right.$$

input
```
Int[x^2*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
Sech[e/2 + (I/4)*Pi + (f*x)/2]*((2*x^2*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f +
 ((4*I)*(((2*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/f - ((4*I)*Sinh[e/2 + (I
/4)*Pi + (f*x)/2])/f^2))/f)*Sqrt[a + I*a*Sinh[e + f*x]]
```

3.120. $\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx$

3.120.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.120.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

method	result	size
risch	$\frac{i\sqrt{2}\,\sqrt{a\left(i\,{\mathrm e}^{2 f x+2 e}-i+2\,{\mathrm e}^{f x+e}\right){\mathrm e}^{-f x-e}}\,\left(i x^{2} f^{2}+f^{2} x^{2} {\mathrm e}^{f x+e}+4 i x f-4 f x\,{\mathrm e}^{f x+e}+8 i+8\,{\mathrm e}^{f x+e}\right)\left({\mathrm e}^{f x+e}-i\right)}{\left(i\,{\mathrm e}^{2 f x+2 e}-i+2\,{\mathrm e}^{f x+e}\right) f^{3}}$	128

input
```
int(x^2*(a+I*a*sinh(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*2^(1/2)*(a*(I*exp(2*f*x+2*e)-I+2*exp(f*x+e))*exp(-f*x-e))^(1/2)/(I*exp(2
*f*x+2*e)-I+2*exp(f*x+e))*(I*x^2*f^2+f^2*x^2*exp(f*x+e)+4*I*x*f-4*f*x*exp(
f*x+e)+8*I+8*exp(f*x+e))*(exp(f*x+e)-I)/f^3
```

3.120.5 Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{a + ia\sinh(e + fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.120.6 Sympy [F]

$$\int x^2\sqrt{a + ia\sinh(e + fx)}\,dx = \int x^2\sqrt{ia\left(\sinh\left(e + fx\right) - i\right)}\,dx$$

input
```
integrate(x**2*(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(x**2*sqrt(I*a*(sinh(e + f*x) - I)), x)
```

3.120.7 Maxima [F]

$$\int x^2\sqrt{a + ia\sinh(e + fx)}\,dx = \int \sqrt{i\,a\sinh\left(fx + e\right) + a}x^2\,dx$$

input
```
integrate(x^2*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)*x^2, x)
```

3.120.8 Giac [F]

$$\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx = \int \sqrt{i\,a\sinh\left(fx+e\right)+a}x^2\,dx$$

input
```
integrate(x^2*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)*x^2, x)
```

3.120.9 Mupad [B] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.83

$$\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx$$
$$= \frac{\sqrt{2}\,\sqrt{a\,\mathrm{e}^{-e-f\,x}\,\left(\mathrm{e}^{e+f\,x}-\mathrm{i}\right)^2\,1\mathrm{i}}\,\left(8\,\mathrm{e}^{e+f\,x}+f\,x\,4\mathrm{i}+f^2\,x^2\,1\mathrm{i}+f^2\,x^2\,\mathrm{e}^{e+f\,x}-4\,f\,x\,\mathrm{e}^{e+f\,x}+8\mathrm{i}\right)}{f^3\,\left(\mathrm{e}^{e+f\,x}-\mathrm{i}\right)}$$

input
```
int(x^2*(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

output
```
(2^(1/2)*(a*exp(- e - f*x)*(exp(e + f*x) - 1i)^2*1i)^(1/2)*(8*exp(e + f*x)
 + f*x*4i + f^2*x^2*1i + f^2*x^2*exp(e + f*x) - 4*f*x*exp(e + f*x) + 8i))/
(f^3*(exp(e + f*x) - 1i))
```

3.121 $\int x\sqrt{a+ia\sinh(e+fx)}\,dx$

3.121.1 Optimal result

Integrand size = 19, antiderivative size = 66

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx=-\frac{4\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{2x\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f}$$

output
```
-4*(a+I*a*sinh(f*x+e))^(1/2)/f^2+2*x*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+
1/4*I*Pi+1/2*f*x)/f
```

3.121.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.32

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx=\frac{2\left((-2+ifx)\cosh\left(\frac{1}{2}(e+fx)\right)+(-2i+fx)\sinh\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a+ia\sinh(e+fx)}}{f^2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[x*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
(2*((-2 + I*f*x)*Cosh[(e + f*x)/2] + (-2*I + f*x)*Sinh[(e + f*x)/2])*Sqrt[
a + I*a*Sinh[e + f*x]])/(f^2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

3.121.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3800, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sqrt{a+a\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{3800}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int -i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2\int \sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2\int -i\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2i\int\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)$$

$$\Big\downarrow \text{ 3118}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}\right)$$

input
```
Int[x*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
Sech[e/2 + (I/4)*Pi + (f*x)/2]*((-4*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/f^2 +
(2*x*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f)*Sqrt[a + I*a*Sinh[e + f*x]]
```

3.121.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.121.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.59

method	result	size
risch	$\frac{i\sqrt{2}\,\sqrt{a\left(i\mathrm{e}^{2fx+2e}-i+2\,\mathrm{e}^{fx+e}\right)\mathrm{e}^{-fx-e}}\,\left(ixf+fx\,\mathrm{e}^{fx+e}+2i-2\,\mathrm{e}^{fx+e}\right)\left(\mathrm{e}^{fx+e}-i\right)}{\left(i\mathrm{e}^{2fx+2e}-i+2\,\mathrm{e}^{fx+e}\right)f^{2}}$	105

input
```
int(x*(a+I*a*sinh(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*2^(1/2)*(a*(I*exp(2*f*x+2*e)-I+2*exp(f*x+e))*exp(-f*x-e))^(1/2)/(I*exp(2
*f*x+2*e)-I+2*exp(f*x+e))*(I*f*x+f*x*exp(f*x+e)+2*I-2*exp(f*x+e))*(exp(f*x
+e)-I)/f^2
```

3.121.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.121.6 Sympy [F]

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx = \int x\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}\,dx$$

input
```
integrate(x*(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(x*sqrt(I*a*(sinh(e + f*x) - I)), x)
```

3.121.7 Maxima [F]

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx=\int\sqrt{i\,a\sinh\left(fx+e\right)+a}x\,dx$$

input
```
integrate(x*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)*x, x)
```

3.121.8 Giac [F]

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx=\int\sqrt{i\,a\sinh\left(fx+e\right)+a}x\,dx$$

input
```
integrate(x*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)*x, x)
```

3.121.9 Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx$$
$$=\frac{\sqrt{2}\left(\mathrm{e}^{e+f\,x}+1\mathrm{i}\right)\,\left(f\,x\,\mathrm{e}^{e+f\,x}+f\,x\,1\mathrm{i}-2\,\mathrm{e}^{e+f\,x}+2\mathrm{i}\right)\,\sqrt{a\,\mathrm{e}^{-e-f\,x}\,{\left(\mathrm{e}^{e+f\,x}-\mathrm{i}\right)}^2\,1\mathrm{i}}}{f^2\,\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)}$$

input
```
int(x*(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

output
```
(2^(1/2)*(exp(e + f*x) + 1i)*(f*x*1i - 2*exp(e + f*x) + f*x*exp(e + f*x) +
 2i)*(a*exp(- e - f*x)*(exp(e + f*x) - 1i)^2*1i)^(1/2))/(f^2*(exp(2*e + 2*
f*x) + 1))
```

3.122 $\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx$

3.122.1 Optimal result

Integrand size = 21, antiderivative size = 125

$$\begin{aligned}\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx = {} & i\mathrm{Chi}\left(\frac{fx}{2}\right)\mathrm{sech}\left(\frac{e}{2}+\frac{i\pi}{4}\right.\\ & \left.+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e-i\pi)\right)\sqrt{a+ia\sinh(e+fx)}\\ & +i\cosh\left(\frac{1}{4}(2e-i\pi)\right)\mathrm{sech}\left(\frac{e}{2}+\frac{i\pi}{4}\right.\\ & \left.+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\mathrm{Shi}\left(\frac{fx}{2}\right)\end{aligned}$$

output
```
sinh(1/2*e+1/4*I*Pi)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*Shi(1/2*f*x)*(a+I*a*sinh
(f*x+e))^(1/2)+Chi(1/2*f*x)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*cosh(1/2*e+1/4*I*
Pi)*(a+I*a*sinh(f*x+e))^(1/2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx = \frac{\sqrt{a+ia\sinh(e+fx)}\left(\mathrm{Chi}\left(\frac{fx}{2}\right)\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)+\left(i\cosh\left(\frac{e}{2}\right)+\sinh\left(\frac{e}{2}\right)\right)\mathrm{Shi}\left(\frac{fx}{2}\right)\right)}{\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)}$$

input
```
Integrate[Sqrt[a + I*a*Sinh[e + f*x]]/x,x]
```

output
```
(Sqrt[a + I*a*Sinh[e + f*x]]*(CoshIntegral[(f*x)/2]*(Cosh[e/2] + I*Sinh[e/
2]) + (I*Cosh[e/2] + Sinh[e/2])*SinhIntegral[(f*x)/2]))/(Cosh[(e + f*x)/2]
 + I*Sinh[(e + f*x)/2])
```

3.122.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.71, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3800, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+a\sin(ie+ifx)}}{x}dx$$

$$\Big\downarrow \text{3800}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)}{x}dx$$

$$\Big\downarrow \text{3784}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int \frac{\cosh\left(\frac{fx}{2}\right)}{x}dx+\cosh\left(\frac{1}{4}(2e-i\pi)\right)\int \frac{i\sinh}{x}\right.$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int \frac{\cosh\left(\frac{fx}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}(2e-i\pi)\right)\int \frac{\sinh}{x}\right.$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}(2e-i\pi)\right)\int -\right.$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+\cosh\left(\frac{1}{4}(2e-i\pi)\right)\int \frac{\sin}{}\right.$$

$$\Big\downarrow \text{3779}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}(2e-i\pi)\right)\operatorname{Shi}\left(\right.\right.$$

$$\Big\downarrow \text{3782}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\operatorname{Chi}\left(\frac{fx}{2}\right)+i\cosh\left(\frac{1}{4}(2e-i\pi)\right)\operatorname{Shi}\left(\frac{fx}{2}\right)\right)$$

input
```
Int[Sqrt[a + I*a*Sinh[e + f*x]]/x,x]
```

output
```
Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*(I*CoshIntegral
[(f*x)/2]*Sinh[(2*e - I*Pi)/4] + I*Cosh[(2*e - I*Pi)/4]*SinhIntegral[(f*x)
/2])
```

3.122.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.122.4 Maple [F]

$$\int \frac{\sqrt{a + ia \sinh{(fx + e)}}}{x} dx$$

input
```
int((a+I*a*sinh(f*x+e))^(1/2)/x,x)
```

output
```
int((a+I*a*sinh(f*x+e))^(1/2)/x,x)
```

3.122.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + ia \sinh(e + fx)}}{x} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.122.6 Sympy [F]

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx=\int \frac{\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))**(1/2)/x,x)
```

output
```
Integral(sqrt(I*a*(sinh(e + f*x) - I))/x, x)
```

3.122.7 Maxima [F]

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx=\int \frac{\sqrt{i\,a\sinh\left(fx+e\right)+a}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)/x, x)
```

3.122.8 Giac [F]

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx=\int \frac{\sqrt{i\,a\sinh\left(fx+e\right)+a}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)/x, x)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx = \int \frac{\sqrt{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}}{x}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^(1/2)/x,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^(1/2)/x, x)
```

3.123 $\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx$

3.123.1 Optimal result

Integrand size = 21, antiderivative size = 149

$$\begin{aligned}\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx = {} & -\frac{\sqrt{a+ia\sinh(e+fx)}}{x}+\frac{1}{2}f\operatorname{Chi}\left(\frac{fx}{2}\right)\operatorname{sech}\left(\frac{e}{2}+\frac{i\pi}{4}\right.\\ & \left.+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e+i\pi)\right)\sqrt{a+ia\sinh(e+fx)}\\ & +\frac{1}{2}f\cosh\left(\frac{1}{4}(2e+i\pi)\right)\operatorname{sech}\left(\frac{e}{2}+\frac{i\pi}{4}\right.\\ & \left.+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\operatorname{Shi}\left(\frac{fx}{2}\right)\end{aligned}$$

output
```
-(a+I*a*sinh(f*x+e))^(1/2)/x+1/2*f*cosh(1/2*e+1/4*I*Pi)*sech(1/2*e+1/4*I*P
i+1/2*f*x)*Shi(1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)+1/2*f*Chi(1/2*f*x)*sech(
1/2*e+1/4*I*Pi+1/2*f*x)*sinh(1/2*e+1/4*I*Pi)*(a+I*a*sinh(f*x+e))^(1/2)
```

3.123.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.89

$$\begin{aligned}&\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx\\ &= \frac{\sqrt{a+ia\sinh(e+fx)}\left(fx\operatorname{Chi}\left(\frac{fx}{2}\right)\left(i\cosh\left(\frac{e}{2}\right)+\sinh\left(\frac{e}{2}\right)\right)-2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)+\ldots}{2x\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}\end{aligned}$$

input
```
Integrate[Sqrt[a + I*a*Sinh[e + f*x]]/x^2,x]
```

output
```
(Sqrt[a + I*a*Sinh[e + f*x]]*(f*x*CoshIntegral[(f*x)/2]*(I*Cosh[e/2] + Sin
h[e/2]) - 2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]) + f*x*(Cosh[e/2] + I
*Sinh[e/2])*SinhIntegral[(f*x)/2]))/(2*x*(Cosh[(e + f*x)/2] + I*Sinh[(e +
f*x)/2]))
```

3.123.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3800, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+a\sin(ie+ifx)}}{x^2}dx$$

$$\Big\downarrow \text{3800}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)}{x^2}dx$$

$$\Big\downarrow \text{3778}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{1}{2}if\int -\frac{i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}dx-\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{1}{2}f\int \frac{\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}dx-\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{1}{2}f\int -\frac{i\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)}{x}dx-\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{2}if\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)}{x}dx-\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}\right)$$

$$\Big\downarrow \text{3784}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e+i\pi)\right)\int \frac{\cosh\left(\frac{fx}{2}\right)}{x}dx+\cosh\left(\frac{1}{4}(2e+i\pi)\right)\int\right.\right.$$

$$\Big\downarrow \text{26}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e+i\pi)\right)\int \frac{\cosh\left(\frac{fx}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}(2e+i\pi)\right)\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e+i\pi)\right)\int \frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}(2e+i\pi\right.\right.\right.$$

$$\Big\downarrow \text{26}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e+i\pi)\right)\int \frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+\cosh\left(\frac{1}{4}(2e+i\pi)\right.\right.\right.$$

$$\Big\downarrow \text{3779}$$

$$\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e+i\pi)\right)\int \frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}(2e+i\pi\right.\right.\right.$$

$$\Big\downarrow \text{3782}$$

3.123. $\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e+i\pi)\right)\operatorname{Chi}\left(\frac{fx}{2}\right)+i\cosh\left(\frac{1}{4}(2e+i\pi)\right)\operatorname{Shi}\left(\frac{f}{2}\right.\right.\right.$$

input
```
Int[Sqrt[a + I*a*Sinh[e + f*x]]/x^2,x]
```

output
```
Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*(-(Cosh[e/2 + (
I/4)*Pi + (f*x)/2]/x) - (I/2)*f*(I*CoshIntegral[(f*x)/2]*Sinh[(2*e + I*Pi)
/4] + I*Cosh[(2*e + I*Pi)/4]*SinhIntegral[(f*x)/2]))
```

3.123.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.123.4 Maple [F]

$$\int \frac{\sqrt{a + ia\sinh\left(fx + e\right)}}{x^2}dx$$

input
```
int((a+I*a*sinh(f*x+e))^(1/2)/x^2,x)
```

output
```
int((a+I*a*sinh(f*x+e))^(1/2)/x^2,x)
```

3.123.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + ia\sinh(e + fx)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.123.6 Sympy [F]

$$\int \frac{\sqrt{a + ia\sinh(e + fx)}}{x^2}\,dx = \int \frac{\sqrt{ia\left(\sinh\left(e + fx\right) - i\right)}}{x^2}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(I*a*(sinh(e + f*x) - I))/x**2, x)
```

3.123.7 Maxima [F]

$$\int \frac{\sqrt{a + ia\sinh(e + fx)}}{x^2}\,dx = \int \frac{\sqrt{i\,a\sinh\left(fx + e\right) + a}}{x^2}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)/x^2, x)
```

3.123.8 Giac [F]

$$\int \frac{\sqrt{a + ia\sinh(e + fx)}}{x^2}\,dx = \int \frac{\sqrt{i\,a\sinh\left(fx + e\right) + a}}{x^2}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)/x^2, x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + ia\sinh(e + fx)}}{x^2}\,dx = \int \frac{\sqrt{a + a\sinh\left(e + f\,x\right)\,1\mathrm{i}}}{x^2}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^(1/2)/x^2,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^(1/2)/x^2, x)
```

3.124 $\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx$

3.124.1 Optimal result

Integrand size = 21, antiderivative size = 204

$$\begin{aligned}\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx = &-\frac{\sqrt{a+ia\sinh(e+fx)}}{2x^2}+\frac{1}{8}if^2\mathrm{Chi}\left(\frac{fx}{2}\right)\mathrm{sech}\left(\frac{e}{2}+\frac{i\pi}{4}\right.\\&\left.+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e-i\pi)\right)\sqrt{a+ia\sinh(e+fx)}\\&+\frac{1}{8}if^2\cosh\left(\frac{1}{4}(2e-i\pi)\right)\mathrm{sech}\left(\frac{e}{2}+\frac{i\pi}{4}\right.\\&\left.+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\mathrm{Shi}\left(\frac{fx}{2}\right)\\&-\frac{f\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{4x}\end{aligned}$$

output
```
-1/2*(a+I*a*sinh(f*x+e))^(1/2)/x^2+1/8*f^2*sinh(1/2*e+1/4*I*Pi)*sech(1/2*e
+1/4*I*Pi+1/2*f*x)*Shi(1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)+1/8*f^2*Chi(1/2*
f*x)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*cosh(1/2*e+1/4*I*Pi)*(a+I*a*sinh(f*x+e))
^(1/2)-1/4*f*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/x
```

3.124.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx$$
$$= \frac{\sqrt{a+ia\sinh(e+fx)}\left(-4\cosh\left(\frac{1}{2}(e+fx)\right)-2ifx\cosh\left(\frac{1}{2}(e+fx)\right)+f^2x^2\mathrm{Chi}\left(\frac{fx}{2}\right)\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right.\right.\right.}{8x^2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\right.}$$

input
```
Integrate[Sqrt[a + I*a*Sinh[e + f*x]]/x^3,x]
```

output
```
(Sqrt[a + I*a*Sinh[e + f*x]]*(-4*Cosh[(e + f*x)/2] - (2*I)*f*x*Cosh[(e + f
*x)/2] + f^2*x^2*CoshIntegral[(f*x)/2]*(Cosh[e/2] + I*Sinh[e/2]) - (4*I)*S
inh[(e + f*x)/2] - 2*f*x*Sinh[(e + f*x)/2] + f^2*x^2*(I*Cosh[e/2] + Sinh[e
/2])*SinhIntegral[(f*x)/2]))/(8*x^2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/
2]))
```

3.124.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.78, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 3800, 3042, 3778, 26, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a+a\sin(ie+ifx)}}{x^3}dx$$
$$\Big\downarrow \text{3800}$$
$$\mathrm{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\mathrm{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)}{x^3}dx$$

$$\Big\downarrow \text{3778}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{1}{4}if\int -\frac{i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x^2}dx-\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{1}{4}f\int \frac{\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x^2}dx-\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{1}{4}f\int -\frac{i\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)}{x^2}dx-\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)}{x^2}dx-\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{3778}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\left(\frac{1}{2}if\int \frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}dx-\frac{i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}\right)-\frac{\mathrm{co}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\left(\frac{1}{2}if\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)}{x}dx-\frac{i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}\right)-\frac{\mathrm{cos}}{}\right.$$

$$\Big\downarrow \text{3784}$$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\left(\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int \frac{\cosh\left(\frac{fx}{2}\right)}{x}dx+\cosh\left(\frac{1}{4}(2e-i\right.\right.\right.\right.$$

$$\Big\downarrow \text{26}$$

3.124. $\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx$

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\left(\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\cosh\left(\frac{fx}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}(2e-\right.\right.\right.\right.$$

$\downarrow$ 3042

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\left(\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}(2\right.\right.\right.\right.$$

$\downarrow$ 26

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\left(\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+\cosh\left(\frac{1}{4}(2e\right.\right.\right.\right.$$

$\downarrow$ 3779

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\left(\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\int\frac{\sin\left(\frac{ifx}{2}+\frac{\pi}{2}\right)}{x}dx+i\cosh\left(\frac{1}{4}(2\right.\right.\right.\right.$$

$\downarrow$ 3782

$$\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{1}{4}if\left(\frac{1}{2}if\left(i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\operatorname{Chi}\left(\frac{fx}{2}\right)+i\cosh\left(\frac{1}{4}(2e-i\pi)\right)\right.\right.\right.$$

input
```
Int[Sqrt[a + I*a*Sinh[e + f*x]]/x^3,x]
```

output
```
Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*(-1/2*Cosh[e/2
+ (I/4)*Pi + (f*x)/2]/x^2 - (I/4)*f*(((-I)*Sinh[e/2 + (I/4)*Pi + (f*x)/2])
/x + (I/2)*f*(I*CoshIntegral[(f*x)/2]*Sinh[(2*e - I*Pi)/4] + I*Cosh[(2*e -
 I*Pi)/4]*SinhIntegral[(f*x)/2])))
```

3.124. $\int\frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx$

3.124.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.124. $\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx$

3.124.4 Maple [F]

$$\int \frac{\sqrt{a+ia\sinh\left(fx+e\right)}}{x^3}dx$$

input
```
int((a+I*a*sinh(f*x+e))^(1/2)/x^3,x)
```

output
```
int((a+I*a*sinh(f*x+e))^(1/2)/x^3,x)
```

3.124.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.124.6 Sympy [F]

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx = \int \frac{\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}{x^3}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(I*a*(sinh(e + f*x) - I))/x**3, x)
```

3.124.7 Maxima [F]

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx = \int \frac{\sqrt{i\,a\sinh\left(fx+e\right)+a}}{x^3}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)/x^3, x)
```

3.124.8 Giac [F]

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx = \int \frac{\sqrt{i\,a\sinh\left(fx+e\right)+a}}{x^3}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*sinh(f*x + e) + a)/x^3, x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx = \int \frac{\sqrt{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}}{x^3}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^(1/2)/x^3,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^(1/2)/x^3, x)
```

3.125 $\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx$

3.125.1 Optimal result

Integrand size = 21, antiderivative size = 377

$$\begin{aligned}\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx=&-\frac{1280a\sqrt{a+ia\sinh(e+fx)}}{9f^4}\\&-\frac{16ax^2\sqrt{a+ia\sinh(e+fx)}}{f^2}-\frac{64a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{27f^4}\\&-\frac{8ax^2\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{3f^2}\\&+\frac{32ax\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^3}\\&+\frac{4ax^3\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{3f}\\&+\frac{640ax\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{9f^3}\\&+\frac{8ax^3\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3f}\end{aligned}$$

output
```
-1280/9*a*(a+I*a*sinh(f*x+e))^(1/2)/f^4-16*a*x^2*(a+I*a*sinh(f*x+e))^(1/2)
/f^2-64/27*a*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^2*(a+I*a*sinh(f*x+e))^(1/2)/f^4-
8/3*a*x^2*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^2*(a+I*a*sinh(f*x+e))^(1/2)/f^2+32/
9*a*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*sinh(1/2*e+1/4*I*Pi+1/2*f*x)*(a+I*a*sin
h(f*x+e))^(1/2)/f^3+4/3*a*x^3*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*sinh(1/2*e+1/4*
I*Pi+1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)/f+640/9*a*x*(a+I*a*sinh(f*x+e))^(1
/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^3+8/3*a*x^3*(a+I*a*sinh(f*x+e))^(1/2)*t
anh(1/2*e+1/4*I*Pi+1/2*f*x)/f
```

3.125.2 Mathematica [A] (verified)

Time = 7.66 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.71

$$\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx =$$
$$-\frac{a(-i+\sinh(e+fx))\sqrt{a+ia\sinh(e+fx)}\left(81(48i+24fx+6if^2x^2+f^3x^3)\cosh\left(\frac{1}{2}(e+fx)\right)+(-16i+\right.}{}$$

input
```
Integrate[x^3*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
-1/27*(a*(-I + Sinh[e + f*x])*Sqrt[a + I*a*Sinh[e + f*x]]*(81*(48*I + 24*f
*x + (6*I)*f^2*x^2 + f^3*x^3)*Cosh[(e + f*x)/2] + (-16*I + 24*f*x - (18*I)
*f^2*x^2 + 9*f^3*x^3)*Cosh[(3*(e + f*x))/2] - 3888*Sinh[(e + f*x)/2] - (19
44*I)*f*x*Sinh[(e + f*x)/2] - 486*f^2*x^2*Sinh[(e + f*x)/2] - (81*I)*f^3*x
^3*Sinh[(e + f*x)/2] - 16*Sinh[(3*(e + f*x))/2] + (24*I)*f*x*Sinh[(3*(e +
f*x))/2] - 18*f^2*x^2*Sinh[(3*(e + f*x))/2] + (9*I)*f^3*x^3*Sinh[(3*(e + f
*x))/2]))/(f^4*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^3)
```

3.125.3 Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.06, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.095$, Rules used = {3042, 3800, 3042, 3792, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 3791, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int x^3(a+a\sin(ie+ifx))^{3/2}dx$$
$$\Big\downarrow \; 3800$$
$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^3\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$
$$\Big\downarrow \; 3042$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^3\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx$$

$$\Big\downarrow \text{3792}$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{3f^2}+\frac{2}{3}\int x^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx\right.$$

$$\Big\downarrow \text{3042}$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\int x^3\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx-\right.$$

$$\Big\downarrow \text{3777}$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{2x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6i\int}{}\right.\right.$$

$$\Big\downarrow \text{26}$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{2x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6\int}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{2x^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{6\int}{}\right.\right.$$

$$\Big\downarrow \text{26}$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6i\int x^2\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\right.\right.$$

$$\Big\downarrow \text{3777}$$

3.125. $\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\int x\,c}{}\right.}{f}\right.\right.$$

$\big\downarrow$ 3042

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\int x\,s}{}\right.}{f}\right.\right.$$

$\big\downarrow$ 3777

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-4i\left(\frac{2x}{}\right.\right.}{}\right.\right.$$

$\big\downarrow$ 26

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-4i\left(\frac{2x}{}\right.\right.}{}\right.\right.$$

$\big\downarrow$ 3042

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-4i\left(\frac{2x}{}\right.\right.}{}\right.\right.$$

$\big\downarrow$ 26

3.125. $\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\frac{2i}{}\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 3118

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{3f^2}-\frac{4x^2\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2}{3}\left(\frac{6i}{}\right.\right.$$

$\downarrow$ 3791

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\left(\frac{2}{3}\int x\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{9f^2}+\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}\right.}{}\right.}{3f^2}\right.$$

$\downarrow$ 3042

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\left(\frac{2}{3}\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{9f^2}+\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\right.}{}\right.}{3f^2}\right.$$

$\downarrow$ 3777

3.125. $\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int -i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\right.}{9f^2}\right.}{3f^2}\right.$$

$$\Big\downarrow \; 26$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2\int \sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right.}{9f^2}\right.}{3f^2}\right.$$

$$\Big\downarrow \; 3042$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2\int -i\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\right.}{9f^2}\right.}{3f^2}\right.$$

$$\Big\downarrow \; 26$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\left(\frac{2}{3}\left(\frac{2i\int \sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right.}{9f^2}\right.}{3f^2}\right.$$

$$\Big\downarrow \; 3118$$

3.125. $\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{4x^2\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3f^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\left(\frac{2x\sin}{}\right.}{}\right.}{f}\right.\right.$$

input
```
Int[x^3*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
2*a*Sech[e/2 + (I/4)*Pi + (f*x)/2]*((-4*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]
^3)/(3*f^2) + (2*x^3*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sinh[e/2 + (I/4)*Pi
+ (f*x)/2])/(3*f) + (8*((-4*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^3)/(9*f^2) + (2
*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f)
+ (2*((-4*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/f^2 + (2*x*Sinh[e/2 + (I/4)*Pi +
 (f*x)/2])/f))/3))/(3*f^2) + (2*((2*x^3*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f
+ ((6*I)*(((2*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/f - ((4*I)*((-4*Cosh[
e/2 + (I/4)*Pi + (f*x)/2])/f^2 + (2*x*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f))/
f))/f))/3)*Sqrt[a + I*a*Sinh[e + f*x]]
```

3.125.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.125.4 Maple [F]

$$\int x^3(a+ia\sinh(fx+e))^{\frac{3}{2}}\,dx$$

input
```
int(x^3*(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
int(x^3*(a+I*a*sinh(f*x+e))^(3/2),x)
```

3.125.5 Fricas [F(-2)]

Exception generated.

$$\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.125.6 Sympy [F]

$$\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx=\int x^3(ia(\sinh(e+fx)-i))^{\frac{3}{2}}\,dx$$

input
```
integrate(x**3*(a+I*a*sinh(f*x+e))**(3/2),x)
```

output
```
Integral(x**3*(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

3.125.7 Maxima [F]

$$\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx=\int(i\,a\sinh(fx+e)+a)^{\frac{3}{2}}x^3\,dx$$

input
```
integrate(x^3*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^3, x)
```

3.125.8 Giac [F]

$$\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx=\int(i\,a\sinh(fx+e)+a)^{\frac{3}{2}}x^3\,dx$$

input
```
integrate(x^3*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^3, x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx=\int x^3\,(a+a\sinh(e+f\,x)\,1\mathrm{i})^{3/2}\,dx$$

input
```
int(x^3*(a + a*sinh(e + f*x)*1i)^(3/2),x)
```

output
```
int(x^3*(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

3.126 $\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx$

3.126.1 Optimal result

Integrand size = 21, antiderivative size = 303

$$\begin{aligned}\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx &= -\frac{32ax\sqrt{a+ia\sinh(e+fx)}}{3f^2} \\ &- \frac{16ax\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2} \\ &+ \frac{4ax^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{3f} \\ &+ \frac{224a\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{9f^3} \\ &+ \frac{8ax^2\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3f} \\ &+ \frac{32a\sinh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{27f^3}\end{aligned}$$

output

```
-32/3*a*x*(a+I*a*sinh(f*x+e))^(1/2)/f^2-16/9*a*x*cosh(1/2*e+1/4*I*Pi+1/2*f
*x)^2*(a+I*a*sinh(f*x+e))^(1/2)/f^2+4/3*a*x^2*cosh(1/2*e+1/4*I*Pi+1/2*f*x)
*sinh(1/2*e+1/4*I*Pi+1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)/f+224/9*a*(a+I*a*s
inh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^3+8/3*a*x^2*(a+I*a*sinh(f
*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f+32/27*a*sinh(1/2*e+1/4*I*Pi+1/
2*f*x)^2*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^3
```

3.126.2 Mathematica [A] (verified)

Time = 7.46 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.57

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx =$$
$$-\frac{a\left(81(8+4ifx+f^2x^2)\cosh\left(\frac{1}{2}(e+fx)\right)+(8-12ifx+9f^2x^2)\cosh\left(\frac{3}{2}(e+fx)\right)+2i(-4(80-42ifx+\right.}{27f^3\left(\cosh\left(\frac{1}{2}(e+fx)\right)\right.}$$

input
```
Integrate[x^2*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
-1/27*(a*(81*(8 + (4*I)*f*x + f^2*x^2)*Cosh[(e + f*x)/2] + (8 - (12*I)*f*x
 + 9*f^2*x^2)*Cosh[(3*(e + f*x))/2] + (2*I)*(-4*(80 - (42*I)*f*x + 9*f^2*x
^2) + (8 + (12*I)*f*x + 9*f^2*x^2)*Cosh[e + f*x])*Sinh[(e + f*x)/2])*(-I +
 Sinh[e + f*x])*Sqrt[a + I*a*Sinh[e + f*x]])/(f^3*(Cosh[(e + f*x)/2] + I*S
inh[(e + f*x)/2])^3)
```

3.126.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.93, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3800, 3042, 3792, 3042, 3113, 2009, 3777, 26, 3042, 26, 3777, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int x^2(a+a\sin(ie+ifx))^{3/2}dx$$
$$\Big\downarrow \text{3800}$$
$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^2\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$
$$\Big\downarrow \text{3042}$$
$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x^2\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx$$

$$\Big\downarrow \color{blue}{3792}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{9f^2}+\frac{2}{3}\int x^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx-\right.$$

$$\Big\downarrow \color{blue}{3042}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{8\int\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{9f^2}+\frac{2}{3}\int x^2\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx-\frac{8x}{}\right.$$

$$\Big\downarrow \color{blue}{3113}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{16i\int\left(\sinh^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)+1\right)d\left(-i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{9f^3}+\frac{2}{3}\right.$$

$$\Big\downarrow \color{blue}{2009}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\int x^2\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx+\frac{16i\left(-\frac{1}{3}i\sinh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)-\right.}{9f^3}\right.$$

$$\Big\downarrow \color{blue}{3777}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4i\int-ix\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)+\right.$$

$$\Big\downarrow \color{blue}{26}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\int x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)+\frac{16}{}\right.$$

$$\Big\downarrow \color{blue}{3042}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\int-ix\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)+\right.$$

$$\Big\downarrow \color{blue}{26}$$

3.126. $\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{4i\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)+\frac{16}{}\right.$$

$$\Big\downarrow \textbf{3777}$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)}{f}+\frac{2x^2\sinh\left(\frac{e}{2}\right.}{f}\right.\right.$$

$$\Big\downarrow \textbf{3042}$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{f}\right)}{f}+\frac{2x^2\sinh\left(\frac{e}{2}+\right.}{f}\right.\right.$$

$$\Big\downarrow \textbf{3118}$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{16i\left(-\frac{1}{3}i\sinh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)-i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{9f^3}+\frac{2}{3}\left(\frac{4i\left(\right.}{}\right.\right.$$

input
```
Int[x^2*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
2*a*Sech[e/2 + (I/4)*Pi + (f*x)/2]*((-8*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^3
)/(9*f^2) + (2*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sinh[e/2 + (I/4)*Pi +
(f*x)/2])/(3*f) + (((16*I)/9)*((-I)*Sinh[e/2 + (I/4)*Pi + (f*x)/2] - (I/3)
*Sinh[e/2 + (I/4)*Pi + (f*x)/2]^3))/f^3 + (2*((2*x^2*Sinh[e/2 + (I/4)*Pi +
 (f*x)/2])/f + ((4*I)*(((2*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/f - ((4*I)
*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/f^2))/f))/3)*Sqrt[a + I*a*Sinh[e + f*x]]
```

3.126. $\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx$

3.126.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.126.4 Maple [F]

$$\int x^2(a+ia\sinh(fx+e))^{\frac{3}{2}}\,dx$$

input
```
int(x^2*(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
int(x^2*(a+I*a*sinh(f*x+e))^(3/2),x)
```

3.126.5 Fricas [F(-2)]

Exception generated.

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.126.6 Sympy [F]

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx=\int x^2(ia(\sinh(e+fx)-i))^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(a+I*a*sinh(f*x+e))**(3/2),x)
```

output
```
Integral(x**2*(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

3.126.7 Maxima [F]

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx=\int (i\,a\sinh(fx+e)+a)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^2, x)
```

3.126.8 Giac [F]

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx=\int (i\,a\sinh(fx+e)+a)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^2, x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx=\int x^2\,(a+a\sinh(e+f\,x)\;1\mathrm{i})^{3/2}\,dx$$

input
```
int(x^2*(a + a*sinh(e + f*x)*1i)^(3/2),x)
```

output
```
int(x^2*(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

3.127 $\int x(a+ia\sinh(e+fx))^{3/2}\,dx$

3.127.1 Optimal result

Integrand size = 19, antiderivative size = 185

$$\int x(a+ia\sinh(e+fx))^{3/2}\,dx=-\frac{16a\sqrt{a+ia\sinh(e+fx)}}{3f^2}-\frac{8a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2}+\frac{4ax\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{3f}+\frac{8ax\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3f}$$

output
```
-16/3*a*(a+I*a*sinh(f*x+e))^(1/2)/f^2-8/9*a*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^2
*(a+I*a*sinh(f*x+e))^(1/2)/f^2+4/3*a*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*sinh(1
/2*e+1/4*I*Pi+1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)/f+8/3*a*x*(a+I*a*sinh(f*x
+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f
```

3.127.2 Mathematica [A] (verified)

Time = 3.41 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.75

$$\int x(a+ia\sinh(e+fx))^{3/2}\,dx =$$
$$-\frac{a\left(27(2i+fx)\cosh\left(\frac{1}{2}(e+fx)\right)+(-2i+3fx)\cosh\left(\frac{3}{2}(e+fx)\right)+2i(28i-12fx+(2i+3fx)\cosh(e+\right.}{9f^2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right.}$$

input
```
Integrate[x*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
-1/9*(a*(27*(2*I + f*x)*Cosh[(e + f*x)/2] + (-2*I + 3*f*x)*Cosh[(3*(e + f*
x))/2] + (2*I)*(28*I - 12*f*x + (2*I + 3*f*x)*Cosh[e + f*x])*Sinh[(e + f*x
)/2])*(-I + Sinh[e + f*x])*Sqrt[a + I*a*Sinh[e + f*x]])/(f^2*(Cosh[(e + f*
x)/2] + I*Sinh[(e + f*x)/2])^3)
```

3.127.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 3800, 3042, 3791, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+ia\sinh(e+fx))^{3/2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int x(a+a\sin(ie+ifx))^{3/2}dx$$
$$\Big\downarrow \; 3800$$
$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$
$$\Big\downarrow \; 3042$$
$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx$$
$$\Big\downarrow \; 3791$$

$$2a\,\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\int x\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{9f^2}+\frac{2x\sin\cdots}{\cdots}\right.$$

$$\Big\downarrow \; 3042$$

$$2a\,\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\int x\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{9f^2}+\frac{2x\sinh\cdots}{\cdots}\right.$$

$$\Big\downarrow \; 3777$$

$$2a\,\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2i\int-i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)-\frac{4\cdots}{\cdots}\right.$$

$$\Big\downarrow \; 26$$

$$2a\,\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2\int\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f}\right)-\frac{4\cos\cdots}{\cdots}\right.$$

$$\Big\downarrow \; 3042$$

$$2a\,\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{2\int-i\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}\right)-\frac{4c\cdots}{\cdots}\right.$$

$$\Big\downarrow \; 26$$

$$2a\,\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{2}{3}\left(\frac{2i\int\sin\left(\frac{ie}{2}+\frac{ifx}{2}-\frac{\pi}{4}\right)dx}{f}+\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)-\frac{4\cos\cdots}{\cdots}\right.$$

$$\Big\downarrow \; 3118$$

$$2a\,\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(-\frac{4\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{9f^2}+\frac{2}{3}\left(\frac{2x\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}-\frac{4\cosh\left(\frac{e}{2}\cdots\right.}{\cdots}\right.\right.$$

input
```
Int[x*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

3.127. $\int x(a+ia\sinh(e+fx))^{3/2}\,dx$

output
```
2*a*Sech[e/2 + (I/4)*Pi + (f*x)/2]*((-4*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^3)/
(9*f^2) + (2*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sinh[e/2 + (I/4)*Pi + (f*x
)/2])/(3*f) + (2*((-4*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/f^2 + (2*x*Sinh[e/2
+ (I/4)*Pi + (f*x)/2])/f))/3)*Sqrt[a + I*a*Sinh[e + f*x]]
```

3.127.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.127.4 Maple [F]

$$\int x(a + ia \sinh (fx + e))^{\frac{3}{2}} dx$$

input
```
int(x*(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
int(x*(a+I*a*sinh(f*x+e))^(3/2),x)
```

3.127.5 Fricas [F(-2)]

Exception generated.

$$\int x(a + ia \sinh(e + fx))^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.127.6 Sympy [F]

$$\int x(a + ia \sinh(e + fx))^{3/2} dx = \int x(ia(\sinh (e + fx) - i))^{\frac{3}{2}} dx$$

input
```
integrate(x*(a+I*a*sinh(f*x+e))**(3/2),x)
```

output
```
Integral(x*(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

3.127.7 Maxima [F]

$$\int x(a+ia\sinh(e+fx))^{3/2}\,dx = \int (i\,a\sinh(fx+e)+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)*x, x)
```

3.127.8 Giac [F]

$$\int x(a+ia\sinh(e+fx))^{3/2}\,dx = \int (i\,a\sinh(fx+e)+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)*x, x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int x(a+ia\sinh(e+fx))^{3/2}\,dx = \int x\,(a+a\sinh(e+f\,x)\ 1\mathrm{i})^{3/2}\,dx$$

input
```
int(x*(a + a*sinh(e + f*x)*1i)^(3/2),x)
```

output
```
int(x*(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

3.128 $\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx$

3.128.1 Optimal result

Integrand size = 21, antiderivative size = 261

$$\begin{aligned}\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx &= \frac{3}{2}ia\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}\right.\\
&+\left.\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e-i\pi)\right)\sqrt{a+ia\sinh(e+fx)}\\
&+\frac{1}{2}ia\text{Chi}\left(\frac{3fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(6e+i\pi)\right)\sqrt{a+ia\sinh(e+fx)}\\
&+\frac{3}{2}ia\cosh\left(\frac{1}{4}(2e-i\pi)\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\text{Shi}\left(\frac{fx}{2}\right)\\
&+\frac{1}{2}ia\cosh\left(\frac{1}{4}(6e+i\pi)\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\text{Shi}\left(\frac{3fx}{2}\right)\end{aligned}$$

output
```
3/2*a*sinh(1/2*e+1/4*I*Pi)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*Shi(1/2*f*x)*(a+I*
a*sinh(f*x+e))^(1/2)+1/2*I*a*cosh(3/2*e+1/4*I*Pi)*sech(1/2*e+1/4*I*Pi+1/2*
f*x)*Shi(3/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)+3/2*a*Chi(1/2*f*x)*sech(1/2*e+
1/4*I*Pi+1/2*f*x)*cosh(1/2*e+1/4*I*Pi)*(a+I*a*sinh(f*x+e))^(1/2)+1/2*I*a*C
hi(3/2*f*x)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*sinh(3/2*e+1/4*I*Pi)*(a+I*a*sinh(
f*x+e))^(1/2)
```

3.128.2 Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.56

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx = \frac{a\sqrt{a+ia\sinh(e+fx)}\left(3\text{Chi}\left(\frac{fx}{2}\right)\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)-\text{Chi}\left(\frac{3fx}{2}\right)\left(\cosh\right.\right.}{2\left(\cosh\left(\frac{1}{2}(e+\right.\right.}$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])^(3/2)/x,x]
```

output
```
(a*Sqrt[a + I*a*Sinh[e + f*x]]*(3*CoshIntegral[(f*x)/2]*(Cosh[e/2] + I*Sin
h[e/2]) - CoshIntegral[(3*f*x)/2]*(Cosh[(3*e)/2] - I*Sinh[(3*e)/2]) + (I*C
osh[e/2] + Sinh[e/2])*(3*SinhIntegral[(f*x)/2] + (1 + (2*I)*Sinh[e])*SinhI
ntegral[(3*f*x)/2])))/(2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

3.128.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.57, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3800, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+a\sin(ie+ifx))^{3/2}}{x}dx$$

$$\Big\downarrow \text{ 3800}$$

$$2a\,\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x}dx$$

$$\Big\downarrow \text{ 3042}$$

$$2a\,\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3}{x}dx$$

$$\Big\downarrow \text{ 3793}$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int\left(\frac{3i\sinh\left(\frac{1}{4}(2e-i\pi)+\frac{fx}{2}\right)}{4x}+\frac{i\sinh\left(\frac{1}{4}(6e+i\pi)+\frac{3fx}{2}\right)}{4x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{3}{4}i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\operatorname{Chi}\left(\frac{fx}{2}\right)+\frac{1}{4}i\sinh\left(\frac{1}{4}(6e+i\pi)\right)\operatorname{Chi}\left(\frac{3f}{2}\right.\right.$$

input
```
Int[(a + I*a*Sinh[e + f*x])^(3/2)/x,x]
```

output
```
2*a*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*(((3*I)/4)*
CoshIntegral[(f*x)/2]*Sinh[(2*e - I*Pi)/4] + (I/4)*CoshIntegral[(3*f*x)/2]
*Sinh[(6*e + I*Pi)/4] + ((3*I)/4)*Cosh[(2*e - I*Pi)/4]*SinhIntegral[(f*x)/
2] + (I/4)*Cosh[(6*e + I*Pi)/4]*SinhIntegral[(3*f*x)/2])
```

3.128.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.128. $\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx$

3.128.4 Maple [F]

$$\int \frac{(a + ia\sinh(fx + e))^{\frac{3}{2}}}{x} dx$$

input
```
int((a+I*a*sinh(f*x+e))^(3/2)/x,x)
```

output
```
int((a+I*a*sinh(f*x+e))^(3/2)/x,x)
```

3.128.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + ia\sinh(e + fx))^{3/2}}{x} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(f*x+e))^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.128.6 Sympy [F]

$$\int \frac{(a + ia\sinh(e + fx))^{3/2}}{x} dx = \int \frac{(ia(\sinh(e + fx) - i))^{\frac{3}{2}}}{x} dx$$

input
```
integrate((a+I*a*sinh(f*x+e))**(3/2)/x,x)
```

output
```
Integral((I*a*(sinh(e + f*x) - I))**(3/2)/x, x)
```

3.128.7 Maxima [F]

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx = \int \frac{(i\,a\sinh\left(fx+e\right)+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)/x, x)
```

3.128.8 Giac [F]

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx = \int \frac{(i\,a\sinh\left(fx+e\right)+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)/x, x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx = \int \frac{(a+a\sinh(e+f\,x)\,\,1\mathrm{i})^{3/2}}{x}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^(3/2)/x,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^(3/2)/x, x)
```

3.129 $\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx$

3.129.1 Optimal result

Integrand size = 21, antiderivative size = 302

$$\begin{aligned}\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx &= -\frac{2a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{x}\\
&- \frac{3}{4}af\operatorname{Chi}\left(\frac{3fx}{2}\right)\operatorname{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(6e-i\pi)\right)\sqrt{a+ia\sinh(e+fx)}\\
&+ \frac{3}{4}af\operatorname{Chi}\left(\frac{fx}{2}\right)\operatorname{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e+i\pi)\right)\sqrt{a+ia\sinh(e+fx)}\\
&+ \frac{3}{4}af\cosh\left(\frac{1}{4}(2e+i\pi)\right)\operatorname{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\operatorname{Shi}\left(\frac{fx}{2}\right)\\
&- \frac{3}{4}af\cosh\left(\frac{1}{4}(6e-i\pi)\right)\operatorname{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\operatorname{Shi}\left(\frac{3fx}{2}\right)\end{aligned}$$

output
```
-2*a*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^2*(a+I*a*sinh(f*x+e))^(1/2)/x+3/4*a*f*co
sh(1/2*e+1/4*I*Pi)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*Shi(1/2*f*x)*(a+I*a*sinh(f
*x+e))^(1/2)+3/4*I*a*f*sinh(3/2*e+1/4*I*Pi)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*S
hi(3/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)+3/4*I*a*f*Chi(3/2*f*x)*sech(1/2*e+1/
4*I*Pi+1/2*f*x)*cosh(3/2*e+1/4*I*Pi)*(a+I*a*sinh(f*x+e))^(1/2)+3/4*a*f*Chi
(1/2*f*x)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*sinh(1/2*e+1/4*I*Pi)*(a+I*a*sinh(f*
x+e))^(1/2)
```

3.129.2 Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.80

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx = \frac{a(-i+\sinh(e+fx))\sqrt{a+ia\sinh(e+fx)}\left(-6i\cosh\left(\frac{1}{2}(e+fx)\right)+2i\cos\right.}{}$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])^(3/2)/x^2,x]
```

output
```
(a*(-I + Sinh[e + f*x])*Sqrt[a + I*a*Sinh[e + f*x]]*((-6*I)*Cosh[(e + f*x)
/2] + (2*I)*Cosh[(3*(e + f*x))/2] - 3*f*x*CoshIntegral[(f*x)/2]*(Cosh[e/2]
 - I*Sinh[e/2]) - 3*f*x*CoshIntegral[(3*f*x)/2]*(Cosh[(3*e)/2] + I*Sinh[(3
*e)/2]) + 6*Sinh[(e + f*x)/2] + 2*Sinh[(3*(e + f*x))/2] + (3*I)*f*x*Cosh[e
/2]*SinhIntegral[(f*x)/2] - 3*f*x*Sinh[e/2]*SinhIntegral[(f*x)/2] - (3*I)*
f*x*Cosh[(3*e)/2]*SinhIntegral[(3*f*x)/2] - 3*f*x*Sinh[(3*e)/2]*SinhIntegr
al[(3*f*x)/2]))/(4*x*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^3)
```

3.129.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.61, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3800, 3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+a\sin(ie+ifx))^{3/2}}{x^2}dx$$

$$\Big\downarrow \; 3800$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{x^2}dx$$

$$\Big\downarrow \; 3042$$

$$2a\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3}{x^2}dx$$

$$\Big\downarrow \text{3794}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{3}{2}if\int\left(\frac{\cosh\left(\frac{3e}{2}+\frac{3fx}{2}+\frac{i\pi}{4}\right)}{4x}-\frac{i\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{4x}\right)dx-\frac{\cos}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$2a\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}\left(\frac{3}{2}if\left(\frac{1}{4}i\sinh\left(\frac{1}{4}(6e-i\pi)\right)\text{Chi}\left(\frac{3fx}{2}\right)-\frac{1}{4}i\sinh\left(\frac{1}{4}(2e+i\pi)\right)\text{C}\right.\right.$$

input
```
Int[(a + I*a*Sinh[e + f*x])^(3/2)/x^2,x]
```

output
```
2*a*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*(-(Cosh[e/2
 + (I/4)*Pi + (f*x)/2]^3/x) + ((3*I)/2)*f*((I/4)*CoshIntegral[(3*f*x)/2]*S
inh[(6*e - I*Pi)/4] - (I/4)*CoshIntegral[(f*x)/2]*Sinh[(2*e + I*Pi)/4] - (
I/4)*Cosh[(2*e + I*Pi)/4]*SinhIntegral[(f*x)/2] + (I/4)*Cosh[(6*e - I*Pi)/
4]*SinhIntegral[(3*f*x)/2]))
```

3.129.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.129.4 Maple [F]

$$\int \frac{(a + ia \sinh (fx + e))^{\frac{3}{2}}}{x^2} dx$$

input
```
int((a+I*a*sinh(f*x+e))^(3/2)/x^2,x)
```

output
```
int((a+I*a*sinh(f*x+e))^(3/2)/x^2,x)
```

3.129.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + ia \sinh(e + fx))^{3/2}}{x^2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(f*x+e))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.129.6 Sympy [F]

$$\int \frac{(a + ia \sinh(e + fx))^{3/2}}{x^2} dx = \int \frac{(ia(\sinh (e + fx) - i))^{\frac{3}{2}}}{x^2} dx$$

input
```
integrate((a+I*a*sinh(f*x+e))**(3/2)/x**2,x)
```

output
```
Integral((I*a*(sinh(e + f*x) - I))**(3/2)/x**2, x)
```

3.129.7 Maxima [F]

$$\int \frac{(a + ia\sinh(e + fx))^{3/2}}{x^2}\,dx = \int \frac{(i\,a\sinh(fx + e) + a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)/x^2, x)
```

3.129.8 Giac [F]

$$\int \frac{(a + ia\sinh(e + fx))^{3/2}}{x^2}\,dx = \int \frac{(i\,a\sinh(fx + e) + a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(3/2)/x^2, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + ia\sinh(e + fx))^{3/2}}{x^2}\,dx = \int \frac{(a + a\sinh(e + f\,x)\ 1\mathrm{i})^{3/2}}{x^2}\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^(3/2)/x^2,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^(3/2)/x^2, x)
```

3.130 $\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx$

3.130.1 Optimal result

Integrand size = 21, antiderivative size = 638

$$\begin{aligned}\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx =& -\frac{265216a^2\sqrt{a+ia\sinh(c+dx)}}{1125d^4}-\frac{128a^2x^2\sqrt{a+ia\sinh(c+dx)}}{5d^2}\\&-\frac{17408a^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{3375d^4}\\&-\frac{64a^2x^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{15d^2}\\&-\frac{384a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{625d^4}\\&-\frac{48a^2x^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}\\&+\frac{8704a^2x\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{1125d^3}\\&+\frac{32a^2x^3\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{15d}\\&+\frac{192a^2x\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{125d^3}\\&+\frac{8a^2x^3\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{5d}\\&+\frac{132608a^2x\sqrt{a+ia\sinh(c+dx)}\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{1125d^3}\\&+\frac{64a^2x^3\sqrt{a+ia\sinh(c+dx)}\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{15d}\end{aligned}$$

output
```
-265216/1125*a^2*(a+I*a*sinh(d*x+c))^(1/2)/d^4-128/5*a^2*x^2*(a+I*a*sinh(d
*x+c))^(1/2)/d^2-17408/3375*a^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^2*(a+I*a*sinh
(d*x+c))^(1/2)/d^4-64/15*a^2*x^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^2*(a+I*a*sin
h(d*x+c))^(1/2)/d^2-384/625*a^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^4*(a+I*a*sinh
(d*x+c))^(1/2)/d^4-48/25*a^2*x^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^4*(a+I*a*sin
h(d*x+c))^(1/2)/d^2+8704/1125*a^2*x*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*sinh(1/2*
c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/d^3+32/15*a^2*x^3*cosh(1/2*c
+1/4*I*Pi+1/2*d*x)*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/
d+192/125*a^2*x*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^3*sinh(1/2*c+1/4*I*Pi+1/2*d*x
)*(a+I*a*sinh(d*x+c))^(1/2)/d^3+8/5*a^2*x^3*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^3
*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/d+132608/1125*a^2*
x*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/d^3+64/15*a^2*x^3
*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/d
```

3.130.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2918 vs. $2(638) = 1276$.

Time = 15.62 (sec) , antiderivative size = 2918, normalized size of antiderivative = 4.57

$$\int x^3(a + ia\sinh(c + dx))^{5/2}\,dx = \text{Result too large to show}$$

input
```
Integrate[x^3*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
(2*(((-1/135000 - I/135000)*Cosh[5*(c/2 + (d*x)/2)])/d^3 + ((1/135000 + I/
135000)*Sinh[5*(c/2 + (d*x)/2)])/d^3)*(1296*I - (3240*I)*c + (4050*I)*c^2
- (3375*I)*c^3 + (6480*I)*(c/2 + (d*x)/2) - (16200*I)*c*(c/2 + (d*x)/2) +
(20250*I)*c^2*(c/2 + (d*x)/2) + (16200*I)*(c/2 + (d*x)/2)^2 - (40500*I)*c*
(c/2 + (d*x)/2)^2 + (27000*I)*(c/2 + (d*x)/2)^3 - 50000*Cosh[2*(c/2 + (d*x
)/2)] + 75000*c*Cosh[2*(c/2 + (d*x)/2)] - 56250*c^2*Cosh[2*(c/2 + (d*x)/2)
] + 28125*c^3*Cosh[2*(c/2 + (d*x)/2)] - 150000*(c/2 + (d*x)/2)*Cosh[2*(c/2
 + (d*x)/2)] + 225000*c*(c/2 + (d*x)/2)*Cosh[2*(c/2 + (d*x)/2)] - 168750*c
^2*(c/2 + (d*x)/2)*Cosh[2*(c/2 + (d*x)/2)] - 225000*(c/2 + (d*x)/2)^2*Cosh
[2*(c/2 + (d*x)/2)] + 337500*c*(c/2 + (d*x)/2)^2*Cosh[2*(c/2 + (d*x)/2)] -
 225000*(c/2 + (d*x)/2)^3*Cosh[2*(c/2 + (d*x)/2)] - (8100000*I)*Cosh[4*(c/
2 + (d*x)/2)] + (4050000*I)*c*Cosh[4*(c/2 + (d*x)/2)] - (1012500*I)*c^2*Co
sh[4*(c/2 + (d*x)/2)] + (168750*I)*c^3*Cosh[4*(c/2 + (d*x)/2)] - (8100000*
I)*(c/2 + (d*x)/2)*Cosh[4*(c/2 + (d*x)/2)] + (4050000*I)*c*(c/2 + (d*x)/2)
*Cosh[4*(c/2 + (d*x)/2)] - (1012500*I)*c^2*(c/2 + (d*x)/2)*Cosh[4*(c/2 + (
d*x)/2)] - (4050000*I)*(c/2 + (d*x)/2)^2*Cosh[4*(c/2 + (d*x)/2)] + (202500
0*I)*c*(c/2 + (d*x)/2)^2*Cosh[4*(c/2 + (d*x)/2)] - (1350000*I)*(c/2 + (d*x
)/2)^3*Cosh[4*(c/2 + (d*x)/2)] + 8100000*Cosh[6*(c/2 + (d*x)/2)] + 4050000
*c*Cosh[6*(c/2 + (d*x)/2)] + 1012500*c^2*Cosh[6*(c/2 + (d*x)/2)] + 168750*
c^3*Cosh[6*(c/2 + (d*x)/2)] - 8100000*(c/2 + (d*x)/2)*Cosh[6*(c/2 + (d*...
```

3.130.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^3(a+a\sin(ic+idx))^{5/2}dx$$

$$\Big\downarrow \text{ 3800}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int x^3\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int x^3\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5dx$$

$$\Big\downarrow \text{ 3792}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\int x\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{25d^2}+\frac{4}{5}\int x^3\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx\right.$$

$\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5dx}{25d^2}+\frac{4}{5}\int x^3\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx-\right.$$

$\downarrow$ 3791

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\left(\frac{4}{5}\int x\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx-\frac{4\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{25d^2}+\frac{2x\sinh\left(\frac{c}{2}+\right.}{}\right.}{25d^2}\right.$$

$\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\left(\frac{4}{5}\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx-\frac{4\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{25d^2}+\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}\right.}{}\right.}{25d^2}\right.$$

$\downarrow$ 3791

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\left(\frac{4}{5}\left(\frac{2}{3}\int x\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{9d^2}+\frac{2x\sinh\left(\frac{c}{2}\right.}{}\right.\right.}{}\right.$$

$\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\left(\frac{4}{5}\left(\frac{2}{3}\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{9d^2}+\frac{2x\sinh\left(\frac{c}{2}+\right.}{}\right.\right.}{}\right.$$

$\downarrow$ 3777

3.130. $\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2i\int -i\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)-\frac{4\cosh^3\left(\frac{c}{2}\right.}{9}\right.\right.}{}\right.$$

$$\Big\downarrow \; 26$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2\int\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)-\frac{4\cosh^3\left(\frac{c}{2}+\frac{d}{2}\right.}{9d^2}\right.\right.}{}\right.$$

$$\Big\downarrow \; 3042$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2\int -i\sin\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)-\frac{4\cosh^3\left(\frac{c}{2}+\right.}{9d^2}\right.\right.}{}\right.$$

$$\Big\downarrow \; 26$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{24\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2i\int\sin\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)-\frac{4\cosh^3\left(\frac{c}{2}+\frac{d}{2}\right.}{9d^2}\right.\right.}{}\right.$$

$$\Big\downarrow \; 3118$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\int x^3\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx-\frac{12x^2\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{25d^2}+\frac{24}{}\right.$$

$$\Big\downarrow \; 3792$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{3d^2}+\frac{2}{3}\int x^3\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)d\right.\right.$$

3.130. $\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx$

$$\Big\downarrow \ 3042$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\int x^3\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx\right.\right.$$

$$\Big\downarrow \ 3777$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6i}{}\right.\right.\right.$$

$$\Big\downarrow \ 26$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6}{}\right.\right.\right.$$

$$\Big\downarrow \ 3042$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6}{}\right.\right.\right.$$

$$\Big\downarrow \ 26$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{6i\int x^2\sin\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right.\right.\right.$$

$$\Big\downarrow \ 3777$$

3.130. $\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4i}{}\right.}{d}\right.\right.\right.$$

$\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4i}{}\right.}{d}\right.\right.\right.$$

$\downarrow$ 3777

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4i}{}\right.}{}\right.\right.\right.$$

$\downarrow$ 26

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4i}{}\right.}{}\right.\right.\right.$$

$\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4i}{}\right.}{}\right.\right.\right.$$

$\downarrow$ 26

3.130. $\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4i\cdots}{}\cdots\right.}{}\cdots\right.\right.\right.$$

$\downarrow$ 3118

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}-\frac{4x^2\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{3d^2}+\frac{2}{3}\left(\cdots\right.\right.\right.$$

$\downarrow$ 3791

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\left(\frac{2}{3}\int x\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{9d^2}+\frac{2x\sinh\left(\frac{c}{2}\cdots\right.}{}\cdots\right.}{3d^2}\cdots\right.\right.$$

$\downarrow$ 3042

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\left(\frac{2}{3}\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{9d^2}+\frac{2x\sinh\left(\frac{c}{2}+\cdots\right.}{}\cdots\right.}{3d^2}\cdots\right.\right.$$

$\downarrow$ 3777

3.130. $\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx$

$$4a^2\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2i\int -i\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)-\frac{4\cosh^3\left(\frac{c}{2}\right.}{9d}\right.}{3d^2}\right.\right.$$

input
```
Int[x^3*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

3.130.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.130.4 Maple [F]

$$\int x^3(a+ia\sinh{(dx+c)})^{\frac{5}{2}}\,dx$$

input
```
int(x^3*(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
int(x^3*(a+I*a*sinh(d*x+c))^(5/2),x)
```

3.130.5 Fricas [F(-2)]

Exception generated.

$$\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.130.6 Sympy [F(-1)]

Timed out.

$$\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+I*a*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.130.7 Maxima [F]

$$\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx = \int (i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}x^3\,dx$$

input
```
integrate(x^3*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^3, x)
```

3.130.8 Giac [F]

$$\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx = \int (i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}x^3\,dx$$

input
```
integrate(x^3*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^3, x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx=\int x^3\,\left(a+a\sinh(c+d\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(x^3*(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

output
```
int(x^3*(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

3.131 $\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx$

3.131.1 Optimal result

Integrand size = 21, antiderivative size = 506

$$\begin{aligned}\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx =& -\frac{256a^2x\sqrt{a+ia\sinh(c+dx)}}{15d^2}\\
&-\frac{128a^2x\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}\\
&-\frac{32a^2x\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}\\
&+\frac{32a^2x^2\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{15d}\\
&+\frac{8a^2x^2\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{5d}\\
&+\frac{9536a^2\sqrt{a+ia\sinh(c+dx)}\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{225d^3}\\
&+\frac{64a^2x^2\sqrt{a+ia\sinh(c+dx)}\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{15d}\\
&+\frac{2432a^2\sinh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{675d^3}\\
&+\frac{64a^2\sinh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{125d^3}\end{aligned}$$

output
```
-256/15*a^2*x*(a+I*a*sinh(d*x+c))^(1/2)/d^2-128/45*a^2*x*cosh(1/2*c+1/4*I*
Pi+1/2*d*x)^2*(a+I*a*sinh(d*x+c))^(1/2)/d^2-32/25*a^2*x*cosh(1/2*c+1/4*I*P
i+1/2*d*x)^4*(a+I*a*sinh(d*x+c))^(1/2)/d^2+32/15*a^2*x^2*cosh(1/2*c+1/4*I*
Pi+1/2*d*x)*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/d+8/5*a
^2*x^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^3*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*
sinh(d*x+c))^(1/2)/d+9536/225*a^2*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4
*I*Pi+1/2*d*x)/d^3+64/15*a^2*x^2*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4*
I*Pi+1/2*d*x)/d+2432/675*a^2*sinh(1/2*c+1/4*I*Pi+1/2*d*x)^2*(a+I*a*sinh(d*
x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/d^3+64/125*a^2*sinh(1/2*c+1/4*I*P
i+1/2*d*x)^4*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/d^3
```

3.131.2 Mathematica [A] (verified)

Time = 9.75 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.59

$$\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx = \frac{a^2\sqrt{a+ia\sinh(c+dx)}\left(33750i(8+4idx+d^2x^2)\cosh\left(\frac{1}{2}(c+dx)\right)+625(8i+12dx+9id^2x^2$$

input
```
Integrate[x^2*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
(a^2*Sqrt[a + I*a*Sinh[c + d*x]]*((33750*I)*(8 + (4*I)*d*x + d^2*x^2)*Cosh
[(c + d*x)/2] + 625*(8*I + 12*d*x + (9*I)*d^2*x^2)*Cosh[(3*(c + d*x))/2] -
 (216*I)*Cosh[(5*(c + d*x))/2] + 540*d*x*Cosh[(5*(c + d*x))/2] - (675*I)*d
^2*x^2*Cosh[(5*(c + d*x))/2] + 270000*Sinh[(c + d*x)/2] - (135000*I)*d*x*S
inh[(c + d*x)/2] + 33750*d^2*x^2*Sinh[(c + d*x)/2] - 5000*Sinh[(3*(c + d*x
))/2] - (7500*I)*d*x*Sinh[(3*(c + d*x))/2] - 5625*d^2*x^2*Sinh[(3*(c + d*x
))/2] - 216*Sinh[(5*(c + d*x))/2] + (540*I)*d*x*Sinh[(5*(c + d*x))/2] - 67
5*d^2*x^2*Sinh[(5*(c + d*x))/2]))/(6750*d^3*(Cosh[(c + d*x)/2] + I*Sinh[(c
 + d*x)/2]))
```

3.131.3 Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.91, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 3800, 3042, 3792, 3042, 3113, 2009, 3792, 3042, 3113, 2009, 3777, 26, 3042, 26, 3777, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2(a+a\sin(ic+idx))^{5/2}dx$$

$$\Big\downarrow \text{3800}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int x^2\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int x^2\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5dx$$

$$\Big\downarrow \text{3792}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{8\int\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{25d^2}+\frac{4}{5}\int x^2\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx-\right.$$

$$\Big\downarrow \text{3042}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{8\int\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5dx}{25d^2}+\frac{4}{5}\int x^2\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx-\frac{8x}{}\right.$$

$$\Big\downarrow \text{3113}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{16i\int\left(\sinh^4\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)+2\sinh^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)+1\right)d\left(-i\sinh\right.}{25d^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\int x^2\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx+\frac{16i\left(-\frac{1}{5}i\sinh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)-\right.}{}\right.$$

$$\Big\downarrow \ 3792$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{9d^2}+\frac{2}{3}\int x^2\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx\right.\right.$$

$$\Big\downarrow \ 3042$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{8\int\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{9d^2}+\frac{2}{3}\int x^2\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx-\right.\right.$$

$$\Big\downarrow \ 3113$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{16i\int\left(\sinh^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)+1\right)d\left(-i\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{9d^3}+\frac{2}{3}\right.\right.$$

$$\Big\downarrow \ 2009$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\int x^2\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx+\frac{16i\left(-\frac{1}{3}i\sinh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right.}{9d^3}\right.\right.$$

$$\Big\downarrow \ 3777$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x^2\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4i\int -ix\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)\right.\right.$$

$$\Big\downarrow \ 26$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x^2\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4\int x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)+\right.\right.$$

$$\Big\downarrow \ 3042$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x^2\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4\int -ix\sin\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)+\right.\right.$$

$$\Big\downarrow \ 26$$

3.131. $\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{4i\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2x^2\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)+\right.\right.\right.$$

$$\Big\downarrow \; 3777$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2i\int\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{d}+\frac{2x^2\sinh\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{4i\left(\frac{2ix\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2i\int\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx}{d}\right)}{d}+\frac{2x^2\sinh(\right.\right.\right.$$

$$\Big\downarrow \; 3118$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{16i\left(-\frac{1}{5}i\sinh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)-\frac{2}{3}i\sinh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)-i\sinh\left(\frac{c}{2}\right.\right.}{25d^3}\right.$$

input
```
Int[x^2*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
4*a^2*Sech[c/2 + (I/4)*Pi + (d*x)/2]*((-8*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]
^5)/(25*d^2) + (2*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sinh[c/2 + (I/4)*Pi
 + (d*x)/2])/(5*d) + (((16*I)/25)*((-I)*Sinh[c/2 + (I/4)*Pi + (d*x)/2] - (
(2*I)/3)*Sinh[c/2 + (I/4)*Pi + (d*x)/2]^3 - (I/5)*Sinh[c/2 + (I/4)*Pi + (d
*x)/2]^5))/d^3 + (4*((-8*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3)/(9*d^2) + (2*
x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^2*Sinh[c/2 + (I/4)*Pi + (d*x)/2])/(3*d)
 + (((16*I)/9)*((-I)*Sinh[c/2 + (I/4)*Pi + (d*x)/2] - (I/3)*Sinh[c/2 + (I/
4)*Pi + (d*x)/2]^3))/d^3 + (2*((2*x^2*Sinh[c/2 + (I/4)*Pi + (d*x)/2])/d +
((4*I)*(((2*I)*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/d - ((4*I)*Sinh[c/2 + (I/
4)*Pi + (d*x)/2])/d^2))/d))/3))/5)*Sqrt[a + I*a*Sinh[c + d*x]]
```

3.131. $\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx$

3.131.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.131. $\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx$

3.131.4 Maple [F]

$$\int x^2 (a + ia \sinh(dx + c))^{\frac{5}{2}} dx$$

input
```
int(x^2*(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
int(x^2*(a+I*a*sinh(d*x+c))^(5/2),x)
```

3.131.5 Fricas [F(-2)]

Exception generated.

$$\int x^2 (a + ia \sinh(c + dx))^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.131.6 Sympy [F(-1)]

Timed out.

$$\int x^2 (a + ia \sinh(c + dx))^{5/2} dx = \text{Timed out}$$

input
```
integrate(x**2*(a+I*a*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.131.7 Maxima [F]

$$\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx=\int(i\,a\sinh(dx+c)+a)^{\frac{5}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^2, x)
```

3.131.8 Giac [F]

$$\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx=\int(i\,a\sinh(dx+c)+a)^{\frac{5}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^2, x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx=\int x^2\,(a+a\sinh(c+d\,x)\;1\mathrm{i})^{5/2}\,dx$$

input
```
int(x^2*(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

output
```
int(x^2*(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

3.132 $\int x(a+ia\sinh(c+dx))^{5/2}\,dx$

3.132.1 Optimal result

Integrand size = 19, antiderivative size = 312

$$\begin{aligned}\int x(a+ia\sinh(c+dx))^{5/2}\,dx &= -\frac{128a^2\sqrt{a+ia\sinh(c+dx)}}{15d^2}\\ &-\frac{64a^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}\\ &-\frac{16a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}\\ &+\frac{32a^2x\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{15d}\\ &+\frac{8a^2x\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{5d}\\ &+\frac{64a^2x\sqrt{a+ia\sinh(c+dx)}\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{15d}\end{aligned}$$

output
```
-128/15*a^2*(a+I*a*sinh(d*x+c))^(1/2)/d^2-64/45*a^2*cosh(1/2*c+1/4*I*Pi+1/
2*d*x)^2*(a+I*a*sinh(d*x+c))^(1/2)/d^2-16/25*a^2*cosh(1/2*c+1/4*I*Pi+1/2*d
*x)^4*(a+I*a*sinh(d*x+c))^(1/2)/d^2+32/15*a^2*x*cosh(1/2*c+1/4*I*Pi+1/2*d*
x)*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/d+8/5*a^2*x*cosh
(1/2*c+1/4*I*Pi+1/2*d*x)^3*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c)
)^(1/2)/d+64/15*a^2*x*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*
x)/d
```

3.132.2 Mathematica [A] (verified)

Time = 9.43 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.70

$$\int x(a+ia\sinh(c+dx))^{5/2}\,dx = \frac{a^2(-i+\sinh(c+dx))^2\sqrt{a+ia\sinh(c+dx)}\left(2250(2-idx)\cosh\left(\frac{1}{2}(c+dx)\right)+(-250-375
$$

input
```
Integrate[x*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
(a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*(2250*(2 - I*d*x)*
Cosh[(c + d*x)/2] + (-250 - (375*I)*d*x)*Cosh[(3*(c + d*x))/2] - 18*Cosh[(
5*(c + d*x))/2] + (45*I)*d*x*Cosh[(5*(c + d*x))/2] + (4500*I)*Sinh[(c + d*
x)/2] - 2250*d*x*Sinh[(c + d*x)/2] + (250*I)*Sinh[(3*(c + d*x))/2] + 375*d
*x*Sinh[(3*(c + d*x))/2] - (18*I)*Sinh[(5*(c + d*x))/2] + 45*d*x*Sinh[(5*(
c + d*x))/2]))/(450*d^2*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^5)
```

3.132.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.84, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3042, 3800, 3042, 3791, 3042, 3791, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+ia\sinh(c+dx))^{5/2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int x(a+a\sin(ic+idx))^{5/2}dx$$
$$\Big\downarrow \text{3800}$$
$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int x\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx$$
$$\Big\downarrow \text{3042}$$
$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5dx$$

$$\Big\downarrow \ 3791$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\int x\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx-\frac{4\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{25d^2}+\frac{2x\sin\ldots}{}\right.$$

$$\Big\downarrow \ 3042$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx-\frac{4\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{25d^2}+\frac{2x\sinh\ldots}{}\right.$$

$$\Big\downarrow \ 3791$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\int x\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{9d^2}+\frac{2x\ldots}{}\right.\right.$$

$$\Big\downarrow \ 3042$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx-\frac{4\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{9d^2}+\frac{2x\,s\ldots}{}\right.\right.$$

$$\Big\downarrow \ 3777$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2i\int -i\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)-\ldots\right.\right.$$

$$\Big\downarrow \ 26$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2\int\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)-\frac{4\,c\ldots}{}\right.\right.$$

$$\Big\downarrow \ 3042$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2\int -i\sin\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)-\frac{4\ldots}{}\right.\right.$$

$$\Big\downarrow \ 26$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\left(\frac{2i\int\sin\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)-\frac{4\,c\ldots}{}\right.\right.$$

$$\Big\downarrow \ 3118$$

3.132. $\int x(a+ia\sinh(c+dx))^{5/2}\,dx$

$$4a^2\mathrm{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(-\frac{4\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{25d^2}+\frac{4}{5}\left(-\frac{4\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{9d^2}+\frac{2}{3}\left(\frac{2x\sin\right.\right.\right.$$

input
```
Int[x*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
4*a^2*Sech[c/2 + (I/4)*Pi + (d*x)/2]*((-4*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^5
)/(25*d^2) + (2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sinh[c/2 + (I/4)*Pi + (
d*x)/2])/(5*d) + (4*((-4*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3)/(9*d^2) + (2*x*
Cosh[c/2 + (I/4)*Pi + (d*x)/2]^2*Sinh[c/2 + (I/4)*Pi + (d*x)/2])/(3*d) + (
2*((-4*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/d^2 + (2*x*Sinh[c/2 + (I/4)*Pi + (d
*x)/2])/d))/3))/5)*Sqrt[a + I*a*Sinh[c + d*x]]
```

3.132.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.132.4 Maple [F]

$$\int x(a + ia\sinh{(dx + c)})^{\frac{5}{2}}\,dx$$

input
```
int(x*(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
int(x*(a+I*a*sinh(d*x+c))^(5/2),x)
```

3.132.5 Fricas [F(-2)]

Exception generated.

$$\int x(a + ia\sinh(c + dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int x(a + ia\sinh(c + dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+I*a*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.132.7 Maxima [F]

$$\int x(a+ia\sinh(c+dx))^{5/2}\,dx=\int (i\,a\sinh{(dx+c)}+a)^{\frac{5}{2}}x\,dx$$

input
```
integrate(x*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)*x, x)
```

3.132.8 Giac [F]

$$\int x(a+ia\sinh(c+dx))^{5/2}\,dx=\int (i\,a\sinh{(dx+c)}+a)^{\frac{5}{2}}x\,dx$$

input
```
integrate(x*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)*x, x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int x(a+ia\sinh(c+dx))^{5/2}\,dx=\int x\,(a+a\sinh(c+d\,x)\;1\mathrm{i})^{5/2}\,dx$$

input
```
int(x*(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

output
```
int(x*(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

3.133 $\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx$

3.133.1 Optimal result

Integrand size = 21, antiderivative size = 403

$$\begin{aligned}
&\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx = \\
&-\frac{1}{4}ia^2\mathrm{Chi}\left(\frac{5dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)} \\
&+\frac{5}{2}ia^2\mathrm{Chi}\left(\frac{dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{1}{4}(2c-i\pi)\right)\sqrt{a+ia\sinh(c+dx)} \\
&+\frac{5}{4}ia^2\mathrm{Chi}\left(\frac{3dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{1}{4}(6c+i\pi)\right)\sqrt{a+ia\sinh(c+dx)} \\
&+\frac{5}{2}ia^2\cosh\left(\frac{1}{4}(2c-i\pi)\right)\mathrm{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\,\mathrm{Shi}\left(\frac{dx}{2}\right) \\
&+\frac{5}{4}ia^2\cosh\left(\frac{1}{4}(6c+i\pi)\right)\mathrm{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\,\mathrm{Shi}\left(\frac{3dx}{2}\right) \\
&-\frac{1}{4}ia^2\cosh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\,\mathrm{Shi}\left(\frac{5dx}{2}\right)
\end{aligned}$$

output
```
5/2*a^2*sinh(1/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*Shi(1/2*d*x)*(a+
I*a*sinh(d*x+c))^(1/2)+5/4*I*a^2*cosh(3/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+
1/2*d*x)*Shi(3/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)-1/4*a^2*sinh(5/2*c+1/4*I*P
i)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*Shi(5/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)-1/4
*a^2*Chi(5/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*cosh(5/2*c+1/4*I*Pi)*(a+I*a
*sinh(d*x+c))^(1/2)+5/2*a^2*Chi(1/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*cosh
(1/2*c+1/4*I*Pi)*(a+I*a*sinh(d*x+c))^(1/2)+5/4*I*a^2*Chi(3/2*d*x)*sech(1/2
*c+1/4*I*Pi+1/2*d*x)*sinh(3/2*c+1/4*I*Pi)*(a+I*a*sinh(d*x+c))^(1/2)
```

3.133.2 Mathematica [A] (verified)

Time = 4.70 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.60

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx = \frac{a^2(-i+\sinh(c+dx))^2\sqrt{a+ia\sinh(c+dx)}\left(\cosh\left(\frac{5c}{2}\right)\text{Chi}\left(\frac{5dx}{2}\right)-10\text{Chi}(\right.}{}$$

input
```
Integrate[(a + I*a*Sinh[c + d*x])^(5/2)/x,x]
```

output
```
(a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*(Cosh[(5*c)/2]*Cos
hIntegral[(5*d*x)/2] - 10*CoshIntegral[(d*x)/2]*(Cosh[c/2] + I*Sinh[c/2])
+ 5*CoshIntegral[(3*d*x)/2]*(Cosh[(3*c)/2] - I*Sinh[(3*c)/2]) + I*CoshInte
gral[(5*d*x)/2]*Sinh[(5*c)/2] - (10*I)*Cosh[c/2]*SinhIntegral[(d*x)/2] - 1
0*Sinh[c/2]*SinhIntegral[(d*x)/2] - (5*I)*Cosh[(3*c)/2]*SinhIntegral[(3*d*
x)/2] + 5*Sinh[(3*c)/2]*SinhIntegral[(3*d*x)/2] + I*Cosh[(5*c)/2]*SinhInte
gral[(5*d*x)/2] + Sinh[(5*c)/2]*SinhIntegral[(5*d*x)/2]))/(4*(Cosh[(c + d*
x)/2] + I*Sinh[(c + d*x)/2])^5)
```

3.133.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3800, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+a\sin(ic+idx))^{5/2}}{x}dx$$

$$\Big\downarrow \text{3800}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int \frac{\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int\frac{\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5}{x}dx$$

$$\Big\downarrow \text{3793}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int\left(\frac{5i\sinh\left(\frac{1}{4}(2c-i\pi)+\frac{dx}{2}\right)}{8x}+\frac{5i\sinh\left(\frac{1}{4}(6c+i\pi)+\frac{3dx}{2}\right)}{16x}-\frac{i\sinh\right.$$

$$\Big\downarrow \text{2009}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(-\frac{1}{16}i\sinh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\text{Chi}\left(\frac{5dx}{2}\right)+\frac{5}{8}i\sinh\left(\frac{1}{4}(2c-i\pi)\right)\text{Chi}\left(\frac{d}{2}\right.\right.$$

input
```
Int[(a + I*a*Sinh[c + d*x])^(5/2)/x,x]
```

output
```
4*a^2*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*((-1/16*I
)*CoshIntegral[(5*d*x)/2]*Sinh[(5*c)/2 - (I/4)*Pi] + ((5*I)/8)*CoshIntegra
l[(d*x)/2]*Sinh[(2*c - I*Pi)/4] + ((5*I)/16)*CoshIntegral[(3*d*x)/2]*Sinh[
(6*c + I*Pi)/4] + ((5*I)/8)*Cosh[(2*c - I*Pi)/4]*SinhIntegral[(d*x)/2] + (
(5*I)/16)*Cosh[(6*c + I*Pi)/4]*SinhIntegral[(3*d*x)/2] - (I/16)*Cosh[(5*c)
/2 - (I/4)*Pi]*SinhIntegral[(5*d*x)/2])
```

3.133.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.133.4 Maple [F]

$$\int \frac{(a + ia\sinh(dx + c))^{\frac{5}{2}}}{x} dx$$

input
```
int((a+I*a*sinh(d*x+c))^(5/2)/x,x)
```

output
```
int((a+I*a*sinh(d*x+c))^(5/2)/x,x)
```

3.133.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + ia\sinh(c + dx))^{5/2}}{x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.133.6 Sympy [F]

$$\int \frac{(a + ia\sinh(c + dx))^{5/2}}{x} \, dx = \int \frac{(ia(\sinh(c + dx) - i))^{\frac{5}{2}}}{x} \, dx$$

input
```
integrate((a+I*a*sinh(d*x+c))**(5/2)/x,x)
```

output
```
Integral((I*a*(sinh(c + d*x) - I))**(5/2)/x, x)
```

3.133.7 Maxima [F]

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx = \int \frac{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)/x, x)
```

3.133.8 Giac [F]

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx = \int \frac{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x,x, algorithm="giac")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)/x, x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx = \int \frac{(a+a\sinh(c+d\,x)\ 1\mathrm{i})^{5/2}}{x}\,dx$$

input
```
int((a + a*sinh(c + d*x)*1i)^(5/2)/x,x)
```

output
```
int((a + a*sinh(c + d*x)*1i)^(5/2)/x, x)
```

3.134 $\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx$

3.134.1 Optimal result

Integrand size = 21, antiderivative size = 444

$$\begin{aligned}
\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx &= -\frac{4a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x} \\
&\quad -\frac{5}{8}a^2d\,\mathrm{Chi}\left(\frac{5dx}{2}\right)\operatorname{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{5c}{2}\right. \\
&\quad \left.+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}-\frac{15}{8}a^2d\,\mathrm{Chi}\left(\frac{3dx}{2}\right)\operatorname{sech}\left(\frac{c}{2}\right. \\
&\quad \left.+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{1}{4}(6c-i\pi)\right)\sqrt{a+ia\sinh(c+dx)} \\
&\quad +\frac{5}{4}a^2d\,\mathrm{Chi}\left(\frac{dx}{2}\right)\operatorname{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{1}{4}(2c+i\pi)\right)\sqrt{a+ia\sinh(c+dx)} \\
&\quad +\frac{5}{4}a^2d\cosh\left(\frac{1}{4}(2c+i\pi)\right)\operatorname{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\,\mathrm{Shi}\left(\frac{dx}{2}\right) \\
&\quad -\frac{15}{8}a^2d\cosh\left(\frac{1}{4}(6c-i\pi)\right)\operatorname{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\,\mathrm{Shi}\left(\frac{3dx}{2}\right) \\
&\quad -\frac{5}{8}a^2d\cosh\left(\frac{5c}{2}+\frac{i\pi}{4}\right)\operatorname{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\,\mathrm{Shi}\left(\frac{5dx}{2}\right)
\end{aligned}$$

output
```
-4*a^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^4*(a+I*a*sinh(d*x+c))^(1/2)/x+5/4*a^2*
d*cosh(1/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*Shi(1/2*d*x)*(a+I*a*si
nh(d*x+c))^(1/2)+15/8*I*a^2*d*sinh(3/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+1/2
*d*x)*Shi(3/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)-5/8*a^2*d*cosh(5/2*c+1/4*I*Pi
)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*Shi(5/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)-5/8*
a^2*d*Chi(5/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*sinh(5/2*c+1/4*I*Pi)*(a+I*
a*sinh(d*x+c))^(1/2)+15/8*I*a^2*d*Chi(3/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x
)*cosh(3/2*c+1/4*I*Pi)*(a+I*a*sinh(d*x+c))^(1/2)+5/4*a^2*d*Chi(1/2*d*x)*se
ch(1/2*c+1/4*I*Pi+1/2*d*x)*sinh(1/2*c+1/4*I*Pi)*(a+I*a*sinh(d*x+c))^(1/2)
```

3.134.2 Mathematica [A] (verified)

Time = 5.36 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.78

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx = \frac{a^2(-i+\sinh(c+dx))^2\sqrt{a+ia\sinh(c+dx)}\left(20\cosh\left(\frac{1}{2}(c+dx)\right)-10\cosh\right.}{}$$

input
```
Integrate[(a + I*a*Sinh[c + d*x])^(5/2)/x^2,x]
```

output
```
(a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*(20*Cosh[(c + d*x)
/2] - 10*Cosh[(3*(c + d*x))/2] - 2*Cosh[(5*(c + d*x))/2] + (5*I)*d*x*Cosh[
(5*c)/2]*CoshIntegral[(5*d*x)/2] - (10*I)*d*x*CoshIntegral[(d*x)/2]*(Cosh[
c/2] - I*Sinh[c/2]) + 15*d*x*CoshIntegral[(3*d*x)/2]*((-I)*Cosh[(3*c)/2] +
 Sinh[(3*c)/2]) + 5*d*x*CoshIntegral[(5*d*x)/2]*Sinh[(5*c)/2] + (20*I)*Sin
h[(c + d*x)/2] + (10*I)*Sinh[(3*(c + d*x))/2] - (2*I)*Sinh[(5*(c + d*x))/2
] - 10*d*x*Cosh[c/2]*SinhIntegral[(d*x)/2] - (10*I)*d*x*Sinh[c/2]*SinhInte
gral[(d*x)/2] + 15*d*x*Cosh[(3*c)/2]*SinhIntegral[(3*d*x)/2] - (15*I)*d*x*
Sinh[(3*c)/2]*SinhIntegral[(3*d*x)/2] + 5*d*x*Cosh[(5*c)/2]*SinhIntegral[(
5*d*x)/2] + (5*I)*d*x*Sinh[(5*c)/2]*SinhIntegral[(5*d*x)/2]))/(8*x*(Cosh[(
c + d*x)/2] + I*Sinh[(c + d*x)/2])^5)
```

3.134.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.54, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3800, 3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+a\sin(ic+idx))^{5/2}}{x^2}dx$$

$$\Big\downarrow \text{3800}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int \frac{\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int \frac{\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5}{x^2}dx$$

$$\Big\downarrow \text{3794}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{5}{2}id\int \left(\frac{3\cosh\left(\frac{3c}{2}+\frac{3dx}{2}+\frac{i\pi}{4}\right)}{16x}-\frac{i\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{8x}+\frac{i\sinh\left(\frac{5}{\,}\right.}{\,}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{5}{2}id\left(\frac{1}{16}i\sinh\left(\frac{5c}{2}+\frac{i\pi}{4}\right)\text{Chi}\left(\frac{5dx}{2}\right)+\frac{3}{16}i\sinh\left(\frac{1}{4}(6c-i\pi)\right)\text{C}\right.\right.$$

input
```
Int[(a + I*a*Sinh[c + d*x])^(5/2)/x^2,x]
```

output
```
4*a^2*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*(-(Cosh[c
/2 + (I/4)*Pi + (d*x)/2]^5/x) + ((5*I)/2)*d*((I/16)*CoshIntegral[(5*d*x)/2
]*Sinh[(5*c)/2 + (I/4)*Pi] + ((3*I)/16)*CoshIntegral[(3*d*x)/2]*Sinh[(6*c
- I*Pi)/4] - (I/8)*CoshIntegral[(d*x)/2]*Sinh[(2*c + I*Pi)/4] - (I/8)*Cosh
[(2*c + I*Pi)/4]*SinhIntegral[(d*x)/2] + ((3*I)/16)*Cosh[(6*c - I*Pi)/4]*S
inhIntegral[(3*d*x)/2] + (I/16)*Cosh[(5*c)/2 + (I/4)*Pi]*SinhIntegral[(5*d
*x)/2]))
```

3.134.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.134.4 Maple [F]

$$\int \frac{(a+ia\sinh(dx+c))^{\frac{5}{2}}}{x^2}dx$$

input
```
int((a+I*a*sinh(d*x+c))^(5/2)/x^2,x)
```

output
```
int((a+I*a*sinh(d*x+c))^(5/2)/x^2,x)
```

3.134.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.134.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*sinh(d*x+c))**(5/2)/x**2,x)
```

output
```
Timed out
```

3.134.7 Maxima [F]

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx = \int \frac{(i\,a\sinh(dx+c)+a)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^2, x)
```

3.134.8 Giac [F]

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx = \int \frac{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x^2,x, algorithm="giac")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^2, x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx = \int \frac{\left(a+a\sinh(c+d\,x)\ 1\mathrm{i}\right)^{5/2}}{x^2}\,dx$$

input
```
int((a + a*sinh(c + d*x)*1i)^(5/2)/x^2,x)
```

output
```
int((a + a*sinh(c + d*x)*1i)^(5/2)/x^2, x)
```

3.135 $\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx$

3.135.1 Optimal result

Integrand size = 21, antiderivative size = 536

$$
\begin{aligned}
\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx =
&-\frac{2a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x^2} \\
&-\frac{25}{32}ia^2d^2\mathrm{Chi}\left(\frac{5dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{5c}{2}\right. \\
&\left.-\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}+\frac{5}{16}ia^2d^2\mathrm{Chi}\left(\frac{dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{i\pi}{4}\right. \\
&\left.+\frac{dx}{2}\right)\sinh\left(\frac{1}{4}(2c-i\pi)\right)\sqrt{a+ia\sinh(c+dx)}+\frac{45}{32}ia^2d^2\mathrm{Chi}\left(\frac{3dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}\right. \\
&\left.+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{1}{4}(6c+i\pi)\right)\sqrt{a+ia\sinh(c+dx)} \\
&-\frac{5a^2d\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x} \\
&+\frac{5}{16}ia^2d^2\cosh\left(\frac{1}{4}(2c-i\pi)\right)\mathrm{sech}\left(\frac{c}{2}+\frac{i\pi}{4}\right. \\
&\left.+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\mathrm{Shi}\left(\frac{dx}{2}\right)+\frac{45}{32}ia^2d^2\cosh\left(\frac{1}{4}(6c+i\pi)\right)\mathrm{sech}\left(\frac{c}{2}\right. \\
&\left.+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\mathrm{Shi}\left(\frac{3dx}{2}\right)-\frac{25}{32}ia^2d^2\cosh\left(\frac{5c}{2}\right. \\
&\left.-\frac{i\pi}{4}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\mathrm{Shi}\left(\frac{5dx}{2}\right)
\end{aligned}
$$

output
```
-2*a^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^4*(a+I*a*sinh(d*x+c))^(1/2)/x^2+5/16*a
^2*d^2*sinh(1/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*Shi(1/2*d*x)*(a+I
*a*sinh(d*x+c))^(1/2)+45/32*I*a^2*d^2*cosh(3/2*c+1/4*I*Pi)*sech(1/2*c+1/4*
I*Pi+1/2*d*x)*Shi(3/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)-25/32*a^2*d^2*sinh(5/
2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*Shi(5/2*d*x)*(a+I*a*sinh(d*x+c)
)^(1/2)-25/32*a^2*d^2*Chi(5/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*cosh(5/2*c
+1/4*I*Pi)*(a+I*a*sinh(d*x+c))^(1/2)+5/16*a^2*d^2*Chi(1/2*d*x)*sech(1/2*c+
1/4*I*Pi+1/2*d*x)*cosh(1/2*c+1/4*I*Pi)*(a+I*a*sinh(d*x+c))^(1/2)+45/32*I*a
^2*d^2*Chi(3/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*sinh(3/2*c+1/4*I*Pi)*(a+I
*a*sinh(d*x+c))^(1/2)-5*a^2*d*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^3*sinh(1/2*c+1/
4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/x
```

3.135.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4751 vs. $2(536) = 1072$.

Time = 10.66 (sec) , antiderivative size = 4751, normalized size of antiderivative = 8.86

$$\int \frac{(a + ia\sinh(c + dx))^{5/2}}{x^3} dx = \text{Result too large to show}$$

input
```
Integrate[(a + I*a*Sinh[c + d*x])^(5/2)/x^3,x]
```

output
```
(2*((1/128 + I/128)*Cosh[5*(c/2 + (d*x)/2)] - (1/128 + I/128)*Sinh[5*(c/2
+ (d*x)/2)])*(a + I*a*Sinh[c + d*x])^(5/2)*((-4*I)*d^3 - (10*I)*c*d^3 + (2
0*I)*d^3*(c/2 + (d*x)/2) + 20*d^3*Cosh[2*(c/2 + (d*x)/2)] + 30*c*d^3*Cosh[
2*(c/2 + (d*x)/2)] - 60*d^3*(c/2 + (d*x)/2)*Cosh[2*(c/2 + (d*x)/2)] + (40*
I)*d^3*Cosh[4*(c/2 + (d*x)/2)] + (20*I)*c*d^3*Cosh[4*(c/2 + (d*x)/2)] - (4
0*I)*d^3*(c/2 + (d*x)/2)*Cosh[4*(c/2 + (d*x)/2)] - 40*d^3*Cosh[6*(c/2 + (d
*x)/2)] + 20*c*d^3*Cosh[6*(c/2 + (d*x)/2)] - 40*d^3*(c/2 + (d*x)/2)*Cosh[6
*(c/2 + (d*x)/2)] - (20*I)*d^3*Cosh[8*(c/2 + (d*x)/2)] + (30*I)*c*d^3*Cosh
[8*(c/2 + (d*x)/2)] - (60*I)*d^3*(c/2 + (d*x)/2)*Cosh[8*(c/2 + (d*x)/2)] +
 4*d^3*Cosh[10*(c/2 + (d*x)/2)] - 10*c*d^3*Cosh[10*(c/2 + (d*x)/2)] + 20*d
^3*(c/2 + (d*x)/2)*Cosh[10*(c/2 + (d*x)/2)] - (10*I)*c^2*d^3*Cosh[c/2 - 5*
(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] + (40*I)*c*d^3*(c/2 + (d*x)/2)*Cosh
[c/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] - (40*I)*d^3*(c/2 + (d*x)/
2)^2*Cosh[c/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] + 10*c^2*d^3*Cosh
[c/2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] - 40*c*d^3*(c/2 + (d*x)/2)
*Cosh[c/2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] + 40*d^3*(c/2 + (d*x)
/2)^2*Cosh[c/2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] - 45*c^2*d^3*Cos
h[(3*c)/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)]
+ 180*c*d^3*(c/2 + (d*x)/2)*Cosh[(3*c)/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral
[(-3*c)/2 + 3*(c/2 + (d*x)/2)] - 180*d^3*(c/2 + (d*x)/2)^2*Cosh[(3*c)/2...
```

3.135.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.76, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3800, 3042, 3795, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+a\sin(ic+idx))^{5/2}}{x^3}dx$$

$$\Big\downarrow \text{3800}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int \frac{\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\int\frac{\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5}{x^3}dx$$

$$\Big\downarrow \text{3795}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(\frac{25}{8}d^2\int\frac{\cosh^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{x}dx-\frac{5}{2}d^2\int\frac{\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{x}dx-\right.$$

$$\Big\downarrow \text{3042}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(-\frac{5}{2}d^2\int\frac{\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3}{x}dx+\frac{25}{8}d^2\int\frac{\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5}{x}dx-\right.$$

$$\Big\downarrow \text{3793}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(-\frac{5}{2}d^2\int\left(\frac{3i\sinh\left(\frac{1}{4}(2c-i\pi)+\frac{dx}{2}\right)}{4x}+\frac{i\sinh\left(\frac{1}{4}(6c+i\pi)+\frac{3dx}{2}\right)}{4x}\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$4a^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}\left(-\frac{5}{2}d^2\left(\frac{3}{4}i\sinh\left(\frac{1}{4}(2c-i\pi)\right)\text{Chi}\left(\frac{dx}{2}\right)+\frac{1}{4}i\sinh\left(\frac{1}{4}(6c+i\pi)\right)\text{C}\right.\right.$$

input
```
Int[(a + I*a*Sinh[c + d*x])^(5/2)/x^3,x]
```

output
```
4*a^2*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*(-1/2*Cos
h[c/2 + (I/4)*Pi + (d*x)/2]^5/x^2 - (5*d*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*
Sinh[c/2 + (I/4)*Pi + (d*x)/2])/(4*x) - (5*d^2*(((3*I)/4)*CoshIntegral[(d*
x)/2]*Sinh[(2*c - I*Pi)/4] + (I/4)*CoshIntegral[(3*d*x)/2]*Sinh[(6*c + I*P
i)/4] + ((3*I)/4)*Cosh[(2*c - I*Pi)/4]*SinhIntegral[(d*x)/2] + (I/4)*Cosh[
(6*c + I*Pi)/4]*SinhIntegral[(3*d*x)/2]))/2 + (25*d^2*((-1/16*I)*CoshInteg
ral[(5*d*x)/2]*Sinh[(5*c)/2 - (I/4)*Pi] + ((5*I)/8)*CoshIntegral[(d*x)/2]*
Sinh[(2*c - I*Pi)/4] + ((5*I)/16)*CoshIntegral[(3*d*x)/2]*Sinh[(6*c + I*Pi
)/4] + ((5*I)/8)*Cosh[(2*c - I*Pi)/4]*SinhIntegral[(d*x)/2] + ((5*I)/16)*C
osh[(6*c + I*Pi)/4]*SinhIntegral[(3*d*x)/2] - (I/16)*Cosh[(5*c)/2 - (I/4)*
Pi]*SinhIntegral[(5*d*x)/2]))/8)
```

3.135. $\int\frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx$

3.135.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.135.4 Maple [F]

$$\int \frac{(a + ia\sinh{(dx + c)})^{\frac{5}{2}}}{x^3} dx$$

input
```
int((a+I*a*sinh(d*x+c))^(5/2)/x^3,x)
```

output
```
int((a+I*a*sinh(d*x+c))^(5/2)/x^3,x)
```

3.135.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.135.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*sinh(d*x+c))**(5/2)/x**3,x)
```

output
```
Timed out
```

3.135.7 Maxima [F]

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx = \int \frac{(i\,a\sinh(dx+c)+a)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^3, x)
```

3.135.8 Giac [F]

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx = \int \frac{(i\,a\sinh(dx+c)+a)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2)/x^3,x, algorithm="giac")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^3, x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx = \int \frac{(a+a\sinh(c+d\,x)\ 1\mathrm{i})^{5/2}}{x^3}\,dx$$

input
```
int((a + a*sinh(c + d*x)*1i)^(5/2)/x^3,x)
```

output
```
int((a + a*sinh(c + d*x)*1i)^(5/2)/x^3, x)
```

3.136 $\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx$

3.136.1 Optimal result

Integrand size = 21, antiderivative size = 493

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx &= \frac{4ix^3\operatorname{arctanh}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} \\ &+ \frac{12ix^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}} \\ &- \frac{12ix^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}} \\ &- \frac{48ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^3\sqrt{a+ia\sinh(e+fx)}} \\ &+ \frac{48ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(3,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^3\sqrt{a+ia\sinh(e+fx)}} \\ &+ \frac{96i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(4,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^4\sqrt{a+ia\sinh(e+fx)}} \\ &- \frac{96i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(4,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^4\sqrt{a+ia\sinh(e+fx)}}\end{aligned}$$

output
```
-4*I*x^3*arctanh(exp(1/2*e+3/4*I*Pi+1/2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)
/f/(a+I*a*sinh(f*x+e))^(1/2)+12*I*x^2*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog
(2,exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I*a*sinh(f*x+e))^(1/2)-12*I*x^2*cos
h(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I
*a*sinh(f*x+e))^(1/2)-48*I*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(3,exp(1/
2*e+3/4*I*Pi+1/2*f*x))/f^3/(a+I*a*sinh(f*x+e))^(1/2)+48*I*x*cosh(1/2*e+1/4
*I*Pi+1/2*f*x)*polylog(3,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^3/(a+I*a*sinh(f*x
+e))^(1/2)+96*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(4,exp(1/2*e+3/4*I*Pi+
1/2*f*x))/f^4/(a+I*a*sinh(f*x+e))^(1/2)-96*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*
polylog(4,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^4/(a+I*a*sinh(f*x+e))^(1/2)
```

3.136.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx$$
$$= \frac{(1-i)(-1)^{3/4}\left(2ie^3\arctan\left(\sqrt[4]{-1}e^{\frac{1}{2}(e+fx)}\right)+e^3\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+f^3x^3\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(e+f}\right.\right.}{}$$

input
```
Integrate[x^3/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
((1 - I)*(-1)^(3/4)*((2*I)*e^3*ArcTan[(-1)^(1/4)*E^((e + f*x)/2)] + e^3*Lo
g[1 - (-1)^(3/4)*E^((e + f*x)/2)] + f^3*x^3*Log[1 - (-1)^(3/4)*E^((e + f*x
)/2)] - e^3*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - f^3*x^3*Log[1 + (-1)^(3/
4)*E^((e + f*x)/2)] - 6*f^2*x^2*PolyLog[2, -((-1)^(3/4)*E^((e + f*x)/2))]
 + 6*f^2*x^2*PolyLog[2, (-1)^(3/4)*E^((e + f*x)/2)] + 24*f*x*PolyLog[3, -((
-1)^(3/4)*E^((e + f*x)/2))] - 24*f*x*PolyLog[3, (-1)^(3/4)*E^((e + f*x)/2)
] - 48*PolyLog[4, -((-1)^(3/4)*E^((e + f*x)/2))] + 48*PolyLog[4, (-1)^(3/4
)*E^((e + f*x)/2)])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))/(f^4*Sqrt[a
 + I*a*Sinh[e + f*x]])
```

3.136.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.58, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3800, 3042, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x^3}{\sqrt{a+a\sin(ie+ifx)}}dx$$

$$\Big\downarrow \text{3800}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x^3\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x^3\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{4670}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{6i\int x^2\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{6i\int x^2\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}+\frac{4ix^3\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}\right)}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{6i\left(\frac{4\int x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{2x^2\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right)}{f}+\frac{6i\left(\frac{4\int x\,\text{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right.}{f}\right.}{}\right.}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{7163}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{6i\left(\frac{4\left(\frac{2x\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}-\frac{2\int \text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}\right)}{f}-\frac{2x^2\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right)}{f}+\right.}{\sqrt{a+ia\,\text{s}}}$$

$\downarrow$ 2720

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{6i\left(\frac{4\left(\frac{2x\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}-\frac{4\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}}{f^2}\right)}{f}-\frac{2x^2\,\text{PolyL}}{}\right.}{f}\right.}{}$$

$\downarrow$ 7143

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{4ix^3\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}-\frac{6i\left(\frac{4\left(\frac{2x\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}-\frac{4\,\text{PolyLog}\left(4,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2}\right)}{f}-\frac{2x^2\,\text{Po}}{}\right.}{f}\right.}{\sqrt{a+ia\sinh(}}$$

input
```
Int[x^3/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

3.136. $\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx$

output
```
(Cosh[e/2 + (I/4)*Pi + (f*x)/2]*(((4*I)*x^3*ArcTanh[E^((2*e - I*Pi)/4 + (f
*x)/2)])/f - ((6*I)*((-2*x^2*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f
+ (4*((2*x*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f - (4*PolyLog[4, -E
^((2*e - I*Pi)/4 + (f*x)/2)])/f^2))/f))/f + ((6*I)*((-2*x^2*PolyLog[2, E^(
(2*e - I*Pi)/4 + (f*x)/2)])/f + (4*((2*x*PolyLog[3, E^((2*e - I*Pi)/4 + (f
*x)/2)])/f - (4*PolyLog[4, E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2))/f))/f))/Sq
rt[a + I*a*Sinh[e + f*x]]
```

3.136.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.136.4 Maple [F]

$$\int \frac{x^3}{\sqrt{a + ia \sinh (fx + e)}} dx$$

input
```
int(x^3/(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
int(x^3/(a+I*a*sinh(f*x+e))^(1/2),x)
```

3.136.5 Fricas [F]

$$\int \frac{x^3}{\sqrt{a + ia \sinh(e + fx)}} dx = \int \frac{x^3}{\sqrt{i a \sinh (fx + e) + a}} dx$$

input
```
integrate(x^3/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(-2*I*sqrt(1/2*I*a*e^(-f*x - e))*x^3*e^(f*x + e)/(a*e^(f*x + e) -
I*a), x)
```

3.136.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{x^3}{\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}\,dx$$

input
```
integrate(x**3/(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(x**3/sqrt(I*a*(sinh(e + f*x) - I)), x)
```

3.136.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{x^3}{\sqrt{i\,a\sinh\left(fx+e\right)+a}}\,dx$$

input
```
integrate(x^3/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(I*a*sinh(f*x + e) + a), x)
```

3.136.8 Giac [F]

$$\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{x^3}{\sqrt{i\,a\sinh\left(fx+e\right)+a}}\,dx$$

input
```
integrate(x^3/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(I*a*sinh(f*x + e) + a), x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx=\int \frac{x^3}{\sqrt{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(x^3/(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

output
```
int(x^3/(a + a*sinh(e + f*x)*1i)^(1/2), x)
```

3.137 $\int \frac{x^2}{\sqrt{a+ia\sinh(e+fx)}}\,dx$

3.137.1 Optimal result

Integrand size = 21, antiderivative size = 349

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+ia\sinh(e+fx)}}\,dx &= \frac{4ix^2\text{arctanh}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} \\ &+ \frac{8ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}} \\ &- \frac{8ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}} \\ &- \frac{16i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^3\sqrt{a+ia\sinh(e+fx)}} \\ &+ \frac{16i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{PolyLog}\left(3,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^3\sqrt{a+ia\sinh(e+fx)}}\end{aligned}$$

output
```
-4*I*x^2*arctanh(exp(1/2*e+3/4*I*Pi+1/2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)
/f/(a+I*a*sinh(f*x+e))^(1/2)+8*I*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,
exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I*a*sinh(f*x+e))^(1/2)-8*I*x*cosh(1/2*
e+1/4*I*Pi+1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I*a*sin
h(f*x+e))^(1/2)-16*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(3,exp(1/2*e+3/4*
I*Pi+1/2*f*x))/f^3/(a+I*a*sinh(f*x+e))^(1/2)+16*I*cosh(1/2*e+1/4*I*Pi+1/2*
f*x)*polylog(3,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^3/(a+I*a*sinh(f*x+e))^(1/2)
```

3.137.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\sqrt{a+ia\sinh(e+fx)}}\,dx$$
$$= \frac{(1+i)(-1)^{3/4}\left(-2ie^2\arctan\left(\sqrt[4]{-1}e^{\frac{1}{2}(e+fx)}\right)-e^2\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+f^2x^2\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(e\cdots}\right.\right.}{}$$

input
```
Integrate[x^2/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
((1 + I)*(-1)^(3/4)*((-2*I)*e^2*ArcTan[(-1)^(1/4)*E^((e + f*x)/2)] - e^2*L
og[1 - (-1)^(3/4)*E^((e + f*x)/2)] + f^2*x^2*Log[1 - (-1)^(3/4)*E^((e + f*
x)/2)] + e^2*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - f^2*x^2*Log[1 + (-1)^(3
/4)*E^((e + f*x)/2)] - 4*f*x*PolyLog[2, -((-1)^(3/4)*E^((e + f*x)/2))] + 4
*f*x*PolyLog[2, (-1)^(3/4)*E^((e + f*x)/2)] + 8*PolyLog[3, -((-1)^(3/4)*E^
((e + f*x)/2))] - 8*PolyLog[3, (-1)^(3/4)*E^((e + f*x)/2)])*((-I)*Cosh[(e
+ f*x)/2] + Sinh[(e + f*x)/2]))/(f^3*Sqrt[a + I*a*Sinh[e + f*x]])
```

3.137.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.60, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3800, 3042, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+ia\sinh(e+fx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{x^2}{\sqrt{a+a\sin(ie+ifx)}}dx$$
$$\Big\downarrow \; 3800$$
$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x^2\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x^2\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \; 4670$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{4i\int x\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{4i\int x\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}+\frac{4ix^2\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}\right)}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \; 3011$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{4i\left(\frac{2\int \text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{2x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right)}{f}+\frac{4i\left(\frac{2\int \text{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\right.}{f}\right.}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \; 2720$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{4i\left(\frac{4\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}}{f^2}-\frac{2x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right)}{f}+\frac{4i\left(\frac{4\int e\right.}{}\right.}{\sqrt{a+ia\sinh(e+}}$$

$$\Big\downarrow \; 7143$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{4ix^2\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}-\frac{4i\left(\frac{4\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2}-\frac{2x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right)}{f}+\frac{4i\left(\frac{4\,\text{P}}{}\right.}{}\right.}{\sqrt{a+ia\sinh(e+fx)}}$$

input
```
Int[x^2/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

3.137. $\int \frac{x^2}{\sqrt{a+ia\sinh(e+fx)}}\,dx$

output
```
(Cosh[e/2 + (I/4)*Pi + (f*x)/2]*(((4*I)*x^2*ArcTanh[E^((2*e - I*Pi)/4 + (f
*x)/2)])/f - ((4*I)*((-2*x*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f +
(4*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2))/f + ((4*I)*((-2*x*Poly
Log[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/f + (4*PolyLog[3, E^((2*e - I*Pi)/4
+ (f*x)/2)])/f^2))/f))/Sqrt[a + I*a*Sinh[e + f*x]]
```

3.137.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.137.4 Maple [F]

$$\int \frac{x^2}{\sqrt{a + ia \sinh\left(fx + e\right)}}dx$$

input
```
int(x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
int(x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

3.137.5 Fricas [F]

$$\int \frac{x^2}{\sqrt{a + ia \sinh(e + fx)}}\, dx = \int \frac{x^2}{\sqrt{i\, a \sinh\left(fx + e\right) + a}}\, dx$$

input
```
integrate(x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(-2*I*sqrt(1/2*I*a*e^(-f*x - e))*x^2*e^(f*x + e)/(a*e^(f*x + e) -
I*a), x)
```

3.137.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a + ia \sinh(e + fx)}}\, dx = \int \frac{x^2}{\sqrt{ia\left(\sinh\left(e + fx\right) - i\right)}}\, dx$$

input
```
integrate(x**2/(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(x**2/sqrt(I*a*(sinh(e + f*x) - I)), x)
```

3.137.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{a + ia\sinh(e + fx)}}\,dx = \int \frac{x^2}{\sqrt{i\,a\sinh\left(fx + e\right) + a}}\,dx$$

input
```
integrate(x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(I*a*sinh(f*x + e) + a), x)
```

3.137.8 Giac [F]

$$\int \frac{x^2}{\sqrt{a + ia\sinh(e + fx)}}\,dx = \int \frac{x^2}{\sqrt{i\,a\sinh\left(fx + e\right) + a}}\,dx$$

input
```
integrate(x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(I*a*sinh(f*x + e) + a), x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a + ia\sinh(e + fx)}}\,dx = \int \frac{x^2}{\sqrt{a + a\sinh\left(e + f\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(x^2/(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

output
```
int(x^2/(a + a*sinh(e + f*x)*1i)^(1/2), x)
```

3.138 $\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx$

3.138.1 Optimal result

Integrand size = 19, antiderivative size = 207

$$\begin{aligned}\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx = {} & \frac{4ix\operatorname{arctanh}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} \\ & + \frac{4i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}} \\ & - \frac{4i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}\end{aligned}$$

output
```
-4*I*x*arctanh(exp(1/2*e+3/4*I*Pi+1/2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/f
/(a+I*a*sinh(f*x+e))^(1/2)+4*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,exp(
1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I*a*sinh(f*x+e))^(1/2)-4*I*cosh(1/2*e+1/4*
I*Pi+1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I*a*sinh(f*x+
e))^(1/2)
```

3.138.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.86

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx$$
$$= \frac{(2-2i)(-1)^{3/4}\left(ie\arctan\left(\sqrt[4]{-1}e^{\frac{1}{2}(e+fx)}\right)+\frac{1}{2}(e+fx)\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)-\frac{1}{2}(e+fx)\log\left(1+(\cdots\right.\right.}{f^2\cdots}$$

input
```
Integrate[x/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
((2 - 2*I)*(-1)^(3/4)*(I*e*ArcTan[(-1)^(1/4)*E^((e + f*x)/2)] + ((e + f*x)
*Log[1 - (-1)^(3/4)*E^((e + f*x)/2)])/2 - ((e + f*x)*Log[1 + (-1)^(3/4)*E^
((e + f*x)/2)])/2 - PolyLog[2, -((-1)^(3/4)*E^((e + f*x)/2))] + PolyLog[2,
 (-1)^(3/4)*E^((e + f*x)/2)])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))/(
f^2*Sqrt[a + I*a*Sinh[e + f*x]])
```

3.138.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3800, 3042, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{x}{\sqrt{a+a\sin(ie+ifx)}}dx$$
$$\Big\downarrow \text{3800}$$
$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{\sqrt{a+ia\sinh(e+fx)}}$$
$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{ 4670}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{2i\int\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{2i\int\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}+\frac{4ix\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}\right)}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{4i\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}}{f^2}-\frac{4i\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}}{f^2}\cdots\right.}{\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{4ix\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}+\frac{4i\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2}-\frac{4i\,\text{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2}\right)}{\sqrt{a+ia\sinh(e+fx)}}$$

input
```
Int[x/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

output
```
(Cosh[e/2 + (I/4)*Pi + (f*x)/2]*(((4*I)*x*ArcTanh[E^((2*e - I*Pi)/4 + (f*x
)/2)])/f + ((4*I)*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2 - ((4*I)*
PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2))/Sqrt[a + I*a*Sinh[e + f*x]
]
```

3.138.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.138.4 Maple [F]

$$\int \frac{x}{\sqrt{a + ia\sinh\left(fx + e\right)}}dx$$

input
```
int(x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
int(x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

3.138.5 Fricas [F]

$$\int \frac{x}{\sqrt{a + ia\sinh(e + fx)}}\,dx = \int \frac{x}{\sqrt{i\,a\sinh\left(fx + e\right) + a}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(-2*I*sqrt(1/2*I*a*e^(-f*x - e))*x*e^(f*x + e)/(a*e^(f*x + e) - I*
a), x)
```

3.138.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{x}{\sqrt{ia\,(\sinh{(e+fx)}-i)}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(x/sqrt(I*a*(sinh(e + f*x) - I)), x)
```

3.138.7 Maxima [F]

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{x}{\sqrt{i\,a\sinh{(fx+e)}+a}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(I*a*sinh(f*x + e) + a), x)
```

3.138.8 Giac [F]

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{x}{\sqrt{i\,a\sinh{(fx+e)}+a}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(I*a*sinh(f*x + e) + a), x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx=\int \frac{x}{\sqrt{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(x/(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

output
```
int(x/(a + a*sinh(e + f*x)*1i)^(1/2), x)
```

3.139 $\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx$

3.139.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx = \operatorname{Int}\left(\frac{1}{x\sqrt{a+ia\sinh(e+fx)}},x\right)$$

output
```
Unintegrable(1/x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

3.139.2 Mathematica [N/A]

Not integrable

Time = 3.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[a + I*a*Sinh[e + f*x]]),x]
```

output
```
Integrate[1/(x*Sqrt[a + I*a*Sinh[e + f*x]]), x]
```

3.139.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{x\sqrt{a+a\sin(ie+ifx)}}dx$$
$$\Big\downarrow \ 3807$$
$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}dx$$

input
```
Int[1/(x*Sqrt[a + I*a*Sinh[e + f*x]]),x]
```

output
```
$Aborted
```

3.139.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.139.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\sqrt{a+ia\sinh\left(fx+e\right)}}dx$$

input
```
int(1/x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
int(1/x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

3.139.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{\sqrt{i\,a\sinh\left(fx+e\right)+ax}}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(-2*I*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(a*x*e^(f*x + e) - I*
a*x), x)
```

3.139.6 Sympy [N/A]

Not integrable

Time = 2.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{x\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(I*a*(sinh(e + f*x) - I))), x)
```

3.139.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{\sqrt{i\,a\sinh\left(fx+e\right)+a}x}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(I*a*sinh(f*x + e) + a)*x), x)
```

3.139.8 Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{\sqrt{i\,a\sinh\left(fx+e\right)+a}x}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(I*a*sinh(f*x + e) + a)*x), x)
```

3.139.9 Mupad [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{x\,\sqrt{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(1/(x*(a + a*sinh(e + f*x)*1i)^(1/2)),x)
```

output
```
int(1/(x*(a + a*sinh(e + f*x)*1i)^(1/2)), x)
```

3.140 $\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx$

3.140.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx = \mathrm{Int}\left(\frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}},x\right)$$

output
```
Unintegrable(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

3.140.2 Mathematica [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[a + I*a*Sinh[e + f*x]]),x]
```

output
```
Integrate[1/(x^2*Sqrt[a + I*a*Sinh[e + f*x]]), x]
```

3.140.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{x^2\sqrt{a+a\sin(ie+ifx)}}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}dx$$

input
```
Int[1/(x^2*Sqrt[a + I*a*Sinh[e + f*x]]),x]
```

output
```
$Aborted
```

3.140.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.140.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^2\sqrt{a+ia\sinh\left(fx+e\right)}}dx$$

input
```
int(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

output
```
int(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

3.140.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.10

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{\sqrt{i\,a\sinh\left(fx+e\right)+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(-2*I*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(a*x^2*e^(f*x + e) -
I*a*x^2), x)
```

3.140.6 Sympy [N/A]

Not integrable

Time = 4.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{x^2\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}\,dx$$

input
```
integrate(1/x**2/(a+I*a*sinh(f*x+e))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(I*a*(sinh(e + f*x) - I))), x)
```

3.140.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx=\int \frac{1}{\sqrt{i\,a\sinh\left(fx+e\right)+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(I*a*sinh(f*x + e) + a)*x^2), x)
```

3.140.8 Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx=\int \frac{1}{\sqrt{i\,a\sinh\left(fx+e\right)+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(I*a*sinh(f*x + e) + a)*x^2), x)
```

3.140.9 Mupad [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx=\int \frac{1}{x^2\,\sqrt{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(1/(x^2*(a + a*sinh(e + f*x)*1i)^(1/2)),x)
```

output
```
int(1/(x^2*(a + a*sinh(e + f*x)*1i)^(1/2)), x)
```

3.141 $\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

3.141.1 Optimal result

Integrand size = 21, antiderivative size = 807

$$
\begin{aligned}
\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx = {} & \frac{3x^2}{af^2\sqrt{a+ia\sinh(e+fx)}} \\
& - \frac{24ix\,\mathrm{arctanh}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} \\
& + \frac{ix^3\,\mathrm{arctanh}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+ia\sinh(e+fx)}} \\
& - \frac{24i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\mathrm{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^4\sqrt{a+ia\sinh(e+fx)}} \\
& + \frac{3ix^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\mathrm{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}} \\
& + \frac{24i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\mathrm{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^4\sqrt{a+ia\sinh(e+fx)}} \\
& - \frac{3ix^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\mathrm{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}} \\
& - \frac{12ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\mathrm{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} \\
& + \frac{12ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\mathrm{PolyLog}\left(3,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} \\
& + \frac{24i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\mathrm{PolyLog}\left(4,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^4\sqrt{a+ia\sinh(e+fx)}} \\
& - \frac{24i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\mathrm{PolyLog}\left(4,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^4\sqrt{a+ia\sinh(e+fx)}} + \frac{x^3\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+ia\sinh(e+fx)}}
\end{aligned}
$$

output
```
3*x^2/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)+24*I*x*arctanh(exp(1/2*e+3/4*I*Pi+1/
2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f^3/(a+I*a*sinh(f*x+e))^(1/2)-I*x^3
*arctanh(exp(1/2*e+3/4*I*Pi+1/2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f/(a+
I*a*sinh(f*x+e))^(1/2)-24*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,exp(1/2
*e+3/4*I*Pi+1/2*f*x))/a/f^4/(a+I*a*sinh(f*x+e))^(1/2)+3*I*x^2*cosh(1/2*e+1
/4*I*Pi+1/2*f*x)*polylog(2,exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^2/(a+I*a*sinh(
f*x+e))^(1/2)+24*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I
*Pi+1/2*f*x))/a/f^4/(a+I*a*sinh(f*x+e))^(1/2)-3*I*x^2*cosh(1/2*e+1/4*I*Pi+
1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^2/(a+I*a*sinh(f*x+e))
^(1/2)-12*I*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(3,exp(1/2*e+3/4*I*Pi+1/
2*f*x))/a/f^3/(a+I*a*sinh(f*x+e))^(1/2)+12*I*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x
)*polylog(3,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^3/(a+I*a*sinh(f*x+e))^(1/2)+
24*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(4,exp(1/2*e+3/4*I*Pi+1/2*f*x))/a
/f^4/(a+I*a*sinh(f*x+e))^(1/2)-24*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(4
,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^4/(a+I*a*sinh(f*x+e))^(1/2)+1/2*x^3*tan
h(1/2*e+1/4*I*Pi+1/2*f*x)/a/f/(a+I*a*sinh(f*x+e))^(1/2)
```

3.141.2 Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 546, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \frac{\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)\left(f^2x^2(6+ifx)\left(\cosh\left(\frac{1}{2}(e+fx)\right)\right.\right.}{}$$

input
```
Integrate[x^3/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
((Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])*(f^2*x^2*(6 + I*f*x)*(Cosh[(e +
 f*x)/2] + I*Sinh[(e + f*x)/2]) + (1/2 - I/2)*(-1)^(3/4)*(-48*e*ArcTanh[(-
1)^(3/4)*E^((e + f*x)/2)] + 2*e^3*ArcTanh[(-1)^(3/4)*E^((e + f*x)/2)] - 24
*e*Log[1 - (-1)^(3/4)*E^((e + f*x)/2)] + e^3*Log[1 - (-1)^(3/4)*E^((e + f*
x)/2)] - 24*f*x*Log[1 - (-1)^(3/4)*E^((e + f*x)/2)] + f^3*x^3*Log[1 - (-1)
^(3/4)*E^((e + f*x)/2)] + 24*e*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - e^3*L
og[1 + (-1)^(3/4)*E^((e + f*x)/2)] + 24*f*x*Log[1 + (-1)^(3/4)*E^((e + f*x
)/2)] - f^3*x^3*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - 6*(-8 + f^2*x^2)*Pol
yLog[2, -((-1)^(3/4)*E^((e + f*x)/2))] + 6*(-8 + f^2*x^2)*PolyLog[2, (-1)^
(3/4)*E^((e + f*x)/2)] + 24*f*x*PolyLog[3, -((-1)^(3/4)*E^((e + f*x)/2))]
- 24*f*x*PolyLog[3, (-1)^(3/4)*E^((e + f*x)/2)] - 48*PolyLog[4, -((-1)^(3/
4)*E^((e + f*x)/2))] + 48*PolyLog[4, (-1)^(3/4)*E^((e + f*x)/2)])*(Cosh[(e
 + f*x)/2] + I*Sinh[(e + f*x)/2])^2 + 2*f^3*x^3*Sinh[(e + f*x)/2]))/(2*f^4
*(a + I*a*Sinh[e + f*x])^(3/2))
```

3.141.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 475, normalized size of antiderivative = 0.59, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3800, 3042, 4674, 3042, 4670, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x^3}{(a+a\sin(ie+ifx))^{3/2}}dx$$

$$\Big\downarrow \text{3800}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x^3\text{sech}^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x^3\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{4674}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{12\int x\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f^2}+\frac{1}{2}\int x^3\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx+\frac{6x^2\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{x^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right.}{}\right.}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{12\int x\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{f^2}+\frac{1}{2}\int x^3\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx+\frac{6x^2\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{x^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\right.}{}\right.}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{4670}$$

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{12\left(\frac{2i\int\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{2i\int\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}+\frac{4ix\,\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}\right)}{f^2}+\frac{1}{2}\left(\frac{6i\int x\right.\right.$$

$$\Big\downarrow \; 2715$$

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{12\left(\frac{4i\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}}{f^2}-\frac{4i\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-i\right.}{f^2}}{f^2}\right.$$

$$\Big\downarrow \; 2838$$

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(\frac{6i\int x^2\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{6i\int x^2\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}+\frac{4ix^3\,\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}\right)\right.$$

$$\Big\downarrow \; 3011$$

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(-\frac{6i\left(\frac{4\int x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{2x^2\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right)}{f}+\frac{6i\left(\frac{4\int x\,\text{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\right.}{f}\right.}{}\right.\right.$$

$$\Big\downarrow \; 7163$$

3.141. $\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(-\frac{6i\left(\frac{4\left(\frac{2x\,\mathrm{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}-\frac{2\int \mathrm{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}\right)}{f}-\frac{2x^2\,\mathrm{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right.}{f}\right.\right.$$

$$\Big\downarrow \; 2720$$

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(-\frac{6i\left(\frac{4\left(\frac{2x\,\mathrm{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}-\frac{4\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\,\mathrm{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}}{f^2}\right)}{f}-\frac{2x^2}{}\right.}{f}\right.\right.$$

$$\Big\downarrow \; 7143$$

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(\frac{4ix^3\operatorname{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}-\frac{6i\left(\frac{4\left(\frac{2x\,\mathrm{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}-\frac{4\,\mathrm{PolyLog}\left(4,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2}\right)}{f}-\frac{2x}{}\right.}{f}\right.\right.$$

input

```
Int[x^3/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

3.141. $\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

output
```
(Cosh[e/2 + (I/4)*Pi + (f*x)/2]*((-12*(((4*I)*x*ArcTanh[E^((2*e - I*Pi)/4
+ (f*x)/2)])/f + ((4*I)*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2 - (
(4*I)*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2))/f^2 + (((4*I)*x^3*Ar
cTanh[E^((2*e - I*Pi)/4 + (f*x)/2)])/f - ((6*I)*((-2*x^2*PolyLog[2, -E^((2
*e - I*Pi)/4 + (f*x)/2)])/f + (4*((2*x*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*
x)/2)])/f - (4*PolyLog[4, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2))/f))/f + ((
6*I)*((-2*x^2*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/f + (4*((2*x*PolyL
og[3, E^((2*e - I*Pi)/4 + (f*x)/2)])/f - (4*PolyLog[4, E^((2*e - I*Pi)/4 +
 (f*x)/2)])/f^2))/f))/f)/2 + (6*x^2*Sech[e/2 + (I/4)*Pi + (f*x)/2])/f^2 +
(x^3*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f))/(2
*a*Sqrt[a + I*a*Sinh[e + f*x]])
```

3.141.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.141. $\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

3.141.4 Maple [F]

$$\int \frac{x^3}{(a+ia\sinh{(fx+e)})^{\frac{3}{2}}}dx$$

input
```
int(x^3/(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
int(x^3/(a+I*a*sinh(f*x+e))^(3/2),x)
```

3.141.5 Fricas [F]

$$\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x^3}{(i\,a\sinh{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
((a^2*f^2*e^(2*f*x + 2*e) - 2*I*a^2*f^2*e^(f*x + e) - a^2*f^2)*integral(1/
2*(-I*f^2*x^3 + 24*I*x)*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(a^2*f^2*e^
(f*x + e) - I*a^2*f^2), x) + ((-I*f*x^3 - 6*I*x^2)*e^(2*f*x + 2*e) + (f*x^
3 - 6*x^2)*e^(f*x + e))*sqrt(1/2*I*a*e^(-f*x - e)))/(a^2*f^2*e^(2*f*x + 2*
e) - 2*I*a^2*f^2*e^(f*x + e) - a^2*f^2)
```

3.141.6 Sympy [F]

$$\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x^3}{(ia\,(\sinh{(e+fx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(a+I*a*sinh(f*x+e))**(3/2),x)
```

output
```
Integral(x**3/(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

3.141.7 Maxima [F]

$$\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x^3}{(i\,a\sinh\left(fx+e\right)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

3.141.8 Giac [F]

$$\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x^3}{(i\,a\sinh\left(fx+e\right)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x^3}{{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

input
```
int(x^3/(a + a*sinh(e + f*x)*1i)^(3/2),x)
```

output
```
int(x^3/(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

3.142 $\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

3.142.1 Optimal result

Integrand size = 21, antiderivative size = 506

$$\begin{aligned}
\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx &= \frac{2x}{af^2\sqrt{a+ia\sinh(e+fx)}} \\
&- \frac{4\arctan\left(\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} \\
&+ \frac{ix^2\operatorname{arctanh}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+ia\sinh(e+fx)}} \\
&+ \frac{2ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}} \\
&- \frac{2ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}} \\
&- \frac{4i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} \\
&+ \frac{4i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(3,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} + \frac{x^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+ia\sinh(e+fx)}}
\end{aligned}$$

output
```
2*x/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)-4*arctan(sinh(1/2*e+1/4*I*Pi+1/2*f*x))
*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f^3/(a+I*a*sinh(f*x+e))^(1/2)-I*x^2*arctan
h(exp(1/2*e+3/4*I*Pi+1/2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f/(a+I*a*sin
h(f*x+e))^(1/2)+2*I*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,exp(1/2*e+3/4
*I*Pi+1/2*f*x))/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)-2*I*x*cosh(1/2*e+1/4*I*Pi+
1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^2/(a+I*a*sinh(f*x+e))
^(1/2)-4*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(3,exp(1/2*e+3/4*I*Pi+1/2*f
*x))/a/f^3/(a+I*a*sinh(f*x+e))^(1/2)+4*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*poly
log(3,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^3/(a+I*a*sinh(f*x+e))^(1/2)+1/2*x^
2*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f/(a+I*a*sinh(f*x+e))^(1/2)
```

3.142.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \frac{\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)\left(fx(4+ifx)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+\right.\right.}{}$$

input
```
Integrate[x^2/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
((Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])*(f*x*(4 + I*f*x)*(Cosh[(e + f*x
)/2] + I*Sinh[(e + f*x)/2]) - (1/2 - I/2)*(-1)^(3/4)*(-16*ArcTanh[(-1)^(3/
4)*E^((e + f*x)/2)] + 2*e^2*ArcTanh[(-1)^(3/4)*E^((e + f*x)/2)] + e^2*Log[
1 - (-1)^(3/4)*E^((e + f*x)/2)] - f^2*x^2*Log[1 - (-1)^(3/4)*E^((e + f*x)/
2)] - e^2*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] + f^2*x^2*Log[1 + (-1)^(3/4)
*E^((e + f*x)/2)] + 4*f*x*PolyLog[2, -((-1)^(3/4)*E^((e + f*x)/2))] - 4*f*
x*PolyLog[2, (-1)^(3/4)*E^((e + f*x)/2)] - 8*PolyLog[3, -((-1)^(3/4)*E^((e
 + f*x)/2))] + 8*PolyLog[3, (-1)^(3/4)*E^((e + f*x)/2)])*(Cosh[(e + f*x)/2
] + I*Sinh[(e + f*x)/2])^2 + 2*f^2*x^2*Sinh[(e + f*x)/2]))/(2*f^3*(a + I*a
*Sinh[e + f*x])^(3/2))
```

3.142.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.63, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3800, 3042, 4674, 3042, 4257, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{x^2}{(a+a\sin(ie+ifx))^{3/2}}dx$$

$$\Big\downarrow \; 3800$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x^2\text{sech}^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x^2\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \; 4674$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{4\int \text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{f^2}+\frac{1}{2}\int x^2\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx+\frac{4x\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{x^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right.}{\cdots}\right.}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{4\int \csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx}{f^2}+\frac{1}{2}\int x^2\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx+\frac{4x\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{x^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f\cdots}\right.}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \; 4257$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\int x^2\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx-\frac{8\arctan\left(\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{f^3}+\frac{4x\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{x^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\cdots\right.}{\cdots}\right.}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \ 4670$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(\frac{4i\int x\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{4i\int x\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}+\frac{4ix^2\operatorname{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}\right)-\right.}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \ 3011$$

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(-\frac{4i\left(\frac{2\int \text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{2x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right)}{f}+\frac{4i\left(\frac{2\int \text{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right.}{f}\right.}{}\right.\right.$$

$$\Big\downarrow \ 2720$$

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(-\frac{4i\left(\frac{4\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}}{f^2}-\frac{2x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f}\right)}{f}+\frac{4i\left(\right.}{}\right.\right.$$

$$\Big\downarrow \ 7143$$

$$\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(-\frac{8\arctan\left(\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{f^3}+\frac{1}{2}\left(\frac{4ix^2\operatorname{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}-\frac{4i\left(\frac{4\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2}-\frac{2x\,\text{P}}{}\right.}{f}\right.\right.$$

input
```
Int[x^2/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

3.142. $\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

output
```
(Cosh[e/2 + (I/4)*Pi + (f*x)/2]*((-8*ArcTan[Sinh[e/2 + (I/4)*Pi + (f*x)/2]
])/f^3 + (((4*I)*x^2*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)])/f - ((4*I)*((-
2*x*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f + (4*PolyLog[3, -E^((2*e
- I*Pi)/4 + (f*x)/2)])/f^2))/f + ((4*I)*((-2*x*PolyLog[2, E^((2*e - I*Pi)/
4 + (f*x)/2)])/f + (4*PolyLog[3, E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2))/f)/2
 + (4*x*Sech[e/2 + (I/4)*Pi + (f*x)/2])/f^2 + (x^2*Sech[e/2 + (I/4)*Pi + (
f*x)/2]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f))/(2*a*Sqrt[a + I*a*Sinh[e + f*x
]])
```

3.142.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.142. $\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.142.4 Maple [F]

$$\int \frac{x^2}{(a+ia\sinh(fx+e))^{\frac{3}{2}}}dx$$

input
```
int(x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
int(x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

3.142.5 Fricas [F]

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x^2}{(i\,a\sinh(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
((a^2*f^2*e^(2*f*x + 2*e) - 2*I*a^2*f^2*e^(f*x + e) - a^2*f^2)*integral(1/
2*(-I*f^2*x^2 + 8*I)*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(a^2*f^2*e^(f*
x + e) - I*a^2*f^2), x) + ((-I*f*x^2 - 4*I*x)*e^(2*f*x + 2*e) + (f*x^2 - 4
*x)*e^(f*x + e))*sqrt(1/2*I*a*e^(-f*x - e)))/(a^2*f^2*e^(2*f*x + 2*e) - 2*
I*a^2*f^2*e^(f*x + e) - a^2*f^2)
```

3.142.6 Sympy [F]

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x^2}{(ia(\sinh(e+fx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(a+I*a*sinh(f*x+e))**(3/2),x)
```

output
```
Integral(x**2/(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

3.142.7 Maxima [F]

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x^2}{(i\,a\sinh(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

3.142.8 Giac [F]

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x^2}{(i\,a\sinh(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x^2}{{\left(a+a\,\sinh\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

input
```
int(x^2/(a + a*sinh(e + f*x)*1i)^(3/2),x)
```

output
```
int(x^2/(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

3.143 $\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

3.143.1 Optimal result

Integrand size = 19, antiderivative size = 288

$$
\begin{aligned}
\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx &= \frac{1}{af^2\sqrt{a+ia\sinh(e+fx)}} \\
&+ \frac{ix\operatorname{arctanh}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+ia\sinh(e+fx)}} \\
&+ \frac{i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}} \\
&- \frac{i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}} + \frac{x\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+ia\sinh(e+fx)}}
\end{aligned}
$$

output
```
1/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)-I*x*arctanh(exp(1/2*e+3/4*I*Pi+1/2*f*x))
*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f/(a+I*a*sinh(f*x+e))^(1/2)+I*cosh(1/2*e+1
/4*I*Pi+1/2*f*x)*polylog(2,exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^2/(a+I*a*sinh(
f*x+e))^(1/2)-I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi
+1/2*f*x))/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)+1/2*x*tanh(1/2*e+1/4*I*Pi+1/2*f
*x)/a/f/(a+I*a*sinh(f*x+e))^(1/2)
```

3.143.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.90

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \frac{\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)\left((2+ifx)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\right.\right.}{}$$

input
```
Integrate[x/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

output
```
((Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])*((2 + I*f*x)*(Cosh[(e + f*x)/2]
 + I*Sinh[(e + f*x)/2]) + (1 - I)*(-1)^(3/4)*(I*e*ArcTan[(-1)^(1/4)*E^((e
+ f*x)/2)] + ((e + f*x)*Log[1 - (-1)^(3/4)*E^((e + f*x)/2)])/2 - ((e + f*x
)*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)])/2 - PolyLog[2, -((-1)^(3/4)*E^((e +
 f*x)/2))] + PolyLog[2, (-1)^(3/4)*E^((e + f*x)/2)])*(Cosh[(e + f*x)/2] +
I*Sinh[(e + f*x)/2])^2 + 2*f*x*Sinh[(e + f*x)/2]))/(2*f^2*(a + I*a*Sinh[e
+ f*x])^(3/2))
```

3.143.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.75, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3800, 3042, 4673, 3042, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{x}{(a+a\sin(ie+ifx))^{3/2}}dx$$

$$\Big\downarrow \; 3800$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x\text{sech}^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\int x\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^3dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{ 4673}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\int x\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx+\frac{2\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{x\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\int x\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)dx+\frac{2\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2}+\frac{x\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f}\right)}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{ 4670}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(\frac{2i\int\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}-\frac{2i\int\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)dx}{f}+\frac{4ix\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}\right)+\frac{2\text{se}}{}\right.}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(\frac{4i\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\log\left(1-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}}{f^2}-\frac{4i\int e^{\frac{1}{4}(i\pi-2e)-\frac{fx}{2}}\log\left(1+e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)de^{\frac{1}{4}(}}{f^2}\right.\right.}{2a\sqrt{a+ia\sinh(e+f}}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\left(\frac{1}{2}\left(\frac{4ix\text{arctanh}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f}+\frac{4i\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2}-\frac{4i\,\text{PolyLog}\left(2,e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2}\right)+\frac{2\text{S}}{}\right.}{2a\sqrt{a+ia\sinh(e+fx)}}$$

input
```
Int[x/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

3.143. $\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

output
```
(Cosh[e/2 + (I/4)*Pi + (f*x)/2]*((((4*I)*x*ArcTanh[E^((2*e - I*Pi)/4 + (f*
x)/2)])/f + ((4*I)*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2 - ((4*I)
*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/f^2)/2 + (2*Sech[e/2 + (I/4)*Pi
 + (f*x)/2])/f^2 + (x*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Tanh[e/2 + (I/4)*Pi +
 (f*x)/2])/f))/(2*a*Sqrt[a + I*a*Sinh[e + f*x]])
```

3.143.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

3.143.4 Maple [F]

$$\int \frac{x}{(a+ia\sinh(fx+e))^{\frac{3}{2}}}dx$$

input
```
int(x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
int(x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

3.143.5 Fricas [F]

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x}{(i\,a\sinh(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
((a^2*f^2*e^(2*f*x + 2*e) - 2*I*a^2*f^2*e^(f*x + e) - a^2*f^2)*integral(-1
/2*I*sqrt(1/2*I*a*e^(-f*x - e))*x*e^(f*x + e)/(a^2*e^(f*x + e) - I*a^2), x
) + ((-I*f*x - 2*I)*e^(2*f*x + 2*e) + (f*x - 2)*e^(f*x + e))*sqrt(1/2*I*a*
e^(-f*x - e)))/(a^2*f^2*e^(2*f*x + 2*e) - 2*I*a^2*f^2*e^(f*x + e) - a^2*f^
2)
```

3.143.6 Sympy [F]

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x}{(ia\,(\sinh(e+fx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(f*x+e))**(3/2),x)
```

output
```
Integral(x/(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

3.143.7 Maxima [F]

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x}{(i\,a\sinh(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

3.143.8 Giac [F]

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x}{(i\,a\sinh(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{x}{(a+a\sinh(e+f\,x)\;1\mathrm{i})^{3/2}}\,dx$$

input
```
int(x/(a + a*sinh(e + f*x)*1i)^(3/2),x)
```

output
```
int(x/(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

3.144 $\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx$

3.144.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{x(a+ia\sinh(e+fx))^{3/2}},x\right)$$

output
```
Unintegrable(1/x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

3.144.2 Mathematica [N/A]

Not integrable

Time = 21.86 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx$$

input
```
Integrate[1/(x*(a + I*a*Sinh[e + f*x])^(3/2)),x]
```

output
```
Integrate[1/(x*(a + I*a*Sinh[e + f*x])^(3/2)), x]
```

3.144.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{x(a+a\sin(ie+ifx))^{3/2}}dx$$
$$\Big\downarrow \text{ 3807}$$
$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}dx$$

input
```
Int[1/(x*(a + I*a*Sinh[e + f*x])^(3/2)),x]
```

output
```
$Aborted
```

3.144.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.144.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\left(a+ia\sinh\left(fx+e\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
int(1/x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

3.144.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 209, normalized size of antiderivative = 9.95

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(i\,a\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
((a^2*f^2*x^2*e^(2*f*x + 2*e) - 2*I*a^2*f^2*x^2*e^(f*x + e) - a^2*f^2*x^2)
*integral(1/2*(-I*f^2*x^2 + 8*I)*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(a
^2*f^2*x^3*e^(f*x + e) - I*a^2*f^2*x^3), x) + ((-I*f*x + 2*I)*e^(2*f*x + 2
*e) + (f*x + 2)*e^(f*x + e))*sqrt(1/2*I*a*e^(-f*x - e)))/(a^2*f^2*x^2*e^(2
*f*x + 2*e) - 2*I*a^2*f^2*x^2*e^(f*x + e) - a^2*f^2*x^2)
```

3.144.6 Sympy [N/A]

Not integrable

Time = 28.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{x\left(ia\left(\sinh\left(e+fx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(f*x+e))**(3/2),x)
```

output
```
Integral(1/(x*(I*a*(sinh(e + f*x) - I))**(3/2)), x)
```

3.144.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(i\,a\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*sinh(f*x + e) + a)^(3/2)*x), x)
```

3.144.8 Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(i\,a\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((I*a*sinh(f*x + e) + a)^(3/2)*x), x)
```

3.144.9 Mupad [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{x\,{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

input
```
int(1/(x*(a + a*sinh(e + f*x)*1i)^(3/2)),x)
```

output
```
int(1/(x*(a + a*sinh(e + f*x)*1i)^(3/2)), x)
```

3.145 $\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx$

3.145.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}},x\right)$$

output
```
Unintegrable(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

3.145.2 Mathematica [N/A]

Not integrable

Time = 23.84 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx$$

input
```
Integrate[1/(x^2*(a + I*a*Sinh[e + f*x])^(3/2)),x]
```

output
```
Integrate[1/(x^2*(a + I*a*Sinh[e + f*x])^(3/2)), x]
```

3.145.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{x^2(a+a\sin(ie+ifx))^{3/2}}dx$$
$$\Big\downarrow \text{ 3807}$$
$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}dx$$

input
```
Int[1/(x^2*(a + I*a*Sinh[e + f*x])^(3/2)),x]
```

output
```
$Aborted
```

3.145.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.145.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^2 (a + ia \sinh (fx + e))^{\frac{3}{2}}} dx$$

input
```
int(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

output
```
int(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

3.145.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 209, normalized size of antiderivative = 9.95

$$\int \frac{1}{x^2(a + ia \sinh(e + fx))^{3/2}} dx = \int \frac{1}{(i\,a \sinh (fx + e) + a)^{\frac{3}{2}} x^2} dx$$

input
```
integrate(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
((a^2*f^2*x^3*e^(2*f*x + 2*e) - 2*I*a^2*f^2*x^3*e^(f*x + e) - a^2*f^2*x^3)
*integral(1/2*(-I*f^2*x^2 + 24*I)*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(
a^2*f^2*x^4*e^(f*x + e) - I*a^2*f^2*x^4), x) + ((-I*f*x + 4*I)*e^(2*f*x +
2*e) + (f*x + 4)*e^(f*x + e))*sqrt(1/2*I*a*e^(-f*x - e)))/(a^2*f^2*x^3*e^(
2*f*x + 2*e) - 2*I*a^2*f^2*x^3*e^(f*x + e) - a^2*f^2*x^3)
```

3.145.6 Sympy [N/A]

Not integrable

Time = 52.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2(a + ia \sinh(e + fx))^{3/2}} dx = \int \frac{1}{x^2 (ia (\sinh (e + fx) - i))^{\frac{3}{2}}} dx$$

input
```
integrate(1/x**2/(a+I*a*sinh(f*x+e))**(3/2),x)
```

output
```
Integral(1/(x**2*(I*a*(sinh(e + f*x) - I))**(3/2)), x)
```

3.145.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{(i\,a\sinh\left(fx+e\right)+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*sinh(f*x + e) + a)^(3/2)*x^2), x)
```

3.145.8 Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{(i\,a\sinh\left(fx+e\right)+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((I*a*sinh(f*x + e) + a)^(3/2)*x^2), x)
```

3.145.9 Mupad [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{x^2\left(a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + a*sinh(e + f*x)*1i)^(3/2)),x)
```

output
```
int(1/(x^2*(a + a*sinh(e + f*x)*1i)^(3/2)), x)
```

3.146 $\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

3.146.1 Optimal result

Integrand size = 21, antiderivative size = 1016

$$
\begin{aligned}
\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx &= -\frac{1}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} \\
&+ \frac{9x^2}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{10ix\operatorname{arctanh}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{a^2d^3\sqrt{a+ia\sinh(c+dx)}} \\
&+ \frac{3ix^3\operatorname{arctanh}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}} \\
&- \frac{10i\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} \\
&+ \frac{9ix^2\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} \\
&+ \frac{10i\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} \\
&- \frac{9ix^2\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} \\
&- \frac{9ix\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(3,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{2a^2d^3\sqrt{a+ia\sinh(c+dx)}} \\
&+ \frac{9ix\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(3,e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{2a^2d^3\sqrt{a+ia\sinh(c+dx)}} \\
&+ \frac{9i\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(4,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} \\
&- \frac{9i\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(4,e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} \\
&+ \frac{x^2\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{x\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{2a^2d^3\sqrt{a+ia\sinh(c+dx)}} \\
&+ \frac{3x^3\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{16a^2d\sqrt{a+ia\sinh(c+dx)}} + \frac{x^3\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}}
\end{aligned}
$$

output
```
-1/a^2/d^4/(a+I*a*sinh(d*x+c))^(1/2)+9/8*x^2/a^2/d^2/(a+I*a*sinh(d*x+c))^(
1/2)+10*I*x*arctanh(exp(1/2*c+3/4*I*Pi+1/2*d*x))*cosh(1/2*c+1/4*I*Pi+1/2*d
*x)/a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)+9/2*I*x*cosh(1/2*c+1/4*I*Pi+1/2*d*x)
*polylog(3,-exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)
+9/8*I*x^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,exp(1/2*c+3/4*I*Pi+1/2*d
*x))/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)-9*I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*po
lylog(4,-exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^4/(a+I*a*sinh(d*x+c))^(1/2)-10
*I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2
/d^4/(a+I*a*sinh(d*x+c))^(1/2)-3/8*I*x^3*arctanh(exp(1/2*c+3/4*I*Pi+1/2*d*
x))*cosh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)+10*I*cosh
(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,-exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^4/(
a+I*a*sinh(d*x+c))^(1/2)+9*I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(4,exp(1/
2*c+3/4*I*Pi+1/2*d*x))/a^2/d^4/(a+I*a*sinh(d*x+c))^(1/2)-9/2*I*x*cosh(1/2*
c+1/4*I*Pi+1/2*d*x)*polylog(3,exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^3/(a+I*a*
sinh(d*x+c))^(1/2)-9/8*I*x^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,-exp(1
/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)+1/4*x^2*sech(1/2
*c+1/4*I*Pi+1/2*d*x)^2/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)-1/2*x*tanh(1/2*c+
1/4*I*Pi+1/2*d*x)/a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)+3/16*x^3*tanh(1/2*c+1/
4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)+1/8*x^3*sech(1/2*c+1/4*I*P
i+1/2*d*x)^2*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/...
```

3.146.2 Mathematica [A] (verified)

Time = 2.99 (sec) , antiderivative size = 1200, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[x^3/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(-48*Cosh[(c + d*x)/2] + (8*I)*
c*Cosh[(c + d*x)/2] + 70*c^2*Cosh[(c + d*x)/2] - (11*I)*c^3*Cosh[(c + d*x)
/2] - (8*I)*(c + d*x)*Cosh[(c + d*x)/2] - 140*c*(c + d*x)*Cosh[(c + d*x)/2
] + (33*I)*c^2*(c + d*x)*Cosh[(c + d*x)/2] + 70*(c + d*x)^2*Cosh[(c + d*x)
/2] - (33*I)*c*(c + d*x)^2*Cosh[(c + d*x)/2] + (11*I)*(c + d*x)^3*Cosh[(c
+ d*x)/2] + 16*Cosh[(3*(c + d*x))/2] + (8*I)*c*Cosh[(3*(c + d*x))/2] - 18*
c^2*Cosh[(3*(c + d*x))/2] - (3*I)*c^3*Cosh[(3*(c + d*x))/2] - (8*I)*(c + d
*x)*Cosh[(3*(c + d*x))/2] + 36*c*(c + d*x)*Cosh[(3*(c + d*x))/2] + (9*I)*c
^2*(c + d*x)*Cosh[(3*(c + d*x))/2] - 18*(c + d*x)^2*Cosh[(3*(c + d*x))/2]
- (9*I)*c*(c + d*x)^2*Cosh[(3*(c + d*x))/2] + (3*I)*(c + d*x)^3*Cosh[(3*(c
 + d*x))/2] + (1 - I)*(-1)^(3/4)*(-160*c*ArcTanh[(-1)^(3/4)*E^((c + d*x)/2
)] + 6*c^3*ArcTanh[(-1)^(3/4)*E^((c + d*x)/2)] - 80*c*Log[1 - (-1)^(3/4)*E
^((c + d*x)/2)] + 3*c^3*Log[1 - (-1)^(3/4)*E^((c + d*x)/2)] - 80*d*x*Log[1
 - (-1)^(3/4)*E^((c + d*x)/2)] + 3*d^3*x^3*Log[1 - (-1)^(3/4)*E^((c + d*x)
/2)] + 80*c*Log[1 + (-1)^(3/4)*E^((c + d*x)/2)] - 3*c^3*Log[1 + (-1)^(3/4)
*E^((c + d*x)/2)] + 80*d*x*Log[1 + (-1)^(3/4)*E^((c + d*x)/2)] - 3*d^3*x^3
*Log[1 + (-1)^(3/4)*E^((c + d*x)/2)] - 2*(-80 + 9*d^2*x^2)*PolyLog[2, -((-
1)^(3/4)*E^((c + d*x)/2))] + 2*(-80 + 9*d^2*x^2)*PolyLog[2, (-1)^(3/4)*E^(
(c + d*x)/2)] + 72*d*x*PolyLog[3, -((-1)^(3/4)*E^((c + d*x)/2))] - 72*d*x*
PolyLog[3, (-1)^(3/4)*E^((c + d*x)/2)] - 144*PolyLog[4, -((-1)^(3/4)*E^...
```

3.146.3 Rubi [A] (verified)

Time = 2.71 (sec) , antiderivative size = 737, normalized size of antiderivative = 0.73, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.905$, Rules used = {3042, 3800, 3042, 4674, 3042, 4673, 3042, 4670, 2715, 2838, 4674, 3042, 4670, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x^3}{(a+a\sin(ic+idx))^{5/2}}dx$$

$$\Big\downarrow \text{3800}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\int x^3\text{sech}^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\int x^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{4674}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\int x\operatorname{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d^2}+\frac{3}{4}\int x^3\operatorname{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx+\frac{x^2\operatorname{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\right.}{}\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\int x\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{d^2}+\frac{3}{4}\int x^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx+\frac{x^2\operatorname{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\right.}{}\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\left(\frac{1}{2}\int x\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx+\frac{2\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{d^2}+\frac{3}{4}\int x^3\csc\left(\frac{ic}{2}+\right.\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\left(\frac{1}{2}\int x\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx+\frac{2\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{d^2}+\frac{3}{4}\int x^3\csc\left(\frac{ic}{2}+\right.\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{4670}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\left(\frac{1}{2}\left(\frac{2i\int\log\left(1-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}-\frac{2i\int\log\left(1+e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}+\frac{4ix\operatorname{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}\right)+\frac{2\operatorname{sech}\left(\frac{c}{2}+\right.}{d^2}\right.}{d^2}\right.}{4a^2\sqrt{}}$$

$$\Big\downarrow \text{2715}$$

3.146. $\displaystyle\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\left(\frac{1}{2}\left(\frac{4i\int e^{\frac{1}{4}(i\pi-2c)-\frac{dx}{2}}\log\left(1-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)de^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}}{d^2}-\frac{4i\int e^{\frac{1}{4}(i\pi-2c)-\frac{dx}{2}}\log\left(1+e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)de^{\frac{1}{4}(2c-\right.}{d^2}\right.\right.}{d^2}\right.$$

$\downarrow$ 2838

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\int x^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx-\frac{2\left(\frac{1}{2}\left(\frac{4ix\,\text{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}+\frac{4i\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d^2}-\frac{4i\,\text{Pol}}{}\right.\right.}{}\right.$$

$4a^2\sqrt{}$

$\downarrow$ 4674

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(-\frac{12\int x\,\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d^2}+\frac{1}{2}\int x^3\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx+\frac{6x^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x^3\tanh\left(\frac{c}{2}+\right.}{}\right.\right.$$

$\downarrow$ 3042

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(-\frac{12\int x\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx}{d^2}+\frac{1}{2}\int x^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx+\frac{6x^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x^3\tanh\left(\frac{c}{2}+\frac{d}{2}\right.}{}\right.\right.$$

$\downarrow$ 4670

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(-\frac{12\left(\frac{2i\int\log\left(1-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}-\frac{2i\int\log\left(1+e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}+\frac{4ix\,\text{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(\frac{6i}{}\right.\right.\right.$$

3.146. $\int\frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \text{2715}$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\Bigg(\frac{3}{4}\Bigg(-\frac{12\Bigg(\frac{4i\int e^{\frac{1}{4}(i\pi-2c)-\frac{dx}{2}}\log\left(1-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)de^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}}{d^2}-\frac{4i\int e^{\frac{1}{4}(i\pi-2c)-\frac{dx}{2}}\log\left(1+e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)de^{\frac{1}{4}(2c}}{d^2}}{d^2}$$

$$\Big\downarrow \text{2838}$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\Bigg(\frac{3}{4}\Bigg(\frac{1}{2}\Bigg(\frac{6i\int x^2\log\left(1-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}-\frac{6i\int x^2\log\left(1+e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}+\frac{4ix^3\text{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}$$

$$\Big\downarrow \text{3011}$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\Bigg(\frac{3}{4}\Bigg(\frac{1}{2}\Bigg(-\frac{6i\left(\frac{4\int x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}-\frac{2x^2\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d}\right)}{d}+\frac{6i\Bigg(\frac{4\int x\,\text{PolyLog}\left(2,e^{\frac{1}{4}(2c-i}\right.}{d}}{}$$

$$\Big\downarrow \text{7163}$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\Bigg(\frac{3}{4}\Bigg(\frac{1}{2}\Bigg(-\frac{6i\Bigg(\frac{4\left(\frac{2x\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d}-\frac{2\int\text{PolyLog}\left(3,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}\right)}{d}-\frac{2x^2\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d}}{d}$$

$$\Big\downarrow \text{2720}$$

3.146. $\displaystyle\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{x^2\operatorname{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\operatorname{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{2d}-\frac{2\left(\frac{1}{2}\left(\frac{4ix\operatorname{arctanh}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d}+\frac{4i\operatorname{Pol}\cdots}{\cdots}\right.\right.}{\cdots}\right.\cdots$$

$\big\downarrow$ 7143

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{4ix^3\operatorname{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}-\frac{6i\left(\frac{4\left(\frac{2x\operatorname{PolyLog}\left(3,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d}-\frac{4\operatorname{PolyLog}\left(4,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d^2}\right)}{d}-\cdots\right.}{d}\cdots\right.\right.\right.$$

input
```
Int[x^3/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

3.146. $\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

output
```
(Cosh[c/2 + (I/4)*Pi + (d*x)/2]*((x^2*Sech[c/2 + (I/4)*Pi + (d*x)/2]^3)/d^
2 + (x^3*Sech[c/2 + (I/4)*Pi + (d*x)/2]^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/
(2*d) - (2*((((4*I)*x*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)])/d + ((4*I)*Po
lyLog[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/d^2 - ((4*I)*PolyLog[2, E^((2*c -
 I*Pi)/4 + (d*x)/2)])/d^2)/2 + (2*Sech[c/2 + (I/4)*Pi + (d*x)/2])/d^2 + (x
*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/d^2 +
(3*((-12*(((4*I)*x*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)])/d + ((4*I)*PolyL
og[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/d^2 - ((4*I)*PolyLog[2, E^((2*c - I*
Pi)/4 + (d*x)/2)])/d^2))/d^2 + (((4*I)*x^3*ArcTanh[E^((2*c - I*Pi)/4 + (d*
x)/2)])/d - ((6*I)*((-2*x^2*PolyLog[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/d +
 (4*((2*x*PolyLog[3, -E^((2*c - I*Pi)/4 + (d*x)/2)])/d - (4*PolyLog[4, -E^
((2*c - I*Pi)/4 + (d*x)/2)])/d^2))/d))/d + ((6*I)*((-2*x^2*PolyLog[2, E^((
2*c - I*Pi)/4 + (d*x)/2)])/d + (4*((2*x*PolyLog[3, E^((2*c - I*Pi)/4 + (d*
x)/2)])/d - (4*PolyLog[4, E^((2*c - I*Pi)/4 + (d*x)/2)])/d^2))/d))/d)/2 +
(6*x^2*Sech[c/2 + (I/4)*Pi + (d*x)/2])/d^2 + (x^3*Sech[c/2 + (I/4)*Pi + (d
*x)/2]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/4))/(4*a^2*Sqrt[a + I*a*Sinh[c
+ d*x]])
```

3.146.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.146. $\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.146.4 Maple [F]

$$\int \frac{x^3}{(a+ia\sinh(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(x^3/(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
int(x^3/(a+I*a*sinh(d*x+c))^(5/2),x)
```

3.146.5 Fricas [F]

$$\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x^3}{(i\,a\sinh(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/8*(8*(a^3*d^4*e^(4*d*x + 4*c) - 4*I*a^3*d^4*e^(3*d*x + 3*c) - 6*a^3*d^4*
e^(2*d*x + 2*c) + 4*I*a^3*d^4*e^(d*x + c) + a^3*d^4)*integral(1/16*(-3*I*d
^2*x^3 + 80*I*x)*sqrt(1/2*I*a*e^(-d*x - c))*e^(d*x + c)/(a^3*d^2*e^(d*x +
c) - I*a^3*d^2), x) + ((-3*I*d^3*x^3 - 18*I*d^2*x^2 + 8*I*d*x + 16*I)*e^(4
*d*x + 4*c) - (11*d^3*x^3 + 70*d^2*x^2 - 8*d*x - 48)*e^(3*d*x + 3*c) + (-1
1*I*d^3*x^3 + 70*I*d^2*x^2 + 8*I*d*x - 48*I)*e^(2*d*x + 2*c) - (3*d^3*x^3
- 18*d^2*x^2 - 8*d*x + 16)*e^(d*x + c))*sqrt(1/2*I*a*e^(-d*x - c)))/(a^3*d
^4*e^(4*d*x + 4*c) - 4*I*a^3*d^4*e^(3*d*x + 3*c) - 6*a^3*d^4*e^(2*d*x + 2*
c) + 4*I*a^3*d^4*e^(d*x + c) + a^3*d^4)
```

3.146.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(a+I*a*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.146.7 Maxima [F]

$$\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x^3}{\left(i\,a\sinh\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

3.146.8 Giac [F]

$$\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x^3}{\left(i\,a\sinh\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(x^3/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x^3}{\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(x^3/(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

output
```
int(x^3/(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

3.147 $\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

3.147.1 Optimal result

Integrand size = 21, antiderivative size = 689

$$\begin{aligned}
\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx &= \frac{3x}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}} \\
&- \frac{5\arctan\left(\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{3a^2d^3\sqrt{a+ia\sinh(c+dx)}} \\
&+ \frac{3ix^2\text{arctanh}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}} \\
&+ \frac{3ix\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}} \\
&- \frac{3ix\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\text{PolyLog}\left(2,e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}} \\
&- \frac{3i\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\text{PolyLog}\left(3,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{2a^2d^3\sqrt{a+ia\sinh(c+dx)}} \\
&+ \frac{3i\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\text{PolyLog}\left(3,e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{2a^2d^3\sqrt{a+ia\sinh(c+dx)}} \\
&+ \frac{x\,\text{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{6a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{6a^2d^3\sqrt{a+ia\sinh(c+dx)}} \\
&+ \frac{3x^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{16a^2d\sqrt{a+ia\sinh(c+dx)}} + \frac{x^2\text{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}}
\end{aligned}$$

output
```
3/4*x/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)-5/3*arctan(sinh(1/2*c+1/4*I*Pi+1/2
*d*x))*cosh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)-3/8*
I*x^2*arctanh(exp(1/2*c+3/4*I*Pi+1/2*d*x))*cosh(1/2*c+1/4*I*Pi+1/2*d*x)/a^
2/d/(a+I*a*sinh(d*x+c))^(1/2)+3/4*I*x*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog
(2,exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)-3/4*I*x*
cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,-exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d
^2/(a+I*a*sinh(d*x+c))^(1/2)-3/2*I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(3,
exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)+3/2*I*cosh(
1/2*c+1/4*I*Pi+1/2*d*x)*polylog(3,-exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^3/(a
+I*a*sinh(d*x+c))^(1/2)+1/6*x*sech(1/2*c+1/4*I*Pi+1/2*d*x)^2/a^2/d^2/(a+I*
a*sinh(d*x+c))^(1/2)-1/6*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d^3/(a+I*a*sinh(
d*x+c))^(1/2)+3/16*x^2*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+
c))^(1/2)+1/8*x^2*sech(1/2*c+1/4*I*Pi+1/2*d*x)^2*tanh(1/2*c+1/4*I*Pi+1/2*d
*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)
```

3.147.2 Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 482, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(4dx(4+3idx)\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right.\right.}{}$$

input
```
Integrate[x^2/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(4*d*x*(4 + (3*I)*d*x)*(Cosh[(c
 + d*x)/2] + I*Sinh[(c + d*x)/2]) + (-8*I + 36*d*x + (9*I)*d^2*x^2)*(Cosh[
(c + d*x)/2] + I*Sinh[(c + d*x)/2])^3 - (1/2 - I/2)*(-1)^(3/4)*(-160*ArcTa
nh[(-1)^(3/4)*E^((c + d*x)/2)] + 18*c^2*ArcTanh[(-1)^(3/4)*E^((c + d*x)/2)
] + 9*c^2*Log[1 - (-1)^(3/4)*E^((c + d*x)/2)] - 9*d^2*x^2*Log[1 - (-1)^(3/
4)*E^((c + d*x)/2)] - 9*c^2*Log[1 + (-1)^(3/4)*E^((c + d*x)/2)] + 9*d^2*x^
2*Log[1 + (-1)^(3/4)*E^((c + d*x)/2)] + 36*d*x*PolyLog[2, -((-1)^(3/4)*E^(
(c + d*x)/2))] - 36*d*x*PolyLog[2, (-1)^(3/4)*E^((c + d*x)/2)] - 72*PolyLo
g[3, -((-1)^(3/4)*E^((c + d*x)/2))] + 72*PolyLog[3, (-1)^(3/4)*E^((c + d*x
)/2)])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^4 + 24*d^2*x^2*Sinh[(c +
d*x)/2] + 2*(-8 + 9*d^2*x^2)*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2*S
inh[(c + d*x)/2]))/(48*d^3*(a + I*a*Sinh[c + d*x])^(5/2))
```

3.147.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.70, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 3800, 3042, 4674, 3042, 4255, 3042, 4257, 4674, 3042, 4257, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x^2}{(a+a\sin(ic+idx))^{5/2}}dx$$

$$\Big\downarrow \text{3800}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\int x^2\text{sech}^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\int x^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{4674}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\int \text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{3d^2}+\frac{3}{4}\int x^2\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx+\frac{2x\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{3d^2}+\frac{x^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\right.}{}\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\int\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx}{3d^2}+\frac{3}{4}\int x^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx+\frac{2x\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{3d^2}+\frac{x^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right.}{}\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\left(\frac{1}{2}\int \text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx+\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{3d^2}+\frac{3}{4}\int x^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx+\frac{2x\text{s}}{}\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(-\frac{2\left(\frac{1}{2}\int \csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx+\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{3d^2}+\frac{3}{4}\int x^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx+\frac{2x\text{se}}{}\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$\downarrow$ 4257

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\int x^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx-\frac{2\left(\frac{\arctan\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{d}+\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{3d^2}+\frac{2x\text{sec}}{}\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$\downarrow$ 4674

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(-\frac{4\int \text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d^2}+\frac{1}{2}\int x^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx+\frac{4x\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\right.}{}\right.\right.}{4a}$$

$\downarrow$ 3042

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(-\frac{4\int \csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx}{d^2}+\frac{1}{2}\int x^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx+\frac{4x\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i}{}\right.}{}\right.\right.}{4a}$$

$\downarrow$ 4257

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\int x^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx-\frac{8\arctan\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{d^3}+\frac{4x\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x^2\tanh\left(\frac{c}{2}+\frac{dx}{2}\right.}{}\right.\right.}{4}$$

$\downarrow$ 4670

3.147. $\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{4i\int x\log\left(1-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}-\frac{4i\int x\log\left(1+e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}+\frac{4ix^2\text{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}\right)\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{4i\left(\frac{2\int \text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}-\frac{2x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d}\right)}{d}+\frac{4i\left(\frac{2\int \text{PolyLog}\left(2,e^{\frac{1}{4}(2c-i\pi)+\cdots}\right.}{d}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{4i\left(\frac{4\int e^{\frac{1}{4}(i\pi-2c)-\frac{dx}{2}}\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)de^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}}{d^2}-\frac{2x\,\text{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d}\right)}{d}+-\frac{4i\cdots}{}\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(-\frac{8\arctan\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{d^3}+\frac{1}{2}\left(\frac{4ix^2\text{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}-\frac{4i\left(\frac{4\,\text{PolyLog}\left(3,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d^2}-\cdots\right.}{d}\right.\right.\right.$$

input
```
Int[x^2/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

3.147. $\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

output
```
(Cosh[c/2 + (I/4)*Pi + (d*x)/2]*((2*x*Sech[c/2 + (I/4)*Pi + (d*x)/2]^3)/(3
*d^2) + (x^2*Sech[c/2 + (I/4)*Pi + (d*x)/2]^3*Tanh[c/2 + (I/4)*Pi + (d*x)/
2])/(2*d) - (2*(ArcTan[Sinh[c/2 + (I/4)*Pi + (d*x)/2]]/d + (Sech[c/2 + (I/
4)*Pi + (d*x)/2]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/(3*d^2) + (3*((-8*Arc
Tan[Sinh[c/2 + (I/4)*Pi + (d*x)/2]])/d^3 + (((4*I)*x^2*ArcTanh[E^((2*c - I
*Pi)/4 + (d*x)/2)])/d - ((4*I)*((-2*x*PolyLog[2, -E^((2*c - I*Pi)/4 + (d*x
)/2)])/d + (4*PolyLog[3, -E^((2*c - I*Pi)/4 + (d*x)/2)])/d^2))/d + ((4*I)*
((-2*x*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/d + (4*PolyLog[3, E^((2*c
 - I*Pi)/4 + (d*x)/2)])/d^2))/d)/2 + (4*x*Sech[c/2 + (I/4)*Pi + (d*x)/2])/
d^2 + (x^2*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/
d))/4))/(4*a^2*Sqrt[a + I*a*Sinh[c + d*x]])
```

3.147.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.147.4 Maple [F]

$$\int \frac{x^2}{(a + ia \sinh(dx + c))^{\frac{5}{2}}} dx$$

input
```
int(x^2/(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
int(x^2/(a+I*a*sinh(d*x+c))^(5/2),x)
```

3.147.5 Fricas [F]

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x^2}{(i\,a\sinh{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/24*(24*(a^3*d^3*e^(4*d*x + 4*c) - 4*I*a^3*d^3*e^(3*d*x + 3*c) - 6*a^3*d^
3*e^(2*d*x + 2*c) + 4*I*a^3*d^3*e^(d*x + c) + a^3*d^3)*integral(1/48*(-9*I
*d^2*x^2 + 80*I)*sqrt(1/2*I*a*e^(-d*x - c))*e^(d*x + c)/(a^3*d^2*e^(d*x +
c) - I*a^3*d^2), x) + ((-9*I*d^2*x^2 - 36*I*d*x + 8*I)*e^(4*d*x + 4*c) - (
33*d^2*x^2 + 140*d*x - 8)*e^(3*d*x + 3*c) + (-33*I*d^2*x^2 + 140*I*d*x + 8
*I)*e^(2*d*x + 2*c) - (9*d^2*x^2 - 36*d*x - 8)*e^(d*x + c))*sqrt(1/2*I*a*e
^(-d*x - c)))/(a^3*d^3*e^(4*d*x + 4*c) - 4*I*a^3*d^3*e^(3*d*x + 3*c) - 6*a
^3*d^3*e^(2*d*x + 2*c) + 4*I*a^3*d^3*e^(d*x + c) + a^3*d^3)
```

3.147.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(a+I*a*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.147.7 Maxima [F]

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x^2}{(i\,a\sinh{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

3.147.8 Giac [F]

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x^2}{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x^2}{{\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

input
```
int(x^2/(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

output
```
int(x^2/(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

3.148 $\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

3.148.1 Optimal result

Integrand size = 19, antiderivative size = 416

$$\begin{aligned}\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx &= \frac{3}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}}\\ &+\frac{3ix\operatorname{arctanh}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}}\\ &+\frac{3i\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}}\\ &-\frac{3i\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}}+\frac{\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{12a^2d^2\sqrt{a+ia\sinh(c+dx)}}\\ &+\frac{3x\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{16a^2d\sqrt{a+ia\sinh(c+dx)}}+\frac{x\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}}\end{aligned}$$

output
```
3/8/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)-3/8*I*x*arctanh(exp(1/2*c+3/4*I*Pi+1
/2*d*x))*cosh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)+3/8*
I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/
d^2/(a+I*a*sinh(d*x+c))^(1/2)-3/8*I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2
,-exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)+1/12*sech
(1/2*c+1/4*I*Pi+1/2*d*x)^2/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)+3/16*x*tanh(1
/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)+1/8*x*sech(1/2*c+1/
4*I*Pi+1/2*d*x)^2*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(
1/2)
```

3.148.2 Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.81

$$\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(4(2+3idx)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\right.\right.}{}$$

input
```
Integrate[x/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(4*(2 + (3*I)*d*x)*(Cosh[(c + d
*x)/2] + I*Sinh[(c + d*x)/2]) + 9*(2 + I*d*x)*(Cosh[(c + d*x)/2] + I*Sinh[
(c + d*x)/2])^3 + (9 - 9*I)*(-1)^(3/4)*(I*c*ArcTan[(-1)^(1/4)*E^((c + d*x)
/2)] + ((c + d*x)*Log[1 - (-1)^(3/4)*E^((c + d*x)/2)])/2 - ((c + d*x)*Log[
1 + (-1)^(3/4)*E^((c + d*x)/2)])/2 - PolyLog[2, -((-1)^(3/4)*E^((c + d*x)/
2))] + PolyLog[2, (-1)^(3/4)*E^((c + d*x)/2)])*(Cosh[(c + d*x)/2] + I*Sinh
[(c + d*x)/2])^4 + 24*d*x*Sinh[(c + d*x)/2] + 18*d*x*(Cosh[(c + d*x)/2] +
I*Sinh[(c + d*x)/2])^2*Sinh[(c + d*x)/2]))/(48*d^2*(a + I*a*Sinh[c + d*x])
^(5/2))
```

3.148.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.72, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 3800, 3042, 4673, 3042, 4673, 3042, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{x}{(a+a\sin(ic+idx))^{5/2}}dx$$

$$\Big\downarrow \; 3800$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\int x\text{sech}^5\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\int x\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^5dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\int x\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx+\frac{\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{3d^2}+\frac{x\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{2d}\right)}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\int x\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^3dx+\frac{\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{3d^2}+\frac{x\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{2d}\right)}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\int x\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx+\frac{2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)+\frac{\text{sech}}{}\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\int x\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)dx+\frac{2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d^2}+\frac{x\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)+\frac{\text{sech}}{}\right.}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$\Big\downarrow \text{4670}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{2i\int\log\left(1-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}-\frac{2i\int\log\left(1+e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)dx}{d}+\frac{4ix\text{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}\right)+\right.\right.}{4a^2\sqrt{a+ia\sin}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{4i\int e^{\frac{1}{4}(i\pi-2c)-\frac{dx}{2}}\log\left(1-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)de^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}}{d^2}-\frac{4i\int e^{\frac{1}{4}(i\pi-2c)-\frac{dx}{2}}\log\left(1+e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)de}{d^2}\right.\right.\right.}{}$$

$$\Big\downarrow \text{2838}$$

3.148. $\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{4ix\operatorname{arctanh}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{d}+\frac{4i\operatorname{PolyLog}\left(2,-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d^2}-\frac{4i\operatorname{PolyLog}\left(2,e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{d^2}\right)+\right.\right.}{4a^2\sqrt{a+ia\sin}}$$

input
```
Int[x/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
(Cosh[c/2 + (I/4)*Pi + (d*x)/2]*(Sech[c/2 + (I/4)*Pi + (d*x)/2]^3/(3*d^2)
+ (x*Sech[c/2 + (I/4)*Pi + (d*x)/2]^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(2*d
) + (3*((((4*I)*x*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)])/d + ((4*I)*PolyLo
g[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/d^2 - ((4*I)*PolyLog[2, E^((2*c - I*P
i)/4 + (d*x)/2)])/d^2)/2 + (2*Sech[c/2 + (I/4)*Pi + (d*x)/2])/d^2 + (x*Sec
h[c/2 + (I/4)*Pi + (d*x)/2]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/4))/(4*a^2
*Sqrt[a + I*a*Sinh[c + d*x]])
```

3.148.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

3.148.4 Maple [F]

$$\int \frac{x}{(a+ia\sinh(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
int(x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

3.148.5 Fricas [F]

$$\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x}{(i\,a\sinh(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/24*(24*(a^3*d^2*e^(4*d*x + 4*c) - 4*I*a^3*d^2*e^(3*d*x + 3*c) - 6*a^3*d^
2*e^(2*d*x + 2*c) + 4*I*a^3*d^2*e^(d*x + c) + a^3*d^2)*integral(-3/16*I*sq
rt(1/2*I*a*e^(-d*x - c))*x*e^(d*x + c)/(a^3*e^(d*x + c) - I*a^3), x) - (9*
(I*d*x + 2*I)*e^(4*d*x + 4*c) + (33*d*x + 70)*e^(3*d*x + 3*c) - (-33*I*d*x
 + 70*I)*e^(2*d*x + 2*c) + 9*(d*x - 2)*e^(d*x + c))*sqrt(1/2*I*a*e^(-d*x -
 c)))/(a^3*d^2*e^(4*d*x + 4*c) - 4*I*a^3*d^2*e^(3*d*x + 3*c) - 6*a^3*d^2*e
^(2*d*x + 2*c) + 4*I*a^3*d^2*e^(d*x + c) + a^3*d^2)
```

3.148.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x/(a+I*a*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.148.7 Maxima [F]

$$\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx=\int \frac{x}{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

3.148.8 Giac [F]

$$\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx=\int \frac{x}{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(x/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{x}{\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(x/(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

output
```
int(x/(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

3.149 $\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx$

3.149.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx = \operatorname{Int}\left(\frac{1}{x(a+ia\sinh(c+dx))^{5/2}},x\right)$$

output
```
Unintegrable(1/x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

3.149.2 Mathematica [N/A]

Not integrable

Time = 36.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx$$

input
```
Integrate[1/(x*(a + I*a*Sinh[c + d*x])^(5/2)),x]
```

output
```
Integrate[1/(x*(a + I*a*Sinh[c + d*x])^(5/2)), x]
```

3.149.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{x(a+a\sin(ic+idx))^{5/2}}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}dx$$

input
```
Int[1/(x*(a + I*a*Sinh[c + d*x])^(5/2)),x]
```

output
```
$Aborted
```

3.149.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.149.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\left(a+ia\sinh\left(dx+c\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
int(1/x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

3.149.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 396, normalized size of antiderivative = 18.86

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(i\,a\sinh\left(dx+c\right)+a\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/24*(24*(a^3*d^4*x^4*e^(4*d*x + 4*c) - 4*I*a^3*d^4*x^4*e^(3*d*x + 3*c) -
6*a^3*d^4*x^4*e^(2*d*x + 2*c) + 4*I*a^3*d^4*x^4*e^(d*x + c) + a^3*d^4*x^4)
*integral(1/48*(-9*I*d^4*x^4 + 80*I*d^2*x^2 - 384*I)*sqrt(1/2*I*a*e^(-d*x
- c))*e^(d*x + c)/(a^3*d^4*x^5*e^(d*x + c) - I*a^3*d^4*x^5), x) + ((-9*I*d
^3*x^3 + 18*I*d^2*x^2 + 8*I*d*x - 48*I)*e^(4*d*x + 4*c) - (33*d^3*x^3 - 70
*d^2*x^2 - 8*d*x + 144)*e^(3*d*x + 3*c) + (-33*I*d^3*x^3 - 70*I*d^2*x^2 +
8*I*d*x + 144*I)*e^(2*d*x + 2*c) - (9*d^3*x^3 + 18*d^2*x^2 - 8*d*x - 48)*e
^(d*x + c))*sqrt(1/2*I*a*e^(-d*x - c)))/(a^3*d^4*x^4*e^(4*d*x + 4*c) - 4*I
*a^3*d^4*x^4*e^(3*d*x + 3*c) - 6*a^3*d^4*x^4*e^(2*d*x + 2*c) + 4*I*a^3*d^4
*x^4*e^(d*x + c) + a^3*d^4*x^4)
```

3.149.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(a+I*a*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.149.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(i\,a\sinh{(dx+c)}+a)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*sinh(d*x + c) + a)^(5/2)*x), x)
```

3.149.8 Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(i\,a\sinh{(dx+c)}+a)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((I*a*sinh(d*x + c) + a)^(5/2)*x), x)
```

3.149.9 Mupad [N/A]

Not integrable

Time = 2.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx=\int \frac{1}{x\,\left(a+a\,\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(1/(x*(a + a*sinh(c + d*x)*1i)^(5/2)),x)
```

output
```
int(1/(x*(a + a*sinh(c + d*x)*1i)^(5/2)), x)
```

3.150 $\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx$

3.150.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx=\operatorname{Int}\left(\frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x},x\right)$$

output
```
Unintegrable((a+I*a*sinh(f*x+e))^(1/3)/x,x)
```

3.150.2 Mathematica [N/A]

Not integrable

Time = 4.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx=\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx$$

input
```
Integrate[(a + I*a*Sinh[e + f*x])^(1/3)/x,x]
```

output
```
Integrate[(a + I*a*Sinh[e + f*x])^(1/3)/x, x]
```

3.150.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sqrt[3]{a+a\sin(ie+ifx)}}{x}dx$$
$$\Big\downarrow \text{ 3807}$$
$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}dx$$

input
```
Int[(a + I*a*Sinh[e + f*x])^(1/3)/x,x]
```

output
```
$Aborted
```

3.150.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.150.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a + ia\sinh(fx + e))^{\frac{1}{3}}}{x} dx$$

input
```
int((a+I*a*sinh(f*x+e))^(1/3)/x,x)
```

output
```
int((a+I*a*sinh(f*x+e))^(1/3)/x,x)
```

3.150.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{a + ia\sinh(e + fx)}}{x} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/3)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.150.6 Sympy [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt[3]{a + ia\sinh(e + fx)}}{x} dx = \int \frac{\sqrt[3]{ia(\sinh(e + fx) - i)}}{x} dx$$

input
```
integrate((a+I*a*sinh(f*x+e))**(1/3)/x,x)
```

output
```
Integral((I*a*(sinh(e + f*x) - I))**(1/3)/x, x)
```

3.150.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt[3]{a + ia\sinh(e + fx)}}{x}\, dx = \int \frac{(i\, a \sinh\left(fx + e\right) + a)^{\frac{1}{3}}}{x}\, dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/3)/x,x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(1/3)/x, x)
```

3.150.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt[3]{a + ia\sinh(e + fx)}}{x}\, dx = \int \frac{(i\, a \sinh\left(fx + e\right) + a)^{\frac{1}{3}}}{x}\, dx$$

input
```
integrate((a+I*a*sinh(f*x+e))^(1/3)/x,x, algorithm="giac")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^(1/3)/x, x)
```

3.150.9 Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt[3]{a + ia\sinh(e + fx)}}{x}\, dx = \int \frac{\left(a + a \sinh(e + f\, x)\ 1\mathrm{i}\right)^{1/3}}{x}\, dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^(1/3)/x,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^(1/3)/x, x)
```

3.151 $\int (c+dx)^m (a+ia\sinh(e+fx))^n \, dx$

3.151.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (c+dx)^m (a+ia\sinh(e+fx))^n \, dx = \text{Int}((c+dx)^m (a+ia\sinh(e+fx))^n, x)$$

output
```
Unintegrable((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x)
```

3.151.2 Mathematica [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (c+dx)^m (a+ia\sinh(e+fx))^n \, dx = \int (c+dx)^m (a+ia\sinh(e+fx))^n \, dx$$

input
```
Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^n,x]
```

output
```
Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^n, x]
```

3.151.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m(a+a\sin(ie+ifx))^ndx$$

$$\Big\downarrow \text{3807}$$

$$\int (c+dx)^m(a+ia\sinh(e+fx))^ndx$$

input
```
Int[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^n,x]
```

output
```
$Aborted
```

3.151.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.151.4 Maple [N/A] (verified)

Not integrable

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (dx+c)^m (a+ia\sinh(fx+e))^n\,dx$$

input
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x)
```

output
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x)
```

3.151.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx = \int (dx+c)^m(i\,a\sinh(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*(1/2*(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*e^
(-f*x - e))^n, x)
```

3.151.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*(a+I*a*sinh(f*x+e))**n,x)
```

output
```
Timed out
```

3.151.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx=\int (dx+c)^m(i\,a\sinh{(fx+e)}+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*(I*a*sinh(f*x + e) + a)^n, x)
```

3.151.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx=\int (dx+c)^m(i\,a\sinh{(fx+e)}+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*(I*a*sinh(f*x + e) + a)^n, x)
```

3.151.9 Mupad [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx=\int {\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^n\,{\left(c+d\,x\right)}^m \,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^n*(c + d*x)^m,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^n*(c + d*x)^m, x)
```

3.152 $\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx$

3.152.1 Optimal result

Integrand size = 23, antiderivative size = 410

$$\begin{aligned}
&\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx \\
&= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)} - \frac{i3^{-1-m}a^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)}{8f} \\
&\quad - \frac{3\ 2^{-3-m}a^3e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f} \\
&\quad + \frac{15ia^3e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{8f} \\
&\quad + \frac{15ia^3e^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{8f} \\
&\quad + \frac{3\ 2^{-3-m}a^3e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{f} \\
&\quad - \frac{i3^{-1-m}a^3e^{-3e+\frac{3cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3f(c+dx)}{d}\right)}{8f}
\end{aligned}$$

output
```
5/2*a^3*(d*x+c)^(1+m)/d/(1+m)-1/8*I*3^(-1-m)*a^3*exp(3*e-3*c*f/d)*(d*x+c)^
m*GAMMA(1+m,-3*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)-3*2^(-3-m)*a^3*exp(2*e-2*
c*f/d)*(d*x+c)^m*GAMMA(1+m,-2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+15/8*I*a^3
*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+15/8*
I*a^3*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)+3
*2^(-3-m)*a^3*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+m,2*f*(d*x+c)/d)/f/((f*(
d*x+c)/d)^m)-1/8*I*3^(-1-m)*a^3*exp(-3*e+3*c*f/d)*(d*x+c)^m*GAMMA(1+m,3*f*
(d*x+c)/d)/f/((f*(d*x+c)/d)^m)
```

3.152.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx \\
&= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)} - \frac{i3^{-1-m}a^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)}{8f} \\
&\quad - \frac{3\,2^{-3-m}a^3e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f} \\
&\quad + \frac{15ia^3e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{8f} \\
&\quad + \frac{15ia^3e^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{8f} \\
&\quad + \frac{3\,2^{-3-m}a^3e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{f} \\
&\quad - \frac{i3^{-1-m}a^3e^{-3e+\frac{3cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3f(c+dx)}{d}\right)}{8f}
\end{aligned}$$

input
```
Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^3,x]
```

output
```
(5*a^3*(c + d*x)^(1 + m))/(2*d*(1 + m)) - ((I/8)*3^(-1 - m)*a^3*E^(3*e - (
3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(c + d*x))/d])/(f*(-((f*(c + d*x)
)/d))^m) - (3*2^(-3 - m)*a^3*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m,
(-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (((15*I)/8)*a^3*E^(e - (
c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/
d))^m) + (((15*I)/8)*a^3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c +
 d*x))/d])/(f*((f*(c + d*x))/d)^m) + (3*2^(-3 - m)*a^3*E^(-2*e + (2*c*f)/d
)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - (
(I/8)*3^(-1 - m)*a^3*E^(-3*e + (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c
 + d*x))/d])/(f*((f*(c + d*x))/d)^m)
```

3.152.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 3799, 25, 25, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)^m(a+a\sin(ie+ifx))^3dx$$

$$\Big\downarrow \; 3799$$

$$8a^3\int -(c+dx)^m\sinh^6\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \; 25$$

$$-8a^3\int -(c+dx)^m\cosh^6\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \; 25$$

$$8a^3\int (c+dx)^m\cosh^6\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)dx$$

$$\Big\downarrow \; 3042$$

$$8a^3\int (c+dx)^m\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^6dx$$

$$\Big\downarrow \text{3793}$$

$$8a^3 \int \left(-\frac{3}{16}\cosh(2e+2fx)(c+dx)^m + \frac{15}{32}i\sinh(e+fx)(c+dx)^m - \frac{1}{32}i\sinh(3e+3fx)(c+dx)^m + \frac{5}{16}(c+dx\right.$$

$$\Big\downarrow \text{2009}$$

$$8a^3\left(-\frac{i3^{-m-1}e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3f(c+dx)}{d}\right)}{64f} - \frac{3\ 2^{-m-6}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}}{f}\right.$$

input
```
Int[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^3,x]
```

output
```
8*a^3*((5*(c + d*x)^(1 + m))/(16*d*(1 + m)) - ((I/64)*3^(-1 - m)*E^(3*e -
(3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(c + d*x))/d])/(f*(-((f*(c + d*x
))/d))^m) - (3*2^(-6 - m)*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2
*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (((15*I)/64)*E^(e - (c*f)/d
)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m)
 + (((15*I)/64)*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d]
)/(f*((f*(c + d*x))/d)^m) + (3*2^(-6 - m)*E^(-2*e + (2*c*f)/d)*(c + d*x)^m
*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - ((I/64)*3^(-1
- m)*E^(-3*e + (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c + d*x))/d])/(f*
((f*(c + d*x))/d)^m))
```

3.152.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.152.4 Maple [F]

$$\int (dx+c)^m (a+ia\sinh(fx+e))^3\,dx$$

input
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x)
```

output
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x)
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.91

$$\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx$$
$$= \frac{(-i\,a^3dm - i\,a^3d)e^{\left(-\frac{dm\log\left(\frac{3f}{d}\right)+3\,de-3\,cf}{d}\right)}\Gamma\left(m+1, \frac{3\,(dfx+cf)}{d}\right) + 9\,(a^3dm + a^3d)e^{\left(-\frac{dm\log\left(\frac{2f}{d}\right)+2\,de-2\,cf}{d}\right)}\Gamma\left(m-\right.}{}$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/24*((-I*a^3*d*m - I*a^3*d)*e^(-(d*m*log(3*f/d) + 3*d*e - 3*c*f)/d)*gamma
(m + 1, 3*(d*f*x + c*f)/d) + 9*(a^3*d*m + a^3*d)*e^(-(d*m*log(2*f/d) + 2*d
*e - 2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) - 45*(-I*a^3*d*m - I*a^3*d)
*e^(-(d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x + c*f)/d) - 45*(-I*
a^3*d*m - I*a^3*d)*e^(-(d*m*log(-f/d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x
 + c*f)/d) - 9*(a^3*d*m + a^3*d)*e^(-(d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*
gamma(m + 1, -2*(d*f*x + c*f)/d) + (-I*a^3*d*m - I*a^3*d)*e^(-(d*m*log(-3*
f/d) - 3*d*e + 3*c*f)/d)*gamma(m + 1, -3*(d*f*x + c*f)/d) + 60*(a^3*d*f*x
+ a^3*c*f)*(d*x + c)^m)/(d*f*m + d*f)
```

3.152.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+I*a*sinh(f*x+e))**3,x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.91

$$\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx =$$
$$-\frac{1}{8}i\left(\frac{(dx+c)^{m+1}e^{\left(-3e+\frac{3cf}{d}\right)}E_{-m}\left(\frac{3(dx+c)f}{d}\right)}{d}-\frac{3(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{3(dx+c)^{m+1}e^{(}}{}\right.$$
$$+\frac{3}{4}\left(\frac{(dx+c)^{m+1}e^{\left(-2e+\frac{2cf}{d}\right)}E_{-m}\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(2e-\frac{2cf}{d}\right)}E_{-m}\left(-\frac{2(dx+c)f}{d}\right)}{d}+\frac{2(dx+c)^{m+1}}{d(m+1)}\right)$$
$$+\frac{3}{2}i\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a^3$$
$$+\frac{(dx+c)^{m+1}a^3}{d(m+1)}$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/8*I*((d*x + c)^(m + 1)*e^(-3*e + 3*c*f/d)*exp_integral_e(-m, 3*(d*x + c
)*f/d)/d - 3*(d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)
*f/d)/d + 3*(d*x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*
f/d)/d - (d*x + c)^(m + 1)*e^(3*e - 3*c*f/d)*exp_integral_e(-m, -3*(d*x +
c)*f/d)/d)*a^3 + 3/4*((d*x + c)^(m + 1)*e^(-2*e + 2*c*f/d)*exp_integral_e(
-m, 2*(d*x + c)*f/d)/d + (d*x + c)^(m + 1)*e^(2*e - 2*c*f/d)*exp_integral_
e(-m, -2*(d*x + c)*f/d)/d + 2*(d*x + c)^(m + 1)/(d*(m + 1)))*a^3 + 3/2*I*(
(d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d - (d*
x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a^3 + (
d*x + c)^(m + 1)*a^3/(d*(m + 1))
```

3.152.8 Giac [F]

$$\int (c+dx)^m (a+ia\sinh(e+fx))^3\,dx = \int (i\,a\sinh(fx+e)+a)^3 (dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^3*(d*x + c)^m, x)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m (a+ia\sinh(e+fx))^3\,dx = \int (a+a\sinh(e+f\,x)\ 1\mathrm{i})^3\,(c+d\,x)^m\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^3*(c + d*x)^m,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^3*(c + d*x)^m, x)
```

3.153 $\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx$

3.153.1 Optimal result

Integrand size = 23, antiderivative size = 268

$$\begin{aligned}
&\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx\\
&=\frac{3a^2(c+dx)^{1+m}}{2d(1+m)}-\frac{2^{-3-m}a^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f}\\
&\quad+\frac{ia^2e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{f}\\
&\quad+\frac{ia^2e^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{f}\\
&\quad+\frac{2^{-3-m}a^2e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{f}
\end{aligned}$$

output
```
3/2*a^2*(d*x+c)^(1+m)/d/(1+m)-2^(-3-m)*a^2*exp(2*e-2*c*f/d)*(d*x+c)^m*GAMM
A(1+m,-2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+I*a^2*exp(e-c*f/d)*(d*x+c)^m*GA
MMA(1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+I*a^2*exp(-e+c*f/d)*(d*x+c)^m*G
AMMA(1+m,f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)+2^(-3-m)*a^2*exp(-2*e+2*c*f/d)*(
d*x+c)^m*GAMMA(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)
```

3.153.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.85

$$\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx = \frac{1}{8}a^2(c+dx)^m\left(\frac{12(c+dx)}{d(1+m)} - \frac{2^{-m}e^{2e-\frac{2cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f} + \frac{8ie^{e-\frac{cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{f} + \frac{8ie^{-e+\frac{cf}{d}}\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{f} + \frac{2^{-m}e^{-2e+\frac{2cf}{d}}\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{f}\right)$$

input
```
Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
(a^2*(c + d*x)^m*((12*(c + d*x))/(d*(1 + m)) - (E^(2*e - (2*c*f)/d)*Gamma[
1 + m, (-2*f*(c + d*x))/d])/(2^m*f*(-((f*(c + d*x))/d))^m) + ((8*I)*E^(e -
 (c*f)/d)*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) + (
(8*I)*E^(-e + (c*f)/d)*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)
^m) + (E^(-2*e + (2*c*f)/d)*Gamma[1 + m, (2*f*(c + d*x))/d])/(2^m*f*((f*(c
 + d*x))/d)^m)))/8
```

3.153.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3799, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m (a+a\sin(ie+ifx))^2 dx$$

$$\Big\downarrow \text{3799}$$

$$4a^2 \int (c+dx)^m \sinh^4\left(\frac{e}{2}+\frac{fx}{2}-\frac{i\pi}{4}\right) dx$$

$$\Big\downarrow \text{3042}$$

$$4a^2 \int (c+dx)^m \sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{4}\right)^4 dx$$

$$\Big\downarrow \text{3793}$$

$$4a^2 \int \left(-\frac{1}{8}\cosh(2e+2fx)(c+dx)^m + \frac{1}{2}i\sinh(e+fx)(c+dx)^m + \frac{3}{8}(c+dx)^m\right) dx$$

$$\Big\downarrow \text{2009}$$

$$4a^2 \left(-\frac{2^{-m-5}e^{2e-\frac{2cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f} + \frac{ie^{e-\frac{cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,\right.}{4f}\right.$$

input
```
Int[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
4*a^2*((3*(c + d*x)^(1 + m))/(8*d*(1 + m)) - (2^(-5 - m)*E^(2*e - (2*c*f)/
d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m
) + ((I/4)*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(
f*(-((f*(c + d*x))/d))^m) + ((I/4)*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 +
m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) + (2^(-5 - m)*E^(-2*e + (2*c*
f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m)
)
```

3.153. $\int (c+dx)^m (a+ia\sinh(e+fx))^2\, dx$

3.153.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.153.4 Maple [F]

$$\int (dx+c)^m (a+ia\sinh(fx+e))^2\,dx$$

input
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x)
```

output
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x)
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.97

$$\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx$$
$$= \frac{(a^2dm+a^2d)e^{\left(-\frac{dm\log\left(\frac{2f}{d}\right)+2de-2cf}{d}\right)}\Gamma\left(m+1,\frac{2(dfx+cf)}{d}\right)-8\left(-i\,a^2dm-i\,a^2d\right)e^{\left(-\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)}\Gamma\left(m+1,\right.}{}$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/8*((a^2*d*m + a^2*d)*e^(-(d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m + 1
, 2*(d*f*x + c*f)/d) - 8*(-I*a^2*d*m - I*a^2*d)*e^(-(d*m*log(f/d) + d*e -
c*f)/d)*gamma(m + 1, (d*f*x + c*f)/d) - 8*(-I*a^2*d*m - I*a^2*d)*e^(-(d*m*
log(-f/d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) - (a^2*d*m + a^2*
d)*e^(-(d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2*(d*f*x + c*f)/
d) + 12*(a^2*d*f*x + a^2*c*f)*(d*x + c)^m)/(d*f*m + d*f)
```

3.153.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+I*a*sinh(f*x+e))**2,x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.78

$$\begin{aligned}
&\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx \\
&= \frac{1}{4}\left(\frac{(dx+c)^{m+1}e^{\left(-2e+\frac{2cf}{d}\right)}E_{-m}\left(\frac{2(dx+c)f}{d}\right)}{d} + \frac{(dx+c)^{m+1}e^{\left(2e-\frac{2cf}{d}\right)}E_{-m}\left(-\frac{2(dx+c)f}{d}\right)}{d} + \frac{2(dx+c)^{m+1}}{d(m+1)}\right)a \\
&\quad + i\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} - \frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a^2 \\
&\quad + \frac{(dx+c)^{m+1}a^2}{d(m+1)}
\end{aligned}$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/4*((d*x + c)^(m + 1)*e^(-2*e + 2*c*f/d)*exp_integral_e(-m, 2*(d*x + c)*f
/d)/d + (d*x + c)^(m + 1)*e^(2*e - 2*c*f/d)*exp_integral_e(-m, -2*(d*x + c
)*f/d)/d + 2*(d*x + c)^(m + 1)/(d*(m + 1)))*a^2 + I*((d*x + c)^(m + 1)*e^(
-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d - (d*x + c)^(m + 1)*e^(e -
 c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a^2 + (d*x + c)^(m + 1)*a^2/
(d*(m + 1))
```

3.153.8 Giac [F]

$$\int (c+dx)^m (a+ia\sinh(e+fx))^2\,dx = \int (i\,a\sinh{(fx+e)}+a)^2(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((I*a*sinh(f*x + e) + a)^2*(d*x + c)^m, x)
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m (a+ia\sinh(e+fx))^2\,dx = \int (a+a\sinh(e+f\,x)\,\,1\mathrm{i})^2\,(c+d\,x)^m\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^m,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^m, x)
```

3.154 $\int (c+dx)^m(a+ia\sinh(e+fx))\,dx$

3.154.1 Optimal result

Integrand size = 21, antiderivative size = 135

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx = \frac{a(c+dx)^{1+m}}{d(1+m)}+\frac{iae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{iae^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{2f}$$

output
```
a*(d*x+c)^(1+m)/d/(1+m)+1/2*I*a*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c
)/d)/f/((-f*(d*x+c)/d)^m)+1/2*I*a*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x
+c)/d)/f/((f*(d*x+c)/d)^m)
```

3.154.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.53

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx = -\frac{ae^{-e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(-2ie^{e+\frac{cf}{d}}f(c+dx)\left(-\frac{f^2(c+dx)^2}{d^2}\right)^m+de^{2e}(1+m)\left(f\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(1\cdots\right.\right.}{2df(1+m)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\cdots\right.}$$

input
```
Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x]),x]
```

output
```
-1/2*(a*E^(-e - (c*f)/d)*(c + d*x)^m*((-2*I)*E^(e + (c*f)/d)*f*(c + d*x)*(
-((f^2*(c + d*x)^2)/d^2))^m + d*E^(2*e)*(1 + m)*(f*(c/d + x))^m*Gamma[1 +
m, -((f*(c + d*x))/d)] + d*E^((2*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Ga
mma[1 + m, (f*(c + d*x))/d])*(-I + Sinh[e + f*x]))/(d*f*(1 + m)*(-((f^2*(c
 + d*x)^2)/d^2))^m*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^2)
```

3.154.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m(a+a\sin(ie+ifx))dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a(c+dx)^m+ia(c+dx)^m\sinh(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{iae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{iae^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{a(c+dx)^{m+1}}{d(m+1)}$$

input
```
Int[(c + d*x)^m*(a + I*a*Sinh[e + f*x]),x]
```

output
```
(a*(c + d*x)^(1 + m))/(d*(1 + m)) + ((I/2)*a*E^(e - (c*f)/d)*(c + d*x)^m*G
amma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) + ((I/2)*a*E^(
-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))
/d)^m)
```

3.154.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.154.4 Maple [F]

$$\int (dx+c)^m (a+ia\sinh(fx+e))\,dx$$

input
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e)),x)
```

output
```
int((d*x+c)^m*(a+I*a*sinh(f*x+e)),x)
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx$$
$$= \frac{(i\,adm+i\,ad)e^{\left(-\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)}\Gamma\left(m+1,\frac{dfx+cf}{d}\right)+(i\,adm+i\,ad)e^{\left(-\frac{dm\log\left(-\frac{f}{d}\right)-de+cf}{d}\right)}\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)}{2\,(dfm+df)}$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*((I*a*d*m + I*a*d)*e^(-(d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f
*x + c*f)/d) + (I*a*d*m + I*a*d)*e^(-(d*m*log(-f/d) - d*e + c*f)/d)*gamma(
m + 1, -(d*f*x + c*f)/d) + 2*(a*d*f*x + a*c*f)*(d*x + c)^m)/(d*f*m + d*f)
```

3.154.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+I*a*sinh(f*x+e)),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.75

$$\begin{aligned}
&\int (c+dx)^m(a+ia\sinh(e+fx))\,dx \\
&= \frac{1}{2}i\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} - \frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a \\
&\quad + \frac{(dx+c)^{m+1}a}{d(m+1)}
\end{aligned}$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*I*((d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/
d - (d*x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*
a + (d*x + c)^(m + 1)*a/(d*(m + 1))
```

3.154.8 Giac [F]

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx = \int (i\,a\sinh\left(fx+e\right)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((I*a*sinh(f*x + e) + a)*(d*x + c)^m, x)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx = \int \left(a+a\sinh(e+f\,x)\ 1\mathrm{i}\right)\,\left(c+d\,x\right)^m\,dx$$

input
```
int((a + a*sinh(e + f*x)*1i)*(c + d*x)^m,x)
```

output
```
int((a + a*sinh(e + f*x)*1i)*(c + d*x)^m, x)
```

3.155 $\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx$

3.155.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx = \operatorname{Int}\left(\frac{(c+dx)^m}{a+ia\sinh(e+fx)},x\right)$$

output
```
Unintegrable((d*x+c)^m/(a+I*a*sinh(f*x+e)),x)
```

3.155.2 Mathematica [N/A]

Not integrable

Time = 3.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + I*a*Sinh[e + f*x]),x]
```

output
```
Integrate[(c + d*x)^m/(a + I*a*Sinh[e + f*x]), x]
```

3.155.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^m}{a+a\sin(ie+ifx)}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}dx$$

input
```
Int[(c + d*x)^m/(a + I*a*Sinh[e + f*x]),x]
```

output
```
$Aborted
```

3.155.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.155.4 Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(dx+c)^m}{a+ia\sinh(fx+e)}dx$$

input
```
int((d*x+c)^m/(a+I*a*sinh(f*x+e)),x)
```

output
```
int((d*x+c)^m/(a+I*a*sinh(f*x+e)),x)
```

3.155.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.74

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx = \int \frac{(dx+c)^m}{i\,a\sinh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
((a*f*e^(f*x + e) - I*a*f)*integral(-2*I*(d*x + c)^m*d*m/(-I*a*d*f*x - I*a
*c*f + (a*d*f*x + a*c*f)*e^(f*x + e)), x) + 2*I*(d*x + c)^m)/(a*f*e^(f*x +
 e) - I*a*f)
```

3.155.6 Sympy [N/A]

Not integrable

Time = 11.91 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx = -\frac{i\int \frac{(c+dx)^m}{\sinh(e+fx)-i}\,dx}{a}$$

input
```
integrate((d*x+c)**m/(a+I*a*sinh(f*x+e)),x)
```

output
```
-I*Integral((c + d*x)**m/(sinh(e + f*x) - I), x)/a
```

3.155.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx = \int \frac{(dx+c)^m}{i\,a\sinh{(fx+e)}+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a), x)
```

3.155.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx = \int \frac{(dx+c)^m}{i\,a\sinh{(fx+e)}+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a), x)
```

3.155.9 Mupad [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx = \int \frac{(c+d\,x)^m}{a+a\sinh{(e+f\,x)}\,1\mathrm{i}}\,dx$$

input
```
int((c + d*x)^m/(a + a*sinh(e + f*x)*1i),x)
```

output
```
int((c + d*x)^m/(a + a*sinh(e + f*x)*1i), x)
```

3.156 $\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx$

3.156.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx=\mathrm{Int}\left(\frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2},x\right)$$

output
```
Unintegrable((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x)
```

3.156.2 Mathematica [N/A]

Not integrable

Time = 13.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx=\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
Integrate[(c + d*x)^m/(a + I*a*Sinh[e + f*x])^2, x]
```

3.156.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^m}{(a+a\sin(ie+ifx))^2}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}dx$$

input
```
Int[(c + d*x)^m/(a + I*a*Sinh[e + f*x])^2,x]
```

output
```
$Aborted
```

3.156.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.156.4 Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(dx+c)^m}{(a+ia\sinh(fx+e))^2}dx$$

input
```
int((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x)
```

output
```
int((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x)
```

3.156.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 696, normalized size of antiderivative = 30.26

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(i\,a\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-(2*(I*d^2*f^2*x^2 + 2*I*c*d*f^2*x + I*c^2*f^2 - I*d^2*m^2 + I*d^2*m + (I*
d^2*f*m*x + I*d^2*m^2 + (I*c*d*f - I*d^2)*m)*e^(2*f*x + 2*e) - (3*d^2*f^2*
x^2 + 3*c^2*f^2 - 2*d^2*m^2 - (c*d*f - 2*d^2)*m + (6*c*d*f^2 - d^2*f*m)*x)
*e^(f*x + e))*(d*x + c)^m + 3*(-I*a^2*d^2*f^3*x^2 - 2*I*a^2*c*d*f^3*x - I*
a^2*c^2*f^3 - (a^2*d^2*f^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2*f^3)*e^(3*f*x +
 3*e) + 3*(I*a^2*d^2*f^3*x^2 + 2*I*a^2*c*d*f^3*x + I*a^2*c^2*f^3)*e^(2*f*x
 + 2*e) + 3*(a^2*d^2*f^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2*f^3)*e^(f*x + e))
*integral(-2*(I*d^3*f^2*m*x^2 + 2*I*c*d^2*f^2*m*x - I*d^3*m^3 + 3*I*d^3*m^
2 + (I*c^2*d*f^2 - 2*I*d^3)*m)*(d*x + c)^m/(-3*I*a^2*d^3*f^3*x^3 - 9*I*a^2
*c*d^2*f^3*x^2 - 9*I*a^2*c^2*d*f^3*x - 3*I*a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^
3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(f*x + e)), x
))/(3*I*a^2*d^2*f^3*x^2 + 6*I*a^2*c*d*f^3*x + 3*I*a^2*c^2*f^3 + 3*(a^2*d^2
*f^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2*f^3)*e^(3*f*x + 3*e) - 9*(I*a^2*d^2*f
^3*x^2 + 2*I*a^2*c*d*f^3*x + I*a^2*c^2*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^2*f
^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2*f^3)*e^(f*x + e))
```

3.156.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m/(a+I*a*sinh(f*x+e))**2,x)
```

output
```
Timed out
```

3.156.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(i\,a\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a)^2, x)
```

3.156.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(i\,a\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a)^2, x)
```

3.156.9 Mupad [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^m}{(a+a\sinh\left(e+f\,x\right)\,1\mathrm{i})^2}\,dx$$

input
```
int((c + d*x)^m/(a + a*sinh(e + f*x)*1i)^2,x)
```

output
```
int((c + d*x)^m/(a + a*sinh(e + f*x)*1i)^2, x)
```

3.157 $\int (c+dx)^3(a+b\sinh(e+fx))\,dx$

3.157.1 Optimal result

Integrand size = 18, antiderivative size = 89

$$\int (c+dx)^3(a+b\sinh(e+fx))\,dx = \frac{a(c+dx)^4}{4d} + \frac{6bd^2(c+dx)\cosh(e+fx)}{f^3} + \frac{b(c+dx)^3\cosh(e+fx)}{f} - \frac{6bd^3\sinh(e+fx)}{f^4} - \frac{3bd(c+dx)^2\sinh(e+fx)}{f^2}$$

output
```
1/4*a*(d*x+c)^4/d+6*b*d^2*(d*x+c)*cosh(f*x+e)/f^3+b*(d*x+c)^3*cosh(f*x+e)/
f-6*b*d^3*sinh(f*x+e)/f^4-3*b*d*(d*x+c)^2*sinh(f*x+e)/f^2
```

3.157.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.38

$$\begin{aligned}&\int (c+dx)^3(a+b\sinh(e+fx))\,dx\\&=\frac{1}{4}ax\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)\\&\quad+\frac{b(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\cosh(e+fx)}{f^3}\\&\quad-\frac{3bd(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\sinh(e+fx)}{f^4}\end{aligned}$$

input
```
Integrate[(c + d*x)^3*(a + b*Sinh[e + f*x]),x]
```

output
```
(a*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 + (b*(c + d*x)*(c^2*f^
2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f*x])/f^3 - (3*b*d*(c^2*f^2
+ 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x])/f^4
```

3.157.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\sinh (e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3(a-ib\sin (ie+ifx))dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a(c+dx)^3+b(c+dx)^3\sinh (e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cosh (e+fx)}{f^3}-\frac{3bd(c+dx)^2\sinh (e+fx)}{f^2}+\frac{b(c+dx)^3\cosh (e+fx)}{f}-\frac{6bd^3\sinh (e+fx)}{f^4}$$

input
```
Int[(c + d*x)^3*(a + b*Sinh[e + f*x]),x]
```

output
```
(a*(c + d*x)^4)/(4*d) + (6*b*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (b*(c + d*
x)^3*Cosh[e + f*x])/f - (6*b*d^3*Sinh[e + f*x])/f^4 - (3*b*d*(c + d*x)^2*S
inh[e + f*x])/f^2
```

3.157.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.157.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

method	result
parallelrisch	$\frac{f\left((dx+c)^2f^2+6d^2\right)b(dx+c)\cosh(fx+e)-3db\left((dx+c)^2f^2+2d^2\right)\sinh(fx+e)+f\left(\left(\frac{dx}{2}+c\right)\left(\frac{1}{2}d^2x^2+cdx+c^2\right)xa\,f^3+b\,c^3\right.}{f^4}$
risch	$\frac{a\,d^3x^4}{4}+a\,d^2c\,x^3+\frac{3ad\,c^2x^2}{2}+a\,c^3x+\frac{a\,c^4}{4d}+\frac{b\left(d^3x^3f^3+3c\,d^2f^3x^2+3c^2d\,f^3x-3d^3f^2x^2+c^3f^3-6c\,d^2f^2x-\right.}{2f^4}$
parts	$\frac{a(dx+c)^4}{4d}+\frac{b\left(\frac{d^3\left((fx+e)^3\cosh(fx+e)-3(fx+e)^2\sinh(fx+e)+6(fx+e)\cosh(fx+e)-6\sinh(fx+e)\right)}{f^3}-\frac{3d^3e\left((fx+e)^2\cosh(fx+e\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{d^3a(fx+e)^4}{4f^3}+\frac{d^3b\left((fx+e)^3\cosh(fx+e)-3(fx+e)^2\sinh(fx+e)+6(fx+e)\cosh(fx+e)-6\sinh(fx+e)\right)}{f^3}-\frac{d^3ea(fx+e)^3}{f^3}-\frac{3d^3eb\left((fx+\right.}{}}{}$
default	$\frac{\frac{d^3a(fx+e)^4}{4f^3}+\frac{d^3b\left((fx+e)^3\cosh(fx+e)-3(fx+e)^2\sinh(fx+e)+6(fx+e)\cosh(fx+e)-6\sinh(fx+e)\right)}{f^3}-\frac{d^3ea(fx+e)^3}{f^3}-\frac{3d^3eb\left((fx+\right.}{}}{}$

input
```
int((d*x+c)^3*(a+b*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
(f*((d*x+c)^2*f^2+6*d^2)*b*(d*x+c)*cosh(f*x+e)-3*d*b*((d*x+c)^2*f^2+2*d^2)
*sinh(f*x+e)+f*((1/2*d*x+c)*(1/2*d^2*x^2+c*d*x+c^2)*x*a*f^3+b*c^3*f^2+6*b*
c*d^2))/f^4
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.89

$$\int (c+dx)^3(a+b\sinh(e+fx))\,dx$$
$$=\frac{ad^3f^4x^4+4\,acd^2f^4x^3+6\,ac^2df^4x^2+4\,ac^3f^4x+4\left(bd^3f^3x^3+3\,bcd^2f^3x^2+bc^3f^3+6\,bcd^2f+3\left(bc^2df^3+\right.\right.}{4\,f^4}$$

input
```
integrate((d*x+c)^3*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
 + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + b*c^3*f^3 + 6*b*c*d^2*f + 3*(b*c
^2*d*f^3 + 2*b*d^3*f)*x)*cosh(f*x + e) - 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2
*x + b*c^2*d*f^2 + 2*b*d^3)*sinh(f*x + e))/f^4
```

3.157.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(88) = 176.

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.97

$$\int (c+dx)^3(a+b\sinh(e+fx))\,dx$$
$$=\begin{cases} ac^3x+\frac{3ac^2dx^2}{2}+acd^2x^3+\frac{ad^3x^4}{4}+\frac{bc^3\cosh{(e+fx)}}{f}+\frac{3bc^2dx\cosh{(e+fx)}}{f}-\frac{3bc^2d\sinh{(e+fx)}}{f^2}+\frac{3bcd^2x^2\cosh{(e+fx)}}{f}-\frac{6 \\ (a+b\sinh{(e)})\left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right) \end{cases}$$

input
```
integrate((d*x+c)**3*(a+b*sinh(f*x+e)),x)
```

output
```
Piecewise((a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 +
b*c**3*cosh(e + f*x)/f + 3*b*c**2*d*x*cosh(e + f*x)/f - 3*b*c**2*d*sinh(e
+ f*x)/f**2 + 3*b*c*d**2*x**2*cosh(e + f*x)/f - 6*b*c*d**2*x*sinh(e + f*x)
/f**2 + 6*b*c*d**2*cosh(e + f*x)/f**3 + b*d**3*x**3*cosh(e + f*x)/f - 3*b*
d**3*x**2*sinh(e + f*x)/f**2 + 6*b*d**3*x*cosh(e + f*x)/f**3 - 6*b*d**3*si
nh(e + f*x)/f**4, Ne(f, 0)), ((a + b*sinh(e))*(c**3*x + 3*c**2*d*x**2/2 +
c*d**2*x**3 + d**3*x**4/4), True))
```

3.157.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(87) = 174$.

Time = 0.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.63

$$\int (c+dx)^3(a+b\sinh(e+fx))\,dx$$
$$= \frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{3}{2}\,bc^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right)$$
$$+ \frac{3}{2}\,bcd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right)$$
$$+ \frac{1}{2}\,bd^3\left(\frac{(f^3x^3e^e - 3\,f^2x^2e^e + 6\,fxe^e - 6\,e^e)e^{(fx)}}{f^4} + \frac{(f^3x^3 + 3\,f^2x^2 + 6\,fx + 6)e^{(-fx-e)}}{f^4}\right)$$
$$+ \frac{bc^3\cosh{(fx+e)}}{f}$$

input
```
integrate((d*x+c)^3*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3/2*b*c^2*d*((f*
x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + 3/2*b*c*d^2*((f^2
*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x
- e)/f^3) + 1/2*b*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6*f*x*e^e - 6*e^e)*e
^(f*x)/f^4 + (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x - e)/f^4) + b*c^3*c
osh(f*x + e)/f
```

3.157.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(87) = 174$.

Time = 0.27 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.90

$$\int (c+dx)^3(a+b\sinh(e+fx))\,dx = \frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x$$
$$+ \frac{(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + 3\,bc^2df^3x - 3\,bd^3f^2x^2 + bc^3f^3 - 6\,bcd^2f^2x - 3\,bc^2df^2 + 6\,bd^3fx + 6\,bcd^2f - 6}{2\,f^4}$$
$$+ \frac{(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + 3\,bc^2df^3x + 3\,bd^3f^2x^2 + bc^3f^3 + 6\,bcd^2f^2x + 3\,bc^2df^2 + 6\,bd^3fx + 6\,bcd^2f + 6}{2\,f^4}$$

input
```
integrate((d*x+c)^3*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 1/2*(b*d^3*f^3*x
^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x - 3*b*d^3*f^2*x^2 + b*c^3*f^3 - 6
*b*c*d^2*f^2*x - 3*b*c^2*d*f^2 + 6*b*d^3*f*x + 6*b*c*d^2*f - 6*b*d^3)*e^(f
*x + e)/f^4 + 1/2*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3
*b*d^3*f^2*x^2 + b*c^3*f^3 + 6*b*c*d^2*f^2*x + 3*b*c^2*d*f^2 + 6*b*d^3*f*x
 + 6*b*c*d^2*f + 6*b*d^3)*e^(-f*x - e)/f^4
```

3.157.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.10

$$\begin{aligned}\int (c+dx)^3(a+b\sinh(e+fx))\,dx = {} & \frac{\cosh(e+f\,x)\,\left(b\,c^3\,f^2+6\,b\,c\,d^2\right)}{f^3} \\ & - \frac{3\sinh(e+f\,x)\,\left(b\,c^2\,d\,f^2+2\,b\,d^3\right)}{f^4} + \frac{a\,d^3\,x^4}{4} \\ & + a\,c^3\,x + \frac{3\,x\cosh(e+f\,x)\,\left(b\,c^2\,d\,f^2+2\,b\,d^3\right)}{f^3} \\ & + \frac{3\,a\,c^2\,d\,x^2}{2} + a\,c\,d^2\,x^3 + \frac{b\,d^3\,x^3\cosh(e+f\,x)}{f} \\ & - \frac{3\,b\,d^3\,x^2\sinh(e+f\,x)}{f^2} - \frac{6\,b\,c\,d^2\,x\sinh(e+f\,x)}{f^2} \\ & + \frac{3\,b\,c\,d^2\,x^2\cosh(e+f\,x)}{f}\end{aligned}$$

input
```
int((a + b*sinh(e + f*x))*(c + d*x)^3,x)
```

output
```
(cosh(e + f*x)*(b*c^3*f^2 + 6*b*c*d^2))/f^3 - (3*sinh(e + f*x)*(2*b*d^3 +
b*c^2*d*f^2))/f^4 + (a*d^3*x^4)/4 + a*c^3*x + (3*x*cosh(e + f*x)*(2*b*d^3
+ b*c^2*d*f^2))/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 + (b*d^3*x^3*cosh(e
+ f*x))/f - (3*b*d^3*x^2*sinh(e + f*x))/f^2 - (6*b*c*d^2*x*sinh(e + f*x))/
f^2 + (3*b*c*d^2*x^2*cosh(e + f*x))/f
```

3.158 $\int (c+dx)^2(a+b\sinh(e+fx))\,dx$

3.158.1 Optimal result

Integrand size = 18, antiderivative size = 67

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx=\frac{a(c+dx)^3}{3d}+\frac{2bd^2\cosh(e+fx)}{f^3}+\frac{b(c+dx)^2\cosh(e+fx)}{f}-\frac{2bd(c+dx)\sinh(e+fx)}{f^2}$$

output
```
1/3*a*(d*x+c)^3/d+2*b*d^2*cosh(f*x+e)/f^3+b*(d*x+c)^2*cosh(f*x+e)/f-2*b*d*
(d*x+c)*sinh(f*x+e)/f^2
```

3.158.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx=\frac{1}{3}ax\left(3c^2+3cdx+d^2x^2\right)+\frac{b(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)}{f^3}-\frac{2bd(c+dx)\sinh(e+fx)}{f^2}$$

input
```
Integrate[(c + d*x)^2*(a + b*Sinh[e + f*x]),x]
```

output
```
(a*x*(3*c^2 + 3*c*d*x + d^2*x^2))/3 + (b*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 +
 f^2*x^2))*Cosh[e + f*x])/f^3 - (2*b*d*(c + d*x)*Sinh[e + f*x])/f^2
```

3.158.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2(a-ib\sin(ie+ifx))dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a(c+dx)^2+b(c+dx)^2\sinh(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\sinh(e+fx)}{f^2}+\frac{b(c+dx)^2\cosh(e+fx)}{f}+\frac{2bd^2\cosh(e+fx)}{f^3}$$

input
```
Int[(c + d*x)^2*(a + b*Sinh[e + f*x]),x]
```

output
```
(a*(c + d*x)^3)/(3*d) + (2*b*d^2*Cosh[e + f*x])/f^3 + (b*(c + d*x)^2*Cosh[
e + f*x])/f - (2*b*d*(c + d*x)*Sinh[e + f*x])/f^2
```

3.158.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.158.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

method	result
parallelrisch	$\frac{b\left((dx+c)^2 f^2+2d^2\right)\cosh(fx+e)-2bdf(dx+c)\sinh(fx+e)+\left(\frac{1}{3}d^2x^2+cdx+c^2\right)xa\,f^3+b\,c^2f^2+2b\,d^2}{f^3}$
risch	$\frac{a\,d^2x^3}{3}+adc\,x^2+ax\,c^2+\frac{a\,c^3}{3d}+\frac{b(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2fx-2cdf+2d^2)\mathrm{e}^{fx+e}}{2f^3}+\frac{b(d^2x^2f^2+2cd\,f^2x}{}$
parts	$\frac{a(dx+c)^3}{3d}+\frac{b\left(\frac{d^2\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{2d^2e((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f^2}+\frac{2dc((f}{}\right.}{f}$
derivativedivides	$\frac{\frac{d^2a(fx+e)^3}{3f^2}+\frac{d^2b\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{d^2ea(fx+e)^2}{f^2}-\frac{2d^2eb((fx+e)\cosh(fx+e)-\sinh(fx}{f^2}}{}$
default	$\frac{\frac{d^2a(fx+e)^3}{3f^2}+\frac{d^2b\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{d^2ea(fx+e)^2}{f^2}-\frac{2d^2eb((fx+e)\cosh(fx+e)-\sinh(fx}{f^2}}{}$

input
```
int((d*x+c)^2*(a+b*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
(b*((d*x+c)^2*f^2+2*d^2)*cosh(f*x+e)-2*b*d*f*(d*x+c)*sinh(f*x+e)+(1/3*d^2*
x^2+c*d*x+c^2)*x*a*f^3+b*c^2*f^2+2*b*d^2)/f^3
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx$$
$$=\frac{ad^2f^3x^3+3\,acdf^3x^2+3\,ac^2f^3x+3\left(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2+2\,bd^2\right)\cosh\left(fx+e\right)-6\left(bd^2fx+bcdf\right.}{3\,f^3}$$

input
```
integrate((d*x+c)^2*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x + 3*(b*d^2*f^2*x^2 +
2*b*c*d*f^2*x + b*c^2*f^2 + 2*b*d^2)*cosh(f*x + e) - 6*(b*d^2*f*x + b*c*d*
f)*sinh(f*x + e))/f^3
```

3.158.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(65) = 130$.

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.25

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx$$
$$= \begin{cases} ac^2x + acdx^2 + \frac{ad^2x^3}{3} + \frac{bc^2\cosh(e+fx)}{f} + \frac{2bcdx\cosh(e+fx)}{f} - \frac{2bcd\sinh(e+fx)}{f^2} + \frac{bd^2x^2\cosh(e+fx)}{f} - \frac{2bd^2x\sinh(e+fx)}{f^2} - \\ (a+b\sinh(e))\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(a+b*sinh(f*x+e)),x)
```

output
```
Piecewise((a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 + b*c**2*cosh(e + f*x)/f
+ 2*b*c*d*x*cosh(e + f*x)/f - 2*b*c*d*sinh(e + f*x)/f**2 + b*d**2*x**2*cos
h(e + f*x)/f - 2*b*d**2*x*sinh(e + f*x)/f**2 + 2*b*d**2*cosh(e + f*x)/f**3
, Ne(f, 0)), ((a + b*sinh(e))*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

3.158.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(65) = 130$.

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx$$
$$= \frac{1}{3}ad^2x^3 + acdx^2 + ac^2x + bcd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{1}{2}bd^2\left(\frac{(f^2x^2e^e - 2fxe^e + 2e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2fx + 2)e^{(-fx-e)}}{f^3}\right) + \frac{bc^2\cosh(fx+e)}{f}$$

input
```
integrate((d*x+c)^2*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + b*c*d*((f*x*e^e - e^e)*e^(f*x)/f^2 +
 (f*x + 1)*e^(-f*x - e)/f^2) + 1/2*b*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e
)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + b*c^2*cosh(f*x +
 e)/f
```

3.158.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(65) = 130$.

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.18

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx$$
$$= \frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x + \frac{(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 - 2\,bd^2fx - 2\,bcdf + 2\,bd^2)e^{(fx+e)}}{2\,f^3}$$
$$+ \frac{(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 + 2\,bd^2fx + 2\,bcdf + 2\,bd^2)e^{(-fx-e)}}{2\,f^3}$$

input
```
integrate((d*x+c)^2*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + 1/2*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x +
 b*c^2*f^2 - 2*b*d^2*f*x - 2*b*c*d*f + 2*b*d^2)*e^(f*x + e)/f^3 + 1/2*(b*d
^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2 + 2*b*d^2*f*x + 2*b*c*d*f + 2*b*d^2
)*e^(-f*x - e)/f^3
```

3.158.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.64

$$\int (c+dx)^2(a+b\sinh(e+fx))\,dx = \frac{a\,d^2\,x^3}{3} + \frac{\cosh(e+f\,x)\,\left(b\,c^2\,f^2+2\,b\,d^2\right)}{f^3} + a\,c^2\,x + a\,c\,d\,x^2 - \frac{2\,b\,d^2\,x\sinh(e+f\,x)}{f^2} + \frac{b\,d^2\,x^2\cosh(e+f\,x)}{f} - \frac{2\,b\,c\,d\sinh(e+f\,x)}{f^2} + \frac{2\,b\,c\,d\,x\cosh(e+f\,x)}{f}$$

input
```
int((a + b*sinh(e + f*x))*(c + d*x)^2,x)
```

output
```
(a*d^2*x^3)/3 + (cosh(e + f*x)*(2*b*d^2 + b*c^2*f^2))/f^3 + a*c^2*x + a*c*
d*x^2 - (2*b*d^2*x*sinh(e + f*x))/f^2 + (b*d^2*x^2*cosh(e + f*x))/f - (2*b
*c*d*sinh(e + f*x))/f^2 + (2*b*c*d*x*cosh(e + f*x))/f
```

3.159 $\int (c + dx)(a + b\sinh(e + fx))\,dx$

3.159.1 Optimal result

Integrand size = 16, antiderivative size = 45

$$\int (c+dx)(a+b\sinh(e+fx))\,dx = \frac{a(c+dx)^2}{2d} + \frac{b(c+dx)\cosh(e+fx)}{f} - \frac{bd\sinh(e+fx)}{f^2}$$

output
```
1/2*a*(d*x+c)^2/d+b*(d*x+c)*cosh(f*x+e)/f-b*d*sinh(f*x+e)/f^2
```

3.159.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int (c+dx)(a+b\sinh(e+fx))\,dx = \frac{1}{2}ax(2c+dx) + \frac{b(c+dx)\cosh(e+fx)}{f} - \frac{bd\sinh(e+fx)}{f^2}$$

input
```
Integrate[(c + d*x)*(a + b*Sinh[e + f*x]),x]
```

output
```
(a*x*(2*c + d*x))/2 + (b*(c + d*x)*Cosh[e + f*x])/f - (b*d*Sinh[e + f*x])/
f^2
```

3.159.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\sinh (e+fx)) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)(a-ib\sin (ie+ifx))dx$$

$$\Big\downarrow \text{3798}$$

$$\int (a(c+dx)+b(c+dx)\sinh (e+fx))dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\cosh (e+fx)}{f}-\frac{bd\sinh (e+fx)}{f^2}$$

input
```
Int[(c + d*x)*(a + b*Sinh[e + f*x]),x]
```

output
```
(a*(c + d*x)^2)/(2*d) + (b*(c + d*x)*Cosh[e + f*x])/f - (b*d*Sinh[e + f*x]
)/f^2
```

3.159.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.159.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
parallelrisch	$\frac{(dx+c)bf\cosh(fx+e)-\sinh(fx+e)bd+f\left(ax\left(\frac{dx}{2}+c\right)f+bc\right)}{f^2}$	46
risch	$\frac{ad\,x^2}{2}+acx+\frac{b(dfx+cf-d)\,\mathrm{e}^{fx+e}}{2f^2}+\frac{b(dfx+cf+d)\,\mathrm{e}^{-fx-e}}{2f^2}$	60
parts	$a\left(\frac{1}{2}d\,x^2+cx\right)+\frac{b\left(\frac{d((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{de\cosh(fx+e)}{f}+c\cosh(fx+e)\right)}{f}$	67
derivativedivides	$\frac{\frac{da(fx+e)^2}{2f}+\frac{db((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{deb\cosh(fx+e)}{f}+ac(fx+e)+bc\cosh(fx+e)}{f}$	91
default	$\frac{\frac{da(fx+e)^2}{2f}+\frac{db((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{deb\cosh(fx+e)}{f}+ac(fx+e)+bc\cosh(fx+e)}{f}$	91

input
```
int((d*x+c)*(a+b*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*b*f*cosh(f*x+e)-sinh(f*x+e)*b*d+f*(a*x*(1/2*d*x+c)*f+b*c))/f^2
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int (c+dx)(a+b\sinh(e+fx))\,dx$$
$$=\frac{adf^2x^2+2\,acf^2x-2\,bd\sinh\left(fx+e\right)+2\left(bdfx+bcf\right)\cosh\left(fx+e\right)}{2\,f^2}$$

input
```
integrate((d*x+c)*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x - 2*b*d*sinh(f*x + e) + 2*(b*d*f*x + b*c*f)
*cosh(f*x + e))/f^2
```

3.159.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.51

$$\int (c+dx)(a+b\sinh(e+fx))\,dx = \begin{cases} acx + \frac{adx^2}{2} + \frac{bc\cosh(e+fx)}{f} + \frac{bdx\cosh(e+fx)}{f} - \frac{bd\sinh(e+fx)}{f^2} & \text{for } f\neq 0 \\ (a+b\sinh(e))\left(cx+\frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*(a+b*sinh(f*x+e)),x)
```

output
```
Piecewise((a*c*x + a*d*x**2/2 + b*c*cosh(e + f*x)/f + b*d*x*cosh(e + f*x)/
f - b*d*sinh(e + f*x)/f**2, Ne(f, 0)), ((a + b*sinh(e))*(c*x + d*x**2/2),
True))
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int (c+dx)(a+b\sinh(e+fx))\,dx = \frac{1}{2}adx^2 + acx + \frac{1}{2}bd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{bc\cosh(fx+e)}{f}$$

input
```
integrate((d*x+c)*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + a*c*x + 1/2*b*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(
-f*x - e)/f^2) + b*c*cosh(f*x + e)/f
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.42

$$\int (c+dx)(a+b\sinh(e+fx))\,dx = \frac{1}{2}adx^2 + acx + \frac{(bdfx+bcf-bd)e^{(fx+e)}}{2f^2} + \frac{(bdfx+bcf+bd)e^{(-fx-e)}}{2f^2}$$

input
```
integrate((d*x+c)*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
1/2*a*d*x^2 + a*c*x + 1/2*(b*d*f*x + b*c*f - b*d)*e^(f*x + e)/f^2 + 1/2*(b
*d*f*x + b*c*f + b*d)*e^(-f*x - e)/f^2
```

3.159.9 Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int (c+dx)(a+b\sinh(e+fx))\,dx = \frac{f\,(b\,c\cosh(e+f\,x)+b\,d\,x\cosh(e+f\,x))-b\,d\sinh(e+f\,x)}{f^2} + a\,c\,x + \frac{a\,d\,x^2}{2}$$

input
```
int((a + b*sinh(e + f*x))*(c + d*x),x)
```

output
```
(f*(b*c*cosh(e + f*x) + b*d*x*cosh(e + f*x)) - b*d*sinh(e + f*x))/f^2 + a*
c*x + (a*d*x^2)/2
```

3.160 $\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx$

3.160.1 Optimal result

Integrand size = 18, antiderivative size = 64

$$\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx=\frac{a\log(c+dx)}{d}+\frac{b\mathrm{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(\frac{cf}{d}+fx\right)}{d}$$

output
```
a*ln(d*x+c)/d+b*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d-b*Chi(c*f/d+f*x)*sinh(-e+c
*f/d)/d
```

3.160.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx=\frac{a\log(c+dx)+b\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+b\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{d}$$

input
```
Integrate[(a + b*Sinh[e + f*x])/(c + d*x),x]
```

output
```
(a*Log[c + d*x] + b*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + b*Cosh[e
 - (c*f)/d]*SinhIntegral[f*(c/d + x)])/d
```

3.160.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sinh(e+fx)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-ib\sin(ie+ifx)}{c+dx}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a}{c+dx}+\frac{b\sinh(e+fx)}{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\log(c+dx)}{d}+\frac{b\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}$$

input
```
Int[(a + b*Sinh[e + f*x])/(c + d*x),x]
```

output
```
(a*Log[c + d*x])/d + (b*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d +
 (b*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

3.160.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.160.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

method	result	size
risch	$\frac{a\ln(dx+c)}{d}+\frac{b\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{Ei}_1\left(fx+e+\frac{cf-de}{d}\right)}{2d}-\frac{b\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{Ei}_1\left(-fx-e-\frac{cf-de}{d}\right)}{2d}$	94

input
```
int((a+b*sinh(f*x+e))/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*ln(d*x+c)/d+1/2*b/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*b/d*exp
(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.73

$$\int\frac{a+b\sinh(e+fx)}{c+dx}\,dx$$
$$=\frac{\left(b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+2\,a\log\left(dx+c\right)-\left(b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{d}{}\right.}{2\,d}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

output
```
1/2*((b*Ei((d*f*x + c*f)/d) - b*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d)
 + 2*a*log(d*x + c) - (b*Ei((d*f*x + c*f)/d) + b*Ei(-(d*f*x + c*f)/d))*sin
h(-(d*e - c*f)/d))/d
```

3.160.6 Sympy [F]

$$\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx=\int \frac{a+b\sinh\left(e+fx\right)}{c+dx}\,dx$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c),x)
```

output
```
Integral((a + b*sinh(e + f*x))/(c + d*x), x)
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx=\frac{1}{2}\,b\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right)+\frac{a\log\left(dx+c\right)}{d}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*b*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(e - c*f/d)*e
xp_integral_e(1, -(d*x + c)*f/d)/d) + a*log(d*x + c)/d
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx=\frac{b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}-b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+2\,a\log\left(dx+c\right)}{2\,d}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - b*Ei(-(d*f*x + c*f)/d)*e^(-e +
c*f/d) + 2*a*log(d*x + c))/d
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx = \int \frac{a+b\sinh(e+f\,x)}{c+d\,x}\,dx$$

input
```
int((a + b*sinh(e + f*x))/(c + d*x),x)
```

output
```
int((a + b*sinh(e + f*x))/(c + d*x), x)
```

3.161 $\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx$

3.161.1 Optimal result

Integrand size = 18, antiderivative size = 87

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx = -\frac{a}{d(c+dx)}+\frac{bf\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(\frac{cf}{d}+fx\right)}{d^2} -\frac{b\sinh(e+fx)}{d(c+dx)}+\frac{bf\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(\frac{cf}{d}+fx\right)}{d^2}$$

output
```
-a/d/(d*x+c)+b*f*Chi(c*f/d+f*x)*cosh(-e+c*f/d)/d^2-b*f*Shi(c*f/d+f*x)*sinh
(-e+c*f/d)/d^2-b*sinh(f*x+e)/d/(d*x+c)
```

3.161.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx = \frac{bf\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(f\left(\frac{c}{d}+x\right)\right)-\frac{d(a+b\sinh(e+fx))}{c+dx}+bf\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{d^2}$$

input
```
Integrate[(a + b*Sinh[e + f*x])/(c + d*x)^2,x]
```

output
```
(b*f*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] - (d*(a + b*Sinh[e + f*x]
))/(c + d*x) + b*f*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)])/d^2
```

3.161.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-ib\sin(ie+ifx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a}{(c+dx)^2}+\frac{b\sinh(e+fx)}{(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{d(c+dx)}+\frac{bf\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{bf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{b\sinh(e+fx)}{d(c+dx)}$$

input
```
Int[(a + b*Sinh[e + f*x])/(c + d*x)^2,x]
```

output
```
-(a/(d*(c + d*x))) + (b*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d
^2 - (b*Sinh[e + f*x])/(d*(c + d*x)) + (b*f*Sinh[e - (c*f)/d]*SinhIntegral
[(c*f)/d + f*x])/d^2
```

3.161.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.161.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.71

method	result	size
risch	$-\frac{a}{d(dx+c)}+\frac{f b\,\mathrm{e}^{-fx-e}}{2d(dfx+cf)}-\frac{f b\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{Ei}_1\left(fx+e+\frac{cf-de}{d}\right)}{2d^2}-\frac{f b\,\mathrm{e}^{fx+e}}{2d^2\left(\frac{cf}{d}+fx\right)}-\frac{f b\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{Ei}_1\left(-fx-e-\frac{cf-de}{d}\right)}{2d^2}$	149

input
```
int((a+b*sinh(f*x+e))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/(d*x+c)+1/2*f*b*exp(-f*x-e)/d/(d*f*x+c*f)-1/2*f*b/d^2*exp((c*f-d*e)/d
)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*f*b/d^2*exp(f*x+e)/(c*f/d+f*x)-1/2*f*b/d^2*e
xp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.86

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx = -\frac{2\,bd\sinh\left(fx+e\right)+2\,ad-\left((bdfx+bcf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+(bdfx+bcf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+\left((bd\right.}{2\left(d^3x+cd^2\right)}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*sinh(f*x + e) + 2*a*d - ((b*d*f*x + b*c*f)*Ei((d*f*x + c*f)/d)
 + (b*d*f*x + b*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) + ((b*d*f*
x + b*c*f)*Ei((d*f*x + c*f)/d) - (b*d*f*x + b*c*f)*Ei(-(d*f*x + c*f)/d))*s
inh(-(d*e - c*f)/d))/(d^3*x + c*d^2)
```

3.161.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c)**2,x)
```

output
```
Timed out
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx = \frac{1}{2}b\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right) - \frac{a}{d^2x+cd}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*b*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/((d*x + c)*d) - e^(
e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d)) - a/(d^2*x + c
*d)
```

3.161.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. 2(90) = 180.

Time = 0.31 (sec) , antiderivative size = 630, normalized size of antiderivative = 7.24

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx$$

$$= \frac{1}{2}b\left(\frac{\left(\left((dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^2\mathrm{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)} - def^2\mathrm{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right.}{d}\right.\right.\right.}{\left((dx+c)d^4\left(\frac{de}{dx+c}-\frac{cf}{dx+c}\right.\right.}\right.$$

$$- \frac{a}{(dx+c)d}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*b*(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d
*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - d*e*
f^2*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((
d*e - c*f)/d) + c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) -
d*e + c*f)/d)*e^((d*e - c*f)/d) - d*f^2*e^((d*x + c)*(d*e/(d*x + c) - c*f/
(d*x + c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x + c) - c*f/(d*x + c) + f)
 - d^5*e + c*d^4*f)*f) + ((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^
2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(
d*e - c*f)/d) - d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)
 - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + c*f^3*Ei(-((d*x + c)*(d*e/(d*x + c)
- c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + d*f^2*e^(-(d*x +
 c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x
+ c) - c*f/(d*x + c) + f) - d^5*e + c*d^4*f)*f)) - a/((d*x + c)*d)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx=\int \frac{a+b\sinh(e+f\,x)}{{(c+d\,x)}^2}\,dx$$

input
```
int((a + b*sinh(e + f*x))/(c + d*x)^2,x)
```

output
```
int((a + b*sinh(e + f*x))/(c + d*x)^2, x)
```

3.162 $\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx$

3.162.1 Optimal result

Integrand size = 18, antiderivative size = 123

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx = -\frac{a}{2d(c+dx)^2}-\frac{bf\cosh(e+fx)}{2d^2(c+dx)}+\frac{bf^2\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{b\sinh(e+fx)}{2d(c+dx)^2}+\frac{bf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{2d^3}$$

output
```
-1/2*a/d/(d*x+c)^2-1/2*b*f*cosh(f*x+e)/d^2/(d*x+c)+1/2*b*f^2*cosh(-e+c*f/d
)*Shi(c*f/d+f*x)/d^3-1/2*b*f^2*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d^3-1/2*b*sin
h(f*x+e)/d/(d*x+c)^2
```

3.162.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx = \frac{bf^2\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)-\frac{d(bf(c+dx)\cosh(e+fx)+d(a+b\sinh(e+fx)))}{(c+dx)^2}+bf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

input
```
Integrate[(a + b*Sinh[e + f*x])/(c + d*x)^3,x]
```

output
```
(b*f^2*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] - (d*(b*f*(c + d*x)*Cos
h[e + f*x] + d*(a + b*Sinh[e + f*x])))/(c + d*x)^2 + b*f^2*Cosh[e - (c*f)/
d]*SinhIntegral[f*(c/d + x)])/(2*d^3)
```

3.162.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{a-ib\sin(ie+ifx)}{(c+dx)^3}dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(\frac{a}{(c+dx)^3}+\frac{b\sinh(e+fx)}{(c+dx)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a}{2d(c+dx)^2}+\frac{bf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{bf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{bf\cosh(e+fx)}{2d^2(c+dx)}-\frac{b\sinh(e+fx)}{2d(c+dx)^2}$$

input
```
Int[(a + b*Sinh[e + f*x])/(c + d*x)^3,x]
```

output
```
-1/2*a/(d*(c + d*x)^2) - (b*f*Cosh[e + f*x])/(2*d^2*(c + d*x)) + (b*f^2*Co
shIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/(2*d^3) - (b*Sinh[e + f*x])/(
2*d*(c + d*x)^2) + (b*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/(
2*d^3)
```

3.162.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.162.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(115) = 230$.

Time = 1.08 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.41

method	result
risch	$-\frac{a}{2d(dx+c)^2} - \frac{f^3 b\,\mathrm{e}^{-fx-e} x}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)} - \frac{f^3 b\,\mathrm{e}^{-fx-e} c}{4d^2(d^2x^2f^2+2cd\,f^2x+c^2f^2)} + \frac{f^2 b\,\mathrm{e}^{-fx-e}}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)} + \frac{f^2 b\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{Ei}_1}{4\cdots}$

input
```
int((a+b*sinh(f*x+e))/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/(d*x+c)^2-1/4*f^3*b*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^
2)*x-1/4*f^3*b*exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/4*f^2
*b*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/4*f^2*b/d^3*exp((c*f-
d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/4*f^2*b/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/4
*f^2*b/d^3*exp(f*x+e)/(c*f/d+f*x)-1/4*f^2*b/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*
x-e-(c*f-d*e)/d)
```

3.162.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(115) = 230$.

Time = 0.25 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.23

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx = -\frac{2\,bd^2\sinh(fx+e)+2\,ad^2+2\,(bd^2fx+bcdf)\cosh(fx+e)-\left((bd^2f^2x^2+2\,bcdf^2x+bc^2f^2)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)\right.}{}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*b*d^2*sinh(f*x + e) + 2*a*d^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(f*x +
 e) - ((b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei((d*f*x + c*f)/d) - (
b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*
e - c*f)/d) + ((b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei((d*f*x + c*f
)/d) + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*s
inh(-(d*e - c*f)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.162.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c)**3,x)
```

output
```
Timed out
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx = \frac{1}{2}\,b\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right) - \frac{a}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/2*b*(e^(-e + c*f/d)*exp_integral_e(3, (d*x + c)*f/d)/((d*x + c)^2*d) - e
^(e - c*f/d)*exp_integral_e(3, -(d*x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a/(d
^3*x^2 + 2*c*d^2*x + c^2*d)
```

3.162.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(115) = 230$.

Time = 0.27 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.59

$$\int \frac{a + b\sinh(e + fx)}{(c + dx)^3}\,dx$$
$$= \frac{bd^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} - bd^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} + 2\,bcdf^2x\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} - 2\,bcdf^2x\mathrm{Ei}\left(-\right.}{}$$

input
```
integrate((a+b*sinh(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(b*d^2*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - b*d^2*f^2*x^2*Ei(-(
d*f*x + c*f)/d)*e^(-e + c*f/d) + 2*b*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e -
c*f/d) - 2*b*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + b*c^2*f^2*Ei(
(d*f*x + c*f)/d)*e^(e - c*f/d) - b*c^2*f^2*Ei(-(d*f*x + c*f)/d)*e^(-e + c*
f/d) - b*d^2*f*x*e^(f*x + e) - b*d^2*f*x*e^(-f*x - e) - b*c*d*f*e^(f*x + e
) - b*c*d*f*e^(-f*x - e) - b*d^2*e^(f*x + e) + b*d^2*e^(-f*x - e) - 2*a*d^
2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sinh(e + fx)}{(c + dx)^3}\,dx = \int \frac{a + b\sinh(e + f\,x)}{(c + d\,x)^3}\,dx$$

input
```
int((a + b*sinh(e + f*x))/(c + d*x)^3,x)
```

output
```
int((a + b*sinh(e + f*x))/(c + d*x)^3, x)
```

3.163 $\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx$

3.163.1 Optimal result

Integrand size = 20, antiderivative size = 250

$$\begin{aligned}\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx = &-\frac{3b^2cd^2x}{4f^2}-\frac{3b^2d^3x^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}\\&-\frac{b^2(c+dx)^4}{8d}+\frac{12abd^2(c+dx)\cosh(e+fx)}{f^3}\\&+\frac{2ab(c+dx)^3\cosh(e+fx)}{f}-\frac{12abd^3\sinh(e+fx)}{f^4}\\&-\frac{6abd(c+dx)^2\sinh(e+fx)}{f^2}\\&+\frac{3b^2d^2(c+dx)\cosh(e+fx)\sinh(e+fx)}{4f^3}\\&+\frac{b^2(c+dx)^3\cosh(e+fx)\sinh(e+fx)}{2f}\\&-\frac{3b^2d^3\sinh^2(e+fx)}{8f^4}-\frac{3b^2d(c+dx)^2\sinh^2(e+fx)}{4f^2}\end{aligned}$$

output
```
-3/4*b^2*c*d^2*x/f^2-3/8*b^2*d^3*x^2/f^2+1/4*a^2*(d*x+c)^4/d-1/8*b^2*(d*x+
c)^4/d+12*a*b*d^2*(d*x+c)*cosh(f*x+e)/f^3+2*a*b*(d*x+c)^3*cosh(f*x+e)/f-12
*a*b*d^3*sinh(f*x+e)/f^4-6*a*b*d*(d*x+c)^2*sinh(f*x+e)/f^2+3/4*b^2*d^2*(d*
x+c)*cosh(f*x+e)*sinh(f*x+e)/f^3+1/2*b^2*(d*x+c)^3*cosh(f*x+e)*sinh(f*x+e)
/f-3/8*b^2*d^3*sinh(f*x+e)^2/f^4-3/4*b^2*d*(d*x+c)^2*sinh(f*x+e)^2/f^2
```

3.163.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.94

$$\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx$$
$$=\frac{32abf(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\cosh(e+fx)-3b^2d(2c^2f^2+4cdf^2x+d^2(1+2f^2x^2))\cosh}{}$$

input
```
Integrate[(c + d*x)^3*(a + b*Sinh[e + f*x])^2,x]
```

output
```
(32*a*b*f*(c + d*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f
*x] - 3*b^2*d*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(1 + 2*f^2*x^2))*Cosh[2*(e +
f*x)] + 2*((2*a^2 - b^2)*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)
 - 48*a*b*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x] + b^
2*f*(c + d*x)*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(3 + 2*f^2*x^2))*Sinh[2*(e +
f*x)]))/(16*f^4)
```

3.163.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^3(a-ib\sin(ie+ifx))^2dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(a^2(c+dx)^3+2ab(c+dx)^3\sinh(e+fx)+b^2(c+dx)^3\sinh^2(e+fx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^4}{4d}+\frac{12abd^2(c+dx)\cosh(e+fx)}{f^3}-\frac{6abd(c+dx)^2\sinh(e+fx)}{f^2}+\frac{2ab(c+dx)^3\cosh(e+fx)}{f}-\frac{12abd^3\sinh(e+fx)}{f^4}+\frac{3b^2d^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{4f^3}-\frac{3b^2d(c+dx)^2\sinh^2(e+fx)}{4f^2}+\frac{b^2(c+dx)^3\sinh(e+fx)\cosh(e+fx)}{2f}-\frac{3b^2d(c+dx)^2}{8f^2}-\frac{b^2(c+dx)^4}{8d}-\frac{3b^2d^3\sinh^2(e+fx)}{8f^4}$$

input
```
Int[(c + d*x)^3*(a + b*Sinh[e + f*x])^2,x]
```

output
```
(-3*b^2*d*(c + d*x)^2)/(8*f^2) + (a^2*(c + d*x)^4)/(4*d) - (b^2*(c + d*x)^
4)/(8*d) + (12*a*b*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (2*a*b*(c + d*x)^3*C
osh[e + f*x])/f - (12*a*b*d^3*Sinh[e + f*x])/f^4 - (6*a*b*d*(c + d*x)^2*Si
nh[e + f*x])/f^2 + (3*b^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(4*f^
3) + (b^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) - (3*b^2*d^3*Sinh
[e + f*x]^2)/(8*f^4) - (3*b^2*d*(c + d*x)^2*Sinh[e + f*x]^2)/(4*f^2)
```

3.163.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.163.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.85

method	result
parallelrisch	$\frac{4f\,b^2(dx+c)\left((dx+c)^2f^2+\frac{3d^2}{2}\right)\sinh(2fx+2e)-6d\,b^2\left((dx+c)^2f^2+\frac{d^2}{2}\right)\cosh(2fx+2e)+32f\left((dx+c)^2f^2+6d^2\right)b(dx+c)}{}$
risch	$\frac{a^2d^3x^4}{4}-\frac{d^3b^2x^4}{8}+a^2c\,d^2x^3-\frac{d^2b^2c\,x^3}{2}+\frac{3a^2c^2d\,x^2}{2}-\frac{3d\,b^2c^2x^2}{4}+c^3a^2x-\frac{b^2c^3x}{2}+\frac{a^2c^4}{4d}-\frac{b^2c^4}{8d}+$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^3*(a+b*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*f*b^2*(d*x+c)*((d*x+c)^2*f^2+3/2*d^2)*sinh(2*f*x+2*e)-6*d*b^2*((d*
x+c)^2*f^2+1/2*d^2)*cosh(2*f*x+2*e)+32*f*((d*x+c)^2*f^2+6*d^2)*b*(d*x+c)*a
*cosh(f*x+e)-96*d*b*a*((d*x+c)^2*f^2+2*d^2)*sinh(f*x+e)+16*(1/2*d*x+c)*(a^
2-1/2*b^2)*(1/2*d^2*x^2+c*d*x+c^2)*x*f^4+32*a*b*c^3*f^3+6*b^2*c^2*d*f^2+19
2*a*b*c*d^2*f+3*d^3*b^2)/f^4
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.67

$$\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx$$
$$=\frac{2\,(2\,a^2-b^2)d^3f^4x^4+8\,(2\,a^2-b^2)cd^2f^4x^3+12\,(2\,a^2-b^2)c^2df^4x^2+8\,(2\,a^2-b^2)c^3f^4x-3\,(2\,b^2d^3f^2x^2+}{}$$

input
```
integrate((d*x+c)^3*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/16*(2*(2*a^2 - b^2)*d^3*f^4*x^4 + 8*(2*a^2 - b^2)*c*d^2*f^4*x^3 + 12*(2*
a^2 - b^2)*c^2*d*f^4*x^2 + 8*(2*a^2 - b^2)*c^3*f^4*x - 3*(2*b^2*d^3*f^2*x^
2 + 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 + b^2*d^3)*cosh(f*x + e)^2 - 3*(2*
b^2*d^3*f^2*x^2 + 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 + b^2*d^3)*sinh(f*x
+ e)^2 + 32*(a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + a*b*c^3*f^3 + 6*a*b*c
*d^2*f + 3*(a*b*c^2*d*f^3 + 2*a*b*d^3*f)*x)*cosh(f*x + e) - 4*(24*a*b*d^3*
f^2*x^2 + 48*a*b*c*d^2*f^2*x + 24*a*b*c^2*d*f^2 + 48*a*b*d^3 - (2*b^2*d^3*
f^3*x^3 + 6*b^2*c*d^2*f^3*x^2 + 2*b^2*c^3*f^3 + 3*b^2*c*d^2*f + 3*(2*b^2*c
^2*d*f^3 + b^2*d^3*f)*x)*cosh(f*x + e))*sinh(f*x + e))/f^4
```

3.163.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(255) = 510$.

Time = 0.43 (sec) , antiderivative size = 779, normalized size of antiderivative = 3.12

$$\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx$$
$$= \begin{cases} a^2c^3x + \frac{3a^2c^2dx^2}{2} + a^2cd^2x^3 + \frac{a^2d^3x^4}{4} + \frac{2abc^3\cosh(e+fx)}{f} + \frac{6abc^2dx\cosh(e+fx)}{f} - \frac{6abc^2d\sinh(e+fx)}{f^2} + \frac{6abcd^2x^2\cosh(e+}{f} \\ (a+b\sinh(e))^2\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) \end{cases}$$

input
```
integrate((d*x+c)**3*(a+b*sinh(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c**3*x + 3*a**2*c**2*d*x**2/2 + a**2*c*d**2*x**3 + a**2*d*
*3*x**4/4 + 2*a*b*c**3*cosh(e + f*x)/f + 6*a*b*c**2*d*x*cosh(e + f*x)/f -
6*a*b*c**2*d*sinh(e + f*x)/f**2 + 6*a*b*c*d**2*x**2*cosh(e + f*x)/f - 12*a
*b*c*d**2*x*sinh(e + f*x)/f**2 + 12*a*b*c*d**2*cosh(e + f*x)/f**3 + 2*a*b*
d**3*x**3*cosh(e + f*x)/f - 6*a*b*d**3*x**2*sinh(e + f*x)/f**2 + 12*a*b*d*
*3*x*cosh(e + f*x)/f**3 - 12*a*b*d**3*sinh(e + f*x)/f**4 + b**2*c**3*x*sin
h(e + f*x)**2/2 - b**2*c**3*x*cosh(e + f*x)**2/2 + b**2*c**3*sinh(e + f*x)
*cosh(e + f*x)/(2*f) + 3*b**2*c**2*d*x**2*sinh(e + f*x)**2/4 - 3*b**2*c**2
*d*x**2*cosh(e + f*x)**2/4 + 3*b**2*c**2*d*x*sinh(e + f*x)*cosh(e + f*x)/(
2*f) - 3*b**2*c**2*d*sinh(e + f*x)**2/(4*f**2) + b**2*c*d**2*x**3*sinh(e +
 f*x)**2/2 - b**2*c*d**2*x**3*cosh(e + f*x)**2/2 + 3*b**2*c*d**2*x**2*sinh
(e + f*x)*cosh(e + f*x)/(2*f) - 3*b**2*c*d**2*x*sinh(e + f*x)**2/(4*f**2)
- 3*b**2*c*d**2*x*cosh(e + f*x)**2/(4*f**2) + 3*b**2*c*d**2*sinh(e + f*x)*
cosh(e + f*x)/(4*f**3) + b**2*d**3*x**4*sinh(e + f*x)**2/8 - b**2*d**3*x**
4*cosh(e + f*x)**2/8 + b**2*d**3*x**3*sinh(e + f*x)*cosh(e + f*x)/(2*f) -
3*b**2*d**3*x**2*sinh(e + f*x)**2/(8*f**2) - 3*b**2*d**3*x**2*cosh(e + f*x
)**2/(8*f**2) + 3*b**2*d**3*x*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) - 3*b**
2*d**3*sinh(e + f*x)**2/(8*f**4), Ne(f, 0)), ((a + b*sinh(e))**2*(c**3*x +
 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), True))
```

3.163.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(234) = 468$.

Time = 0.22 (sec) , antiderivative size = 520, normalized size of antiderivative = 2.08

$$
\begin{aligned}
\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx &= \frac{1}{4}\,a^2d^3x^4 + a^2cd^2x^3 + \frac{3}{2}\,a^2c^2dx^2 \\
&- \frac{3}{16}\left(4\,x^2 - \frac{\left(2\,fxe^{(2\,e)} - e^{(2\,e)}\right)e^{(2\,fx)}}{f^2} + \frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)b^2c^2d \\
&- \frac{1}{16}\left(8\,x^3 - \frac{3\left(2\,f^2x^2e^{(2\,e)} - 2\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(2\,fx)}}{f^3} + \frac{3\,(2\,f^2x^2+2\,fx+1)e^{(-2\,fx-2\,e)}}{f^3}\right)b^2cd^2 \\
&- \frac{1}{32}\left(4\,x^4 - \frac{\left(4\,f^3x^3e^{(2\,e)} - 6\,f^2x^2e^{(2\,e)} + 6\,fxe^{(2\,e)} - 3\,e^{(2\,e)}\right)e^{(2\,fx)}}{f^4} + \frac{(4\,f^3x^3+6\,f^2x^2+6\,fx+3)e^{(-2\,f}}{f^4}\right. \\
&- \frac{1}{8}\,b^2c^3\left(4\,x - \frac{e^{(2\,fx+2\,e)}}{f} + \frac{e^{(-2\,fx-2\,e)}}{f}\right) + a^2c^3x \\
&+ 3\,abc^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) \\
&+ 3\,abcd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2+2\,fx+2)e^{(-fx-e)}}{f^3}\right) \\
&+ abd^3\left(\frac{(f^3x^3e^e - 3\,f^2x^2e^e + 6\,fxe^e - 6\,e^e)e^{(fx)}}{f^4} + \frac{(f^3x^3+3\,f^2x^2+6\,fx+6)e^{(-fx-e)}}{f^4}\right) \\
&+ \frac{2\,abc^3\cosh(fx+e)}{f}
\end{aligned}
$$

input
```
integrate((d*x+c)^3*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 - 3/16*(4*x^2 - (2*f*x
*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*
c^2*d - 1/16*(8*x^3 - 3*(2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2
*f*x)/f^3 + 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*b^2*c*d^2 - 1/
32*(4*x^4 - (4*f^3*x^3*e^(2*e) - 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) - 3*e^(
2*e))*e^(2*f*x)/f^4 + (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x - 2*e)
/f^4)*b^2*d^3 - 1/8*b^2*c^3*(4*x - e^(2*f*x + 2*e)/f + e^(-2*f*x - 2*e)/f)
 + a^2*c^3*x + 3*a*b*c^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-f*
x - e)/f^2) + 3*a*b*c*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 +
 (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + a*b*d^3*((f^3*x^3*e^e - 3*f^2*x
^2*e^e + 6*f*x*e^e - 6*e^e)*e^(f*x)/f^4 + (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6
)*e^(-f*x - e)/f^4) + 2*a*b*c^3*cosh(f*x + e)/f
```

3.163. $\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx$

3.163.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(234) = 468$.

Time = 0.29 (sec) , antiderivative size = 598, normalized size of antiderivative = 2.39

$$\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx$$
$$= \frac{1}{4}a^2d^3x^4 - \frac{1}{8}b^2d^3x^4 + a^2cd^2x^3 - \frac{1}{2}b^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 - \frac{3}{4}b^2c^2dx^2 + a^2c^3x - \frac{1}{2}b^2c^3x$$
$$+ \frac{(4\,b^2d^3f^3x^3 + 12\,b^2cd^2f^3x^2 + 12\,b^2c^2df^3x - 6\,b^2d^3f^2x^2 + 4\,b^2c^3f^3 - 12\,b^2cd^2f^2x - 6\,b^2c^2df^2 + 6\,b^2d^3fx}{32\,f^4}$$
$$+ \frac{(abd^3f^3x^3 + 3\,abcd^2f^3x^2 + 3\,abc^2df^3x - 3\,abd^3f^2x^2 + abc^3f^3 - 6\,abcd^2f^2x - 3\,abc^2df^2 + 6\,abd^3fx + 6}{f^4}$$
$$+ \frac{(abd^3f^3x^3 + 3\,abcd^2f^3x^2 + 3\,abc^2df^3x + 3\,abd^3f^2x^2 + abc^3f^3 + 6\,abcd^2f^2x + 3\,abc^2df^2 + 6\,abd^3fx + 6}{f^4}$$
$$- \frac{(4\,b^2d^3f^3x^3 + 12\,b^2cd^2f^3x^2 + 12\,b^2c^2df^3x + 6\,b^2d^3f^2x^2 + 4\,b^2c^3f^3 + 12\,b^2cd^2f^2x + 6\,b^2c^2df^2 + 6\,b^2d^3fx}{32\,f^4}$$

input
```
integrate((d*x+c)^3*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/4*a^2*d^3*x^4 - 1/8*b^2*d^3*x^4 + a^2*c*d^2*x^3 - 1/2*b^2*c*d^2*x^3 + 3/
2*a^2*c^2*d*x^2 - 3/4*b^2*c^2*d*x^2 + a^2*c^3*x - 1/2*b^2*c^3*x + 1/32*(4*
b^2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x - 6*b^2*d^3*f^
2*x^2 + 4*b^2*c^3*f^3 - 12*b^2*c*d^2*f^2*x - 6*b^2*c^2*d*f^2 + 6*b^2*d^3*f
*x + 6*b^2*c*d^2*f - 3*b^2*d^3)*e^(2*f*x + 2*e)/f^4 + (a*b*d^3*f^3*x^3 + 3
*a*b*c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3*x - 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 -
 6*a*b*c*d^2*f^2*x - 3*a*b*c^2*d*f^2 + 6*a*b*d^3*f*x + 6*a*b*c*d^2*f - 6*a
*b*d^3)*e^(f*x + e)/f^4 + (a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + 3*a*b*c
^2*d*f^3*x + 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 + 6*a*b*c*d^2*f^2*x + 3*a*b*c
^2*d*f^2 + 6*a*b*d^3*f*x + 6*a*b*c*d^2*f + 6*a*b*d^3)*e^(-f*x - e)/f^4 - 1
/32*(4*b^2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x + 6*b^2
*d^3*f^2*x^2 + 4*b^2*c^3*f^3 + 12*b^2*c*d^2*f^2*x + 6*b^2*c^2*d*f^2 + 6*b^
2*d^3*f*x + 6*b^2*c*d^2*f + 3*b^2*d^3)*e^(-2*f*x - 2*e)/f^4
```

3.163.9 Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.92

$$\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx = a^2\,c^3\,x-\frac{b^2\,c^3\,x}{2}+\frac{a^2\,d^3\,x^4}{4}-\frac{b^2\,d^3\,x^4}{8}+\frac{3\,a^2\,c^2\,d\,x^2}{2}+a^2\,c\,d^2\,x^3-\frac{3\,b^2\,c^2\,d\,x^2}{4}-\frac{b^2\,c\,d^2\,x^3}{2}-\frac{3\,b^2\,d^3\cosh(2\,e+2\,f\,x)}{16\,f^4}+\frac{b^2\,c^3\sinh(2\,e+2\,f\,x)}{4\,f}+\frac{2\,a\,b\,c^3\cosh(e+f\,x)}{f}-\frac{12\,a\,b\,d^3\sinh(e+f\,x)}{f^4}-\frac{3\,b^2\,d^3\,x^2\cosh(2\,e+2\,f\,x)}{8\,f^2}+\frac{b^2\,d^3\,x^3\sinh(2\,e+2\,f\,x)}{4\,f}-\frac{3\,b^2\,c^2\,d\cosh(2\,e+2\,f\,x)}{8\,f^2}+\frac{3\,b^2\,c\,d^2\sinh(2\,e+2\,f\,x)}{8\,f^3}+\frac{3\,b^2\,d^3\,x\sinh(2\,e+2\,f\,x)}{8\,f^3}-\frac{3\,b^2\,c\,d^2\,x\cosh(2\,e+2\,f\,x)}{4\,f^2}+\frac{3\,b^2\,c^2\,d\,x\sinh(2\,e+2\,f\,x)}{4\,f}+\frac{12\,a\,b\,c\,d^2\cosh(e+f\,x)}{f^3}-\frac{6\,a\,b\,c^2\,d\sinh(e+f\,x)}{f^2}+\frac{12\,a\,b\,d^3\,x\cosh(e+f\,x)}{f^3}+\frac{3\,b^2\,c\,d^2\,x^2\sinh(2\,e+2\,f\,x)}{4\,f}+\frac{2\,a\,b\,d^3\,x^3\cosh(e+f\,x)}{f}-\frac{6\,a\,b\,d^3\,x^2\sinh(e+f\,x)}{f^2}+\frac{6\,a\,b\,c\,d^2\,x^2\cosh(e+f\,x)}{f}+\frac{6\,a\,b\,c^2\,d\,x\cosh(e+f\,x)}{f}-\frac{12\,a\,b\,c\,d^2\,x\sinh(e+f\,x)}{f^2}$$

input
```
int((a + b*sinh(e + f*x))^2*(c + d*x)^3,x)
```

output
```
a^2*c^3*x - (b^2*c^3*x)/2 + (a^2*d^3*x^4)/4 - (b^2*d^3*x^4)/8 + (3*a^2*c^2
*d*x^2)/2 + a^2*c*d^2*x^3 - (3*b^2*c^2*d*x^2)/4 - (b^2*c*d^2*x^3)/2 - (3*b
^2*d^3*cosh(2*e + 2*f*x))/(16*f^4) + (b^2*c^3*sinh(2*e + 2*f*x))/(4*f) + (
2*a*b*c^3*cosh(e + f*x))/f - (12*a*b*d^3*sinh(e + f*x))/f^4 - (3*b^2*d^3*x
^2*cosh(2*e + 2*f*x))/(8*f^2) + (b^2*d^3*x^3*sinh(2*e + 2*f*x))/(4*f) - (3
*b^2*c^2*d*cosh(2*e + 2*f*x))/(8*f^2) + (3*b^2*c*d^2*sinh(2*e + 2*f*x))/(8
*f^3) + (3*b^2*d^3*x*sinh(2*e + 2*f*x))/(8*f^3) - (3*b^2*c*d^2*x*cosh(2*e
+ 2*f*x))/(4*f^2) + (3*b^2*c^2*d*x*sinh(2*e + 2*f*x))/(4*f) + (12*a*b*c*d^
2*cosh(e + f*x))/f^3 - (6*a*b*c^2*d*sinh(e + f*x))/f^2 + (12*a*b*d^3*x*cos
h(e + f*x))/f^3 + (3*b^2*c*d^2*x^2*sinh(2*e + 2*f*x))/(4*f) + (2*a*b*d^3*x
^3*cosh(e + f*x))/f - (6*a*b*d^3*x^2*sinh(e + f*x))/f^2 + (6*a*b*c*d^2*x^2
*cosh(e + f*x))/f + (6*a*b*c^2*d*x*cosh(e + f*x))/f - (12*a*b*c*d^2*x*sinh
(e + f*x))/f^2
```

3.163. $\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx$

3.164 $\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx$

3.164.1 Optimal result

Integrand size = 20, antiderivative size = 182

$$\begin{aligned}\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx = & -\frac{b^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{3d}-\frac{b^2(c+dx)^3}{6d} \\ & +\frac{4abd^2\cosh(e+fx)}{f^3}+\frac{2ab(c+dx)^2\cosh(e+fx)}{f} \\ & -\frac{4abd(c+dx)\sinh(e+fx)}{f^2} \\ & +\frac{b^2d^2\cosh(e+fx)\sinh(e+fx)}{4f^3} \\ & +\frac{b^2(c+dx)^2\cosh(e+fx)\sinh(e+fx)}{2f} \\ & -\frac{b^2d(c+dx)\sinh^2(e+fx)}{2f^2}\end{aligned}$$

output
```
-1/4*b^2*d^2*x/f^2+1/3*a^2*(d*x+c)^3/d-1/6*b^2*(d*x+c)^3/d+4*a*b*d^2*cosh(
f*x+e)/f^3+2*a*b*(d*x+c)^2*cosh(f*x+e)/f-4*a*b*d*(d*x+c)*sinh(f*x+e)/f^2+1
/4*b^2*d^2*cosh(f*x+e)*sinh(f*x+e)/f^3+1/2*b^2*(d*x+c)^2*cosh(f*x+e)*sinh(
f*x+e)/f-1/2*b^2*d*(d*x+c)*sinh(f*x+e)^2/f^2
```

3.164.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.37

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx$$
$$= \frac{24a^2c^2f^3x-12b^2c^2f^3x+24a^2cdf^3x^2-12b^2cdf^3x^2+8a^2d^2f^3x^3-4b^2d^2f^3x^3+48ab(c^2f^2+2cdf^2x+d^2(}{}$$

input
```
Integrate[(c + d*x)^2*(a + b*Sinh[e + f*x])^2,x]
```

output
```
(24*a^2*c^2*f^3*x - 12*b^2*c^2*f^3*x + 24*a^2*c*d*f^3*x^2 - 12*b^2*c*d*f^3
*x^2 + 8*a^2*d^2*f^3*x^3 - 4*b^2*d^2*f^3*x^3 + 48*a*b*(c^2*f^2 + 2*c*d*f^2
*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x] - 6*b^2*d*f*(c + d*x)*Cosh[2*(e + f*
x)] - 96*a*b*c*d*f*Sinh[e + f*x] - 96*a*b*d^2*f*x*Sinh[e + f*x] + 3*b^2*d^
2*Sinh[2*(e + f*x)] + 6*b^2*c^2*f^2*Sinh[2*(e + f*x)] + 12*b^2*c*d*f^2*x*S
inh[2*(e + f*x)] + 6*b^2*d^2*f^2*x^2*Sinh[2*(e + f*x)])/(24*f^3)
```

3.164.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^2(a-ib\sin(ie+ifx))^2dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(a^2(c+dx)^2+2ab(c+dx)^2\sinh(e+fx)+b^2(c+dx)^2\sinh^2(e+fx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^3}{3d}-\frac{4abd(c+dx)\sinh(e+fx)}{f^2}+\frac{2ab(c+dx)^2\cosh(e+fx)}{f}+\frac{4abd^2\cosh(e+fx)}{f^3}-\frac{b^2d(c+dx)\sinh^2(e+fx)}{2f^2}+\frac{b^2(c+dx)^2\sinh(e+fx)\cosh(e+fx)}{2f}-\frac{b^2(c+dx)^3}{6d}+\frac{b^2d^2\sinh(e+fx)\cosh(e+fx)}{4f^3}-\frac{b^2d^2x}{4f^2}$$

input
```
Int[(c + d*x)^2*(a + b*Sinh[e + f*x])^2,x]
```

output
```
-1/4*(b^2*d^2*x)/f^2 + (a^2*(c + d*x)^3)/(3*d) - (b^2*(c + d*x)^3)/(6*d) +
 (4*a*b*d^2*Cosh[e + f*x])/f^3 + (2*a*b*(c + d*x)^2*Cosh[e + f*x])/f - (4*
a*b*d*(c + d*x)*Sinh[e + f*x])/f^2 + (b^2*d^2*Cosh[e + f*x]*Sinh[e + f*x])
/(4*f^3) + (b^2*(c + d*x)^2*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) - (b^2*d*(c
 + d*x)*Sinh[e + f*x]^2)/(2*f^2)
```

3.164.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.164.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.86

method	result
parallelrisch	$\frac{b^2\left((dx+c)^2 f^2+\frac{d^2}{2}\right)\sinh(2fx+2e)-b^2 df(dx+c)\cosh(2fx+2e)+8ba\left((dx+c)^2 f^2+2d^2\right)\cosh(fx+e)-16abdf(dx+c)\sin}{4f^3}$
risch	$\frac{a^2d^2x^3}{3}-\frac{d^2b^2x^3}{6}+d\,a^2c\,x^2-\frac{d\,b^2c\,x^2}{2}+a^2c^2x-\frac{b^2c^2x}{2}+\frac{a^2c^3}{3d}-\frac{b^2c^3}{6d}+\frac{b^2\left(2d^2x^2f^2+4cd\,f^2x+2c^2f\right.}{16}$
parts	$\frac{a^2(dx+c)^3}{3d}+\frac{b^2\left(\frac{d^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^3}{6}-\frac{(fx+e)\cosh(fx+e)^2}{2}+\frac{\cosh(fx+e)\sinh(fx+e)}{4}+\frac{fx}{4}+\frac{e}{4}\right)}{f^2}-\right.}{}$
derivativedivides	$\frac{d^2a^2(fx+e)^3}{3f^2}+\frac{2d^2ab\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}+\frac{d^2b^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e}{6}\right.}{}$
default	$\frac{d^2a^2(fx+e)^3}{3f^2}+\frac{2d^2ab\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}+\frac{d^2b^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e}{6}\right.}{}$

input
```
int((d*x+c)^2*(a+b*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b^2*((d*x+c)^2*f^2+1/2*d^2)*sinh(2*f*x+2*e)-b^2*d*f*(d*x+c)*cosh(2*f*
x+2*e)+8*b*a*((d*x+c)^2*f^2+2*d^2)*cosh(f*x+e)-16*a*b*d*f*(d*x+c)*sinh(f*x
+e)+4*(1/3*d^2*x^2+c*d*x+c^2)*(a^2-1/2*b^2)*x*f^3+8*a*b*c^2*f^2+b^2*c*d*f+
16*a*d^2*b)/f^3
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.36

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx$$
$$=\frac{2\,(2\,a^2-b^2)d^2f^3x^3+6\,(2\,a^2-b^2)cdf^3x^2+6\,(2\,a^2-b^2)c^2f^3x-3\,(b^2d^2fx+b^2cdf)\cosh\left(fx+e\right)^2-3\,(b^2}{}$$

input
```
integrate((d*x+c)^2*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/12*(2*(2*a^2 - b^2)*d^2*f^3*x^3 + 6*(2*a^2 - b^2)*c*d*f^3*x^2 + 6*(2*a^2
 - b^2)*c^2*f^3*x - 3*(b^2*d^2*f*x + b^2*c*d*f)*cosh(f*x + e)^2 - 3*(b^2*d
^2*f*x + b^2*c*d*f)*sinh(f*x + e)^2 + 24*(a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*
x + a*b*c^2*f^2 + 2*a*b*d^2)*cosh(f*x + e) - 3*(16*a*b*d^2*f*x + 16*a*b*c*
d*f - (2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 + b^2*d^2)*cosh
(f*x + e))*sinh(f*x + e))/f^3
```

3.164.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(177) = 354$.

Time = 0.33 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.51

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx$$
$$= \begin{cases} a^2c^2x + a^2cdx^2 + \frac{a^2d^2x^3}{3} + \frac{2abc^2\cosh(e+fx)}{f} + \frac{4abcdx\cosh(e+fx)}{f} - \frac{4abcd\sinh(e+fx)}{f^2} + \frac{2abd^2x^2\cosh(e+fx)}{f} - \frac{4abd^2x\sinh}{f} \\ (a+b\sinh(e))^2\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(a+b*sinh(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c**2*x + a**2*c*d*x**2 + a**2*d**2*x**3/3 + 2*a*b*c**2*cos
h(e + f*x)/f + 4*a*b*c*d*x*cosh(e + f*x)/f - 4*a*b*c*d*sinh(e + f*x)/f**2
+ 2*a*b*d**2*x**2*cosh(e + f*x)/f - 4*a*b*d**2*x*sinh(e + f*x)/f**2 + 4*a*
b*d**2*cosh(e + f*x)/f**3 + b**2*c**2*x*sinh(e + f*x)**2/2 - b**2*c**2*x*c
osh(e + f*x)**2/2 + b**2*c**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) + b**2*c*d
*x**2*sinh(e + f*x)**2/2 - b**2*c*d*x**2*cosh(e + f*x)**2/2 + b**2*c*d*x*s
inh(e + f*x)*cosh(e + f*x)/f - b**2*c*d*sinh(e + f*x)**2/(2*f**2) + b**2*d
**2*x**3*sinh(e + f*x)**2/6 - b**2*d**2*x**3*cosh(e + f*x)**2/6 + b**2*d**
2*x**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d**2*x*sinh(e + f*x)**2/(4
*f**2) - b**2*d**2*x*cosh(e + f*x)**2/(4*f**2) + b**2*d**2*sinh(e + f*x)*c
osh(e + f*x)/(4*f**3), Ne(f, 0)), ((a + b*sinh(e))**2*(c**2*x + c*d*x**2 +
 d**2*x**3/3), True))
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.77

$$
\begin{aligned}
&\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx \\
&= \frac{1}{3}\,a^2d^2x^3 + a^2cdx^2 - \frac{1}{8}\left(4\,x^2 - \frac{\left(2\,fxe^{(2\,e)} - e^{(2\,e)}\right)e^{(2\,fx)}}{f^2} + \frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)b^2cd \\
&\quad - \frac{1}{48}\left(8\,x^3 - \frac{3\left(2\,f^2x^2e^{(2\,e)} - 2\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(2\,fx)}}{f^3} + \frac{3\left(2\,f^2x^2 + 2\,fx + 1\right)e^{(-2\,fx-2\,e)}}{f^3}\right)b^2d^2 \\
&\quad - \frac{1}{8}\,b^2c^2\left(4\,x - \frac{e^{(2\,fx+2\,e)}}{f} + \frac{e^{(-2\,fx-2\,e)}}{f}\right) + a^2c^2x \\
&\quad + 2\,abcd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) \\
&\quad + abd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right) \\
&\quad + \frac{2\,abc^2\cosh\left(fx+e\right)}{f}
\end{aligned}
$$

input
```
integrate((d*x+c)^2*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*d^2*x^3 + a^2*c*d*x^2 - 1/8*(4*x^2 - (2*f*x*e^(2*e) - e^(2*e))*e^(
2*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*c*d - 1/48*(8*x^3 - 3*(
2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x)/f^3 + 3*(2*f^2*x^2
+ 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*b^2*d^2 - 1/8*b^2*c^2*(4*x - e^(2*f*x +
 2*e)/f + e^(-2*f*x - 2*e)/f) + a^2*c^2*x + 2*a*b*c*d*((f*x*e^e - e^e)*e^(
f*x)/f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + a*b*d^2*((f^2*x^2*e^e - 2*f*x*e^e
 + 2*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + 2*a*b*c^
2*cosh(f*x + e)/f
```

3.164.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(170) = 340$.

Time = 0.27 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.89

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx$$
$$= \frac{1}{3}a^2d^2x^3 - \frac{1}{6}b^2d^2x^3 + a^2cdx^2 - \frac{1}{2}b^2cdx^2 + a^2c^2x - \frac{1}{2}b^2c^2x$$
$$+ \frac{(2b^2d^2f^2x^2 + 4b^2cdf^2x + 2b^2c^2f^2 - 2b^2d^2fx - 2b^2cdf + b^2d^2)e^{(2fx+2e)}}{16f^3}$$
$$+ \frac{(abd^2f^2x^2 + 2abcdf^2x + abc^2f^2 - 2abd^2fx - 2abcdf + 2abd^2)e^{(fx+e)}}{f^3}$$
$$+ \frac{(abd^2f^2x^2 + 2abcdf^2x + abc^2f^2 + 2abd^2fx + 2abcdf + 2abd^2)e^{(-fx-e)}}{f^3}$$
$$- \frac{(2b^2d^2f^2x^2 + 4b^2cdf^2x + 2b^2c^2f^2 + 2b^2d^2fx + 2b^2cdf + b^2d^2)e^{(-2fx-2e)}}{16f^3}$$

input
```
integrate((d*x+c)^2*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*a^2*d^2*x^3 - 1/6*b^2*d^2*x^3 + a^2*c*d*x^2 - 1/2*b^2*c*d*x^2 + a^2*c^
2*x - 1/2*b^2*c^2*x + 1/16*(2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^
2*f^2 - 2*b^2*d^2*f*x - 2*b^2*c*d*f + b^2*d^2)*e^(2*f*x + 2*e)/f^3 + (a*b*
d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 - 2*a*b*d^2*f*x - 2*a*b*c*d*f
+ 2*a*b*d^2)*e^(f*x + e)/f^3 + (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^
2*f^2 + 2*a*b*d^2*f*x + 2*a*b*c*d*f + 2*a*b*d^2)*e^(-f*x - e)/f^3 - 1/16*(
2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 + 2*b^2*d^2*f*x + 2*b^
2*c*d*f + b^2*d^2)*e^(-2*f*x - 2*e)/f^3
```

3.164. $\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx$

3.164.9 Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.54

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx = a^2\,c^2\,x-\frac{b^2\,c^2\,x}{2}+\frac{a^2\,d^2\,x^3}{3}-\frac{b^2\,d^2\,x^3}{6}+\frac{b^2\,c^2\,\sinh(2\,e+2\,f\,x)}{4\,f}+\frac{b^2\,d^2\,\sinh(2\,e+2\,f\,x)}{8\,f^3}+a^2\,c\,d\,x^2-\frac{b^2\,c\,d\,x^2}{2}+\frac{2\,a\,b\,c^2\,\cosh(e+f\,x)}{f}+\frac{4\,a\,b\,d^2\,\cosh(e+f\,x)}{f^3}+\frac{b^2\,d^2\,x^2\,\sinh(2\,e+2\,f\,x)}{4\,f}-\frac{b^2\,c\,d\,\cosh(2\,e+2\,f\,x)}{4\,f^2}-\frac{b^2\,d^2\,x\,\cosh(2\,e+2\,f\,x)}{4\,f^2}-\frac{4\,a\,b\,c\,d\,\sinh(e+f\,x)}{f^2}-\frac{4\,a\,b\,d^2\,x\,\sinh(e+f\,x)}{f^2}+\frac{2\,a\,b\,d^2\,x^2\,\cosh(e+f\,x)}{f}+\frac{b^2\,c\,d\,x\,\sinh(2\,e+2\,f\,x)}{2\,f}+\frac{4\,a\,b\,c\,d\,x\,\cosh(e+f\,x)}{f}$$

input
```
int((a + b*sinh(e + f*x))^2*(c + d*x)^2,x)
```

output
```
a^2*c^2*x - (b^2*c^2*x)/2 + (a^2*d^2*x^3)/3 - (b^2*d^2*x^3)/6 + (b^2*c^2*s
inh(2*e + 2*f*x))/(4*f) + (b^2*d^2*sinh(2*e + 2*f*x))/(8*f^3) + a^2*c*d*x^
2 - (b^2*c*d*x^2)/2 + (2*a*b*c^2*cosh(e + f*x))/f + (4*a*b*d^2*cosh(e + f*
x))/f^3 + (b^2*d^2*x^2*sinh(2*e + 2*f*x))/(4*f) - (b^2*c*d*cosh(2*e + 2*f*
x))/(4*f^2) - (b^2*d^2*x*cosh(2*e + 2*f*x))/(4*f^2) - (4*a*b*c*d*sinh(e +
f*x))/f^2 - (4*a*b*d^2*x*sinh(e + f*x))/f^2 + (2*a*b*d^2*x^2*cosh(e + f*x)
)/f + (b^2*c*d*x*sinh(2*e + 2*f*x))/(2*f) + (4*a*b*c*d*x*cosh(e + f*x))/f
```

3.165 $\int (c + dx)(a + b\sinh(e + fx))^2 \, dx$

3.165.1 Optimal result

Integrand size = 18, antiderivative size = 116

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx = -\frac{1}{2}b^2cx - \frac{1}{4}b^2dx^2 + \frac{a^2(c+dx)^2}{2d} + \frac{2ab(c+dx)\cosh(e+fx)}{f} - \frac{2abd\sinh(e+fx)}{f^2} + \frac{b^2(c+dx)\cosh(e+fx)\sinh(e+fx)}{2f} - \frac{b^2d\sinh^2(e+fx)}{4f^2}$$

output
```
-1/2*b^2*c*x-1/4*b^2*d*x^2+1/2*a^2*(d*x+c)^2/d+2*a*b*(d*x+c)*cosh(f*x+e)/f
-2*a*b*d*sinh(f*x+e)/f^2+1/2*b^2*(d*x+c)*cosh(f*x+e)*sinh(f*x+e)/f-1/4*b^2
*d*sinh(f*x+e)^2/f^2
```

3.165.2 Mathematica [A] (verified)

Time = 4.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx = -\frac{2(2a^2-b^2)(e+fx)(-2cf+d(e-fx)) - 16abf(c+dx)\cosh(e+fx) + b^2d\cosh(2(e+fx)) + 16abd}{8f^2}$$

input
```
Integrate[(c + d*x)*(a + b*Sinh[e + f*x])^2,x]
```

output
```
-1/8*(2*(2*a^2 - b^2)*(e + f*x)*(-2*c*f + d*(e - f*x)) - 16*a*b*f*(c + d*x
)*Cosh[e + f*x] + b^2*d*Cosh[2*(e + f*x)] + 16*a*b*d*Sinh[e + f*x] - 2*b^2
*f*(c + d*x)*Sinh[2*(e + f*x)])/f^2
```

3.165.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx$$
$$\Big\downarrow \ 3042$$
$$\int (c+dx)(a-ib\sin(ie+ifx))^2dx$$
$$\Big\downarrow \ 3798$$
$$\int \left(a^2(c+dx)+2ab(c+dx)\sinh(e+fx)+b^2(c+dx)\sinh^2(e+fx)\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\cosh(e+fx)}{f}-\frac{2abd\sinh(e+fx)}{f^2}+\frac{b^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}-\frac{b^2(c+dx)^2}{4d}-\frac{b^2d\sinh^2(e+fx)}{4f^2}$$

input
```
Int[(c + d*x)*(a + b*Sinh[e + f*x])^2,x]
```

output
```
(a^2*(c + d*x)^2)/(2*d) - (b^2*(c + d*x)^2)/(4*d) + (2*a*b*(c + d*x)*Cosh[
e + f*x])/f - (2*a*b*d*Sinh[e + f*x])/f^2 + (b^2*(c + d*x)*Cosh[e + f*x]*S
inh[e + f*x])/(2*f) - (b^2*d*Sinh[e + f*x]^2)/(4*f^2)
```

3.165.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.165.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

method	result
parallelrisch	$\frac{2b^2f(dx+c)\sinh(2fx+2e)-b^2d\cosh(2fx+2e)+16abf(dx+c)\cosh(fx+e)-16adb\sinh(fx+e)+\left(\left(-2dx^2-4cx\right)f^2+d\right)b^2}{8f^2}$
risch	$\frac{a^2dx^2}{2}+a^2cx-\frac{dx^2b^2}{4}-\frac{b^2cx}{2}+\frac{b^2(2dfx+2cf-d)e^{2fx+2e}}{16f^2}+\frac{ab(dfx+cf-d)e^{fx+e}}{f^2}+\frac{ab(dfx+cf+d)e^{-fx-}}{f^2}$
parts	$a^2\left(\frac{1}{2}dx^2+cx\right)+\frac{b^2\left(\frac{d\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)}{f}-\frac{de\left(\frac{\cosh(fx+e)\sinh(fx+e)}{2}-\frac{fx}{2}-\right)}{f}\right)}{f}$
derivativedivides	$\frac{\frac{da^2(fx+e)^2}{2f}+\frac{2dab((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}+\frac{db^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)}{f}-\frac{dea^2(f}{f}}{}$
default	$\frac{\frac{da^2(fx+e)^2}{2f}+\frac{2dab((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}+\frac{db^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)}{f}-\frac{dea^2(f}{f}}{}$

input
```
int((d*x+c)*(a+b*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*b^2*f*(d*x+c)*sinh(2*f*x+2*e)-b^2*d*cosh(2*f*x+2*e)+16*a*b*f*(d*x+c
)*cosh(f*x+e)-16*a*d*b*sinh(f*x+e)+((-2*d*x^2-4*c*x)*f^2+d)*b^2+16*a*b*c*f
+8*f^2*(1/2*d*x+c)*x*a^2)/f^2
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx$$
$$= \frac{2\,(2\,a^2-b^2)df^2x^2+4\,(2\,a^2-b^2)cf^2x-b^2d\cosh\left(fx+e\right)^2-b^2d\sinh\left(fx+e\right)^2+16\,(abdfx+abcf)\cosh}{8\,f^2}$$

input
```
integrate((d*x+c)*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^2 - b^2)*d*f^2*x^2 + 4*(2*a^2 - b^2)*c*f^2*x - b^2*d*cosh(f*x
+ e)^2 - b^2*d*sinh(f*x + e)^2 + 16*(a*b*d*f*x + a*b*c*f)*cosh(f*x + e) -
4*(4*a*b*d - (b^2*d*f*x + b^2*c*f)*cosh(f*x + e))*sinh(f*x + e))/f^2
```

3.165.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.89

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx$$
$$= \begin{cases} a^2cx+\frac{a^2dx^2}{2}+\frac{2abc\cosh\left(e+fx\right)}{f}+\frac{2abdx\cosh\left(e+fx\right)}{f}-\frac{2abd\sinh\left(e+fx\right)}{f^2}+\frac{b^2cx\sinh^2\left(e+fx\right)}{2}-\frac{b^2cx\cosh^2\left(e+fx\right)}{2}+\frac{b^2c\sinh}{} \\ \left(a+b\sinh\left(e\right)\right)^2\left(cx+\frac{dx^2}{2}\right) \end{cases}$$

input
```
integrate((d*x+c)*(a+b*sinh(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c*x + a**2*d*x**2/2 + 2*a*b*c*cosh(e + f*x)/f + 2*a*b*d*x*
cosh(e + f*x)/f - 2*a*b*d*sinh(e + f*x)/f**2 + b**2*c*x*sinh(e + f*x)**2/2
 - b**2*c*x*cosh(e + f*x)**2/2 + b**2*c*sinh(e + f*x)*cosh(e + f*x)/(2*f)
+ b**2*d*x**2*sinh(e + f*x)**2/4 - b**2*d*x**2*cosh(e + f*x)**2/4 + b**2*d
*x*sinh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d*sinh(e + f*x)**2/(4*f**2), N
e(f, 0)), ((a + b*sinh(e))**2*(c*x + d*x**2/2), True))
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.41

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx$$
$$= \frac{1}{2}a^2dx^2 - \frac{1}{16}\left(4x^2 - \frac{(2fxe^{(2e)} - e^{(2e)})e^{(2fx)}}{f^2} + \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)b^2d$$
$$- \frac{1}{8}b^2c\left(4x - \frac{e^{(2fx+2e)}}{f} + \frac{e^{(-2fx-2e)}}{f}\right) + a^2cx$$
$$+ abd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{2abc\cosh(fx+e)}{f}$$

input
```
integrate((d*x+c)*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*d*x^2 - 1/16*(4*x^2 - (2*f*x*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2
*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*d - 1/8*b^2*c*(4*x - e^(2*f*x + 2*e)/f
 + e^(-2*f*x - 2*e)/f) + a^2*c*x + a*b*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f
*x + 1)*e^(-f*x - e)/f^2) + 2*a*b*c*cosh(f*x + e)/f
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.37

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx = \frac{1}{2}a^2dx^2 - \frac{1}{4}b^2dx^2 + a^2cx - \frac{1}{2}b^2cx$$
$$+ \frac{(2b^2dfx + 2b^2cf - b^2d)e^{(2fx+2e)}}{16f^2}$$
$$+ \frac{(abdfx + abcf - abd)e^{(fx+e)}}{f^2}$$
$$+ \frac{(abdfx + abcf + abd)e^{(-fx-e)}}{f^2}$$
$$- \frac{(2b^2dfx + 2b^2cf + b^2d)e^{(-2fx-2e)}}{16f^2}$$

input
```
integrate((d*x+c)*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/2*a^2*d*x^2 - 1/4*b^2*d*x^2 + a^2*c*x - 1/2*b^2*c*x + 1/16*(2*b^2*d*f*x
+ 2*b^2*c*f - b^2*d)*e^(2*f*x + 2*e)/f^2 + (a*b*d*f*x + a*b*c*f - a*b*d)*e
^(f*x + e)/f^2 + (a*b*d*f*x + a*b*c*f + a*b*d)*e^(-f*x - e)/f^2 - 1/16*(2*
b^2*d*f*x + 2*b^2*c*f + b^2*d)*e^(-2*f*x - 2*e)/f^2
```

3.165.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx = \frac{a^2\,d\,x^2}{2} - \frac{b^2\,d\,x^2}{4} + a^2\,c\,x - \frac{b^2\,c\,x}{2} - \frac{b^2\,d\cosh(e+f\,x)^2}{4\,f^2} + \frac{b^2\,c\cosh(e+f\,x)\,\sinh(e+f\,x)}{2\,f} + \frac{2\,a\,b\,c\cosh(e+f\,x)}{f} - \frac{2\,a\,b\,d\sinh(e+f\,x)}{f^2} + \frac{2\,a\,b\,d\,x\cosh(e+f\,x)}{f} + \frac{b^2\,d\,x\cosh(e+f\,x)\,\sinh(e+f\,x)}{2\,f}$$

input
```
int((a + b*sinh(e + f*x))^2*(c + d*x),x)
```

output
```
(a^2*d*x^2)/2 - (b^2*d*x^2)/4 + a^2*c*x - (b^2*c*x)/2 - (b^2*d*cosh(e + f*
x)^2)/(4*f^2) + (b^2*c*cosh(e + f*x)*sinh(e + f*x))/(2*f) + (2*a*b*c*cosh(
e + f*x))/f - (2*a*b*d*sinh(e + f*x))/f^2 + (2*a*b*d*x*cosh(e + f*x))/f +
(b^2*d*x*cosh(e + f*x)*sinh(e + f*x))/(2*f)
```

3.166 $\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx$

3.166.1 Optimal result

Integrand size = 20, antiderivative size = 156

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx = \frac{b^2\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{a^2\log(c+dx)}{d} - \frac{b^2\log(c+dx)}{2d}+\frac{2ab\mathrm{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d} + \frac{2ab\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(\frac{cf}{d}+fx\right)}{d} + \frac{b^2\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2cf}{d}+2fx\right)}{2d}$$

output
```
1/2*b^2*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/d+a^2*ln(d*x+c)/d-1/2*b^2*ln
(d*x+c)/d+2*a*b*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d-1/2*b^2*Shi(2*c*f/d+2*f*x)
*sinh(-2*e+2*c*f/d)/d-2*a*b*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d
```

3.166.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx = \frac{b^2\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2f(c+dx)}{d}\right)+2a^2\log(c+dx)-b^2\log(c+dx)+4ab\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+}{2d}$$

input
```
Integrate[(a + b*Sinh[e + f*x])^2/(c + d*x),x]
```

output
```
(b^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d] + 2*a^2*Log[c +
 d*x] - b^2*Log[c + d*x] + 4*a*b*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/
d] + 4*a*b*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + b^2*Sinh[2*e - (2
*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d])/(2*d)
```

3.166.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a-ib\sin(ie+ifx))^2}{c+dx}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a^2}{c+dx}+\frac{2ab\sinh(e+fx)}{c+dx}+\frac{b^2\sinh^2(e+fx)}{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\log(c+dx)}{d}+\frac{2ab\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{b^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2d}+\frac{b^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{2d}-\frac{b^2\log(c+dx)}{2d}$$

input
```
Int[(a + b*Sinh[e + f*x])^2/(c + d*x),x]
```

output
```
(b^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (a^2*L
og[c + d*x])/d - (b^2*Log[c + d*x])/(2*d) + (2*a*b*CoshIntegral[(c*f)/d +
f*x]*Sinh[e - (c*f)/d])/d + (2*a*b*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d
+ f*x])/d + (b^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2
*d)
```

3.166.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.166.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.29

method	result
risch	$-\frac{ab\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{Ei}_1\left(-fx-e-\frac{cf-de}{d}\right)}{d}+\frac{a^2\ln(dx+c)}{d}-\frac{b^2\ln(dx+c)}{2d}-\frac{b^2\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{Ei}_1\left(2fx+2e+\frac{2cf-2de}{d}\right)}{4d}-\frac{b^2\mathrm{e}^{-\frac{2(cf-de)}{d}}\,\mathrm{E}}{}$

input
```
int((a+b*sinh(f*x+e))^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-a*b/d*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)+a^2*ln(d*x+c)/d-1/2*b^2*
ln(d*x+c)/d-1/4*b^2/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/4
*b^2/d*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)+a*b/d*exp((c*f-d
*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.49

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx$$
$$= \frac{4\left(ab\mathrm{Ei}\left(\frac{dfx+cf}{d}\right) - ab\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right) + \left(b^2\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right) + b^2\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\right)\cosh\left(-\frac{2\,(de-c}{d}\right.}{}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

output
```
1/4*(4*(a*b*Ei((d*f*x + c*f)/d) - a*b*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c
*f)/d) + (b^2*Ei(2*(d*f*x + c*f)/d) + b^2*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*
(d*e - c*f)/d) + 2*(2*a^2 - b^2)*log(d*x + c) - 4*(a*b*Ei((d*f*x + c*f)/d)
 + a*b*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) - (b^2*Ei(2*(d*f*x + c*f
)/d) - b^2*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/d
```

3.166.6 Sympy [F]

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx = \int \frac{(a+b\sinh{(e+fx)})^2}{c+dx}\,dx$$

input
```
integrate((a+b*sinh(f*x+e))**2/(d*x+c),x)
```

output
```
Integral((a + b*sinh(e + f*x))**2/(c + d*x), x)
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx$$
$$= -\frac{1}{4}b^2\left(\frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_1\left(\frac{2\,(dx+c)f}{d}\right)}{d} + \frac{e^{\left(2\,e-\frac{2\,cf}{d}\right)}E_1\left(-\frac{2\,(dx+c)f}{d}\right)}{d} + \frac{2\,\log\left(dx+c\right)}{d}\right)$$
$$+ ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right) + \frac{a^2\log\left(dx+c\right)}{d}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

output
```
-1/4*b^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/d + e^(2*e
 - 2*c*f/d)*exp_integral_e(1, -2*(d*x + c)*f/d)/d + 2*log(d*x + c)/d) + a*
b*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(e - c*f/d)*exp_i
ntegral_e(1, -(d*x + c)*f/d)/d) + a^2*log(d*x + c)/d
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx$$
$$= \frac{b^2\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(2\,e-\frac{2\,cf}{d}\right)}+4\,ab\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}-4\,ab\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+b^2\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(-2\,e\right.}}{4\,d}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

output
```
1/4*(b^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a*b*Ei((d*f*x + c*f)/
d)*e^(e - c*f/d) - 4*a*b*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + b^2*Ei(-2*(
d*f*x + c*f)/d)*e^(-2*e + 2*c*f/d) + 4*a^2*log(d*x + c) - 2*b^2*log(d*x +
c))/d
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx=\int \frac{{\left(a+b\,\mathrm{sinh}\left(e+f\,x\right)\right)}^2}{c+d\,x}\,dx$$

input
```
int((a + b*sinh(e + f*x))^2/(c + d*x),x)
```

output
```
int((a + b*sinh(e + f*x))^2/(c + d*x), x)
```

3.167 $\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx$

3.167.1 Optimal result

Integrand size = 20, antiderivative size = 183

$$\begin{aligned}\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx = &-\frac{a^2}{d(c+dx)}+\frac{2abf\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(\frac{cf}{d}+fx\right)}{d^2}\\ &+\frac{b^2f\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}-\frac{2ab\sinh(e+fx)}{d(c+dx)}\\ &-\frac{b^2\sinh^2(e+fx)}{d(c+dx)}+\frac{2abf\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(\frac{cf}{d}+fx\right)}{d^2}\\ &+\frac{b^2f\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{d^2}\end{aligned}$$

output
```
-a^2/d/(d*x+c)+2*a*b*f*Chi(c*f/d+f*x)*cosh(-e+c*f/d)/d^2+b^2*f*cosh(-2*e+2
*c*f/d)*Shi(2*c*f/d+2*f*x)/d^2-b^2*f*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)
/d^2-2*a*b*f*Shi(c*f/d+f*x)*sinh(-e+c*f/d)/d^2-2*a*b*sinh(f*x+e)/d/(d*x+c)
-b^2*sinh(f*x+e)^2/d/(d*x+c)
```

3.167.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.27

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx$$
$$= \frac{-2a^2d+b^2d-b^2d\cosh(2(e+fx))+4abf(c+dx)\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(f\left(\frac{c}{d}+x\right)\right)+2b^2f(c+dx)\operatorname{Chi}\Big(\frac{2f(c}{}}{}$$

input
```
Integrate[(a + b*Sinh[e + f*x])^2/(c + d*x)^2,x]
```

output
```
(-2*a^2*d + b^2*d - b^2*d*Cosh[2*(e + f*x)] + 4*a*b*f*(c + d*x)*Cosh[e - (
c*f)/d]*CoshIntegral[f*(c/d + x)] + 2*b^2*f*(c + d*x)*CoshIntegral[(2*f*(c
 + d*x))/d]*Sinh[2*e - (2*c*f)/d] - 4*a*b*d*Sinh[e + f*x] + 4*a*b*c*f*Sinh
[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + 4*a*b*d*f*x*Sinh[e - (c*f)/d]*Si
nhIntegral[f*(c/d + x)] + 2*b^2*c*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*
f*(c + d*x))/d] + 2*b^2*d*f*x*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c +
 d*x))/d])/(2*d^2*(c + d*x))
```

3.167.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a-ib\sin(ie+ifx))^2}{(c+dx)^2}dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(\frac{a^2}{(c+dx)^2}+\frac{2ab\sinh(e+fx)}{(c+dx)^2}+\frac{b^2\sinh^2(e+fx)}{(c+dx)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{d(c+dx)}+\frac{2abf\operatorname{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{2abf\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{2ab\sinh(e+fx)}{d(c+dx)}+\frac{b^2f\operatorname{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{b^2f\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(2xf+\frac{2cf}{d}\right)}{d^2}-\frac{b^2\sinh^2(e+fx)}{d(c+dx)}$$

input
```
Int[(a + b*Sinh[e + f*x])^2/(c + d*x)^2,x]
```

output
```
-(a^2/(d*(c + d*x))) + (2*a*b*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f
*x])/d^2 + (b^2*f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d
^2 - (2*a*b*Sinh[e + f*x])/(d*(c + d*x)) - (b^2*Sinh[e + f*x]^2)/(d*(c + d
*x)) + (2*a*b*f*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^2 + (b^2*
f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d^2
```

3.167.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.167.4 Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.74

method	result
risch	$-\frac{fab\,\mathrm{e}^{fx+e}}{d^2\left(\frac{cf}{d}+fx\right)}-\frac{fab\,\mathrm{e}^{-\frac{cf-de}{d}}\operatorname{Ei}_1\left(-fx-e-\frac{cf-de}{d}\right)}{d^2}-\frac{a^2}{d(dx+c)}+\frac{b^2}{2(dx+c)d}-\frac{f\,b^2\mathrm{e}^{-2fx-2e}}{4d(dfx+cf)}+\frac{f\,b^2\mathrm{e}^{\frac{2cf-2de}{d}}\operatorname{Ei}_1\left(2fx+2\right.}{2d^2}$

input
```
int((a+b*sinh(f*x+e))^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d^2*f*a*b*exp(f*x+e)/(c*f/d+f*x)-1/d^2*f*a*b*exp(-(c*f-d*e)/d)*Ei(1,-f*
x-e-(c*f-d*e)/d)-a^2/d/(d*x+c)+1/2*b^2/(d*x+c)/d-1/4*f*b^2*exp(-2*f*x-2*e)
/d/(d*f*x+c*f)+1/2*f*b^2/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)
/d)-1/4*f*b^2/d^2*exp(2*f*x+2*e)/(c*f/d+f*x)-1/2*f*b^2/d^2*exp(-2*(c*f-d*e
)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)+f*a*b*exp(-f*x-e)/d/(d*f*x+c*f)-f*a*b/
d^2*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.95

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx =$$

$$-\frac{b^2d\cosh\left(fx+e\right)^2+b^2d\sinh\left(fx+e\right)^2+4\,abd\sinh\left(fx+e\right)+(2\,a^2-b^2)d-2\left((abdfx+abcf)\mathrm{Ei}\left(\frac{dfx\cdots}{d}\right.\right.}{}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(b^2*d*cosh(f*x + e)^2 + b^2*d*sinh(f*x + e)^2 + 4*a*b*d*sinh(f*x + e
) + (2*a^2 - b^2)*d - 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/d) + (a*b*
d*f*x + a*b*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - ((b^2*d*f*x
+ b^2*c*f)*Ei(2*(d*f*x + c*f)/d) - (b^2*d*f*x + b^2*c*f)*Ei(-2*(d*f*x + c*
f)/d))*cosh(-2*(d*e - c*f)/d) + 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/
d) - (a*b*d*f*x + a*b*c*f)*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) + ((
b^2*d*f*x + b^2*c*f)*Ei(2*(d*f*x + c*f)/d) + (b^2*d*f*x + b^2*c*f)*Ei(-2*(
d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^3*x + c*d^2)
```

3.167.6 Sympy [F]

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{\left(a+b\sinh\left(e+fx\right)\right)^2}{\left(c+dx\right)^2}\,dx$$

input
```
integrate((a+b*sinh(f*x+e))**2/(d*x+c)**2,x)
```

output
```
Integral((a + b*sinh(e + f*x))**2/(c + d*x)**2, x)
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.99

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx$$
$$= -\frac{1}{4}\,b^2\left(\frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_2\left(\frac{2\,(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(2\,e-\frac{2\,cf}{d}\right)}E_2\left(-\frac{2\,(dx+c)f}{d}\right)}{(dx+c)d}-\frac{2}{d^2x+cd}\right)$$
$$+ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)-\frac{a^2}{d^2x+cd}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*b^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)
*d) + e^(2*e - 2*c*f/d)*exp_integral_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d)
- 2/(d^2*x + c*d)) + a*b*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/
((d*x + c)*d) - e^(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)
*d)) - a^2/(d^2*x + c*d)
```

3.167.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(186) = 372$.

Time = 0.35 (sec) , antiderivative size = 1135, normalized size of antiderivative = 6.20

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*(2*(d*x + c)*b^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(2*((d*x +
c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d)
 - 2*b^2*d*e*f^2*Ei(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e
 + c*f)/d)*e^(2*(d*e - c*f)/d) + 2*b^2*c*f^3*Ei(2*((d*x + c)*(d*e/(d*x + c
) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d) + 4*(d*x + c)*a
*b*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d*e/(d*x + c) -
c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - 4*a*b*d*e*f^2*Ei(((
d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f
)/d) + 4*a*b*c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e
 + c*f)/d)*e^((d*e - c*f)/d) + 4*(d*x + c)*a*b*(d*e/(d*x + c) - c*f/(d*x +
 c) + f)*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*
f)/d)*e^(-(d*e - c*f)/d) - 4*a*b*d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c
*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + 4*a*b*c*f^3*Ei(-((d
*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f
)/d) - 2*(d*x + c)*b^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x
 + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f
)/d) + 2*b^2*d*e*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)
- d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) - 2*b^2*c*f^3*Ei(-2*((d*x + c)*(d*e/(
d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) - b^2*d
*f^2*e^(2*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) - 4*a*b*d*f^...
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{\left(a+b\sinh(e+f\,x)\right)^2}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + b*sinh(e + f*x))^2/(c + d*x)^2,x)
```

output
```
int((a + b*sinh(e + f*x))^2/(c + d*x)^2, x)
```

3.168 $\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx$

3.168.1 Optimal result

Integrand size = 20, antiderivative size = 242

$$\begin{aligned}\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx = &-\frac{a^2}{2d(c+dx)^2}-\frac{abf\cosh(e+fx)}{d^2(c+dx)} \\ &+\frac{b^2f^2\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)}{d^3} \\ &+\frac{abf^2\operatorname{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^3}-\frac{ab\sinh(e+fx)}{d(c+dx)^2} \\ &-\frac{b^2f\cosh(e+fx)\sinh(e+fx)}{d^2(c+dx)}-\frac{b^2\sinh^2(e+fx)}{2d(c+dx)^2} \\ &+\frac{abf^2\cosh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(\frac{cf}{d}+fx\right)}{d^3} \\ &+\frac{b^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{d^3}\end{aligned}$$

output
```
-1/2*a^2/d/(d*x+c)^2+b^2*f^2*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/d^3-a*b
*f*cosh(f*x+e)/d^2/(d*x+c)+a*b*f^2*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d^3-b^2*f
^2*Shi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/d^3-a*b*f^2*Chi(c*f/d+f*x)*sinh(-
e+c*f/d)/d^3-a*b*sinh(f*x+e)/d/(d*x+c)^2-b^2*f*cosh(f*x+e)*sinh(f*x+e)/d^2
/(d*x+c)-1/2*b^2*sinh(f*x+e)^2/d/(d*x+c)^2
```

3.168.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.63

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx$$
$$= \frac{-2a^2d^2+b^2d^2-4abcdf\cosh(e+fx)-4abd^2fx\cosh(e+fx)-b^2d^2\cosh(2(e+fx))+4b^2f^2(c+dx)^2\,\mathrm{c}}{}$$

input
```
Integrate[(a + b*Sinh[e + f*x])^2/(c + d*x)^3,x]
```

output
```
(-2*a^2*d^2 + b^2*d^2 - 4*a*b*c*d*f*Cosh[e + f*x] - 4*a*b*d^2*f*x*Cosh[e +
 f*x] - b^2*d^2*Cosh[2*(e + f*x)] + 4*b^2*f^2*(c + d*x)^2*Cosh[2*e - (2*c*
f)/d]*CoshIntegral[(2*f*(c + d*x))/d] + 4*a*b*f^2*(c + d*x)^2*CoshIntegral
[f*(c/d + x)]*Sinh[e - (c*f)/d] - 4*a*b*d^2*Sinh[e + f*x] - 2*b^2*c*d*f*Si
nh[2*(e + f*x)] - 2*b^2*d^2*f*x*Sinh[2*(e + f*x)] + 4*a*b*c^2*f^2*Cosh[e -
 (c*f)/d]*SinhIntegral[f*(c/d + x)] + 8*a*b*c*d*f^2*x*Cosh[e - (c*f)/d]*Si
nhIntegral[f*(c/d + x)] + 4*a*b*d^2*f^2*x^2*Cosh[e - (c*f)/d]*SinhIntegral
[f*(c/d + x)] + 4*b^2*c^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c +
 d*x))/d] + 8*b^2*c*d*f^2*x*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d
*x))/d] + 4*b^2*d^2*f^2*x^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d
*x))/d])/(4*d^3*(c + d*x)^2)
```

3.168.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(a-ib\sin(ie+ifx))^2}{(c+dx)^3}dx$$
$$\Big\downarrow \; 3798$$

$$\int \left(\frac{a^2}{(c+dx)^3} + \frac{2ab\sinh(e+fx)}{(c+dx)^3} + \frac{b^2\sinh^2(e+fx)}{(c+dx)^3}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{2d(c+dx)^2} + \frac{abf^2\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^3} + \frac{abf^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{d^3} - \frac{abf\cosh(e+fx)}{d^2(c+dx)} - \frac{ab\sinh(e+fx)}{d(c+dx)^2} + \frac{b^2f^2\mathrm{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d^3} + \frac{b^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(2xf+\frac{2cf}{d}\right)}{d^3} - \frac{b^2f\sinh(e+fx)\cosh(e+fx)}{d^2(c+dx)} - \frac{b^2\sinh^2(e+fx)}{2d(c+dx)^2}$$

input
```
Int[(a + b*Sinh[e + f*x])^2/(c + d*x)^3,x]
```

output
```
-1/2*a^2/(d*(c + d*x)^2) - (a*b*f*Cosh[e + f*x])/(d^2*(c + d*x)) + (b^2*f^
2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d^3 + (a*b*f^2*Co
shIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - (a*b*Sinh[e + f*x])/(d*
(c + d*x)^2) - (b^2*f*Cosh[e + f*x]*Sinh[e + f*x])/(d^2*(c + d*x)) - (b^2*
Sinh[e + f*x]^2)/(2*d*(c + d*x)^2) + (a*b*f^2*Cosh[e - (c*f)/d]*SinhIntegr
al[(c*f)/d + f*x])/d^3 + (b^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*
f)/d + 2*f*x])/d^3
```

3.168.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.168.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(242) = 484$.

Time = 4.36 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.59

method	result
risch	$-\frac{f^{2} b a \,\mathrm{e}^{f x+e}}{2 d^{3} \left(\frac{c f}{d}+f x \right)^{2}}-\frac{f^{2} b a \,\mathrm{e}^{f x+e}}{2 d^{3} \left(\frac{c f}{d}+f x \right)}-\frac{f^{2} b a \,\mathrm{e}^{-\frac{c f -d e}{d}} \operatorname{Ei}_{1}\left(-f x -e -\frac{c f -d e}{d}\right)}{2 d^{3}}-\frac{a^{2}}{2 d \left(d x +c \right)^{2}}+\frac{b^{2}}{4 \left(d x +c \right)^{2} d}+\frac{f^{3} b^{2} \mathrm{e}^{-2 f x -2}}{4 d \left(d^{2} x^{2} f^{2}+2 c d \,f^{2}\right.}$

input
```
int((a+b*sinh(f*x+e))^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d^3*f^2*b*a*exp(f*x+e)/(c*f/d+f*x)^2-1/2/d^3*f^2*b*a*exp(f*x+e)/(c*f/
d+f*x)-1/2/d^3*f^2*b*a*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)-1/2*a^2/
d/(d*x+c)^2+1/4*b^2/(d*x+c)^2/d+1/4*f^3*b^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2
+2*c*d*f^2*x+c^2*f^2)*x+1/4*f^3*b^2*exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d
*f^2*x+c^2*f^2)*c-1/8*f^2*b^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c
^2*f^2)-1/2*f^2*b^2/d^3*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1
/8*f^2*b^2/d^3*exp(2*f*x+2*e)/(c*f/d+f*x)^2-1/4*f^2*b^2/d^3*exp(2*f*x+2*e)
/(c*f/d+f*x)-1/2*f^2*b^2/d^3*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*
e)/d)-1/2*f^3*a*b*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x-1/2*f^
3*a*b*exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/2*f^2*a*b*exp(
-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/2*f^2*a*b/d^3*exp((c*f-d*e)/
d)*Ei(1,f*x+e+(c*f-d*e)/d)
```

3.168.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(242) = 484$.

Time = 0.25 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.44

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx =$$
$$-\frac{b^{2} d^{2} \cosh\left(fx+e\right)^{2}+b^{2} d^{2} \sinh\left(fx+e\right)^{2}+\left(2\,a^{2}-b^{2}\right) d^{2}+4\left(abd^{2} fx+abcdf\right)\cosh\left(fx+e\right)-2\left(\left(abd^{2}\right.\right.}{}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(b^2*d^2*cosh(f*x + e)^2 + b^2*d^2*sinh(f*x + e)^2 + (2*a^2 - b^2)*d^
2 + 4*(a*b*d^2*f*x + a*b*c*d*f)*cosh(f*x + e) - 2*((a*b*d^2*f^2*x^2 + 2*a*
b*c*d*f^2*x + a*b*c^2*f^2)*Ei((d*f*x + c*f)/d) - (a*b*d^2*f^2*x^2 + 2*a*b*
c*d*f^2*x + a*b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - 2*((
b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d) + (
b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*c
osh(-2*(d*e - c*f)/d) + 4*(a*b*d^2 + (b^2*d^2*f*x + b^2*c*d*f)*cosh(f*x +
e))*sinh(f*x + e) + 2*((a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*E
i((d*f*x + c*f)/d) + (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*Ei(
-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) + 2*((b^2*d^2*f^2*x^2 + 2*b^2*c*d*
f^2*x + b^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d) - (b^2*d^2*f^2*x^2 + 2*b^2*c*d*
f^2*x + b^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^5*
x^2 + 2*c*d^4*x + c^2*d^3)
```

3.168.6 Sympy [F]

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx = \int \frac{(a+b\sinh{(e+fx)})^2}{(c+dx)^3}\,dx$$

input
```
integrate((a+b*sinh(f*x+e))**2/(d*x+c)**3,x)
```

output
```
Integral((a + b*sinh(e + f*x))**2/(c + d*x)**3, x)
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx = \frac{1}{4}b^2\left(\frac{1}{d^3x^2+2\,cd^2x+c^2d} - \frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_3\left(\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d} - \frac{e^{\left(2\,e-\frac{2\,cf}{d}\right)}E_3\left(-\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d}\right) + ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right) - \frac{a^2}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*b^2*(1/(d^3*x^2 + 2*c*d^2*x + c^2*d) - e^(-2*e + 2*c*f/d)*exp_integral
_e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*d) - e^(2*e - 2*c*f/d)*exp_integral_e(
3, -2*(d*x + c)*f/d)/((d*x + c)^2*d)) + a*b*(e^(-e + c*f/d)*exp_integral_e
(3, (d*x + c)*f/d)/((d*x + c)^2*d) - e^(e - c*f/d)*exp_integral_e(3, -(d*x
 + c)*f/d)/((d*x + c)^2*d)) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

3.168.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(242) = 484$.

Time = 0.28 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.80

$$\int \frac{(a + b\sinh(e + fx))^2}{(c + dx)^3}\,dx$$
$$= \frac{4\,b^2d^2f^2x^2\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(2\,e-\frac{2\,cf}{d}\right)} + 4\,abd^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} - 4\,abd^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} + 4}{}$$

input
```
integrate((a+b*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*(4*b^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a*b*d^2
*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - 4*a*b*d^2*f^2*x^2*Ei(-(d*f*x
+ c*f)/d)*e^(-e + c*f/d) + 4*b^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*
e + 2*c*f/d) + 8*b^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 8
*a*b*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - 8*a*b*c*d*f^2*x*Ei(-(d*
f*x + c*f)/d)*e^(-e + c*f/d) + 8*b^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f)/d)*e^(-
2*e + 2*c*f/d) + 4*b^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4
*a*b*c^2*f^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - 4*a*b*c^2*f^2*Ei(-(d*f*x
+ c*f)/d)*e^(-e + c*f/d) + 4*b^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*e +
2*c*f/d) - 2*b^2*d^2*f*x*e^(2*f*x + 2*e) - 4*a*b*d^2*f*x*e^(f*x + e) - 4*a
*b*d^2*f*x*e^(-f*x - e) + 2*b^2*d^2*f*x*e^(-2*f*x - 2*e) - 2*b^2*c*d*f*e^(
2*f*x + 2*e) - 4*a*b*c*d*f*e^(f*x + e) - 4*a*b*c*d*f*e^(-f*x - e) + 2*b^2*
c*d*f*e^(-2*f*x - 2*e) - b^2*d^2*e^(2*f*x + 2*e) - 4*a*b*d^2*e^(f*x + e) +
 4*a*b*d^2*e^(-f*x - e) - b^2*d^2*e^(-2*f*x - 2*e) - 4*a^2*d^2 + 2*b^2*d^2
)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx = \int \frac{\left(a+b\sinh(e+f\,x)\right)^2}{\left(c+d\,x\right)^3}\,dx$$

input
```
int((a + b*sinh(e + f*x))^2/(c + d*x)^3,x)
```

output
```
int((a + b*sinh(e + f*x))^2/(c + d*x)^3, x)
```

3.169 $\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx$

3.169.1 Optimal result

Integrand size = 20, antiderivative size = 404

$$\begin{aligned}\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx &= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f}-\frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f}\\
&+\frac{3d(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f^2}\\
&-\frac{3d(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f^2}\\
&-\frac{6d^2(c+dx)\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f^3}\\
&+\frac{6d^2(c+dx)\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f^3}\\
&+\frac{6d^3\operatorname{PolyLog}\left(4,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f^4}-\frac{6d^3\operatorname{PolyLog}\left(4,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f^4}\end{aligned}$$

output
```
(d*x+c)^3*ln(1+b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/f/(a^2+b^2)^(1/2)-(d*x+c)
^3*ln(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/f/(a^2+b^2)^(1/2)+3*d*(d*x+c)^2*
polylog(2,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/f^2/(a^2+b^2)^(1/2)-3*d*(d*x+
c)^2*polylog(2,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/f^2/(a^2+b^2)^(1/2)-6*d^
2*(d*x+c)*polylog(3,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/f^3/(a^2+b^2)^(1/2)
+6*d^2*(d*x+c)*polylog(3,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/f^3/(a^2+b^2)^
(1/2)+6*d^3*polylog(4,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/f^4/(a^2+b^2)^(1/
2)-6*d^3*polylog(4,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/f^4/(a^2+b^2)^(1/2)
```

3.169.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)-(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)+\frac{3d\left(f^2(c+dx)^2\operatorname{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)-2df(c+dx)\operatorname{Po}\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(c + d*x)^3/(a + b*Sinh[e + f*x]),x]
```

output
```
((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])] - (c + d*x)^3*
Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])] + (3*d*(f^2*(c + d*x)^2*Pol
yLog[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(c + d*x)*PolyLog[
3, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] + 2*d^2*PolyLog[4, (b*E^(e + f*
x))/(-a + Sqrt[a^2 + b^2])]))/f^3 - (3*d*(f^2*(c + d*x)^2*PolyLog[2, -((b*
E^(e + f*x))/(a + Sqrt[a^2 + b^2]))] - 2*d*f*(c + d*x)*PolyLog[3, -((b*E^(
e + f*x))/(a + Sqrt[a^2 + b^2]))] + 2*d^2*PolyLog[4, -((b*E^(e + f*x))/(a
+ Sqrt[a^2 + b^2]))]))/f^3)/(Sqrt[a^2 + b^2]*f)
```

3.169.3 Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3803, 25, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{(c+dx)^3}{a-ib\sin(ie+ifx)}dx$$

$\Big\downarrow$ 3803

$$2\int -\frac{e^{e+fx}(c+dx)^3}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx$$

$\Big\downarrow$ 25

$$-2\int \frac{e^{e+fx}(c+dx)^3}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx$$

$\Big\downarrow$ 2694

$$-2\left(\frac{b\int -\frac{e^{e+fx}(c+dx)^3}{2\left(a+be^{e+fx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{e+fx}(c+dx)^3}{2\left(a+be^{e+fx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)$$

$\Big\downarrow$ 27

$$-2\left(\frac{b\int \frac{e^{e+fx}(c+dx)^3}{a+be^{e+fx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)^3}{a+be^{e+fx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)$$

$\Big\downarrow$ 2620

$$-2\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{3d\int (c+dx)^2\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\frac{3d\int (c+dx)^2\log\left(\frac{\ldots}{a-\ldots}\right.}{bf}\right.}{2\sqrt{a^2+b^2}}\right.$$

$\Big\downarrow$ 3011

3.169. $\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx$

$$-2\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{3d\left(\frac{2d\int(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)dx}{f}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^3\log\right.}{}\right.}{}\right.$$

$\downarrow$ 7163

$$-2\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}-\frac{d\int\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)dx}{f}\right)}{f}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{be^{e}}{a+\sqrt{a}}\right.}{f}\right.}{bf}\right.}{2\sqrt{a^2+b^2}}\right.$$

$\downarrow$ 2720

3.169. $\int\frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx$

$$-2\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\,\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}-\frac{d\int e^{-e-fx}\,\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)de^{e+fx}}{f^2}\right)}{f}-\frac{(c+dx)^2\,\text{PolyL}\cdots}{\cdots}\cdots}{bf}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right.$$

$$\Big\downarrow \; 7143$$

$$-2\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\,\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}-\frac{d\,\text{PolyLog}\left(4,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f^2}\right)}{f}-\frac{(c+dx)^2\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b\cdots}}\right.}{f}\cdots}{bf}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right.$$

input
```
Int[(c + d*x)^3/(a + b*Sinh[e + f*x]),x]
```

3.169. $\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx$

output
```
-2*(-1/2*(b*(((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/
(b*f) - (3*d*(-(((c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 +
b^2]))])/f) + (2*d*(((c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2
+ b^2]))])/f - (d*PolyLog[4, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/f^
2))/f))/(b*f)))/Sqrt[a^2 + b^2] + (b*(((c + d*x)^3*Log[1 + (b*E^(e + f*x))
/(a + Sqrt[a^2 + b^2])])/(b*f) - (3*d*(-(((c + d*x)^2*PolyLog[2, -((b*E^(e
 + f*x))/(a + Sqrt[a^2 + b^2]))])/f) + (2*d*(((c + d*x)*PolyLog[3, -((b*E^
(e + f*x))/(a + Sqrt[a^2 + b^2]))])/f - (d*PolyLog[4, -((b*E^(e + f*x))/(a
 + Sqrt[a^2 + b^2]))])/f^2))/f))/(b*f)))/(2*Sqrt[a^2 + b^2]))
```

3.169.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.169.4 Maple [F]

$$\int \frac{(dx+c)^3}{a+b\sinh(fx+e)}dx$$

input
```
int((d*x+c)^3/(a+b*sinh(f*x+e)),x)
```

output
```
int((d*x+c)^3/(a+b*sinh(f*x+e)),x)
```

3.169.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1004 vs. $2(362) = 724$.

Time = 0.28 (sec) , antiderivative size = 1004, normalized size of antiderivative = 2.49

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
(6*b*d^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(f*x + e) + a*sinh(f*x +
e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))/b) - 6*b*d
^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(f*x + e) + a*sinh(f*x + e) - (
b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))/b) + 3*(b*d^3*f^
2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh
(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^
2 + b^2)/b^2) - b)/b + 1) - 3*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f
^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e) - (b*co
sh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b*d^3*
e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)
*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) + 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) - (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt((a^
2 + b^2)/b^2)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 +
b^2)/b^2) + 2*a) + (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x +
b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*sqrt((a^2 + b^2)/b^2)*log(-
(a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*s
qrt((a^2 + b^2)/b^2) - b)/b) - (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^
2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*sqrt((a^2 + b^2
)/b^2)*log(-(a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh
(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b) - 6*(b*d^3*f*x + b*c*d^2*f)*sq...
```

3.169.6 Sympy [F]

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx = \int \frac{(c+dx)^3}{a+b\sinh{(e+fx)}}\,dx$$

input
```
integrate((d*x+c)**3/(a+b*sinh(f*x+e)),x)
```

output
```
Integral((c + d*x)**3/(a + b*sinh(e + f*x)), x)
```

3.169.7 Maxima [F]

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx = \int \frac{(dx+c)^3}{b\sinh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
c^3*log((b*e^(-f*x - e) - a - sqrt(a^2 + b^2))/(b*e^(-f*x - e) - a + sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*f) + integrate(2*d^3*x^3/(b*(e^(f*x + e) - e
^(-f*x - e)) + 2*a) + 6*c*d^2*x^2/(b*(e^(f*x + e) - e^(-f*x - e)) + 2*a) +
 6*c^2*d*x/(b*(e^(f*x + e) - e^(-f*x - e)) + 2*a), x)
```

3.169.8 Giac [F]

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx = \int \frac{(dx+c)^3}{b\sinh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(b*sinh(f*x + e) + a), x)
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx = \int \frac{(c+d\,x)^3}{a+b\sinh(e+f\,x)}\,dx$$

input
```
int((c + d*x)^3/(a + b*sinh(e + f*x)),x)
```

output
```
int((c + d*x)^3/(a + b*sinh(e + f*x)), x)
```

3.170 $\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx$

3.170.1 Optimal result

Integrand size = 20, antiderivative size = 296

$$\begin{aligned}\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx &= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f}-\frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f}\\ &\quad+\frac{2d(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2}\\ &\quad-\frac{2d(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2}\\ &\quad-\frac{2d^2\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^3}+\frac{2d^2\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^3}\end{aligned}$$

output
```
(d*x+c)^2*ln(1+b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/f/(a^2+b^2)^(1/2)-(d*x+c)
^2*ln(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/f/(a^2+b^2)^(1/2)+2*d*(d*x+c)*po
lylog(2,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/f^2/(a^2+b^2)^(1/2)-2*d*(d*x+c)
*polylog(2,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/f^2/(a^2+b^2)^(1/2)-2*d^2*po
lylog(3,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/f^3/(a^2+b^2)^(1/2)+2*d^2*polyl
og(3,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/f^3/(a^2+b^2)^(1/2)
```

3.170.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx$$
$$=\frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)-(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)+\frac{2d\left(f(c+dx)\operatorname{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)-d\operatorname{PolyLog}\left(3,-\right.\right.}{f^2}}{\sqrt{a^2+b^2}f}$$

input
```
Integrate[(c + d*x)^2/(a + b*Sinh[e + f*x]),x]
```

output
```
((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])] - (c + d*x)^2*
Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])] + (2*d*(f*(c + d*x)*PolyLog
[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] - d*PolyLog[3, (b*E^(e + f*x))
/(-a + Sqrt[a^2 + b^2])]))/f^2 - (2*d*(f*(c + d*x)*PolyLog[2, -((b*E^(e +
f*x))/(a + Sqrt[a^2 + b^2]))] - d*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a
^2 + b^2]))]))/f^2)/(Sqrt[a^2 + b^2]*f)
```

3.170.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(c+dx)^2}{a-ib\sin(ie+ifx)}dx$$
$$\Big\downarrow \; 3803$$
$$2\int -\frac{e^{e+fx}(c+dx)^2}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx$$
$$\Big\downarrow \; 25$$

$$-2\int \frac{e^{e+fx}(c+dx)^2}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx$$

$$\Big\downarrow \; 2694$$

$$-2\left(\frac{b\int -\frac{e^{e+fx}(c+dx)^2}{2\left(a+be^{e+fx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{e+fx}(c+dx)^2}{2\left(a+be^{e+fx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)$$

$$\Big\downarrow \; 27$$

$$-2\left(\frac{b\int \frac{e^{e+fx}(c+dx)^2}{a+be^{e+fx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)^2}{a+be^{e+fx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)$$

$$\Big\downarrow \; 2620$$

$$-2\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\int (c+dx)\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\frac{2d\int (c+dx)\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2+b^2}}\right)$$

$$\Big\downarrow \; 3011$$

$$-2\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\int \text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)dx}{f}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\cdots\right)}{\cdots}\right)$$

$$\Big\downarrow \; 2720$$

3.170. $\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx$

$$-2\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{7143}$$

$$-2\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\,\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b}\right.}{bf}\right.}{}\right.$$

input
```
Int[(c + d*x)^2/(a + b*Sinh[e + f*x]),x]
```

output
```
-2*(-1/2*(b*(((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/
(b*f) - (2*d*(-(((c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^
2]))])/f) + (d*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/f^2))
/(b*f)))/Sqrt[a^2 + b^2] + (b*(((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a + S
qrt[a^2 + b^2])])/(b*f) - (2*d*(-(((c + d*x)*PolyLog[2, -((b*E^(e + f*x))/
(a + Sqrt[a^2 + b^2]))])/f) + (d*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^
2 + b^2]))])/f^2))/(b*f)))/(2*Sqrt[a^2 + b^2]))
```

3.170. $\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx$

3.170.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.170. $\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx$

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.170.4 Maple [F]

$$\int \frac{(dx+c)^2}{a+b\sinh(fx+e)}dx$$

input
```
int((d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

output
```
int((d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

3.170.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(264) = 528$.

Time = 0.26 (sec) , antiderivative size = 708, normalized size of antiderivative = 2.39

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx = -\frac{2\,bd^2\sqrt{\frac{a^2+b^2}{b^2}}\operatorname{polylog}\left(3, \frac{a\cosh(fx+e)+a\sinh(fx+e)+(b\cosh(fx+e)+b\sinh(fx+e))\sqrt{\frac{a^2+b^2}{b^2}}}{b}\right) - 2\,bd^2\sqrt{\frac{a^2+b^2}{b^2}}\operatorname{polylog}\Big(3,}{}$$

input
```
integrate((d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
-(2*b*d^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(f*x + e) + a*sinh(f*x +
 e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))/b) - 2*b*
d^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(f*x + e) + a*sinh(f*x + e) -
(b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))/b) - 2*(b*d^2*f
*x + b*c*d*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x +
e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
 + 2*(b*d^2*f*x + b*c*d*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) +
a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2
) - b)/b + 1) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt((a^2 + b^2)/b^2
)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) + 2*b*sqrt((a^2 + b^2)/b^2) +
2*a) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(2*b
*cosh(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b
*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*sqrt((a^2 + b^2)/b
^2)*log(-(a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*
x + e))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b
*d^2*e^2 + 2*b*c*d*e*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + e) + a*si
nh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) -
b)/b))/((a^2 + b^2)*f^3)
```

3.170.6 Sympy [F]

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx = \int \frac{(c+dx)^2}{a+b\sinh{(e+fx)}}\,dx$$

input
```
integrate((d*x+c)**2/(a+b*sinh(f*x+e)),x)
```

output
```
Integral((c + d*x)**2/(a + b*sinh(e + f*x)), x)
```

3.170.7 Maxima [F]

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx = \int \frac{(dx+c)^2}{b\sinh{(fx+e)}+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
c^2*log((b*e^(-f*x - e) - a - sqrt(a^2 + b^2))/(b*e^(-f*x - e) - a + sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*f) + integrate(2*d^2*x^2/(b*(e^(f*x + e) - e
^(-f*x - e)) + 2*a) + 4*c*d*x/(b*(e^(f*x + e) - e^(-f*x - e)) + 2*a), x)
```

3.170.8 Giac [F]

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx = \int \frac{(dx+c)^2}{b\sinh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*sinh(f*x + e) + a), x)
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx = \int \frac{(c+d\,x)^2}{a+b\sinh(e+f\,x)}\,dx$$

input
```
int((c + d*x)^2/(a + b*sinh(e + f*x)),x)
```

output
```
int((c + d*x)^2/(a + b*sinh(e + f*x)), x)
```

3.171 $\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx$

3.171.1 Optimal result

Integrand size = 18, antiderivative size = 187

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx=\frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f}-\frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f}+\frac{d\,\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f^2}-\frac{d\,\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f^2}$$

output
```
(d*x+c)*ln(1+b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/f/(a^2+b^2)^(1/2)-(d*x+c)*l
n(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/f/(a^2+b^2)^(1/2)+d*polylog(2,-b*exp
(f*x+e)/(a-(a^2+b^2)^(1/2)))/f^2/(a^2+b^2)^(1/2)-d*polylog(2,-b*exp(f*x+e)
/(a+(a^2+b^2)^(1/2)))/f^2/(a^2+b^2)^(1/2)
```

3.171.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.76

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx =\frac{f(c+dx)\left(\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)-\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)\right)+d\,\mathrm{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)-d\,\mathrm{PolyLog}\left(2,-\frac{be}{a+}\right.}{\sqrt{a^2+b^2}\,f^2}$$

input
```
Integrate[(c + d*x)/(a + b*Sinh[e + f*x]),x]
```

output
```
(f*(c + d*x)*(Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])] - Log[1 + (b*
E^(e + f*x))/(a + Sqrt[a^2 + b^2])]) + d*PolyLog[2, (b*E^(e + f*x))/(-a +
Sqrt[a^2 + b^2])] - d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))]
)/(Sqrt[a^2 + b^2]*f^2)
```

3.171.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a+b\sinh(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{a-ib\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{3803}$$

$$2\int -\frac{e^{e+fx}(c+dx)}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{e^{e+fx}(c+dx)}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx$$

$$\Big\downarrow \text{2694}$$

$$-2\left(\frac{b\int -\frac{e^{e+fx}(c+dx)}{2\left(a+be^{e+fx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{e+fx}(c+dx)}{2\left(a+be^{e+fx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$-2\left(\frac{b\int \frac{e^{e+fx}(c+dx)}{a+be^{e+fx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)}{a+be^{e+fx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)$$

$$\Big\downarrow \text{2620}$$

$$-2\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{d\int \log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\frac{d\int \log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2+b^2}}\right)$$

$$\Big\downarrow \text{2715}$$

$$-2\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{d\int e^{-e-fx}\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{e+fx}}{bf^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\frac{d\int e^{-e-fx}\log\left(\cdots\right.}{b\cdots}\right.}{2\sqrt{a^2+b^2}\cdots}\right.$$

$$\Big\downarrow \text{2838}$$

$$-2\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{bf^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{bf^2}\right)}{2\sqrt{a^2+b^2}}\right.$$

input
```
Int[(c + d*x)/(a + b*Sinh[e + f*x]),x]
```

output
```
-2*(-1/2*(b*(((c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(b
*f) + (d*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(b*f^2)))/S
qrt[a^2 + b^2] + (b*(((c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^
2])])/(b*f) + (d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(b*
f^2)))/(2*Sqrt[a^2 + b^2]))
```

3.171.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.171.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(167) = 334$.

Time = 1.28 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.10

method	result
risch	$-\frac{2c \operatorname{arctanh}\left(\frac{2b\,e^{fx+e}+2a}{2\sqrt{a^2+b^2}}\right)}{f\sqrt{a^2+b^2}}+\frac{d\ln\left(\frac{-b\,e^{fx+e}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{f\sqrt{a^2+b^2}}-\frac{d\ln\left(\frac{b\,e^{fx+e}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)x}{f\sqrt{a^2+b^2}}+\frac{d\ln\left(\frac{-b\,e^{fx+e}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)e}{f^2\sqrt{a^2+b^2}}$

input
```
int((d*x+c)/(a+b*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2/f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(f*x+e)+2*a)/(a^2+b^2)^(1/2))+1
/f*d/(a^2+b^2)^(1/2)*ln((-b*exp(f*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1
/2)))*x-1/f*d/(a^2+b^2)^(1/2)*ln((b*exp(f*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+
b^2)^(1/2)))*x+1/f^2*d/(a^2+b^2)^(1/2)*ln((-b*exp(f*x+e)+(a^2+b^2)^(1/2)-a
)/(-a+(a^2+b^2)^(1/2)))*e-1/f^2*d/(a^2+b^2)^(1/2)*ln((b*exp(f*x+e)+(a^2+b^
2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*e+1/f^2*d/(a^2+b^2)^(1/2)*dilog((-b*exp(f
*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/f^2*d/(a^2+b^2)^(1/2)*dil
og((b*exp(f*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/f^2*d*e/(a^2+b^
2)^(1/2)*arctanh(1/2*(2*b*exp(f*x+e)+2*a)/(a^2+b^2)^(1/2))
```

3.171.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(165) = 330$.

Time = 0.26 (sec) , antiderivative size = 455, normalized size of antiderivative = 2.43

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx = \frac{bd\sqrt{\frac{a^2+b^2}{b^2}}\mathrm{Li}_2\left(\frac{a\cosh(fx+e)+a\sinh(fx+e)+(b\cosh(fx+e)+b\sinh(fx+e))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)-bd\sqrt{\frac{a^2+b^2}{b^2}}\mathrm{Li}_2\left(\frac{a\cosh(fx+e)+a\sin}{}\right.}{}$$

input
```
integrate((d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
(b*d*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e) + (b*c
osh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - b*d*sq
rt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x
 + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b*d*e - b*c*
f)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) + 2*b*s
qrt((a^2 + b^2)/b^2) + 2*a) - (b*d*e - b*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*
b*cosh(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (
b*d*f*x + b*d*e)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + e) + a*sinh(f*x
+ e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b) -
 (b*d*f*x + b*d*e)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + e) + a*sinh(f*
x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b)
)/((a^2 + b^2)*f^2)
```

3.171.6 Sympy [F]

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx = \int \frac{c+dx}{a+b\sinh{(e+fx)}}\,dx$$

input
```
integrate((d*x+c)/(a+b*sinh(f*x+e)),x)
```

output
```
Integral((c + d*x)/(a + b*sinh(e + f*x)), x)
```

3.171.7 Maxima [F]

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx = \int \frac{dx+c}{b\sinh{(fx+e)}+a}\,dx$$

input
```
integrate((d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
d*integrate(2*x/(b*(e^(f*x + e) - e^(-f*x - e)) + 2*a), x) + c*log((b*e^(-
f*x - e) - a - sqrt(a^2 + b^2))/(b*e^(-f*x - e) - a + sqrt(a^2 + b^2)))/(s
qrt(a^2 + b^2)*f)
```

3.171.8 Giac [F]

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx = \int \frac{dx+c}{b\sinh{(fx+e)}+a}\,dx$$

input
```
integrate((d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*sinh(f*x + e) + a), x)
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx = \int \frac{c+d\,x}{a+b\sinh\left(e+f\,x\right)}\,dx$$

input
```
int((c + d*x)/(a + b*sinh(e + f*x)),x)
```

output
```
int((c + d*x)/(a + b*sinh(e + f*x)), x)
```

3.172 $\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx$

3.172.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \mathrm{Int}\left(\frac{1}{(c+dx)(a+b\sinh(e+fx))},x\right)$$

output
```
Unintegrable(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
```

3.172.2 Mathematica [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*Sinh[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*Sinh[e + f*x])), x]
```

3.172.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)(a-ib\sin(ie+ifx))}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}dx$$

input
```
Int[1/((c + d*x)*(a + b*Sinh[e + f*x])),x]
```

output
```
$Aborted
```

3.172.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.172.4 Maple [N/A] (verified)

Not integrable

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\left(a+b\sinh\left(fx+e\right)\right)}dx$$

input
```
int(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
```

output
```
int(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
```

3.172.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)(b\sinh\left(fx+e\right)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d*x + a*c + (b*d*x + b*c)*sinh(f*x + e)), x)
```

3.172.6 Sympy [N/A]

Not integrable

Time = 10.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \int \frac{1}{\left(a+b\sinh\left(e+fx\right)\right)(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
```

output
```
Integral(1/((a + b*sinh(e + f*x))*(c + d*x)), x)
```

3.172.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)(b\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)*(b*sinh(f*x + e) + a)), x)
```

3.172.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)(b\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(b*sinh(f*x + e) + a)), x)
```

3.172.9 Mupad [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(a+b\sinh(e+f\,x))\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*sinh(e + f*x))*(c + d*x)),x)
```

output
```
int(1/((a + b*sinh(e + f*x))*(c + d*x)), x)
```

3.173 $\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx$

3.173.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx=\mathrm{Int}\left(\frac{1}{(c+dx)^2(a+b\sinh(e+fx))},x\right)$$

output
```
Unintegrable(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

3.173.2 Mathematica [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx=\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*Sinh[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*Sinh[e + f*x])), x]
```

3.173.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)^2(a-ib\sin(ie+ifx))}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}dx$$

input
```
Int[1/((c + d*x)^2*(a + b*Sinh[e + f*x])),x]
```

output
```
$Aborted
```

3.173.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.173.4 Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2\,(a+b\sinh(fx+e))}dx$$

input
```
int(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

output
```
int(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

3.173.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2
)*sinh(f*x + e)), x)
```

3.173.6 Sympy [N/A]

Not integrable

Time = 59.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(a+b\sinh(e+fx))\,(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(a+b*sinh(f*x+e)),x)
```

output
```
Integral(1/((a + b*sinh(e + f*x))*(c + d*x)**2), x)
```

3.173.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)^2*(b*sinh(f*x + e) + a)), x)
```

3.173.8 Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\sinh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(b*sinh(f*x + e) + a)), x)
```

3.173.9 Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(a+b\sinh(e+f\,x))\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + b*sinh(e + f*x))*(c + d*x)^2),x)
```

output
```
int(1/((a + b*sinh(e + f*x))*(c + d*x)^2), x)
```

3.174 $\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx$

3.174.1 Optimal result

Integrand size = 20, antiderivative size = 549

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx = -\frac{(c+dx)^2}{(a^2+b^2)\,f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,f^2} + \frac{a(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,f^2} - \frac{a(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f} + \frac{2d^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,f^3} + \frac{2ad(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f^2} + \frac{2d^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,f^3} - \frac{2ad(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f^2} - \frac{2ad^2\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f^3} + \frac{2ad^2\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f^3} - \frac{b(c+dx)^2\cosh(e+fx)}{(a^2+b^2)\,f(a+b\sinh(e+fx))}$$

output
```
-(d*x+c)^2/(a^2+b^2)/f+2*d*(d*x+c)*ln(1+b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/
(a^2+b^2)/f^2+a*(d*x+c)^2*ln(1+b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)
^(3/2)/f+2*d*(d*x+c)*ln(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/f^2-
a*(d*x+c)^2*ln(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f+2*d^2
*polylog(2,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/f^3+2*a*d*(d*x+c)*
polylog(2,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f^2+2*d^2*pol
ylog(2,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/f^3-2*a*d*(d*x+c)*poly
log(2,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f^2-2*a*d^2*polyl
og(3,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f^3+2*a*d^2*polylo
g(3,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f^3-b*(d*x+c)^2*cos
h(f*x+e)/(a^2+b^2)/f/(a+b*sinh(f*x+e))
```

3.174.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.78

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx$$

$$= \frac{-f^2(c+dx)^2 + 2df(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right) + 2df(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right) + 2d^2\operatorname{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\cdots}\right.}{}$$

input
```
Integrate[(c + d*x)^2/(a + b*Sinh[e + f*x])^2,x]
```

output
```
(-(f^2*(c + d*x)^2) + 2*d*f*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^
2 + b^2])] + 2*d*f*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])
] + 2*d^2*PolyLog[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] + 2*d^2*PolyL
og[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))] - (a*(-(f^2*(c + d*x)^2*Lo
g[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])]) + f^2*(c + d*x)^2*Log[1 + (b
*E^(e + f*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(c + d*x)*PolyLog[2, (b*E^(e
+ f*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(c + d*x)*PolyLog[2, -((b*E^(e + f
*x))/(a + Sqrt[a^2 + b^2]))] + 2*d^2*PolyLog[3, (b*E^(e + f*x))/(-a + Sqrt
[a^2 + b^2])] - 2*d^2*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))]
))/Sqrt[a^2 + b^2] - (b*f^2*(c + d*x)^2*Cosh[e + f*x])/(a + b*Sinh[e + f*x
]))/((a^2 + b^2)*f^3)
```

3.174.3 Rubi [A] (verified)

Time = 2.68 (sec) , antiderivative size = 520, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3805, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 6095, 2620, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^2}{(a-ib\sin(ie+ifx))^2}dx$$

$$\Big\downarrow \text{3805}$$

$$\frac{a\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}dx}{a^2+b^2}+\frac{2bd\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f(a^2+b^2)}-\frac{b(c+dx)^2\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{(c+dx)^2}{a-ib\sin(ie+ifx)}dx}{a^2+b^2}+\frac{2bd\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f(a^2+b^2)}-\frac{b(c+dx)^2\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{3803}$$

$$\frac{2a\int -\frac{e^{e+fx}(c+dx)^2}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx}{a^2+b^2}+\frac{2bd\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f(a^2+b^2)}-\frac{b(c+dx)^2\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2a\int \frac{e^{e+fx}(c+dx)^2}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx}{a^2+b^2}+\frac{2bd\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f(a^2+b^2)}-\frac{b(c+dx)^2\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{2a\left(\frac{b\int -\frac{e^{e+fx}(c+dx)^2}{2\left(a+be^{e+fx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{e+fx}(c+dx)^2}{2\left(a+be^{e+fx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{2bd\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f(a^2+b^2)}-\frac{b(c+dx)^2\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2a\left(\frac{b\int \frac{e^{e+fx}(c+dx)^2}{a+be^{e+fx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)^2}{a+be^{e+fx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{2bd\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f\left(a^2+b^2\right)}-\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$\downarrow$ 2620

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\int (c+dx)\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\frac{2d\int (c+dx)\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$$\frac{2bd\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f\left(a^2+b^2\right)}-\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$\downarrow$ 3011

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\int \text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)dx}{f}-\frac{(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right.}{bf}\right.}{}\right.}{a^2+b^2}$$

$$\frac{2bd\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f\left(a^2+b^2\right)}-\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$\downarrow$ 2720

3.174. $\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx$

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{a^2+b^2}$$

$$\frac{2bd\int\frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f\left(a^2+b^2\right)}-\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{6095}$$

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{a^2+b^2}$$

$$\frac{2bd\left(\int\frac{e^{e+fx}(c+dx)}{a+be^{e+fx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{e+fx}(c+dx)}{a+be^{e+fx}+\sqrt{a^2+b^2}}dx-\frac{(c+dx)^2}{2bd}\right)}{f\left(a^2+b^2\right)}-\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{2620}$$

3.174. $\int\frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx$

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right)}{a^2+b^2}\cdots$$

$$\frac{2bd\left(-\frac{d\int\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bf}-\frac{d\int\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bf}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{(c+dx)^2}{2bd}\cdots\right)}{f\left(a^2+b^2\right)}\cdots$$

$$\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \; 2715$$

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right)}{a^2+b^2}\cdots$$

$$\frac{2bd\left(-\frac{d\int e^{-e-fx}\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{e+fx}}{bf^2}-\frac{d\int e^{-e-fx}\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{e+fx}}{bf^2}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}+\frac{(c+dx)\log\left(\frac{b\cdots}{\sqrt{a\cdots}}\right.}{bf}\cdots\right)}{f\left(a^2+b^2\right)}\cdots$$

$$\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \; 2838$$

3.174. $\int\frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx$

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\cdots\right)}{\cdots}\right.}{\cdots}\right.}{a^2+b^2}$$

$$\frac{2bd\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}+\frac{d\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{bf^2}+\frac{d\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{bf^2}-\frac{(c+dx)^2}{2bd}\cdots\right.}{f\left(a^2+b^2\right)}$$

$$\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \; 7143$$

$$\frac{2bd\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}+\frac{d\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{bf^2}+\frac{d\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{bf^2}-\frac{(c+dx)^2}{2bd}\cdots\right.}{f\left(a^2+b^2\right)}$$

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{2d\left(\frac{d\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\frac{2d\cdots}{\cdots}\right.}{\cdots}\right.}{a^2+b^2}$$

$$\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

input
```
Int[(c + d*x)^2/(a + b*Sinh[e + f*x])^2,x]
```

3.174. $\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx$

output
```
(2*b*d*(-1/2*(c + d*x)^2/(b*d) + ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a - S
qrt[a^2 + b^2])])/(b*f) + ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2
 + b^2])])/(b*f) + (d*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))]
)/(b*f^2) + (d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(b*f^
2)))/((a^2 + b^2)*f) - (2*a*(-1/2*(b*(((c + d*x)^2*Log[1 + (b*E^(e + f*x))
/(a - Sqrt[a^2 + b^2])])/(b*f) - (2*d*(-(((c + d*x)*PolyLog[2, -((b*E^(e +
 f*x))/(a - Sqrt[a^2 + b^2]))])/f) + (d*PolyLog[3, -((b*E^(e + f*x))/(a -
Sqrt[a^2 + b^2]))])/f^2))/(b*f)))/Sqrt[a^2 + b^2] + (b*(((c + d*x)^2*Log[1
 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(b*f) - (2*d*(-(((c + d*x)*Poly
Log[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/f) + (d*PolyLog[3, -((b*
E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/f^2))/(b*f)))/(2*Sqrt[a^2 + b^2])))/
(a^2 + b^2) - (b*(c + d*x)^2*Cosh[e + f*x])/((a^2 + b^2)*f*(a + b*Sinh[e +
 f*x]))
```

3.174.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

3.174. $\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.174.4 Maple [F]

$$\int \frac{(dx+c)^2}{(a+b\sinh(fx+e))^2}dx$$

input
```
int((d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

output
```
int((d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

3.174.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3957 vs. $2(503) = 1006$.

Time = 0.33 (sec) , antiderivative size = 3957, normalized size of antiderivative = 7.21

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-(2*(a^2*b + b^3)*d^2*e^2 - 4*(a^2*b + b^3)*c*d*e*f + 2*(a^2*b + b^3)*c^2*
f^2 + 2*((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*f^2*x - (a^2*b +
b^3)*d^2*e^2 + 2*(a^2*b + b^3)*c*d*e*f)*cosh(f*x + e)^2 + 2*((a^2*b + b^3)
*d^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*f^2*x - (a^2*b + b^3)*d^2*e^2 + 2*(a^2*
b + b^3)*c*d*e*f)*sinh(f*x + e)^2 + 2*(a*b^2*d^2*cosh(f*x + e)^2 + a*b^2*d
^2*sinh(f*x + e)^2 + 2*a^2*b*d^2*cosh(f*x + e) - a*b^2*d^2 + 2*(a*b^2*d^2*
cosh(f*x + e) + a^2*b*d^2)*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2)*polylog(3,
 (a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*
sqrt((a^2 + b^2)/b^2))/b) - 2*(a*b^2*d^2*cosh(f*x + e)^2 + a*b^2*d^2*sinh(
f*x + e)^2 + 2*a^2*b*d^2*cosh(f*x + e) - a*b^2*d^2 + 2*(a*b^2*d^2*cosh(f*x
 + e) + a^2*b*d^2)*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh
(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^
2 + b^2)/b^2))/b) + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*c*d*f^2
*x - 2*(a^3 + a*b^2)*d^2*e^2 + 4*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2
*f^2)*cosh(f*x + e) - 2*((a^2*b + b^3)*d^2*cosh(f*x + e)^2 + (a^2*b + b^3)
*d^2*sinh(f*x + e)^2 + 2*(a^3 + a*b^2)*d^2*cosh(f*x + e) - (a^2*b + b^3)*d
^2 + 2*((a^2*b + b^3)*d^2*cosh(f*x + e) + (a^3 + a*b^2)*d^2)*sinh(f*x + e)
 - (a*b^2*d^2*f*x + a*b^2*c*d*f - (a*b^2*d^2*f*x + a*b^2*c*d*f)*cosh(f*x +
 e)^2 - (a*b^2*d^2*f*x + a*b^2*c*d*f)*sinh(f*x + e)^2 - 2*(a^2*b*d^2*f*x +
 a^2*b*c*d*f)*cosh(f*x + e) - 2*(a^2*b*d^2*f*x + a^2*b*c*d*f + (a*b^2*d...
```

3.174.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2/(a+b*sinh(f*x+e))**2,x)
```

output
```
Timed out
```

3.174.7 Maxima [F]

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
2*a*d^2*f*integrate(x^2*e^(f*x + e)/(a^2*b*f*e^(2*f*x + 2*e) + b^3*f*e^(2*
f*x + 2*e) + 2*a^3*f*e^(f*x + e) + 2*a*b^2*f*e^(f*x + e) - a^2*b*f - b^3*f
), x) + 4*a*c*d*f*integrate(x*e^(f*x + e)/(a^2*b*f*e^(2*f*x + 2*e) + b^3*f
*e^(2*f*x + 2*e) + 2*a^3*f*e^(f*x + e) + 2*a*b^2*f*e^(f*x + e) - a^2*b*f -
 b^3*f), x) + 2*b*c*d*(a*log((b*e^(f*x + e) + a - sqrt(a^2 + b^2))/(b*e^(f
*x + e) + a + sqrt(a^2 + b^2)))/((a^2*b + b^3)*sqrt(a^2 + b^2)*f^2) - 2*(f
*x + e)/((a^2*b + b^3)*f^2) + log(b*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - b)
/((a^2*b + b^3)*f^2)) - 4*a*d^2*integrate(x*e^(f*x + e)/(a^2*b*f*e^(2*f*x
+ 2*e) + b^3*f*e^(2*f*x + 2*e) + 2*a^3*f*e^(f*x + e) + 2*a*b^2*f*e^(f*x +
e) - a^2*b*f - b^3*f), x) + 4*b*d^2*integrate(x/(a^2*b*f*e^(2*f*x + 2*e) +
 b^3*f*e^(2*f*x + 2*e) + 2*a^3*f*e^(f*x + e) + 2*a*b^2*f*e^(f*x + e) - a^2
*b*f - b^3*f), x) + c^2*(a*log((b*e^(-f*x - e) - a - sqrt(a^2 + b^2))/(b*e
^(-f*x - e) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*f) - 2*(a*e^(-f*x -
 e) + b)/((a^2*b + b^3 + 2*(a^3 + a*b^2)*e^(-f*x - e) - (a^2*b + b^3)*e^(-
2*f*x - 2*e))*f)) - 2*a*c*d*log((b*e^(f*x + e) + a - sqrt(a^2 + b^2))/(b*e
^(f*x + e) + a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*f^2) + 2*(b*d^2*x^2
+ 2*b*c*d*x - (a*d^2*x^2*e^e + 2*a*c*d*x*e^e)*e^(f*x))/(a^2*b*f + b^3*f -
(a^2*b*f*e^(2*e) + b^3*f*e^(2*e))*e^(2*f*x) - 2*(a^3*f*e^e + a*b^2*f*e^e)*
e^(f*x))
```

3.174.8 Giac [F]

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*sinh(f*x + e) + a)^2, x)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^2}{(a+b\sinh\left(e+f\,x\right))^2}\,dx$$

input
```
int((c + d*x)^2/(a + b*sinh(e + f*x))^2,x)
```

output
```
int((c + d*x)^2/(a + b*sinh(e + f*x))^2, x)
```

3.175 $\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx$

3.175.1 Optimal result

Integrand size = 18, antiderivative size = 254

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx = \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f} - \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f} + \frac{d\log(a+b\sinh(e+fx))}{(a^2+b^2)f^2} + \frac{ad\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f^2} - \frac{ad\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f^2} - \frac{b(c+dx)\cosh(e+fx)}{(a^2+b^2)f(a+b\sinh(e+fx))}$$

output
```
d*ln(a+b*sinh(f*x+e))/(a^2+b^2)/f^2+a*(d*x+c)*ln(1+b*exp(f*x+e)/(a-(a^2+b^
2)^(1/2)))/(a^2+b^2)^(3/2)/f-a*(d*x+c)*ln(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2
)))/(a^2+b^2)^(3/2)/f+a*d*polylog(2,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^
2+b^2)^(3/2)/f^2-a*d*polylog(2,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2
)^(3/2)/f^2-b*(d*x+c)*cosh(f*x+e)/(a^2+b^2)/f/(a+b*sinh(f*x+e))
```

3.175.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.76

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx$$
$$= \frac{d\log(a+b\sinh(e+fx))+\frac{a\left(f(c+dx)\left(\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)-\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)\right)+d\,\mathrm{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)-d\,\mathrm{PolyLog}\left(2,-\cdots\right.}{\sqrt{a^2+b^2}}}{(a^2+b^2)\,f^2}$$

input
```
Integrate[(c + d*x)/(a + b*Sinh[e + f*x])^2,x]
```

output
```
(d*Log[a + b*Sinh[e + f*x]] + (a*(f*(c + d*x)*(Log[1 + (b*E^(e + f*x))/(a
- Sqrt[a^2 + b^2])] - Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])]) + d*
PolyLog[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] - d*PolyLog[2, -((b*E^(
e + f*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] - (b*f*(c + d*x)*Cosh[
e + f*x])/(a + b*Sinh[e + f*x]))/((a^2 + b^2)*f^2)
```

3.175.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3805, 3042, 3147, 16, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{c+dx}{(a-ib\sin(ie+ifx))^2}dx$$
$$\Big\downarrow \; 3805$$
$$\frac{a\int \frac{c+dx}{a+b\sinh(e+fx)}dx}{a^2+b^2}+\frac{bd\int \frac{\cosh(e+fx)}{a+b\sinh(e+fx)}dx}{f(a^2+b^2)}-\frac{b(c+dx)\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}$$
$$\Big\downarrow \; 3042$$
$$\frac{a\int \frac{c+dx}{a-ib\sin(ie+ifx)}dx}{a^2+b^2}+\frac{bd\int \frac{\cos(ie+ifx)}{a-ib\sin(ie+ifx)}dx}{f(a^2+b^2)}-\frac{b(c+dx)\cosh(e+fx)}{f(a^2+b^2)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{3147}$$

$$\frac{a\int \frac{c+dx}{a-ib\sin(ie+ifx)}dx}{a^2+b^2}+\frac{d\int \frac{1}{a+b\sinh(e+fx)}d(b\sinh(e+fx))}{f^2\left(a^2+b^2\right)}-\frac{b(c+dx)\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\int \frac{c+dx}{a-ib\sin(ie+ifx)}dx}{a^2+b^2}-\frac{b(c+dx)\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}+\frac{d\log(a+b\sinh(e+fx))}{f^2\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3803}$$

$$\frac{2a\int -\frac{e^{e+fx}(c+dx)}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx}{a^2+b^2}-\frac{b(c+dx)\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}+\frac{d\log(a+b\sinh(e+fx))}{f^2\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2a\int \frac{e^{e+fx}(c+dx)}{-2e^{e+fx}a-be^{2(e+fx)}+b}dx}{a^2+b^2}-\frac{b(c+dx)\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}+\frac{d\log(a+b\sinh(e+fx))}{f^2\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{2a\left(\frac{b\int -\frac{e^{e+fx}(c+dx)}{2\left(a+be^{e+fx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{e+fx}(c+dx)}{2\left(a+be^{e+fx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}-\frac{b(c+dx)\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}+\frac{d\log(a+b\sinh(e+fx))}{f^2\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2a\left(\frac{b\int \frac{e^{e+fx}(c+dx)}{a+be^{e+fx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)}{a+be^{e+fx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}-\frac{b(c+dx)\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}+\frac{d\log(a+b\sinh(e+fx))}{f^2\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{d\int \log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\frac{d\int \log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}-\frac{b(c+dx)\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}+\frac{d\log(a+b\sinh(e+fx))}{f^2\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{2715}$$

3.175. $\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx$

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}-\frac{d\int e^{-e-fx}\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{e+fx}}{bf^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}-\frac{d\int e^{-e-fx}\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2+b^2}}+1\right)}{bf^2}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$$\frac{b(c+dx)\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}+\frac{d\log(a+b\sinh(e+fx))}{f^2\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2a\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{bf}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{bf^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{bf}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{bf^2}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}-$$

$$\frac{b(c+dx)\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}+\frac{d\log(a+b\sinh(e+fx))}{f^2\left(a^2+b^2\right)}$$

input
```
Int[(c + d*x)/(a + b*Sinh[e + f*x])^2,x]
```

output
```
(d*Log[a + b*Sinh[e + f*x]])/((a^2 + b^2)*f^2) - (2*a*(-1/2*(b*(((c + d*x)
*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(b*f) + (d*PolyLog[2, -((
b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(b*f^2)))/Sqrt[a^2 + b^2] + (b*(((
c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(b*f) + (d*PolyLo
g[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(b*f^2)))/(2*Sqrt[a^2 + b^
2])))/(a^2 + b^2) - (b*(c + d*x)*Cosh[e + f*x])/((a^2 + b^2)*f*(a + b*Sinh
[e + f*x]))
```

3.175.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.175. $\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx$

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

3.175.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(234) = 468$.

Time = 1.53 (sec) , antiderivative size = 519, normalized size of antiderivative = 2.04

method	result
risch	$\frac{2(dx+c)\left(a\,\mathrm{e}^{fx+e}-b\right)}{f(a^2+b^2)\left(b\,\mathrm{e}^{2fx+2e}+2a\,\mathrm{e}^{fx+e}-b\right)}-\frac{2d\ln\left(\mathrm{e}^{fx+e}\right)}{f^2(a^2+b^2)}+\frac{d\ln\left(b\,\mathrm{e}^{2fx+2e}+2a\,\mathrm{e}^{fx+e}-b\right)}{f^2(a^2+b^2)}-\frac{2ac\,\mathrm{arctanh}\left(\frac{2b\,\mathrm{e}^{fx+e}+2a}{2\sqrt{a^2+b^2}}\right)}{f(a^2+b^2)^{\frac{3}{2}}}+\frac{ad\ln\left(\right.}{}$

input
```
int((d*x+c)/(a+b*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(d*x+c)*(a*exp(f*x+e)-b)/f/(a^2+b^2)/(b*exp(2*f*x+2*e)+2*a*exp(f*x+e)-b)
-2/f^2/(a^2+b^2)*d*ln(exp(f*x+e))+1/f^2/(a^2+b^2)*d*ln(b*exp(2*f*x+2*e)+2*
a*exp(f*x+e)-b)-2/f/(a^2+b^2)^(3/2)*a*c*arctanh(1/2*(2*b*exp(f*x+e)+2*a)/(
a^2+b^2)^(1/2))+1/f/(a^2+b^2)^(3/2)*a*d*ln((-b*exp(f*x+e)+(a^2+b^2)^(1/2)-
a)/(-a+(a^2+b^2)^(1/2)))*x-1/f/(a^2+b^2)^(3/2)*a*d*ln((b*exp(f*x+e)+(a^2+b
^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/f^2/(a^2+b^2)^(3/2)*a*d*ln((-b*exp(f
*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*e-1/f^2/(a^2+b^2)^(3/2)*a*d
*ln((b*exp(f*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*e+1/f^2/(a^2+b^2
)^(3/2)*a*d*dilog((-b*exp(f*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-
1/f^2/(a^2+b^2)^(3/2)*a*d*dilog((b*exp(f*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b
^2)^(1/2)))+2/f^2/(a^2+b^2)^(3/2)*a*d*e*arctanh(1/2*(2*b*exp(f*x+e)+2*a)/(
a^2+b^2)^(1/2))
```

3.175.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1717 vs. $2(232) = 464$.

Time = 0.27 (sec) , antiderivative size = 1717, normalized size of antiderivative = 6.76

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
(2*(a^2*b + b^3)*d*e - 2*(a^2*b + b^3)*c*f - 2*((a^2*b + b^3)*d*f*x + (a^2
*b + b^3)*d*e)*cosh(f*x + e)^2 - 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*
e)*sinh(f*x + e)^2 + (a*b^2*d*cosh(f*x + e)^2 + a*b^2*d*sinh(f*x + e)^2 +
2*a^2*b*d*cosh(f*x + e) - a*b^2*d + 2*(a*b^2*d*cosh(f*x + e) + a^2*b*d)*si
nh(f*x + e))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e
) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
- (a*b^2*d*cosh(f*x + e)^2 + a*b^2*d*sinh(f*x + e)^2 + 2*a^2*b*d*cosh(f*x
+ e) - a*b^2*d + 2*(a*b^2*d*cosh(f*x + e) + a^2*b*d)*sinh(f*x + e))*sqrt((
a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e
) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (a*b^2*d*f*x + a*
b^2*d*e - (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e)^2 - (a*b^2*d*f*x + a*b^2
*d*e)*sinh(f*x + e)^2 - 2*(a^2*b*d*f*x + a^2*b*d*e)*cosh(f*x + e) - 2*(a^2
*b*d*f*x + a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e))*sinh(f*x +
 e))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + e) + a*sinh(f*x + e) + (b*co
sh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a*b^2*d*f*
x + a*b^2*d*e - (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e)^2 - (a*b^2*d*f*x +
 a*b^2*d*e)*sinh(f*x + e)^2 - 2*(a^2*b*d*f*x + a^2*b*d*e)*cosh(f*x + e) -
2*(a^2*b*d*f*x + a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e))*sinh
(f*x + e))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + e) + a*sinh(f*x + e) -
 (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*...
```

3.175.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)/(a+b*sinh(f*x+e))**2,x)
```

output
```
Timed out
```

3.175.7 Maxima [F]

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{dx+c}{(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
(2*a*f*integrate(x*e^(f*x + e)/(a^2*b*f*e^(2*f*x + 2*e) + b^3*f*e^(2*f*x +
 2*e) + 2*a^3*f*e^(f*x + e) + 2*a*b^2*f*e^(f*x + e) - a^2*b*f - b^3*f), x)
 + b*(a*log((b*e^(f*x + e) + a - sqrt(a^2 + b^2))/(b*e^(f*x + e) + a + sqr
t(a^2 + b^2)))/((a^2*b + b^3)*sqrt(a^2 + b^2)*f^2) - 2*(f*x + e)/((a^2*b +
 b^3)*f^2) + log(b*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - b)/((a^2*b + b^3)*f
^2)) - 2*(a*x*e^(f*x + e) - b*x)/(a^2*b*f + b^3*f - (a^2*b*f*e^(2*e) + b^3
*f*e^(2*e))*e^(2*f*x) - 2*(a^3*f*e^e + a*b^2*f*e^e)*e^(f*x)) - a*log((b*e^
(f*x + e) + a - sqrt(a^2 + b^2))/(b*e^(f*x + e) + a + sqrt(a^2 + b^2)))/((
a^2 + b^2)^(3/2)*f^2))*d + c*(a*log((b*e^(-f*x - e) - a - sqrt(a^2 + b^2))
/(b*e^(-f*x - e) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*f) - 2*(a*e^(-
f*x - e) + b)/((a^2*b + b^3 + 2*(a^3 + a*b^2)*e^(-f*x - e) - (a^2*b + b^3)
*e^(-2*f*x - 2*e))*f))
```

3.175.8 Giac [F]

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{dx+c}{(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*sinh(f*x + e) + a)^2, x)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{c+d\,x}{\left(a+b\sinh\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int((c + d*x)/(a + b*sinh(e + f*x))^2,x)
```

output
```
int((c + d*x)/(a + b*sinh(e + f*x))^2, x)
```

3.176 $\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx$

3.176.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx = \operatorname{Int}\left(\frac{1}{(c+dx)(a+b\sinh(e+fx))^2},x\right)$$

output
```
Unintegrable(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x)
```

3.176.2 Mathematica [N/A]

Not integrable

Time = 26.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*Sinh[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*Sinh[e + f*x])^2), x]
```

3.176.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)(a-ib\sin(ie+ifx))^2}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}dx$$

input
```
Int[1/((c + d*x)*(a + b*Sinh[e + f*x])^2),x]
```

output
```
$Aborted
```

3.176.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.176.4 Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\left(a+b\sinh\left(fx+e\right)\right)^{2}}dx$$

input
```
int(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x)
```

3.176.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\sinh\left(fx+e\right)+a)^{2}}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*sinh(f*x + e)^2 + 2*(a*b*d
*x + a*b*c)*sinh(f*x + e)), x)
```

3.176.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(a+b*sinh(f*x+e))**2,x)
```

output
```
Timed out
```

3.176.7 Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 406, normalized size of antiderivative = 20.30

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-2*(a*e^(f*x + e) - b)/(a^2*b*c*f + b^3*c*f + (a^2*b*d*f + b^3*d*f)*x - (a
^2*b*c*f*e^(2*e) + b^3*c*f*e^(2*e) + (a^2*b*d*f*e^(2*e) + b^3*d*f*e^(2*e))
*x)*e^(2*f*x) - 2*(a^3*c*f*e^e + a*b^2*c*f*e^e + (a^3*d*f*e^e + a*b^2*d*f*
e^e)*x)*e^(f*x)) + integrate(2*(b*d - (a*d*f*x*e^e + (c*f*e^e + d*e^e)*a)*
e^(f*x))/(a^2*b*c^2*f + b^3*c^2*f + (a^2*b*d^2*f + b^3*d^2*f)*x^2 + 2*(a^2
*b*c*d*f + b^3*c*d*f)*x - (a^2*b*c^2*f*e^(2*e) + b^3*c^2*f*e^(2*e) + (a^2*
b*d^2*f*e^(2*e) + b^3*d^2*f*e^(2*e))*x^2 + 2*(a^2*b*c*d*f*e^(2*e) + b^3*c*
d*f*e^(2*e))*x)*e^(2*f*x) - 2*(a^3*c^2*f*e^e + a*b^2*c^2*f*e^e + (a^3*d^2*
f*e^e + a*b^2*d^2*f*e^e)*x^2 + 2*(a^3*c*d*f*e^e + a*b^2*c*d*f*e^e)*x)*e^(f
*x)), x)
```

3.176.8 Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(b*sinh(f*x + e) + a)^2), x)
```

3.176.9 Mupad [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx = \int \frac{1}{(a+b\sinh(e+f\,x))^2\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*sinh(e + f*x))^2*(c + d*x)),x)
```

output
```
int(1/((a + b*sinh(e + f*x))^2*(c + d*x)), x)
```

3.177 $\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2} dx$

3.177.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2} dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2},x\right)$$

output
```
Unintegrable(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

3.177.2 Mathematica [N/A]

Not integrable

Time = 26.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2} dx = \int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2} dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*Sinh[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*Sinh[e + f*x])^2), x]
```

3.177.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)^2(a-ib\sin(ie+ifx))^2}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}dx$$

input
```
Int[1/((c + d*x)^2*(a + b*Sinh[e + f*x])^2),x]
```

output
```
$Aborted
```

3.177.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.177.4 Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2 (a+b\sinh(fx+e))^2} dx$$

input
```
int(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

3.177.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.80

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2} dx = \int \frac{1}{(dx+c)^2(b\sinh(fx+e)+a)^2} dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*sinh(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*si
nh(f*x + e)), x)
```

3.177.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2} dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**2/(a+b*sinh(f*x+e))**2,x)
```

output
```
Timed out
```

3.177.7 Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 606, normalized size of antiderivative = 30.30

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2} \, dx = \int \frac{1}{(dx+c)^2(b\sinh(fx+e)+a)^2} \, dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-2*(a*e^(f*x + e) - b)/(a^2*b*c^2*f + b^3*c^2*f + (a^2*b*d^2*f + b^3*d^2*f
)*x^2 + 2*(a^2*b*c*d*f + b^3*c*d*f)*x - (a^2*b*c^2*f*e^(2*e) + b^3*c^2*f*e
^(2*e) + (a^2*b*d^2*f*e^(2*e) + b^3*d^2*f*e^(2*e))*x^2 + 2*(a^2*b*c*d*f*e^
(2*e) + b^3*c*d*f*e^(2*e))*x)*e^(2*f*x) - 2*(a^3*c^2*f*e^e + a*b^2*c^2*f*e
^e + (a^3*d^2*f*e^e + a*b^2*d^2*f*e^e)*x^2 + 2*(a^3*c*d*f*e^e + a*b^2*c*d*
f*e^e)*x)*e^(f*x)) + integrate(2*(2*b*d - (a*d*f*x*e^e + (c*f*e^e + 2*d*e^
e)*a)*e^(f*x))/(a^2*b*c^3*f + b^3*c^3*f + (a^2*b*d^3*f + b^3*d^3*f)*x^3 +
3*(a^2*b*c*d^2*f + b^3*c*d^2*f)*x^2 + 3*(a^2*b*c^2*d*f + b^3*c^2*d*f)*x -
(a^2*b*c^3*f*e^(2*e) + b^3*c^3*f*e^(2*e) + (a^2*b*d^3*f*e^(2*e) + b^3*d^3*
f*e^(2*e))*x^3 + 3*(a^2*b*c*d^2*f*e^(2*e) + b^3*c*d^2*f*e^(2*e))*x^2 + 3*(
a^2*b*c^2*d*f*e^(2*e) + b^3*c^2*d*f*e^(2*e))*x)*e^(2*f*x) - 2*(a^3*c^3*f*e
^e + a*b^2*c^3*f*e^e + (a^3*d^3*f*e^e + a*b^2*d^3*f*e^e)*x^3 + 3*(a^3*c*d^
2*f*e^e + a*b^2*c*d^2*f*e^e)*x^2 + 3*(a^3*c^2*d*f*e^e + a*b^2*c^2*d*f*e^e)
*x)*e^(f*x)), x)
```

3.177.8 Giac [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2} \, dx = \int \frac{1}{(dx+c)^2(b\sinh(fx+e)+a)^2} \, dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(b*sinh(f*x + e) + a)^2), x)
```

3.177.9 Mupad [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx = \int \frac{1}{\left(a+b\sinh\left(e+f\,x\right)\right)^2\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((a + b*sinh(e + f*x))^2*(c + d*x)^2),x)
```

output
```
int(1/((a + b*sinh(e + f*x))^2*(c + d*x)^2), x)
```

3.178 $\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx$

3.178.1 Optimal result

Integrand size = 18, antiderivative size = 544

$$\begin{aligned}\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx &= \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{5/2}d}-\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}d}\\
&\quad-\frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{5/2}d}\\
&\quad+\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}d}+\frac{3af\log(a+b\sinh(c+dx))}{2(a^2+b^2)^2d^2}\\
&\quad+\frac{3a^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{5/2}d^2}-\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}d^2}\\
&\quad-\frac{3a^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{5/2}d^2}+\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}d^2}\\
&\quad-\frac{b(e+fx)\cosh(c+dx)}{2(a^2+b^2)d(a+b\sinh(c+dx))^2}\\
&\quad-\frac{f}{2(a^2+b^2)d^2(a+b\sinh(c+dx))}\\
&\quad-\frac{3ab(e+fx)\cosh(c+dx)}{2(a^2+b^2)^2d(a+b\sinh(c+dx))}\end{aligned}$$

output
```
3/2*a*f*ln(a+b*sinh(d*x+c))/(a^2+b^2)^2/d^2+3/2*a^2*(f*x+e)*ln(1+b*exp(d*x
+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(5/2)/d-1/2*(f*x+e)*ln(1+b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-3/2*a^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a
+(a^2+b^2)^(1/2)))/(a^2+b^2)^(5/2)/d+1/2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+3/2*a^2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/(a^2+b^2)^(5/2)/d^2-1/2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/(a^2+b^2)^(3/2)/d^2-3/2*a^2*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/(a^2+b^2)^(5/2)/d^2+1/2*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))/(a^2+b^2)^(3/2)/d^2-1/2*b*(f*x+e)*cosh(d*x+c)/(a^2+b^2)/d/(a+b*sin
h(d*x+c))^2-1/2*f/(a^2+b^2)/d^2/(a+b*sinh(d*x+c))-3/2*a*b*(f*x+e)*cosh(d*x
+c)/(a^2+b^2)^2/d/(a+b*sinh(d*x+c))
```

3.178.2 Mathematica [A] (verified)

Time = 5.03 (sec) , antiderivative size = 773, normalized size of antiderivative = 1.42

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx =$$

$$-\frac{-\frac{-3a\sqrt{-(a^2+b^2)^2}f(c+dx)+6a^2\sqrt{a^2+b^2}f\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)-4a^2\sqrt{-a^2-b^2}de\,\mathrm{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2b^2\sqrt{-a^2-b^2}de\,\mathrm{arctanh}\left(\frac{a+b}{\sqrt{a}}\right.}{}}{}$$

input
```
Integrate[(e + f*x)/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*(-((-3*a*Sqrt[-(a^2 + b^2)^2]*f*(c + d*x) + 6*a^2*Sqrt[a^2 + b^2]*f*A
rcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]] - 4*a^2*Sqrt[-a^2 - b^2]*d*e*A
rcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*b^2*Sqrt[-a^2 - b^2]*d*e*A
rcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 6*a^2*Sqrt[-a^2 - b^2]*f*Arc
Tanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 4*a^2*Sqrt[-a^2 - b^2]*c*f*Arc
Tanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*b^2*Sqrt[-a^2 - b^2]*c*f*Arc
Tanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*a^2*Sqrt[-a^2 - b^2]*f*(c +
d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - b^2*Sqrt[-a^2 - b^2]
*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*a^2*Sqrt[-
a^2 - b^2]*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + b^
2*Sqrt[-a^2 - b^2]*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])] + 3*a*Sqrt[-(a^2 + b^2)^2]*f*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d
*x)))] + Sqrt[-a^2 - b^2]*(2*a^2 - b^2)*f*PolyLog[2, (b*E^(c + d*x))/(-a +
 Sqrt[a^2 + b^2])] + Sqrt[-a^2 - b^2]*(-2*a^2 + b^2)*f*PolyLog[2, -((b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2]))])/Sqrt[-(a^2 + b^2)^2]) + (b*(a^2 + b^2)*
d*(e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2 + ((a^2 + b^2)*f + 3*a*
b*d*(e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x]))/((a^2 + b^2)^2*d^2)
```

3.178.3 Rubi [A] (verified)

Time = 3.63 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.53, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.056, Rules used = {3042, 3806, 26, 3042, 3147, 17, 3805, 3042, 3147, 16, 3803, 25, 2694, 27, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{e+fx}{(a-ib\sin(ic+idx))^3}dx$$

$$\Big\downarrow \text{3806}$$

$$\frac{a\int \frac{e+fx}{(a+b\sinh(c+dx))^2}dx}{a^2+b^2}+\frac{ib\int \frac{i(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}+\frac{bf\int \frac{\cosh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{a\int \frac{e+fx}{(a+b\sinh(c+dx))^2}dx}{a^2+b^2}-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}+\frac{bf\int \frac{\cosh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{e+fx}{(a-ib\sin(ic+idx))^2}dx}{a^2+b^2}-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}+\frac{bf\int \frac{\cos(ic+idx)}{(a-ib\sin(ic+idx))^2}dx}{2d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3147}$$

$$\frac{f\int \frac{1}{(a+b\sinh(c+dx))^2}d(b\sinh(c+dx))}{2d^2(a^2+b^2)}+\frac{a\int \frac{e+fx}{(a-ib\sin(ic+idx))^2}dx}{a^2+b^2}-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{17}$$

$$\frac{a\int \frac{e+fx}{(a-ib\sin(ic+idx))^2}dx}{a^2+b^2}-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}-\frac{f}{2d^2(a^2+b^2)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3805}$$

$$-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}+\frac{a\left(\frac{a\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{bf\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{a^2+b^2}-\frac{f}{2d^2(a^2+b^2)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}+\frac{a\left(\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{bf\int \frac{\cos(ic+idx)}{a-ib\sin(ic+idx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{a^2+b^2}-\frac{f}{2d^2(a^2+b^2)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3147}$$

$$\frac{a\left(\frac{f\int \frac{1}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{d^2(a^2+b^2)}+\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{a^2+b^2}-\frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}-\frac{f}{2d^2(a^2+b^2)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

3.178. $\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx$

$$\Big\downarrow \; 16$$

$$\frac{a\left(\frac{a\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{a^2+b^2}-\frac{b\int\frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}-\frac{f}{2d^2(a^2+b^2)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 3803$$

$$\frac{a\left(\frac{2a\int-\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{a^2+b^2}-\frac{b\int\frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}-\frac{f}{2d^2(a^2+b^2)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{a\left(-\frac{2a\int\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{a^2+b^2}-\frac{b\int\frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}-\frac{f}{2d^2(a^2+b^2)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 2694$$

$$\frac{a\left(-\frac{2a\left(\frac{b\int-\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{a^2+b^2}-\frac{b\int\frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}-\frac{f}{2d^2(a^2+b^2)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{a\left(-\frac{2a\left(\frac{b\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{a^2+b^2}-\frac{b\int\frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}-\frac{f}{2d^2(a^2+b^2)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

3.178. $\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx$

$$\Big\downarrow \text{2620}$$

$$\frac{a\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a+b}{d^2(c}\right.}{a^2+b^2}$$

$$\frac{b\int\frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2\left(a^2+b^2\right)}-\frac{f}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{2715}$$

$$\frac{a\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d}{bd^2}\right.}{2\sqrt{a^2+b^2}}}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{b\int\frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2\left(a^2+b^2\right)}-\frac{f}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\int\frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2\left(a^2+b^2\right)}+$$

$$\frac{a\left(-\frac{2a\left(\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a}{d}\right.}{a^2+b^2}$$

$$\frac{f}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

3.178. $\int\frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\int\left(\frac{e+fx}{b(a+b\sinh(c+dx))}-\frac{a(e+fx)}{b(a+b\sinh(c+dx))^2}\right)dx}{2\left(a^2+b^2\right)}+$$

$$\frac{a\left(-\frac{2a\left(\frac{b\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a}{d}\right.}{a^2+b^2}$$

$$\frac{f}{2d^2\left(a^2+b^2\right)\left(a+b\sinh(c+dx)\right)}-\frac{b(e+fx)\cosh(c+dx)}{2d\left(a^2+b^2\right)\left(a+b\sinh(c+dx)\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{f}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}-$$

$$\frac{b\left(-\frac{(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)a^2}{b(a^2+b^2)^{3/2}d}+\frac{(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)a^2}{b(a^2+b^2)^{3/2}d}-\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)a^2}{b(a^2+b^2)^{3/2}d^2}+\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)a^2}{b(a^2+b^2)^{3/2}d^2}-\right.}{}$$

$$\frac{a\left(-\frac{b(e+fx)\cosh(c+dx)}{(a^2+b^2)d(a+b\sinh(c+dx))}+\frac{f\log(a+b\sinh(c+dx))}{(a^2+b^2)d^2}-\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log(}{}\right.}{}\right.}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{b(e+fx)\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2}$$

input
```
Int[(e + f*x)/(a + b*Sinh[c + d*x])^3,x]
```

3.178. $\int\frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx$

output
```
-1/2*(b*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x])^2) -
 f/(2*(a^2 + b^2)*d^2*(a + b*Sinh[c + d*x])) - (b*(-((a^2*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d)) + ((e +
 f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d
) + (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2
 + b^2)^(3/2)*d) - ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(b*Sqrt[a^2 + b^2]*d) - (a*f*Log[a + b*Sinh[c + d*x]])/(b*(a^2 + b^2)*
d^2) - (a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^
2 + b^2)^(3/2)*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
))])/(b*Sqrt[a^2 + b^2]*d^2) + (a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^2) - (f*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (a*(e + f*x)*Cosh[c
 + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x]))))/(2*(a^2 + b^2)) + (a*((f*
Log[a + b*Sinh[c + d*x]])/((a^2 + b^2)*d^2) - (2*a*(-1/2*(b*(((e + f*x)*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e +
 f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])
))/(a^2 + b^2) - (b*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c
+ d*x]))))/(a^2 + b^2)
```

3.178.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.178. $\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx$

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 3806
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(-b)*(c + d*x)^m*Cos[e + f*x]*((a + b*Sin[e + f*x])^(n +
 1)/(f*(n + 1)*(a^2 - b^2))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m*(a
 + b*Sin[e + f*x])^(n + 1), x], x] - Simp[b*((n + 2)/((n + 1)*(a^2 - b^2)))
   Int[(c + d*x)^m*Sin[e + f*x]*(a + b*Sin[e + f*x])^(n + 1), x], x] + Simp
[b*d*(m/(f*(n + 1)*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*Cos[e + f*x]*(a +
b*Sin[e + f*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2
 - b^2, 0] && ILtQ[n, -2] && IGtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.178.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1231 vs. $2(480) = 960$.

Time = 2.05 (sec) , antiderivative size = 1232, normalized size of antiderivative = 2.26

method	result	size
risch	Expression too large to display	1232

input
```
int((f*x+e)/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
(2*a^2*b*d*f*x*exp(3*d*x+3*c)-b^3*d*f*x*exp(3*d*x+3*c)+6*a^3*d*f*x*exp(2*d
*x+2*c)+2*a^2*b*d*e*exp(3*d*x+3*c)-3*a*b^2*d*f*x*exp(2*d*x+2*c)-b^3*d*e*ex
p(3*d*x+3*c)+6*a^3*d*e*exp(2*d*x+2*c)-10*a^2*b*d*f*x*exp(d*x+c)-a^2*b*f*ex
p(3*d*x+3*c)-3*a*b^2*d*e*exp(2*d*x+2*c)-b^3*d*f*x*exp(d*x+c)-b^3*f*exp(3*d
*x+3*c)-2*a^3*f*exp(2*d*x+2*c)-10*a^2*b*d*e*exp(d*x+c)+3*a*b^2*d*f*x-2*a*b
^2*f*exp(2*d*x+2*c)-b^3*d*e*exp(d*x+c)+a^2*b*f*exp(d*x+c)+3*d*e*a*b^2+b^3*
f*exp(d*x+c))/d^2/(a^2+b^2)^2/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)^2-1/(a^2
+b^2)^(5/2)/d^2*b^2*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-
3/(a^2+b^2)^2/d^2*a*f*ln(exp(d*x+c))+3/2/(a^2+b^2)^2/d^2*a*f*ln(b*exp(2*d*
x+2*c)+2*a*exp(d*x+c)-b)-2/(a^2+b^2)^(5/2)/d*a^2*e*arctanh(1/2*(2*b*exp(d*
x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(5/2)/d^2*a^2*f*c*arctanh(1/2*(2*b*
exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(5/2)/d*a^2*f*ln((-b*exp(d*x+
c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(5/2)/d*a^2*f*ln
((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(5/2)
/d^2*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/
(a^2+b^2)^(5/2)/d^2*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)
^(1/2)))*c+1/(a^2+b^2)^(5/2)/d^2*a^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2
)-a)/(-a+(a^2+b^2)^(1/2)))-1/(a^2+b^2)^(5/2)/d^2*a^2*f*dilog((b*exp(d*x+c)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/(a^2+b^2)^(5/2)/d*b^2*e*arctanh
(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/2/(a^2+b^2)^(5/2)/d*b^2*f*...
```

3.178.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6396 vs. $2(476) = 952$.

Time = 0.35 (sec) , antiderivative size = 6396, normalized size of antiderivative = 11.76

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.178.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

3.178.7 Maxima [F]

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{fx+e}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(4*a^2*d*integrate(x*e^(d*x + c)/(a^4*b*d*e^(2*d*x + 2*c) + 2*a^2*b^3*
d*e^(2*d*x + 2*c) + b^5*d*e^(2*d*x + 2*c) + 2*a^5*d*e^(d*x + c) + 4*a^3*b^
2*d*e^(d*x + c) + 2*a*b^4*d*e^(d*x + c) - a^4*b*d - 2*a^2*b^3*d - b^5*d),
x) - 2*b^2*d*integrate(x*e^(d*x + c)/(a^4*b*d*e^(2*d*x + 2*c) + 2*a^2*b^3*
d*e^(2*d*x + 2*c) + b^5*d*e^(2*d*x + 2*c) + 2*a^5*d*e^(d*x + c) + 4*a^3*b^
2*d*e^(d*x + c) + 2*a*b^4*d*e^(d*x + c) - a^4*b*d - 2*a^2*b^3*d - b^5*d),
x) + 3*a*b*(a*log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a
 + sqrt(a^2 + b^2)))/((a^4*b + 2*a^2*b^3 + b^5)*sqrt(a^2 + b^2)*d^2) - 2*(
d*x + c)/((a^4*b + 2*a^2*b^3 + b^5)*d^2) + log(b*e^(2*d*x + 2*c) + 2*a*e^(
d*x + c) - b)/((a^4*b + 2*a^2*b^3 + b^5)*d^2)) + 2*(3*a*b^2*d*x - (a^2*b*e
^(3*c) + b^3*e^(3*c) - (2*a^2*b*d*e^(3*c) - b^3*d*e^(3*c))*x)*e^(3*d*x) -
(2*a^3*e^(2*c) + 2*a*b^2*e^(2*c) - 3*(2*a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*x
)*e^(2*d*x) + (a^2*b*e^c + b^3*e^c - (10*a^2*b*d*e^c + b^3*d*e^c)*x)*e^(d*
x))/(a^4*b^2*d^2 + 2*a^2*b^4*d^2 + b^6*d^2 + (a^4*b^2*d^2*e^(4*c) + 2*a^2*
b^4*d^2*e^(4*c) + b^6*d^2*e^(4*c))*e^(4*d*x) + 4*(a^5*b*d^2*e^(3*c) + 2*a^
3*b^3*d^2*e^(3*c) + a*b^5*d^2*e^(3*c))*e^(3*d*x) + 2*(2*a^6*d^2*e^(2*c) +
3*a^4*b^2*d^2*e^(2*c) - b^6*d^2*e^(2*c))*e^(2*d*x) - 4*(a^5*b*d^2*e^c + 2*
a^3*b^3*d^2*e^c + a*b^5*d^2*e^c)*e^(d*x)) - 3*a^2*log((b*e^(d*x + c) + a -
 sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2
 + b^4)*sqrt(a^2 + b^2)*d^2))*f + 1/2*e*((2*a^2 - b^2)*log((b*e^(-d*x -...
```

3.178.8 Giac [F]

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx=\int \frac{fx+e}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)/(b*sinh(d*x + c) + a)^3, x)
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx=\int \frac{e+f\,x}{(a+b\sinh(c+d\,x))^3}\,dx$$

input
```
int((e + f*x)/(a + b*sinh(c + d*x))^3,x)
```

output
```
int((e + f*x)/(a + b*sinh(c + d*x))^3, x)
```

3.179 $\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx$

3.179.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx = \text{Int}\left(\frac{1}{(e+fx)(a+b\sinh(c+dx))^3},x\right)$$

output
```
Unintegrable(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x)
```

3.179.2 Mathematica [N/A]

Not integrable

Time = 46.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx$$

input
```
Integrate[1/((e + f*x)*(a + b*Sinh[c + d*x])^3),x]
```

output
```
Integrate[1/((e + f*x)*(a + b*Sinh[c + d*x])^3), x]
```

3.179.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(e+fx)(a-ib\sin(ic+idx))^3}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}dx$$

input
```
Int[1/((e + f*x)*(a + b*Sinh[c + d*x])^3),x]
```

output
```
$Aborted
```

3.179.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.179.4 Maple [N/A] (verified)

Not integrable

Time = 0.74 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(fx+e)\left(a+b\sinh\left(dx+c\right)\right)^{3}}dx$$

input
```
int(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x)
```

output
```
int(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x)
```

3.179.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 4.15

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(fx+e)(b\sinh\left(dx+c\right)+a)^{3}}\,dx$$

input
```
integrate(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(1/(a^3*f*x + a^3*e + (b^3*f*x + b^3*e)*sinh(d*x + c)^3 + 3*(a*b^2
*f*x + a*b^2*e)*sinh(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*sinh(d*x + c)),
x)
```

3.179.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(f*x+e)/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

3.179.7 Maxima [N/A]

Not integrable

Time = 2.41 (sec) , antiderivative size = 1651, normalized size of antiderivative = 82.55

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(fx+e)(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
(3*a*b^2*d*f*x + 3*a*b^2*d*e + ((2*d*e + f)*a^2*b*e^(3*c) - (d*e - f)*b^3*
e^(3*c) + (2*a^2*b*d*f*e^(3*c) - b^3*d*f*e^(3*c))*x)*e^(3*d*x) + (2*(3*d*e
 + f)*a^3*e^(2*c) - (3*d*e - 2*f)*a*b^2*e^(2*c) + 3*(2*a^3*d*f*e^(2*c) - a
*b^2*d*f*e^(2*c))*x)*e^(2*d*x) - ((10*d*e + f)*a^2*b*e^c + (d*e + f)*b^3*e
^c + (10*a^2*b*d*f*e^c + b^3*d*f*e^c)*x)*e^(d*x))/(a^4*b^2*d^2*e^2 + 2*a^2
*b^4*d^2*e^2 + b^6*d^2*e^2 + (a^4*b^2*d^2*f^2 + 2*a^2*b^4*d^2*f^2 + b^6*d^
2*f^2)*x^2 + 2*(a^4*b^2*d^2*e*f + 2*a^2*b^4*d^2*e*f + b^6*d^2*e*f)*x + (a^
4*b^2*d^2*e^2*e^(4*c) + 2*a^2*b^4*d^2*e^2*e^(4*c) + b^6*d^2*e^2*e^(4*c) +
(a^4*b^2*d^2*f^2*e^(4*c) + 2*a^2*b^4*d^2*f^2*e^(4*c) + b^6*d^2*f^2*e^(4*c)
)*x^2 + 2*(a^4*b^2*d^2*e*f*e^(4*c) + 2*a^2*b^4*d^2*e*f*e^(4*c) + b^6*d^2*e
*f*e^(4*c))*x)*e^(4*d*x) + 4*(a^5*b*d^2*e^2*e^(3*c) + 2*a^3*b^3*d^2*e^2*e^
(3*c) + a*b^5*d^2*e^2*e^(3*c) + (a^5*b*d^2*f^2*e^(3*c) + 2*a^3*b^3*d^2*f^2
*e^(3*c) + a*b^5*d^2*f^2*e^(3*c))*x^2 + 2*(a^5*b*d^2*e*f*e^(3*c) + 2*a^3*b
^3*d^2*e*f*e^(3*c) + a*b^5*d^2*e*f*e^(3*c))*x)*e^(3*d*x) + 2*(2*a^6*d^2*e^
2*e^(2*c) + 3*a^4*b^2*d^2*e^2*e^(2*c) - b^6*d^2*e^2*e^(2*c) + (2*a^6*d^2*f
^2*e^(2*c) + 3*a^4*b^2*d^2*f^2*e^(2*c) - b^6*d^2*f^2*e^(2*c))*x^2 + 2*(2*a
^6*d^2*e*f*e^(2*c) + 3*a^4*b^2*d^2*e*f*e^(2*c) - b^6*d^2*e*f*e^(2*c))*x)*e
^(2*d*x) - 4*(a^5*b*d^2*e^2*e^c + 2*a^3*b^3*d^2*e^2*e^c + a*b^5*d^2*e^2*e^
c + (a^5*b*d^2*f^2*e^c + 2*a^3*b^3*d^2*f^2*e^c + a*b^5*d^2*f^2*e^c)*x^2 +
2*(a^5*b*d^2*e*f*e^c + 2*a^3*b^3*d^2*e*f*e^c + a*b^5*d^2*e*f*e^c)*x)*e^...
```

3.179.8 Giac [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(fx+e)(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((f*x + e)*(b*sinh(d*x + c) + a)^3), x)
```

3.179.9 Mupad [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(1/((e + f*x)*(a + b*sinh(c + d*x))^3),x)
```

output
```
int(1/((e + f*x)*(a + b*sinh(c + d*x))^3), x)
```

3.180 $\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx$

3.180.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx = \mathrm{Int}\left(\frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3},x\right)$$

output
```
Unintegrable(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x)
```

3.180.2 Mathematica [N/A]

Not integrable

Time = 46.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx$$

input
```
Integrate[1/((e + f*x)^2*(a + b*Sinh[c + d*x])^3),x]
```

output
```
Integrate[1/((e + f*x)^2*(a + b*Sinh[c + d*x])^3), x]
```

3.180.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(e+fx)^2(a-ib\sin(ic+idx))^3}dx$$
$$\Big\downarrow \text{ 3807}$$
$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}dx$$

input
```
Int[1/((e + f*x)^2*(a + b*Sinh[c + d*x])^3),x]
```

output
```
$Aborted
```

3.180.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.180.4 Maple [N/A] (verified)

Not integrable

Time = 0.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(fx+e)^2 (a+b\sinh (dx+c))^3} dx$$

input
```
int(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x)
```

output
```
int(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x)
```

3.180.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 7.05

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\, dx = \int \frac{1}{(fx+e)^2 (b\sinh (dx+c)+a)^3}\, dx$$

input
```
integrate(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(1/(a^3*f^2*x^2 + 2*a^3*e*f*x + a^3*e^2 + (b^3*f^2*x^2 + 2*b^3*e*f
*x + b^3*e^2)*sinh(d*x + c)^3 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*e*f*x + a*b^2*e
^2)*sinh(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x + a^2*b*e^2)*sinh(d
*x + c)), x)
```

3.180.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\, dx = \text{Timed out}$$

input
```
integrate(1/(f*x+e)**2/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

3.180.7 Maxima [N/A]

Not integrable

Time = 3.62 (sec) , antiderivative size = 2122, normalized size of antiderivative = 106.10

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(fx+e)^2(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
(3*a*b^2*d*f*x + 3*a*b^2*d*e + (2*(d*e + f)*a^2*b*e^(3*c) - (d*e - 2*f)*b^
3*e^(3*c) + (2*a^2*b*d*f*e^(3*c) - b^3*d*f*e^(3*c))*x)*e^(3*d*x) + (2*(3*d
*e + 2*f)*a^3*e^(2*c) - (3*d*e - 4*f)*a*b^2*e^(2*c) + 3*(2*a^3*d*f*e^(2*c)
 - a*b^2*d*f*e^(2*c))*x)*e^(2*d*x) - (2*(5*d*e + f)*a^2*b*e^c + (d*e + 2*f
)*b^3*e^c + (10*a^2*b*d*f*e^c + b^3*d*f*e^c)*x)*e^(d*x))/(a^4*b^2*d^2*e^3
+ 2*a^2*b^4*d^2*e^3 + b^6*d^2*e^3 + (a^4*b^2*d^2*f^3 + 2*a^2*b^4*d^2*f^3 +
 b^6*d^2*f^3)*x^3 + 3*(a^4*b^2*d^2*e*f^2 + 2*a^2*b^4*d^2*e*f^2 + b^6*d^2*e
*f^2)*x^2 + 3*(a^4*b^2*d^2*e^2*f + 2*a^2*b^4*d^2*e^2*f + b^6*d^2*e^2*f)*x
+ (a^4*b^2*d^2*e^3*e^(4*c) + 2*a^2*b^4*d^2*e^3*e^(4*c) + b^6*d^2*e^3*e^(4*
c) + (a^4*b^2*d^2*f^3*e^(4*c) + 2*a^2*b^4*d^2*f^3*e^(4*c) + b^6*d^2*f^3*e^
(4*c))*x^3 + 3*(a^4*b^2*d^2*e*f^2*e^(4*c) + 2*a^2*b^4*d^2*e*f^2*e^(4*c) +
b^6*d^2*e*f^2*e^(4*c))*x^2 + 3*(a^4*b^2*d^2*e^2*f*e^(4*c) + 2*a^2*b^4*d^2*
e^2*f*e^(4*c) + b^6*d^2*e^2*f*e^(4*c))*x)*e^(4*d*x) + 4*(a^5*b*d^2*e^3*e^(
3*c) + 2*a^3*b^3*d^2*e^3*e^(3*c) + a*b^5*d^2*e^3*e^(3*c) + (a^5*b*d^2*f^3*
e^(3*c) + 2*a^3*b^3*d^2*f^3*e^(3*c) + a*b^5*d^2*f^3*e^(3*c))*x^3 + 3*(a^5*
b*d^2*e*f^2*e^(3*c) + 2*a^3*b^3*d^2*e*f^2*e^(3*c) + a*b^5*d^2*e*f^2*e^(3*c
))*x^2 + 3*(a^5*b*d^2*e^2*f*e^(3*c) + 2*a^3*b^3*d^2*e^2*f*e^(3*c) + a*b^5*
d^2*e^2*f*e^(3*c))*x)*e^(3*d*x) + 2*(2*a^6*d^2*e^3*e^(2*c) + 3*a^4*b^2*d^2
*e^3*e^(2*c) - b^6*d^2*e^3*e^(2*c) + (2*a^6*d^2*f^3*e^(2*c) + 3*a^4*b^2*d^
2*f^3*e^(2*c) - b^6*d^2*f^3*e^(2*c))*x^3 + 3*(2*a^6*d^2*e*f^2*e^(2*c) +...
```

3.180.8 Giac [N/A]

Not integrable

Time = 60.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(fx+e)^2(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((f*x + e)^2*(b*sinh(d*x + c) + a)^3), x)
```

3.180.9 Mupad [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx=\int \frac{1}{{\left(e+f\,x\right)}^2\,{\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^3} \,d x$$

input
```
int(1/((e + f*x)^2*(a + b*sinh(c + d*x))^3),x)
```

output
```
int(1/((e + f*x)^2*(a + b*sinh(c + d*x))^3), x)
```

3.181 $\int (c+dx)^m (a+b\sinh(e+fx))^n \, dx$

3.181.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m (a+b\sinh(e+fx))^n \, dx = \text{Int}((c+dx)^m (a+b\sinh(e+fx))^n, x)$$

output
```
Unintegrable((d*x+c)^m*(a+b*sinh(f*x+e))^n,x)
```

3.181.2 Mathematica [N/A]

Not integrable

Time = 3.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m (a+b\sinh(e+fx))^n \, dx = \int (c+dx)^m (a+b\sinh(e+fx))^n \, dx$$

input
```
Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x])^n,x]
```

output
```
Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x])^n, x]
```

3.181.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+b\sinh (e+fx))^n\, dx$$
$$\Big\downarrow \ 3042$$
$$\int (c+dx)^m(a-ib\sin (ie+ifx))^ndx$$
$$\Big\downarrow \ 3807$$
$$\int (c+dx)^m(a+b\sinh (e+fx))^ndx$$

input
```
Int[(c + d*x)^m*(a + b*Sinh[e + f*x])^n,x]
```

output
```
$Aborted
```

3.181.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.181.4 Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^m (a+b\sinh(fx+e))^n\, dx$$

input
```
int((d*x+c)^m*(a+b*sinh(f*x+e))^n,x)
```

output
```
int((d*x+c)^m*(a+b*sinh(f*x+e))^n,x)
```

3.181.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\sinh(e+fx))^n\, dx = \int (dx+c)^m(b\sinh(fx+e)+a)^n\, dx$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*(b*sinh(f*x + e) + a)^n, x)
```

3.181.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m(a+b\sinh(e+fx))^n\, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*(a+b*sinh(f*x+e))**n,x)
```

output
```
Timed out
```

3.181.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\sinh(e+fx))^n\,dx=\int (dx+c)^m(b\sinh(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*(b*sinh(f*x + e) + a)^n, x)
```

3.181.8 Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\sinh(e+fx))^n\,dx=\int (dx+c)^m(b\sinh(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*(b*sinh(f*x + e) + a)^n, x)
```

3.181.9 Mupad [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\sinh(e+fx))^n\,dx=\int (a+b\sinh(e+f\,x))^n\,(c+d\,x)^m\,dx$$

input
```
int((a + b*sinh(e + f*x))^n*(c + d*x)^m,x)
```

output
```
int((a + b*sinh(e + f*x))^n*(c + d*x)^m, x)
```

3.182 $\int (c+dx)^m (a+b\sinh(e+fx))^3 \, dx$

3.182.1 Optimal result

Integrand size = 20, antiderivative size = 543

$$
\begin{aligned}
&\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx \\
&= \frac{a^3(c+dx)^{1+m}}{d(1+m)} - \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} \\
&+ \frac{3^{-1-m}b^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)}{8f} \\
&+ \frac{3\ 2^{-3-m}ab^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f} \\
&+ \frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f} \\
&- \frac{3b^3e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{8f} \\
&+ \frac{3a^2be^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{2f} \\
&- \frac{3b^3e^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{8f} \\
&- \frac{3\ 2^{-3-m}ab^2e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{f} \\
&+ \frac{3^{-1-m}b^3e^{-3e+\frac{3cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3f(c+dx)}{d}\right)}{8f}
\end{aligned}
$$

output
```
a^3*(d*x+c)^(1+m)/d/(1+m)-3/2*a*b^2*(d*x+c)^(1+m)/d/(1+m)+1/8*3^(-1-m)*b^3
*exp(3*e-3*c*f/d)*(d*x+c)^m*GAMMA(1+m,-3*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)
+3*2^(-3-m)*a*b^2*exp(2*e-2*c*f/d)*(d*x+c)^m*GAMMA(1+m,-2*f*(d*x+c)/d)/f/(
(-f*(d*x+c)/d)^m)+3/2*a^2*b*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)
/f/((-f*(d*x+c)/d)^m)-3/8*b^3*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/
d)/f/((-f*(d*x+c)/d)^m)+3/2*a^2*b*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x
+c)/d)/f/((f*(d*x+c)/d)^m)-3/8*b^3*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*
x+c)/d)/f/((f*(d*x+c)/d)^m)-3*2^(-3-m)*a*b^2*exp(-2*e+2*c*f/d)*(d*x+c)^m*G
AMMA(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)+1/8*3^(-1-m)*b^3*exp(-3*e+3*c*
f/d)*(d*x+c)^m*GAMMA(1+m,3*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)
```

3.182.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.83

$$\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx$$
$$= \frac{2^{-3-m}3^{-1-m}e^{-3\left(e+\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(2^mb^3de^{6e}(1+m)\left(\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)+3^2\right.}{}$$

input
```
Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x])^3,x]
```

output
```
(2^(-3 - m)*3^(-1 - m)*(c + d*x)^m*(2^m*b^3*d*E^(6*e)*(1 + m)*((f*(c + d*x
))/d)^m*Gamma[1 + m, (-3*f*(c + d*x))/d] + 3^(2 + m)*a*b^2*d*E^(5*e + (c*f
)/d)*(1 + m)*(f*(c/d + x))^m*Gamma[1 + m, (-2*f*(c + d*x))/d] - 2^m*3^(2 +
 m)*b*(-4*a^2 + b^2)*d*E^(4*e + (2*c*f)/d)*(1 + m)*((f*(c + d*x))/d)^m*Gam
ma[1 + m, -((f*(c + d*x))/d)] - 2^m*3^(2 + m)*b*(-4*a^2 + b^2)*d*E^(2*e +
(4*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (f*(c + d*x))/d] -
3^(2 + m)*a*b^2*d*E^(e + (5*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1
 + m, (2*f*(c + d*x))/d] + 2^m*E^((3*c*f)/d)*(4*3^(1 + m)*a*(2*a^2 - 3*b^2
)*E^(3*e)*f*(c + d*x)*(-((f^2*(c + d*x)^2)/d^2))^m + b^3*d*E^((3*c*f)/d)*(
1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (3*f*(c + d*x))/d])))/(d*E^(3*(
e + (c*f)/d))*f*(1 + m)*(-((f^2*(c + d*x)^2)/d^2))^m)
```

3.182.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)^m(a-ib\sin(ie+ifx))^3dx$$

$$\Big\downarrow \; 3798$$

$$\int \left(a^3(c+dx)^m + 3a^2b(c+dx)^m \sinh(e+fx) + 3ab^2(c+dx)^m \sinh^2(e+fx) + b^3(c+dx)^m \sinh^3(e+fx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(c+dx)^{m+1}}{d(m+1)} + \frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f} + \frac{3a^2be^{\frac{cf}{d}-e}(c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{2f} + \frac{3ab^2 2^{-m-3}e^{2e-\frac{2cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f} - \frac{3ab^2 2^{-m-3}e^{\frac{2cf}{d}-2e}(c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{f} - \frac{3ab^2(c+dx)^{m+1}}{2d(m+1)} + \frac{b^3 3^{-m-1}e^{3e-\frac{3cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{3f(c+dx)}{d}\right)}{8f} - \frac{3b^3e^{e-\frac{cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{8f} - \frac{3b^3e^{\frac{cf}{d}-e}(c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{8f} + \frac{b^3 3^{-m-1}e^{\frac{3cf}{d}-3e}(c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,\frac{3f(c+dx)}{d}\right)}{8f}$$

input
```
Int[(c + d*x)^m*(a + b*Sinh[e + f*x])^3,x]
```

output
```
(a^3*(c + d*x)^(1 + m))/(d*(1 + m)) - (3*a*b^2*(c + d*x)^(1 + m))/(2*d*(1
+ m)) + (3^(-1 - m)*b^3*E^(3*e - (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-3*f
*(c + d*x))/d])/(8*f*(-((f*(c + d*x))/d))^m) + (3*2^(-3 - m)*a*b^2*E^(2*e
- (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d
*x))/d))^m) + (3*a^2*b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c +
d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) - (3*b^3*E^(e - (c*f)/d)*(c + d*x)
^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(8*f*(-((f*(c + d*x))/d))^m) + (3*a^2
*b*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c
 + d*x))/d)^m) - (3*b^3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c +
d*x))/d])/(8*f*((f*(c + d*x))/d)^m) - (3*2^(-3 - m)*a*b^2*E^(-2*e + (2*c*f
)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m)
+ (3^(-1 - m)*b^3*E^(-3*e + (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c +
d*x))/d])/(8*f*((f*(c + d*x))/d)^m)
```

3.182. $\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx$

3.182.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.182.4 Maple [F]

$$\int (dx+c)^m (a+b\sinh(fx+e))^3 dx$$

input
```
int((d*x+c)^m*(a+b*sinh(f*x+e))^3,x)
```

output
```
int((d*x+c)^m*(a+b*sinh(f*x+e))^3,x)
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.53

$$\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx$$
$$= \frac{(b^3dm+b^3d)\cosh\left(\frac{dm\log\left(\frac{3f}{d}\right)+3de-3cf}{d}\right)\Gamma\left(m+1,\frac{3(dfx+cf)}{d}\right)-9(ab^2dm+ab^2d)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2de-2c}{d}\right)}{}$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/24*((b^3*d*m + b^3*d)*cosh((d*m*log(3*f/d) + 3*d*e - 3*c*f)/d)*gamma(m +
 1, 3*(d*f*x + c*f)/d) - 9*(a*b^2*d*m + a*b^2*d)*cosh((d*m*log(2*f/d) + 2*
d*e - 2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) + 9*((4*a^2*b - b^3)*d*m +
 (4*a^2*b - b^3)*d)*cosh((d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x
 + c*f)/d) + 9*((4*a^2*b - b^3)*d*m + (4*a^2*b - b^3)*d)*cosh((d*m*log(-f/
d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) + 9*(a*b^2*d*m + a*b^2*d
)*cosh((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2*(d*f*x + c*f)/
d) + (b^3*d*m + b^3*d)*cosh((d*m*log(-3*f/d) - 3*d*e + 3*c*f)/d)*gamma(m +
 1, -3*(d*f*x + c*f)/d) - (b^3*d*m + b^3*d)*gamma(m + 1, 3*(d*f*x + c*f)/d
)*sinh((d*m*log(3*f/d) + 3*d*e - 3*c*f)/d) + 9*(a*b^2*d*m + a*b^2*d)*gamma
(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d) - 9*((
4*a^2*b - b^3)*d*m + (4*a^2*b - b^3)*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh
((d*m*log(f/d) + d*e - c*f)/d) - 9*((4*a^2*b - b^3)*d*m + (4*a^2*b - b^3)*
d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh((d*m*log(-f/d) - d*e + c*f)/d) - 9*
(a*b^2*d*m + a*b^2*d)*gamma(m + 1, -2*(d*f*x + c*f)/d)*sinh((d*m*log(-2*f/
d) - 2*d*e + 2*c*f)/d) - (b^3*d*m + b^3*d)*gamma(m + 1, -3*(d*f*x + c*f)/d
)*sinh((d*m*log(-3*f/d) - 3*d*e + 3*c*f)/d) + 12*((2*a^3 - 3*a*b^2)*d*f*x
+ (2*a^3 - 3*a*b^2)*c*f)*cosh(m*log(d*x + c)) + 12*((2*a^3 - 3*a*b^2)*d*f*
x + (2*a^3 - 3*a*b^2)*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

3.182.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+b*sinh(f*x+e))**3,x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.69

$$\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx$$
$$= \frac{3}{2}\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} - \frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a^2b$$
$$-\frac{3}{4}\left(\frac{(dx+c)^{m+1}e^{\left(-2e+\frac{2cf}{d}\right)}E_{-m}\left(\frac{2(dx+c)f}{d}\right)}{d} + \frac{(dx+c)^{m+1}e^{\left(2e-\frac{2cf}{d}\right)}E_{-m}\left(-\frac{2(dx+c)f}{d}\right)}{d} + \frac{2(dx+c)^{m+1}}{d(m+1)}\right.$$
$$+\frac{1}{8}\left(\frac{(dx+c)^{m+1}e^{\left(-3e+\frac{3cf}{d}\right)}E_{-m}\left(\frac{3(dx+c)f}{d}\right)}{d} - \frac{3(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} + \frac{3(dx+c)^{m+1}e^{\left(e\right.}}{}\right.$$
$$+\frac{(dx+c)^{m+1}a^3}{d(m+1)}$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^3,x, algorithm="maxima")
```

output
```
3/2*((d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d
- (d*x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a^
2*b - 3/4*((d*x + c)^(m + 1)*e^(-2*e + 2*c*f/d)*exp_integral_e(-m, 2*(d*x
+ c)*f/d)/d + (d*x + c)^(m + 1)*e^(2*e - 2*c*f/d)*exp_integral_e(-m, -2*(d
*x + c)*f/d)/d + 2*(d*x + c)^(m + 1)/(d*(m + 1)))*a*b^2 + 1/8*((d*x + c)^(
m + 1)*e^(-3*e + 3*c*f/d)*exp_integral_e(-m, 3*(d*x + c)*f/d)/d - 3*(d*x +
 c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d + 3*(d*x +
c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d - (d*x + c)^
(m + 1)*e^(3*e - 3*c*f/d)*exp_integral_e(-m, -3*(d*x + c)*f/d)/d)*b^3 + (d
*x + c)^(m + 1)*a^3/(d*(m + 1))
```

3.182.8 Giac [F]

$$\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx = \int (b\sinh(fx+e)+a)^3(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e) + a)^3*(d*x + c)^m, x)
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx = \int (a+b\sinh(e+f\,x))^3\,(c+d\,x)^m\,dx$$

input
```
int((a + b*sinh(e + f*x))^3*(c + d*x)^m,x)
```

output
```
int((a + b*sinh(e + f*x))^3*(c + d*x)^m, x)
```

3.183 $\int (c+dx)^m (a+b\sinh(e+fx))^2\,dx$

3.183.1 Optimal result

Integrand size = 20, antiderivative size = 281

$$
\begin{aligned}
&\int (c+dx)^m (a+b\sinh(e+fx))^2\,dx \\
&= \frac{a^2(c+dx)^{1+m}}{d(1+m)} - \frac{b^2(c+dx)^{1+m}}{2d(1+m)} \\
&\quad + \frac{2^{-3-m}b^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f} \\
&\quad + \frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{f} \\
&\quad + \frac{abe^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{f} \\
&\quad - \frac{2^{-3-m}b^2e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{f}
\end{aligned}
$$

output
```
a^2*(d*x+c)^(1+m)/d/(1+m)-1/2*b^2*(d*x+c)^(1+m)/d/(1+m)+2^(-3-m)*b^2*exp(2
*e-2*c*f/d)*(d*x+c)^m*GAMMA(1+m,-2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+a*b*e
xp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+a*b*exp
(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-2^(-3-m)*b
^2*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m
)
```

3.183.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.90

$$\int (c+dx)^m(a+b\sinh(e+fx))^2\,dx$$
$$=\frac{(c+dx)^m\left(8a^2f(c+dx)-4b^2f(c+dx)+2^{-m}b^2de^{2e-\frac{2cf}{d}}(1+m)\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)+\right.}{}$$

input
```
Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x])^2,x]
```

output
```
((c + d*x)^m*(8*a^2*f*(c + d*x) - 4*b^2*f*(c + d*x) + (b^2*d*E^(2*e - (2*c
*f)/d)*(1 + m)*Gamma[1 + m, (-2*f*(c + d*x))/d])/(2^m*(-((f*(c + d*x))/d))
^m) + (8*a*b*d*E^(e - (c*f)/d)*(1 + m)*Gamma[1 + m, -((f*(c + d*x))/d)])/(
-((f*(c + d*x))/d))^m + (8*a*b*d*E^(-e + (c*f)/d)*(1 + m)*Gamma[1 + m, (f*
(c + d*x))/d])/((f*(c + d*x))/d)^m - (b^2*d*E^(-2*e + (2*c*f)/d)*(1 + m)*G
amma[1 + m, (2*f*(c + d*x))/d])/(2^m*((f*(c + d*x))/d)^m)))/(8*d*f*(1 + m)
)
```

3.183.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+b\sinh(e+fx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^m(a-ib\sin(ie+ifx))^2dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(a^2(c+dx)^m+2ab(c+dx)^m\sinh(e+fx)+b^2(c+dx)^m\sinh^2(e+fx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^{m+1}}{d(m+1)}+\frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{f}+\frac{abe^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{f}+\frac{b^2 2^{-m-3}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}-\frac{b^2 2^{-m-3}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{f}-\frac{b^2(c+dx)^{m+1}}{2d(m+1)}$$

input
```
Int[(c + d*x)^m*(a + b*Sinh[e + f*x])^2,x]
```

output
```
(a^2*(c + d*x)^(1 + m))/(d*(1 + m)) - (b^2*(c + d*x)^(1 + m))/(2*d*(1 + m)
) + (2^(-3 - m)*b^2*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c
+ d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (a*b*E^(e - (c*f)/d)*(c + d*x)^m*
Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) + (a*b*E^(-e
+ (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)
^m) - (2^(-3 - m)*b^2*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(
c + d*x))/d])/(f*((f*(c + d*x))/d)^m)
```

3.183.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.183.4 Maple [F]

$$\int (dx+c)^m (a+b\sinh(fx+e))^2 \, dx$$

input
```
int((d*x+c)^m*(a+b*sinh(f*x+e))^2,x)
```

output
```
int((d*x+c)^m*(a+b*sinh(f*x+e))^2,x)
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.84

$$\int (c+dx)^m (a+b\sinh(e+fx))^2 \, dx =$$
$$-\frac{(b^2dm+b^2d)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2\,de-2\,cf}{d}\right)\Gamma\left(m+1,\frac{2\,(dfx+cf)}{d}\right)-8\,(abdm+abd)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)}{}$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/8*((b^2*d*m + b^2*d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m +
 1, 2*(d*f*x + c*f)/d) - 8*(a*b*d*m + a*b*d)*cosh((d*m*log(f/d) + d*e - c*
f)/d)*gamma(m + 1, (d*f*x + c*f)/d) - 8*(a*b*d*m + a*b*d)*cosh((d*m*log(-f
/d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) - (b^2*d*m + b^2*d)*cos
h((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2*(d*f*x + c*f)/d) -
(b^2*d*m + b^2*d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2
*d*e - 2*c*f)/d) + 8*(a*b*d*m + a*b*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh(
(d*m*log(f/d) + d*e - c*f)/d) + 8*(a*b*d*m + a*b*d)*gamma(m + 1, -(d*f*x +
 c*f)/d)*sinh((d*m*log(-f/d) - d*e + c*f)/d) + (b^2*d*m + b^2*d)*gamma(m +
 1, -2*(d*f*x + c*f)/d)*sinh((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d) - 4*((2*
a^2 - b^2)*d*f*x + (2*a^2 - b^2)*c*f)*cosh(m*log(d*x + c)) - 4*((2*a^2 - b
^2)*d*f*x + (2*a^2 - b^2)*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

3.183.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m(a+b\sinh(e+fx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+b*sinh(f*x+e))**2,x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.74

$$\int (c+dx)^m(a+b\sinh(e+fx))^2\,dx$$
$$= \left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} - \frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)ab$$
$$-\frac{1}{4}\left(\frac{(dx+c)^{m+1}e^{\left(-2e+\frac{2cf}{d}\right)}E_{-m}\left(\frac{2(dx+c)f}{d}\right)}{d} + \frac{(dx+c)^{m+1}e^{\left(2e-\frac{2cf}{d}\right)}E_{-m}\left(-\frac{2(dx+c)f}{d}\right)}{d} + \frac{2(dx+c)^{m+1}}{d(m+1)}\right)$$
$$+\frac{(dx+c)^{m+1}a^2}{d(m+1)}$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
((d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d - (d
*x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a*b -
1/4*((d*x + c)^(m + 1)*e^(-2*e + 2*c*f/d)*exp_integral_e(-m, 2*(d*x + c)*f
/d)/d + (d*x + c)^(m + 1)*e^(2*e - 2*c*f/d)*exp_integral_e(-m, -2*(d*x + c
)*f/d)/d + 2*(d*x + c)^(m + 1)/(d*(m + 1)))*b^2 + (d*x + c)^(m + 1)*a^2/(d
*(m + 1))
```

3.183.8 Giac [F]

$$\int (c+dx)^m(a+b\sinh(e+fx))^2\,dx = \int (b\sinh(fx+e)+a)^2(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e) + a)^2*(d*x + c)^m, x)
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+b\sinh(e+fx))^2\,dx = \int (a+b\sinh(e+f\,x))^2\,(c+d\,x)^m\,dx$$

input
```
int((a + b*sinh(e + f*x))^2*(c + d*x)^m,x)
```

output
```
int((a + b*sinh(e + f*x))^2*(c + d*x)^m, x)
```

3.184 $\int (c+dx)^m(a+b\sinh(e+fx))\,dx$

3.184.1 Optimal result

Integrand size = 18, antiderivative size = 131

$$\int (c+dx)^m(a+b\sinh(e+fx))\,dx=\frac{a(c+dx)^{1+m}}{d(1+m)}+\frac{be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{be^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{2f}$$

output
```
a*(d*x+c)^(1+m)/d/(1+m)+1/2*b*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/
d)/f/((-f*(d*x+c)/d)^m)+1/2*b*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/
d)/f/((f*(d*x+c)/d)^m)
```

3.184.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.90

$$\int (c+dx)^m(a+b\sinh(e+fx))\,dx = \frac{1}{2}(c+dx)^m\left(\frac{2a(c+dx)}{d(1+m)} + \frac{be^{e-\frac{cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{f} + \frac{be^{-e+\frac{cf}{d}}\left(f\left(\frac{c}{d}+x\right)\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{f}\right)$$

input
```
Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x]),x]
```

output
```
((c + d*x)^m*((2*a*(c + d*x))/(d*(1 + m)) + (b*E^(e - (c*f)/d)*Gamma[1 + m
, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) + (b*E^(-e + (c*f)/d)*Ga
mma[1 + m, (f*(c + d*x))/d])/(f*(f*(c/d + x))^m)))/2
```

3.184.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+b\sinh(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^m(a-ib\sin(ie+ifx))dx$$

$$\Big\downarrow \text{ 3798}$$

$$\int \left(a(c+dx)^m+b(c+dx)^m\sinh(e+fx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a(c+dx)^{m+1}}{d(m+1)}+\frac{be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}$$

input
```
Int[(c + d*x)^m*(a + b*Sinh[e + f*x]),x]
```

output
```
(a*(c + d*x)^(1 + m))/(d*(1 + m)) + (b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1
 + m, -((f*(c + d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) + (b*E^(-e + (c*f)
/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))/d)^m)
```

3.184.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.184.4 Maple [F]

$$\int (dx+c)^m\,(a+b\sinh(fx+e))\,dx$$

input
```
int((d*x+c)^m*(a+b*sinh(f*x+e)),x)
```

output
```
int((d*x+c)^m*(a+b*sinh(f*x+e)),x)
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.90

$$\int (c+dx)^m(a+b\sinh(e+fx))\,dx$$
$$=\frac{(bdm+bd)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)+(bdm+bd)\cosh\left(\frac{dm\log\left(-\frac{f}{d}\right)-de+cf}{d}\right)\Gamma\left(m+1,-\frac{df}{}\right.}{}$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*((b*d*m + b*d)*cosh((d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x
+ c*f)/d) + (b*d*m + b*d)*cosh((d*m*log(-f/d) - d*e + c*f)/d)*gamma(m + 1,
 -(d*f*x + c*f)/d) - (b*d*m + b*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m
*log(f/d) + d*e - c*f)/d) - (b*d*m + b*d)*gamma(m + 1, -(d*f*x + c*f)/d)*s
inh((d*m*log(-f/d) - d*e + c*f)/d) + 2*(a*d*f*x + a*c*f)*cosh(m*log(d*x +
c)) + 2*(a*d*f*x + a*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

3.184.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m(a+b\sinh(e+fx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+b*sinh(f*x+e)),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.77

$$\int (c+dx)^m(a+b\sinh(e+fx))\,dx$$
$$=\frac{1}{2}\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)b$$
$$+\frac{(dx+c)^{m+1}a}{d(m+1)}$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*((d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d
- (d*x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*b
+ (d*x + c)^(m + 1)*a/(d*(m + 1))
```

3.184.8 Giac [F]

$$\int (c+dx)^m(a+b\sinh(e+fx))\,dx=\int (b\sinh(fx+e)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e) + a)*(d*x + c)^m, x)
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+b\sinh(e+fx))\,dx=\int (a+b\sinh(e+f\,x))\,\left(c+d\,x\right)^m\,dx$$

input
```
int((a + b*sinh(e + f*x))*(c + d*x)^m,x)
```

output
```
int((a + b*sinh(e + f*x))*(c + d*x)^m, x)
```

3.185 $\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx$

3.185.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx = \operatorname{Int}\left(\frac{(c+dx)^m}{a+b\sinh(e+fx)},x\right)$$

output
```
Unintegrable((d*x+c)^m/(a+b*sinh(f*x+e)),x)
```

3.185.2 Mathematica [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + b*Sinh[e + f*x]),x]
```

output
```
Integrate[(c + d*x)^m/(a + b*Sinh[e + f*x]), x]
```

3.185.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)} dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(c+dx)^m}{a-ib\sin(ie+ifx)}dx$$
$$\Big\downarrow \text{ 3807}$$
$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}dx$$

input
```
Int[(c + d*x)^m/(a + b*Sinh[e + f*x]),x]
```

output
```
$Aborted
```

3.185.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.185.4 Maple [N/A] (verified)

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^m}{a+b\sinh(fx+e)}dx$$

input
```
int((d*x+c)^m/(a+b*sinh(f*x+e)),x)
```

output
```
int((d*x+c)^m/(a+b*sinh(f*x+e)),x)
```

3.185.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx = \int \frac{(dx+c)^m}{b\sinh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/(b*sinh(f*x + e) + a), x)
```

3.185.6 Sympy [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx$$

input
```
integrate((d*x+c)**m/(a+b*sinh(f*x+e)),x)
```

output
```
Integral((c + d*x)**m/(a + b*sinh(e + f*x)), x)
```

3.185.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx = \int \frac{(dx+c)^m}{b\sinh\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(b*sinh(f*x + e) + a), x)
```

3.185.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx = \int \frac{(dx+c)^m}{b\sinh\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(b*sinh(f*x + e) + a), x)
```

3.185.9 Mupad [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx = \int \frac{\left(c+d\,x\right)^m}{a+b\,\mathrm{sinh}\left(e+f\,x\right)}\,dx$$

input
```
int((c + d*x)^m/(a + b*sinh(e + f*x)),x)
```

output
```
int((c + d*x)^m/(a + b*sinh(e + f*x)), x)
```

3.186 $\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx$

3.186.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx = \text{Int}\left(\frac{(c+dx)^m}{(a+b\sinh(e+fx))^2},x\right)$$

output
```
Unintegrable((d*x+c)^m/(a+b*sinh(f*x+e))^2,x)
```

3.186.2 Mathematica [N/A]

Not integrable

Time = 4.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + b*Sinh[e + f*x])^2,x]
```

output
```
Integrate[(c + d*x)^m/(a + b*Sinh[e + f*x])^2, x]
```

3.186.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^m}{(a-ib\sin(ie+ifx))^2}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}dx$$

input
```
Int[(c + d*x)^m/(a + b*Sinh[e + f*x])^2,x]
```

output
```
$Aborted
```

3.186.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.186.4 Maple [N/A] (verified)

Not integrable

Time = 0.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^m}{(a+b\sinh(fx+e))^2}dx$$

input
```
int((d*x+c)^m/(a+b*sinh(f*x+e))^2,x)
```

output
```
int((d*x+c)^m/(a+b*sinh(f*x+e))^2,x)
```

3.186.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/(b^2*sinh(f*x + e)^2 + 2*a*b*sinh(f*x + e) + a^2), x)
```

3.186.6 Sympy [N/A]

Not integrable

Time = 27.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx$$

input
```
integrate((d*x+c)**m/(a+b*sinh(f*x+e))**2,x)
```

output
```
Integral((c + d*x)**m/(a + b*sinh(e + f*x))**2, x)
```

3.186.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(b*sinh(f*x + e) + a)^2, x)
```

3.186.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(b\sinh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(b*sinh(f*x + e) + a)^2, x)
```

3.186.9 Mupad [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^m}{(a+b\sinh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)^m/(a + b*sinh(e + f*x))^2,x)
```

output
```
int((c + d*x)^m/(a + b*sinh(e + f*x))^2, x)
```

3.187 $\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.187.1 Optimal result

Integrand size = 29, antiderivative size = 163

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i(e+fx)^3}{ad} - \frac{i(e+fx)^4}{4af} - \frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2} - \frac{12if^2(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3} + \frac{12if^3\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4} + \frac{i(e+fx)^3\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
I*(f*x+e)^3/a/d-1/4*I*(f*x+e)^4/a/f-6*I*f*(f*x+e)^2*ln(1+I*exp(d*x+c))/a/d
^2-12*I*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^3+12*I*f^3*polylog(3,-I*e
xp(d*x+c))/a/d^4+I*(f*x+e)^3*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

3.187.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.31

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{-\frac{8(e+fx)^3}{d(-i+e^c)} - ix(4e^3+6e^2fx+4ef^2x^2+f^3x^3) - \frac{24if(e+fx)^2\log(1-ie^{-c-dx})}{d^2} + \frac{48if^2(d(e+fx)\operatorname{PolyLog}(2,ie^{-c-dx})+f\operatorname{Po}}{d^4}}{4a}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-8*(e + f*x)^3)/(d*(-I + E^c)) - I*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 +
f^3*x^3) - ((24*I)*f*(e + f*x)^2*Log[1 - I*E^(-c - d*x)])/d^2 + ((48*I)*f^
2*(d*(e + f*x)*PolyLog[2, I*E^(-c - d*x)] + f*PolyLog[3, I*E^(-c - d*x)]))
/d^4 + ((8*I)*(e + f*x)^3*Sinh[(d*x)/2])/(d*(Cosh[c/2] + I*Sinh[c/2])*(Cos
h[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/(4*a)
```

3.187.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.586$, Rules used = {6091, 17, 3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 4199, 26, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \sinh(c+dx)}{a+ia\sinh(c+dx)} \, dx$$

$$\Big\downarrow \text{6091}$$

$$i\int \frac{(e+fx)^3}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^3dx}{a}$$

$$\Big\downarrow \text{17}$$

$$i\int \frac{(e+fx)^3}{i\sinh(c+dx)a+a}dx-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{3042}$$

$$i\int \frac{(e+fx)^3}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{3799}$$

$$\frac{i\int -(e+fx)^3\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i\int -(e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int(e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{i\int(e+fx)^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \textbf{4672}$$

$$\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6if\int -i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \textbf{26}$$

$$\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6f\int(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6f\int -i(e+fx)^2\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \textbf{26}$$

$$\frac{i\left(\frac{6if\int(e+fx)^2\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \textbf{4199}$$

$$\frac{i\left(\frac{6if\left(2i\int\frac{ie^{c+dx}(e+fx)^2}{1+ie^{c+dx}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \textbf{26}$$

$$\frac{i\left(\frac{6if\left(-2\int\frac{e^{c+dx}(e+fx)^2}{1+ie^{c+dx}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \textbf{2620}$$

3.187. $\int\frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}dx$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\int(e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{3011}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\int \operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{2720}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{7143}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}$$

3.187. $\displaystyle\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^3*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-1/4*I)*(e + f*x)^4)/(a*f) + ((I/2)*(((6*I)*f*(((-1/3*I)*(e + f*x)^3)/f
- 2*(((-I)*(e + f*x)^2*Log[1 + I*E^(c + d*x)])/d + ((2*I)*f*(-(((e + f*x)*
PolyLog[2, (-I)*E^(c + d*x)])/d) + (f*PolyLog[3, (-I)*E^(c + d*x)])/d^2))/
d)))/d + (2*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/a
```

3.187.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.187. $\int \frac{(e+fx)^3 \sinh(c+dx)}{a+ia \sinh(c+dx)} \, dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.187.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(142) = 284$.

Time = 1.98 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.55

method	result
risch	$\frac{6e^2 f \arctan(e^{dx+c})}{a\,d^2} + \frac{6c^2 f^3 \arctan(e^{dx+c})}{a\,d^4} - \frac{12ic\,f^2 e \ln(e^{dx+c})}{a\,d^3} + \frac{6ic\,f^2 e \ln(1+e^{2dx+2c})}{a\,d^3} + \frac{12if^2 ecx}{a\,d^2} - \frac{12c\,f^2 e \arctan(e^{d}}{a\,d^3}$

input
```
int((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(d*x+c)-I)+6*I/a/d^4*f^3*ln
(1+I*exp(d*x+c))*c^2-6*I/a/d^3*f^3*x*c^2-12*I/a/d^3*f^3*polylog(2,-I*exp(d
*x+c))*x+6*I/a/d*f^2*e*x^2-6*I/a/d^2*f^3*ln(1+I*exp(d*x+c))*x^2+6/a/d^2*e^
2*f*arctan(exp(d*x+c))+6/a/d^4*c^2*f^3*arctan(exp(d*x+c))+2*I/a/d*f^3*x^3-
4*I/a/d^4*f^3*c^3+12*I*f^3*polylog(3,-I*exp(d*x+c))/a/d^4-12*I/a/d^2*f^2*e
*ln(1+I*exp(d*x+c))*x-12*I/a/d^3*f^2*e*ln(1+I*exp(d*x+c))*c-12*I/a/d^3*c*f
^2*e*ln(exp(d*x+c))+6*I/a/d^3*c*f^2*e*ln(1+exp(2*d*x+2*c))+12*I/a/d^2*f^2*
e*c*x-3*I/a/d^4*c^2*f^3*ln(1+exp(2*d*x+2*c))+6*I/a/d^4*c^2*f^3*ln(exp(d*x+
c))-3*I/a/d^2*e^2*f*ln(1+exp(2*d*x+2*c))+6*I/a/d^2*e^2*f*ln(exp(d*x+c))-12
*I/a/d^3*f^2*e*polylog(2,-I*exp(d*x+c))+6*I/a/d^3*f^2*e*c^2-1/4*I/a*f^3*x^
4-12/a/d^3*c*f^2*e*arctan(exp(d*x+c))-I/a*e^3*x-1/4*I/a/f*e^4-I/a*f^2*e*x^
3-3/2*I/a*f*e^2*x^2
```

3.187.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(131) = 262$.

Time = 0.26 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.80

$$\int \frac{(e+fx)^3 \sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx =$$
$$-\frac{d^4 f^3 x^4 + 4\,d^4 e f^2 x^3 + 6\,d^4 e^2 f x^2 + 4\,d^4 e^3 x + 8\,d^3 e^3 - 24\,cd^2 e^2 f + 24\,c^2 d e f^2 - 8\,c^3 f^3 + 48\,\left(df^3 x + def^2\right.}{}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2 + 4*d^4*e^3*x + 8*d^
3*e^3 - 24*c*d^2*e^2*f + 24*c^2*d*e*f^2 - 8*c^3*f^3 + 48*(d*f^3*x + d*e*f^
2 + (I*d*f^3*x + I*d*e*f^2)*e^(d*x + c))*dilog(-I*e^(d*x + c)) - (-I*d^4*f
^3*x^4 + 24*I*c*d^2*e^2*f - 24*I*c^2*d*e*f^2 + 8*I*c^3*f^3 - 4*(I*d^4*e*f^
2 - 2*I*d^3*f^3)*x^3 - 6*(I*d^4*e^2*f - 4*I*d^3*e*f^2)*x^2 - 4*(I*d^4*e^3
- 6*I*d^3*e^2*f)*x)*e^(d*x + c) + 24*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3 +
(I*d^2*e^2*f - 2*I*c*d*e*f^2 + I*c^2*f^3)*e^(d*x + c))*log(e^(d*x + c) - I
) + 24*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3 + (I*d^2*f^3*x
^2 + 2*I*d^2*e*f^2*x + 2*I*c*d*e*f^2 - I*c^2*f^3)*e^(d*x + c))*log(I*e^(d*
x + c) + 1) + 48*(-I*f^3*e^(d*x + c) - f^3)*polylog(3, -I*e^(d*x + c)))/(a
*d^4*e^(d*x + c) - I*a*d^4)
```

3.187.6 Sympy [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-2e^3-6e^2fx-6ef^2x^2-2f^3x^3}{ade^ce^{dx}-iad} - \frac{i\left(\int\left(-\frac{ide^3}{e^ce^{dx}-i}\right)dx+\int\frac{6ie^2f}{e^ce^{dx}-i}\,dx+\int\frac{6if^3x^2}{e^ce^{dx}-i}\,dx+\int\left(-\frac{idf^3x^3}{e^ce^{dx}-i}\right)dx+\int\frac{12ief^2x}{e^ce^{dx}-i}\,dx+\int\frac{de^3e^ce^{dx}}{e^ce^{dx}-i}\,dx+\int\Big(\cdots\right)}{ad}$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(-2*e**3 - 6*e**2*f*x - 6*e*f**2*x**2 - 2*f**3*x**3)/(a*d*exp(c)*exp(d*x)
- I*a*d) - I*(Integral(-I*d*e**3/(exp(c)*exp(d*x) - I), x) + Integral(6*I*
e**2*f/(exp(c)*exp(d*x) - I), x) + Integral(6*I*f**3*x**2/(exp(c)*exp(d*x)
 - I), x) + Integral(-I*d*f**3*x**3/(exp(c)*exp(d*x) - I), x) + Integral(1
2*I*e*f**2*x/(exp(c)*exp(d*x) - I), x) + Integral(d*e**3*exp(c)*exp(d*x)/(
exp(c)*exp(d*x) - I), x) + Integral(-3*I*d*e*f**2*x**2/(exp(c)*exp(d*x) -
I), x) + Integral(-3*I*d*e**2*f*x/(exp(c)*exp(d*x) - I), x) + Integral(d*f
**3*x**3*exp(c)*exp(d*x)/(exp(c)*exp(d*x) - I), x) + Integral(3*d*e*f**2*x
**2*exp(c)*exp(d*x)/(exp(c)*exp(d*x) - I), x) + Integral(3*d*e**2*f*x*exp(
c)*exp(d*x)/(exp(c)*exp(d*x) - I), x))/(a*d)
```

3.187.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(131) = 262.

Time = 0.29 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.95

$$
\begin{aligned}
&\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx \\
&= \frac{3}{2}e^2f\left(\frac{-i\,dx^2+(dx^2e^c-4\,xe^c)e^{(dx)}}{i\,ade^{(dx+c)}+ad}-\frac{4i\,\log\left(\left(e^{(dx+c)}-i\right)e^{(-c)}\right)}{ad^2}\right) \\
&\quad - e^3\left(\frac{i\,(dx+c)}{ad}+\frac{2}{(ae^{(-dx-c)}+i\,a)d}\right) \\
&\quad - \frac{df^3x^4+24\,ef^2x^2+4\,(def^2+2\,f^3)x^3+(i\,df^3x^4e^c+4i\,def^2x^3e^c)e^{(dx)}}{4\,(ade^{(dx+c)}-i\,ad)} \\
&\quad - \frac{12i\,\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)\right)ef^2}{ad^3} \\
&\quad - \frac{6i\,\left(d^2x^2\log\left(i\,e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-2\,\mathrm{Li}_3(-i\,e^{(dx+c)})\right)f^3}{ad^4} \\
&\quad - \frac{2\,(-i\,d^3f^3x^3-3i\,d^3ef^2x^2)}{ad^4}
\end{aligned}
$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
3/2*e^2*f*((-I*d*x^2 + (d*x^2*e^c - 4*x*e^c)*e^(d*x))/(I*a*d*e^(d*x + c) +
 a*d) - 4*I*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) - e^3*(I*(d*x + c)/(a*d
) + 2/((a*e^(-d*x - c) + I*a)*d)) - 1/4*(d*f^3*x^4 + 24*e*f^2*x^2 + 4*(d*e
*f^2 + 2*f^3)*x^3 + (I*d*f^3*x^4*e^c + 4*I*d*e*f^2*x^3*e^c)*e^(d*x))/(a*d*
e^(d*x + c) - I*a*d) - 12*I*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x
+ c)))*e*f^2/(a*d^3) - 6*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-
I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4) - 2*(-I*d^3*f^3
*x^3 - 3*I*d^3*e*f^2*x^2)/(a*d^4)
```

3.187.8 Giac [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\sinh(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sinh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,\left(e+f\,x\right)^3}{a+a\sinh\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

input
```
int((sinh(c + d*x)*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((sinh(c + d*x)*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i), x)
```

3.188 $\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.188.1 Optimal result

Integrand size = 29, antiderivative size = 130

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i(e+fx)^2}{ad}-\frac{i(e+fx)^3}{3af}-\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2} -\frac{4if^2\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
I*(f*x+e)^2/a/d-1/3*I*(f*x+e)^3/a/f-4*I*f*(f*x+e)*ln(1+I*exp(d*x+c))/a/d^2
-4*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3+I*(f*x+e)^2*tanh(1/2*c+1/4*I*Pi+1/
2*d*x)/a/d
```

3.188.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.44

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{-ix(3e^2+3efx+f^2x^2)+\frac{-6d(e+fx)(d(e+fx)+2(1+ie^c)f\log(1-ie^{-c-dx}))+12(1+ie^c)f^2\,\mathrm{PolyLog}(2,ie^{-c-dx})}{d^3(-i+e^c)}+\frac{\cdots}{d(\cosh(\frac{c}{2})+i\,\mathrm{s}\cdots}}{3a}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-I)*x*(3*e^2 + 3*e*f*x + f^2*x^2) + (-6*d*(e + f*x)*(d*(e + f*x) + 2*(1
+ I*E^c)*f*Log[1 - I*E^(-c - d*x)]) + 12*(1 + I*E^c)*f^2*PolyLog[2, I*E^(-
c - d*x)])/(d^3*(-I + E^c)) + ((6*I)*(e + f*x)^2*Sinh[(d*x)/2])/(d*(Cosh[c
/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/(3*a)
```

3.188.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.552$, Rules used = {6091, 17, 3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 4199, 26, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 \sinh(c+dx)}{a+ia\sinh(c+dx)} \, dx$$

$$\Big\downarrow \text{6091}$$

$$i\int \frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^2dx}{a}$$

$$\Big\downarrow \text{17}$$

$$i\int \frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{3042}$$

$$i\int \frac{(e+fx)^2}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{3799}$$

$$\frac{i\int -(e+fx)^2\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i\int -(e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int (e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\int (e+fx)^2 \csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2 dx}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{4672}$$

$$\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4if\int -i(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\int (e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\int -i(e+fx)\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(\frac{4if\int (e+fx)\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{4199}$$

$$\frac{i\left(\frac{4if\left(2i\int \frac{ie^{c+dx}(e+fx)}{1+ie^{c+dx}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(\frac{4if\left(-2\int \frac{e^{c+dx}(e+fx)}{1+ie^{c+dx}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{2620}$$

$$\frac{i\left(\frac{4if\left(-2\left(\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{2715}$$

3.188. $\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{i\left(\frac{4if\left(-2\left(\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \; 2838$$

$$\frac{i\left(\frac{4if\left(-2\left(-\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}$$

input
```
Int[((e + f*x)^2*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-1/3*I)*(e + f*x)^3)/(a*f) + ((I/2)*(((4*I)*f*(((-1/2*I)*(e + f*x)^2)/f
- 2*(((-I)*(e + f*x)*Log[1 + I*E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c
 + d*x)])/d^2)))/d + (2*(e + f*x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/a
```

3.188.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

3.188. $\int \frac{(e+fx)^2 \sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.188.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(112) = 224.

Time = 1.66 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.47

method	result
risch	$\frac{2if^2x^2}{ad}+\frac{2if^2c^2}{a\,d^3}-\frac{4if^2\,\mathrm{polylog}\left(2,-i\mathrm{e}^{dx+c}\right)}{a\,d^3}+\frac{2ic\,f^2\ln\left(1+\mathrm{e}^{2dx+2c}\right)}{a\,d^3}-\frac{2\left(x^2f^2+2efx+e^2\right)}{da\left(\mathrm{e}^{dx+c}-i\right)}+\frac{4ief\ln\left(\mathrm{e}^{dx+c}\right)}{a\,d^2}+\frac{4ef\arctan}{a\,d}$

input
```
int((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*I/a/d*f^2*x^2+2*I/a/d^3*f^2*c^2-4*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3+2
*I/a/d^3*c*f^2*ln(1+exp(2*d*x+2*c))-2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(d*x+c
)-I)+4*I/a/d^2*e*f*ln(exp(d*x+c))+4/a/d^2*e*f*arctan(exp(d*x+c))-4*I/a/d^3
*f^2*ln(1+I*exp(d*x+c))*c-2*I/a/d^2*e*f*ln(1+exp(2*d*x+2*c))+4*I/a/d^2*f^2
*c*x-1/3*I/a*f^2*x^3-I/a*f*e*x^2-4*I/a/d^3*c*f^2*ln(exp(d*x+c))-4*I/a/d^2*
f^2*ln(1+I*exp(d*x+c))*x-1/3*I/a/f*e^3-4/a/d^3*c*f^2*arctan(exp(d*x+c))-I/
a*e^2*x
```

3.188.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(103) = 206.

Time = 0.26 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.01

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx =$$
$$-\frac{d^3f^2x^3+3\,d^3efx^2+3\,d^3e^2x+6\,d^2e^2-12\,cdef+6\,c^2f^2+12\left(i\,f^2e^{(dx+c)}+f^2\right)\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-\left(-i\,d\right.}{}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x + 6*d^2*e^2 - 12*c*d*e*f +
 6*c^2*f^2 + 12*(I*f^2*e^(d*x + c) + f^2)*dilog(-I*e^(d*x + c)) - (-I*d^3*
f^2*x^3 + 12*I*c*d*e*f - 6*I*c^2*f^2 - 3*(I*d^3*e*f - 2*I*d^2*f^2)*x^2 - 3
*(I*d^3*e^2 - 4*I*d^2*e*f)*x)*e^(d*x + c) + 12*(d*e*f - c*f^2 + (I*d*e*f -
 I*c*f^2)*e^(d*x + c))*log(e^(d*x + c) - I) + 12*(d*f^2*x + c*f^2 + (I*d*f
^2*x + I*c*f^2)*e^(d*x + c))*log(I*e^(d*x + c) + 1))/(a*d^3*e^(d*x + c) -
I*a*d^3)
```

3.188.6 Sympy [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-2e^2-4efx-2f^2x^2}{ade^ce^{dx}-iad} - \frac{i\left(\int\left(-\frac{ide^2}{e^ce^{dx}-i}\right)dx+\int\frac{4ief}{e^ce^{dx}-i}\,dx+\int\frac{4if^2x}{e^ce^{dx}-i}\,dx+\int\left(-\frac{idf^2x^2}{e^ce^{dx}-i}\right)dx+\int\frac{de^2e^ce^{dx}}{e^ce^{dx}-i}\,dx+\int\left(-\frac{2idefx}{e^ce^{dx}-i}\right)dx\ \cdots\right)}{ad}$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(-2*e**2 - 4*e*f*x - 2*f**2*x**2)/(a*d*exp(c)*exp(d*x) - I*a*d) - I*(Integ
ral(-I*d*e**2/(exp(c)*exp(d*x) - I), x) + Integral(4*I*e*f/(exp(c)*exp(d*x
) - I), x) + Integral(4*I*f**2*x/(exp(c)*exp(d*x) - I), x) + Integral(-I*d
*f**2*x**2/(exp(c)*exp(d*x) - I), x) + Integral(d*e**2*exp(c)*exp(d*x)/(ex
p(c)*exp(d*x) - I), x) + Integral(-2*I*d*e*f*x/(exp(c)*exp(d*x) - I), x) +
 Integral(d*f**2*x**2*exp(c)*exp(d*x)/(exp(c)*exp(d*x) - I), x) + Integral
(2*d*e*f*x*exp(c)*exp(d*x)/(exp(c)*exp(d*x) - I), x))/(a*d)
```

3.188.7 Maxima [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\sinh(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*f^2*((-I*d*x^3*e^(d*x + c) - d*x^3 - 6*x^2)/(a*d*e^(d*x + c) - I*a*d)
+ 12*integrate(x/(a*d*e^(d*x + c) - I*a*d), x)) + e*f*((-I*d*x^2 + (d*x^2*
e^c - 4*x*e^c)*e^(d*x))/(I*a*d*e^(d*x + c) + a*d) - 4*I*log((e^(d*x + c) -
 I)*e^(-c))/(a*d^2)) - e^2*(I*(d*x + c)/(a*d) + 2/((a*e^(-d*x - c) + I*a)*
d))
```

3.188.8 Giac [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\sinh(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sinh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{\sinh(c+d\,x)\,\left(e+f\,x\right)^2}{a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((sinh(c + d*x)*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((sinh(c + d*x)*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i), x)
```

3.189 $\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.189.1 Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{iex}{a}-\frac{ifx^2}{2a}-\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}+\frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
-I*e*x/a-1/2*I*f*x^2/a-2*I*f*ln(cosh(1/2*c+1/4*I*Pi+1/2*d*x))/a/d^2+I*(f*x
+e)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

3.189.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 239 vs. 2(90) = 180.

Time = 0.75 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.66

$$\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-2dfx\cosh\left(c+\frac{dx}{2}\right)-i\cosh\left(\frac{dx}{2}\right)\left(d^2x(2e+fx)+4if\arctan\left(\operatorname{sech}\left(c+\frac{dx}{2}\right)\sinh\left(\frac{dx}{2}\right)\right)+2f\log(\cosh(c+\right.}{}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-2*d*f*x*Cosh[c + (d*x)/2] - I*Cosh[(d*x)/2]*(d^2*x*(2*e + f*x) + (4*I)*f
*ArcTan[Sech[c + (d*x)/2]*Sinh[(d*x)/2]] + 2*f*Log[Cosh[c + d*x]]) + (4*I)
*d*e*Sinh[(d*x)/2] + (2*I)*d*f*x*Sinh[(d*x)/2] + 2*d^2*e*x*Sinh[c + (d*x)/
2] + d^2*f*x^2*Sinh[c + (d*x)/2] + (4*I)*f*ArcTan[Sech[c + (d*x)/2]*Sinh[(
d*x)/2]]*Sinh[c + (d*x)/2] + 2*f*Log[Cosh[c + d*x]]*Sinh[c + (d*x)/2])/(2*
a*d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))
```

3.189.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6091, 17, 3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6091}$$

$$i\int \frac{e+fx}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)dx}{a}$$

$$\Big\downarrow \text{17}$$

$$i\int \frac{e+fx}{i\sinh(c+dx)a+a}dx-\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \text{3042}$$

$$i\int \frac{e+fx}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \text{3799}$$

$$\frac{i\int -\left((e+fx)\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)\right)dx}{2a}-\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i\int -\left((e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)dx}{2a}-\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int (e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\int (e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \; 4672$$

$$\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2if\int -i\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \; 26$$

$$\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int \tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int -i\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \; 26$$

$$\frac{i\left(\frac{2if\int \tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \; 3956$$

$$\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{d^2}\right)}{2a}-\frac{i(e+fx)^2}{2af}$$

input
```
Int[((e + f*x)*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-1/2*I)*(e + f*x)^2)/(a*f) + ((I/2)*((-4*f*Log[Cosh[c/2 + (I/4)*Pi + (d*
x)/2]])/d^2 + (2*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/a
```

3.189. $\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.189.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

3.189. $\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.189.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{if\,x^{2}}{2a}-\frac{iex}{a}+\frac{2ifx}{ad}+\frac{2ifc}{a\,d^{2}}-\frac{2(fx+e)}{da\left(\mathrm{e}^{dx+c}-i\right)}-\frac{2if\ln\left(\mathrm{e}^{dx+c}-i\right)}{a\,d^{2}}$
parallelrisch	$\frac{2f\left(-1-i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(1-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2f\left(i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\ln\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+d\left(x\left(\left(\frac{1}{2}ixf+ie\right)d+(-1-i)f\right.\right.}{d^{2}a\left(i-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

input
```
int((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*f*x^2/a-I*e*x/a+2*I*f/a/d*x+2*I*f/a/d^2*c-2*(f*x+e)/d/a/(exp(d*x+c)
-I)-2*I*f/a/d^2*ln(exp(d*x+c)-I)
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07

$$\int\frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx=$$
$$-\frac{d^{2}fx^{2}+2\,d^{2}ex+4\,de-(-i\,d^{2}fx^{2}-2\,(i\,d^{2}e-2i\,df)x)e^{(dx+c)}+4\left(i\,fe^{(dx+c)}+f\right)\log\left(e^{(dx+c)}-i\right)}{2\left(ad^{2}e^{(dx+c)}-i\,ad^{2}\right)}$$

input
```
integrate((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(d^2*f*x^2 + 2*d^2*e*x + 4*d*e - (-I*d^2*f*x^2 - 2*(I*d^2*e - 2*I*d*f
)*x)*e^(d*x + c) + 4*(I*f*e^(d*x + c) + f)*log(e^(d*x + c) - I))/(a*d^2*e^
(d*x + c) - I*a*d^2)
```

3.189.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81

$$\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-2e-2fx}{ade^{c}e^{dx}-iad} - \frac{ifx^{2}}{2a} + \frac{x(-ide+2if)}{ad} - \frac{2if\log\left(e^{dx}-ie^{-c}\right)}{ad^{2}}$$

input
```
integrate((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
(-2*e - 2*f*x)/(a*d*exp(c)*exp(d*x) - I*a*d) - I*f*x**2/(2*a) + x*(-I*d*e
+ 2*I*f)/(a*d) - 2*I*f*log(exp(d*x) - I*exp(-c))/(a*d**2)
```

3.189.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{1}{2}\,f\left(\frac{-i\,dx^{2}+(dx^{2}e^{c}-4\,xe^{c})e^{(dx)}}{i\,ade^{(dx+c)}+ad} - \frac{4i\,\log\left(\left(e^{(dx+c)}-i\right)e^{(-c)}\right)}{ad^{2}}\right)\\ &\quad - e\left(\frac{i\,(dx+c)}{ad} + \frac{2}{\left(ae^{(-dx-c)}+i\,a\right)d}\right)\end{aligned}$$

input
```
integrate((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*f*((-I*d*x^2 + (d*x^2*e^c - 4*x*e^c)*e^(d*x))/(I*a*d*e^(d*x + c) + a*d
) - 4*I*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) - e*(I*(d*x + c)/(a*d) + 2/
((a*e^(-d*x - c) + I*a)*d))
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.23

$$\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx =$$
$$-\frac{i\,d^{2}fx^{2}e^{(dx+c)}+d^{2}fx^{2}+2i\,d^{2}exe^{(dx+c)}+2\,d^{2}ex-4i\,dfxe^{(dx+c)}+4i\,fe^{(dx+c)}\log\left(e^{(dx+c)}-i\right)+4\,de+}{2\left(ad^{2}e^{(dx+c)}-i\,ad^{2}\right)}$$

input
```
integrate((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(I*d^2*f*x^2*e^(d*x + c) + d^2*f*x^2 + 2*I*d^2*e*x*e^(d*x + c) + 2*d^
2*e*x - 4*I*d*f*x*e^(d*x + c) + 4*I*f*e^(d*x + c)*log(e^(d*x + c) - I) + 4
*d*e + 4*f*log(e^(d*x + c) - I))/(a*d^2*e^(d*x + c) - I*a*d^2)
```

3.189.9 Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{f\,x^2\,1\mathrm{i}}{2\,a} - \frac{2\,(e+f\,x)}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)} + \frac{x\,(2\,f-d\,e)\,1\mathrm{i}}{a\,d} - \frac{f\,\ln\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-\mathrm{i}\right)\,2\mathrm{i}}{a\,d^2}$$

input
```
int((sinh(c + d*x)*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

output
```
(x*(2*f - d*e)*1i)/(a*d) - (2*(e + f*x))/(a*d*(exp(c + d*x) - 1i)) - (f*x^
2*1i)/(2*a) - (f*log(exp(d*x)*exp(c) - 1i)*2i)/(a*d^2)
```

3.190 $\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.190.1 Optimal result

Integrand size = 22, antiderivative size = 35

$$\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx=-\frac{ix}{a}-\frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}$$

output
```
-I*x/a-cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))
```

3.190.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{i\cosh(c+dx)\left(1-\frac{\operatorname{arcsinh}(\sinh(c+dx))(-i+\sinh(c+dx))}{\sqrt{\cosh^2(c+dx)}}\right)}{ad(-i+\sinh(c+dx))}$$

input
```
Integrate[Sinh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(I*Cosh[c + d*x]*(1 - (ArcSinh[Sinh[c + d*x]]*(-I + Sinh[c + d*x]))/Sqrt[C
osh[c + d*x]^2]))/(a*d*(-I + Sinh[c + d*x]))
```

3.190.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 26, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i\sin(ic+idx)}{a+a\sin(ic+idx)}dx$$
$$\Big\downarrow \text{26}$$
$$-i\int \frac{\sin(ic+idx)}{\sin(ic+idx)a+a}dx$$
$$\Big\downarrow \text{3214}$$
$$-i\left(\frac{x}{a}-\int \frac{1}{i\sinh(c+dx)a+a}dx\right)$$
$$\Big\downarrow \text{3042}$$
$$-i\left(\frac{x}{a}-\int \frac{1}{\sin(ic+idx)a+a}dx\right)$$
$$\Big\downarrow \text{3127}$$
$$-i\left(\frac{x}{a}-\frac{i\cosh(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

input
```
Int[Sinh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-I)*(x/a - (I*Cosh[c + d*x])/(d*(a + I*a*Sinh[c + d*x])))
```

3.190.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.190.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{ix}{a}-\frac{2}{da\left(\mathrm{e}^{dx+c}-i\right)}$	28
parallelrisch	$\frac{ix\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)d+dx-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(i-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	53
derivativedivides	$\frac{i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{4i}{-2i+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	57
default	$\frac{i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{4i}{-2i+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	57

input
```
int(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-I*x/a-2/d/a/(exp(d*x+c)-I)
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-i\,dxe^{(dx+c)} - dx - 2}{ade^{(dx+c)} - i\,ad}$$

input
```
integrate(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(-I*d*x*e^(d*x + c) - d*x - 2)/(a*d*e^(d*x + c) - I*a*d)
```

3.190.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{2}{ade^{c}e^{dx} - iad} - \frac{ix}{a}$$

input
```
integrate(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-2/(a*d*exp(c)*exp(d*x) - I*a*d) - I*x/a
```

3.190.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\,(dx+c)}{ad} - \frac{2}{(ae^{(-dx-c)} + i\,a)d}$$

input
```
integrate(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-I*(d*x + c)/(a*d) - 2/((a*e^(-d*x - c) + I*a)*d)
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\frac{i\,(dx+c)}{a}+\frac{2i}{a\left(i\,e^{(dx+c)}+1\right)}}{d}$$

input
```
integrate(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(I*(d*x + c)/a + 2*I/(a*(I*e^(d*x + c) + 1)))/d
```

3.190.9 Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{x\,1\mathrm{i}}{a}-\frac{2}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}$$

input
```
int(sinh(c + d*x)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
- (x*1i)/a - 2/(a*d*(exp(c + d*x) - 1i))
```

3.191 $\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

3.191.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.191.2 Mathematica [N/A]

Not integrable

Time = 37.76 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

3.191.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.191.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.191.4 Maple [N/A] (verified)

Not integrable

Time = 0.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sinh\left(dx+c\right)}{\left(fx+e\right)\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.191.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 5.31

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral(-(d*f*x +
 d*e - (-I*d*f*x - I*d*e)*e^(d*x + c) + 2*f)/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f
*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c)), x) -
2)/(-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))
```

3.191.6 Sympy [N/A]

Not integrable

Time = 12.08 (sec) , antiderivative size = 425, normalized size of antiderivative = 14.66

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{2}{-iade-iadfx+(adee^c+adfxe^c)\,e^{dx}} - \frac{i\left(\int\left(-\frac{2if}{e^2e^ce^{dx}-ie^2+2efxe^ce^{dx}-2iefx+f^2x^2e^ce^{dx}-if^2x^2}\right)dx+\int\left(-\frac{ide}{e^2e^ce^{dx}-ie^2+2efxe^ce^{dx}-2iefx+f^2x^2e^ce^{dx}-if^2x^2}\right)dx}{}$$

input
```
integrate(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-2/(-I*a*d*e - I*a*d*f*x + (a*d*e*exp(c) + a*d*f*x*exp(c))*exp(d*x)) - I*(
Integral(-2*I*f/(e**2*exp(c)*exp(d*x) - I*e**2 + 2*e*f*x*exp(c)*exp(d*x) -
 2*I*e*f*x + f**2*x**2*exp(c)*exp(d*x) - I*f**2*x**2), x) + Integral(-I*d*
e/(e**2*exp(c)*exp(d*x) - I*e**2 + 2*e*f*x*exp(c)*exp(d*x) - 2*I*e*f*x + f
**2*x**2*exp(c)*exp(d*x) - I*f**2*x**2), x) + Integral(-I*d*f*x/(e**2*exp(
c)*exp(d*x) - I*e**2 + 2*e*f*x*exp(c)*exp(d*x) - 2*I*e*f*x + f**2*x**2*exp
(c)*exp(d*x) - I*f**2*x**2), x) + Integral(d*e*exp(c)*exp(d*x)/(e**2*exp(c
)*exp(d*x) - I*e**2 + 2*e*f*x*exp(c)*exp(d*x) - 2*I*e*f*x + f**2*x**2*exp(
c)*exp(d*x) - I*f**2*x**2), x) + Integral(d*f*x*exp(c)*exp(d*x)/(e**2*exp(
c)*exp(d*x) - I*e**2 + 2*e*f*x*exp(c)*exp(d*x) - 2*I*e*f*x + f**2*x**2*exp
(c)*exp(d*x) - I*f**2*x**2), x))/(a*d)
```

3.191.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.00

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*f*integrate(1/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^
2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 2/(-I*a*d*f*x - I*a*
d*e + (a*d*f*x*e^c + a*d*e*e^c)*e^(d*x)) - I*log(f*x + e)/(a*f)
```

3.191.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

3.191.9 Mupad [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)}{(e+f\,x)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(sinh(c + d*x)/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(sinh(c + d*x)/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.192 $\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

3.192.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.192.2 Mathematica [N/A]

Not integrable

Time = 29.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

3.192.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.192.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.192.4 Maple [N/A] (verified)

Not integrable

Time = 0.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sinh(dx+c)}{(fx+e)^2(a+ia\sinh(dx+c))}dx$$

input
```
int(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.192.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 232, normalized size of antiderivative = 8.00

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(fx+e\right)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x
+ a*d*e^2)*e^(d*x + c))*integral(-(d*f*x + d*e - (-I*d*f*x - I*d*e)*e^(d*x
 + c) + 4*f)/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x - I*a*d
*e^3 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^(d*x +
c)), x) - 2)/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 +
2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

3.192.6 Sympy [N/A]

Not integrable

Time = 22.54 (sec) , antiderivative size = 631, normalized size of antiderivative = 21.76

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$= -\frac{2}{-iade^2 - 2iadefx - iadf^2x^2 + \left(ade^2e^c + 2adefxe^c + adf^2x^2e^c\right)e^{dx}}$$
$$- \frac{i\left(\int \left(-\frac{4if}{e^3e^ce^{dx} - ie^3 + 3e^2fxe^ce^{dx} - 3ie^2fx + 3ef^2x^2e^ce^{dx} - 3ief^2x^2 + f^3x^3e^ce^{dx} - if^3x^3}\right)dx + \int \left(-\frac{}{e^3e^ce^{dx} - ie^3 + 3e^2fxe^ce^{dx} - 3ie^2 \cdots}\right.\right.}{}$$

input
```
integrate(sinh(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-2/(-I*a*d*e**2 - 2*I*a*d*e*f*x - I*a*d*f**2*x**2 + (a*d*e**2*exp(c) + 2*a
*d*e*f*x*exp(c) + a*d*f**2*x**2*exp(c))*exp(d*x)) - I*(Integral(-4*I*f/(e*
*3*exp(c)*exp(d*x) - I*e**3 + 3*e**2*f*x*exp(c)*exp(d*x) - 3*I*e**2*f*x +
3*e*f**2*x**2*exp(c)*exp(d*x) - 3*I*e*f**2*x**2 + f**3*x**3*exp(c)*exp(d*x
) - I*f**3*x**3), x) + Integral(-I*d*e/(e**3*exp(c)*exp(d*x) - I*e**3 + 3*
e**2*f*x*exp(c)*exp(d*x) - 3*I*e**2*f*x + 3*e*f**2*x**2*exp(c)*exp(d*x) -
3*I*e*f**2*x**2 + f**3*x**3*exp(c)*exp(d*x) - I*f**3*x**3), x) + Integral(
-I*d*f*x/(e**3*exp(c)*exp(d*x) - I*e**3 + 3*e**2*f*x*exp(c)*exp(d*x) - 3*I
*e**2*f*x + 3*e*f**2*x**2*exp(c)*exp(d*x) - 3*I*e*f**2*x**2 + f**3*x**3*ex
p(c)*exp(d*x) - I*f**3*x**3), x) + Integral(d*e*exp(c)*exp(d*x)/(e**3*exp(
c)*exp(d*x) - I*e**3 + 3*e**2*f*x*exp(c)*exp(d*x) - 3*I*e**2*f*x + 3*e*f**
2*x**2*exp(c)*exp(d*x) - 3*I*e*f**2*x**2 + f**3*x**3*exp(c)*exp(d*x) - I*f
**3*x**3), x) + Integral(d*f*x*exp(c)*exp(d*x)/(e**3*exp(c)*exp(d*x) - I*e
**3 + 3*e**2*f*x*exp(c)*exp(d*x) - 3*I*e**2*f*x + 3*e*f**2*x**2*exp(c)*exp
(d*x) - 3*I*e*f**2*x**2 + f**3*x**3*exp(c)*exp(d*x) - I*f**3*x**3), x))/(a
*d)
```

3.192.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 6.69

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\sinh\left(dx+c\right)}{(fx+e)^2(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-4*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x - I
*a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*
d*e^3*e^c)*e^(d*x)), x) + (d*f*x + d*e - (-I*d*f*x*e^c - I*d*e*e^c)*e^(d*x
) - 2*f)/(-I*a*d*f^3*x^2 - 2*I*a*d*e*f^2*x - I*a*d*e^2*f + (a*d*f^3*x^2*e^
c + 2*a*d*e*f^2*x*e^c + a*d*e^2*f*e^c)*e^(d*x))
```

3.192.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

3.192.9 Mupad [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)}{(e+f\,x)^2\,(a+a\sinh(c+d\,x)\,1\mathrm{i})}\,dx$$

input
```
int(sinh(c + d*x)/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(sinh(c + d*x)/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.193 $\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.193.1 Optimal result

Integrand size = 31, antiderivative size = 241

$$\begin{aligned}\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx =& -\frac{(e+fx)^3}{ad}+\frac{(e+fx)^4}{4af}-\frac{6if^2(e+fx)\cosh(c+dx)}{ad^3}\\ &-\frac{i(e+fx)^3\cosh(c+dx)}{ad}+\frac{6f(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}\\ &+\frac{12f^2(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}\\ &-\frac{12f^3\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4}+\frac{6if^3\sinh(c+dx)}{ad^4}\\ &+\frac{3if(e+fx)^2\sinh(c+dx)}{ad^2}-\frac{(e+fx)^3\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\end{aligned}$$

output
```
-(f*x+e)^3/a/d+1/4*(f*x+e)^4/a/f-6*I*f^2*(f*x+e)*cosh(d*x+c)/a/d^3-I*(f*x+
e)^3*cosh(d*x+c)/a/d+6*f*(f*x+e)^2*ln(1+I*exp(d*x+c))/a/d^2+12*f^2*(f*x+e)
*polylog(2,-I*exp(d*x+c))/a/d^3-12*f^3*polylog(3,-I*exp(d*x+c))/a/d^4+6*I*
f^3*sinh(d*x+c)/a/d^4+3*I*f*(f*x+e)^2*sinh(d*x+c)/a/d^2-(f*x+e)^3*tanh(1/2
*c+1/4*I*Pi+1/2*d*x)/a/d
```

3.193.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2619 vs. $2(241) = 482$.

Time = 3.58 (sec) , antiderivative size = 2619, normalized size of antiderivative = 10.87

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-10*I)*d^3*e^3*E^c*Cosh[(d*x)/2] - 2*d^3*e^3*E^(2*c)*Cosh[(d*x)/2] - (6*
I)*d^2*e^2*E^c*f*Cosh[(d*x)/2] + 6*d^2*e^2*E^(2*c)*f*Cosh[(d*x)/2] - (12*I
)*d*e*E^c*f^2*Cosh[(d*x)/2] - 12*d*e*E^(2*c)*f^2*Cosh[(d*x)/2] - (12*I)*E^
c*f^3*Cosh[(d*x)/2] + 12*E^(2*c)*f^3*Cosh[(d*x)/2] - (4*I)*d^4*e^3*E^c*x*C
osh[(d*x)/2] + 4*d^4*e^3*E^(2*c)*x*Cosh[(d*x)/2] - (30*I)*d^3*e^2*E^c*f*x*
Cosh[(d*x)/2] - 6*d^3*e^2*E^(2*c)*f*x*Cosh[(d*x)/2] - (12*I)*d^2*e*E^c*f^2
*x*Cosh[(d*x)/2] + 12*d^2*e*E^(2*c)*f^2*x*Cosh[(d*x)/2] - (12*I)*d*E^c*f^3
*x*Cosh[(d*x)/2] - 12*d*E^(2*c)*f^3*x*Cosh[(d*x)/2] - (6*I)*d^4*e^2*E^c*f*
x^2*Cosh[(d*x)/2] + 6*d^4*e^2*E^(2*c)*f*x^2*Cosh[(d*x)/2] - (30*I)*d^3*e*E
^c*f^2*x^2*Cosh[(d*x)/2] - 6*d^3*e*E^(2*c)*f^2*x^2*Cosh[(d*x)/2] - (6*I)*d
^2*E^c*f^3*x^2*Cosh[(d*x)/2] + 6*d^2*E^(2*c)*f^3*x^2*Cosh[(d*x)/2] - (4*I)
*d^4*e*E^c*f^2*x^3*Cosh[(d*x)/2] + 4*d^4*e*E^(2*c)*f^2*x^3*Cosh[(d*x)/2] -
 (10*I)*d^3*E^c*f^3*x^3*Cosh[(d*x)/2] - 2*d^3*E^(2*c)*f^3*x^3*Cosh[(d*x)/2
] - I*d^4*E^c*f^3*x^4*Cosh[(d*x)/2] + d^4*E^(2*c)*f^3*x^4*Cosh[(d*x)/2] -
2*d^3*e^3*Cosh[(3*d*x)/2] - (2*I)*d^3*e^3*E^(3*c)*Cosh[(3*d*x)/2] - 6*d^2*
e^2*f*Cosh[(3*d*x)/2] + (6*I)*d^2*e^2*E^(3*c)*f*Cosh[(3*d*x)/2] - 12*d*e*f
^2*Cosh[(3*d*x)/2] - (12*I)*d*e*E^(3*c)*f^2*Cosh[(3*d*x)/2] - 12*f^3*Cosh[
(3*d*x)/2] + (12*I)*E^(3*c)*f^3*Cosh[(3*d*x)/2] - 6*d^3*e^2*f*x*Cosh[(3*d*
x)/2] - (6*I)*d^3*e^2*E^(3*c)*f*x*Cosh[(3*d*x)/2] - 12*d^2*e*f^2*x*Cosh[(3
*d*x)/2] + (12*I)*d^2*e*E^(3*c)*f^2*x*Cosh[(3*d*x)/2] - 12*d*f^3*x*Cosh...
```

3.193.3 Rubi [A] (verified)

Time = 1.97 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.10, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.935$, Rules used = {6091, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 6091, 17, 3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 4199, 26, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.193. $\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6091}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^3\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -i(e+fx)^3\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\int (e+fx)^3\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\cosh(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \text{3777}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}\right)}{d}}{a}$$

3.193. $\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 3777$$

$$\frac{i \int \frac{(e+fx)^3 \sinh(c+dx)}{i \sinh(c+dx)a+a}dx - \frac{i(e+fx)^3 \cosh(c+dx)}{d} - \frac{3if\left(\frac{(e+fx)^2 \sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{i \int \frac{(e+fx)^3 \sinh(c+dx)}{i \sinh(c+dx)a+a}dx - \frac{i(e+fx)^3 \cosh(c+dx)}{d} - \frac{3if\left(\frac{(e+fx)^2 \sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \; 3117$$

$$\frac{i \int \frac{(e+fx)^3 \sinh(c+dx)}{i \sinh(c+dx)a+a}dx - \frac{i(e+fx)^3 \cosh(c+dx)}{d} - \frac{3if\left(\frac{(e+fx)^2 \sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \; 6091$$

$$\frac{i\left(i \int \frac{(e+fx)^3}{i \sinh(c+dx)a+a}dx - \frac{i\int (e+fx)^3dx}{a}\right) - \frac{i(e+fx)^3 \cosh(c+dx)}{d} - \frac{3if\left(\frac{(e+fx)^2 \sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \; 17$$

$$\frac{i\left(i \int \frac{(e+fx)^3}{i \sinh(c+dx)a+a}dx - \frac{i(e+fx)^4}{4af}\right) - \frac{i(e+fx)^3 \cosh(c+dx)}{d} - \frac{3if\left(\frac{(e+fx)^2 \sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \; 3042$$

3.193. $\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{i\left(i\int\frac{(e+fx)^3}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^4}{4af}\right)-\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \ 3799$$

$$\frac{i\left(\frac{i\int-(e+fx)^3\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \ 25$$

$$\frac{i\left(-\frac{i\int-(e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \ 25$$

$$\frac{i\left(\frac{i\int(e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{i\left(\frac{i\int(e+fx)^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \ 4672$$

3.193. $\int\frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$i\left(\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6if\int -i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6f\int (e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6f\int -i(e+fx)^2\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{i\left(\frac{6if\int (e+fx)^2\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \; 4199$$

3.193. $\displaystyle\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$i\left(\frac{i\left(\frac{6if\left(2i\int\frac{ie^{c+dx}(e+fx)^2}{1+ie^{c+dx}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\downarrow 26$$

$$i\left(\frac{i\left(\frac{6if\left(-2\int\frac{e^{c+dx}(e+fx)^2}{1+ie^{c+dx}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\downarrow 2620$$

$$i\left(\frac{i\left(\frac{6if\left(-2\left(\frac{2if\int(e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\downarrow 3011$$

3.193. $\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$i\left(\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\cdots\right)}{2a}\cdots\right.$$

$$\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{2720}$$

$$i\left(\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\cdots}{\cdots}\right)}{2a}\cdots\right.$$

$$\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{7143}$$

3.193. $\displaystyle\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$i\left(\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right.}{2a}\right.$$

$$\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

input
```
Int[((e + f*x)^3*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-(((I*(e + f*x)^3*Cosh[c + d*x])/d - ((3*I)*f*(((e + f*x)^2*Sinh[c + d*x])
/d + ((2*I)*f*((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/d
))/d)/a) + I*(((-1/4*I)*(e + f*x)^4)/(a*f) + ((I/2)*(((6*I)*f*(((-1/3*I)*(
e + f*x)^3)/f - 2*(((-I)*(e + f*x)^2*Log[1 + I*E^(c + d*x)])/d + ((2*I)*f*
(-(((e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d) + (f*PolyLog[3, (-I)*E^(c +
 d*x)])/d^2))/d)))/d + (2*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/
a)
```

3.193.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.193. $\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.193.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 698 vs. $2(222) = 444$.

Time = 2.90 (sec) , antiderivative size = 699, normalized size of antiderivative = 2.90

method	result
risch	$\frac{f^3x^4}{4a}+\frac{e^4}{4af}+\frac{12f^2e\ln\left(1+i\mathrm{e}^{dx+c}\right)x}{a\,d^2}-\frac{12f^2ec\ln\left(\mathrm{e}^{dx+c}-i\right)}{a\,d^3}+\frac{12f^2e\ln\left(1+i\mathrm{e}^{dx+c}\right)c}{a\,d^3}-\frac{6f^2e\,c^2}{a\,d^3}-\frac{6f\ln\left(\mathrm{e}^{dx+c}\right)e^2}{a\,d^2}-\frac{6f^2e}{ad}$

input
```
int((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-12*f^3*polylog(3,-I*exp(d*x+c))/a/d^4+1/4/a*f^3*x^4+1/4/a/f*e^4-1/2*I*(d^
3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x-3*d^2*f^3*x^2+d^3*e^3-6*d^2*e*f^2*
x-3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-6*f^3)/d^4/a*exp(d*x+c)-2*I*(f^3*x^3+3*e
*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(d*x+c)-I)+6/a/d^2*f^3*ln(1+I*exp(d*x+c))*
x^2+12/a/d^3*f^2*e*polylog(2,-I*exp(d*x+c))-6/a/d^3*f^2*e*c^2+6/a/d^2*f*ln
(exp(d*x+c)-I)*e^2-6/a/d^2*f*ln(exp(d*x+c))*e^2+6/a/d^4*f^3*c^2*ln(exp(d*x
+c)-I)+12/a/d^3*f^3*polylog(2,-I*exp(d*x+c))*x-6/a/d*f^2*e*x^2-6/a/d^4*f^3
*c^2*ln(exp(d*x+c))-6/a/d^4*f^3*ln(1+I*exp(d*x+c))*c^2+6/a/d^3*f^3*x*c^2+1
/a*f^2*e*x^3+3/2/a*f*e^2*x^2+1/a*e^3*x+4/a/d^4*f^3*c^3-2/a/d*f^3*x^3-1/2*I
*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x+3*d^2*f^3*x^2+d^3*e^3+6*d^2*e*
f^2*x+3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+6*f^3)/d^4/a*exp(-d*x-c)+12/a/d^2*f^
2*e*ln(1+I*exp(d*x+c))*x-12/a/d^2*f^2*e*c*x-12/a/d^3*f^2*e*c*ln(exp(d*x+c)
-I)+12/a/d^3*f^2*e*c*ln(exp(d*x+c))+12/a/d^3*f^2*e*ln(1+I*exp(d*x+c))*c
```

3.193.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(213) = 426$.

Time = 0.26 (sec) , antiderivative size = 823, normalized size of antiderivative = 3.41

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx =$$

$$-\frac{2\,d^3f^3x^3 + 2\,d^3e^3 + 6\,d^2e^2f + 12\,def^2 + 12\,f^3 + 6\,(d^3ef^2 + d^2f^3)x^2 + 6\,(d^3e^2f + 2\,d^2ef^2 + 2\,df^3)x - 4}{}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-1/4*(2*d^3*f^3*x^3 + 2*d^3*e^3 + 6*d^2*e^2*f + 12*d*e*f^2 + 12*f^3 + 6*(d
^3*e*f^2 + d^2*f^3)*x^2 + 6*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*x - 48*((d
*f^3*x + d*e*f^2)*e^(2*d*x + 2*c) - (I*d*f^3*x + I*d*e*f^2)*e^(d*x + c))*d
ilog(-I*e^(d*x + c)) + 2*(I*d^3*f^3*x^3 + I*d^3*e^3 - 3*I*d^2*e^2*f + 6*I*
d*e*f^2 - 6*I*f^3 + 3*(I*d^3*e*f^2 - I*d^2*f^3)*x^2 + 3*(I*d^3*e^2*f - 2*I
*d^2*e*f^2 + 2*I*d*f^3)*x)*e^(3*d*x + 3*c) - (d^4*f^3*x^4 - 2*d^3*e^3 - 6*
(4*c - 1)*d^2*e^2*f + 12*(2*c^2 - 1)*d*e*f^2 - 4*(2*c^3 - 3)*f^3 + 2*(2*d^
4*e*f^2 - 5*d^3*f^3)*x^3 + 6*(d^4*e^2*f - 5*d^3*e*f^2 + d^2*f^3)*x^2 + 2*(
2*d^4*e^3 - 15*d^3*e^2*f + 6*d^2*e*f^2 - 6*d*f^3)*x)*e^(2*d*x + 2*c) - (-I
*d^4*f^3*x^4 - 10*I*d^3*e^3 - 6*(-4*I*c + I)*d^2*e^2*f - 12*(2*I*c^2 + I)*
d*e*f^2 - 4*(-2*I*c^3 + 3*I)*f^3 - 2*(2*I*d^4*e*f^2 + I*d^3*f^3)*x^3 - 6*(
I*d^4*e^2*f + I*d^3*e*f^2 + I*d^2*f^3)*x^2 - 2*(2*I*d^4*e^3 + 3*I*d^3*e^2*
f + 6*I*d^2*e*f^2 + 6*I*d*f^3)*x)*e^(d*x + c) - 24*((d^2*e^2*f - 2*c*d*e*f
^2 + c^2*f^3)*e^(2*d*x + 2*c) - (I*d^2*e^2*f - 2*I*c*d*e*f^2 + I*c^2*f^3)*
e^(d*x + c))*log(e^(d*x + c) - I) - 24*((d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c
*d*e*f^2 - c^2*f^3)*e^(2*d*x + 2*c) - (I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x + 2
*I*c*d*e*f^2 - I*c^2*f^3)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + 48*(f^3*e^
(2*d*x + 2*c) - I*f^3*e^(d*x + c))*polylog(3, -I*e^(d*x + c)))/(a*d^4*e^(2
*d*x + 2*c) - I*a*d^4*e^(d*x + c))
```

3.193.6 Sympy [F]

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-2ie^3-6ie^2fx-6ief^2x^2-2if^3x^3}{ade^ce^{dx}-iad}$$
$$-\frac{i\left(\int \frac{ide^3}{e^ce^{2dx}-ie^{dx}}\,dx+\int \frac{idf^3x^3}{e^ce^{2dx}-ie^{dx}}\,dx+\int \frac{de^3e^ce^{dx}}{e^ce^{2dx}-ie^{dx}}\,dx+\int \frac{de^3e^{3c}e^{3dx}}{e^ce^{2dx}-ie^{dx}}\,dx+\int \left(-\frac{12e^2fe^ce^{dx}}{e^ce^{2dx}-ie^{dx}}\right)dx+\int \left(-\frac{12}{e^c}\right.\right.}{}$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
(-2*I*e**3 - 6*I*e**2*f*x - 6*I*e*f**2*x**2 - 2*I*f**3*x**3)/(a*d*exp(c)*e
xp(d*x) - I*a*d) - I*(Integral(I*d*e**3/(exp(c)*exp(2*d*x) - I*exp(d*x)),
x) + Integral(I*d*f**3*x**3/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integra
l(d*e**3*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(d
*e**3*exp(3*c)*exp(3*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(
-12*e**2*f*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral
(-12*f**3*x**2*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Inte
gral(3*I*d*e*f**2*x**2/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(3*I
*d*e**2*f*x/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(I*d*e**3*exp(2
*c)*exp(2*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(d*f**3*x**3
*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(d*f**3*x*
*3*exp(3*c)*exp(3*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(-24
*e*f**2*x*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(
I*d*f**3*x**3*exp(2*c)*exp(2*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + I
ntegral(3*d*e*f**2*x**2*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)),
x) + Integral(3*d*e*f**2*x**2*exp(3*c)*exp(3*d*x)/(exp(c)*exp(2*d*x) - I*e
xp(d*x)), x) + Integral(3*d*e**2*f*x*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) -
I*exp(d*x)), x) + Integral(3*d*e**2*f*x*exp(3*c)*exp(3*d*x)/(exp(c)*exp(2*
d*x) - I*exp(d*x)), x) + Integral(3*I*d*e*f**2*x**2*exp(2*c)*exp(2*d*x)/(e
xp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(3*I*d*e**2*f*x*exp(2*c)*e...
```

3.193.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(213) = 426$.

Time = 0.38 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.78

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx =$$

$$-\frac{3}{2}e^2f\left(\frac{2\,xe^{(dx+c)}}{ade^{(dx+c)}-i\,ad}+\frac{i\,d^2x^2e^c+i\,dxe^c-\left(-i\,dxe^{(3\,c)}+i\,e^{(3\,c)}\right)e^{(2\,dx)}-\left(d^2x^2e^{(2\,c)}-3\,dxe^{(2\,c)}+e^{(2\,c}\cdots\right.}{ad^2e^{(dx+2\,c)}-i\,ad^2e^c}\right.$$

$$+\frac{1}{2}e^3\left(\frac{2\,(dx+c)}{ad}+\frac{-5i\,e^{(-dx-c)}+1}{\left(i\,ae^{(-dx-c)}+ae^{(-2\,dx-2\,c)}\right)d}-\frac{i\,e^{(-dx-c)}}{ad}\right)$$

$$+\frac{-i\,d^4f^3x^4-12i\,def^2+2\left(-2i\,d^4ef^2-5i\,d^3f^3\right)x^3-12i\,f^3+6\left(-5i\,d^3ef^2-i\,d^2f^3\right)x^2+12\left(-i\,d^2ef^2-\cdots\right.}{\cdots}$$

$$+\frac{12\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)\right)ef^2}{ad^3}$$

$$+\frac{6\left(d^2x^2\log\left(i\,e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-2\,\mathrm{Li}_3\left(-i\,e^{(dx+c)}\right)\right)f^3}{ad^4}$$

$$-\frac{2\left(d^3f^3x^3+3\,d^3ef^2x^2\right)}{ad^4}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-3/2*e^2*f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) + (I*d^2*x^2*e^c + I
*d*x*e^c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d
*x*e^(2*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x
+ 2*c) - I*a*d^2*e^c) - 4*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) + 1/2*e^3
*(2*(d*x + c)/(a*d) + (-5*I*e^(-d*x - c) + 1)/((I*a*e^(-d*x - c) + a*e^(-2
*d*x - 2*c))*d) - I*e^(-d*x - c)/(a*d)) + 1/4*(-I*d^4*f^3*x^4 - 12*I*d*e*f
^2 + 2*(-2*I*d^4*e*f^2 - 5*I*d^3*f^3)*x^3 - 12*I*f^3 + 6*(-5*I*d^3*e*f^2 -
 I*d^2*f^3)*x^2 + 12*(-I*d^2*e*f^2 - I*d*f^3)*x + 2*(-I*d^3*f^3*x^3*e^(2*c
) + 3*(-I*d^3*e*f^2 + I*d^2*f^3)*x^2*e^(2*c) + 6*(I*d^2*e*f^2 - I*d*f^3)*x
*e^(2*c) + 6*(-I*d*e*f^2 + I*f^3)*e^(2*c))*e^(2*d*x) + (d^4*f^3*x^4*e^c +
2*(2*d^4*e*f^2 - d^3*f^3)*x^3*e^c - 6*(d^3*e*f^2 - d^2*f^3)*x^2*e^c + 12*(
d^2*e*f^2 - d*f^3)*x*e^c - 12*(d*e*f^2 - f^3)*e^c)*e^(d*x))/(a*d^4*e^(d*x
+ c) - I*a*d^4) + 12*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*
e*f^2/(a*d^3) + 6*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x
+ c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4) - 2*(d^3*f^3*x^3 + 3*d^3
*e*f^2*x^2)/(a*d^4)
```

3.193.8 Giac [F]

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \sinh(dx+c)^2}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sinh(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)^2\,(e+f\,x)^3}{a+a\sinh(c+d\,x)\ 1\mathrm{i}}\,dx$$

input
```
int((sinh(c + d*x)^2*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((sinh(c + d*x)^2*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i), x)
```

3.193. $\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.194 $\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.194.1 Optimal result

Integrand size = 31, antiderivative size = 184

$$\begin{aligned}\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = &-\frac{(e+fx)^2}{ad}+\frac{(e+fx)^3}{3af}-\frac{2if^2\cosh(c+dx)}{ad^3}\\&-\frac{i(e+fx)^2\cosh(c+dx)}{ad}+\frac{4f(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}\\&+\frac{4f^2\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{2if(e+fx)\sinh(c+dx)}{ad^2}\\&-\frac{(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\end{aligned}$$

output
```
-(f*x+e)^2/a/d+1/3*(f*x+e)^3/a/f-2*I*f^2*cosh(d*x+c)/a/d^3-I*(f*x+e)^2*cos
h(d*x+c)/a/d+4*f*(f*x+e)*ln(1+I*exp(d*x+c))/a/d^2+4*f^2*polylog(2,-I*exp(d
*x+c))/a/d^3+2*I*f*(f*x+e)*sinh(d*x+c)/a/d^2-(f*x+e)^2*tanh(1/2*c+1/4*I*Pi
+1/2*d*x)/a/d
```

3.194.2 Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.41

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{x(3e^2+3efx+f^2x^2)+\frac{6\left(\frac{d(e+fx)\left(-id(e+fx)+2(-i+e^c)f\log\left(1-ie^{-c-dx}\right)\right)}{-i+e^c}-2f^2\operatorname{PolyLog}\left(2,ie^{-c-dx}\right)\right)}{d^3}-\frac{3i\cosh(dx)\left(\left(2f^2+d^2(e+\cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2) + (6*((d*(e + f*x)*((-I)*d*(e + f*x) + 2*(-
I + E^c)*f*Log[1 - I*E^(-c - d*x)]))/(-I + E^c) - 2*f^2*PolyLog[2, I*E^(-c
 - d*x)]))/d^3 - ((3*I)*Cosh[d*x]*((2*f^2 + d^2*(e + f*x)^2)*Cosh[c] - 2*d
*f*(e + f*x)*Sinh[c]))/d^3 - ((3*I)*(-2*d*f*(e + f*x)*Cosh[c] + (2*f^2 + d
^2*(e + f*x)^2)*Sinh[c])*Sinh[d*x])/d^3 - (6*(e + f*x)^2*Sinh[(d*x)/2])/(d
*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/(3*
a)
```

3.194.3 Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.11, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.839, Rules used = {6091, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 6091, 17, 3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 4199, 26, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$\Big\downarrow \ 6091$$
$$i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^2\sinh(c+dx)dx}{a}$$
$$\Big\downarrow \ 3042$$
$$i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -i(e+fx)^2\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \; 26$$

$$i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx - \frac{\int (e+fx)^2\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \; 3777$$

$$i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx - \frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \; 3042$$

$$i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx - \frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \; 3777$$

$$i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx - \frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \; 26$$

$$i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx - \frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \; 3042$$

$$i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx - \frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \; 26$$

$$i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx - \frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \; 3118$$

$$i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx - \frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \; 6091$$

$$i\left(i\int \frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx - \frac{i\int (e+fx)^2dx}{a}\right) - \frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \; 17$$

3.194. $\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$i\left(i\int \frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$i\left(i\int \frac{(e+fx)^2}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 3799}$$

$$i\left(\frac{i\int -(e+fx)^2\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 25}$$

$$i\left(-\frac{i\int -(e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 25}$$

$$i\left(\frac{i\int (e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$i\left(\frac{i\int (e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 4672}$$

$$i\left(\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4if\int -i(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

3.194. $\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\Big\downarrow 26$$

$$i\left(\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\int(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow 3042$$

$$i\left(\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\int-i(e+fx)\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow 26$$

$$i\left(\frac{i\left(\frac{4if\int(e+fx)\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow 4199$$

$$i\left(\frac{i\left(\frac{4if\left(2i\int\frac{ie^{c+dx}(e+fx)}{1+ie^{c+dx}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow 26$$

3.194. $\int\frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$i\left(\frac{i\left(\frac{4if\left(-2\int\frac{e^{c+dx}(e+fx)}{1+ie^{c+dx}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{2620}$$

$$i\left(\frac{i\left(\frac{4if\left(-2\left(\frac{if\int\log\left(1+ie^{c+dx}\right)dx}{d}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{2715}$$

$$i\left(\frac{i\left(\frac{4if\left(-2\left(\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)$$

$$-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{2838}$$

$$i\left(\frac{i\left(\frac{4if\left(-2\left(-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

3.194. $\int\frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^2*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-(((I*(e + f*x)^2*Cosh[c + d*x])/d - ((2*I)*f*(-((f*Cosh[c + d*x])/d^2) +
((e + f*x)*Sinh[c + d*x])/d))/d)/a) + I*(((-1/3*I)*(e + f*x)^3)/(a*f) + ((
I/2)*(((4*I)*f*(((-1/2*I)*(e + f*x)^2)/f - 2*(((-I)*(e + f*x)*Log[1 + I*E^
(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2)))/d + (2*(e + f*x)
^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/a)
```

3.194.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

3.194.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(168) = 336$.

Time = 2.50 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.09

3.194. $\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+ia \sinh(c+dx)} dx$

method	result
risch	$\frac{f^2x^3}{3a}+\frac{fe\,x^2}{a}+\frac{e^2x}{a}+\frac{e^3}{3af}-\frac{2i(x^2f^2+2efx+e^2)}{da(\mathrm{e}^{dx+c}-i)}-\frac{i(d^2x^2f^2+2d^2efx+d^2e^2-2xdf^2-2def+2f^2)\mathrm{e}^{dx+c}}{2d^3a}-\frac{i(d^2x^2f^2+2d$

input
```
int((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*f^2*x^3+1/a*f*e*x^2+1/a*e^2*x+1/3/a/f*e^3-2*I*(f^2*x^2+2*e*f*x+e^2)/
d/a/(exp(d*x+c)-I)-1/2*I*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2-2*d*f^2*x-2*d*e*
f+2*f^2)/d^3/a*exp(d*x+c)-1/2*I*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2+2*d*f^2*x
+2*d*e*f+2*f^2)/d^3/a*exp(-d*x-c)+4/a/d^2*f*ln(exp(d*x+c)-I)*e-4/a/d^2*f*l
n(exp(d*x+c))*e-2*f^2*x^2/a/d-4/a/d^2*f^2*c*x-2/a/d^3*f^2*c^2+4/a/d^2*f^2*
ln(1+I*exp(d*x+c))*x+4/a/d^3*f^2*ln(1+I*exp(d*x+c))*c+4*f^2*polylog(2,-I*e
xp(d*x+c))/a/d^3-4/a/d^3*f^2*c*ln(exp(d*x+c)-I)+4/a/d^3*f^2*c*ln(exp(d*x+c
))
```

3.194.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(161) = 322.

Time = 0.26 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.58

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{3\,d^2f^2x^2+3\,d^2e^2+6\,def+6\,f^2+6\,(d^2ef+df^2)x-24\,\left(f^2e^{(2\,dx+2\,c)}-i\,f^2e^{(dx+c)}\right)\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)+3\,(}{}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-1/6*(3*d^2*f^2*x^2 + 3*d^2*e^2 + 6*d*e*f + 6*f^2 + 6*(d^2*e*f + d*f^2)*x
- 24*(f^2*e^(2*d*x + 2*c) - I*f^2*e^(d*x + c))*dilog(-I*e^(d*x + c)) + 3*(
I*d^2*f^2*x^2 + I*d^2*e^2 - 2*I*d*e*f + 2*I*f^2 + 2*(I*d^2*e*f - I*d*f^2)*
x)*e^(3*d*x + 3*c) - (2*d^3*f^2*x^3 - 3*d^2*e^2 - 6*(4*c - 1)*d*e*f + 6*(2
*c^2 - 1)*f^2 + 3*(2*d^3*e*f - 5*d^2*f^2)*x^2 + 6*(d^3*e^2 - 5*d^2*e*f + d
*f^2)*x)*e^(2*d*x + 2*c) - (-2*I*d^3*f^2*x^3 - 15*I*d^2*e^2 - 6*(-4*I*c +
I)*d*e*f - 6*(2*I*c^2 + I)*f^2 - 3*(2*I*d^3*e*f + I*d^2*f^2)*x^2 - 6*(I*d^
3*e^2 + I*d^2*e*f + I*d*f^2)*x)*e^(d*x + c) - 24*((d*e*f - c*f^2)*e^(2*d*x
 + 2*c) - (I*d*e*f - I*c*f^2)*e^(d*x + c))*log(e^(d*x + c) - I) - 24*((d*f
^2*x + c*f^2)*e^(2*d*x + 2*c) - (I*d*f^2*x + I*c*f^2)*e^(d*x + c))*log(I*e
^(d*x + c) + 1))/(a*d^3*e^(2*d*x + 2*c) - I*a*d^3*e^(d*x + c))
```

3.194.6 Sympy [F]

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-2ie^2-4iefx-2if^2x^2}{ade^ce^{dx}-iad}$$
$$-\frac{i\left(\int \frac{ide^2}{e^ce^{2dx}-ie^{dx}}\,dx+\int \frac{idf^2x^2}{e^ce^{2dx}-ie^{dx}}\,dx+\int \frac{de^2e^ce^{dx}}{e^ce^{2dx}-ie^{dx}}\,dx+\int \frac{de^2e^{3c}e^{3dx}}{e^ce^{2dx}-ie^{dx}}\,dx+\int\left(-\frac{8efe^ce^{dx}}{e^ce^{2dx}-ie^{dx}}\right)dx+\int\left(-\frac{8}{e^c}\right.\right.}{}$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
(-2*I*e**2 - 4*I*e*f*x - 2*I*f**2*x**2)/(a*d*exp(c)*exp(d*x) - I*a*d) - I*
(Integral(I*d*e**2/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(I*d*f**
2*x**2/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(d*e**2*exp(c)*exp(d
*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(d*e**2*exp(3*c)*exp(3*
d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(-8*e*f*exp(c)*exp(d*x
)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(-8*f**2*x*exp(c)*exp(d*x
)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(2*I*d*e*f*x/(exp(c)*exp(
2*d*x) - I*exp(d*x)), x) + Integral(I*d*e**2*exp(2*c)*exp(2*d*x)/(exp(c)*e
xp(2*d*x) - I*exp(d*x)), x) + Integral(d*f**2*x**2*exp(c)*exp(d*x)/(exp(c)
*exp(2*d*x) - I*exp(d*x)), x) + Integral(d*f**2*x**2*exp(3*c)*exp(3*d*x)/(
exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(I*d*f**2*x**2*exp(2*c)*exp(
2*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(2*d*e*f*x*exp(c)*ex
p(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(2*d*e*f*x*exp(3*c)*
exp(3*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(2*I*d*e*f*x*exp
(2*c)*exp(2*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x))*exp(-c)/(2*a*d)
```

3.194.7 Maxima [F]

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\sinh{(dx+c)}^2}{i\,a\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-e*f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) + (I*d^2*x^2*e^c + I*d*x*e
^c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d*x*e^(
2*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x + 2*c)
 - I*a*d^2*e^c) - 4*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) - 1/6*f^2*((2*I
*d^3*x^3 + 15*I*d^2*x^2 + 6*I*d*x - 3*(-I*d^2*x^2*e^(2*c) + 2*I*d*x*e^(2*c
) - 2*I*e^(2*c))*e^(2*d*x) - (2*d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c -
6*e^c)*e^(d*x) + 6*I)/(a*d^3*e^(d*x + c) - I*a*d^3) - 24*I*integrate(x/(a*
d*e^(d*x + c) - I*a*d), x)) + 1/2*e^2*(2*(d*x + c)/(a*d) + (-5*I*e^(-d*x -
 c) + 1)/((I*a*e^(-d*x - c) + a*e^(-2*d*x - 2*c))*d) - I*e^(-d*x - c)/(a*d
))
```

3.194.8 Giac [F]

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\sinh{(dx+c)}^2}{i\,a\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sinh(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{\sinh(c+d\,x)^2\,(e+f\,x)^2}{a+a\sinh{(c+d\,x)}\;1\mathrm{i}}\,dx$$

input
```
int((sinh(c + d*x)^2*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((sinh(c + d*x)^2*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i), x)
```

3.194. $\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\, dx$

3.195 $\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.195.1 Optimal result

Integrand size = 29, antiderivative size = 119

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{ex}{a}+\frac{fx^2}{2a}-\frac{i(e+fx)\cosh(c+dx)}{ad} + \frac{2f\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}+\frac{if\sinh(c+dx)}{ad^2} - \frac{(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
e*x/a+1/2*f*x^2/a-I*(f*x+e)*cosh(d*x+c)/a/d+2*f*ln(cosh(1/2*c+1/4*I*Pi+1/2
*d*x))/a/d^2+I*f*sinh(d*x+c)/a/d^2-(f*x+e)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/
d
```

3.195.2 Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.00

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(-i\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\sinh\left(\frac{1}{2}(c+dx)\right)\left(i(2i+c+dx)(2de-cf+dfx)-4f\arctan\right.\right.}{}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])*(Sinh[(c + d*x)/2]*(I*(2*I +
 c + d*x)*(2*d*e - c*f + d*f*x) - 4*f*ArcTan[Tanh[(c + d*x)/2]] + 2*d*(e +
 f*x)*Cosh[c + d*x] + (2*I)*f*Log[Cosh[c + d*x]] - 2*f*Sinh[c + d*x]) + Co
sh[(c + d*x)/2]*(2*c*d*e - (2*I)*c*f - c^2*f + 2*d^2*e*x - (2*I)*d*f*x + d
^2*f*x^2 + (4*I)*f*ArcTan[Tanh[(c + d*x)/2]] - (2*I)*d*(e + f*x)*Cosh[c +
d*x] + 2*f*Log[Cosh[c + d*x]] + (2*I)*f*Sinh[c + d*x]))/(2*a*d^2*(-I + Si
nh[c + d*x]))
```

3.195.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.621$, Rules used = {6091, 3042, 26, 3777, 3042, 3117, 6091, 17, 3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6091}$$

$$i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -i(e+fx)\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\int (e+fx)\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \text{3117}$$

$$i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \; 6091$$

$$i\left(i\int \frac{e+fx}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)dx}{a}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \; 17$$

$$i\left(i\int \frac{e+fx}{i\sinh(c+dx)a+a}dx-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \; 3042$$

$$i\left(i\int \frac{e+fx}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \; 3799$$

$$i\left(\frac{i\int -\left((e+fx)\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)\right)dx}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \; 25$$

$$i\left(-\frac{i\int -\left((e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)dx}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \; 25$$

$$i\left(\frac{i\int (e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{i\int (e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \; 4672$$

$$i\left(\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2if\int -i\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \; 26$$

3.195. $\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}dx$

$$i\left(\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int-i\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{i\left(\frac{2if\int\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

$$\Big\downarrow \; 3956$$

$$i\left(\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{d^2}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

input
```
Int[((e + f*x)*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-(((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2)/a) + I*(((-1/2
*I)*(e + f*x)^2)/(a*f) + ((I/2)*((-4*f*Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2]]
)/d^2 + (2*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/a)
```

3.195. $\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.195.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.195. $\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

3.195.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13

method	result
risch	$\frac{f\,x^{2}}{2a}+\frac{ex}{a}-\frac{i(dfx+de-f)\mathrm{e}^{dx+c}}{2a\,d^{2}}-\frac{i(dfx+de+f)\mathrm{e}^{-dx-c}}{2a\,d^{2}}-\frac{2fx}{ad}-\frac{2fc}{a\,d^{2}}-\frac{2i(fx+e)}{da(\mathrm{e}^{dx+c}-i)}+\frac{2f\ln(\mathrm{e}^{dx+c}-i)}{a\,d^{2}}$
parallelrisch	$\frac{-4f\left(i\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(1-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4f\left(i\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+((i}{}$

input
```
int((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*f*x^2/a+e*x/a-1/2*I*(d*f*x+d*e-f)/a/d^2*exp(d*x+c)-1/2*I*(d*f*x+d*e+f)
/a/d^2*exp(-d*x-c)-2*f*x/a/d-2*f/a/d^2*c-2*I*(f*x+e)/d/a/(exp(d*x+c)-I)+2*
f/a/d^2*ln(exp(d*x+c)-I)
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.45

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx =$$
$$-\frac{dfx+de-(-i\,dfx-i\,de+i\,f)e^{(3\,dx+3\,c)}-(d^2fx^2-de+(2\,d^2e-5\,df)x+f)e^{(2\,dx+2\,c)}-(-i\,d^2fx^2-}{2\,(ad^2e^{(2\,dx+2\,c)}-i\,ad^2}$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(d*f*x + d*e - (-I*d*f*x - I*d*e + I*f)*e^(3*d*x + 3*c) - (d^2*f*x^2
- d*e + (2*d^2*e - 5*d*f)*x + f)*e^(2*d*x + 2*c) - (-I*d^2*f*x^2 - 5*I*d*e
 + (-2*I*d^2*e - I*d*f)*x - I*f)*e^(d*x + c) - 4*(f*e^(2*d*x + 2*c) - I*f*
e^(d*x + c))*log(e^(d*x + c) - I) + f)/(a*d^2*e^(2*d*x + 2*c) - I*a*d^2*e^
(d*x + c))
```

3.195.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.88

$$
\begin{aligned}
&\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx \\
&= \frac{-2ie-2ifx}{ade^ce^{dx}-iad} \\
&\quad+\begin{cases} \frac{((-2iad^3e-2iad^3fx-2iad^2f)e^{-dx}+(-2iad^3ee^{2c}-2iad^3fxe^{2c}+2iad^2fe^{2c})e^{dx})e^{-c}}{4a^2d^4} & \text{for } a^2d^4e^c\neq 0 \\ \frac{x^2(-ife^{2c}+if)e^{-c}}{4a}+\frac{x(-iee^{2c}+ie)e^{-c}}{2a} & \text{otherwise} \end{cases} \\
&\quad+\frac{fx^2}{2a}+\frac{x(de-2f)}{ad}+\frac{2f\log\left(e^{dx}-ie^{-c}\right)}{ad^2}
\end{aligned}
$$

input
```
integrate((f*x+e)*sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
(-2*I*e - 2*I*f*x)/(a*d*exp(c)*exp(d*x) - I*a*d) + Piecewise((((-2*I*a*d**
3*e - 2*I*a*d**3*f*x - 2*I*a*d**2*f)*exp(-d*x) + (-2*I*a*d**3*e*exp(2*c) -
 2*I*a*d**3*f*x*exp(2*c) + 2*I*a*d**2*f*exp(2*c))*exp(d*x))*exp(-c)/(4*a**
2*d**4), Ne(a**2*d**4*exp(c), 0)), (x**2*(-I*f*exp(2*c) + I*f)*exp(-c)/(4*
a) + x*(-I*e*exp(2*c) + I*e)*exp(-c)/(2*a), True)) + f*x**2/(2*a) + x*(d*e
 - 2*f)/(a*d) + 2*f*log(exp(d*x) - I*exp(-c))/(a*d**2)
```

3.195.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(100) = 200.

Time = 0.25 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.00

$$
\begin{aligned}
&\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \\
&-\frac{1}{2}f\left(\frac{2\,xe^{(dx+c)}}{ade^{(dx+c)}-i\,ad}+\frac{i\,d^2x^2e^c+i\,dxe^c-\left(-i\,dxe^{(3\,c)}+i\,e^{(3\,c)}\right)e^{(2\,dx)}-\left(d^2x^2e^{(2\,c)}-3\,dxe^{(2\,c)}+e^{(2\,c)}\right)}{ad^2e^{(dx+2\,c)}-i\,ad^2e^c}\right. \\
&+\frac{1}{2}e\left(\frac{2\,(dx+c)}{ad}+\frac{-5i\,e^{(-dx-c)}+1}{\left(i\,ae^{(-dx-c)}+ae^{(-2\,dx-2\,c)}\right)d}-\frac{i\,e^{(-dx-c)}}{ad}\right)
\end{aligned}
$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) + (I*d^2*x^2*e^c + I*d*x
*e^c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d*x*e
^(2*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x + 2*
c) - I*a*d^2*e^c) - 4*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) + 1/2*e*(2*(d
*x + c)/(a*d) + (-5*I*e^(-d*x - c) + 1)/((I*a*e^(-d*x - c) + a*e^(-2*d*x -
 2*c))*d) - I*e^(-d*x - c)/(a*d))
```

3.195.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(100) = 200.

Time = 0.28 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.11

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{d^2fx^2e^{(2\,dx+2\,c)} - i\,d^2fx^2e^{(dx+c)} + 2\,d^2exe^{(2\,dx+2\,c)} - 2i\,d^2exe^{(dx+c)} - i\,dfxe^{(3\,dx+3\,c)} - 5\,dfxe^{(2\,dx+2\,c)} - i\,df$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(d^2*f*x^2*e^(2*d*x + 2*c) - I*d^2*f*x^2*e^(d*x + c) + 2*d^2*e*x*e^(2*
d*x + 2*c) - 2*I*d^2*e*x*e^(d*x + c) - I*d*f*x*e^(3*d*x + 3*c) - 5*d*f*x*e
^(2*d*x + 2*c) - I*d*f*x*e^(d*x + c) - d*f*x - I*d*e*e^(3*d*x + 3*c) - d*e
*e^(2*d*x + 2*c) - 5*I*d*e*e^(d*x + c) + 4*f*e^(2*d*x + 2*c)*log(e^(d*x +
c) - I) - 4*I*f*e^(d*x + c)*log(e^(d*x + c) - I) - d*e + I*f*e^(3*d*x + 3*
c) + f*e^(2*d*x + 2*c) - I*f*e^(d*x + c) - f)/(a*d^2*e^(2*d*x + 2*c) - I*a
*d^2*e^(d*x + c))
```

3.195.9 Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{f\,x^2}{2\,a} + \mathrm{e}^{c+d\,x}\left(\frac{(f-d\,e)\ 1\mathrm{i}}{2\,a\,d^2} - \frac{f\,x\,1\mathrm{i}}{2\,a\,d}\right) \\ &\quad - \mathrm{e}^{-c-d\,x}\left(\frac{(f+d\,e)\ 1\mathrm{i}}{2\,a\,d^2} + \frac{f\,x\,1\mathrm{i}}{2\,a\,d}\right) - \frac{(e+f\,x)\ 2\mathrm{i}}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)} \\ &\quad - \frac{x\,(2\,f-d\,e)}{a\,d} + \frac{2\,f\,\ln\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-\mathrm{i}\right)}{a\,d^2}\end{aligned}$$

input
```
int((sinh(c + d*x)^2*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

output
```
exp(c + d*x)*(((f - d*e)*1i)/(2*a*d^2) - (f*x*1i)/(2*a*d)) - exp(- c - d*x
)*(((f + d*e)*1i)/(2*a*d^2) + (f*x*1i)/(2*a*d)) + (f*x^2)/(2*a) - ((e + f*
x)*2i)/(a*d*(exp(c + d*x) - 1i)) - (x*(2*f - d*e))/(a*d) + (2*f*log(exp(d*
x)*exp(c) - 1i))/(a*d^2)
```

3.195. $\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.196 $\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.196.1 Optimal result

Integrand size = 24, antiderivative size = 52

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{x}{a}-\frac{i\cosh(c+dx)}{ad}-\frac{i\cosh(c+dx)}{ad(1+i\sinh(c+dx))}$$

output
```
x/a-I*cosh(d*x+c)/a/d-I*cosh(d*x+c)/a/d/(1+I*sinh(d*x+c))
```

3.196.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{\cosh(c+dx)\left(\frac{\operatorname{arcsinh}(\sinh(c+dx))}{\sqrt{\cosh^2(c+dx)}}+\frac{-2-i\sinh(c+dx)}{-i+\sinh(c+dx)}\right)}{ad}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(Cosh[c + d*x]*(ArcSinh[Sinh[c + d*x]]/Sqrt[Cosh[c + d*x]^2] + (-2 - I*Sin
h[c + d*x])/(-I + Sinh[c + d*x])))/(a*d)
```

3.196.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 25, 3225, 26, 3042, 26, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2}{a+a\sin(ic+idx)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2}{\sin(ic+idx)a+a}dx$$

$$\Big\downarrow \text{3225}$$

$$-\frac{\int -\frac{i\sinh(c+dx)}{i\sinh(c+dx)+1}dx}{a}-\frac{i\cosh(c+dx)}{ad}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \frac{\sinh(c+dx)}{i\sinh(c+dx)+1}dx}{a}-\frac{i\cosh(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\int -\frac{i\sin(ic+idx)}{\sin(ic+idx)+1}dx}{a}-\frac{i\cosh(c+dx)}{ad}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\sin(ic+idx)}{\sin(ic+idx)+1}dx}{a}-\frac{i\cosh(c+dx)}{ad}$$

$$\Big\downarrow \text{3214}$$

$$\frac{x-\int \frac{1}{i\sinh(c+dx)+1}dx}{a}-\frac{i\cosh(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x-\int \frac{1}{\sin(ic+idx)+1}dx}{a}-\frac{i\cosh(c+dx)}{ad}$$

$$\Big\downarrow \text{3127}$$

$$\frac{x-\frac{i\cosh(c+dx)}{d(1+i\sinh(c+dx))}}{a}-\frac{i\cosh(c+dx)}{ad}$$

input
```
Int[Sinh[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-I)*Cosh[c + d*x])/(a*d) + (x - (I*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x
])))/a
```

3.196.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.196. $\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.196.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

method	result	size
risch	$\frac{x}{a}-\frac{i\mathrm{e}^{dx+c}}{2ad}-\frac{i\mathrm{e}^{-dx-c}}{2ad}-\frac{2i}{da(\mathrm{e}^{dx+c}-i)}$	60
derivativedivides	$\frac{-\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{i}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{8i}{8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-8}-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}$	86
default	$\frac{-\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{i}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{8i}{8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-8}-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}$	86
parallelrisch	$\frac{(-2dx-3i)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)+2idx\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\sinh\left(\frac{3dx}{2}+\frac{3c}{2}\right)+3\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+\cosh\left(\frac{3dx}{2}+\frac{3c}{2}\right)}{2ad\left(i\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	99

input
```
int(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
x/a-1/2*I/a/d*exp(d*x+c)-1/2*I/a/d*exp(-d*x-c)-2*I/d/a/(exp(d*x+c)-I)
```

3.196.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{(2\,dx-1)e^{(2\,dx+2\,c)}+(-2i\,dx-5i)e^{(dx+c)}-i\,e^{(3\,dx+3\,c)}-1}{2\left(ade^{(2\,dx+2\,c)}-i\,ade^{(dx+c)}\right)}$$

input
```
integrate(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((2*d*x - 1)*e^(2*d*x + 2*c) + (-2*I*d*x - 5*I)*e^(d*x + c) - I*e^(3*d
*x + 3*c) - 1)/(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c))
```

3.196.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.90

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\begin{cases}\frac{(-2iade^{2c}e^{dx}-2iade^{-dx})e^{-c}}{4a^2d^2} & \text{for } a^2d^2e^c\neq 0\\ x\left(\frac{(-ie^{2c}+2e^c+i)e^{-c}}{2a}-\frac{1}{a}\right) & \text{otherwise}\end{cases}-\frac{2i}{ade^ce^{dx}-iad}+\frac{x}{a}$$

input
```
integrate(sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise(((-2*I*a*d*exp(2*c)*exp(d*x) - 2*I*a*d*exp(-d*x))*exp(-c)/(4*a**
2*d**2), Ne(a**2*d**2*exp(c), 0)), (x*((-I*exp(2*c) + 2*exp(c) + I)*exp(-c
)/(2*a) - 1/a), True)) - 2*I/(a*d*exp(c)*exp(d*x) - I*a*d) + x/a
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{dx+c}{ad} + \frac{-5i\,e^{(-dx-c)}+1}{2\,(i\,ae^{(-dx-c)}+ae^{(-2\,dx-2\,c)})d} - \frac{i\,e^{(-dx-c)}}{2\,ad}$$

input
```
integrate(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
(d*x + c)/(a*d) + 1/2*(-5*I*e^(-d*x - c) + 1)/((I*a*e^(-d*x - c) + a*e^(-2
*d*x - 2*c))*d) - 1/2*I*e^(-d*x - c)/(a*d)
```

3.196.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\frac{2\,(dx+c)}{a} - \frac{i\,e^{(dx+c)}}{a} - \frac{(5i\,e^{(dx+c)}+1)e^{(-dx-c)}}{a(e^{(dx+c)}-i)}}{2\,d}$$

input
```
integrate(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)/a - I*e^(d*x + c)/a - (5*I*e^(d*x + c) + 1)*e^(-d*x - c)/
(a*(e^(d*x + c) - I)))/d
```

3.196.9 Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{x}{a} - \frac{2\mathrm{i}}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)} - \frac{\mathrm{e}^{c+d\,x}\,1\mathrm{i}}{2\,a\,d} - \frac{\mathrm{e}^{-c-d\,x}\,1\mathrm{i}}{2\,a\,d}$$

input
```
int(sinh(c + d*x)^2/(a + a*sinh(c + d*x)*1i),x)
```

output
```
x/a - 2i/(a*d*(exp(c + d*x) - 1i)) - (exp(c + d*x)*1i)/(2*a*d) - (exp(- c
- d*x)*1i)/(2*a*d)
```

3.197 $\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

3.197.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.197.2 Mathematica [N/A]

Not integrable

Time = 21.38 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

3.197.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.197.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.197.4 Maple [N/A] (verified)

Not integrable

Time = 1.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\sinh\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.197.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 203, normalized size of antiderivative = 6.55

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^2}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral(1/2*(d*f*
x + d*e + (-I*d*f*x - I*d*e)*e^(3*d*x + 3*c) + (d*f*x + d*e)*e^(2*d*x + 2*
c) + (-I*d*f*x - I*d*e - 4*I*f)*e^(d*x + c))/((a*d*f^2*x^2 + 2*a*d*e*f*x +
 a*d*e^2)*e^(2*d*x + 2*c) - (I*a*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2)*e^
(d*x + c)), x) - 2*I)/(-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c
))
```

3.197.6 Sympy [N/A]

Not integrable

Time = 74.64 (sec) , antiderivative size = 950, normalized size of antiderivative = 30.65

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{2i}{-iade-iadfx+\left(adee^{c}+adfxe^{c}\right)e^{dx}}$$
$$-\frac{i\left(\int \frac{ide}{e^2e^ce^{2dx}-ie^2e^{dx}+2efxe^ce^{2dx}-2iefxe^{dx}+f^2x^2e^ce^{2dx}-if^2x^2e^{dx}}\,dx+\int \frac{4fe^ce^{dx}}{e^2e^ce^{2dx}-ie^2e^{dx}+2efxe^ce^{2dx}-2iefxe^{dx}+f^2x^2e^ce^{2dx}-i}\right.}{}$$

input
```
integrate(sinh(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-2*I/(-I*a*d*e - I*a*d*f*x + (a*d*e*exp(c) + a*d*f*x*exp(c))*exp(d*x)) - I
*(Integral(I*d*e/(e**2*exp(c)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e*f*x*exp(c
)*exp(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2*x**2*exp(c)*exp(2*d*x) - I*f**2*x
**2*exp(d*x)), x) + Integral(4*f*exp(c)*exp(d*x)/(e**2*exp(c)*exp(2*d*x) -
 I*e**2*exp(d*x) + 2*e*f*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2*x
**2*exp(c)*exp(2*d*x) - I*f**2*x**2*exp(d*x)), x) + Integral(I*d*f*x/(e**2
*exp(c)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e*f*x*exp(c)*exp(2*d*x) - 2*I*e*f
*x*exp(d*x) + f**2*x**2*exp(c)*exp(2*d*x) - I*f**2*x**2*exp(d*x)), x) + In
tegral(d*e*exp(c)*exp(d*x)/(e**2*exp(c)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e
*f*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2*x**2*exp(c)*exp(2*d*x)
- I*f**2*x**2*exp(d*x)), x) + Integral(d*e*exp(3*c)*exp(3*d*x)/(e**2*exp(c
)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e*f*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp
(d*x) + f**2*x**2*exp(c)*exp(2*d*x) - I*f**2*x**2*exp(d*x)), x) + Integral
(I*d*e*exp(2*c)*exp(2*d*x)/(e**2*exp(c)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e
*f*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2*x**2*exp(c)*exp(2*d*x)
- I*f**2*x**2*exp(d*x)), x) + Integral(d*f*x*exp(c)*exp(d*x)/(e**2*exp(c)*
exp(2*d*x) - I*e**2*exp(d*x) + 2*e*f*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp(d
*x) + f**2*x**2*exp(c)*exp(2*d*x) - I*f**2*x**2*exp(d*x)), x) + Integral(d
*f*x*exp(3*c)*exp(3*d*x)/(e**2*exp(c)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e*f
*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2*x**2*exp(c)*exp(2*d*x)...
```

3.197.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 5.71

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^2}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*I*f*integrate(1/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*
x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 1/2*I*e^(-c + d*e/
f)*exp_integral_e(1, (f*x + e)*d/f)/(a*f) + 1/2*I*e^(c - d*e/f)*exp_integr
al_e(1, -(f*x + e)*d/f)/(a*f) - 2*I/(-I*a*d*f*x - I*a*d*e + (a*d*f*x*e^c +
 a*d*e*e^c)*e^(d*x)) + log(f*x + e)/(a*f)
```

3.197.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^2}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^2/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

3.197.9 Mupad [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)^2}{(e+f\,x)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(sinh(c + d*x)^2/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(sinh(c + d*x)^2/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.198 $\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

3.198.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \mathrm{Int}\left(\frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.198.2 Mathematica [N/A]

Not integrable

Time = 22.82 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

3.198.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.198.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.198.4 Maple [N/A] (verified)

Not integrable

Time = 1.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\sinh \left(dx+c\right)^{2}}{\left(fx+e\right)^{2} \left(a+ia\sinh \left(dx+c\right)\right)}dx$$

input
```
int(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.198.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 281, normalized size of antiderivative = 9.06

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^2}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
((-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x
+ a*d*e^2)*e^(d*x + c))*integral(1/2*(d*f*x + d*e + (-I*d*f*x - I*d*e)*e^(
3*d*x + 3*c) + (d*f*x + d*e)*e^(2*d*x + 2*c) + (-I*d*f*x - I*d*e - 8*I*f)*
e^(d*x + c))/((a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^
(2*d*x + 2*c) - (I*a*d*f^3*x^3 + 3*I*a*d*e*f^2*x^2 + 3*I*a*d*e^2*f*x + I*a
*d*e^3)*e^(d*x + c)), x) - 2*I)/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^
2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

3.198.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.198.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 270, normalized size of antiderivative = 8.71

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^2}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-4*I*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x -
 I*a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c +
a*d*e^3*e^c)*e^(d*x)), x) - (-I*d*f*x - I*d*e + (d*f*x*e^c + d*e*e^c)*e^(d
*x) + 2*I*f)/(-I*a*d*f^3*x^2 - 2*I*a*d*e*f^2*x - I*a*d*e^2*f + (a*d*f^3*x^
2*e^c + 2*a*d*e*f^2*x*e^c + a*d*e^2*f*e^c)*e^(d*x)) - 1/2*I*e^(-c + d*e/f)
*exp_integral_e(2, (f*x + e)*d/f)/((f*x + e)*a*f) + 1/2*I*e^(c - d*e/f)*ex
p_integral_e(2, -(f*x + e)*d/f)/((f*x + e)*a*f)
```

3.198.8 Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^2}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^2/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

3.198.9 Mupad [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)^2}{(e+f\,x)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(sinh(c + d*x)^2/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(sinh(c + d*x)^2/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.199 $\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.199.1 Optimal result

Integrand size = 31, antiderivative size = 393

$$\begin{aligned}\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{3ief^2x}{4ad^2}+\frac{3if^3x^2}{8ad^2}-\frac{i(e+fx)^3}{ad}+\frac{3i(e+fx)^4}{8af}\\
&+\frac{6f^2(e+fx)\cosh(c+dx)}{ad^3}+\frac{(e+fx)^3\cosh(c+dx)}{ad}\\
&+\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}\\
&+\frac{12if^2(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}\\
&-\frac{12if^3\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4}\\
&-\frac{6f^3\sinh(c+dx)}{ad^4}-\frac{3f(e+fx)^2\sinh(c+dx)}{ad^2}\\
&-\frac{3if^2(e+fx)\cosh(c+dx)\sinh(c+dx)}{4ad^3}\\
&-\frac{i(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{2ad}\\
&+\frac{3if^3\sinh^2(c+dx)}{8ad^4}+\frac{3if(e+fx)^2\sinh^2(c+dx)}{4ad^2}\\
&-\frac{i(e+fx)^3\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\end{aligned}$$

output
```
3/4*I*e*f^2*x/a/d^2+6*I*f*(f*x+e)^2*ln(1+I*exp(d*x+c))/a/d^2-3/4*I*f^2*(f*
x+e)*cosh(d*x+c)*sinh(d*x+c)/a/d^3-1/2*I*(f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)
/a/d+6*f^2*(f*x+e)*cosh(d*x+c)/a/d^3+(f*x+e)^3*cosh(d*x+c)/a/d+3/8*I*f^3*s
inh(d*x+c)^2/a/d^4+3/4*I*f*(f*x+e)^2*sinh(d*x+c)^2/a/d^2-I*(f*x+e)^3/a/d-6
*f^3*sinh(d*x+c)/a/d^4-3*f*(f*x+e)^2*sinh(d*x+c)/a/d^2+12*I*f^2*(f*x+e)*po
lylog(2,-I*exp(d*x+c))/a/d^3+3/8*I*(f*x+e)^4/a/f+3/8*I*f^3*x^2/a/d^2-12*I*
f^3*polylog(3,-I*exp(d*x+c))/a/d^4-I*(f*x+e)^3*tanh(1/2*c+1/4*I*Pi+1/2*d*x
)/a/d
```

3.199.2 Mathematica [A] (verified)

Time = 4.70 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.96

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$= \frac{24ie^3x+36ie^2fx^2+24ief^2x^3+6if^3x^4+\frac{32(e+fx)^3}{d(-i+e^c)}+\frac{96f^2(e+fx)\cosh(c+dx)}{d^3}+\frac{16(e+fx)^3\cosh(c+dx)}{d}+\frac{3if^3\cosh(2(c}{d^4}}{}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((24*I)*e^3*x + (36*I)*e^2*f*x^2 + (24*I)*e*f^2*x^3 + (6*I)*f^3*x^4 + (32*
(e + f*x)^3)/(d*(-I + E^c)) + (96*f^2*(e + f*x)*Cosh[c + d*x])/d^3 + (16*(
e + f*x)^3*Cosh[c + d*x])/d + ((3*I)*f^3*Cosh[2*(c + d*x)])/d^4 + ((6*I)*f
*(e + f*x)^2*Cosh[2*(c + d*x)])/d^2 + ((96*I)*f*(e + f*x)^2*Log[1 - I*E^(-
c - d*x)])/d^2 - ((192*I)*f^2*(d*(e + f*x)*PolyLog[2, I*E^(-c - d*x)] + f*
PolyLog[3, I*E^(-c - d*x)]))/d^4 - ((32*I)*(e + f*x)^3*Sinh[(d*x)/2])/(d*(
Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])) - (96*
f^3*Sinh[c + d*x])/d^4 - (48*f*(e + f*x)^2*Sinh[c + d*x])/d^2 - ((6*I)*f^2
*(e + f*x)*Sinh[2*(c + d*x)])/d^3 - ((4*I)*(e + f*x)^3*Sinh[2*(c + d*x)])/
d)/(16*a)
```

3.199.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6091}$$

$$i\int \frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^3\sinh^2(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$i\int \frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -(e+fx)^3\sin(ic+idx)^2dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{i\int (e+fx)^3\sin(ic+idx)^2dx}{a}+i\int \frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3792}$$

$$\frac{i\left(\frac{3f^2\int -((e+fx)\sinh^2(c+dx))dx}{2d^2}+\frac{1}{2}\int (e+fx)^3dx+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{a}+$$
$$i\int \frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{i\left(\frac{3f^2\int -((e+fx)\sinh^2(c+dx))dx}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}+$$
$$i\int \frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{i\left(-\frac{3f^2\int (e+fx)\sinh^2(c+dx)dx}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}+$$
$$i\int \frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{i\left(-\frac{3f^2\int -((e+fx)\sin(ic+idx)^2)dx}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}+$$
$$i\int\frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \; 25$$

$$\frac{i\left(\frac{3f^2\int (e+fx)\sin(ic+idx)^2dx}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}+$$
$$i\int\frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \; 3791$$

$$\frac{i\left(\frac{3f^2\left(\frac{1}{2}\int(e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx}{8f}\right.}{a}$$
$$i\int\frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \; 17$$

$$i\int\frac{(e+fx)^3\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx+$$
$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \; 6091$$

$$i\left(i\int\frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int(e+fx)^3\sinh(c+dx)dx}{a}\right)+$$
$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$i\left(i\int\frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -i(e+fx)^3\sin(ic+idx)dx}{a}\right)+$$
$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \; 26$$

3.199. $\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\int (e+fx)^3\sin(ic+idx)dx}{a}\right)+ i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\cosh(c+dx)dx}{d}}{a}\right)+ i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}\right)+ i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}}{a}\right)+ i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}\right)}{d}}{a}\right)+ i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

3.199. $\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}}{a}\right)+i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}\right)}{d}}{a}\right)+i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}\right)}{d}}{a}\right)+i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}}{a}\right)+i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

3.199. $\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3117}$$

$$\frac{i\left(i\int \frac{(e+fx)^3\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}\right)+i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \text{6091}$$

$$\frac{i\left(i\left(i\int \frac{(e+fx)^3}{i\sinh(c+dx)a+a}dx-\frac{i\int(e+fx)^3dx}{a}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\cdots\right.}{d}\cdots\right.}{d}}{a}\cdots\right.+i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \text{17}$$

$$\frac{i\left(i\left(i\int \frac{(e+fx)^3}{i\sinh(c+dx)a+a}dx-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sin\cdots}{\cdots}\right.}{d}\cdots\right.}{d}}{a}\cdots\right.+i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

3.199. $\displaystyle\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{i\left(i\left(i\int \frac{(e+fx)^3}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh}{d}\right.}{d}\right.}{d}}{a}\right.}{}$$

$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$\downarrow$ 3799

$$\frac{i\left(i\left(\frac{i\int -(e+fx)^3\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cos}{d}\right.}{}\right.}{d}}{a}\right.}{}$$

$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$\downarrow$ 25

$$\frac{i\left(i\left(-\frac{i\int -(e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)}{}\right.}{}\right.}{d}}{a}\right.}{}$$

$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$\downarrow$ 25

3.199. $\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{i\left(i\left(\frac{i\int(e+fx)^3\mathrm{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c}{d}\right.}{}\right.}{d}}{a}\right.}{}$$

$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(i\left(\frac{i\int(e+fx)^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c}{d}\right.}{}\right.}{d}}{a}\right.}{}$$

$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{i\left(i\left(\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6if\int-i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^4}{4af}\right)-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left((e+f\right.}{}}{}\right.}{}$$

$$\frac{i\left(\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}\right)}{a}$$

input
```
Int[((e + f*x)^3*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.199. $\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.199.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.199. $\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

3.199.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1005 vs. 2(354) = 708.

Time = 2.53 (sec) , antiderivative size = 1006, normalized size of antiderivative = 2.56

method	result
risch	$-\frac{6e^2 f \arctan(\mathrm{e}^{dx+c})}{a\,d^2}-\frac{6c^2 f^3 \arctan(\mathrm{e}^{dx+c})}{a\,d^4}+\frac{3if^3x^4}{8a}+\frac{3ie^4}{8af}+\frac{3ie^3x}{2a}+\frac{12ic\,f^2 e\ln(\mathrm{e}^{dx+c})}{a\,d^3}-\frac{6ic\,f^2 e\ln(1+\mathrm{e}^{2dx+2c})}{a\,d^3}-$

input
```
int((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(d*x+c)-I)-6/a/d^2*e^2*f*arc
tan(exp(d*x+c))-6/a/d^4*c^2*f^3*arctan(exp(d*x+c))+12*I/a/d^3*e*f^2*ln(1+I
*exp(d*x+c))*c-6*I/a/d^3*c*e*f^2*ln(1+exp(2*d*x+2*c))+12*I/a/d^3*c*e*f^2*l
n(exp(d*x+c))-12*I/a/d^2*e*f^2*c*x+12*I/a/d^2*e*f^2*ln(1+I*exp(d*x+c))*x+1
/2*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x+3*d^2*f^3*x^2+d^3*e^3+6*d^2*
e*f^2*x+3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+6*f^3)/d^4/a*exp(-d*x-c)+12/a/d^3*
c*f^2*e*arctan(exp(d*x+c))-12*I*f^3*polylog(3,-I*exp(d*x+c))/a/d^4+1/32*I*
(4*d^3*f^3*x^3+12*d^3*e*f^2*x^2+12*d^3*e^2*f*x+6*d^2*f^3*x^2+4*d^3*e^3+12*
d^2*e*f^2*x+6*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+3*f^3)/d^4/a*exp(-2*d*x-2*c)-1
/32*I*(4*d^3*f^3*x^3+12*d^3*e*f^2*x^2+12*d^3*e^2*f*x-6*d^2*f^3*x^2+4*d^3*e
^3-12*d^2*e*f^2*x-6*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-3*f^3)/d^4/a*exp(2*d*x+2
*c)+4*I/a/d^4*f^3*c^3+3/8*I/a*f^3*x^4+3/8*I/a/f*e^4-6*I/a/d*e*f^2*x^2+6*I/
a/d^3*f^3*x*c^2+6*I/a/d^2*f^3*ln(1+I*exp(d*x+c))*x^2+3*I/a/d^4*c^2*f^3*ln(
1+exp(2*d*x+2*c))+1/2*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x-3*d^2*f^3
*x^2+d^3*e^3-6*d^2*e*f^2*x-3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-6*f^3)/d^4/a*ex
p(d*x+c)+3/2*I/a*e^3*x+3/2*I/a*f^2*e*x^3+9/4*I/a*f*e^2*x^2-6*I/a/d^3*e*f^2
*c^2-6*I/a/d^4*f^3*ln(1+I*exp(d*x+c))*c^2-6*I/a/d^4*c^2*f^3*ln(exp(d*x+c))
-6*I/a/d^2*e^2*f*ln(exp(d*x+c))+12*I/a/d^3*e*f^2*polylog(2,-I*exp(d*x+c))+
12*I/a/d^3*f^3*polylog(2,-I*exp(d*x+c))*x+3*I/a/d^2*e^2*f*ln(1+exp(2*d*x+2
*c))-2*I/a/d*f^3*x^3
```

3.199.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1044 vs. $2(337) = 674$.

Time = 0.31 (sec) , antiderivative size = 1044, normalized size of antiderivative = 2.66

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/32*(4*d^3*f^3*x^3 + 4*d^3*e^3 + 6*d^2*e^2*f + 6*d*e*f^2 + 3*f^3 + 6*(2*d
^3*e*f^2 + d^2*f^3)*x^2 + 6*(2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*x - 384*((
-I*d*f^3*x - I*d*e*f^2)*e^(3*d*x + 3*c) - (d*f^3*x + d*e*f^2)*e^(2*d*x + 2
*c))*dilog(-I*e^(d*x + c)) + (-4*I*d^3*f^3*x^3 - 4*I*d^3*e^3 + 6*I*d^2*e^2
*f - 6*I*d*e*f^2 + 3*I*f^3 - 6*(2*I*d^3*e*f^2 - I*d^2*f^3)*x^2 - 6*(2*I*d^
3*e^2*f - 2*I*d^2*e*f^2 + I*d*f^3)*x)*e^(5*d*x + 5*c) + 3*(4*d^3*f^3*x^3 +
 4*d^3*e^3 - 14*d^2*e^2*f + 30*d*e*f^2 - 31*f^3 + 2*(6*d^3*e*f^2 - 7*d^2*f
^3)*x^2 + 2*(6*d^3*e^2*f - 14*d^2*e*f^2 + 15*d*f^3)*x)*e^(4*d*x + 4*c) - 4
*(-3*I*d^4*f^3*x^4 + 4*I*d^3*e^3 + 12*(4*I*c - I)*d^2*e^2*f + 24*(-2*I*c^2
 + I)*d*e*f^2 + 8*(2*I*c^3 - 3*I)*f^3 + 4*(-3*I*d^4*e*f^2 + 5*I*d^3*f^3)*x
^3 + 6*(-3*I*d^4*e^2*f + 10*I*d^3*e*f^2 - 2*I*d^2*f^3)*x^2 + 12*(-I*d^4*e^
3 + 5*I*d^3*e^2*f - 2*I*d^2*e*f^2 + 2*I*d*f^3)*x)*e^(3*d*x + 3*c) + 4*(3*d
^4*f^3*x^4 + 20*d^3*e^3 - 12*(4*c - 1)*d^2*e^2*f + 24*(2*c^2 + 1)*d*e*f^2
- 8*(2*c^3 - 3)*f^3 + 4*(3*d^4*e*f^2 + d^3*f^3)*x^3 + 6*(3*d^4*e^2*f + 2*d
^3*e*f^2 + 2*d^2*f^3)*x^2 + 12*(d^4*e^3 + d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^
3)*x)*e^(2*d*x + 2*c) - 3*(4*I*d^3*f^3*x^3 + 4*I*d^3*e^3 + 14*I*d^2*e^2*f
+ 30*I*d*e*f^2 + 31*I*f^3 + 2*(6*I*d^3*e*f^2 + 7*I*d^2*f^3)*x^2 + 2*(6*I*d
^3*e^2*f + 14*I*d^2*e*f^2 + 15*I*d*f^3)*x)*e^(d*x + c) - 192*((-I*d^2*e^2*
f + 2*I*c*d*e*f^2 - I*c^2*f^3)*e^(3*d*x + 3*c) - (d^2*e^2*f - 2*c*d*e*f^2
+ c^2*f^3)*e^(2*d*x + 2*c))*log(e^(d*x + c) - I) - 192*((-I*d^2*f^3*x^2...
```

3.199.6 Sympy [F]

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{2e^3+6e^2fx+6ef^2x^2+2f^3x^3}{ade^ce^{dx}-iad} - \frac{i\left(\int\left(-\frac{ide^3}{e^ce^{3dx}-ie^{2dx}}\right)dx+\int\left(-\frac{idf^3x^3}{e^ce^{3dx}-ie^{2dx}}\right)dx+\int\left(-\frac{de^3e^ce^{dx}}{e^ce^{3dx}-ie^{2dx}}\right)dx+\int\left(-\frac{4de^3e^{3c}e^{3dx}}{e^ce^{3dx}-ie^{2dx}}\right)dx+\int\frac{de^3e\ldots}{e^ce^{3d\ldots}}\right.}{\ldots}$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(2*e**3 + 6*e**2*f*x + 6*e*f**2*x**2 + 2*f**3*x**3)/(a*d*exp(c)*exp(d*x) -
 I*a*d) - I*(Integral(-I*d*e**3/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + I
ntegral(-I*d*f**3*x**3/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-
d*e**3*exp(c)*exp(d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-
4*d*e**3*exp(3*c)*exp(3*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Inte
gral(d*e**3*exp(5*c)*exp(5*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + I
ntegral(-3*I*d*e*f**2*x**2/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integr
al(-3*I*d*e**2*f*x/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(4*I*d
*e**3*exp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integra
l(I*d*e**3*exp(4*c)*exp(4*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + In
tegral(-24*I*e**2*f*exp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x))
, x) + Integral(-24*I*f**3*x**2*exp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I
*exp(2*d*x)), x) + Integral(-d*f**3*x**3*exp(c)*exp(d*x)/(exp(c)*exp(3*d*x
) - I*exp(2*d*x)), x) + Integral(-4*d*f**3*x**3*exp(3*c)*exp(3*d*x)/(exp(c
)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(d*f**3*x**3*exp(5*c)*exp(5*d*x
)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(4*I*d*f**3*x**3*exp(2*
c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(I*d*f**3*x
**3*exp(4*c)*exp(4*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(
-48*I*e*f**2*x*exp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x)
+ Integral(-3*d*e*f**2*x**2*exp(c)*exp(d*x)/(exp(c)*exp(3*d*x) - I*exp(...
```

3.199.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+ia\sinh(c+dx)} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.199.8 Giac [F]

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\sinh{(dx+c)}^3}{i\,a\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sinh(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{\sinh(c+d\,x)^3\,(e+f\,x)^3}{a+a\sinh{(c+d\,x)}\ 1\mathrm{i}}\,dx$$

input
```
int((sinh(c + d*x)^3*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((sinh(c + d*x)^3*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i), x)
```

3.200 $\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.200.1 Optimal result

Integrand size = 31, antiderivative size = 287

$$\begin{aligned}\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{if^2x}{4ad^2}-\frac{i(e+fx)^2}{ad}+\frac{i(e+fx)^3}{2af}\\ &\quad+\frac{2f^2\cosh(c+dx)}{ad^3}+\frac{(e+fx)^2\cosh(c+dx)}{ad}\\ &\quad+\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{4if^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}\\ &\quad-\frac{2f(e+fx)\sinh(c+dx)}{ad^2}-\frac{if^2\cosh(c+dx)\sinh(c+dx)}{4ad^3}\\ &\quad-\frac{i(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{2ad}\\ &\quad+\frac{if(e+fx)\sinh^2(c+dx)}{2ad^2}-\frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\end{aligned}$$

output
```
1/4*I*f^2*x/a/d^2-I*(f*x+e)^2/a/d+1/2*I*(f*x+e)^3/a/f+2*f^2*cosh(d*x+c)/a/
d^3+(f*x+e)^2*cosh(d*x+c)/a/d+4*I*f*(f*x+e)*ln(1+I*exp(d*x+c))/a/d^2+4*I*f
^2*polylog(2,-I*exp(d*x+c))/a/d^3-2*f*(f*x+e)*sinh(d*x+c)/a/d^2-1/4*I*f^2*
cosh(d*x+c)*sinh(d*x+c)/a/d^3-1/2*I*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/a/d+
1/2*I*f*(f*x+e)*sinh(d*x+c)^2/a/d^2-I*(f*x+e)^2*tanh(1/2*c+1/4*I*Pi+1/2*d*
x)/a/d
```

3.200.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1661 vs. 2(287) = 574.

Time = 3.72 (sec) , antiderivative size = 1661, normalized size of antiderivative = 5.79

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-6*I)*d^2*e^2*E^c*Cosh[(3*d*x)/2] + 6*d^2*e^2*E^(4*c)*Cosh[(3*d*x)/2] -
(14*I)*d*e*E^c*f*Cosh[(3*d*x)/2] - 14*d*e*E^(4*c)*f*Cosh[(3*d*x)/2] - (15*
I)*E^c*f^2*Cosh[(3*d*x)/2] + 15*E^(4*c)*f^2*Cosh[(3*d*x)/2] - (12*I)*d^2*e
*E^c*f*x*Cosh[(3*d*x)/2] + 12*d^2*e*E^(4*c)*f*x*Cosh[(3*d*x)/2] - (14*I)*d
*E^c*f^2*x*Cosh[(3*d*x)/2] - 14*d*E^(4*c)*f^2*x*Cosh[(3*d*x)/2] - (6*I)*d^
2*E^c*f^2*x^2*Cosh[(3*d*x)/2] + 6*d^2*E^(4*c)*f^2*x^2*Cosh[(3*d*x)/2] + 2*
d^2*e^2*Cosh[(5*d*x)/2] - (2*I)*d^2*e^2*E^(5*c)*Cosh[(5*d*x)/2] + 2*d*e*f*
Cosh[(5*d*x)/2] + (2*I)*d*e*E^(5*c)*f*Cosh[(5*d*x)/2] + f^2*Cosh[(5*d*x)/2
] - I*E^(5*c)*f^2*Cosh[(5*d*x)/2] + 4*d^2*e*f*x*Cosh[(5*d*x)/2] - (4*I)*d^
2*e*E^(5*c)*f*x*Cosh[(5*d*x)/2] + 2*d*f^2*x*Cosh[(5*d*x)/2] + (2*I)*d*E^(5
*c)*f^2*x*Cosh[(5*d*x)/2] + 2*d^2*f^2*x^2*Cosh[(5*d*x)/2] - (2*I)*d^2*E^(5
*c)*f^2*x^2*Cosh[(5*d*x)/2] + 8*E^(2*c)*Cosh[(d*x)/2]*(2*(1 - I*E^c)*f^2 +
 2*d*(1 + I*E^c)*f*(e + f*x) + d^2*(5 - I*E^c)*(e + f*x)^2 + d^3*(1 + I*E^
c)*x*(3*e^2 + 3*e*f*x + f^2*x^2) + 8*d*(1 + I*E^c)*f*(e + f*x)*Log[1 - I*E
^(-c - d*x)]) - 40*d^2*e^2*E^(2*c)*Sinh[(d*x)/2] - (8*I)*d^2*e^2*E^(3*c)*S
inh[(d*x)/2] - 16*d*e*E^(2*c)*f*Sinh[(d*x)/2] + (16*I)*d*e*E^(3*c)*f*Sinh[
(d*x)/2] - 16*E^(2*c)*f^2*Sinh[(d*x)/2] - (16*I)*E^(3*c)*f^2*Sinh[(d*x)/2]
 - 24*d^3*e^2*E^(2*c)*x*Sinh[(d*x)/2] + (24*I)*d^3*e^2*E^(3*c)*x*Sinh[(d*x
)/2] - 80*d^2*e*E^(2*c)*f*x*Sinh[(d*x)/2] - (16*I)*d^2*e*E^(3*c)*f*x*Sinh[
(d*x)/2] - 16*d*E^(2*c)*f^2*x*Sinh[(d*x)/2] + (16*I)*d*E^(3*c)*f^2*x*Si...
```

3.200.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow 6091$$

$$i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^2\sinh^2(c+dx)dx}{a}$$

$$\Big\downarrow 3042$$

$$i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -(e+fx)^2\sin(ic+idx)^2dx}{a}$$

$$\Big\downarrow 25$$

$$\frac{i\int (e+fx)^2\sin(ic+idx)^2dx}{a}+i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow 3792$$

$$\frac{i\left(\frac{f^2\int -\sinh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^2dx+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{a}+i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow 17$$

$$\frac{i\left(\frac{f^2\int -\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}+i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow 25$$

$$\frac{i\left(-\frac{f^2\int \sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}+i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow 3042$$

$$\frac{i\left(-\frac{f^2\int -\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}+i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow 25$$

$$\frac{i\left(\frac{f^2\int \sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}+i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

3.200. $\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 3115$$

$$\frac{i\left(\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}+$$

$$i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \; 24$$

$$i\int \frac{(e+fx)^2\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx+$$

$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \; 6091$$

$$i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^2\sinh(c+dx)dx}{a}\right)+$$

$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -i(e+fx)^2\sin(ic+idx)dx}{a}\right)+$$

$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\int (e+fx)^2\sin(ic+idx)dx}{a}\right)+$$

$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \; 3777$$

$$i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}}{a}\right)+$$

$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

3.200. $\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\downarrow \textbf{3042}$$

$$\frac{i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}\right)+i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\downarrow \textbf{3777}$$

$$\frac{i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}}{a}\right)+i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\downarrow \textbf{26}$$

$$\frac{i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}}{a}\right)+i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\downarrow \textbf{3042}$$

$$\frac{i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}}{a}\right)+i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\downarrow \textbf{26}$$

$$\frac{i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}}{a}\right)+i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\downarrow \textbf{3118}$$

3.200. $\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$i\left(i\int \frac{(e+fx)^2\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}\right)+\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{6091}$$

$$i\left(i\left(i\int \frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)^2dx}{a}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}\right)+\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{17}$$

$$i\left(i\left(i\int \frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}\right)+\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$i\left(i\left(i\int \frac{(e+fx)^2}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}\right)+\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{3799}$$

$$i\left(i\left(\frac{i\int -(e+fx)^2\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}\right)+\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

3.200. $\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 25$$

$$i\left(i\left(-\frac{i\int -(e+fx)^2\operatorname{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}\right.$$
$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \; 25$$

$$i\left(i\left(\frac{i\int (e+fx)^2\operatorname{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}\right)+$$
$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$i\left(i\left(\frac{i\int (e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}\right)+$$
$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \; 4672$$

$$i\left(i\left(\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4if\int -i(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)}{}\right.}{}}{a}\right.$$
$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \; 26$$

3.200. $\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$i\left(i\left(\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\int (e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-2if\left(\frac{(e+fx)\sinh}{d}\right.}{a}\right.$$

$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$i\left(i\left(\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\int -i(e+fx)\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^3}{3af}\right)-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-2if\left(\frac{(e+fx)\,\mathrm{s}}{}\right.}{a}\right.$$

$$\frac{i\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{a}$$

input
```
Int[((e + f*x)^2*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.200.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

3.200. $\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.200. $\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

3.200.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(257) = 514$.

Time = 2.42 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.95

method	result
risch	$-\frac{2if^2x^2}{ad}-\frac{4if^2cx}{a\,d^2}+\frac{i(2d^2x^2f^2+4d^2efx+2d^2e^2+2xd\,f^2+2def+f^2)\mathrm{e}^{-2dx-2c}}{16d^3a}+\frac{2ief\ln\left(1+\mathrm{e}^{2dx+2c}\right)}{a\,d^2}-\frac{2ic\,f^2\ln\left(1+\mathrm{e}^{2dx+2}\right.}{a\,d^3}$

input
```
int((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*I/a/d*f^2*x^2-4*I/a/d^2*f^2*c*x+1/16*I*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2
*e^2+2*d*f^2*x+2*d*e*f+f^2)/d^3/a*exp(-2*d*x-2*c)+2*I/a/d^2*e*f*ln(1+exp(2
*d*x+2*c))-2*I/a/d^3*c*f^2*ln(1+exp(2*d*x+2*c))+1/2*(d^2*f^2*x^2+2*d^2*e*f
*x+d^2*e^2-2*d*f^2*x-2*d*e*f+2*f^2)/d^3/a*exp(d*x+c)+1/2*(d^2*f^2*x^2+2*d^
2*e*f*x+d^2*e^2+2*d*f^2*x+2*d*e*f+2*f^2)/d^3/a*exp(-d*x-c)+4*I/a/d^2*f^2*l
n(1+I*exp(d*x+c))*x+2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(d*x+c)-I)+4*I/a/d^3*c
*f^2*ln(exp(d*x+c))-4/a/d^2*e*f*arctan(exp(d*x+c))+4*I*f^2*polylog(2,-I*ex
p(d*x+c))/a/d^3+4*I/a/d^3*f^2*ln(1+I*exp(d*x+c))*c+3/2*I/a*f*e*x^2+1/2*I/a
/f*e^3-1/16*I*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2-2*d*f^2*x-2*d*e*f+f^2)/
d^3/a*exp(2*d*x+2*c)-2*I/a/d^3*f^2*c^2+3/2*I/a*e^2*x-4*I/a/d^2*e*f*ln(exp(
d*x+c))+4/a/d^3*c*f^2*arctan(exp(d*x+c))+1/2*I/a*f^2*x^3
```

3.200.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(244) = 488.

Time = 0.26 (sec) , antiderivative size = 595, normalized size of antiderivative = 2.07

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{2\,d^2f^2x^2+2\,d^2e^2+2\,def+f^2+2\,(2\,d^2ef+df^2)x-64\left(-i\,f^2e^{(3\,dx+3\,c)}-f^2e^{(2\,dx+2\,c)}\right)\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)+(}{}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/16*(2*d^2*f^2*x^2 + 2*d^2*e^2 + 2*d*e*f + f^2 + 2*(2*d^2*e*f + d*f^2)*x
- 64*(-I*f^2*e^(3*d*x + 3*c) - f^2*e^(2*d*x + 2*c))*dilog(-I*e^(d*x + c))
+ (-2*I*d^2*f^2*x^2 - 2*I*d^2*e^2 + 2*I*d*e*f - I*f^2 - 2*(2*I*d^2*e*f - I
*d*f^2)*x)*e^(5*d*x + 5*c) + (6*d^2*f^2*x^2 + 6*d^2*e^2 - 14*d*e*f + 15*f^
2 + 2*(6*d^2*e*f - 7*d*f^2)*x)*e^(4*d*x + 4*c) - 8*(-I*d^3*f^2*x^3 + I*d^2
*e^2 + 2*(4*I*c - I)*d*e*f + 2*(-2*I*c^2 + I)*f^2 + (-3*I*d^3*e*f + 5*I*d^
2*f^2)*x^2 + (-3*I*d^3*e^2 + 10*I*d^2*e*f - 2*I*d*f^2)*x)*e^(3*d*x + 3*c)
+ 8*(d^3*f^2*x^3 + 5*d^2*e^2 - 2*(4*c - 1)*d*e*f + 2*(2*c^2 + 1)*f^2 + (3*
d^3*e*f + d^2*f^2)*x^2 + (3*d^3*e^2 + 2*d^2*e*f + 2*d*f^2)*x)*e^(2*d*x + 2
*c) + (-6*I*d^2*f^2*x^2 - 6*I*d^2*e^2 - 14*I*d*e*f - 15*I*f^2 - 2*(6*I*d^2
*e*f + 7*I*d*f^2)*x)*e^(d*x + c) - 64*((-I*d*e*f + I*c*f^2)*e^(3*d*x + 3*c
) - (d*e*f - c*f^2)*e^(2*d*x + 2*c))*log(e^(d*x + c) - I) - 64*((-I*d*f^2*
x - I*c*f^2)*e^(3*d*x + 3*c) - (d*f^2*x + c*f^2)*e^(2*d*x + 2*c))*log(I*e^
(d*x + c) + 1))/(a*d^3*e^(3*d*x + 3*c) - I*a*d^3*e^(2*d*x + 2*c))
```

3.200.6 Sympy [F]

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{2e^2+4efx+2f^2x^2}{ade^ce^{dx}-iad}$$
$$-\frac{i\left(\int\left(-\frac{ide^2}{e^ce^{3dx}-ie^{2dx}}\right)dx+\int\left(-\frac{idf^2x^2}{e^ce^{3dx}-ie^{2dx}}\right)dx+\int\left(-\frac{de^2e^ce^{dx}}{e^ce^{3dx}-ie^{2dx}}\right)dx+\int\left(-\frac{4de^2e^{3c}e^{3dx}}{e^ce^{3dx}-ie^{2dx}}\right)dx+\int\frac{de^2e}{e^ce^{3dx}}\right.}{}$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(2*e**2 + 4*e*f*x + 2*f**2*x**2)/(a*d*exp(c)*exp(d*x) - I*a*d) - I*(Integr
al(-I*d*e**2/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-I*d*f**2*x
**2/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-d*e**2*exp(c)*exp(d
*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-4*d*e**2*exp(3*c)*e
xp(3*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(d*e**2*exp(5*c
)*exp(5*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-2*I*d*e*f*
x/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(4*I*d*e**2*exp(2*c)*ex
p(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(I*d*e**2*exp(4*
c)*exp(4*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-16*I*e*f*
exp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-16*
I*f**2*x*exp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Inte
gral(-d*f**2*x**2*exp(c)*exp(d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) +
 Integral(-4*d*f**2*x**2*exp(3*c)*exp(3*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*
d*x)), x) + Integral(d*f**2*x**2*exp(5*c)*exp(5*d*x)/(exp(c)*exp(3*d*x) -
I*exp(2*d*x)), x) + Integral(4*I*d*f**2*x**2*exp(2*c)*exp(2*d*x)/(exp(c)*e
xp(3*d*x) - I*exp(2*d*x)), x) + Integral(I*d*f**2*x**2*exp(4*c)*exp(4*d*x)
/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-2*d*e*f*x*exp(c)*exp(d
*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-8*d*e*f*x*exp(3*c)*
exp(3*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(2*d*e*f*x*exp
(5*c)*exp(5*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(8*I*...
```

3.200.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.200.8 Giac [F]

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\sinh\left(dx+c\right)^3}{i\,a\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sinh(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)^3\,\left(e+f\,x\right)^2}{a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((sinh(c + d*x)^3*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((sinh(c + d*x)^3*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i), x)
```

3.201 $\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.201.1 Optimal result

Integrand size = 29, antiderivative size = 175

$$\begin{aligned}\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{3iex}{2a}+\frac{3ifx^2}{4a}+\frac{(e+fx)\cosh(c+dx)}{ad}\\&\quad+\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}-\frac{f\sinh(c+dx)}{ad^2}\\&\quad-\frac{i(e+fx)\cosh(c+dx)\sinh(c+dx)}{2ad}\\&\quad+\frac{if\sinh^2(c+dx)}{4ad^2}-\frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\end{aligned}$$

output
```
3/2*I*e*x/a+3/4*I*f*x^2/a+(f*x+e)*cosh(d*x+c)/a/d+2*I*f*ln(cosh(1/2*c+1/4*
I*Pi+1/2*d*x))/a/d^2-f*sinh(d*x+c)/a/d^2-1/2*I*(f*x+e)*cosh(d*x+c)*sinh(d*
x+c)/a/d+1/4*I*f*sinh(d*x+c)^2/a/d^2-I*(f*x+e)*tanh(1/2*c+1/4*I*Pi+1/2*d*x
)/a/d
```

3.201.2 Mathematica [A] (verified)

Time = 3.20 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.86

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)\left(-8id(e+fx)\cosh(c+dx)+f\cosh(2(c+dx\right.\right.}{}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(Cosh[(c + d*x)/2]*((-8*I)*d*(e
 + f*x)*Cosh[c + d*x] + f*Cosh[2*(c + d*x)] + 2*(6*c*d*e - (4*I)*c*f - 3*c
^2*f + 6*d^2*e*x - (4*I)*d*f*x + 3*d^2*f*x^2 + (8*I)*f*ArcTan[Tanh[(c + d*
x)/2]] + 4*f*Log[Cosh[c + d*x]] + (4*I)*f*Sinh[c + d*x] - d*(e + f*x)*Sinh
[2*(c + d*x)])) + Sinh[(c + d*x)/2]*(8*d*(e + f*x)*Cosh[c + d*x] + I*(f*Co
sh[2*(c + d*x)] + 2*((8*I)*d*e + 6*c*d*e - (4*I)*c*f - 3*c^2*f + 6*d^2*e*x
 + (4*I)*d*f*x + 3*d^2*f*x^2 + (8*I)*f*ArcTan[Tanh[(c + d*x)/2]] + 4*f*Log
[Cosh[c + d*x]] + (4*I)*f*Sinh[c + d*x] - d*(e + f*x)*Sinh[2*(c + d*x)])))
))/(8*a*d^2*(-I + Sinh[c + d*x]))
```

3.201.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.793$, Rules used = {6091, 3042, 25, 3791, 17, 6091, 3042, 26, 3777, 3042, 3117, 6091, 17, 3042, 3799, 25, 25, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{6091}$$
$$i\int \frac{(e+fx)\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)\sinh^2(c+dx)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$i\int \frac{(e+fx)\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -\left((e+fx)\sin(ic+idx)^2\right)dx}{a}$$

$$\Big\downarrow \ 25$$

$$\frac{i\int (e+fx)\sin(ic+idx)^2dx}{a}+i\int \frac{(e+fx)\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \ 3791$$

$$\frac{i\left(\frac{1}{2}\int (e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{a}+i\int \frac{(e+fx)\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \ 17$$

$$i\int \frac{(e+fx)\sinh^2(c+dx)}{i\sinh(c+dx)a+a}dx+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \ 6091$$

$$i\left(i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int (e+fx)\sinh(c+dx)dx}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \ 3042$$

$$i\left(i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{i\int -i(e+fx)\sin(ic+idx)dx}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \ 26$$

$$i\left(i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\int (e+fx)\sin(ic+idx)dx}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \ 3777$$

$$i\left(i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \ 3042$$

3.201. $\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$i\left(\frac{i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}}{}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \text{3117}$$

$$i\left(i\int \frac{(e+fx)\sinh(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \text{6091}$$

$$i\left(i\left(i\int \frac{e+fx}{i\sinh(c+dx)a+a}dx-\frac{i\int(e+fx)dx}{a}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \text{17}$$

$$i\left(i\left(i\int \frac{e+fx}{i\sinh(c+dx)a+a}dx-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$i\left(i\left(i\int \frac{e+fx}{\sin(ic+idx)a+a}dx-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \text{3799}$$

$$i\left(i\left(\frac{i\int -\left((e+fx)\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)\right)dx}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \text{25}$$

3.201. $\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$i\left(i\left(-\frac{i\int -\left((e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)dx}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \; 25$$

$$i\left(i\left(\frac{i\int (e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$i\left(i\left(\frac{i\int (e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \; 4672$$

$$i\left(i\left(\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2if\int -i\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$i\left(i\left(\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int \tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \; 3042$$

3.201. $\displaystyle\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$i\left(i\left(\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int -i\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$i\left(i\left(\frac{i\left(\frac{2if\int\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)+\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}$$

$$\Big\downarrow \text{3956}$$

$$\frac{i\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{a}+i\left(i\left(\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{d^2}\right)}{2a}-\frac{i(e+fx)^2}{2af}\right)-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}\right)$$

input
```
Int[((e + f*x)*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(I*((e + f*x)^2/(4*f) - ((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (f
*Sinh[c + d*x]^2)/(4*d^2)))/a + I*(-(((I*(e + f*x)*Cosh[c + d*x])/d - (I*f
*Sinh[c + d*x])/d^2)/a) + I*(((-1/2*I)*(e + f*x)^2)/(a*f) + ((I/2)*((-4*f*
Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2]])/d^2 + (2*(e + f*x)*Tanh[c/2 + (I/4)*P
i + (d*x)/2])/d))/a))
```

3.201. $\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.201.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.201. $\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

3.201.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13

method	result
risch	$\frac{3if\,x^2}{4a}+\frac{3iex}{2a}-\frac{i(2dfx+2de-f)\mathrm{e}^{2dx+2c}}{16a\,d^2}+\frac{(dfx+de-f)\mathrm{e}^{dx+c}}{2a\,d^2}+\frac{(dfx+de+f)\mathrm{e}^{-dx-c}}{2a\,d^2}+\frac{i(2dfx+2de+f)\mathrm{e}^{-2dx-2c}}{16a\,d^2}-$
parallelrisch	$32f\left(i\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)+\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(1-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-32f\left(i\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)+\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\Big(($

input
```
int((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3/4*I*f*x^2/a+3/2*I*e*x/a-1/16*I*(2*d*f*x+2*d*e-f)/a/d^2*exp(2*d*x+2*c)+1/
2*(d*f*x+d*e-f)/a/d^2*exp(d*x+c)+1/2*(d*f*x+d*e+f)/a/d^2*exp(-d*x-c)+1/16*
I*(2*d*f*x+2*d*e+f)/a/d^2*exp(-2*d*x-2*c)-2*I*f/a/d*x-2*I*f/a/d^2*c+2*(f*x
+e)/d/a/(exp(d*x+c)-I)+2*I*f/a/d^2*ln(exp(d*x+c)-I)
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.31

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{2\,dfx+2\,de+(-2i\,dfx-2i\,de+i\,f)e^{(5\,dx+5\,c)}+(6\,dfx+6\,de-7\,f)e^{(4\,dx+4\,c)}-4\,(-3i\,d^2fx^2+2i\,de+2}{}$$

input
```
integrate((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*(2*d*f*x + 2*d*e + (-2*I*d*f*x - 2*I*d*e + I*f)*e^(5*d*x + 5*c) + (6*
d*f*x + 6*d*e - 7*f)*e^(4*d*x + 4*c) - 4*(-3*I*d^2*f*x^2 + 2*I*d*e + 2*(-3
*I*d^2*e + 5*I*d*f)*x - 2*I*f)*e^(3*d*x + 3*c) + 4*(3*d^2*f*x^2 + 10*d*e +
 2*(3*d^2*e + d*f)*x + 2*f)*e^(2*d*x + 2*c) + (-6*I*d*f*x - 6*I*d*e - 7*I*
f)*e^(d*x + c) - 32*(-I*f*e^(3*d*x + 3*c) - f*e^(2*d*x + 2*c))*log(e^(d*x
+ c) - I) + f)/(a*d^2*e^(3*d*x + 3*c) - I*a*d^2*e^(2*d*x + 2*c))
```

3.201.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.26

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{2e+2fx}{ade^ce^{dx}-iad}$$
$$+\begin{cases} \frac{((512a^3d^7ee^{2c}+512a^3d^7fxe^{2c}+512a^3d^6fe^{2c})e^{-dx}+(512a^3d^7ee^{4c}+512a^3d^7fxe^{4c}-512a^3d^6fe^{4c})e^{dx}+(128ia^3d^7ee^c+128ia^3d^7fxe^c+64i}{1024a^4d^8} \\ \frac{x^2(-ife^{4c}+2fe^{3c}-2fe^c-if)e^{-2c}}{8a}+\frac{x(-iee^{4c}+2ee^{3c}-2ee^c-ie)e^{-2c}}{4a} \end{cases}$$
$$+\frac{3ifx^2}{4a}+\frac{x(3ide-4if)}{2ad}+\frac{2if\log\left(e^{dx}-ie^{-c}\right)}{ad^2}$$

input
```
integrate((f*x+e)*sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
(2*e + 2*f*x)/(a*d*exp(c)*exp(d*x) - I*a*d) + Piecewise((((512*a**3*d**7*e
*exp(2*c) + 512*a**3*d**7*f*x*exp(2*c) + 512*a**3*d**6*f*exp(2*c))*exp(-d*
x) + (512*a**3*d**7*e*exp(4*c) + 512*a**3*d**7*f*x*exp(4*c) - 512*a**3*d**
6*f*exp(4*c))*exp(d*x) + (128*I*a**3*d**7*e*exp(c) + 128*I*a**3*d**7*f*x*e
xp(c) + 64*I*a**3*d**6*f*exp(c))*exp(-2*d*x) + (-128*I*a**3*d**7*e*exp(5*c
) - 128*I*a**3*d**7*f*x*exp(5*c) + 64*I*a**3*d**6*f*exp(5*c))*exp(2*d*x))*
exp(-3*c)/(1024*a**4*d**8), Ne(a**4*d**8*exp(3*c), 0)), (x**2*(-I*f*exp(4*
c) + 2*f*exp(3*c) - 2*f*exp(c) - I*f)*exp(-2*c)/(8*a) + x*(-I*e*exp(4*c) +
 2*e*exp(3*c) - 2*e*exp(c) - I*e)*exp(-2*c)/(4*a), True)) + 3*I*f*x**2/(4*
a) + x*(3*I*d*e - 4*I*f)/(2*a*d) + 2*I*f*log(exp(d*x) - I*exp(-c))/(a*d**2
)
```

3.201.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.201.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(145) = 290.

Time = 0.28 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.95

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{12i\,d^2fx^2e^{(3\,dx+3\,c)} + 12\,d^2fx^2e^{(2\,dx+2\,c)} + 24i\,d^2exe^{(3\,dx+3\,c)} + 24\,d^2exe^{(2\,dx+2\,c)} - 2i\,dfxe^{(5\,dx+5\,c)} + 6\,dfxe^{(4}}{}$$

input
```
integrate((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/16*(12*I*d^2*f*x^2*e^(3*d*x + 3*c) + 12*d^2*f*x^2*e^(2*d*x + 2*c) + 24*I
*d^2*e*x*e^(3*d*x + 3*c) + 24*d^2*e*x*e^(2*d*x + 2*c) - 2*I*d*f*x*e^(5*d*x
 + 5*c) + 6*d*f*x*e^(4*d*x + 4*c) - 40*I*d*f*x*e^(3*d*x + 3*c) + 8*d*f*x*e
^(2*d*x + 2*c) - 6*I*d*f*x*e^(d*x + c) + 2*d*f*x - 2*I*d*e*e^(5*d*x + 5*c)
 + 6*d*e*e^(4*d*x + 4*c) - 8*I*d*e*e^(3*d*x + 3*c) + 40*d*e*e^(2*d*x + 2*c
) - 6*I*d*e*e^(d*x + c) + 32*I*f*e^(3*d*x + 3*c)*log(e^(d*x + c) - I) + 32
*f*e^(2*d*x + 2*c)*log(e^(d*x + c) - I) + 2*d*e + I*f*e^(5*d*x + 5*c) - 7*
f*e^(4*d*x + 4*c) + 8*I*f*e^(3*d*x + 3*c) + 8*f*e^(2*d*x + 2*c) - 7*I*f*e^
(d*x + c) + f)/(a*d^2*e^(3*d*x + 3*c) - I*a*d^2*e^(2*d*x + 2*c))
```

3.201.9 Mupad [B] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.23

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \mathrm{e}^{-c-d\,x}\left(\frac{f+d\,e}{2\,a\,d^2}+\frac{f\,x}{2\,a\,d}\right) + \mathrm{e}^{-2\,c-2\,d\,x}\left(\frac{(f+2\,d\,e)\ 1\mathrm{i}}{16\,a\,d^2}+\frac{f\,x\,1\mathrm{i}}{8\,a\,d}\right) + \mathrm{e}^{2\,c+2\,d\,x}\left(\frac{(f-2\,d\,e)\ 1\mathrm{i}}{16\,a\,d^2}-\frac{f\,x\,1\mathrm{i}}{8\,a\,d}\right) - \mathrm{e}^{c+d\,x}\left(\frac{f-d\,e}{2\,a\,d^2}-\frac{f\,x}{2\,a\,d}\right)+\frac{f\,x^2\,3\mathrm{i}}{4\,a}+\frac{2\,(e+f\,x)}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)} - \frac{x\,(4\,f-3\,d\,e)\ 1\mathrm{i}}{2\,a\,d}+\frac{f\,\ln\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-\mathrm{i}\right)\ 2\mathrm{i}}{a\,d^2}$$

input
```
int((sinh(c + d*x)^3*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

output
```
exp(- c - d*x)*((f + d*e)/(2*a*d^2) + (f*x)/(2*a*d)) + exp(- 2*c - 2*d*x)*
(((f + 2*d*e)*1i)/(16*a*d^2) + (f*x*1i)/(8*a*d)) + exp(2*c + 2*d*x)*(((f -
 2*d*e)*1i)/(16*a*d^2) - (f*x*1i)/(8*a*d)) - exp(c + d*x)*((f - d*e)/(2*a*
d^2) - (f*x)/(2*a*d)) + (f*x^2*3i)/(4*a) + (2*(e + f*x))/(a*d*(exp(c + d*x
) - 1i)) - (x*(4*f - 3*d*e)*1i)/(2*a*d) + (f*log(exp(d*x)*exp(c) - 1i)*2i)
/(a*d^2)
```

3.202 $\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.202.1 Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{3ix}{2a}+\frac{2\cosh(c+dx)}{ad}-\frac{3i\cosh(c+dx)\sinh(c+dx)}{2ad}-\frac{\cosh(c+dx)\sinh^2(c+dx)}{d(a+ia\sinh(c+dx))}$$

output
```
3/2*I*x/a+2*cosh(d*x+c)/a/d-3/2*I*cosh(d*x+c)*sinh(d*x+c)/a/d-cosh(d*x+c)*
sinh(d*x+c)^2/d/(a+I*a*sinh(d*x+c))
```

3.202.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.31

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\cosh(c+dx)\left(3\text{arcsinh}(\sinh(c+dx))\sqrt{1+i\sinh(c+dx)}+\sqrt{1-i\sinh(c+dx)}(-4i+\sinh(c+dx)-i\right.}{2ad\sqrt{1-i\sinh(c+dx)}(-i+\sinh(c+dx))}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(Cosh[c + d*x]*(3*ArcSinh[Sinh[c + d*x]]*Sqrt[1 + I*Sinh[c + d*x]] + Sqrt[
1 - I*Sinh[c + d*x]]*(-4*I + Sinh[c + d*x] - I*Sinh[c + d*x]^2)))/(2*a*d*S
qrt[1 - I*Sinh[c + d*x]]*(-I + Sinh[c + d*x]))
```

3.202.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 26, 3246, 26, 3042, 26, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ic+idx)^3}{a+a\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ic+idx)^3}{\sin(ic+idx)a+a}dx$$

$$\Big\downarrow \text{3246}$$

$$i\left(\frac{i\sinh^2(c+dx)\cosh(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\int i\sinh(c+dx)(2a-3ia\sinh(c+dx))dx}{a^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\sinh^2(c+dx)\cosh(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{i\int \sinh(c+dx)(2a-3ia\sinh(c+dx))dx}{a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\sinh^2(c+dx)\cosh(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{i\int -i\sin(ic+idx)(2a-3a\sin(ic+idx))dx}{a^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\sinh^2(c+dx)\cosh(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\int \sin(ic+idx)(2a-3a\sin(ic+idx))dx}{a^2}\right)$$

$$\Big\downarrow \text{3213}$$

$$i\left(\frac{i\sinh^2(c+dx)\cosh(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\frac{2ia\cosh(c+dx)}{d}+\frac{3a\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{3ax}{2}}{a^2}\right)$$

input
```
Int[Sinh[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

output
```
I*((I*Cosh[c + d*x]*Sinh[c + d*x]^2)/(d*(a + I*a*Sinh[c + d*x])) - ((-3*a*
x)/2 + ((2*I)*a*Cosh[c + d*x])/d + (3*a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
)/a^2)
```

3.202.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3246
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Simp[d/(a*b)   Int[(c + d*
Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[
e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] ||
EqQ[c, 0])
```

3.202. $\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.202.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

method	result
risch	$\frac{3ix}{2a} - \frac{i\mathrm{e}^{2dx+2c}}{8ad} + \frac{\mathrm{e}^{dx+c}}{2ad} + \frac{\mathrm{e}^{-dx-c}}{2ad} + \frac{i\mathrm{e}^{-2dx-2c}}{8ad} + \frac{2}{da(\mathrm{e}^{dx+c}-i)}$
parallelrisch	$\frac{(-12dx-4i)\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+(-12ixd+28)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)+3i\cosh\left(\frac{3dx}{2}+\frac{3c}{2}\right)+i\cosh\left(\frac{5dx}{2}+\frac{5c}{2}\right)+\sinh\left(\frac{5dx}{2}+\frac{5c}{2}\right)-3\sinh}{8\left(i\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ad}$
derivativedivides	$\frac{\frac{i}{2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{16\left(\frac{1}{16}-\frac{i}{32}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{16\left(-\frac{1}{16}-\right.}{\tanh\left(\frac{dx}{2}+\right.}}{ad}$
default	$\frac{\frac{i}{2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{16\left(\frac{1}{16}-\frac{i}{32}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{16\left(-\frac{1}{16}-\right.}{\tanh\left(\frac{dx}{2}+\right.}}{ad}$

input
```
int(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3/2*I*x/a-1/8*I/a/d*exp(2*d*x+2*c)+1/2/a/d*exp(d*x+c)+1/2/a/d*exp(-d*x-c)+
1/8*I/a/d*exp(-2*d*x-2*c)+2/d/a/(exp(d*x+c)-I)
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{4(-3i\,dx+i)e^{(3\,dx+3\,c)}-4(3\,dx+5)e^{(2\,dx+2\,c)}+i\,e^{(5\,dx+5\,c)}-3\,e^{(4\,dx+4\,c)}+3i\,e^{(dx+c)}-1}{8\left(ade^{(3\,dx+3\,c)}-i\,ade^{(2\,dx+2\,c)}\right)}$$

input
```
integrate(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/8*(4*(-3*I*d*x + I)*e^(3*d*x + 3*c) - 4*(3*d*x + 5)*e^(2*d*x + 2*c) + I
*e^(5*d*x + 5*c) - 3*e^(4*d*x + 4*c) + 3*I*e^(d*x + c) - 1)/(a*d*e^(3*d*x
+ 3*c) - I*a*d*e^(2*d*x + 2*c))
```

3.202.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.11

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \begin{cases} \frac{(-32ia^3d^3e^{5c}e^{2dx}+128a^3d^3e^{4c}e^{dx}+128a^3d^3e^{2c}e^{-dx}+32ia^3d^3e^{c}e^{-2dx})e^{-3c}}{256a^4d^4} & \text{for } a^4d^4e^{3c}\neq 0 \\ x\left(\frac{(-ie^{4c}+2e^{3c}+6ie^{2c}-2e^{c}-i)e^{-2c}}{4a}-\frac{3i}{2a}\right) & \text{otherwise} \end{cases} + \frac{2}{ade^ce^{dx}-iad}+\frac{3ix}{2a}$$

input
```
integrate(sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise(((-32*I*a**3*d**3*exp(5*c)*exp(2*d*x) + 128*a**3*d**3*exp(4*c)*e
xp(d*x) + 128*a**3*d**3*exp(2*c)*exp(-d*x) + 32*I*a**3*d**3*exp(c)*exp(-2*
d*x))*exp(-3*c)/(256*a**4*d**4), Ne(a**4*d**4*exp(3*c), 0)), (x*((-I*exp(4
*c) + 2*exp(3*c) + 6*I*exp(2*c) - 2*exp(c) - I)*exp(-2*c)/(4*a) - 3*I/(2*a
)), True)) + 2/(a*d*exp(c)*exp(d*x) - I*a*d) + 3*I*x/(2*a)
```

3.202.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{3i\,(dx+c)}{2\,ad}+\frac{3i\,e^{(-dx-c)}+20\,e^{(-2\,dx-2\,c)}+1}{8\,(i\,ae^{(-2\,dx-2\,c)}+ae^{(-3\,dx-3\,c)})d}+\frac{i\,(-4i\,e^{(-dx-c)}+e^{(-2\,dx-2\,c)})}{8\,ad}$$

input
```
integrate(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
3/2*I*(d*x + c)/(a*d) + 1/8*(3*I*e^(-d*x - c) + 20*e^(-2*d*x - 2*c) + 1)/(
(I*a*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c))*d) + 1/8*I*(-4*I*e^(-d*x - c)
+ e^(-2*d*x - 2*c))/(a*d)
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{-\frac{12i\,(dx+c)}{a} - \frac{\left(20\,e^{(2\,dx+2\,c)}-3i\,e^{(dx+c)}+1\right)e^{(-2\,dx-2\,c)}}{a\left(e^{(dx+c)}-i\right)} + \frac{i\,ae^{(2\,dx+2\,c)}-4\,ae^{(dx+c)}}{a^2}}{8\,d}$$

input
```
integrate(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/8*(-12*I*(d*x + c)/a - (20*e^(2*d*x + 2*c) - 3*I*e^(d*x + c) + 1)*e^(-2
*d*x - 2*c)/(a*(e^(d*x + c) - I)) + (I*a*e^(2*d*x + 2*c) - 4*a*e^(d*x + c)
)/a^2)/d
```

3.202.9 Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{x\,3\mathrm{i}}{2\,a} + \frac{2}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)} + \frac{\mathrm{e}^{c+d\,x}}{2\,a\,d} + \frac{\mathrm{e}^{-c-d\,x}}{2\,a\,d} + \frac{\mathrm{e}^{-2\,c-2\,d\,x}\,1\mathrm{i}}{8\,a\,d} - \frac{\mathrm{e}^{2\,c+2\,d\,x}\,1\mathrm{i}}{8\,a\,d}$$

input
```
int(sinh(c + d*x)^3/(a + a*sinh(c + d*x)*1i),x)
```

output
```
(x*3i)/(2*a) + 2/(a*d*(exp(c + d*x) - 1i)) + exp(c + d*x)/(2*a*d) + exp(-
c - d*x)/(2*a*d) + (exp(- 2*c - 2*d*x)*1i)/(8*a*d) - (exp(2*c + 2*d*x)*1i)
/(8*a*d)
```

3.203 $\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

3.203.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.203.2 Mathematica [N/A]

Not integrable

Time = 32.68 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

3.203.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))} \, dx$$
$$\Big\downarrow \; 6111$$
$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.203.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.203.4 Maple [N/A] (verified)

Not integrable

Time = 0.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\sinh\left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.203.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 249, normalized size of antiderivative = 8.03

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh{(dx+c)}^3}{(fx+e)(i\,a\sinh{(dx+c)}+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral(-1/4*(d*f
*x + d*e - (-I*d*f*x - I*d*e)*e^(5*d*x + 5*c) - (d*f*x + d*e)*e^(4*d*x + 4
*c) + 4*(-I*d*f*x - I*d*e)*e^(3*d*x + 3*c) - 4*(d*f*x + d*e + 2*f)*e^(2*d*
x + 2*c) - (I*d*f*x + I*d*e)*e^(d*x + c))/((a*d*f^2*x^2 + 2*a*d*e*f*x + a*
d*e^2)*e^(3*d*x + 3*c) - (I*a*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2)*e^(2*
d*x + 2*c)), x) + 2)/(-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c)
)
```

3.203.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.203.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.203.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\sinh\left(dx+c\right)^3}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^3/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

3.203.9 Mupad [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\sinh(c+d\,x)^3}{(e+f\,x)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(sinh(c + d*x)^3/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(sinh(c + d*x)^3/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.204 $\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} dx$

3.204.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} dx = \text{Int}\left(\frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.204.2 Mathematica [N/A]

Not integrable

Time = 15.87 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} dx = \int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} dx$$

input
```
Integrate[Sinh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

3.204.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx$$
$$\Big\downarrow \; 6111$$
$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.204.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.204.4 Maple [N/A] (verified)

Not integrable

Time = 0.91 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\sinh \left(dx+c\right)^{3}}{\left(fx+e\right)^{2} \left(a+ia\sinh \left(dx+c\right)\right)}dx$$

input
```
int(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.204.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 327, normalized size of antiderivative = 10.55

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^3}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
((-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x
+ a*d*e^2)*e^(d*x + c))*integral(-1/4*(d*f*x + d*e - (-I*d*f*x - I*d*e)*e^
(5*d*x + 5*c) - (d*f*x + d*e)*e^(4*d*x + 4*c) + 4*(-I*d*f*x - I*d*e)*e^(3*
d*x + 3*c) - 4*(d*f*x + d*e + 4*f)*e^(2*d*x + 2*c) - (I*d*f*x + I*d*e)*e^(
d*x + c))/((a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^(3*
d*x + 3*c) - (I*a*d*f^3*x^3 + 3*I*a*d*e*f^2*x^2 + 3*I*a*d*e^2*f*x + I*a*d*
e^3)*e^(2*d*x + 2*c)), x) + 2)/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2
 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

3.204.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.204.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.204.8 Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh{(dx+c)}^3}{(fx+e)^2(i\,a\sinh{(dx+c)}+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^3/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

3.204.9 Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)^3}{(e+f\,x)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(sinh(c + d*x)^3/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(sinh(c + d*x)^3/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.205 $\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.205.1 Optimal result

Integrand size = 29, antiderivative size = 313

$$\begin{aligned}\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx =& -\frac{i(e+fx)^3}{ad}-\frac{2(e+fx)^3\text{arctanh}\left(e^{c+dx}\right)}{ad}\\&+\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}\\&-\frac{3f(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}\\&+\frac{12if^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}\\&+\frac{3f(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}\\&+\frac{6f^2(e+fx)\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}\\&-\frac{12if^3\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4}\\&-\frac{6f^2(e+fx)\text{PolyLog}\left(3,e^{c+dx}\right)}{ad^3}-\frac{6f^3\text{PolyLog}\left(4,-e^{c+dx}\right)}{ad^4}\\&+\frac{6f^3\text{PolyLog}\left(4,e^{c+dx}\right)}{ad^4}-\frac{i(e+fx)^3\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\end{aligned}$$

output
```
-I*(f*x+e)^3/a/d-2*(f*x+e)^3*arctanh(exp(d*x+c))/a/d+6*I*f*(f*x+e)^2*ln(1+
I*exp(d*x+c))/a/d^2-3*f*(f*x+e)^2*polylog(2,-exp(d*x+c))/a/d^2+12*I*f^2*(f
*x+e)*polylog(2,-I*exp(d*x+c))/a/d^3+3*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a
/d^2+6*f^2*(f*x+e)*polylog(3,-exp(d*x+c))/a/d^3-12*I*f^3*polylog(3,-I*exp(
d*x+c))/a/d^4-6*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a/d^3-6*f^3*polylog(4,-e
xp(d*x+c))/a/d^4+6*f^3*polylog(4,exp(d*x+c))/a/d^4-I*(f*x+e)^3*tanh(1/2*c+
1/4*I*Pi+1/2*d*x)/a/d
```

3.205.2 Mathematica [A] (verified)

Time = 2.59 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\frac{2(e+fx)^3}{-i+e^c}+\frac{6if(e+fx)^2\log(1-ie^{-c-dx})}{d}+(e+fx)^3\log\left(1-e^{c+dx}\right)-(e+fx)^3\log\left(1+e^{c+dx}\right)-\frac{3f(e+fx)^2\,\text{PolyLog}}{d}}{}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((2*(e + f*x)^3)/(-I + E^c) + ((6*I)*f*(e + f*x)^2*Log[1 - I*E^(-c - d*x)]
)/d + (e + f*x)^3*Log[1 - E^(c + d*x)] - (e + f*x)^3*Log[1 + E^(c + d*x)]
- (3*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x)])/d + (3*f*(e + f*x)^2*PolyLog[
2, E^(c + d*x)])/d - ((12*I)*f^2*(d*(e + f*x)*PolyLog[2, I*E^(-c - d*x)] +
 f*PolyLog[3, I*E^(-c - d*x)]))/d^3 + (6*f^2*(e + f*x)*PolyLog[3, -E^(c +
d*x)])/d^2 - (6*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/d^2 - (6*f^3*PolyLo
g[4, -E^(c + d*x)])/d^3 + (6*f^3*PolyLog[4, E^(c + d*x)])/d^3 - ((2*I)*(e
+ f*x)^3*Sinh[(d*x)/2])/((Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*
Sinh[(c + d*x)/2])))/(a*d)
```

3.205.3 Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.04, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.759$, Rules used = {6109, 3042, 26, 3799, 25, 25, 3042, 4670, 3011, 4672, 26, 3042, 26, 4199, 26, 2620, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)^3\text{csch}(c+dx)dx}{a}-i\int \frac{(e+fx)^3}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int i(e+fx)^3\csc(ic+idx)dx}{a}-i\int \frac{(e+fx)^3}{\sin(ic+idx)a+a}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}-i\int \frac{(e+fx)^3}{\sin(ic+idx)a+a}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}-\frac{i\int -(e+fx)^3\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}+\frac{i\int -(e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}-\frac{i\int (e+fx)^3\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}-\frac{i\int (e+fx)^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4670}$$

$$\frac{i\left(\frac{3if\int (e+fx)^2\log\left(1-e^{c+dx}\right)dx}{d}-\frac{3if\int (e+fx)^2\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^3\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{a}-\frac{i\int (e+fx)^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{3011}$$

3.205. $\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{a}$$

$$\frac{i\int(e+fx)^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{a}$$

$$\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6if\int-i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{a}$$

$$\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6f\int(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{a}$$

$$\frac{i\left(\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{6f\int-i(e+fx)^2\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}$$

$$\Big\downarrow \text{26}$$

3.205. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{i\left(\frac{6if\int(e+fx)^2\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\downarrow 4199$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{i\left(\frac{6if\left(2i\int\frac{ie^{c+dx}(e+fx)^2}{1+ie^{c+dx}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\downarrow 26$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{i\left(\frac{6if\left(-2\int\frac{e^{c+dx}(e+fx)^2}{1+ie^{c+dx}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\downarrow 2620$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\int(e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\downarrow 3011$$

3.205. $\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{a}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\int\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right.}{2a}$$

$\downarrow$ 2720

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{a}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tan\right.}{2a}$$

$\downarrow$ 7143

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{a}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$\downarrow$ 7163

3.205. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-e^{c+dx}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{d}-\frac{f\int \mathrm{P}}{}\right)}{d}\right.}{}\right.}{a}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\Big\downarrow \; 2720$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-e^{c+dx}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,e\right.}{d}\right.}{}\right.}{}\right.}{a}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\Big\downarrow \; 7143$$

3.205. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{i\left(\frac{2i(e+fx)^3\operatorname{arctanh}(e^{c+dx})}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-e^{c+dx}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+\cdots}{}\right.}{}\right.}{}\right.}{a}$$

$$\frac{i\left(\frac{6if\left(-2\left(\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{2(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

input
```
Int[((e + f*x)^3*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(I*(((2*I)*(e + f*x)^3*ArcTanh[E^(c + d*x)])/d - ((3*I)*f*(-(((e + f*x)^2*
PolyLog[2, -E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, -E^(c + d*x)])/
d - (f*PolyLog[4, -E^(c + d*x)])/d^2))/d))/d + ((3*I)*f*(-(((e + f*x)^2*Po
lyLog[2, E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, E^(c + d*x)])/d -
(f*PolyLog[4, E^(c + d*x)])/d^2))/d))/d))/a - ((I/2)*(((6*I)*f*(((-1/3*I)*
(e + f*x)^3)/f - 2*(((-I)*(e + f*x)^2*Log[1 + I*E^(c + d*x)])/d + ((2*I)*f
*(-(((e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d) + (f*PolyLog[3, (-I)*E^(c
+ d*x)])/d^2))/d)))/d + (2*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))
/a
```

3.205.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

3.205. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

3.205. $\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.205.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1033 vs. $2(288) = 576$.

Time = 1.98 (sec) , antiderivative size = 1034, normalized size of antiderivative = 3.30

method	result	size
risch	Expression too large to display	1034

input
```
int((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(d*x+c)-I)-12*I/a/d^2*f^2*e*
c*x+12*I/a/d^3*c*f^2*e*ln(exp(d*x+c))-12*I*f^3*polylog(3,-I*exp(d*x+c))/a/
d^4-3/a/d*f^2*e*ln(exp(d*x+c)+1)*x^2-6/a/d^2*f^2*e*polylog(2,-exp(d*x+c))*
x-6*f^3*polylog(4,-exp(d*x+c))/a/d^4+6*f^3*polylog(4,exp(d*x+c))/a/d^4-12*
I/a/d^3*c*f^2*e*ln(exp(d*x+c)-I)+12*I/a/d^3*f^2*e*ln(1+I*exp(d*x+c))*c+12*
I/a/d^2*f^2*e*ln(1+I*exp(d*x+c))*x-1/a/d*f^3*ln(exp(d*x+c)+1)*x^3-1/a/d^4*
c^3*f^3*ln(exp(d*x+c)-1)+3/a/d^2*e^2*f*polylog(2,exp(d*x+c))-3/a/d^2*e^2*f
*polylog(2,-exp(d*x+c))-3/a/d^2*f^3*polylog(2,-exp(d*x+c))*x^2+6/a/d^3*f^3
*polylog(3,-exp(d*x+c))*x+1/a/d*f^3*ln(1-exp(d*x+c))*x^3+3/a/d^2*f^3*polyl
og(2,exp(d*x+c))*x^2-6/a/d^3*f^3*polylog(3,exp(d*x+c))*x+1/a/d^4*f^3*ln(1-
exp(d*x+c))*c^3-6/a/d^3*f^2*e*polylog(3,exp(d*x+c))+6/a/d^3*f^2*e*polylog(
3,-exp(d*x+c))-2*I/a/d*f^3*x^3+4*I/a/d^4*f^3*c^3+12*I/a/d^3*f^2*e*polylog(
2,-I*exp(d*x+c))+3/a/d*f^2*e*ln(1-exp(d*x+c))*x^2+6/a/d^2*f^2*e*polylog(2,
exp(d*x+c))*x-3/a/d^2*e^2*c*f*ln(exp(d*x+c)-1)+3/a/d^3*c^2*f^2*e*ln(exp(d*
x+c)-1)-3/a/d^3*c^2*f^2*e*ln(1-exp(d*x+c))-3/a/d*e^2*f*ln(exp(d*x+c)+1)*x+
3/a/d*e^2*f*ln(1-exp(d*x+c))*x+3/a/d^2*e^2*f*ln(1-exp(d*x+c))*c-6*I/a/d^4*
f^3*ln(1+I*exp(d*x+c))*c^2+6*I/a/d^3*f^3*x*c^2+6*I/a/d^2*f^3*ln(1+I*exp(d*
x+c))*x^2+12*I/a/d^3*f^3*polylog(2,-I*exp(d*x+c))*x-6*I/a/d*f^2*e*x^2+6*I/
a/d^4*c^2*f^3*ln(exp(d*x+c)-I)-6*I/a/d^3*c^2*f^2*e-6*I/a/d^4*c^2*f^3*ln(ex
p(d*x+c))-6*I/a/d^2*e^2*f*ln(exp(d*x+c))+6*I/a/d^2*e^2*f*ln(exp(d*x+c)-...
```

3.205.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1000 vs. $2(276) = 552$.

Time = 0.27 (sec) , antiderivative size = 1000, normalized size of antiderivative = 3.19

$$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*d^3*e^3 - 6*c*d^2*e^2*f + 6*c^2*d*e*f^2 - 2*c^3*f^3 + 12*(d*f^3*x + d*e
*f^2 - (-I*d*f^3*x - I*d*e*f^2)*e^(d*x + c))*dilog(-I*e^(d*x + c)) - 3*(-I
*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x - I*d^2*e^2*f + (d^2*f^3*x^2 + 2*d^2*e*f^2*
x + d^2*e^2*f)*e^(d*x + c))*dilog(-e^(d*x + c)) - 3*(I*d^2*f^3*x^2 + 2*I*d
^2*e*f^2*x + I*d^2*e^2*f - (d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*e^(d*
x + c))*dilog(e^(d*x + c)) - 2*(I*d^3*f^3*x^3 + 3*I*d^3*e*f^2*x^2 + 3*I*d^
3*e^2*f*x + 3*I*c*d^2*e^2*f - 3*I*c^2*d*e*f^2 + I*c^3*f^3)*e^(d*x + c) + (
I*d^3*f^3*x^3 + 3*I*d^3*e*f^2*x^2 + 3*I*d^3*e^2*f*x + I*d^3*e^3 - (d^3*f^3
*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3)*e^(d*x + c))*log(e^(d*x
+ c) + 1) + 6*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3 - (-I*d^2*e^2*f + 2*I*c*d
*e*f^2 - I*c^2*f^3)*e^(d*x + c))*log(e^(d*x + c) - I) + (-I*d^3*e^3 + 3*I*
c*d^2*e^2*f - 3*I*c^2*d*e*f^2 + I*c^3*f^3 + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c
^2*d*e*f^2 - c^3*f^3)*e^(d*x + c))*log(e^(d*x + c) - 1) + 6*(d^2*f^3*x^2 +
 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3 - (-I*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x
 - 2*I*c*d*e*f^2 + I*c^2*f^3)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (-I*d^
3*f^3*x^3 - 3*I*d^3*e*f^2*x^2 - 3*I*d^3*e^2*f*x - 3*I*c*d^2*e^2*f + 3*I*c^
2*d*e*f^2 - I*c^3*f^3 + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3
*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*e^(d*x + c))*log(-e^(d*x + c) + 1)
 - 6*(f^3*e^(d*x + c) - I*f^3)*polylog(4, -e^(d*x + c)) + 6*(f^3*e^(d*x +
c) - I*f^3)*polylog(4, e^(d*x + c)) - 12*(I*f^3*e^(d*x + c) + f^3)*poly...
```

3.205.6 Sympy [F]

$$\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^3\operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{f^3x^3\operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{3ef^2x^2\operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{3e^2fx\operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**3*csch(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**3*x**
*3*csch(c + d*x)/(sinh(c + d*x) - I), x) + Integral(3*e*f**2*x**2*csch(c +
 d*x)/(sinh(c + d*x) - I), x) + Integral(3*e**2*f*x*csch(c + d*x)/(sinh(c
+ d*x) - I), x))/a
```

3.205.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(276) = 552$.

Time = 0.38 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.85

$$\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= -e^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{ad} - \frac{\log\left(e^{(-dx-c)}-1\right)}{ad} - \frac{2}{(ae^{(-dx-c)}+i\,a)d}\right)$$
$$- \frac{6i\,e^2fx}{ad} - \frac{3\left(dx\log\left(e^{(dx+c)}+1\right)+\operatorname{Li}_2\left(-e^{(dx+c)}\right)\right)e^2f}{ad^2}$$
$$+ \frac{3\left(dx\log\left(-e^{(dx+c)}+1\right)+\operatorname{Li}_2\left(e^{(dx+c)}\right)\right)e^2f}{ad^2}$$
$$+ \frac{6i\,e^2f\log\left(i\,e^{(dx+c)}+1\right)}{ad^2} + \frac{2\left(f^3x^3+3\,ef^2x^2+3\,e^2fx\right)}{ade^{(dx+c)}-i\,ad}$$
$$- \frac{3\left(d^2x^2\log\left(e^{(dx+c)}+1\right)+2\,dx\operatorname{Li}_2\left(-e^{(dx+c)}\right)-2\operatorname{Li}_3(-e^{(dx+c)})\right)ef^2}{ad^3}$$
$$+ \frac{3\left(d^2x^2\log\left(-e^{(dx+c)}+1\right)+2\,dx\operatorname{Li}_2\left(e^{(dx+c)}\right)-2\operatorname{Li}_3(e^{(dx+c)})\right)ef^2}{ad^3}$$
$$+ \frac{12i\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\operatorname{Li}_2\left(-i\,e^{(dx+c)}\right)\right)ef^2}{ad^3}$$
$$- \frac{\left(d^3x^3\log\left(e^{(dx+c)}+1\right)+3\,d^2x^2\operatorname{Li}_2\left(-e^{(dx+c)}\right)-6\,dx\operatorname{Li}_3(-e^{(dx+c)})+6\operatorname{Li}_4(-e^{(dx+c)})\right)f^3}{ad^4}$$
$$+ \frac{\left(d^3x^3\log\left(-e^{(dx+c)}+1\right)+3\,d^2x^2\operatorname{Li}_2\left(e^{(dx+c)}\right)-6\,dx\operatorname{Li}_3(e^{(dx+c)})+6\operatorname{Li}_4(e^{(dx+c)})\right)f^3}{ad^4}$$
$$+ \frac{6i\left(d^2x^2\log\left(i\,e^{(dx+c)}+1\right)+2\,dx\operatorname{Li}_2\left(-i\,e^{(dx+c)}\right)-2\operatorname{Li}_3(-i\,e^{(dx+c)})\right)f^3}{ad^4}$$
$$+ \frac{2\left(-i\,d^3f^3x^3-3i\,d^3ef^2x^2\right)}{ad^4}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^3*(log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d) - 2/((a*e^
(-d*x - c) + I*a)*d)) - 6*I*e^2*f*x/(a*d) - 3*(d*x*log(e^(d*x + c) + 1) +
dilog(-e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d*x*log(-e^(d*x + c) + 1) + dilog(
e^(d*x + c)))*e^2*f/(a*d^2) + 6*I*e^2*f*log(I*e^(d*x + c) + 1)/(a*d^2) + 2
*(f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x)/(a*d*e^(d*x + c) - I*a*d) - 3*(d^2*x^
2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x
+ c)))*e*f^2/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d
*x + c)) - 2*polylog(3, e^(d*x + c)))*e*f^2/(a*d^3) + 12*I*(d*x*log(I*e^(d
*x + c) + 1) + dilog(-I*e^(d*x + c)))*e*f^2/(a*d^3) - (d^3*x^3*log(e^(d*x
+ c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c))
 + 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) + 1
) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polyl
og(4, e^(d*x + c)))*f^3/(a*d^4) + 6*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*
d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4) + 2*
(-I*d^3*f^3*x^3 - 3*I*d^3*e*f^2*x^2)/(a*d^4)
```

3.205.8 Giac [F]

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\,\text{csch}\,(dx+c)}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*csch(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(e+f\,x)^3}{\sinh\,(c+d\,x)\,\,(a+a\sinh\,(c+d\,x)\,\,1\text{i})}\,dx$$

input
```
int((e + f*x)^3/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^3/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.206 $\int \frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.206.1 Optimal result

Integrand size = 29, antiderivative size = 224

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i(e+fx)^2}{ad}-\frac{2(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{ad}+\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}-\frac{2f(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{4if^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{2f(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}+\frac{2f^2\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}-\frac{2f^2\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3}-\frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
-I*(f*x+e)^2/a/d-2*(f*x+e)^2*arctanh(exp(d*x+c))/a/d+4*I*f*(f*x+e)*ln(1+I*
exp(d*x+c))/a/d^2-2*f*(f*x+e)*polylog(2,-exp(d*x+c))/a/d^2+4*I*f^2*polylog
(2,-I*exp(d*x+c))/a/d^3+2*f*(f*x+e)*polylog(2,exp(d*x+c))/a/d^2+2*f^2*poly
log(3,-exp(d*x+c))/a/d^3-2*f^2*polylog(3,exp(d*x+c))/a/d^3-I*(f*x+e)^2*tan
h(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

3.206.2 Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.23

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{(e+fx)^2\log\left(1-e^{c+dx}\right)-(e+fx)^2\log\left(1+e^{c+dx}\right)+\frac{2d(e+fx)\left(-id(e+fx)+2(-i+e^c)f\log\left(1-ie^{-c-dx}\right)\right)-4(-i+e^c)f^2}{d^2(-1-ie^c)}}{}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((e + f*x)^2*Log[1 - E^(c + d*x)] - (e + f*x)^2*Log[1 + E^(c + d*x)] + (2*
d*(e + f*x)*((-I)*d*(e + f*x) + 2*(-I + E^c)*f*Log[1 - I*E^(-c - d*x)]) -
4*(-I + E^c)*f^2*PolyLog[2, I*E^(-c - d*x)])/(d^2*(-1 - I*E^c)) - (2*f*(d*
(e + f*x)*PolyLog[2, -E^(c + d*x)] - f*PolyLog[3, -E^(c + d*x)]))/d^2 + (2
*f*(d*(e + f*x)*PolyLog[2, E^(c + d*x)] - f*PolyLog[3, E^(c + d*x)]))/d^2
- ((2*I)*(e + f*x)^2*Sinh[(d*x)/2])/((Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c +
d*x)/2] + I*Sinh[(c + d*x)/2])))/(a*d)
```

3.206.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.05, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.690, Rules used = {6109, 3042, 26, 3799, 25, 25, 3042, 4670, 3011, 2720, 4672, 26, 3042, 26, 4199, 26, 2620, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$\Big\downarrow \; 6109$$
$$\frac{\int (e+fx)^2\operatorname{csch}(c+dx)dx}{a}-i\int \frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx$$
$$\Big\downarrow \; 3042$$
$$\frac{\int i(e+fx)^2\csc(ic+idx)dx}{a}-i\int \frac{(e+fx)^2}{\sin(ic+idx)a+a}dx$$
$$\Big\downarrow \; 26$$

$$\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}-i\int \frac{(e+fx)^2}{\sin(ic+idx)a+a}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int -(e+fx)^2\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}+\frac{i\int -(e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int (e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int (e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4670}$$

$$\frac{i\left(\frac{2if\int (e+fx)\log(1-e^{c+dx})dx}{d}-\frac{2if\int (e+fx)\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^2\text{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int (e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int \text{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)\text{PolyLog}(2,e^{c+dx})}{d}\right)}{d}+\frac{2i(e+fx)^2\text{arc}}{d}\right.}{a}$$
$$\frac{i\int (e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \text{2720}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}(2,-e^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}(2,e^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}(2,e^{c+}}{d}\right.}{d}\right.}{a}$$
$$\frac{i\int (e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

3.206. $\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+ia\sinh(c+dx)}dx$

$\downarrow$ 4672

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+\right.}}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4if\int -i(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}$$

$\downarrow$ 26

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+\right.}}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\int (e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}$$

$\downarrow$ 3042

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+\right.}}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\int -i(e+fx)\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}$$

$\downarrow$ 26

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+\right.}}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{4if\int (e+fx)\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$\downarrow$ 4199

3.206. $\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{4if\left(2i\int\frac{ie^{c+dx}(e+fx)}{1+ie^{c+dx}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{4if\left(-2\int\frac{e^{c+dx}(e+fx)}{1+ie^{c+dx}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{4if\left(-2\left(\frac{if\int\log\left(1+ie^{c+dx}\right)dx}{d}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{4if\left(-2\left(\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\Big\downarrow \text{2838}$$

3.206. $\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{4if\left(-2\left(-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\Big\downarrow \text{7143}$$

$$\frac{i\left(\frac{2i(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,e^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{4if\left(-2\left(-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{2(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

input
```
Int[((e + f*x)^2*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(I*(((2*I)*(e + f*x)^2*ArcTanh[E^(c + d*x)])/d - ((2*I)*f*(-(((e + f*x)*Po
lyLog[2, -E^(c + d*x)])/d) + (f*PolyLog[3, -E^(c + d*x)])/d^2))/d + ((2*I)
*f*(-(((e + f*x)*PolyLog[2, E^(c + d*x)])/d) + (f*PolyLog[3, E^(c + d*x)])
/d^2))/d))/a - ((I/2)*(((4*I)*f*(((-1/2*I)*(e + f*x)^2)/f - 2*(((-I)*(e +
f*x)*Log[1 + I*E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2)))
/d + (2*(e + f*x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/d))/a
```

3.206.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

3.206. $\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.206. $\int \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.206.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(204) = 408$.

Time = 1.78 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.56

method	result
risch	$-\frac{2if^2x^2}{ad}+\frac{4if^2\,\mathrm{polylog}\left(2,-i\mathrm{e}^{dx+c}\right)}{a\,d^3}+\frac{2f^2\,\mathrm{polylog}\left(3,-\mathrm{e}^{dx+c}\right)}{a\,d^3}-\frac{2f^2\,\mathrm{polylog}\left(3,\mathrm{e}^{dx+c}\right)}{a\,d^3}-\frac{4ief\ln\left(\mathrm{e}^{dx+c}\right)}{a\,d^2}+\frac{e^2\ln\left(\mathrm{e}^{dx+c}-1\right.}{ad}$

input
```
int((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-4*I/a/d^2*e*f*ln(exp(d*x+c))-4*I/a/d^2*f^2*c*x+4*I/a/d^2*f^2*ln(1+I*exp(d
*x+c))*x+4*I/a/d^3*f^2*ln(1+I*exp(d*x+c))*c+4*I/a/d^3*c*f^2*ln(exp(d*x+c))
+4*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3-2*I/a/d*f^2*x^2-2*I/a/d^3*f^2*c^2+
2*f^2*polylog(3,-exp(d*x+c))/a/d^3-2*f^2*polylog(3,exp(d*x+c))/a/d^3+1/a/d
*e^2*ln(exp(d*x+c)-1)-1/a/d*e^2*ln(exp(d*x+c)+1)+2/a/d^2*e*f*ln(1-exp(d*x+
c))*c+2/a/d*e*f*ln(1-exp(d*x+c))*x-2/a/d*e*f*ln(exp(d*x+c)+1)*x-2/a/d^2*e*
c*f*ln(exp(d*x+c)-1)+4*I/a/d^2*e*f*ln(exp(d*x+c)-I)-4*I/a/d^3*c*f^2*ln(exp
(d*x+c)-I)+2/a/d^2*e*f*polylog(2,exp(d*x+c))-2/a/d^2*e*f*polylog(2,-exp(d*
x+c))-1/a/d^3*f^2*ln(1-exp(d*x+c))*c^2+1/a/d*f^2*ln(1-exp(d*x+c))*x^2+2/a/
d^2*f^2*polylog(2,exp(d*x+c))*x-1/a/d*f^2*ln(exp(d*x+c)+1)*x^2-2/a/d^2*f^2
*polylog(2,-exp(d*x+c))*x+2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(d*x+c)-I)+1/a/d
^3*c^2*f^2*ln(exp(d*x+c)-1)
```

3.206.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(194) = 388.

Time = 0.26 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.50

$$\int \frac{(e+fx)^2\mathrm{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{2\,d^2e^2 - 4\,cdef + 2\,c^2f^2 - 4\left(-i\,f^2e^{(dx+c)} - f^2\right)\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right) - 2\left(-i\,df^2x - i\,def + (df^2x + def)e^{(dx+c)}\right.}{}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*d^2*e^2 - 4*c*d*e*f + 2*c^2*f^2 - 4*(-I*f^2*e^(d*x + c) - f^2)*dilog(-I
*e^(d*x + c)) - 2*(-I*d*f^2*x - I*d*e*f + (d*f^2*x + d*e*f)*e^(d*x + c))*d
ilog(-e^(d*x + c)) - 2*(I*d*f^2*x + I*d*e*f - (d*f^2*x + d*e*f)*e^(d*x + c
))*dilog(e^(d*x + c)) - 2*(I*d^2*f^2*x^2 + 2*I*d^2*e*f*x + 2*I*c*d*e*f - I
*c^2*f^2)*e^(d*x + c) + (I*d^2*f^2*x^2 + 2*I*d^2*e*f*x + I*d^2*e^2 - (d^2*
f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*e^(d*x + c))*log(e^(d*x + c) + 1) + 4*(d*
e*f - c*f^2 - (-I*d*e*f + I*c*f^2)*e^(d*x + c))*log(e^(d*x + c) - I) + (-I
*d^2*e^2 + 2*I*c*d*e*f - I*c^2*f^2 + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*e^(d*
x + c))*log(e^(d*x + c) - 1) + 4*(d*f^2*x + c*f^2 - (-I*d*f^2*x - I*c*f^2)
*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (-I*d^2*f^2*x^2 - 2*I*d^2*e*f*x - 2
*I*c*d*e*f + I*c^2*f^2 + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)
*e^(d*x + c))*log(-e^(d*x + c) + 1) + 2*(f^2*e^(d*x + c) - I*f^2)*polylog(
3, -e^(d*x + c)) - 2*(f^2*e^(d*x + c) - I*f^2)*polylog(3, e^(d*x + c)))/(a
*d^3*e^(d*x + c) - I*a*d^3)
```

3.206.6 Sympy [F]

$$\int \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^2\operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{f^2x^2\operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{2efx\operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**2*csch(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**2*x**
*2*csch(c + d*x)/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*csch(c + d*x)/
(sinh(c + d*x) - I), x))/a
```

3.206.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.55

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= -e^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{ad} - \frac{\log\left(e^{(-dx-c)}-1\right)}{ad} - \frac{2}{(ae^{(-dx-c)}+i\,a)d}\right) - \frac{2i\,f^2x^2}{ad}$$
$$- \frac{4i\,efx}{ad} + \frac{2\,(f^2x^2+2\,efx)}{ade^{(dx+c)}-i\,ad} - \frac{2\left(dx\log\left(e^{(dx+c)}+1\right)+\operatorname{Li}_2\left(-e^{(dx+c)}\right)\right)ef}{ad^2}$$
$$+ \frac{2\left(dx\log\left(-e^{(dx+c)}+1\right)+\operatorname{Li}_2\left(e^{(dx+c)}\right)\right)ef}{ad^2} + \frac{4i\,ef\log\left(i\,e^{(dx+c)}+1\right)}{ad^2}$$
$$- \frac{\left(d^2x^2\log\left(e^{(dx+c)}+1\right)+2\,dx\operatorname{Li}_2\left(-e^{(dx+c)}\right)-2\operatorname{Li}_3\left(-e^{(dx+c)}\right)\right)f^2}{ad^3}$$
$$+ \frac{\left(d^2x^2\log\left(-e^{(dx+c)}+1\right)+2\,dx\operatorname{Li}_2\left(e^{(dx+c)}\right)-2\operatorname{Li}_3\left(e^{(dx+c)}\right)\right)f^2}{ad^3}$$
$$+ \frac{4i\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\operatorname{Li}_2\left(-i\,e^{(dx+c)}\right)\right)f^2}{ad^3}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^2*(log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d) - 2/((a*e^
(-d*x - c) + I*a)*d)) - 2*I*f^2*x^2/(a*d) - 4*I*e*f*x/(a*d) + 2*(f^2*x^2 +
 2*e*f*x)/(a*d*e^(d*x + c) - I*a*d) - 2*(d*x*log(e^(d*x + c) + 1) + dilog(
-e^(d*x + c))*e*f/(a*d^2) + 2*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x +
 c)))*e*f/(a*d^2) + 4*I*e*f*log(I*e^(d*x + c) + 1)/(a*d^2) - (d^2*x^2*log(
e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))
*f^2/(a*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) -
 2*polylog(3, e^(d*x + c)))*f^2/(a*d^3) + 4*I*(d*x*log(I*e^(d*x + c) + 1)
+ dilog(-I*e^(d*x + c)))*f^2/(a*d^3)
```

3.206.8 Giac [F]

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\operatorname{csch}(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*csch(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\sinh(c+d\,x)\,\left(a+a\sinh(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)^2/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^2/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.207 $\int \frac{(e+fx)\operatorname{\mathbf{csch}}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.207.1 Optimal result

Integrand size = 27, antiderivative size = 126

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{2(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{ad}+\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2} - \frac{f\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{f\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} - \frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
-2*(f*x+e)*arctanh(exp(d*x+c))/a/d+2*I*f*ln(cosh(1/2*c+1/4*I*Pi+1/2*d*x))/
a/d^2-f*polylog(2,-exp(d*x+c))/a/d^2+f*polylog(2,exp(d*x+c))/a/d^2-I*(f*x+
e)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

3.207.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 257 vs. 2(126) = 252.

Time = 2.22 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.04

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(f(c+dx)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)-2f\arctan\left(\right.\right.}{}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(f*(c + d*x)*(Cosh[(c + d*x)/2]
 + I*Sinh[(c + d*x)/2]) - 2*f*ArcTan[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2]
 + I*Sinh[(c + d*x)/2]) + I*f*Log[Cosh[c + d*x]]*(Cosh[(c + d*x)/2] + I*Si
nh[(c + d*x)/2]) + (d*(e + f*x)*(Log[1 - E^(c + d*x)] - Log[1 + E^(c + d*x
)]) - f*PolyLog[2, -E^(c + d*x)] + f*PolyLog[2, E^(c + d*x)])*(Cosh[(c + d
*x)/2] + I*Sinh[(c + d*x)/2]) - (2*I)*d*(e + f*x)*Sinh[(c + d*x)/2]))/(d^2
*(a + I*a*Sinh[c + d*x]))
```

3.207.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6109, 3042, 26, 3799, 25, 25, 3042, 4670, 2715, 2838, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)\text{csch}(c+dx)dx}{a}-i\int \frac{e+fx}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int i(e+fx)\csc(ic+idx)dx}{a}-i\int \frac{e+fx}{\sin(ic+idx)a+a}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-i\int \frac{e+fx}{\sin(ic+idx)a+a}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-\frac{i\int -\left((e+fx)\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)\right)dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int (e+fx)\csc(ic+idx)dx}{a}+\frac{i\int -\left((e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)dx}{2a}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{i\int(e+fx)\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{i\int(e+fx)\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \color{blue}{4670}$$

$$\frac{i\left(\frac{if\int\log(1-e^{c+dx})dx}{d}-\frac{if\int\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int(e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \color{blue}{2715}$$

$$\frac{i\left(\frac{if\int e^{-c-dx}\log(1-e^{c+dx})de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log(1+e^{c+dx})de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int(e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \color{blue}{2838}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\int(e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}$$

$$\Big\downarrow \color{blue}{4672}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2if\int-i\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}$$

$$\Big\downarrow \color{blue}{26}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{d}\right)}{2a}$$

3.207. $\int\frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}dx$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int -i\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}\right)}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{a}-\frac{i\left(\frac{2if\int \tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}\right)}{2a}$$

$$\Big\downarrow \text{3956}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{d^2}\right)}{2a}$$

input
```
Int[((e + f*x)*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(I*(((2*I)*(e + f*x)*ArcTanh[E^(c + d*x)])/d + (I*f*PolyLog[2, -E^(c + d*x
)])/d^2 - (I*f*PolyLog[2, E^(c + d*x)])/d^2))/a - ((I/2)*((-4*f*Log[Cosh[c
/2 + (I/4)*Pi + (d*x)/2]])/d^2 + (2*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/
2])/d))/a
```

3.207.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

3.207. $\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

3.207. $\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.207.4 Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.67

method	result
risch	$\frac{2fx+2e}{da\left(\mathrm{e}^{dx+c}-i\right)}+\frac{2if\ln\left(\mathrm{e}^{dx+c}-i\right)}{a\,d^{2}}-\frac{2if\ln\left(\mathrm{e}^{dx+c}\right)}{a\,d^{2}}+\frac{e\ln\left(\mathrm{e}^{dx+c}-1\right)}{ad}-\frac{e\ln\left(\mathrm{e}^{dx+c}+1\right)}{ad}+\frac{f\ln\left(1-\mathrm{e}^{dx+c}\right)c}{a\,d^{2}}-\frac{cf\ln\left(\mathrm{e}^{dx+c}-1\right)}{a\,d^{2}}$

input
```
int((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*(f*x+e)/d/a/(exp(d*x+c)-I)+2*I*f/a/d^2*ln(exp(d*x+c)-I)-2*I/a/d^2*f*ln(e
xp(d*x+c))+1/a/d*e*ln(exp(d*x+c)-1)-1/a/d*e*ln(exp(d*x+c)+1)+1/a/d^2*f*ln(
1-exp(d*x+c))*c-1/a/d^2*c*f*ln(exp(d*x+c)-1)+f*polylog(2,exp(d*x+c))/a/d^2
-f*polylog(2,-exp(d*x+c))/a/d^2+1/a/d*f*ln(1-exp(d*x+c))*x-1/a/d*f*ln(exp(
d*x+c)+1)*x
```

3.207.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(104) = 208.

Time = 0.26 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.67

$$\int \frac{(e+fx)\mathrm{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{-2i\,dfxe^{(dx+c)}+2\,de-\left(fe^{(dx+c)}-i\,f\right)\mathrm{Li}_2\left(-e^{(dx+c)}\right)+\left(fe^{(dx+c)}-i\,f\right)\mathrm{Li}_2\left(e^{(dx+c)}\right)+\left(i\,dfx+i\,de-(df\right.}{}$$

input
```
integrate((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(-2*I*d*f*x*e^(d*x + c) + 2*d*e - (f*e^(d*x + c) - I*f)*dilog(-e^(d*x + c)
) + (f*e^(d*x + c) - I*f)*dilog(e^(d*x + c)) + (I*d*f*x + I*d*e - (d*f*x +
 d*e)*e^(d*x + c))*log(e^(d*x + c) + 1) - 2*(-I*f*e^(d*x + c) - f)*log(e^(
d*x + c) - I) + (-I*d*e + I*c*f + (d*e - c*f)*e^(d*x + c))*log(e^(d*x + c)
 - 1) + (-I*d*f*x - I*c*f + (d*f*x + c*f)*e^(d*x + c))*log(-e^(d*x + c) +
1))/(a*d^2*e^(d*x + c) - I*a*d^2)
```

3.207.6 Sympy [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e\,\text{csch}\,(c+dx)}{\sinh\,(c+dx)-i}\,dx + \int \frac{fx\,\text{csch}\,(c+dx)}{\sinh\,(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e*csch(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f*x*csch(c
 + d*x)/(sinh(c + d*x) - I), x))/a
```

3.207.7 Maxima [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2*f*(x*e^(d*x + c)/(I*a*d*e^(d*x + c) + a*d) + I*log((e^(d*x + c) - I)*e^(
-c))/(a*d^2) + integrate(1/2*x/(a*e^(d*x + c) + a), x) + integrate(1/2*x/(
a*e^(d*x + c) - a), x)) - e*(log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c
) - 1)/(a*d) - 2/((a*e^(-d*x - c) + I*a)*d))
```

3.207.8 Giac [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*csch(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{e+f\,x}{\sinh\left(c+d\,x\right)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.208 $\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.208.1 Optimal result

Integrand size = 22, antiderivative size = 41

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\cosh(c+dx))}{ad} + \frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}$$

output
```
-arctanh(cosh(d*x+c))/a/d+cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))
```

3.208.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\operatorname{sech}(c+dx)\left(-1+\operatorname{arctanh}\left(\sqrt{\cosh^2(c+dx)}\right)\sqrt{\cosh^2(c+dx)}+i\sinh(c+dx)\right)}{ad}$$

input
```
Integrate[Csch[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-((Sech[c + d*x]*(-1 + ArcTanh[Sqrt[Cosh[c + d*x]^2]]*Sqrt[Cosh[c + d*x]^2
] + I*Sinh[c + d*x]))/(a*d))
```

3.208.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 26, 3226, 26, 3042, 26, 3127, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)(a+a\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ic+idx)(\sin(ic+idx)a+a)}dx$$

$$\Big\downarrow \text{3226}$$

$$i\left(\frac{\int -i\operatorname{csch}(c+dx)dx}{a}-\int \frac{1}{i\sinh(c+dx)a+a}dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\int \frac{1}{i\sinh(c+dx)a+a}dx-\frac{i\int \operatorname{csch}(c+dx)dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\int \frac{1}{\sin(ic+idx)a+a}dx-\frac{i\int i\csc(ic+idx)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\int \csc(ic+idx)dx}{a}-\int \frac{1}{\sin(ic+idx)a+a}dx\right)$$

$$\Big\downarrow \text{3127}$$

$$i\left(\frac{\int \csc(ic+idx)dx}{a}-\frac{i\cosh(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{4257}$$

$$i\left(\frac{i\operatorname{arctanh}(\cosh(c+dx))}{ad}-\frac{i\cosh(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

input
```
Int[Csch[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

output
```
I*((I*ArcTanh[Cosh[c + d*x]])/(a*d) - (I*Cosh[c + d*x])/(d*(a + I*a*Sinh[c
 + d*x])))
```

3.208.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.208. $\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.208.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2i}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{da}$	36
default	$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2i}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{da}$	36
risch	$\frac{2}{da\left(e^{dx+c}-i\right)}-\frac{\ln\left(e^{dx+c}+1\right)}{da}+\frac{\ln\left(e^{dx+c}-1\right)}{da}$	54
parallelrisch	$\frac{\left(i-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(i-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	61

input
```
int(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(ln(tanh(1/2*d*x+1/2*c))-2*I/(-I+tanh(1/2*d*x+1/2*c)))
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\left(e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}+1\right)-\left(e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}-1\right)-2}{ade^{(dx+c)}-i\,ad}$$

input
```
integrate(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-((e^(d*x + c) - I)*log(e^(d*x + c) + 1) - (e^(d*x + c) - I)*log(e^(d*x +
c) - 1) - 2)/(a*d*e^(d*x + c) - I*a*d)
```

3.208.6 Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx=-\frac{i\int\frac{\operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

input
```
integrate(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(csch(c + d*x)/(sinh(c + d*x) - I), x)/a
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.51

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx=-\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}+\frac{2}{(ae^{(-dx-c)}+i\,a)d}$$

input
```
integrate(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d) + 2/((a*e^(-d*x
 - c) + I*a)*d)
```

3.208.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx=-\frac{\frac{\log\left(e^{(dx+c)}+1\right)}{a}-\frac{\log\left(e^{(dx+c)}-1\right)}{a}-\frac{2}{a\left(e^{(dx+c)}-i\right)}}{d}$$

input
```
integrate(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(log(e^(d*x + c) + 1)/a - log(e^(d*x + c) - 1)/a - 2/(a*(e^(d*x + c) - I)
))/d
```

3.208.9 Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-a^2\,d^2}}{a\,d}\right)}{\sqrt{-a^2\,d^2}} + \frac{2}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}$$

input
```
int(1/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
2/(a*d*(exp(c + d*x) - 1i)) - (2*atan((exp(d*x)*exp(c)*(-a^2*d^2)^(1/2))/(
a*d)))/(-a^2*d^2)^(1/2)
```

3.209 $\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

3.209.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.209.2 Mathematica [N/A]

Not integrable

Time = 35.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Csch[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Csch[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

3.209.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\text{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.209.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.209.4 Maple [N/A] (verified)

Not integrable

Time = 0.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\text{csch}\,(dx+c)}{(fx+e)\,(a+ia\sinh\,(dx+c))}dx$$

input
```
int(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.209.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 235, normalized size of antiderivative = 8.10

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral(2*((d*f*x
 + d*e + f)*e^(2*d*x + 2*c) - (I*d*f*x + I*d*e)*e^(d*x + c) - f)/(I*a*d*f^
2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*
e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2)*e^(2*d*x +
2*c) - (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c)), x) + 2)/(-I*a*d
*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))
```

3.209.6 Sympy [N/A]

Not integrable

Time = 14.90 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\operatorname{csch}(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

input
```
integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(csch(c + d*x)/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) - I*f
*x), x)/a
```

3.209.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 169, normalized size of antiderivative = 5.83

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2*f*integrate(1/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2
*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) + 2/(-I*a*d*f*x - I*a*d
*e + (a*d*f*x*e^c + a*d*e*e^c)*e^(d*x)) + 2*integrate(1/2/(a*f*x + a*e + (
a*f*x*e^c + a*e*e^c)*e^(d*x)), x) + 2*integrate(-1/2/(a*f*x + a*e - (a*f*x
*e^c + a*e*e^c)*e^(d*x)), x)
```

3.209.8 Giac [N/A]

Not integrable

Time = 17.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(csch(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

3.209.9 Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\sinh(c+d\,x)\,(e+f\,x)\,(a+a\sinh(c+d\,x)\,1\mathrm{i})}\,dx$$

input
```
int(1/(sinh(c + d*x)*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(sinh(c + d*x)*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.210 $\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

3.210.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.210.2 Mathematica [N/A]

Not integrable

Time = 43.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Csch[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Csch[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

3.210.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.210.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.210.4 Maple [N/A] (verified)

Not integrable

Time = 0.86 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{csch}\left(dx+c\right)}{\left(fx+e\right)^{2}\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.210.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 11.76

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)}{\left(fx+e\right)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x
+ a*d*e^2)*e^(d*x + c))*integral(2*((d*f*x + d*e + 2*f)*e^(2*d*x + 2*c) -
(I*d*f*x + I*d*e)*e^(d*x + c) - 2*f)/(I*a*d*f^3*x^3 + 3*I*a*d*e*f^2*x^2 +
3*I*a*d*e^2*f*x + I*a*d*e^3 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f
*x + a*d*e^3)*e^(3*d*x + 3*c) + (-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*
a*d*e^2*f*x - I*a*d*e^3)*e^(2*d*x + 2*c) - (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2
+ 3*a*d*e^2*f*x + a*d*e^3)*e^(d*x + c)), x) + 2)/(-I*a*d*f^2*x^2 - 2*I*a*d
*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

3.210.6 Sympy [N/A]

Not integrable

Time = 168.50 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.45

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\operatorname{csch}\left(c+dx\right)}{e^2\sinh\left(c+dx\right)-ie^2+2efx\sinh\left(c+dx\right)-2iefx+f^2x^2\sinh\left(c+dx\right)-if^2x^2}\,dx}{a}$$

input
```
integrate(csch(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(csch(c + d*x)/(e**2*sinh(c + d*x) - I*e**2 + 2*e*f*x*sinh(c +
d*x) - 2*I*e*f*x + f**2*x**2*sinh(c + d*x) - I*f**2*x**2), x)/a
```

3.210.7 Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 277, normalized size of antiderivative = 9.55

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)}{(fx+e)^2(ia\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
4*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x - I*
a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*d
*e^3*e^c)*e^(d*x)), x) + 2/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (
a*d*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)) + 2*integrate(1/
2/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2*e^c + 2*a*e*f*x*e^c + a*e^2*
e^c)*e^(d*x)), x) + 2*integrate(-1/2/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 - (a*f
^2*x^2*e^c + 2*a*e*f*x*e^c + a*e^2*e^c)*e^(d*x)), x)
```

3.210.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.210.9 Mupad [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)\,\left(e+f\,x\right)^2\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int(1/(sinh(c + d*x)*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(sinh(c + d*x)*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.210. $\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

3.211 $\int \frac{(e+fx)^3\mathbf{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.211.1 Optimal result

Integrand size = 31, antiderivative size = 419

$$\begin{aligned}\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = &-\frac{2(e+fx)^3}{ad}+\frac{2i(e+fx)^3\operatorname{arctanh}\left(e^{c+dx}\right)}{ad}\\
&-\frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{6f(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}\\
&+\frac{3f(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad^2}\\
&+\frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}\\
&+\frac{12f^2(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}\\
&-\frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}\\
&+\frac{3f^2(e+fx)\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3}\\
&-\frac{6if^2(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}\\
&-\frac{12f^3\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4}\\
&+\frac{6if^2(e+fx)\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3}\\
&-\frac{3f^3\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^4}+\frac{6if^3\operatorname{PolyLog}\left(4,-e^{c+dx}\right)}{ad^4}\\
&-\frac{6if^3\operatorname{PolyLog}\left(4,e^{c+dx}\right)}{ad^4}-\frac{(e+fx)^3\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\end{aligned}$$

output
```
-2*(f*x+e)^3/a/d+3*I*f*(f*x+e)^2*polylog(2,-exp(d*x+c))/a/d^2-(f*x+e)^3*co
th(d*x+c)/a/d+6*f*(f*x+e)^2*ln(1+I*exp(d*x+c))/a/d^2+3*f*(f*x+e)^2*ln(1-ex
p(2*d*x+2*c))/a/d^2-3*I*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a/d^2+12*f^2*(f*
x+e)*polylog(2,-I*exp(d*x+c))/a/d^3-6*I*f^2*(f*x+e)*polylog(3,-exp(d*x+c))
/a/d^3+3*f^2*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^3-6*I*f^3*polylog(4,exp
(d*x+c))/a/d^4-12*f^3*polylog(3,-I*exp(d*x+c))/a/d^4+6*I*f^3*polylog(4,-ex
p(d*x+c))/a/d^4-3/2*f^3*polylog(3,exp(2*d*x+2*c))/a/d^4+2*I*(f*x+e)^3*arct
anh(exp(d*x+c))/a/d+6*I*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a/d^3-(f*x+e)^3*
tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

3.211.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1205 vs. $2(419) = 838$.

Time = 8.70 (sec) , antiderivative size = 1205, normalized size of antiderivative = 2.88

$$\int \frac{(e+fx)^3 \text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-6*I)*E^c*f*((e + f*x)^3/(3*E^c*f) + ((I + E^(-c))*(e + f*x)^2*Log[1 - I
*E^(-c - d*x)])/d - ((2*I)*(-I + E^c)*f*(d*(e + f*x)*PolyLog[2, I*E^(-c -
d*x)] + f*PolyLog[3, I*E^(-c - d*x)]))/(d^3*E^c)))/(a*d*(-I + E^c)) + (I*d
^3*e^2*(-1 + E^(2*c))*(d*e + (3*I)*f)*x + d^3*e^2*(1 - E^(2*c))*(I*d*e + 3
*f)*x - 2*d^3*(e + f*x)^3 + 3*d^2*e*(-1 + E^(2*c))*f*((-I)*d*e + 2*f)*x*Lo
g[1 - E^(-c - d*x)] + 3*d^2*(-1 + E^(2*c))*f^2*((-I)*d*e + f)*x^2*Log[1 -
E^(-c - d*x)] - I*d^3*(-1 + E^(2*c))*f^3*x^3*Log[1 - E^(-c - d*x)] + 3*d^2
*e*(-1 + E^(2*c))*f*(I*d*e + 2*f)*x*Log[1 + E^(-c - d*x)] + 3*d^2*(-1 + E^
(2*c))*f^2*(I*d*e + f)*x^2*Log[1 + E^(-c - d*x)] + I*d^3*(-1 + E^(2*c))*f^
3*x^3*Log[1 + E^(-c - d*x)] + d^2*e^2*(-1 + E^(2*c))*((-I)*d*e + 3*f)*Log[
1 - E^(c + d*x)] + d^2*e^2*(-1 + E^(2*c))*(I*d*e + 3*f)*Log[1 + E^(c + d*x
)] + 3*d*e*(1 - E^(2*c))*f*(I*d*e + 2*f)*PolyLog[2, -E^(-c - d*x)] + 6*d*(
1 - E^(2*c))*f^2*(I*d*e + f)*x*PolyLog[2, -E^(-c - d*x)] - (3*I)*d^2*(-1 +
 E^(2*c))*f^3*x^2*PolyLog[2, -E^(-c - d*x)] + (3*I)*d*e*(-1 + E^(2*c))*(d*
e + (2*I)*f)*f*PolyLog[2, E^(-c - d*x)] + (6*I)*d*(-1 + E^(2*c))*(d*e + I*
f)*f^2*x*PolyLog[2, E^(-c - d*x)] + (3*I)*d^2*(-1 + E^(2*c))*f^3*x^2*PolyL
og[2, E^(-c - d*x)] - 6*(-1 + E^(2*c))*f^2*(I*d*e + f)*PolyLog[3, -E^(-c -
 d*x)] - (6*I)*d*(-1 + E^(2*c))*f^3*x*PolyLog[3, -E^(-c - d*x)] + (6*I)*(-
1 + E^(2*c))*(d*e + I*f)*f^2*PolyLog[3, E^(-c - d*x)] + (6*I)*d*(-1 + E^(2
*c))*f^3*x*PolyLog[3, E^(-c - d*x)] - (6*I)*(-1 + E^(2*c))*f^3*PolyLog[...
```

3.211.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}dx$$
$$\Big\downarrow \text{ 6109}$$
$$\frac{\int (e+fx)^3\text{csch}^2(c+dx)dx}{a}-i\int \frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int -(e+fx)^3\csc(ic+idx)^2dx}{a}-i\int \frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$
$$\Big\downarrow \text{ 25}$$
$$-\frac{\int (e+fx)^3\csc(ic+idx)^2dx}{a}-i\int \frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$
$$\Big\downarrow \text{ 4672}$$

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\coth(c+dx)dx}{d}}{a}-i\int\frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3f\int(e+fx)^2\coth(c+dx)dx}{d}}{a}-i\int\frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}-i\int\frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\int(e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}-i\int\frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{4201}$$

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}-i\int\frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{2620}$$

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}-i\int\frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\Bigg(2i\Bigg(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\Bigg)-\frac{i(e+fx}{3f}}{d}}{a}$$
$$i\int\frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{2720}$$

3.211. $\int\frac{(e+fx)^3\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$i\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \textbf{6109}$$

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$i\left(\frac{\int(e+fx)^3\operatorname{csch}(c+dx)dx}{a}-i\int\frac{(e+fx)^3}{i\sinh(c+dx)a+a}dx\right)$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$i\left(\frac{\int i(e+fx)^3\csc(ic+idx)dx}{a}-i\int\frac{(e+fx)^3}{\sin(ic+idx)a+a}dx\right)$$

$$\Big\downarrow \textbf{26}$$

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$i\left(\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}-i\int\frac{(e+fx)^3}{\sin(ic+idx)a+a}dx\right)$$

$$\Big\downarrow \textbf{3799}$$

3.211. $\int\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$i\left(\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}-\frac{i\int-(e+fx)^3\operatorname{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}\right)$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$i\left(\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}+\frac{i\int-(e+fx)^3\operatorname{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}\right)$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$i\left(\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)^3\operatorname{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}\right)$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$i\left(\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right)$$

$$\Big\downarrow \; 4670$$

3.211. $\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-i\left(\frac{i\left(\frac{3if\int(e+fx)^2\log(1-e^{c+dx})dx}{d}-\frac{3if\int(e+fx)^2\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^3\text{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int(e+fx)^3\csc\left(\frac{ic}{2}+\frac{i\dots}{}\right.}{2a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+\dots}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$\Big\downarrow \text{3011}$$

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+d\dots}\right.}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+\dots}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$\Big\downarrow \text{4672}$$

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+d\dots}\right.}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+\dots}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$\Big\downarrow \text{26}$$

3.211. $\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+d\cdots}\right.}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+\cdots}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+d\cdots}\right.}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+\cdots}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+d\cdots}\right.}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+\cdots}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$\Big\downarrow \text{4199}$$

3.211. $\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,e^{c+d}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}(2,-e^{2c+2dx-i\pi})de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}(2,-e^{2c+}}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$\Big\downarrow$ 26

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,e^{c+d}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}(2,-e^{2c+2dx-i\pi})de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}(2,-e^{2c+}}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$\Big\downarrow$ 2620

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,e^{c+d}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}(2,-e^{2c+2dx-i\pi})de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}(2,-e^{2c+}}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$\Big\downarrow$ 3011

3.211. $\displaystyle\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+d}\right.}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$\downarrow$ 2720

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+d}\right.}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$\downarrow$ 7143

3.211. $\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-i\left(\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right.}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\operatorname{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

input
```
Int[((e + f*x)^3*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.211.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.211. $\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.211.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1603 vs. $2(390) = 780$.

Time = 3.17 (sec) , antiderivative size = 1604, normalized size of antiderivative = 3.83

method	result	size
risch	Expression too large to display	1604

input
```
int((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-6*I*f^3*polylog(4,exp(d*x+c))/a/d^4-6*f^3*polylog(3,-exp(d*x+c))/a/d^4-6*
f^3*polylog(3,exp(d*x+c))/a/d^4-3*I/d/a*e*f^2*ln(1-exp(d*x+c))*x^2+6*I*f^3
*polylog(4,-exp(d*x+c))/a/d^4-12*f^3*polylog(3,-I*exp(d*x+c))/a/d^4-2*I*(f
^3*x^3*exp(2*d*x+2*c)+3*e*f^2*x^2*exp(2*d*x+2*c)+3*e^2*f*x*exp(2*d*x+2*c)-
2*f^3*x^3-I*exp(d*x+c)*f^3*x^3+e^3*exp(2*d*x+2*c)-6*e*f^2*x^2-3*I*exp(d*x+
c)*e*f^2*x^2-6*e^2*f*x-3*I*exp(d*x+c)*e^2*f*x-2*e^3-I*exp(d*x+c)*e^3)/(exp
(2*d*x+2*c)-1)/(exp(d*x+c)-I)/a/d-12/a/d^3*f^2*e*c^2-12/a/d*f^2*e*x^2-6*I/
d^2/a*e*f^2*polylog(2,exp(d*x+c))*x+3*I/d/a*e*f^2*ln(exp(d*x+c)+1)*x^2+6*I
/d^2/a*e*f^2*polylog(2,-exp(d*x+c))*x+3*I/d^3/a*c^2*e*f^2*ln(1-exp(d*x+c))
-3*I/d^3/a*c^2*e*f^2*ln(exp(d*x+c)-1)-12*I/d^3/a*c*e*f^2*arctan(exp(d*x+c)
)+I/d/a*f^3*ln(exp(d*x+c)+1)*x^3+3*I/d^2/a*f^3*polylog(2,-exp(d*x+c))*x^2-
6*I/d^3/a*f^3*polylog(3,-exp(d*x+c))*x-I/d/a*f^3*ln(1-exp(d*x+c))*x^3-3*I/
d^2/a*e^2*f*polylog(2,exp(d*x+c))+3*I/d^2/a*e^2*f*polylog(2,-exp(d*x+c))+6
*I/d^3/a*e*f^2*polylog(3,exp(d*x+c))-6*I/d^3/a*e*f^2*polylog(3,-exp(d*x+c)
)+6*I/d^2/a*e^2*f*arctan(exp(d*x+c))+6*I/d^4/a*c^2*f^3*arctan(exp(d*x+c))-
3*I/d^2/a*f^3*polylog(2,exp(d*x+c))*x^2+6*I/d^3/a*f^3*polylog(3,exp(d*x+c)
)*x-I/d^4/a*f^3*ln(1-exp(d*x+c))*c^3-4/a/d*f^3*x^3-24/a/d^2*f^2*e*c*x-3*I/
d/a*e^2*f*ln(1-exp(d*x+c))*x+3*I/d/a*e^2*f*ln(exp(d*x+c)+1)*x-3*I/d^2/a*e^
2*f*ln(1-exp(d*x+c))*c+3*I/d^2/a*e^2*c*f*ln(exp(d*x+c)-1)+24/a/d^3*f^2*e*c
*ln(exp(d*x+c))+12/d^3/a*f^2*e*polylog(2,-I*exp(d*x+c))+6/d^3/a*f^2*e*p...
```

3.211.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2562 vs. 2(375) = 750.

Time = 0.30 (sec) , antiderivative size = 2562, normalized size of antiderivative = 6.11

$$\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
(4*I*d^3*e^3 - 12*I*c*d^2*e^2*f + 12*I*c^2*d*e*f^2 - 4*I*c^3*f^3 - 12*(-I*
d*f^3*x - I*d*e*f^2 - (d*f^3*x + d*e*f^2)*e^(3*d*x + 3*c) + (I*d*f^3*x + I
*d*e*f^2)*e^(2*d*x + 2*c) + (d*f^3*x + d*e*f^2)*e^(d*x + c))*dilog(-I*e^(d
*x + c)) - 3*(d^2*f^3*x^2 + d^2*e^2*f - 2*I*d*e*f^2 + 2*(d^2*e*f^2 - I*d*f
^3)*x + (-I*d^2*f^3*x^2 - I*d^2*e^2*f - 2*d*e*f^2 + 2*(-I*d^2*e*f^2 - d*f^
3)*x)*e^(3*d*x + 3*c) - (d^2*f^3*x^2 + d^2*e^2*f - 2*I*d*e*f^2 + 2*(d^2*e*
f^2 - I*d*f^3)*x)*e^(2*d*x + 2*c) + (I*d^2*f^3*x^2 + I*d^2*e^2*f + 2*d*e*f
^2 + 2*(I*d^2*e*f^2 + d*f^3)*x)*e^(d*x + c))*dilog(-e^(d*x + c)) + 3*(d^2*
f^3*x^2 + d^2*e^2*f + 2*I*d*e*f^2 + 2*(d^2*e*f^2 + I*d*f^3)*x - (I*d^2*f^3
*x^2 + I*d^2*e^2*f - 2*d*e*f^2 + 2*(I*d^2*e*f^2 - d*f^3)*x)*e^(3*d*x + 3*c
) - (d^2*f^3*x^2 + d^2*e^2*f + 2*I*d*e*f^2 + 2*(d^2*e*f^2 + I*d*f^3)*x)*e^
(2*d*x + 2*c) - (-I*d^2*f^3*x^2 - I*d^2*e^2*f + 2*d*e*f^2 + 2*(-I*d^2*e*f^
2 + d*f^3)*x)*e^(d*x + c))*dilog(e^(d*x + c)) - 4*(d^3*f^3*x^3 + 3*d^3*e*f
^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*e^(3*d*x
 + 3*c) - 2*(-I*d^3*f^3*x^3 - 3*I*d^3*e*f^2*x^2 - 3*I*d^3*e^2*f*x + I*d^3*
e^3 - 6*I*c*d^2*e^2*f + 6*I*c^2*d*e*f^2 - 2*I*c^3*f^3)*e^(2*d*x + 2*c) + 2
*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x - d^3*e^3 + 6*c*d^2*e^2*f
- 6*c^2*d*e*f^2 + 2*c^3*f^3)*e^(d*x + c) - (d^3*f^3*x^3 + d^3*e^3 - 3*I*d^
2*e^2*f + 3*(d^3*e*f^2 - I*d^2*f^3)*x^2 + 3*(d^3*e^2*f - 2*I*d^2*e*f^2)*x
- (I*d^3*f^3*x^3 + I*d^3*e^3 + 3*d^2*e^2*f - 3*(-I*d^3*e*f^2 - d^2*f^3)...
```

3.211.6 Sympy [F]

$$\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= -\frac{i\left(\int \frac{e^3\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{f^3x^3\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{3ef^2x^2\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{3e^2fx\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**3*csch(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(f**3
*x**3*csch(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(3*e*f**2*x**2*cs
ch(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(3*e**2*f*x*csch(c + d*x)
**2/(sinh(c + d*x) - I), x))/a
```

3.211.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 939 vs. $2(375) = 750$.

Time = 0.42 (sec) , antiderivative size = 939, normalized size of antiderivative = 2.24

$$\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-e^3*(2*(e^(-d*x - c) - I*e^(-2*d*x - 2*c) + 2*I)/((a*e^(-d*x - c) - I*a*e
^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c) + I*a)*d) - I*log(e^(-d*x - c) + 1)/(
a*d) + I*log(e^(-d*x - c) - 1)/(a*d)) - 12*e^2*f*x/(a*d) + 3*e^2*f*log(e^(
d*x + c) + 1)/(a*d^2) + 6*e^2*f*log(e^(d*x + c) - I)/(a*d^2) + 3*e^2*f*log
(e^(d*x + c) - 1)/(a*d^2) - 2*(-2*I*f^3*x^3 - 6*I*e*f^2*x^2 - 6*I*e^2*f*x
- (-I*f^3*x^3*e^(2*c) - 3*I*e*f^2*x^2*e^(2*c) - 3*I*e^2*f*x*e^(2*c))*e^(2*
d*x) + (f^3*x^3*e^c + 3*e*f^2*x^2*e^c + 3*e^2*f*x*e^c)*e^(d*x))/(a*d*e^(3*
d*x + 3*c) - I*a*d*e^(2*d*x + 2*c) - a*d*e^(d*x + c) + I*a*d) + 12*(d*x*lo
g(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*e*f^2/(a*d^3) + I*(d^3*x^3*l
og(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^
(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) - I*(d^3*x^3*log(-e^(
d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c
)) + 6*polylog(4, e^(d*x + c)))*f^3/(a*d^4) + 6*(d^2*x^2*log(I*e^(d*x + c)
 + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a
*d^4) - 3*(-I*d*e^2*f - 2*e*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x
 + c)))/(a*d^3) + 3*(-I*d*e^2*f + 2*e*f^2)*(d*x*log(-e^(d*x + c) + 1) + di
log(e^(d*x + c)))/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog
(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*(-I*d*e*f^2 + f^3)/(a*d^4) - 3*
(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -
e^(d*x + c)))*(-I*d*e*f^2 - f^3)/(a*d^4) + 1/4*(I*d^4*f^3*x^4 - 4*(-I*d...
```

3.211.8 Giac [F]

$$\int \frac{(e+fx)^3 \text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\text{csch}\,(dx+c)^2}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*csch(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\sinh\,(c+d\,x)^2\,\left(a+a\sinh\,(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)^3/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^3/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.212 $\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.212.1 Optimal result

Integrand size = 31, antiderivative size = 296

$$\begin{aligned}\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{2(e+fx)^2}{ad}+\frac{2i(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{ad}\\ &-\frac{(e+fx)^2\coth(c+dx)}{ad}+\frac{4f(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}\\ &+\frac{2f(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad^2}\\ &+\frac{2if(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}\\ &+\frac{4f^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}\\ &-\frac{2if(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}\\ &+\frac{f^2\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3}-\frac{2if^2\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}\\ &+\frac{2if^2\,\text{PolyLog}\left(3,e^{c+dx}\right)}{ad^3}-\frac{(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\end{aligned}$$

output
```
-2*(f*x+e)^2/a/d+2*I*(f*x+e)^2*arctanh(exp(d*x+c))/a/d-(f*x+e)^2*coth(d*x+
c)/a/d+4*f*(f*x+e)*ln(1+I*exp(d*x+c))/a/d^2+2*f*(f*x+e)*ln(1-exp(2*d*x+2*c
))/a/d^2+2*I*f*(f*x+e)*polylog(2,-exp(d*x+c))/a/d^2+4*f^2*polylog(2,-I*exp
(d*x+c))/a/d^3-2*I*f*(f*x+e)*polylog(2,exp(d*x+c))/a/d^2+f^2*polylog(2,exp
(2*d*x+2*c))/a/d^3-2*I*f^2*polylog(3,-exp(d*x+c))/a/d^3+2*I*f^2*polylog(3,
exp(d*x+c))/a/d^3-(f*x+e)^2*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

3.212.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 803 vs. $2(296) = 592$.

Time = 8.08 (sec) , antiderivative size = 803, normalized size of antiderivative = 2.71

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= -\frac{4ie^c f\left(\frac{e^{-c}(e+fx)^2}{2f}+\frac{(i+e^{-c})(e+fx)\log(1-ie^{-c-dx})}{d}-\frac{e^{-c}(1+ie^c)f\,\text{PolyLog}(2,ie^{-c-dx})}{d^2}\right)}{ad(-i+e^c)}$$
$$+\frac{id^2e(-1+e^{2c})(de+2if)x+d^2e(1-e^{2c})(ide+2f)x-2d^2(e+fx)^2+2d(-1+e^{2c})f(-ide+f)x\log}{}$$
$$+\frac{\text{sech}\left(\frac{c}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-e^2\sinh\left(\frac{dx}{2}\right)-2efx\sinh\left(\frac{dx}{2}\right)-f^2x^2\sinh\left(\frac{dx}{2}\right)\right)}{2ad}$$
$$+\frac{\text{csch}\left(\frac{c}{2}\right)\text{csch}\left(\frac{c}{2}+\frac{dx}{2}\right)\left(e^2\sinh\left(\frac{dx}{2}\right)+2efx\sinh\left(\frac{dx}{2}\right)+f^2x^2\sinh\left(\frac{dx}{2}\right)\right)}{2ad}$$
$$-\frac{2\left(e^2\sinh\left(\frac{dx}{2}\right)+2efx\sinh\left(\frac{dx}{2}\right)+f^2x^2\sinh\left(\frac{dx}{2}\right)\right)}{ad\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)+i\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-4*I)*E^c*f*((e + f*x)^2/(2*E^c*f) + ((I + E^(-c))*(e + f*x)*Log[1 - I*E
^(-c - d*x)])/d - ((1 + I*E^c)*f*PolyLog[2, I*E^(-c - d*x)])/(d^2*E^c)))/(
a*d*(-I + E^c)) + (I*d^2*e*(-1 + E^(2*c))*(d*e + (2*I)*f)*x + d^2*e*(1 - E
^(2*c))*(I*d*e + 2*f)*x - 2*d^2*(e + f*x)^2 + 2*d*(-1 + E^(2*c))*f*((-I)*d
*e + f)*x*Log[1 - E^(-c - d*x)] - I*d^2*(-1 + E^(2*c))*f^2*x^2*Log[1 - E^(
-c - d*x)] + 2*d*(-1 + E^(2*c))*f*(I*d*e + f)*x*Log[1 + E^(-c - d*x)] + I*
d^2*(-1 + E^(2*c))*f^2*x^2*Log[1 + E^(-c - d*x)] + d*e*(-1 + E^(2*c))*((-I
)*d*e + 2*f)*Log[1 - E^(c + d*x)] + d*e*(-1 + E^(2*c))*(I*d*e + 2*f)*Log[1
 + E^(c + d*x)] - 2*(-1 + E^(2*c))*f*(I*d*e + f)*PolyLog[2, -E^(-c - d*x)]
 - (2*I)*d*(-1 + E^(2*c))*f^2*x*PolyLog[2, -E^(-c - d*x)] + (2*I)*(-1 + E^
(2*c))*(d*e + I*f)*f*PolyLog[2, E^(-c - d*x)] + (2*I)*d*(-1 + E^(2*c))*f^2
*x*PolyLog[2, E^(-c - d*x)] - (2*I)*(-1 + E^(2*c))*f^2*PolyLog[3, -E^(-c -
 d*x)] + (2*I)*(-1 + E^(2*c))*f^2*PolyLog[3, E^(-c - d*x)])/(a*d^3*(-1 + E
^(2*c))) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(e^2*Sinh[(d*x)/2]) - 2*e*f*x*
Sinh[(d*x)/2] - f^2*x^2*Sinh[(d*x)/2]))/(2*a*d) + (Csch[c/2]*Csch[c/2 + (d
*x)/2]*(e^2*Sinh[(d*x)/2] + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*Sinh[(d*x)/2])
)/(2*a*d) - (2*(e^2*Sinh[(d*x)/2] + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*Sinh[(
d*x)/2]))/(a*d*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2
 + (d*x)/2]))
```

3.212.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)^2\text{csch}^2(c+dx)dx}{a}-i\int \frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -(e+fx)^2\csc(ic+idx)^2dx}{a}-i\int \frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int (e+fx)^2\csc(ic+idx)^2dx}{a}-i\int \frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{4672}$$

$$-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2if\int -i(e+fx)\coth(c+dx)dx}{d}}{a}-i\int \frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int (e+fx)\coth(c+dx)dx}{d}}{a}-i\int \frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}-i\int \frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}-i\int \frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{4201}$$

$$-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}-i\int \frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{2620}$$

$$-\frac{-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}-i\int\frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{2715}$$

$$-\frac{-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}-i\int\frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{2838}$$

$$-i\int\frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{6109}$$

$$-i\left(\frac{\int(e+fx)^2\text{csch}(c+dx)dx}{a}-i\int\frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx\right)-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{\int i(e+fx)^2\csc(ic+idx)dx}{a}-i\int\frac{(e+fx)^2}{\sin(ic+idx)a+a}dx\right)-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-i\int\frac{(e+fx)^2}{\sin(ic+idx)a+a}dx\right)-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{3799}$$

3.212. $\int\frac{(e+fx)^2\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}dx$

$$-\frac{-i\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int-(e+fx)^2\operatorname{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}\right)-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \color{blue}{25}$$

$$-\frac{-i\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}+\frac{i\int-(e+fx)^2\operatorname{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}\right)-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \color{blue}{25}$$

$$-\frac{-i\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)^2\operatorname{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}\right)-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \color{blue}{3042}$$

$$-\frac{-i\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right)-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \color{blue}{4670}$$

$$-\frac{-i\left(\frac{i\left(\frac{2if\int(e+fx)\log\left(1-e^{c+dx}\right)dx}{d}-\frac{2if\int(e+fx)\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{d}\right)}{a}-\frac{i\int(e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}\right.}{2a}\right.-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \color{blue}{3011}$$

3.212. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}+\frac{2i(e+fx\cdots}{\cdots}\right.\cdots}{a}\right.\cdots$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 2720$$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\cdots}{d}\right.\cdots}{d}\right.\cdots}{a}\right.\cdots$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 4672$$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\cdots}{d}\right.\cdots}{d}\right.\cdots}{a}\right.\cdots$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 26$$

3.212. $\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \textbf{3042}$$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \textbf{26}$$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \textbf{4199}$$

3.212. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{2620}$$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{2715}$$

3.212. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 2838$$

$$-i\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{d}\right.}{d}\right.}{a}\right.$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.212.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.212. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

3.212.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 888 vs. $2(275) = 550$.

Time = 2.52 (sec) , antiderivative size = 889, normalized size of antiderivative = 3.00

method	result
risch	$-\frac{4f^2x^2}{ad}+\frac{8f^2c\ln(\mathrm{e}^{dx+c})}{a\,d^3}+\frac{2f^2\,\mathrm{polylog}(2,-\mathrm{e}^{dx+c})}{a\,d^3}+\frac{2f^2\,\mathrm{polylog}(2,\mathrm{e}^{dx+c})}{a\,d^3}-\frac{8f\ln(\mathrm{e}^{dx+c})e}{a\,d^2}-\frac{8f^2cx}{a\,d^2}+\frac{2f^2\ln(1-\mathrm{e}^{dx+}}{d^2a}$

input
```
int((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-4*f^2*x^2/a/d+8/a/d^3*f^2*c*ln(exp(d*x+c))-2*I*f^2*polylog(3,-exp(d*x+c))
/a/d^3+2*f^2*polylog(2,-exp(d*x+c))/a/d^3+2*f^2*polylog(2,exp(d*x+c))/a/d^
3+4*f^2*polylog(2,-I*exp(d*x+c))/a/d^3-8/a/d^2*f*ln(exp(d*x+c))*e-8/a/d^2*
f^2*c*x+4/a/d^2*f^2*ln(1+I*exp(d*x+c))*x+4/a/d^3*f^2*ln(1+I*exp(d*x+c))*c+
2*I*f^2*polylog(3,exp(d*x+c))/a/d^3+2/d^2/a*f^2*ln(1-exp(d*x+c))*x+2/d^2/a
*f^2*ln(exp(d*x+c)+1)*x+2/d^3/a*f^2*ln(1-exp(d*x+c))*c-2/d^3/a*f^2*c*ln(1+
exp(2*d*x+2*c))-2/d^3/a*f^2*c*ln(exp(d*x+c)-1)+I/d/a*e^2*ln(exp(d*x+c)+1)+
2/d^2/a*e*f*ln(exp(d*x+c)-1)+2/d^2/a*e*f*ln(exp(d*x+c)+1)+2/d^2/a*e*f*ln(1
+exp(2*d*x+2*c))-I/d/a*e^2*ln(exp(d*x+c)-1)+I/d/a*f^2*ln(exp(d*x+c)+1)*x^2
+2*I/d^2/a*f^2*polylog(2,-exp(d*x+c))*x-4*I/d^3/a*c*f^2*arctan(exp(d*x+c))
-I/d^3/a*c^2*f^2*ln(exp(d*x+c)-1)+4*I/d^2/a*e*f*arctan(exp(d*x+c))+2*I/d^2
/a*e*f*polylog(2,-exp(d*x+c))-2*I/d^2/a*e*f*polylog(2,exp(d*x+c))-4/a/d^3*
f^2*c^2-2*I/d/a*e*f*ln(1-exp(d*x+c))*x+2*I/d/a*e*f*ln(exp(d*x+c)+1)*x-2*I/
d^2/a*e*f*ln(1-exp(d*x+c))*c+2*I/d^2/a*e*c*f*ln(exp(d*x+c)-1)-2*I*(f^2*x^2
*exp(2*d*x+2*c)+2*e*f*x*exp(2*d*x+2*c)+e^2*exp(2*d*x+2*c)-2*x^2*f^2-I*exp(
d*x+c)*f^2*x^2-4*e*f*x-2*I*exp(d*x+c)*e*f*x-2*e^2-I*exp(d*x+c)*e^2)/(exp(2
*d*x+2*c)-1)/(exp(d*x+c)-I)/a/d+I/d^3/a*f^2*ln(1-exp(d*x+c))*c^2-I/d/a*f^2
*ln(1-exp(d*x+c))*x^2-2*I/d^2/a*f^2*polylog(2,exp(d*x+c))*x
```

3.212.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1355 vs. $2(263) = 526$.

Time = 0.28 (sec) , antiderivative size = 1355, normalized size of antiderivative = 4.58

$$\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
(4*I*d^2*e^2 - 8*I*c*d*e*f + 4*I*c^2*f^2 + 4*(f^2*e^(3*d*x + 3*c) - I*f^2*
e^(2*d*x + 2*c) - f^2*e^(d*x + c) + I*f^2)*dilog(-I*e^(d*x + c)) - 2*(d*f^
2*x + d*e*f - I*f^2 + (-I*d*f^2*x - I*d*e*f - f^2)*e^(3*d*x + 3*c) - (d*f^
2*x + d*e*f - I*f^2)*e^(2*d*x + 2*c) + (I*d*f^2*x + I*d*e*f + f^2)*e^(d*x
+ c))*dilog(-e^(d*x + c)) + 2*(d*f^2*x + d*e*f + I*f^2 - (I*d*f^2*x + I*d*
e*f - f^2)*e^(3*d*x + 3*c) - (d*f^2*x + d*e*f + I*f^2)*e^(2*d*x + 2*c) - (
-I*d*f^2*x - I*d*e*f + f^2)*e^(d*x + c))*dilog(e^(d*x + c)) - 4*(d^2*f^2*x
^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(3*d*x + 3*c) - 2*(-I*d^2*f^2*x^
2 - 2*I*d^2*e*f*x + I*d^2*e^2 - 4*I*c*d*e*f + 2*I*c^2*f^2)*e^(2*d*x + 2*c)
 + 2*(d^2*f^2*x^2 + 2*d^2*e*f*x - d^2*e^2 + 4*c*d*e*f - 2*c^2*f^2)*e^(d*x
+ c) - (d^2*f^2*x^2 + d^2*e^2 - 2*I*d*e*f + 2*(d^2*e*f - I*d*f^2)*x - (I*d
^2*f^2*x^2 + I*d^2*e^2 + 2*d*e*f - 2*(-I*d^2*e*f - d*f^2)*x)*e^(3*d*x + 3*
c) - (d^2*f^2*x^2 + d^2*e^2 - 2*I*d*e*f + 2*(d^2*e*f - I*d*f^2)*x)*e^(2*d*
x + 2*c) - (-I*d^2*f^2*x^2 - I*d^2*e^2 - 2*d*e*f - 2*(I*d^2*e*f + d*f^2)*x
)*e^(d*x + c))*log(e^(d*x + c) + 1) - 4*(-I*d*e*f + I*c*f^2 - (d*e*f - c*f
^2)*e^(3*d*x + 3*c) + (I*d*e*f - I*c*f^2)*e^(2*d*x + 2*c) + (d*e*f - c*f^2
)*e^(d*x + c))*log(e^(d*x + c) - I) + (d^2*e^2 - 2*(c - I)*d*e*f + (c^2 -
2*I*c)*f^2 + (-I*d^2*e^2 - 2*(-I*c - 1)*d*e*f + (-I*c^2 - 2*c)*f^2)*e^(3*d
*x + 3*c) - (d^2*e^2 - 2*(c - I)*d*e*f + (c^2 - 2*I*c)*f^2)*e^(2*d*x + 2*c
) + (I*d^2*e^2 - 2*(I*c + 1)*d*e*f + (I*c^2 + 2*c)*f^2)*e^(d*x + c))*lo...
```

3.212.6 Sympy [F]

$$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^2\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{f^2x^2\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{2efx\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**2*csch(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(f**2
*x**2*csch(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*csch(c +
 d*x)**2/(sinh(c + d*x) - I), x))/a
```

3.212.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 601 vs. 2(263) = 526.

Time = 0.42 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.03

$$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx =$$

$$-e^2\left(\frac{2\left(e^{(-dx-c)}-i\,e^{(-2\,dx-2\,c)}+2i\right)}{(ae^{(-dx-c)}-i\,ae^{(-2\,dx-2\,c)}-ae^{(-3\,dx-3\,c)}+i\,a)d}-\frac{i\,\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{i\,\log\left(e^{(-dx-c)}-1\right)}{ad}\right)$$
$$-\frac{2\,f^2x^2}{ad}-\frac{8\,efx}{ad}$$
$$-\frac{2\left(-2i\,f^2x^2-4i\,efx-\left(-i\,f^2x^2e^{(2\,c)}-2i\,efxe^{(2\,c)}\right)e^{(2\,dx)}+(f^2x^2e^c+2\,efxe^c)e^{(dx)}\right)}{ade^{(3\,dx+3\,c)}-i\,ade^{(2\,dx+2\,c)}-ade^{(dx+c)}+i\,ad}$$
$$+\frac{2\,ef\log\left(e^{(dx+c)}+1\right)}{ad^2}+\frac{4\,ef\log\left(e^{(dx+c)}-i\right)}{ad^2}+\frac{2\,ef\log\left(e^{(dx+c)}-1\right)}{ad^2}$$
$$+\frac{i\left(d^2x^2\log\left(e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(-e^{(dx+c)}\right)-2\,\mathrm{Li}_3(-e^{(dx+c)})\right)f^2}{ad^3}$$
$$-\frac{i\left(d^2x^2\log\left(-e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(e^{(dx+c)}\right)-2\,\mathrm{Li}_3(e^{(dx+c)})\right)f^2}{ad^3}$$
$$+\frac{4\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)\right)f^2}{ad^3}$$
$$-\frac{2\,(-i\,def-f^2)\left(dx\log\left(e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-e^{(dx+c)}\right)\right)}{ad^3}$$
$$+\frac{2\,(-i\,def+f^2)\left(dx\log\left(-e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(e^{(dx+c)}\right)\right)}{ad^3}$$
$$+\frac{i\,d^3f^2x^3-3\,(-i\,def+f^2)d^2x^2}{3\,ad^3}-\frac{i\,d^3f^2x^3-3\,(-i\,def-f^2)d^2x^2}{3\,ad^3}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-e^2*(2*(e^(-d*x - c) - I*e^(-2*d*x - 2*c) + 2*I)/((a*e^(-d*x - c) - I*a*e
^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c) + I*a)*d) - I*log(e^(-d*x - c) + 1)/(
a*d) + I*log(e^(-d*x - c) - 1)/(a*d)) - 2*f^2*x^2/(a*d) - 8*e*f*x/(a*d) -
2*(-2*I*f^2*x^2 - 4*I*e*f*x - (-I*f^2*x^2*e^(2*c) - 2*I*e*f*x*e^(2*c))*e^(
2*d*x) + (f^2*x^2*e^c + 2*e*f*x*e^c)*e^(d*x))/(a*d*e^(3*d*x + 3*c) - I*a*d
*e^(2*d*x + 2*c) - a*d*e^(d*x + c) + I*a*d) + 2*e*f*log(e^(d*x + c) + 1)/(
a*d^2) + 4*e*f*log(e^(d*x + c) - I)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/(
a*d^2) + I*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*p
olylog(3, -e^(d*x + c)))*f^2/(a*d^3) - I*(d^2*x^2*log(-e^(d*x + c) + 1) +
2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^2/(a*d^3) + 4*(d*x
*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*f^2/(a*d^3) - 2*(-I*d*e*f
 - f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a*d^3) + 2*(-I*d
*e*f + f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a*d^3) + 1/3
*(I*d^3*f^2*x^3 - 3*(-I*d*e*f + f^2)*d^2*x^2)/(a*d^3) - 1/3*(I*d^3*f^2*x^3
 - 3*(-I*d*e*f - f^2)*d^2*x^2)/(a*d^3)
```

3.212.8 Giac [F]

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{csch}\,(dx+c)^2}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*csch(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\sinh\,(c+d\,x)^2\,\left(a+a\sinh\,(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)^2/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^2/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.213 $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.213.1 Optimal result

Integrand size = 29, antiderivative size = 163

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{2f\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2} + \frac{f\log(\sinh(c+dx))}{ad^2} + \frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} - \frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} - \frac{(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

output
```
2*I*(f*x+e)*arctanh(exp(d*x+c))/a/d-(f*x+e)*coth(d*x+c)/a/d+2*f*ln(cosh(1/
2*c+1/4*I*Pi+1/2*d*x))/a/d^2+f*ln(sinh(d*x+c))/a/d^2+I*f*polylog(2,-exp(d*
x+c))/a/d^2-I*f*polylog(2,exp(d*x+c))/a/d^2-(f*x+e)*tanh(1/2*c+1/4*I*Pi+1/
2*d*x)/a/d
```

3.213.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 366 vs. 2(163) = 326.

Time = 3.08 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.25

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(-d(e+fx)\cosh\left(\frac{1}{2}(c+dx)\right)\left(i+\coth\left(\frac{1}{2}(c+dx)\right)\right)+4if\operatorname{arcta}\right.}{}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(-(d*(e + f*x)*Cosh[(c + d*x)/2
]*(I + Coth[(c + d*x)/2])) + (4*I)*f*ArcTan[Tanh[(c + d*x)/2]]*(Cosh[(c +
d*x)/2] + I*Sinh[(c + d*x)/2]) + 2*f*Log[Cosh[c + d*x]]*(Cosh[(c + d*x)/2]
 + I*Sinh[(c + d*x)/2]) + 2*(f*(c + d*x) + (f - I*d*(e + f*x))*Log[1 - E^(
-c - d*x)] + (f + I*d*(e + f*x))*Log[1 + E^(-c - d*x)] - I*f*PolyLog[2, -E
^(-c - d*x)] + I*f*PolyLog[2, E^(-c - d*x)])*(Cosh[(c + d*x)/2] + I*Sinh[(
c + d*x)/2]) - 4*d*(e + f*x)*Sinh[(c + d*x)/2] + 2*f*(c + d*x)*((-I)*Cosh[
(c + d*x)/2] + Sinh[(c + d*x)/2]) - I*d*(e + f*x)*Sinh[(c + d*x)/2]*(-I +
Tanh[(c + d*x)/2])))/(2*d^2*(a + I*a*Sinh[c + d*x]))
```

3.213.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.07, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.793$, Rules used = {6109, 3042, 25, 4672, 26, 3042, 26, 3956, 6109, 3042, 26, 3799, 25, 25, 3042, 4670, 2715, 2838, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$\Big\downarrow \ 6109$$
$$\frac{\int (e+fx)\operatorname{csch}^2(c+dx)dx}{a}-i\int \frac{(e+fx)\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \ 3042$$

$$\frac{\int -\left((e+fx)\csc(ic+idx)^2\right)dx}{a} - i\int \frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \ 25$$

$$-\frac{\int (e+fx)\csc(ic+idx)^2dx}{a} - i\int \frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \ 4672$$

$$-\frac{\frac{(e+fx)\coth(c+dx)}{d} - \frac{if\int -i\coth(c+dx)dx}{d}}{a} - i\int \frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \ 26$$

$$-\frac{\frac{(e+fx)\coth(c+dx)}{d} - \frac{f\int \coth(c+dx)dx}{d}}{a} - i\int \frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{(e+fx)\coth(c+dx)}{d} - \frac{f\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a} - i\int \frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \ 26$$

$$-\frac{\frac{(e+fx)\coth(c+dx)}{d} + \frac{if\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a} - i\int \frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \ 3956$$

$$-i\int \frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx - \frac{\frac{(e+fx)\coth(c+dx)}{d} - \frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \ 6109$$

$$-i\left(\frac{\int (e+fx)\text{csch}(c+dx)dx}{a} - i\int \frac{e+fx}{i\sinh(c+dx)a+a}dx\right) - \frac{\frac{(e+fx)\coth(c+dx)}{d} - \frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \ 3042$$

$$-i\left(\frac{\int i(e+fx)\csc(ic+idx)dx}{a} - i\int \frac{e+fx}{\sin(ic+idx)a+a}dx\right) - \frac{\frac{(e+fx)\coth(c+dx)}{d} - \frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \ 26$$

3.213. $\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-i\left(\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-i\int \frac{e+fx}{\sin(ic+idx)a+a}dx\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{3799}$$

$$-i\left(\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-\frac{i\int -\left((e+fx)\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)\right)dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{25}$$

$$-i\left(\frac{i\int (e+fx)\csc(ic+idx)dx}{a}+\frac{i\int -\left((e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{25}$$

$$-i\left(\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-\frac{i\int (e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-\frac{i\int (e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{4670}$$

$$-i\left(\frac{i\left(\frac{if\int \log(1-e^{c+dx})dx}{d}-\frac{if\int \log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int (e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{2715}$$

3.213. $\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-i\left(\frac{i\left(\frac{if\int e^{-c-dx}\log(1-e^{c+dx})de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log(1+e^{c+dx})de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int(e+fx)\csc\left(\frac{ic}{2}+\right.}{2a}\right.$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow 2838$$

$$-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\int(e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right)-$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow 4672$$

$$-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2if\int-i\tanh\right.}{2a}\right.$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow 26$$

$$-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int\tanh\left(\frac{c}{2}+\right.}{d}\right.}{2a}\right.$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow 3042$$

$$-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int-i\tan\left(\right.}{}\right.}{2a}\right.$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

3.213. $\int\frac{(e+fx)\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\Big\downarrow \color{blue}{26}$$

$$-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2if\int\tan\left(\frac{ic}{2}+\frac{idx}{2}-\frac{\pi}{4}\right)dx}{d}+\frac{2(e+fx)\tanh(}{d}\right.}{2a}\right.$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \color{blue}{3956}$$

$$-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{4f\log\left(\cosh\left(\right.\right.}{d^2}\right.}{2a}\right.$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

input
```
Int[((e + f*x)*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-((((e + f*x)*Coth[c + d*x])/d - (f*Log[(-I)*Sinh[c + d*x]])/d^2)/a) - I*(
(I*(((2*I)*(e + f*x)*ArcTanh[E^(c + d*x)])/d + (I*f*PolyLog[2, -E^(c + d*x
)])/d^2 - (I*f*PolyLog[2, E^(c + d*x)])/d^2))/a - ((I/2)*((-4*f*Log[Cosh[c
/2 + (I/4)*Pi + (d*x)/2]])/d^2 + (2*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/
2])/d))/a)
```

3.213.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.213. $\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

3.213. $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.213.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(143) = 286$.

Time = 2.07 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.05

method	result
risch	$-\frac{2i(fx\,e^{2dx+2c}+e\,e^{2dx+2c}-2fx-ie^{dx+c}fx-2e-ie^{dx+c}e)}{(e^{2dx+2c}-1)(e^{dx+c}-i)ad}+\frac{icf\ln(e^{dx+c}-1)}{d^2a}-\frac{ie\ln(e^{dx+c}-1)}{da}+\frac{ie\ln(e^{dx+c}+1)}{da}-\frac{if\ln(1$

input
```
int((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*I*(f*x*exp(2*d*x+2*c)+e*exp(2*d*x+2*c)-2*f*x-I*exp(d*x+c)*f*x-2*e-I*exp
(d*x+c)*e)/(exp(2*d*x+2*c)-1)/(exp(d*x+c)-I)/a/d+I/d^2/a*c*f*ln(exp(d*x+c)
-1)-I/d/a*e*ln(exp(d*x+c)-1)+I/d/a*e*ln(exp(d*x+c)+1)-I/d^2/a*f*ln(1-exp(d
*x+c))*c+1/d^2/a*f*ln(exp(d*x+c)-1)+1/d^2/a*f*ln(exp(d*x+c)+1)+1/d^2/a*f*l
n(1+exp(2*d*x+2*c))-4/d^2/a*f*ln(exp(d*x+c))-I/d/a*f*ln(1-exp(d*x+c))*x+I/
d/a*f*ln(exp(d*x+c)+1)*x+2*I/d^2/a*f*arctan(exp(d*x+c))-I*f*polylog(2,exp(
d*x+c))/a/d^2+I*f*polylog(2,-exp(d*x+c))/a/d^2
```

3.213.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(139) = 278$.

Time = 0.27 (sec) , antiderivative size = 506, normalized size of antiderivative = 3.10

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{4i\,de-2i\,cf+\left(i\,fe^{(3\,dx+3\,c)}+fe^{(2\,dx+2\,c)}-i\,fe^{(dx+c)}-f\right)\text{Li}_2\left(-e^{(dx+c)}\right)+\left(-i\,fe^{(3\,dx+3\,c)}-fe^{(2\,dx+2\,c)}+\right.}{}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(4*I*d*e - 2*I*c*f + (I*f*e^(3*d*x + 3*c) + f*e^(2*d*x + 2*c) - I*f*e^(d*x
 + c) - f)*dilog(-e^(d*x + c)) + (-I*f*e^(3*d*x + 3*c) - f*e^(2*d*x + 2*c)
 + I*f*e^(d*x + c) + f)*dilog(e^(d*x + c)) - 2*(2*d*f*x + c*f)*e^(3*d*x +
3*c) - 2*(-I*d*f*x + I*d*e - I*c*f)*e^(2*d*x + 2*c) + 2*(d*f*x - d*e + c*f
)*e^(d*x + c) - (d*f*x + d*e - (I*d*f*x + I*d*e + f)*e^(3*d*x + 3*c) - (d*
f*x + d*e - I*f)*e^(2*d*x + 2*c) - (-I*d*f*x - I*d*e - f)*e^(d*x + c) - I*
f)*log(e^(d*x + c) + 1) + 2*(f*e^(3*d*x + 3*c) - I*f*e^(2*d*x + 2*c) - f*e
^(d*x + c) + I*f)*log(e^(d*x + c) - I) + (d*e - (c - I)*f + (-I*d*e + (I*c
 + 1)*f)*e^(3*d*x + 3*c) - (d*e - (c - I)*f)*e^(2*d*x + 2*c) + (I*d*e + (-
I*c - 1)*f)*e^(d*x + c))*log(e^(d*x + c) - 1) + (d*f*x + c*f + (-I*d*f*x -
 I*c*f)*e^(3*d*x + 3*c) - (d*f*x + c*f)*e^(2*d*x + 2*c) + (I*d*f*x + I*c*f
)*e^(d*x + c))*log(-e^(d*x + c) + 1))/(a*d^2*e^(3*d*x + 3*c) - I*a*d^2*e^(
2*d*x + 2*c) - a*d^2*e^(d*x + c) + I*a*d^2)
```

3.213.6 Sympy [F]

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{fx\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)*csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e*csch(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(f*x*csc
h(c + d*x)**2/(sinh(c + d*x) - I), x))/a
```

3.213.7 Maxima [F]

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)\operatorname{csch}(dx+c)^2}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(4*I*d*integrate(1/4*x/(a*d*e^(d*x + c) + a*d), x) + 4*I*d*integrate(1/4*
x/(a*d*e^(d*x + c) - a*d), x) + 2*(x*e^(3*d*x + 3*c) - I*x)/(a*d*e^(3*d*x
+ 3*c) - I*a*d*e^(2*d*x + 2*c) - a*d*e^(d*x + c) + I*a*d) + 2*(d*x + c)/(a
*d^2) - 2*log((e^(d*x + c) - I)*e^(-c))/(a*d^2) - log(e^(d*x + c) + 1)/(a*
d^2) - log(e^(d*x + c) - 1)/(a*d^2))*f - e*(2*(e^(-d*x - c) - I*e^(-2*d*x
- 2*c) + 2*I)/((a*e^(-d*x - c) - I*a*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c)
 + I*a)*d) - I*log(e^(-d*x - c) + 1)/(a*d) + I*log(e^(-d*x - c) - 1)/(a*d)
)
```

3.213.8 Giac [F]

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)\operatorname{csch}(dx+c)^2}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*csch(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\sinh(c+d\,x)^2\,\left(a+a\sinh(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.214 $\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.214.1 Optimal result

Integrand size = 24, antiderivative size = 57

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{i\operatorname{arctanh}(\cosh(c+dx))}{ad}-\frac{2\coth(c+dx)}{ad}+\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}$$

output
```
I*arctanh(cosh(d*x+c))/a/d-2*coth(d*x+c)/a/d+coth(d*x+c)/d/(a+I*a*sinh(d*x
+c))
```

3.214.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\operatorname{sech}(c+dx)\left(i-i\operatorname{arctanh}\left(\sqrt{\cosh^2(c+dx)}\right)\sqrt{\cosh^2(c+dx)}+\operatorname{csch}(c+dx)+2\sinh(c+dx)\right)}{ad}$$

input
```
Integrate[Csch[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-((Sech[c + d*x]*(I - I*ArcTanh[Sqrt[Cosh[c + d*x]^2]]*Sqrt[Cosh[c + d*x]^
2] + Csch[c + d*x] + 2*Sinh[c + d*x]))/(a*d))
```

3.214.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 25, 3247, 3042, 25, 3227, 25, 26, 3042, 25, 26, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ic+idx)^2(a+a\sin(ic+idx))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sin(ic+idx)^2(\sin(ic+idx)a+a)}dx$$

$$\Big\downarrow \text{3247}$$

$$\frac{\int \operatorname{csch}^2(c+dx)(2a-ia\sinh(c+dx))dx}{a^2}+\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\frac{2a-a\sin(ic+idx)}{\sin(ic+idx)^2}dx}{a^2}+\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\int \frac{2a-a\sin(ic+idx)}{\sin(ic+idx)^2}dx}{a^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{2a\int -\operatorname{csch}^2(c+dx)dx-a\int -i\operatorname{csch}(c+dx)dx}{a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{-2a\int \operatorname{csch}^2(c+dx)dx-a\int -i\operatorname{csch}(c+dx)dx}{a^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{-2a\int \operatorname{csch}^2(c+dx)dx+ia\int \operatorname{csch}(c+dx)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{ia\int i\csc(ic+idx)dx-2a\int-\csc(ic+idx)^2dx}{a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{ia\int i\csc(ic+idx)dx+2a\int\csc(ic+idx)^2dx}{a^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{2a\int\csc(ic+idx)^2dx-a\int\csc(ic+idx)dx}{a^2}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\frac{2ia\int 1d(-i\coth(c+dx))}{d}-a\int\csc(ic+idx)dx}{a^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\frac{2a\coth(c+dx)}{d}-a\int\csc(ic+idx)dx}{a^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\frac{2a\coth(c+dx)}{d}-\frac{ia\text{arctanh}(\cosh(c+dx))}{d}}{a^2}$$

input
```
Int[Csch[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-((((-I)*a*ArcTanh[Cosh[c + d*x]])/d + (2*a*Coth[c + d*x])/d)/a^2) + Coth[
c + d*x]/(d*(a + I*a*Sinh[c + d*x]))
```

3.214.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

3.214. $\int\frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.214.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-2i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}$	63
default	$\frac{-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-2i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}$	63
parallelrisch	$\frac{\left(2i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-i\coth\left(\frac{dx}{2}+\frac{c}{2}\right)-6i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2\left(i-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)da}$	86
risch	$-\frac{2i\left(-i\mathrm{e}^{dx+c}+\mathrm{e}^{2dx+2c}-2\right)}{\left(\mathrm{e}^{2dx+2c}-1\right)\left(\mathrm{e}^{dx+c}-i\right)ad}+\frac{i\ln\left(\mathrm{e}^{dx+c}+1\right)}{ad}-\frac{i\ln\left(\mathrm{e}^{dx+c}-1\right)}{ad}$	91

input
```
int(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a*(-tanh(1/2*d*x+1/2*c)-4/(-I+tanh(1/2*d*x+1/2*c))-1/tanh(1/2*d*x+1/
2*c)-2*I*ln(tanh(1/2*d*x+1/2*c)))
```

3.214.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(53) = 106$.

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.56

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(i\,e^{(3\,dx+3\,c)}+e^{(2\,dx+2\,c)}-i\,e^{(dx+c)}-1\right)\log\left(e^{(dx+c)}+1\right)+\left(-i\,e^{(3\,dx+3\,c)}-e^{(2\,dx+2\,c)}+i\,e^{(dx+c)}+1\right)\log\left(e^{(}\right.}{ade^{(3\,dx+3\,c)}-i\,ade^{(2\,dx+2\,c)}-ade^{(dx+c)}+i\,ad}$$

input
```
integrate(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((I*e^(3*d*x + 3*c) + e^(2*d*x + 2*c) - I*e^(d*x + c) - 1)*log(e^(d*x + c)
 + 1) + (-I*e^(3*d*x + 3*c) - e^(2*d*x + 2*c) + I*e^(d*x + c) + 1)*log(e^(
d*x + c) - 1) - 2*I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + 4*I)/(a*d*e^(3*d*x +
 3*c) - I*a*d*e^(2*d*x + 2*c) - a*d*e^(d*x + c) + I*a*d)
```

3.214.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int \frac{\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

input
```
integrate(csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(csch(c + d*x)**2/(sinh(c + d*x) - I), x)/a
```

3.214.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(53) = 106$.

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.91

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{2\left(e^{(-dx-c)}-i\,e^{(-2\,dx-2\,c)}+2i\right)}{\left(ae^{(-dx-c)}-i\,ae^{(-2\,dx-2\,c)}-ae^{(-3\,dx-3\,c)}+i\,a\right)d} + \frac{i\,\log\left(e^{(-dx-c)}+1\right)}{ad} - \frac{i\,\log\left(e^{(-dx-c)}-1\right)}{ad}$$

input
```
integrate(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*(e^(-d*x - c) - I*e^(-2*d*x - 2*c) + 2*I)/((a*e^(-d*x - c) - I*a*e^(-2*
d*x - 2*c) - a*e^(-3*d*x - 3*c) + I*a)*d) + I*log(e^(-d*x - c) + 1)/(a*d)
- I*log(e^(-d*x - c) - 1)/(a*d)
```

3.214.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{-\frac{i\,\log\left(e^{(dx+c)}+1\right)}{a} + \frac{i\,\log\left(e^{(dx+c)}-1\right)}{a} - \frac{2\left(e^{(2\,dx+2\,c)}-i\,e^{(dx+c)}-2\right)}{a\left(i\,e^{(3\,dx+3\,c)}+e^{(2\,dx+2\,c)}-i\,e^{(dx+c)}-1\right)}}{d}$$

input
```
integrate(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(-I*log(e^(d*x + c) + 1)/a + I*log(e^(d*x + c) - 1)/a - 2*(e^(2*d*x + 2*c
) - I*e^(d*x + c) - 2)/(a*(I*e^(3*d*x + 3*c) + e^(2*d*x + 2*c) - I*e^(d*x
+ c) - 1)))/d
```

3.214.9 Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.14

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\frac{2\,\mathrm{e}^{c+d\,x}}{a\,d}-\frac{4\mathrm{i}}{a\,d}+\frac{\mathrm{e}^{2\,c+2\,d\,x}\,2\mathrm{i}}{a\,d}}{\mathrm{e}^{c+d\,x}+\mathrm{e}^{2\,c+2\,d\,x}\,1\mathrm{i}-\mathrm{e}^{3\,c+3\,d\,x}-\mathrm{i}} - \frac{\ln\left(\mathrm{e}^{c+d\,x}\,2\mathrm{i}-2\mathrm{i}\right)\,1\mathrm{i}}{a\,d} + \frac{\ln\left(\mathrm{e}^{c+d\,x}\,2\mathrm{i}+2\mathrm{i}\right)\,1\mathrm{i}}{a\,d}$$

input
```
int(1/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
((2*exp(c + d*x))/(a*d) - 4i/(a*d) + (exp(2*c + 2*d*x)*2i)/(a*d))/(exp(c +
 d*x) + exp(2*c + 2*d*x)*1i - exp(3*c + 3*d*x) - 1i) - (log(exp(c + d*x)*2
i - 2i)*1i)/(a*d) + (log(exp(c + d*x)*2i + 2i)*1i)/(a*d)
```

3.215 $\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

3.215.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.215.2 Mathematica [N/A]

Not integrable

Time = 70.54 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Csch[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Csch[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

3.215.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.215.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.215.4 Maple [N/A] (verified)

Not integrable

Time = 0.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}(dx+c)^2}{(fx+e)\,(a+ia\sinh(dx+c))}dx$$

input
```
int(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.215.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 342, normalized size of antiderivative = 11.03

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^2}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((I*a*d*f*x + I*a*d*e + (a*d*f*x + a*d*e)*e^(3*d*x + 3*c) + (-I*a*d*f*x -
I*a*d*e)*e^(2*d*x + 2*c) - (a*d*f*x + a*d*e)*e^(d*x + c))*integral(-2*((I*
d*f*x + I*d*e + I*f)*e^(2*d*x + 2*c) + (d*f*x + d*e + f)*e^(d*x + c) - 2*I
*f)/(I*a*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*
x + a*d*e^2)*e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2
)*e^(2*d*x + 2*c) - (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c)), x)
 - 2*I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + 4*I)/(I*a*d*f*x + I*a*d*e + (a*d*
f*x + a*d*e)*e^(3*d*x + 3*c) + (-I*a*d*f*x - I*a*d*e)*e^(2*d*x + 2*c) - (a
*d*f*x + a*d*e)*e^(d*x + c))
```

3.215.6 Sympy [N/A]

Not integrable

Time = 15.97 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\operatorname{csch}^2(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

input
```
integrate(csch(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(csch(c + d*x)**2/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) -
I*f*x), x)/a
```

3.215.7 Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 330, normalized size of antiderivative = 10.65

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^2}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-4*I*f*integrate(1/(-2*I*a*d*f^2*x^2 - 4*I*a*d*e*f*x - 2*I*a*d*e^2 + 2*(a*
d*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 4*(I*e^(2*d*
x + 2*c) + e^(d*x + c) - 2*I)/(2*I*a*d*f*x + 2*I*a*d*e + 2*(a*d*f*x*e^(3*c
) + a*d*e*e^(3*c))*e^(3*d*x) - 2*(I*a*d*f*x*e^(2*c) + I*a*d*e*e^(2*c))*e^(
2*d*x) - 2*(a*d*f*x*e^c + a*d*e*e^c)*e^(d*x)) - 4*integrate(-1/4*(I*d*f*x
+ I*d*e + f)/(a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2 - (a*d*f^2*x^2*e^c + 2*a
*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 4*integrate(1/4*(I*d*f*x + I*d*
e - f)/(a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2 + (a*d*f^2*x^2*e^c + 2*a*d*e*f
*x*e^c + a*d*e^2*e^c)*e^(d*x)), x)
```

3.215.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.215.9 Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/(sinh(c + d*x)^2*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(sinh(c + d*x)^2*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.216 $\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

3.216.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.216.2 Mathematica [N/A]

Not integrable

Time = 82.98 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Csch[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Csch[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

3.216.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

$$\Big\downarrow \; 6111$$

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.216.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.216.4 Maple [N/A] (verified)

Not integrable

Time = 0.85 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}\left(dx+c\right)^{2}}{\left(fx+e\right)^{2}\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.216.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 500, normalized size of antiderivative = 16.13

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^2}{\left(fx+e\right)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
((I*a*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x +
 a*d*e^2)*e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2)*e
^(2*d*x + 2*c) - (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))*integr
al(-2*((I*d*f*x + I*d*e + 2*I*f)*e^(2*d*x + 2*c) + (d*f*x + d*e + 2*f)*e^(
d*x + c) - 4*I*f)/(I*a*d*f^3*x^3 + 3*I*a*d*e*f^2*x^2 + 3*I*a*d*e^2*f*x + I
*a*d*e^3 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^(3*
d*x + 3*c) + (-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x - I*a*d
*e^3)*e^(2*d*x + 2*c) - (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a
*d*e^3)*e^(d*x + c)), x) - 2*I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + 4*I)/(I*a
*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*
e^2)*e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2)*e^(2*d
*x + 2*c) - (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

3.216.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.216.7 Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 477, normalized size of antiderivative = 15.39

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^2}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-4*I*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x -
 I*a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c +
a*d*e^3*e^c)*e^(d*x)), x) - 4*(I*e^(2*d*x + 2*c) + e^(d*x + c) - 2*I)/(2*I
*a*d*f^2*x^2 + 4*I*a*d*e*f*x + 2*I*a*d*e^2 + 2*(a*d*f^2*x^2*e^(3*c) + 2*a*
d*e*f*x*e^(3*c) + a*d*e^2*e^(3*c))*e^(3*d*x) - 2*(I*a*d*f^2*x^2*e^(2*c) +
2*I*a*d*e*f*x*e^(2*c) + I*a*d*e^2*e^(2*c))*e^(2*d*x) - 2*(a*d*f^2*x^2*e^c
+ 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)) - 4*integrate(-1/4*(I*d*f*x + I*
d*e + 2*f)/(a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3 - (a*d
*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*d*e^3*e^c)*e^(d
*x)), x) - 4*integrate(1/4*(I*d*f*x + I*d*e - 2*f)/(a*d*f^3*x^3 + 3*a*d*e*
f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c
 + 3*a*d*e^2*f*x*e^c + a*d*e^3*e^c)*e^(d*x)), x)
```

3.216.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.216.9 Mupad [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$=\int \frac{1}{\sinh\left(c+d\,x\right)^2\,{\left(e+f\,x\right)}^2\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int(1/(sinh(c + d*x)^2*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(sinh(c + d*x)^2*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.217 $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.217.1 Optimal result

Integrand size = 31, antiderivative size = 546

$$\begin{aligned}\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = {} & \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx)\operatorname{arctanh}(e^{c+dx})}{ad^3} \\ & + \frac{3(e+fx)^3\operatorname{arctanh}(e^{c+dx})}{ad} \\ & + \frac{i(e+fx)^3\coth(c+dx)}{ad} - \frac{3f(e+fx)^2\operatorname{csch}(c+dx)}{2ad^2} \\ & - \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} \\ & - \frac{6if(e+fx)^2\log(1+ie^{c+dx})}{ad^2} \\ & - \frac{3if(e+fx)^2\log(1-e^{2(c+dx)})}{ad^2} - \frac{3f^3\operatorname{PolyLog}(2,-e^{c+dx})}{ad^4} \\ & + \frac{9f(e+fx)^2\operatorname{PolyLog}(2,-e^{c+dx})}{2ad^2} \\ & - \frac{12if^2(e+fx)\operatorname{PolyLog}(2,-ie^{c+dx})}{ad^3} \\ & + \frac{3f^3\operatorname{PolyLog}(2,e^{c+dx})}{ad^4} - \frac{9f(e+fx)^2\operatorname{PolyLog}(2,e^{c+dx})}{2ad^2} \\ & - \frac{3if^2(e+fx)\operatorname{PolyLog}(2,e^{2(c+dx)})}{ad^3} \\ & - \frac{9f^2(e+fx)\operatorname{PolyLog}(3,-e^{c+dx})}{ad^3} \\ & + \frac{12if^3\operatorname{PolyLog}(3,-ie^{c+dx})}{ad^4} \\ & + \frac{9f^2(e+fx)\operatorname{PolyLog}(3,e^{c+dx})}{ad^3} \\ & + \frac{3if^3\operatorname{PolyLog}(3,e^{2(c+dx)})}{2ad^4} + \frac{9f^3\operatorname{PolyLog}(4,-e^{c+dx})}{ad^4} \\ & - \frac{9f^3\operatorname{PolyLog}(4,e^{c+dx})}{ad^4} + \frac{i(e+fx)^3\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\end{aligned}$$

output
```
-6*I*f*(f*x+e)^2*ln(1+I*exp(d*x+c))/a/d^2-6*f^2*(f*x+e)*arctanh(exp(d*x+c)
)/a/d^3+3*(f*x+e)^3*arctanh(exp(d*x+c))/a/d-3*I*f^2*(f*x+e)*polylog(2,exp(
2*d*x+2*c))/a/d^3-3/2*f*(f*x+e)^2*csch(d*x+c)/a/d^2-1/2*(f*x+e)^3*coth(d*x
+c)*csch(d*x+c)/a/d-12*I*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^3+3/2*I*
f^3*polylog(3,exp(2*d*x+2*c))/a/d^4-3*f^3*polylog(2,-exp(d*x+c))/a/d^4+9/2
*f*(f*x+e)^2*polylog(2,-exp(d*x+c))/a/d^2+12*I*f^3*polylog(3,-I*exp(d*x+c)
)/a/d^4+3*f^3*polylog(2,exp(d*x+c))/a/d^4-9/2*f*(f*x+e)^2*polylog(2,exp(d*
x+c))/a/d^2+I*(f*x+e)^3*coth(d*x+c)/a/d-9*f^2*(f*x+e)*polylog(3,-exp(d*x+c
))/a/d^3-3*I*f*(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a/d^2+9*f^2*(f*x+e)*polylog(
3,exp(d*x+c))/a/d^3+2*I*(f*x+e)^3/a/d+9*f^3*polylog(4,-exp(d*x+c))/a/d^4-9
*f^3*polylog(4,exp(d*x+c))/a/d^4+I*(f*x+e)^3*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/
a/d
```

3.217.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2585 vs. 2(546) = 1092.

Time = 90.16 (sec) , antiderivative size = 2585, normalized size of antiderivative = 4.73

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-6*E^c*f*((e + f*x)^3/(3*E^c*f) + ((I + E^(-c))*(e + f*x)^2*Log[1 - I*E^(
-c - d*x)])/d - ((2*I)*(-I + E^c)*f*(d*(e + f*x)*PolyLog[2, I*E^(-c - d*x)
] + f*PolyLog[3, I*E^(-c - d*x)]))/(d^3*E^c))/(a*d*(-I + E^c)) + ((12*I)*
d^3*e^2*E^(2*c)*f*x + (12*I)*d^3*e*E^(2*c)*f^2*x^2 + (4*I)*d^3*E^(2*c)*f^3
*x^3 - 6*d^3*e^3*ArcTanh[E^(c + d*x)] + 6*d^3*e^3*E^(2*c)*ArcTanh[E^(c + d
*x)] + 12*d*e*f^2*ArcTanh[E^(c + d*x)] - 12*d*e*E^(2*c)*f^2*ArcTanh[E^(c +
 d*x)] + 9*d^3*e^2*f*x*Log[1 - E^(c + d*x)] - 9*d^3*e^2*E^(2*c)*f*x*Log[1
- E^(c + d*x)] - 6*d*f^3*x*Log[1 - E^(c + d*x)] + 6*d*E^(2*c)*f^3*x*Log[1
- E^(c + d*x)] + 9*d^3*e*f^2*x^2*Log[1 - E^(c + d*x)] - 9*d^3*e*E^(2*c)*f^
2*x^2*Log[1 - E^(c + d*x)] + 3*d^3*f^3*x^3*Log[1 - E^(c + d*x)] - 3*d^3*E^
(2*c)*f^3*x^3*Log[1 - E^(c + d*x)] - 9*d^3*e^2*f*x*Log[1 + E^(c + d*x)] +
9*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(c + d*x)] + 6*d*f^3*x*Log[1 + E^(c + d*x)
] - 6*d*E^(2*c)*f^3*x*Log[1 + E^(c + d*x)] - 9*d^3*e*f^2*x^2*Log[1 + E^(c
+ d*x)] + 9*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] - 3*d^3*f^3*x^3*Log
[1 + E^(c + d*x)] + 3*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(c + d*x)] + (6*I)*d^2
*e^2*f*Log[1 - E^(2*(c + d*x))] - (6*I)*d^2*e^2*E^(2*c)*f*Log[1 - E^(2*(c
+ d*x))] + (12*I)*d^2*e*f^2*x*Log[1 - E^(2*(c + d*x))] - (12*I)*d^2*e*E^(2
*c)*f^2*x*Log[1 - E^(2*(c + d*x))] + (6*I)*d^2*f^3*x^2*Log[1 - E^(2*(c + d
*x))] - (6*I)*d^2*E^(2*c)*f^3*x^2*Log[1 - E^(2*(c + d*x))] + 3*(-1 + E^(2*
c))*f*(-2*f^2 + 3*d^2*(e + f*x)^2)*PolyLog[2, -E^(c + d*x)] - 3*(-1 + E...
```

3.217.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)^3\text{csch}^3(c+dx)dx}{a}-i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -i(e+fx)^3\csc(ic+idx)^3dx}{a}-i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int (e+fx)^3\csc(ic+idx)^3dx}{a}-i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{4674}$$

$$-\frac{i\left(-\frac{3f^2\int -i(e+fx)\text{csch}(c+dx)dx}{d^2}+\frac{1}{2}\int -i(e+fx)^3\text{csch}(c+dx)dx-\frac{3if(e+fx)^2\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)^3\coth(c+dx)\text{csch}(c-}{2d}\right.}{a}$$

$$i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \; 26$$

$$-\frac{i\left(\frac{3if^2\int (e+fx)\text{csch}(c+dx)dx}{d^2}-\frac{1}{2}i\int (e+fx)^3\text{csch}(c+dx)dx-\frac{3if(e+fx)^2\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \; 3042$$

$$-\frac{i\left(\frac{3if^2\int i(e+fx)\csc(ic+idx)dx}{d^2}-\frac{1}{2}i\int i(e+fx)^3\csc(ic+idx)dx-\frac{3if(e+fx)^2\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \; 26$$

$$-\frac{i\left(-\frac{3f^2\int (e+fx)\csc(ic+idx)dx}{d^2}+\frac{1}{2}\int (e+fx)^3\csc(ic+idx)dx-\frac{3if(e+fx)^2\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \; 4670$$

$$-\frac{i\left(-\frac{3f^2\left(\frac{if\int \log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(\frac{3if\int (e+fx)^2\log(1-e^{c+dx})dx}{d}-\frac{3if\int (e+fx)^2\,\text{l}}{}\right.\right.}{a}$$

$$i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \; 2715$$

3.217. $\displaystyle\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-\frac{i\left(-\frac{3f^2\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(\frac{3if\int(e+fx)^2\log(1-e^{c+\cdots}}{d}\right.\right.}{}$$

$$i\int\frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{2838}$$

$$-\frac{i\left(\frac{1}{2}\left(\frac{3if\int(e+fx)^2\log(1-e^{c+dx})dx}{d}-\frac{3if\int(e+fx)^2\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^3\text{arctanh}(e^{c+dx})}{d}\right)-\frac{3f^2\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+\cdots}\right)}{d}\right.}{a}\right.}{}$$

$$i\int\frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{3011}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.\right.}{}$$

$$i\int\frac{(e+fx)^3\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{6109}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.\right.}{}$$

$$i\left(\frac{\int(e+fx)^3\text{csch}^2(c+dx)dx}{a}-i\int\frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.\right.}{}$$

$$i\left(\frac{\int-(e+fx)^3\csc(ic+idx)^2dx}{a}-i\int\frac{(e+fx)^3\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)$$

3.217. $\int\frac{(e+fx)^3\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 25$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+c}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{\int(e+fx)^3\csc(ic+idx)^2dx}{a}-i\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \; 4672$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+c}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3if\int-i(e+fx)^2\coth(c+dx)dx}{d}}{a}-i\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \; 26$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+c}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3f\int(e+fx)^2\coth(c+dx)dx}{d}}{a}-i\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+c}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3f\int-i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}-i\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \; 26$$

3.217. $\int\frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\cdots\right.}{d}\cdots\right.\right.}{i\left(-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\int(e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}-i\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \; 4201$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\cdots\right.}{d}\cdots\right.\right.}{i\left(-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}-i\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \; 2620$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\cdots\right.}{d}\cdots\right.\right.}{i\left(-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}-i\int\frac{(e+fx)^3\operatorname{csch}(c\cdots}{i\sinh(c+dx)\cdots}\right.}$$

$$\Big\downarrow \; 3011$$

3.217. $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-\frac{i\Bigg(\frac{1}{2}\Bigg(-\frac{3if\Big(\frac{2f\int(e+fx)\,\mathrm{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}(2,-e^{c+dx})}{d}\Big)}{d}+\frac{3if\Big(\frac{2f\int(e+fx)\,\mathrm{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}(2,e^{c+\cdots}}{d}\cdots}{d}\cdots}{i\Bigg(-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{\frac{3if\Big(2i\Big(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\Big(\frac{f\int\mathrm{PolyLog}(2,-e^{2c+2dx-i\pi})dx}{2d}-\frac{(e+fx)\,\mathrm{PolyLog}(2,-e^{2c+2dx-i\pi})}{2d}\Big)}{d}\Big)-\frac{i(e\cdots}{}\cdots}{d}}{a}\cdots}$$

$$\Big\downarrow \; 2720$$

$$-\frac{i\Bigg(\frac{1}{2}\Bigg(-\frac{3if\Big(\frac{2f\int(e+fx)\,\mathrm{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}(2,-e^{c+dx})}{d}\Big)}{d}+\frac{3if\Big(\frac{2f\int(e+fx)\,\mathrm{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}(2,e^{c+\cdots}}{d}\cdots}{d}\cdots}{i\Bigg(-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{\frac{3if\Big(2i\Big(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\Big(\frac{f\int e^{-2c-2dx+i\pi}\,\mathrm{PolyLog}(2,-e^{2c+2dx-i\pi})de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\mathrm{PolyLog}(2,\cdots}{2d}\cdots}{d}\cdots}{d}}{a}\cdots}$$

$$\Big\downarrow \; 6109$$

$$-\frac{i\Bigg(\frac{1}{2}\Bigg(-\frac{3if\Big(\frac{2f\int(e+fx)\,\mathrm{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}(2,-e^{c+dx})}{d}\Big)}{d}+\frac{3if\Big(\frac{2f\int(e+fx)\,\mathrm{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}(2,e^{c+\cdots}}{d}\cdots}{d}\cdots}{i\Bigg(-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{\frac{3if\Big(2i\Big(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\Big(\frac{f\int e^{-2c-2dx+i\pi}\,\mathrm{PolyLog}(2,-e^{2c+2dx-i\pi})de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\mathrm{PolyLog}(2,\cdots}{2d}\cdots}{d}\cdots}{d}}{a}\cdots}$$

$$\Big\downarrow \; 3042$$

3.217. $\displaystyle\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\cdots\right.}{d}\cdots\right.\right.}{i\left(-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\cdots\right)}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots\right.}{a}\cdots}$$

$$\Big\downarrow \; 26$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\cdots\right.}{d}\cdots\right.\right.}{i\left(-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\cdots\right)}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots\right.}{a}\cdots}$$

$$\Big\downarrow \; 3799$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\cdots\right.}{d}\cdots\right.\right.}{i\left(-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\cdots\right)}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots\right.}{a}\cdots}$$

$$\Big\downarrow \; 25$$

3.217. $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\cdots\right)}{d}\cdots\right)\right)}{i\left(-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\cdots\right)}{2d}\cdots\right)}{d}\cdots\right)\right)}{d}\cdots\right)\cdots}{a}$$

$\downarrow$ 25

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\cdots\right)}{d}\cdots\right)\right)}{i\left(-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\cdots\right)}{2d}\cdots\right)}{d}\cdots\right)\right)}{d}\cdots\right)\cdots}{a}$$

$\downarrow$ 3042

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\cdots\right)}{d}\cdots\right)\right)}{i\left(-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\cdots\right)}{2d}\cdots\right)}{d}\cdots\right)\right)}{d}\cdots\right)\cdots}{a}$$

$\downarrow$ 4670

3.217. $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,e^{c+c}\right.}{d}\right.}{d}\right.\right.}{i\left(-i\left(\frac{i\left(\frac{3if\int(e+fx)^2\log(1-e^{c+dx})dx}{d}-\frac{3if\int(e+fx)^2\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^3\mathrm{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int(e+fx)^3\csc\left(\frac{ic}{2}\right.}{2a}\right.\right.}$$

input
```
Int[((e + f*x)^3*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.217.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.217. $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

3.217.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2123 vs. 2(504) = 1008.

Time = 3.28 (sec) , antiderivative size = 2124, normalized size of antiderivative = 3.89

method	result	size
risch	Expression too large to display	2124

input
```
int((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
6/a/d^2*e^2*f*arctan(exp(d*x+c))+6/a/d^4*c^2*f^3*arctan(exp(d*x+c))+12*I*f
^3*polylog(3,-I*exp(d*x+c))/a/d^4-12/a/d^3*c*f^2*e*arctan(exp(d*x+c))-6*I/
a/d^3*e*f^2*polylog(2,-exp(d*x+c))+6*I/a/d^3*c*f^2*e*ln(1+exp(2*d*x+2*c))-
6*I/a/d^3*e*f^2*ln(1-exp(d*x+c))*c+9*f^3*polylog(4,-exp(d*x+c))/a/d^4-9*f^
3*polylog(4,exp(d*x+c))/a/d^4-3*f^3*polylog(2,-exp(d*x+c))/a/d^4+3*f^3*pol
ylog(2,exp(d*x+c))/a/d^4-3/2/a/d*e^3*ln(exp(d*x+c)-1)+3/2/a/d*e^3*ln(exp(d
*x+c)+1)-(-9*I*d*e^2*f*x*exp(3*d*x+3*c)-3*I*d*e^3*exp(3*d*x+3*c)-3*I*e^2*f
*exp(3*d*x+3*c)+6*I*e*f^2*x*exp(d*x+c)+I*d*f^3*x^3*exp(d*x+c)+3*d*e^3*exp(
4*d*x+4*c)+3*e^2*f*exp(4*d*x+4*c)+4*d*f^3*x^3+I*e^3*d*exp(d*x+c)-9*I*d*e*f
^2*x^2*exp(3*d*x+3*c)+3*I*d*e*f^2*x^2*exp(d*x+c)+3*I*d*e^2*f*x*exp(d*x+c)+
4*d*e^3+12*d*e*f^2*x^2+12*d*e^2*f*x+3*I*f^3*x^2*exp(d*x+c)+3*I*exp(d*x+c)*
e^2*f+3*f^3*x^2*exp(4*d*x+4*c)-6*e*f^2*x*exp(2*d*x+2*c)+9*d*e*f^2*x^2*exp(
4*d*x+4*c)+9*d*e^2*f*x*exp(4*d*x+4*c)-6*I*e*f^2*x*exp(3*d*x+3*c)-3*I*d*f^3
*x^3*exp(3*d*x+3*c)-3*f^3*x^2*exp(2*d*x+2*c)-5*f^3*x^3*d*exp(2*d*x+2*c)-3*
e^2*f*exp(2*d*x+2*c)-5*e^3*d*exp(2*d*x+2*c)+3*d*f^3*x^3*exp(4*d*x+4*c)+6*e
*f^2*x*exp(4*d*x+4*c)-3*I*f^3*x^2*exp(3*d*x+3*c)-15*e*f^2*x^2*d*exp(2*d*x+
2*c)-15*e^2*f*x*d*exp(2*d*x+2*c))/(exp(2*d*x+2*c)-1)^2/d^2/(exp(d*x+c)-I)/
a-12*I/a/d^3*e*f^2*ln(1+I*exp(d*x+c))*c-24*I/a/d^3*c*e*f^2*ln(exp(d*x+c))-
6*I/a/d^2*e*f^2*ln(1-exp(d*x+c))*x-6*I/a/d^2*e*f^2*ln(exp(d*x+c)+1)*x-12*I
/a/d^2*e*f^2*ln(1+I*exp(d*x+c))*x+24*I/a/d^2*e*f^2*c*x+6*I/a/d^3*c*f^2*...
```

3.217.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4252 vs. $2(485) = 970$.

Time = 0.32 (sec) , antiderivative size = 4252, normalized size of antiderivative = 7.79

$$\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-1/2*(8*d^3*e^3 - 24*c*d^2*e^2*f + 24*c^2*d*e*f^2 - 8*c^3*f^3 + 24*(d*f^3*
x + d*e*f^2 + (I*d*f^3*x + I*d*e*f^2)*e^(5*d*x + 5*c) + (d*f^3*x + d*e*f^2
)*e^(4*d*x + 4*c) + 2*(-I*d*f^3*x - I*d*e*f^2)*e^(3*d*x + 3*c) - 2*(d*f^3*
x + d*e*f^2)*e^(2*d*x + 2*c) + (I*d*f^3*x + I*d*e*f^2)*e^(d*x + c))*dilog(
-I*e^(d*x + c)) + 3*(3*I*d^2*f^3*x^2 + 3*I*d^2*e^2*f + 4*d*e*f^2 - 2*I*f^3
 + 2*(3*I*d^2*e*f^2 + 2*d*f^3)*x - (3*d^2*f^3*x^2 + 3*d^2*e^2*f - 4*I*d*e*
f^2 - 2*f^3 + 2*(3*d^2*e*f^2 - 2*I*d*f^3)*x)*e^(5*d*x + 5*c) + (3*I*d^2*f^
3*x^2 + 3*I*d^2*e^2*f + 4*d*e*f^2 - 2*I*f^3 + 2*(3*I*d^2*e*f^2 + 2*d*f^3)*
x)*e^(4*d*x + 4*c) + 2*(3*d^2*f^3*x^2 + 3*d^2*e^2*f - 4*I*d*e*f^2 - 2*f^3
+ 2*(3*d^2*e*f^2 - 2*I*d*f^3)*x)*e^(3*d*x + 3*c) + 2*(-3*I*d^2*f^3*x^2 - 3
*I*d^2*e^2*f - 4*d*e*f^2 + 2*I*f^3 + 2*(-3*I*d^2*e*f^2 - 2*d*f^3)*x)*e^(2*
d*x + 2*c) - (3*d^2*f^3*x^2 + 3*d^2*e^2*f - 4*I*d*e*f^2 - 2*f^3 + 2*(3*d^2
*e*f^2 - 2*I*d*f^3)*x)*e^(d*x + c))*dilog(-e^(d*x + c)) + 3*(-3*I*d^2*f^3*
x^2 - 3*I*d^2*e^2*f + 4*d*e*f^2 + 2*I*f^3 + 2*(-3*I*d^2*e*f^2 + 2*d*f^3)*x
 + (3*d^2*f^3*x^2 + 3*d^2*e^2*f + 4*I*d*e*f^2 - 2*f^3 + 2*(3*d^2*e*f^2 + 2
*I*d*f^3)*x)*e^(5*d*x + 5*c) + (-3*I*d^2*f^3*x^2 - 3*I*d^2*e^2*f + 4*d*e*f
^2 + 2*I*f^3 + 2*(-3*I*d^2*e*f^2 + 2*d*f^3)*x)*e^(4*d*x + 4*c) - 2*(3*d^2*
f^3*x^2 + 3*d^2*e^2*f + 4*I*d*e*f^2 - 2*f^3 + 2*(3*d^2*e*f^2 + 2*I*d*f^3)*
x)*e^(3*d*x + 3*c) + 2*(3*I*d^2*f^3*x^2 + 3*I*d^2*e^2*f - 4*d*e*f^2 - 2*I*
f^3 + 2*(3*I*d^2*e*f^2 - 2*d*f^3)*x)*e^(2*d*x + 2*c) + (3*d^2*f^3*x^2 +...
```

3.217.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.217.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1320 vs. 2(485) = 970.

Time = 0.52 (sec) , antiderivative size = 1320, normalized size of antiderivative = 2.42

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-1/2*e^3*(2*(-I*e^(-d*x - c) - 5*e^(-2*d*x - 2*c) + 3*I*e^(-3*d*x - 3*c) +
 3*e^(-4*d*x - 4*c) + 4)/((a*e^(-d*x - c) - 2*I*a*e^(-2*d*x - 2*c) - 2*a*e
^(-3*d*x - 3*c) + I*a*e^(-4*d*x - 4*c) + a*e^(-5*d*x - 5*c) + I*a)*d) - 3*
log(e^(-d*x - c) + 1)/(a*d) + 3*log(e^(-d*x - c) - 1)/(a*d)) + 6*I*e^2*f*x
/(a*d) - 6*I*e^2*f*log(I*e^(d*x + c) + 1)/(a*d^2) - (4*d*f^3*x^3 + 12*d*e*
f^2*x^2 + 12*d*e^2*f*x + 3*(d*f^3*x^3*e^(4*c) + e^2*f*e^(4*c) + (3*d*e*f^2
 + f^3)*x^2*e^(4*c) + (3*d*e^2*f + 2*e*f^2)*x*e^(4*c))*e^(4*d*x) - 3*(I*d*
f^3*x^3*e^(3*c) + I*e^2*f*e^(3*c) + (3*I*d*e*f^2 + I*f^3)*x^2*e^(3*c) + (3
*I*d*e^2*f + 2*I*e*f^2)*x*e^(3*c))*e^(3*d*x) - (5*d*f^3*x^3*e^(2*c) + 3*e^
2*f*e^(2*c) + 3*(5*d*e*f^2 + f^3)*x^2*e^(2*c) + 3*(5*d*e^2*f + 2*e*f^2)*x*
e^(2*c))*e^(2*d*x) + (I*d*f^3*x^3*e^c + 3*I*e^2*f*e^c - 3*(-I*d*e*f^2 - I*
f^3)*x^2*e^c - 3*(-I*d*e^2*f - 2*I*e*f^2)*x*e^c)*e^(d*x))/(a*d^2*e^(5*d*x
+ 5*c) - I*a*d^2*e^(4*d*x + 4*c) - 2*a*d^2*e^(3*d*x + 3*c) + 2*I*a*d^2*e^(
2*d*x + 2*c) + a*d^2*e^(d*x + c) - I*a*d^2) - 12*I*(d*x*log(I*e^(d*x + c)
+ 1) + dilog(-I*e^(d*x + c)))*e*f^2/(a*d^3) + 3/2*(d^3*x^3*log(e^(d*x + c)
 + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6
*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) - 3/2*(d^3*x^3*log(-e^(d*x + c) + 1
) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polyl
og(4, e^(d*x + c)))*f^3/(a*d^4) - 6*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*
d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4) -...
```

3.217.8 Giac [F]

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\,\text{csch}\,(dx+c)^3}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*csch(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(e+f\,x)^3}{\sinh\,(c+d\,x)^3\,\left(a+a\sinh\,(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)^3/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^3/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

3.218 $\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.218.1 Optimal result

Integrand size = 31, antiderivative size = 368

$$\begin{aligned}\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{2i(e+fx)^2}{ad}+\frac{3(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{ad}\\
&-\frac{f^2\text{arctanh}(\cosh(c+dx))}{ad^3}\\
&+\frac{i(e+fx)^2\coth(c+dx)}{ad}-\frac{f(e+fx)\text{csch}(c+dx)}{ad^2}\\
&-\frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2ad}\\
&-\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}\\
&-\frac{2if(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad^2}\\
&+\frac{3f(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}\\
&-\frac{4if^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}\\
&-\frac{3f(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}\\
&-\frac{if^2\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3}-\frac{3f^2\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}\\
&+\frac{3f^2\,\text{PolyLog}\left(3,e^{c+dx}\right)}{ad^3}+\frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\end{aligned}$$

output
```
2*I*(f*x+e)^2/a/d+3*(f*x+e)^2*arctanh(exp(d*x+c))/a/d-f^2*arctanh(cosh(d*x
+c))/a/d^3+I*(f*x+e)^2*coth(d*x+c)/a/d-f*(f*x+e)*csch(d*x+c)/a/d^2-1/2*(f*
x+e)^2*coth(d*x+c)*csch(d*x+c)/a/d-4*I*f*(f*x+e)*ln(1+I*exp(d*x+c))/a/d^2-
2*I*f*(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d^2+3*f*(f*x+e)*polylog(2,-exp(d*x+c)
)/a/d^2-4*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3-3*f*(f*x+e)*polylog(2,exp(d
*x+c))/a/d^2-I*f^2*polylog(2,exp(2*d*x+2*c))/a/d^3-3*f^2*polylog(3,-exp(d*
x+c))/a/d^3+3*f^2*polylog(3,exp(d*x+c))/a/d^3+I*(f*x+e)^2*tanh(1/2*c+1/4*I
*Pi+1/2*d*x)/a/d
```

3.218.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1496 vs. $2(368) = 736$.

Time = 8.80 (sec) , antiderivative size = 1496, normalized size of antiderivative = 4.07

$$\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-4*E^c*f*((e + f*x)^2/(2*E^c*f) + ((I + E^(-c))*(e + f*x)*Log[1 - I*E^(-c
 - d*x)])/d - ((1 + I*E^c)*f*PolyLog[2, I*E^(-c - d*x)])/(d^2*E^c)))/(a*d*
(-I + E^c)) + (-(d*(-1 + E^(2*c))*(3*d^2*e^2 - (4*I)*d*e*f - 2*f^2)*x) + d
*(-1 + E^(2*c))*(3*d^2*e^2 + (4*I)*d*e*f - 2*f^2)*x + (4*I)*d^2*(e + f*x)^
2 - 2*d*(-1 + E^(2*c))*(3*d*e + (2*I)*f)*f*x*Log[1 - E^(-c - d*x)] - 3*d^2
*(-1 + E^(2*c))*f^2*x^2*Log[1 - E^(-c - d*x)] + 2*d*(-1 + E^(2*c))*(3*d*e
- (2*I)*f)*f*x*Log[1 + E^(-c - d*x)] + 3*d^2*(-1 + E^(2*c))*f^2*x^2*Log[1
+ E^(-c - d*x)] - (-1 + E^(2*c))*(3*d^2*e^2 + (4*I)*d*e*f - 2*f^2)*Log[1 -
 E^(c + d*x)] + (-1 + E^(2*c))*(3*d^2*e^2 - (4*I)*d*e*f - 2*f^2)*Log[1 + E
^(c + d*x)] - 2*(-1 + E^(2*c))*(3*d*e - (2*I)*f)*f*PolyLog[2, -E^(-c - d*x
)] - 6*d*(-1 + E^(2*c))*f^2*x*PolyLog[2, -E^(-c - d*x)] + 2*(-1 + E^(2*c))
*(3*d*e + (2*I)*f)*f*PolyLog[2, E^(-c - d*x)] + 6*d*(-1 + E^(2*c))*f^2*x*P
olyLog[2, E^(-c - d*x)] - 6*(-1 + E^(2*c))*f^2*PolyLog[3, -E^(-c - d*x)] +
 6*(-1 + E^(2*c))*f^2*PolyLog[3, E^(-c - d*x)])/(2*a*d^3*(-1 + E^(2*c))) +
 (Csch[c]*Csch[c + d*x]^2*(2*e*f*Cosh[(d*x)/2] + 2*f^2*x*Cosh[(d*x)/2] + 2
*e*f*Cosh[(3*d*x)/2] + 2*f^2*x*Cosh[(3*d*x)/2] + (5*I)*d*e^2*Cosh[c - (d*x
)/2] + (10*I)*d*e*f*x*Cosh[c - (d*x)/2] + (5*I)*d*f^2*x^2*Cosh[c - (d*x)/2
] - I*d*e^2*Cosh[c + (d*x)/2] - (2*I)*d*e*f*x*Cosh[c + (d*x)/2] - I*d*f^2*
x^2*Cosh[c + (d*x)/2] - 2*e*f*Cosh[2*c + (d*x)/2] - 2*f^2*x*Cosh[2*c + (d*
x)/2] + I*d*e^2*Cosh[c + (3*d*x)/2] + (2*I)*d*e*f*x*Cosh[c + (3*d*x)/2]...
```

3.218.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6109}$$

$$\frac{\int (e+fx)^2\text{csch}^3(c+dx)dx}{a}-i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -i(e+fx)^2\csc(ic+idx)^3dx}{a}-i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{i\int (e+fx)^2\csc(ic+idx)^3dx}{a}-i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 4674}$$

$$-\frac{i\left(-\frac{f^2\int -i\text{csch}(c+dx)dx}{d^2}+\frac{1}{2}\int -i(e+fx)^2\text{csch}(c+dx)dx-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}-i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{i\left(\frac{if^2\int \text{csch}(c+dx)dx}{d^2}-\frac{1}{2}i\int (e+fx)^2\text{csch}(c+dx)dx-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}-i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{i\left(\frac{if^2\int i\csc(ic+idx)dx}{d^2}-\frac{1}{2}i\int i(e+fx)^2\csc(ic+idx)dx-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}-i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{i\left(-\frac{f^2\int \csc(ic+idx)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\csc(ic+idx)dx-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}-$$
$$i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$
$$\Big\downarrow \; 4257$$
$$-\frac{i\left(\frac{1}{2}\int (e+fx)^2\csc(ic+idx)dx-\frac{if^2\text{arctanh}(\cosh(c+dx))}{d^3}-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}-$$
$$i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$
$$\Big\downarrow \; 4670$$
$$-\frac{i\left(\frac{1}{2}\left(\frac{2if\int (e+fx)\log\left(1-e^{c+dx}\right)dx}{d}-\frac{2if\int (e+fx)\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-\frac{if^2\text{arctanh}(\cosh(c+dx))}{d^3}-i\right.}{a}$$
$$i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$
$$\Big\downarrow \; 3011$$
$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}+2i(e+fx\right.\right.}{a}$$
$$i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$
$$\Big\downarrow \; 2720$$
$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLo}}{d}\right.}{d}\right.\right.}{}$$
$$i\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$
$$\Big\downarrow \; 6109$$

3.218. $\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\cdots}{d}\cdots\right.}{d}\cdots\right.\right.}{i\left(\frac{\int(e+fx)^2\operatorname{csch}^2(c+dx)dx}{a}-i\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\cdots}{d}\cdots\right.}{d}\cdots\right.\right.}{i\left(\frac{\int-(e+fx)^2\csc(ic+idx)^2dx}{a}-i\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \; 25$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\cdots}{d}\cdots\right.}{d}\cdots\right.\right.}{i\left(-\frac{\int(e+fx)^2\csc(ic+idx)^2dx}{a}-i\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \; 4672$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\cdots}{d}\cdots\right.}{d}\cdots\right.\right.}{i\left(-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2if\int-i(e+fx)\coth(c+dx)dx}{d}}{a}-i\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \; 26$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\cdots}{d}\cdots\right.}{d}\cdots\right.\right.}{i\left(-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int(e+fx)\coth(c+dx)dx}{d}}{a}-i\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

3.218. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}}{d}\right.}{d}\right.\right.}{i\left(-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}-i\int\frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}}{d}\right.}{d}\right.\right.}{i\left(-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\int(e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}-i\int\frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}}{d}\right.}{d}\right.\right.}{i\left(-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}-i\int\frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}}{d}\right.}{d}\right.\right.}{i\left(-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}-i\int\frac{(e+fx)^2\text{csch}(c+dx)}{i\sinh(c+dx)a+a}\right.}$$

$$\Big\downarrow \text{2715}$$

3.218. $\int\frac{(e+fx)^2\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{i\left(-\frac{-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}-i\int\frac{(e}{i\,s}\right.}$$

$$\Big\downarrow \ 2838$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{i\left(-i\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+f}{2f}\right.}{d}}{a}\right.}$$

$$\Big\downarrow \ 6109$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{i\left(-i\left(\frac{\int(e+fx)^2\operatorname{csch}(c+dx)dx}{a}-i\int\frac{(e+fx)^2}{i\sinh(c+dx)a+a}dx\right)-\frac{-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right.}{4d^2}\right.\right.}{}}{a}\right.}$$

$$\Big\downarrow \ 3042$$

3.218. $\int\frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{i\left(-i\left(\frac{\int i(e+fx)^2\csc(ic+idx)dx}{a}-i\int\frac{(e+fx)^2}{\sin(ic+idx)a+a}dx\right)-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-}\right.}{4d^2}\right.\right.}{}}{a}\right.}$$

$$\Big\downarrow \ 26$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{i\left(-i\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-i\int\frac{(e+fx)^2}{\sin(ic+idx)a+a}dx\right)-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-}\right.}{4d^2}\right.\right.}{}}{a}\right.}$$

$$\Big\downarrow \ 3799$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{i\left(-i\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int-(e+fx)^2\operatorname{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)dx}{2a}\right)-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyL}}{}\right.\right.}{}}{}\right.}$$

$$\Big\downarrow \ 25$$

3.218. $\int\frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}}{d}\right.}{d}\right.\right.}{i\left(-i\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}+\frac{i\int-(e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}\right)-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyL}}{}\right.\right.}{}\right.}$$

$$\Big\downarrow 25$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}}{d}\right.}{d}\right.\right.}{i\left(-i\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)^2\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}\right)-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}(}{}\right.\right.}{}\right.}$$

$$\Big\downarrow 3042$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}}{d}\right.}{d}\right.\right.}{i\left(-i\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-\frac{i\int(e+fx)^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right)-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}(}{}\right.\right.}{}\right.}$$

$$\Big\downarrow 4670$$

3.218. $\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-i\left(-i\left(\frac{i\left(\frac{2if\int(e+fx)\log(1-e^{c+dx})dx}{d}-\frac{2if\int(e+fx)\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^2\operatorname{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int(e+fx)^2\csc\left(\frac{ic}{2}\right.}{2a}\right.\right.$$

$$\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}(2,-e^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}(2,e^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}(\right.}{d}\right.}{d}\right.\right.}{a}$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.218.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.218. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.218. $\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

3.218.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1146 vs. 2(343) = 686.

Time = 2.78 (sec) , antiderivative size = 1147, normalized size of antiderivative = 3.12

method	result	size
risch	Expression too large to display	1147

input
```
int((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-4*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3-3*f^2*polylog(3,-exp(d*x+c))/a/d^3
+3*f^2*polylog(3,exp(d*x+c))/a/d^3-3/2/a/d*e^2*ln(exp(d*x+c)-1)+3/2/a/d*e^
2*ln(exp(d*x+c)+1)-3/a/d^2*e*f*ln(1-exp(d*x+c))*c-3/a/d*e*f*ln(1-exp(d*x+c
))*x+3/a/d*e*f*ln(exp(d*x+c)+1)*x+3/a/d^2*e*c*f*ln(exp(d*x+c)-1)+2*I/a/d^3
*f^2*c*ln(1+exp(2*d*x+2*c))+8*I/a/d^2*e*f*ln(exp(d*x+c))-2*I/a/d^2*e*f*ln(
exp(d*x+c)-1)-2*I/a/d^2*e*f*ln(exp(d*x+c)+1)-2*I/a/d^2*e*f*ln(1+exp(2*d*x+
2*c))-2*I/a/d^2*f^2*ln(1-exp(d*x+c))*x-2*I/a/d^2*f^2*ln(exp(d*x+c)+1)*x-4*
I/a/d^2*f^2*ln(1+I*exp(d*x+c))*x+8*I/a/d^2*f^2*c*x-8*I/a/d^3*c*f^2*ln(exp(
d*x+c))-4*I/a/d^3*f^2*ln(1+I*exp(d*x+c))*c-2*I/a/d^3*f^2*ln(1-exp(d*x+c))*
c+2*I/a/d^3*f^2*c*ln(exp(d*x+c)-1)-3/a/d^2*e*f*polylog(2,exp(d*x+c))+3/a/d
^2*e*f*polylog(2,-exp(d*x+c))+3/2/a/d^3*f^2*ln(1-exp(d*x+c))*c^2-3/2/a/d*f
^2*ln(1-exp(d*x+c))*x^2-3/a/d^2*f^2*polylog(2,exp(d*x+c))*x+3/2/a/d*f^2*ln
(exp(d*x+c)+1)*x^2+3/a/d^2*f^2*polylog(2,-exp(d*x+c))*x-4/a/d^3*c*f^2*arct
an(exp(d*x+c))+4/a/d^2*e*f*arctan(exp(d*x+c))-2*I/a/d^3*f^2*polylog(2,-exp
(d*x+c))+4*I/a/d^3*c^2*f^2+4*I/a/d*f^2*x^2-2*I/a/d^3*f^2*polylog(2,exp(d*x
+c))-3/2/a/d^3*c^2*f^2*ln(exp(d*x+c)-1)-(4*d*e^2+3*d*e^2*exp(4*d*x+4*c)+2*
f^2*x*exp(4*d*x+4*c)+2*e*f*exp(4*d*x+4*c)-2*f^2*x*exp(2*d*x+2*c)-2*e*f*exp
(2*d*x+2*c)+4*d*f^2*x^2+3*d*f^2*x^2*exp(4*d*x+4*c)+2*I*f^2*x*exp(d*x+c)-5*
f^2*x^2*d*exp(2*d*x+2*c)+6*d*e*f*x*exp(4*d*x+4*c)-3*I*d*f^2*x^2*exp(3*d*x+
3*c)-2*I*e*f*exp(3*d*x+3*c)+2*I*d*e*f*x*exp(d*x+c)-3*I*d*e^2*exp(3*d*x+...
```

3.218.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2215 vs. 2(329) = 658.

Time = 0.28 (sec) , antiderivative size = 2215, normalized size of antiderivative = 6.02

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-1/2*(8*d^2*e^2 - 16*c*d*e*f + 8*c^2*f^2 + 8*(I*f^2*e^(5*d*x + 5*c) + f^2*
e^(4*d*x + 4*c) - 2*I*f^2*e^(3*d*x + 3*c) - 2*f^2*e^(2*d*x + 2*c) + I*f^2*
e^(d*x + c) + f^2)*dilog(-I*e^(d*x + c)) + 2*(3*I*d*f^2*x + 3*I*d*e*f + 2*
f^2 - (3*d*f^2*x + 3*d*e*f - 2*I*f^2)*e^(5*d*x + 5*c) + (3*I*d*f^2*x + 3*I
*d*e*f + 2*f^2)*e^(4*d*x + 4*c) + 2*(3*d*f^2*x + 3*d*e*f - 2*I*f^2)*e^(3*d
*x + 3*c) + 2*(-3*I*d*f^2*x - 3*I*d*e*f - 2*f^2)*e^(2*d*x + 2*c) - (3*d*f^
2*x + 3*d*e*f - 2*I*f^2)*e^(d*x + c))*dilog(-e^(d*x + c)) + 2*(-3*I*d*f^2*
x - 3*I*d*e*f + 2*f^2 + (3*d*f^2*x + 3*d*e*f + 2*I*f^2)*e^(5*d*x + 5*c) +
(-3*I*d*f^2*x - 3*I*d*e*f + 2*f^2)*e^(4*d*x + 4*c) - 2*(3*d*f^2*x + 3*d*e*
f + 2*I*f^2)*e^(3*d*x + 3*c) + 2*(3*I*d*f^2*x + 3*I*d*e*f - 2*f^2)*e^(2*d*
x + 2*c) + (3*d*f^2*x + 3*d*e*f + 2*I*f^2)*e^(d*x + c))*dilog(e^(d*x + c))
 + 8*(-I*d^2*f^2*x^2 - 2*I*d^2*e*f*x - 2*I*c*d*e*f + I*c^2*f^2)*e^(5*d*x +
 5*c) - 2*(d^2*f^2*x^2 - 3*d^2*e^2 + 2*(4*c - 1)*d*e*f - 4*c^2*f^2 + 2*(d^
2*e*f - d*f^2)*x)*e^(4*d*x + 4*c) + 2*(5*I*d^2*f^2*x^2 - 3*I*d^2*e^2 + 2*(
8*I*c - I)*d*e*f - 8*I*c^2*f^2 + 2*(5*I*d^2*e*f - I*d*f^2)*x)*e^(3*d*x + 3
*c) + 2*(3*d^2*f^2*x^2 - 5*d^2*e^2 + 2*(8*c - 1)*d*e*f - 8*c^2*f^2 + 2*(3*
d^2*e*f - d*f^2)*x)*e^(2*d*x + 2*c) + 2*(-3*I*d^2*f^2*x^2 + I*d^2*e^2 + 2*
(-4*I*c + I)*d*e*f + 4*I*c^2*f^2 + 2*(-3*I*d^2*e*f + I*d*f^2)*x)*e^(d*x +
c) - (-3*I*d^2*f^2*x^2 - 3*I*d^2*e^2 - 4*d*e*f + 2*I*f^2 - 2*(3*I*d^2*e*f
+ 2*d*f^2)*x + (3*d^2*f^2*x^2 + 3*d^2*e^2 - 4*I*d*e*f - 2*f^2 + 2*(3*d^...
```

3.218.6 Sympy [F]

$$\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^2\operatorname{csch}^3(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{f^2x^2\operatorname{csch}^3(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{2efx\operatorname{csch}^3(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**2*csch(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(f**2
*x**2*csch(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*csch(c +
 d*x)**3/(sinh(c + d*x) - I), x))/a
```

3.218.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 863 vs. $2(329) = 658$.

Time = 0.46 (sec) , antiderivative size = 863, normalized size of antiderivative = 2.35

$$\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-1/2*e^2*(2*(-I*e^(-d*x - c) - 5*e^(-2*d*x - 2*c) + 3*I*e^(-3*d*x - 3*c) +
 3*e^(-4*d*x - 4*c) + 4)/((a*e^(-d*x - c) - 2*I*a*e^(-2*d*x - 2*c) - 2*a*e
^(-3*d*x - 3*c) + I*a*e^(-4*d*x - 4*c) + a*e^(-5*d*x - 5*c) + I*a)*d) - 3*
log(e^(-d*x - c) + 1)/(a*d) + 3*log(e^(-d*x - c) - 1)/(a*d)) + 2*I*f^2*x^2
/(a*d) + 4*I*e*f*x/(a*d) - (4*d*f^2*x^2 + 8*d*e*f*x + (3*d*f^2*x^2*e^(4*c)
 + 2*e*f*e^(4*c) + 2*(3*d*e*f + f^2)*x*e^(4*c))*e^(4*d*x) + (-3*I*d*f^2*x^
2*e^(3*c) - 2*I*e*f*e^(3*c) - 2*(3*I*d*e*f + I*f^2)*x*e^(3*c))*e^(3*d*x) -
 (5*d*f^2*x^2*e^(2*c) + 2*e*f*e^(2*c) + 2*(5*d*e*f + f^2)*x*e^(2*c))*e^(2*
d*x) + (I*d*f^2*x^2*e^c + 2*I*e*f*e^c - 2*(-I*d*e*f - I*f^2)*x*e^c)*e^(d*x
))/(a*d^2*e^(5*d*x + 5*c) - I*a*d^2*e^(4*d*x + 4*c) - 2*a*d^2*e^(3*d*x + 3
*c) + 2*I*a*d^2*e^(2*d*x + 2*c) + a*d^2*e^(d*x + c) - I*a*d^2) - 4*I*e*f*l
og(I*e^(d*x + c) + 1)/(a*d^2) + 3/2*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*
dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3) - 3/2*(d^2*x
^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x
+ c)))*f^2/(a*d^3) - 4*I*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c
)))*f^2/(a*d^3) + (2*I*d*e*f + f^2)*x/(a*d^2) + (2*I*d*e*f - f^2)*x/(a*d^2
) + (3*d*e*f - 2*I*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(
a*d^3) - (3*d*e*f + 2*I*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c
)))/(a*d^3) - (2*I*d*e*f + f^2)*log(e^(d*x + c) + 1)/(a*d^3) - (2*I*d*e*f
- f^2)*log(e^(d*x + c) - 1)/(a*d^3) + 1/2*(d^3*f^2*x^3 + (3*d*e*f + 2*I...
```

3.218.8 Giac [F]

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{csch}\,(dx+c)^3}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*csch(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\sinh\,(c+d\,x)^3\,\,(a+a\sinh\,(c+d\,x)\,\,1\text{i})}\,dx$$

input
```
int((e + f*x)^2/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^2/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

3.219 $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.219.1 Optimal result

Integrand size = 29, antiderivative size = 214

$$\begin{aligned}\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = {} & \frac{3(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)\coth(c+dx)}{ad} \\ & -\frac{f\operatorname{csch}(c+dx)}{2ad^2}-\frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} \\ & -\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}-\frac{if\log(\sinh(c+dx))}{ad^2} \\ & +\frac{3f\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{2ad^2}-\frac{3f\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{2ad^2} \\ & +\frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\end{aligned}$$

output
```
3*(f*x+e)*arctanh(exp(d*x+c))/a/d+I*(f*x+e)*coth(d*x+c)/a/d-1/2*f*csch(d*x
+c)/a/d^2-1/2*(f*x+e)*coth(d*x+c)*csch(d*x+c)/a/d-2*I*f*ln(cosh(1/2*c+1/4*
I*Pi+1/2*d*x))/a/d^2-I*f*ln(sinh(d*x+c))/a/d^2+3/2*f*polylog(2,-exp(d*x+c)
)/a/d^2-3/2*f*polylog(2,exp(d*x+c))/a/d^2+I*(f*x+e)*tanh(1/2*c+1/4*I*Pi+1/
2*d*x)/a/d
```

3.219.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 461 vs. 2(214) = 428.

Time = 3.54 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.15

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(2i(if+2d(e+fx))\cosh\left(\frac{1}{2}(c+dx)\right)\left(i+\coth\left(\frac{1}{2}(c+dx)\right)\right)-c\right.}{}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*((2*I)*(I*f + 2*d*(e + f*x))*Co
sh[(c + d*x)/2]*(I + Coth[(c + d*x)/2]) - d*(e + f*x)*(I + Coth[(c + d*x)/
2])*Csch[(c + d*x)/2] - 8*f*(c + d*x)*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x
)/2]) + 16*f*ArcTan[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*
x)/2]) + 4*((-2*I)*f*(c + d*x) - ((2*I)*f + 3*d*(e + f*x))*Log[1 - E^(-c -
 d*x)] + ((-2*I)*f + 3*d*(e + f*x))*Log[1 + E^(-c - d*x)] - 3*f*PolyLog[2,
 -E^(-c - d*x)] + 3*f*PolyLog[2, E^(-c - d*x)])*(Cosh[(c + d*x)/2] + I*Sin
h[(c + d*x)/2]) + (16*I)*d*(e + f*x)*Sinh[(c + d*x)/2] + 8*f*Log[Cosh[c +
d*x]]*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]) + 2*(f + (2*I)*d*(e + f
*x))*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*Tanh[(c + d*x)/2] - I*d*(e
+ f*x)*Sech[(c + d*x)/2]*(-I + Tanh[(c + d*x)/2])))/(8*d^2*(a + I*a*Sinh[c
 + d*x]))
```

3.219.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{ 6109}$$
$$\frac{\int (e+fx)\operatorname{csch}^3(c+dx)dx}{a}-i\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$
$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -i(e+fx)\csc(ic+idx)^3dx}{a} - i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow 26$$

$$-\frac{i\int (e+fx)\csc(ic+idx)^3dx}{a} - i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow 4673$$

$$-\frac{i\left(\frac{1}{2}\int -i(e+fx)\text{csch}(c+dx)dx - \frac{if\text{csch}(c+dx)}{2d^2} - \frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a} - i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow 26$$

$$-\frac{i\left(-\frac{1}{2}i\int (e+fx)\text{csch}(c+dx)dx - \frac{if\text{csch}(c+dx)}{2d^2} - \frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a} - i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow 3042$$

$$-\frac{i\left(-\frac{1}{2}i\int i(e+fx)\csc(ic+idx)dx - \frac{if\text{csch}(c+dx)}{2d^2} - \frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a} - i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow 26$$

$$-\frac{i\left(\frac{1}{2}\int (e+fx)\csc(ic+idx)dx - \frac{if\text{csch}(c+dx)}{2d^2} - \frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a} - i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow 4670$$

$$-\frac{i\left(\frac{1}{2}\left(\frac{if\int \log(1-e^{c+dx})dx}{d} - \frac{if\int \log(1+e^{c+dx})dx}{d} + \frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right) - \frac{if\text{csch}(c+dx)}{2d^2} - \frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a} - i\int \frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow 2715$$

3.219. $\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(\frac{if\int e^{-c-dx}\log(1-e^{c+dx})de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log(1+e^{c+dx})de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+f}{}\right.}{a}$$

$$i\int\frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx$$

$$\Big\downarrow \text{2838}$$

$$-i\int\frac{(e+fx)\text{csch}^2(c+dx)}{i\sinh(c+dx)a+a}dx-$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\text{ PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\text{ PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{6109}$$

$$-i\left(\frac{\int(e+fx)\text{csch}^2(c+dx)dx}{a}-i\int\frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)-$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\text{ PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\text{ PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{\int-\left((e+fx)\csc(ic+idx)^2\right)dx}{a}-i\int\frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)-$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\text{ PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\text{ PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{25}$$

$$-i\left(-\frac{\int(e+fx)\csc(ic+idx)^2dx}{a}-i\int\frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)-$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\text{ PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\text{ PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{4672}$$

$$-i\left(-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{if\int-i\coth(c+dx)dx}{d}}{a}-i\int\frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)-$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\text{ PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\text{ PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{26}$$

3.219. $\int\frac{(e+fx)\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}dx$

$$-i\left(-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int\coth(c+dx)dx}{d}}{a}-i\int\frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int-i\tan(ic+idx+\frac{\pi}{2})dx}{d}}{a}-i\int\frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{\frac{(e+fx)\coth(c+dx)}{d}+\frac{if\int\tan(\frac{1}{2}(2ic+\pi)+idx)dx}{d}}{a}-i\int\frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx\right)-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{3956}$$

$$-i\left(-i\int\frac{(e+fx)\text{csch}(c+dx)}{i\sinh(c+dx)a+a}dx-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{6109}$$

$$-i\left(-i\left(\frac{\int(e+fx)\text{csch}(c+dx)dx}{a}-i\int\frac{e+fx}{i\sinh(c+dx)a+a}dx\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-i\left(\frac{\int i(e+fx)\csc(ic+idx)dx}{a}-i\int\frac{e+fx}{\sin(ic+idx)a+a}dx\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

3.219. $\int\frac{(e+fx)\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 26$$

$$-i\left(-i\left(\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-i\int \frac{e+fx}{\sin(ic+idx)a+a}dx\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx}{2d}}{a}$$

$$\Big\downarrow \; 3799$$

$$-i\left(-i\left(\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-\frac{i\int -\left((e+fx)\text{csch}^2\left(\frac{c}{2}+\frac{dx}{2}-\frac{i\pi}{4}\right)\right)dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c}{d^2}}{a}\right.$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx}{2d}}{a}$$

$$\Big\downarrow \; 25$$

$$-i\left(-i\left(\frac{i\int (e+fx)\csc(ic+idx)dx}{a}+\frac{i\int -\left((e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c}{d^2}}{a}\right.$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx}{2d}}{a}$$

$$\Big\downarrow \; 25$$

$$-i\left(-i\left(\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-\frac{i\int (e+fx)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right.$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx}{2d}}{a}$$

$$\Big\downarrow \; 3042$$

$$-i\left(-i\left(\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-\frac{i\int (e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right)-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right.$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx}{2d}}{a}$$

$$\Big\downarrow \; 4670$$

3.219. $\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-i\left(-i\left(\frac{i\left(\frac{if\int\log(1-e^{c+dx})dx}{d}-\frac{if\int\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int(e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right.\right.$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{ 2715}$$

$$-i\left(-i\left(\frac{i\left(\frac{if\int e^{-c-dx}\log(1-e^{c+dx})de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log(1+e^{c+dx})de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)}{a}-\frac{i\int(e+fx)\csc\right.\right.$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{ 2838}$$

$$-i\left(-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\int(e+fx)\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{4}\right)^2dx}{2a}\right.\right.$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{ 4672}$$

$$-i\left(-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2if\int -\right.}{2a}\right.\right.$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{ 26}$$

3.219. $\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$-i\left(-i\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)}{a}-\frac{i\left(\frac{2(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{d}-\frac{2f\int\tan\right.}{2a}\right.\right.$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

input
```
Int[((e + f*x)*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.219.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

3.219.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(185) = 370$.

Time = 2.38 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.06

method	result
risch	$-\frac{-3ide\,\mathrm{e}^{3dx+3c}-5\,\mathrm{e}^{2dx+2c}dfx+3dfx\,\mathrm{e}^{4dx+4c}+ide\,\mathrm{e}^{dx+c}-5\,\mathrm{e}^{2dx+2c}de+3de\,\mathrm{e}^{4dx+4c}+idfx\,\mathrm{e}^{dx+c}+4dfx+f\,\mathrm{e}^{4dx+4c}-i\mathrm{e}^{3dx+3c}f+i\mathrm{e}}{\left(\mathrm{e}^{2dx+2c}-1\right)^{2}d^{2}\left(\mathrm{e}^{dx+c}-i\right)a}$

input
```
int((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(-3*I*d*e*exp(3*d*x+3*c)-5*exp(2*d*x+2*c)*d*f*x+3*d*f*x*exp(4*d*x+4*c)+I*
d*e*exp(d*x+c)-5*exp(2*d*x+2*c)*d*e+3*d*e*exp(4*d*x+4*c)+I*d*f*x*exp(d*x+c
)+4*d*f*x+f*exp(4*d*x+4*c)-I*exp(3*d*x+3*c)*f+I*exp(d*x+c)*f+4*d*e-f*exp(2
*d*x+2*c)-3*I*d*f*x*exp(3*d*x+3*c))/(exp(2*d*x+2*c)-1)^2/d^2/(exp(d*x+c)-I
)/a-3/2/a/d*f*ln(1-exp(d*x+c))*x+3/2/a/d*f*ln(exp(d*x+c)+1)*x-3/2/a/d*e*ln
(exp(d*x+c)-1)+4*I/a/d^2*f*ln(exp(d*x+c))-I/a/d^2*f*ln(exp(d*x+c)-1)-I/a/d
^2*f*ln(exp(d*x+c)+1)-I/a/d^2*f*ln(1+exp(2*d*x+2*c))+3/2/a/d*e*ln(exp(d*x+
c)+1)-3/2/a/d^2*f*ln(1-exp(d*x+c))*c+3/2/a/d^2*c*f*ln(exp(d*x+c)-1)+2/a/d^
2*f*arctan(exp(d*x+c))+3/2*f*polylog(2,-exp(d*x+c))/a/d^2-3/2*f*polylog(2,
exp(d*x+c))/a/d^2
```

3.219.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 817 vs. 2(181) = 362.

Time = 0.28 (sec) , antiderivative size = 817, normalized size of antiderivative = 3.82

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(8*d*e - 4*c*f - 3*(f*e^(5*d*x + 5*c) - I*f*e^(4*d*x + 4*c) - 2*f*e^(
3*d*x + 3*c) + 2*I*f*e^(2*d*x + 2*c) + f*e^(d*x + c) - I*f)*dilog(-e^(d*x
+ c)) + 3*(f*e^(5*d*x + 5*c) - I*f*e^(4*d*x + 4*c) - 2*f*e^(3*d*x + 3*c) +
 2*I*f*e^(2*d*x + 2*c) + f*e^(d*x + c) - I*f)*dilog(e^(d*x + c)) + 4*(-2*I
*d*f*x - I*c*f)*e^(5*d*x + 5*c) - 2*(d*f*x - 3*d*e + (2*c - 1)*f)*e^(4*d*x
 + 4*c) + 2*(5*I*d*f*x - 3*I*d*e + (4*I*c - I)*f)*e^(3*d*x + 3*c) + 2*(3*d
*f*x - 5*d*e + (4*c - 1)*f)*e^(2*d*x + 2*c) + 2*(-3*I*d*f*x + I*d*e + (-2*
I*c + I)*f)*e^(d*x + c) - (-3*I*d*f*x - 3*I*d*e + (3*d*f*x + 3*d*e - 2*I*f
)*e^(5*d*x + 5*c) + (-3*I*d*f*x - 3*I*d*e - 2*f)*e^(4*d*x + 4*c) - 2*(3*d*
f*x + 3*d*e - 2*I*f)*e^(3*d*x + 3*c) - 2*(-3*I*d*f*x - 3*I*d*e - 2*f)*e^(2
*d*x + 2*c) + (3*d*f*x + 3*d*e - 2*I*f)*e^(d*x + c) - 2*f)*log(e^(d*x + c)
 + 1) + 4*(I*f*e^(5*d*x + 5*c) + f*e^(4*d*x + 4*c) - 2*I*f*e^(3*d*x + 3*c)
 - 2*f*e^(2*d*x + 2*c) + I*f*e^(d*x + c) + f)*log(e^(d*x + c) - I) - (3*I*
d*e + (-3*I*c - 2)*f - (3*d*e - (3*c - 2*I)*f)*e^(5*d*x + 5*c) + (3*I*d*e
+ (-3*I*c - 2)*f)*e^(4*d*x + 4*c) + 2*(3*d*e - (3*c - 2*I)*f)*e^(3*d*x + 3
*c) - 2*(3*I*d*e + (-3*I*c - 2)*f)*e^(2*d*x + 2*c) - (3*d*e - (3*c - 2*I)*
f)*e^(d*x + c))*log(e^(d*x + c) - 1) + 3*(-I*d*f*x - I*c*f + (d*f*x + c*f)
*e^(5*d*x + 5*c) + (-I*d*f*x - I*c*f)*e^(4*d*x + 4*c) - 2*(d*f*x + c*f)*e^
(3*d*x + 3*c) + 2*(I*d*f*x + I*c*f)*e^(2*d*x + 2*c) + (d*f*x + c*f)*e^(d*x
 + c))*log(-e^(d*x + c) + 1))/(a*d^2*e^(5*d*x + 5*c) - I*a*d^2*e^(4*d*x...
```

3.219.6 Sympy [F]

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e\operatorname{csch}^3(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{fx\operatorname{csch}^3(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)*csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e*csch(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(f*x*csc
h(c + d*x)**3/(sinh(c + d*x) - I), x))/a
```

3.219.7 Maxima [F]

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)\,\text{csch}\,(dx+c)^3}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(24*d*integrate(1/16*x/(a*d*e^(d*x + c) + a*d), x) + 24*d*integrate(1/16*
x/(a*d*e^(d*x + c) - a*d), x) + 8*(2*d*x*e^(5*d*x + 5*c) + 2*I*d*x + (I*d*
x*e^(4*c) + I*e^(4*c))*e^(4*d*x) - (d*x*e^(3*c) - e^(3*c))*e^(3*d*x) + (-I
*d*x*e^(2*c) - I*e^(2*c))*e^(2*d*x) + (d*x*e^c - e^c)*e^(d*x))/(8*I*a*d^2*
e^(5*d*x + 5*c) + 8*a*d^2*e^(4*d*x + 4*c) - 16*I*a*d^2*e^(3*d*x + 3*c) - 1
6*a*d^2*e^(2*d*x + 2*c) + 8*I*a*d^2*e^(d*x + c) + 8*a*d^2) - 2*I*(d*x + c)
/(a*d^2) + 2*I*log((e^(d*x + c) - I)*e^(-c))/(a*d^2) + I*log(e^(d*x + c) +
 1)/(a*d^2) + I*log(e^(d*x + c) - 1)/(a*d^2))*f - 1/2*e*(2*(-I*e^(-d*x - c
) - 5*e^(-2*d*x - 2*c) + 3*I*e^(-3*d*x - 3*c) + 3*e^(-4*d*x - 4*c) + 4)/((
a*e^(-d*x - c) - 2*I*a*e^(-2*d*x - 2*c) - 2*a*e^(-3*d*x - 3*c) + I*a*e^(-4
*d*x - 4*c) + a*e^(-5*d*x - 5*c) + I*a)*d) - 3*log(e^(-d*x - c) + 1)/(a*d)
 + 3*log(e^(-d*x - c) - 1)/(a*d))
```

3.219.8 Giac [F]

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)\,\text{csch}\,(dx+c)^3}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*csch(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\sinh\left(c+d\,x\right)^3\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

3.220 $\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.220.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{3\operatorname{arctanh}(\cosh(c+dx))}{2ad}+\frac{2i\coth(c+dx)}{ad} - \frac{3\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}+\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))}$$

output
```
3/2*arctanh(cosh(d*x+c))/a/d+2*I*coth(d*x+c)/a/d-3/2*coth(d*x+c)*csch(d*x+
c)/a/d+coth(d*x+c)*csch(d*x+c)/d/(a+I*a*sinh(d*x+c))
```

3.220.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{4i\operatorname{csch}(2(c+dx))-3\operatorname{sech}(c+dx)+3\operatorname{arctanh}\left(\sqrt{\cosh^2(c+dx)}\right)\sqrt{\cosh^2(c+dx)}\operatorname{sech}(c+dx)-\operatorname{csch}^2(c}{2ad}$$

input
```
Integrate[Csch[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((4*I)*Csch[2*(c + d*x)] - 3*Sech[c + d*x] + 3*ArcTanh[Sqrt[Cosh[c + d*x]^
2]]*Sqrt[Cosh[c + d*x]^2]*Sech[c + d*x] - Csch[c + d*x]^2*Sech[c + d*x] +
(4*I)*Tanh[c + d*x])/(2*a*d)
```

3.220.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.11, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {3042, 26, 3247, 26, 3042, 26, 3227, 25, 26, 3042, 25, 26, 4254, 24, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\sin(ic+idx)^3(a+a\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\sin(ic+idx)^3(\sin(ic+idx)a+a)}dx$$

$$\Big\downarrow \text{3247}$$

$$-i\left(\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\int -i\text{csch}^3(c+dx)(3a-2ia\sinh(c+dx))dx}{a^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\int \text{csch}^3(c+dx)(3a-2ia\sinh(c+dx))dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\int -\frac{i(3a-2a\sin(ic+idx))}{\sin(ic+idx)^3}dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{\int \frac{3a-2a\sin(ic+idx)}{\sin(ic+idx)^3}dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{3227}$$

$$-i\left(\frac{-2a\int -\text{csch}^2(c+dx)dx+3a\int i\text{csch}^3(c+dx)dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \textbf{25}$$

$$-i\left(\frac{2a\int \text{csch}^2(c+dx)dx+3a\int i\text{csch}^3(c+dx)dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \textbf{26}$$

$$-i\left(\frac{2a\int \text{csch}^2(c+dx)dx+3ia\int \text{csch}^3(c+dx)dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$-i\left(\frac{2a\int -\csc(ic+idx)^2dx+3ia\int -i\csc(ic+idx)^3dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \textbf{25}$$

$$-i\left(\frac{3ia\int -i\csc(ic+idx)^3dx-2a\int \csc(ic+idx)^2dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \textbf{26}$$

$$-i\left(\frac{3a\int \csc(ic+idx)^3dx-2a\int \csc(ic+idx)^2dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \textbf{4254}$$

$$-i\left(\frac{3a\int \csc(ic+idx)^3dx-\frac{2ia\int 1d(-i\coth(c+dx))}{d}}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \textbf{24}$$

$$-i\left(\frac{-\frac{2a\coth(c+dx)}{d}+3a\int \csc(ic+idx)^3dx}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \textbf{4255}$$

$$-i\left(\frac{-\frac{2a\coth(c+dx)}{d}+3a\left(\frac{1}{2}\int -i\text{csch}(c+dx)dx-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \textbf{26}$$

$$-i\left(\frac{-\frac{2a\coth(c+dx)}{d}+3a\left(-\frac{1}{2}i\int \text{csch}(c+dx)dx-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

3.220. $\int \frac{\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{-\frac{2a\coth(c+dx)}{d}+3a\left(-\frac{1}{2}i\int i\csc(ic+idx)dx-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{-\frac{2a\coth(c+dx)}{d}+3a\left(\frac{1}{2}\int\csc(ic+idx)dx-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

$$\Big\downarrow \text{4257}$$

$$-i\left(\frac{-\frac{2a\coth(c+dx)}{d}+3a\left(\frac{i\text{arctanh}(\cosh(c+dx))}{2d}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a^2}+\frac{i\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))}\right)$$

input
```
Int[Csch[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-I)*(((-2*a*Coth[c + d*x])/d + 3*a*(((I/2)*ArcTanh[Cosh[c + d*x]])/d - ((
I/2)*Coth[c + d*x]*Csch[c + d*x])/d))/a^2 + (I*Coth[c + d*x]*Csch[c + d*x]
)/(d*(a + I*a*Sinh[c + d*x])))
```

3.220.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.220. $\int \frac{\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.220.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{2i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2}+\frac{8i}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{2i}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}$
default	$\frac{2i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2}+\frac{8i}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{2i}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}$
risch	$-\frac{i e^{dx+c}-3i e^{3dx+3c}+3\,e^{4dx+4c}-5\,e^{2dx+2c}+4}{\left(e^{2dx+2c}-1\right)^{2}\left(e^{dx+c}-i\right)ad}-\frac{3\ln\left(e^{dx+c}-1\right)}{2da}+\frac{3\ln\left(e^{dx+c}+1\right)}{2da}$
parallelrisch	$\frac{12\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-i\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-3i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-3\coth\left(\frac{dx}{2}+\frac{c}{2}\right)-24\,\mathrm{t}}{8da\left(i-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

input
```
int(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a*(2*I*tanh(1/2*d*x+1/2*c)+1/2*tanh(1/2*d*x+1/2*c)^2+8*I/(-I+tanh(1/
2*d*x+1/2*c))-1/2/tanh(1/2*d*x+1/2*c)^2+2*I/tanh(1/2*d*x+1/2*c)-6*ln(tanh(
1/2*d*x+1/2*c)))
```

3.220.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(79) = 158.

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.69

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{3\left(e^{(5\,dx+5\,c)}-i\,e^{(4\,dx+4\,c)}-2\,e^{(3\,dx+3\,c)}+2i\,e^{(2\,dx+2\,c)}+e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}+1\right)-3\left(e^{(5\,dx+5\,c)}-i\,e^{(4\,dx}\right.}{2\left(ade^{(5\,dx+5\,c)}-i\,ade^{(4\,dx+4\,c)}-2\right.}$$

input
```
integrate(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(3*(e^(5*d*x + 5*c) - I*e^(4*d*x + 4*c) - 2*e^(3*d*x + 3*c) + 2*I*e^(2
*d*x + 2*c) + e^(d*x + c) - I)*log(e^(d*x + c) + 1) - 3*(e^(5*d*x + 5*c) -
 I*e^(4*d*x + 4*c) - 2*e^(3*d*x + 3*c) + 2*I*e^(2*d*x + 2*c) + e^(d*x + c)
 - I)*log(e^(d*x + c) - 1) - 6*e^(4*d*x + 4*c) + 6*I*e^(3*d*x + 3*c) + 10*
e^(2*d*x + 2*c) - 2*I*e^(d*x + c) - 8)/(a*d*e^(5*d*x + 5*c) - I*a*d*e^(4*d
*x + 4*c) - 2*a*d*e^(3*d*x + 3*c) + 2*I*a*d*e^(2*d*x + 2*c) + a*d*e^(d*x +
 c) - I*a*d)
```

3.220.6 Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int \frac{\operatorname{csch}^3(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

input
```
integrate(csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(csch(c + d*x)**3/(sinh(c + d*x) - I), x)/a
```

3.220.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.79

$$\begin{aligned}\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{-i\,e^{(-dx-c)}-5\,e^{(-2\,dx-2\,c)}+3i\,e^{(-3\,dx-3\,c)}+3\,e^{(-4\,dx-4\,c)}+4}{(ae^{(-dx-c)}-2i\,ae^{(-2\,dx-2\,c)}-2\,ae^{(-3\,dx-3\,c)}+i\,ae^{(-4\,dx-4\,c)}+ae^{(-5\,dx-5\,c)}+i\,a)d}\\&\quad+\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{2\,ad}-\frac{3\,\log\left(e^{(-dx-c)}-1\right)}{2\,ad}\end{aligned}$$

input
```
integrate(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(-I*e^(-d*x - c) - 5*e^(-2*d*x - 2*c) + 3*I*e^(-3*d*x - 3*c) + 3*e^(-4*d*
x - 4*c) + 4)/((a*e^(-d*x - c) - 2*I*a*e^(-2*d*x - 2*c) - 2*a*e^(-3*d*x -
3*c) + I*a*e^(-4*d*x - 4*c) + a*e^(-5*d*x - 5*c) + I*a)*d) + 3/2*log(e^(-d
*x - c) + 1)/(a*d) - 3/2*log(e^(-d*x - c) - 1)/(a*d)
```

3.220.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\frac{3\,\log\left(e^{(dx+c)}+1\right)}{a}-\frac{3\,\log\left(e^{(dx+c)}-1\right)}{a}-\frac{2\left(e^{(3\,dx+3\,c)}-2i\,e^{(2\,dx+2\,c)}+e^{(dx+c)}+2i\right)}{a\left(e^{(2\,dx+2\,c)}-1\right)^2}-\frac{4i}{a\left(i\,e^{(dx+c)}+1\right)}}{2\,d}$$

input
```
integrate(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(3*log(e^(d*x + c) + 1)/a - 3*log(e^(d*x + c) - 1)/a - 2*(e^(3*d*x + 3
*c) - 2*I*e^(2*d*x + 2*c) + e^(d*x + c) + 2*I)/(a*(e^(2*d*x + 2*c) - 1)^2)
 - 4*I/(a*(I*e^(d*x + c) + 1)))/d
```

3.220.9 Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.52

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-a^2\,d^2}}{a\,d}\right)}{\sqrt{-a^2\,d^2}} - \frac{2}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)} - \frac{\mathrm{e}^{c+d\,x}}{a\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)} - \frac{2\,\mathrm{e}^{c+d\,x}}{a\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)^2} + \frac{2\mathrm{i}}{a\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

input
```
int(1/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
(3*atan((exp(d*x)*exp(c)*(-a^2*d^2)^(1/2))/(a*d)))/(-a^2*d^2)^(1/2) - 2/(a
*d*(exp(c + d*x) - 1i)) + 2i/(a*d*(exp(2*c + 2*d*x) - 1)) - exp(c + d*x)/(
a*d*(exp(2*c + 2*d*x) - 1)) - (2*exp(c + d*x))/(a*d*(exp(2*c + 2*d*x) - 1)
^2)
```

3.221 $\int \frac{\text{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

3.221.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\text{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\text{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.221.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{\text{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \$\text{Aborted}$$

input
```
Integrate[Csch[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.221.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\text{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.221.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.221.4 Maple [N/A] (verified)

Not integrable

Time = 0.91 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}\left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.221.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 903, normalized size of antiderivative = 29.13

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^3}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(4*d*f*x + 4*d*e + (3*d*f*x + 3*d*e - f)*e^(4*d*x + 4*c) - (3*I*d*f*x + 3
*I*d*e - I*f)*e^(3*d*x + 3*c) - (5*d*f*x + 5*d*e - f)*e^(2*d*x + 2*c) - (-
I*d*f*x - I*d*e + I*f)*e^(d*x + c) - (-I*a*d^2*f^2*x^2 - 2*I*a*d^2*e*f*x -
 I*a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^(5*d*x + 5*c)
 + (-I*a*d^2*f^2*x^2 - 2*I*a*d^2*e*f*x - I*a*d^2*e^2)*e^(4*d*x + 4*c) - 2*
(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^(3*d*x + 3*c) - 2*(-I*a*d^2*
f^2*x^2 - 2*I*a*d^2*e*f*x - I*a*d^2*e^2)*e^(2*d*x + 2*c) + (a*d^2*f^2*x^2
+ 2*a*d^2*e*f*x + a*d^2*e^2)*e^(d*x + c))*integral((4*d*f^2*x + 4*d*e*f -
(3*d^2*f^2*x^2 + 3*d^2*e^2 + 2*d*e*f - 2*f^2 + 2*(3*d^2*e*f + d*f^2)*x)*e^
(2*d*x + 2*c) + (3*I*d^2*f^2*x^2 + 3*I*d^2*e^2 + 2*I*d*e*f - 2*I*f^2 - 2*(
-3*I*d^2*e*f - I*d*f^2)*x)*e^(d*x + c))/(I*a*d^2*f^3*x^3 + 3*I*a*d^2*e*f^2
*x^2 + 3*I*a*d^2*e^2*f*x + I*a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^
2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(3*d*x + 3*c) + (-I*a*d^2*f^3*x^3 - 3*I
*a*d^2*e*f^2*x^2 - 3*I*a*d^2*e^2*f*x - I*a*d^2*e^3)*e^(2*d*x + 2*c) - (a*d
^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(d*x + c))
, x))/(-I*a*d^2*f^2*x^2 - 2*I*a*d^2*e*f*x - I*a*d^2*e^2 + (a*d^2*f^2*x^2 +
 2*a*d^2*e*f*x + a*d^2*e^2)*e^(5*d*x + 5*c) + (-I*a*d^2*f^2*x^2 - 2*I*a*d^
2*e*f*x - I*a*d^2*e^2)*e^(4*d*x + 4*c) - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x
+ a*d^2*e^2)*e^(3*d*x + 3*c) - 2*(-I*a*d^2*f^2*x^2 - 2*I*a*d^2*e*f*x - I*a
*d^2*e^2)*e^(2*d*x + 2*c) + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)...
```

3.221.6 Sympy [N/A]

Not integrable

Time = 42.90 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\operatorname{csch}^3\left(c+dx\right)}{e\sinh\left(c+dx\right)-ie+fx\sinh\left(c+dx\right)-ifx}\,dx}{a}$$

input
```
integrate(csch(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(csch(c + d*x)**3/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) -
I*f*x), x)/a
```

3.221.7 Maxima [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 775, normalized size of antiderivative = 25.00

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{(fx+e)(ia\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-8*f*integrate(1/(-4*I*a*d*f^2*x^2 - 8*I*a*d*e*f*x - 4*I*a*d*e^2 + 4*(a*d*
f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 8*(4*d*f*x + 4
*d*e + (3*d*f*x*e^(4*c) + (3*d*e - f)*e^(4*c))*e^(4*d*x) + (-3*I*d*f*x*e^(
3*c) + (-3*I*d*e + I*f)*e^(3*c))*e^(3*d*x) - (5*d*f*x*e^(2*c) + (5*d*e - f
)*e^(2*c))*e^(2*d*x) + (I*d*f*x*e^c + (I*d*e - I*f)*e^c)*e^(d*x))/(-8*I*a*
d^2*f^2*x^2 - 16*I*a*d^2*e*f*x - 8*I*a*d^2*e^2 + 8*(a*d^2*f^2*x^2*e^(5*c)
+ 2*a*d^2*e*f*x*e^(5*c) + a*d^2*e^2*e^(5*c))*e^(5*d*x) - 8*(I*a*d^2*f^2*x^
2*e^(4*c) + 2*I*a*d^2*e*f*x*e^(4*c) + I*a*d^2*e^2*e^(4*c))*e^(4*d*x) - 16*
(a*d^2*f^2*x^2*e^(3*c) + 2*a*d^2*e*f*x*e^(3*c) + a*d^2*e^2*e^(3*c))*e^(3*d
*x) - 16*(-I*a*d^2*f^2*x^2*e^(2*c) - 2*I*a*d^2*e*f*x*e^(2*c) - I*a*d^2*e^2
*e^(2*c))*e^(2*d*x) + 8*(a*d^2*f^2*x^2*e^c + 2*a*d^2*e*f*x*e^c + a*d^2*e^2
*e^c)*e^(d*x)) - 8*integrate(1/16*(3*d^2*f^2*x^2 + 3*d^2*e^2 + 2*I*d*e*f -
 2*f^2 + 2*(3*d^2*e*f + I*d*f^2)*x)/(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3
*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c +
3*a*d^2*e^2*f*x*e^c + a*d^2*e^3*e^c)*e^(d*x)), x) - 8*integrate(-1/16*(3*d
^2*f^2*x^2 + 3*d^2*e^2 - 2*I*d*e*f - 2*f^2 + 2*(3*d^2*e*f - I*d*f^2)*x)/(a
*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 - (a*d^2*f^
3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c + 3*a*d^2*e^2*f*x*e^c + a*d^2*e^3*e^c)*e
^(d*x)), x)
```

3.221.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.221.9 Mupad [N/A]

Not integrable

Time = 4.64 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\sinh(c+d\,x)^3\,(e+f\,x)\,(a+a\sinh(c+d\,x)\,1\mathrm{i})}\,dx$$

input
```
int(1/(sinh(c + d*x)^3*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(sinh(c + d*x)^3*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.222 $\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

3.222.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.222.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \$\text{Aborted}$$

input
```
Integrate[Csch[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.222.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.222.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.222.4 Maple [N/A] (verified)

Not integrable

Time = 0.95 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}\left(dx+c\right)^{3}}{\left(fx+e\right)^{2}\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.222.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 1144, normalized size of antiderivative = 36.90

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^3}{\left(fx+e\right)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-(4*d*f*x + 4*d*e + (3*d*f*x + 3*d*e - 2*f)*e^(4*d*x + 4*c) - (3*I*d*f*x +
 3*I*d*e - 2*I*f)*e^(3*d*x + 3*c) - (5*d*f*x + 5*d*e - 2*f)*e^(2*d*x + 2*c
) - (-I*d*f*x - I*d*e + 2*I*f)*e^(d*x + c) - (-I*a*d^2*f^3*x^3 - 3*I*a*d^2
*e*f^2*x^2 - 3*I*a*d^2*e^2*f*x - I*a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*
f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(5*d*x + 5*c) + (-I*a*d^2*f^3*x^3
 - 3*I*a*d^2*e*f^2*x^2 - 3*I*a*d^2*e^2*f*x - I*a*d^2*e^3)*e^(4*d*x + 4*c)
- 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(3
*d*x + 3*c) - 2*(-I*a*d^2*f^3*x^3 - 3*I*a*d^2*e*f^2*x^2 - 3*I*a*d^2*e^2*f*
x - I*a*d^2*e^3)*e^(2*d*x + 2*c) + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*
a*d^2*e^2*f*x + a*d^2*e^3)*e^(d*x + c))*integral((8*d*f^2*x + 8*d*e*f - (3
*d^2*f^2*x^2 + 3*d^2*e^2 + 4*d*e*f - 6*f^2 + 2*(3*d^2*e*f + 2*d*f^2)*x)*e^
(2*d*x + 2*c) + (3*I*d^2*f^2*x^2 + 3*I*d^2*e^2 + 4*I*d*e*f - 6*I*f^2 - 2*(
-3*I*d^2*e*f - 2*I*d*f^2)*x)*e^(d*x + c))/(I*a*d^2*f^4*x^4 + 4*I*a*d^2*e*f
^3*x^3 + 6*I*a*d^2*e^2*f^2*x^2 + 4*I*a*d^2*e^3*f*x + I*a*d^2*e^4 + (a*d^2*
f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^
2*e^4)*e^(3*d*x + 3*c) + (-I*a*d^2*f^4*x^4 - 4*I*a*d^2*e*f^3*x^3 - 6*I*a*d
^2*e^2*f^2*x^2 - 4*I*a*d^2*e^3*f*x - I*a*d^2*e^4)*e^(2*d*x + 2*c) - (a*d^2
*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d
^2*e^4)*e^(d*x + c)), x))/(-I*a*d^2*f^3*x^3 - 3*I*a*d^2*e*f^2*x^2 - 3*I*a*
d^2*e^2*f*x - I*a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^...
```

3.222.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.222.7 Maxima [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 975, normalized size of antiderivative = 31.45

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-8*f*integrate(1/(-2*I*a*d*f^3*x^3 - 6*I*a*d*e*f^2*x^2 - 6*I*a*d*e^2*f*x -
 2*I*a*d*e^3 + 2*(a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^
c + a*d*e^3*e^c)*e^(d*x)), x) - 8*(4*d*f*x + 4*d*e + (3*d*f*x*e^(4*c) + (3
*d*e - 2*f)*e^(4*c))*e^(4*d*x) + (-3*I*d*f*x*e^(3*c) + (-3*I*d*e + 2*I*f)*
e^(3*c))*e^(3*d*x) - (5*d*f*x*e^(2*c) + (5*d*e - 2*f)*e^(2*c))*e^(2*d*x) +
 (I*d*f*x*e^c + (I*d*e - 2*I*f)*e^c)*e^(d*x))/(-8*I*a*d^2*f^3*x^3 - 24*I*a
*d^2*e*f^2*x^2 - 24*I*a*d^2*e^2*f*x - 8*I*a*d^2*e^3 + 8*(a*d^2*f^3*x^3*e^(
5*c) + 3*a*d^2*e*f^2*x^2*e^(5*c) + 3*a*d^2*e^2*f*x*e^(5*c) + a*d^2*e^3*e^(
5*c))*e^(5*d*x) - 8*(I*a*d^2*f^3*x^3*e^(4*c) + 3*I*a*d^2*e*f^2*x^2*e^(4*c)
 + 3*I*a*d^2*e^2*f*x*e^(4*c) + I*a*d^2*e^3*e^(4*c))*e^(4*d*x) - 16*(a*d^2*
f^3*x^3*e^(3*c) + 3*a*d^2*e*f^2*x^2*e^(3*c) + 3*a*d^2*e^2*f*x*e^(3*c) + a*
d^2*e^3*e^(3*c))*e^(3*d*x) - 16*(-I*a*d^2*f^3*x^3*e^(2*c) - 3*I*a*d^2*e*f^
2*x^2*e^(2*c) - 3*I*a*d^2*e^2*f*x*e^(2*c) - I*a*d^2*e^3*e^(2*c))*e^(2*d*x)
 + 8*(a*d^2*f^3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c + 3*a*d^2*e^2*f*x*e^c + a*
d^2*e^3*e^c)*e^(d*x)) - 8*integrate(1/16*(3*d^2*f^2*x^2 + 3*d^2*e^2 + 4*I*
d*e*f - 6*f^2 + 2*(3*d^2*e*f + 2*I*d*f^2)*x)/(a*d^2*f^4*x^4 + 4*a*d^2*e*f^
3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4 + (a*d^2*f^4*x^4
*e^c + 4*a*d^2*e*f^3*x^3*e^c + 6*a*d^2*e^2*f^2*x^2*e^c + 4*a*d^2*e^3*f*x*e
^c + a*d^2*e^4*e^c)*e^(d*x)), x) - 8*integrate(-1/16*(3*d^2*f^2*x^2 + 3*d^
2*e^2 - 4*I*d*e*f - 6*f^2 + 2*(3*d^2*e*f - 2*I*d*f^2)*x)/(a*d^2*f^4*x^4...
```

3.222.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.222.9 Mupad [N/A]

Not integrable

Time = 7.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^3\,{\left(e+f\,x\right)}^2\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int(1/(sinh(c + d*x)^3*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(sinh(c + d*x)^3*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.223 $\int \frac{(e+fx)^3 \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.223.1 Optimal result

Integrand size = 26, antiderivative size = 453

$$\begin{aligned}\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{(e+fx)^4}{4bf}-\frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d}\\
&+\frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d}\\
&-\frac{3af(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}\\
&+\frac{3af(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}\\
&+\frac{6af^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^3}\\
&-\frac{6af^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^3}\\
&-\frac{6af^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^4}\\
&+\frac{6af^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^4}\end{aligned}$$

output
```
1/4*(f*x+e)^4/b/f-a*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d/(
a^2+b^2)^(1/2)+a*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d/(a^2
+b^2)^(1/2)-3*a*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b
/d^2/(a^2+b^2)^(1/2)+3*a*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))/b/d^2/(a^2+b^2)^(1/2)+6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))/b/d^3/(a^2+b^2)^(1/2)-6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))/b/d^3/(a^2+b^2)^(1/2)-6*a*f^3*polylog(4,-b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/b/d^4/(a^2+b^2)^(1/2)+6*a*f^3*polylog(4,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d^4/(a^2+b^2)^(1/2)
```

3.223.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.34

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x(4e^3+6e^2fx+4ef^2x^2+f^3x^3)}{4b} - \frac{a\left(-2d^3e^3\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+3d^3e^2fx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+3d^3ef^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^3f^3x^3\,\mathrm{lo}\right.}{}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(4*b) - (a*(-2*d^3*e^3*Arc
Tanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])] - 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] -
3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^3*f^3*x
^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*Po
lyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*e*f^2*PolyLog[3, (
b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*d*f^3*x*PolyLog[3, (b*E^(c + d*
x))/(-a + Sqrt[a^2 + b^2])] + 6*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))] + 6*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))] + 6*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*f
^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*Sqrt[a^2 + b^
2]*d^4)
```

3.223.3 Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.88, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6091, 17, 3042, 3803, 25, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6091}$$

$$\frac{\int (e+fx)^3dx}{b}-\frac{a\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{(e+fx)^4}{4bf}-\frac{a\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^4}{4bf}-\frac{a\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b}$$

$$\Big\downarrow \text{3803}$$

$$\frac{(e+fx)^4}{4bf}-\frac{2a\int -\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{2a\int \frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \text{2694}$$

$$\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right.}{2\sqrt{a^2+b^2}}}{b}$$

$$\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+}}{a-\sqrt{a^2}}\right.}{bd}\right.}{}}{b}$$

$$\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \text{7163}$$

3.223. $\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\cdots\right.}{d}\cdots\right.}{bd}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.$$

$$\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \; 2720$$

$$2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,\cdots\right.}{d}\cdots\right.}{bd}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.$$

$$\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \; 7143$$

3.223. $\displaystyle\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right.$$

$$\frac{(e+fx)^4}{4bf}$$

input
```
Int[((e + f*x)^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(e + f*x)^4/(4*b*f) + (2*a*(-1/2*(b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^3
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x
)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e +
 f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog
[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(2*Sqrt[a
^2 + b^2])))/b
```

3.223. $\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.223.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.223. $\int \frac{(e+fx)^3 \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.223.4 Maple [F]

$$\int \frac{(fx+e)^3 \sinh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.223.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. $2(409) = 818$.

Time = 0.28 (sec) , antiderivative size = 1112, normalized size of antiderivative = 2.45

$$\int \frac{(e+fx)^3 \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*((a^2 + b^2)*d^4*f^3*x^4 + 4*(a^2 + b^2)*d^4*e*f^2*x^3 + 6*(a^2 + b^2)
*d^4*e^2*f*x^2 + 4*(a^2 + b^2)*d^4*e^3*x - 24*a*b*f^3*sqrt((a^2 + b^2)/b^2
)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*a*b*f^3*sqrt((a^2 + b^2)/b^2)*p
olylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2
*x + a*b*d^2*e^2*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(
d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/
b + 1) + 12*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x + a*b*d^2*e^2*f)*sqrt((a^
2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 4*(a*b*d^3*e^3 - 3*
a*b*c*d^2*e^2*f + 3*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*l
og(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a
) - 4*(a*b*d^3*e^3 - 3*a*b*c*d^2*e^2*f + 3*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*
sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt
((a^2 + b^2)/b^2) + 2*a) - 4*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*
b*d^3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3)*sqrt(
(a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(a*b*d^3*f^3*x^3 +
3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2...
```

3.223.6 Sympy [F]

$$\int \frac{(e+fx)^3 \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3 \sinh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*sinh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.223.7 Maxima [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^3*(a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + s
qrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d) - (d*x + c)/(b*d)) + 1/4*(f^3*x^4 +
 4*e*f^2*x^3 + 6*e^2*f*x^2)/b - integrate(2*(a*f^3*x^3*e^c + 3*a*e*f^2*x^2
*e^c + 3*a*e^2*f*x*e^c)*e^(d*x)/(b^2*e^(2*d*x + 2*c) + 2*a*b*e^(d*x + c) -
 b^2), x)
```

3.223.8 Giac [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,(e+f\,x)^3}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.224 $\int \frac{(e+fx)^2 \sinh(c+dx)}{a+b\sinh(c+dx)} dx$

3.224.1 Optimal result

Integrand size = 26, antiderivative size = 337

$$\begin{aligned}\int \frac{(e+fx)^2 \sinh(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{(e+fx)^3}{3bf} - \frac{a(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} \\ &+ \frac{a(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} \\ &- \frac{2af(e+fx) \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} \\ &+ \frac{2af(e+fx) \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} \\ &+ \frac{2af^2 \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} \\ &- \frac{2af^2 \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3}\end{aligned}$$

output
```
1/3*(f*x+e)^3/b/f-a*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d/(
a^2+b^2)^(1/2)+a*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d/(a^2
+b^2)^(1/2)-2*a*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d
^2/(a^2+b^2)^(1/2)+2*a*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))/b/d^2/(a^2+b^2)^(1/2)+2*a*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/b/d^3/(a^2+b^2)^(1/2)-2*a*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/b/d^3/(a^2+b^2)^(1/2)
```

3.224.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.09

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x(3e^2+3efx+f^2x^2)}{3b}$$
$$-\frac{a\left(-2d^2e^2\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^2f^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2d^2efx\log\left(\right.\right.}{}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) - (a*(-2*d^2*e^2*ArcTanh[(a + b*E^(c
 + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]
- 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^2*f^2*x^2
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[
2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*f^2*PolyLog[3, (b*E^(c + d*x))
/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))]))/(b*Sqrt[a^2 + b^2]*d^3)
```

3.224.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {6091, 17, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6091}$$

$$\frac{\int (e+fx)^2dx}{b}-\frac{a\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{(e+fx)^3}{3bf}-\frac{a\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3}{3bf}-\frac{a\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b}$$

$$\Big\downarrow \text{3803}$$

$$\frac{(e+fx)^3}{3bf}-\frac{2a\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{2a\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^3}{3bf}$$

$$\Big\downarrow \text{2694}$$

$$\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^3}{3bf}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^3}{3bf}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right.}{b}$$

$$\frac{(e+fx)^3}{3bf}$$

$$\Big\downarrow \text{3011}$$

3.224. $\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots$$

$$\frac{(e+fx)^3}{3bf}$$

$$\Big\downarrow \; 2720$$

$$\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\cdots\right.}{bd}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots$$

$$\frac{(e+fx)^3}{3bf}$$

$$\Big\downarrow \; 7143$$

$$\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\cdots\right.}{\cdots}\cdots\right.}{b}\cdots$$

$$\frac{(e+fx)^3}{3bf}$$

input
```
Int[((e + f*x)^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.224. $\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(e + f*x)^3/(3*b*f) + (2*a*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyL
og[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/b
```

3.224.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.224. $\int \frac{(e+fx)^2 \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.224.4 Maple [F]

$$\int \frac{(fx+e)^2 \sinh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.224.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(303) = 606$.

Time = 0.26 (sec) , antiderivative size = 782, normalized size of antiderivative = 2.32

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{(a^2+b^2)d^3f^2x^3 + 3\,(a^2+b^2)d^3efx^2 + 3\,(a^2+b^2)d^3e^2x + 6\,abf^2\sqrt{\frac{a^2+b^2}{b^2}}\,\mathrm{polylog}\left(3, \frac{a\cosh(dx+c)+a\sinh(dx+c)+}{}\right.}{}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*((a^2 + b^2)*d^3*f^2*x^3 + 3*(a^2 + b^2)*d^3*e*f*x^2 + 3*(a^2 + b^2)*d
^3*e^2*x + 6*a*b*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a
*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
)/b) - 6*a*b*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sin
h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b)
 - 6*(a*b*d*f^2*x + a*b*d*e*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c
) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2) - b)/b + 1) + 6*(a*b*d*f^2*x + a*b*d*e*f)*sqrt((a^2 + b^2)/b^2)*dilo
g((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3*(a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*
c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c
^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) -
2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x +
2*a*b*c*d*e*f - a*b*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) +
 a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b) + 3*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^
2*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b))/((a^2*b +
b^3)*d^3)
```

3.224.6 Sympy [F]

$$\int \frac{(e+fx)^2 \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2 \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sinh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.224.7 Maxima [F]

$$\int \frac{(e+fx)^2 \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^2*(a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + s
qrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d) - (d*x + c)/(b*d)) + 1/3*(f^2*x^3 +
 3*e*f*x^2)/b - integrate(2*(a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x)/(b^2*e
^(2*d*x + 2*c) + 2*a*b*e^(d*x + c) - b^2), x)
```

3.224.8 Giac [F]

$$\int \frac{(e+fx)^2 \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.225 $\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.225.1 Optimal result

Integrand size = 24, antiderivative size = 220

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{ex}{b}+\frac{fx^2}{2b}-\frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d}+\frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d}-\frac{af\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}+\frac{af\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}$$

output
```
e*x/b+1/2*f*x^2/b-a*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d/(a^
2+b^2)^(1/2)+a*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d/(a^2+b^2
)^(1/2)-a*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d^2/(a^2+b^2)^(
1/2)+a*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d^2/(a^2+b^2)^(1/2
)
```

3.225.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.86

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x(2e+fx)}{2b} + \frac{a\left(d\left(2e\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-fx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+fx\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)-f\,\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2\cdots}}\right.\right.}{b\sqrt{a^2+b^2}d^2}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(x*(2*e + f*x))/(2*b) + (a*(d*(2*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
b^2]] - f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*x*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]) - f*PolyLog[2, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)]))/(b*Sqrt[a^2 + b^2]*d^2)
```

3.225.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6091, 17, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6091}$$

$$\frac{\int (e+fx)dx}{b} - \frac{a\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{(e+fx)^2}{2bf} - \frac{a\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^2}{2bf} - \frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b}$$

$$\Big\downarrow \text{3803}$$

$$\frac{(e+fx)^2}{2bf}-\frac{2a\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{2a\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{2694}$$

$$\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{2838}$$

3.225. $\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{2a\left(\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}$$

input
```
Int[((e + f*x)*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(e + f*x)^2/(2*b*f) + (2*a*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/b
```

3.225.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

3.225.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(198) = 396$.

Time = 1.52 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.00

method	result
risch	$\frac{f\,x^2}{2b}+\frac{ex}{b}+\frac{2ae\,\operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{db\sqrt{a^2+b^2}}-\frac{af\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{db\sqrt{a^2+b^2}}+\frac{af\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)x}{db\sqrt{a^2+b^2}}-\frac{af\ln\left(\frac{-b\,\mathrm{e}^{dx}}{-}\right.}{d^2b}$

3.225. $\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
int((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*f*x^2/b+e*x/b+2/d*a/b*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))-1/d*a/b*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^
(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d*a/b*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c
)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a/b*f/(a^2+b^2)^(1/2)*ln
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2*a/b*f/(a^
2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/
d^2*a/b*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2
+b^2)^(1/2)))+1/d^2*a/b*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))-2/d^2*a/b*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b
*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

3.225.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(196) = 392$.

Time = 0.27 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.27

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{(a^2+b^2)d^2fx^2 + 2\,(a^2+b^2)d^2ex - 2\,abf\sqrt{\frac{a^2+b^2}{b^2}}\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}\right.}{}$$

input
```
integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((a^2 + b^2)*d^2*f*x^2 + 2*(a^2 + b^2)*d^2*e*x - 2*a*b*f*sqrt((a^2 + b
^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*s
inh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*a*b*f*sqrt((a^2 + b^2)
/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a*b*d*e - a*b*c*f)*sqrt(
(a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2
 + b^2)/b^2) + 2*a) - 2*(a*b*d*e - a*b*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*b*
cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(
a*b*d*f*x + a*b*c*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(
d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/
b) + 2*(a*b*d*f*x + a*b*c*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) +
 a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b))/((a^2*b + b^3)*d^2)
```

3.225.6 Sympy [F]

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\sinh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*sinh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.225.7 Maxima [F]

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\sinh{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(4*a*integrate(x*e^(d*x + c)/(b^2*e^(2*d*x + 2*c) + 2*a*b*e^(d*x + c)
 - b^2), x) - x^2/b)*f - e*(a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(
b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d) - (d*x + c)/(
b*d))
```

3.225.8 Giac [F]

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\sinh{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,\left(e+f\,x\right)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.226 $\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.226.1 Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x}{b} + \frac{2a\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d}$$

output
```
x/b+2*a*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/b/d/(a^2+b^2)^(
1/2)
```

3.226.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{c}{d} + x - \frac{2a\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}d}}{b}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[
-a^2 - b^2]*d))/b
```

3.226.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 26, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i\sin(ic+idx)}{a-ib\sin(ic+idx)}dx$$
$$\Big\downarrow \text{26}$$
$$-i\int \frac{\sin(ic+idx)}{a-ib\sin(ic+idx)}dx$$
$$\Big\downarrow \text{3214}$$
$$-i\left(\frac{ix}{b}-\frac{ia\int \frac{1}{a+b\sinh(c+dx)}dx}{b}\right)$$
$$\Big\downarrow \text{3042}$$
$$-i\left(\frac{ix}{b}-\frac{ia\int \frac{1}{a-ib\sin(ic+idx)}dx}{b}\right)$$
$$\Big\downarrow \text{3139}$$
$$-i\left(\frac{ix}{b}-\frac{2a\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\right)$$
$$\Big\downarrow \text{1083}$$
$$-i\left(\frac{4a\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{bd}+\frac{ix}{b}\right)$$
$$\Big\downarrow \text{217}$$
$$-i\left(\frac{ix}{b}-\frac{2ia\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}}\right)$$

input
```
Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
(-I)*((I*x)/b - ((2*I)*a*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(
b*Sqrt[a^2 + b^2]*d))
```

3.226.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.226.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

method	result	size
derivativedivides	$\frac{-\frac{2a\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}}{d}$	82
default	$\frac{-\frac{2a\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}}{d}$	82
risch	$\frac{x}{b}+\frac{a\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,db}-\frac{a\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,db}$	124

input
```
int(sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2
+b^2)^(1/2))-1/b*ln(tanh(1/2*d*x+1/2*c)-1)+1/b*ln(tanh(1/2*d*x+1/2*c)+1))
```

3.226.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(51) = 102$.

Time = 0.25 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.44

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{(a^2+b^2)dx+\sqrt{a^2+b^2}a\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)+2\,\sqrt{a^2}}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(}\right.}{(a^2b+b^3)d}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((a^2 + b^2)*d*x + sqrt(a^2 + b^2)*a*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d
*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b
)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a
))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(
d*x + c) + a)*sinh(d*x + c) - b)))/((a^2*b + b^3)*d)
```

3.226.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 31.85 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.98

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{x}{b} & \text{for } a=0 \\ \frac{\cosh(c+dx)}{ad} & \text{for } b=0 \\ \frac{x\sinh(c)}{a+b\sinh(c)} & \text{for } d=0 \\ \frac{dx\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-ibd} - \frac{idx}{bd\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-ibd} - \frac{2}{bd\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-ibd} & \text{for } a=-ib \\ \frac{dx\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+ibd} + \frac{idx}{bd\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+ibd} - \frac{2}{bd\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+ibd} & \text{for } a=ib \\ \frac{a\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{bd\sqrt{a^2+b^2}} - \frac{a\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{bd\sqrt{a^2+b^2}} + \frac{x}{b} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/b, Eq(a, 0)), (cosh(
c + d*x)/(a*d), Eq(b, 0)), (x*sinh(c)/(a + b*sinh(c)), Eq(d, 0)), (d*x*tan
h(c/2 + d*x/2)/(b*d*tanh(c/2 + d*x/2) - I*b*d) - I*d*x/(b*d*tanh(c/2 + d*x
/2) - I*b*d) - 2/(b*d*tanh(c/2 + d*x/2) - I*b*d), Eq(a, -I*b)), (d*x*tanh(
c/2 + d*x/2)/(b*d*tanh(c/2 + d*x/2) + I*b*d) + I*d*x/(b*d*tanh(c/2 + d*x/2
) + I*b*d) - 2/(b*d*tanh(c/2 + d*x/2) + I*b*d), Eq(a, I*b)), (a*log(tanh(c
/2 + d*x/2) - b/a - sqrt(a**2 + b**2)/a)/(b*d*sqrt(a**2 + b**2)) - a*log(t
anh(c/2 + d*x/2) - b/a + sqrt(a**2 + b**2)/a)/(b*d*sqrt(a**2 + b**2)) + x/
b, True))
```

3.226.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.57

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}bd} + \frac{dx+c}{bd}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b*d) + (d*x + c)/(b*d)
```

3.226.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.56

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{a\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b} - \frac{dx+c}{b}}{d}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(a*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
 + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b) - (d*x + c)/b)/d
```

3.226.9 Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.24

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x}{b} - \frac{a\,\ln\left(\frac{2\,a\,e^{c+d\,x}}{b^2} - \frac{2\,a\,(b-a\,e^{c+d\,x})}{b^2\,\sqrt{a^2+b^2}}\right)}{b\,d\,\sqrt{a^2+b^2}} + \frac{a\,\ln\left(\frac{2\,a\,e^{c+d\,x}}{b^2} + \frac{2\,a\,(b-a\,e^{c+d\,x})}{b^2\,\sqrt{a^2+b^2}}\right)}{b\,d\,\sqrt{a^2+b^2}}$$

input
```
int(sinh(c + d*x)/(a + b*sinh(c + d*x)),x)
```

output
```
x/b - (a*log((2*a*exp(c + d*x))/b^2 - (2*a*(b - a*exp(c + d*x)))/(b^2*(a^2
 + b^2)^(1/2))))/(b*d*(a^2 + b^2)^(1/2)) + (a*log((2*a*exp(c + d*x))/b^2 +
 (2*a*(b - a*exp(c + d*x)))/(b^2*(a^2 + b^2)^(1/2))))/(b*d*(a^2 + b^2)^(1/
2))
```

3.227 $\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.227.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.227.2 Mathematica [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.227.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.227.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.227.4 Maple [N/A] (verified)

Not integrable

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.227.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.227.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.227.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.38

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*a*integrate(-e^(d*x + c)/(b^2*f*x + b^2*e - (b^2*f*x*e^(2*c) + b^2*e*e^
(2*c))*e^(2*d*x) - 2*(a*b*f*x*e^c + a*b*e*e^c)*e^(d*x)), x) + log(f*x + e)
/(b*f)
```

3.227.8 Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.227.9 Mupad [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

input
```
int(sinh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(sinh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.228 $\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.228.1 Optimal result

Integrand size = 28, antiderivative size = 551

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{a^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d} - \frac{a^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d} + \frac{3a^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d^2} - \frac{3a^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d^2} - \frac{6a^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d^3} + \frac{6a^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d^3} + \frac{6a^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d^4} - \frac{6a^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d^4} - \frac{6f^3\sinh(c+dx)}{bd^4} - \frac{3f(e+fx)^2\sinh(c+dx)}{bd^2}$$

output
```
-1/4*a*(f*x+e)^4/b^2/f+6*f^2*(f*x+e)*cosh(d*x+c)/b/d^3+(f*x+e)^3*cosh(d*x+
c)/b/d-6*f^3*sinh(d*x+c)/b/d^4-3*f*(f*x+e)^2*sinh(d*x+c)/b/d^2+a^2*(f*x+e)
^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d/(a^2+b^2)^(1/2)-a^2*(f*x+e
)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d/(a^2+b^2)^(1/2)+3*a^2*f*(
f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^2/(a^2+b^2)^(1
/2)-3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^2
/(a^2+b^2)^(1/2)-6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/b^2/d^3/(a^2+b^2)^(1/2)+6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a
+(a^2+b^2)^(1/2)))/b^2/d^3/(a^2+b^2)^(1/2)+6*a^2*f^3*polylog(4,-b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/b^2/d^4/(a^2+b^2)^(1/2)-6*a^2*f^3*polylog(4,-b*exp
(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^4/(a^2+b^2)^(1/2)
```

3.228.2 Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 979, normalized size of antiderivative = 1.78

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-4a\sqrt{a^2+b^2}d^4e^3x - 6a\sqrt{a^2+b^2}d^4e^2fx^2 - 4a\sqrt{a^2+b^2}d^4ef^2x^3 - a\sqrt{a^2+b^2}d^4f^3x^4 - 8a^2d^3e^3\text{arctanh}\Big(\cdots}{}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-4*a*Sqrt[a^2 + b^2]*d^4*e^3*x - 6*a*Sqrt[a^2 + b^2]*d^4*e^2*f*x^2 - 4*a*
Sqrt[a^2 + b^2]*d^4*e*f^2*x^3 - a*Sqrt[a^2 + b^2]*d^4*f^3*x^4 - 8*a^2*d^3*
e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 4*b*Sqrt[a^2 + b^2]*d^3
*e^3*Cosh[c + d*x] + 24*b*Sqrt[a^2 + b^2]*d*e*f^2*Cosh[c + d*x] + 12*b*Sqr
t[a^2 + b^2]*d^3*e^2*f*x*Cosh[c + d*x] + 24*b*Sqrt[a^2 + b^2]*d*f^3*x*Cosh
[c + d*x] + 12*b*Sqrt[a^2 + b^2]*d^3*e*f^2*x^2*Cosh[c + d*x] + 4*b*Sqrt[a^
2 + b^2]*d^3*f^3*x^3*Cosh[c + d*x] + 12*a^2*d^3*e^2*f*x*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])] + 12*a^2*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])] + 4*a^2*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2])] - 12*a^2*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])] - 12*a^2*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])] - 4*a^2*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
 + 12*a^2*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])] - 12*a^2*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2]))] - 24*a^2*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])] - 24*a^2*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
 24*a^2*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 24*
a^2*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 24*a^2*
f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 24*a^2*f^3*PolyLo
g[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 12*b*Sqrt[a^2 + b^2]*d...
```

3.228.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.85 (sec) , antiderivative size = 501, normalized size of antiderivative = 0.91, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6091, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 6091, 17, 3042, 3803, 25, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6091}$$

$$\frac{\int (e+fx)^3\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)^3\sin(ic+idx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)^3\sin(ic+idx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\cosh(c+dx)dx}{d}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \textbf{26}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \textbf{3777}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \textbf{3117}$$

3.228. $\displaystyle\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \textbf{6091}$$

$$-\frac{a\left(\frac{\int (e+fx)^3dx}{b}-\frac{a\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \textbf{17}$$

$$-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{a\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{a\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \textbf{3803}$$

3.228. $\displaystyle\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\int -\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{a\left(\frac{2a\int \frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^4}{4bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \textbf{2694}$$

$$-\frac{a\left(\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^4}{4bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \textbf{27}$$

3.228. $\displaystyle\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^4}{4bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d}{bd}\right)}{2\sqrt{a^2+b^2}}}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 3011$$

3.228. $\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c}\cdots}{a-\sqrt{\cdots}}\right.}{bd}\cdots\right.}{\cdots}\cdots\right)}{b}\cdots\right)}{b}\cdots$$

$$\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 7163$$

3.228. $\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+\cdots}}{a+\sqrt{a\cdots}}\right)}{d}\cdots}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\cdots\right)$$

$$-\frac{\cdots}{\cdots}$$

$$\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{ 2720}$$

3.228. $\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLo}}{}}{bd}\right)}{2\sqrt{a^2+b^2}}}{}\right)}{} \cdots$$

$$\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\downarrow 7143$$

3.228. $\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{}\right)}{\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}}$$

input
```
Int[((e + f*x)^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

3.228. $\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
-((a*((e + f*x)^4/(4*b*f) + (2*a*(-1/2*(b*(((e + f*x)^3*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b
*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e +
f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e
 + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*
(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*P
olyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(2*
Sqrt[a^2 + b^2])))/b))/b) - (I*((I*(e + f*x)^3*Cosh[c + d*x])/d - ((3*I)*f
*(((e + f*x)^2*Sinh[c + d*x])/d + ((2*I)*f*((I*(e + f*x)*Cosh[c + d*x])/d
- (I*f*Sinh[c + d*x])/d^2))/d))/d))/b
```

3.228.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.228. $\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.228. $\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.228.4 Maple [F]

$$\int \frac{(fx+e)^3 \sinh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.228.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2612 vs. $2(507) = 1014$.

Time = 0.34 (sec) , antiderivative size = 2612, normalized size of antiderivative = 4.74

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*(a^2*b + b^3)*d^3*f^3*x^3 + 2*(a^2*b + b^3)*d^3*e^3 + 6*(a^2*b + b^
3)*d^2*e^2*f + 12*(a^2*b + b^3)*d*e*f^2 + 12*(a^2*b + b^3)*f^3 + 6*((a^2*b
 + b^3)*d^3*e*f^2 + (a^2*b + b^3)*d^2*f^3)*x^2 + 2*((a^2*b + b^3)*d^3*f^3*
x^3 + (a^2*b + b^3)*d^3*e^3 - 3*(a^2*b + b^3)*d^2*e^2*f + 6*(a^2*b + b^3)*
d*e*f^2 - 6*(a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^2*b + b^3)
*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^2*b + b^3)*d^2*e*f^2 + 2
*(a^2*b + b^3)*d*f^3)*x)*cosh(d*x + c)^2 + 2*((a^2*b + b^3)*d^3*f^3*x^3 +
(a^2*b + b^3)*d^3*e^3 - 3*(a^2*b + b^3)*d^2*e^2*f + 6*(a^2*b + b^3)*d*e*f^
2 - 6*(a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^2*b + b^3)*d^2*f
^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^2*b + b^3)*d^2*e*f^2 + 2*(a^2*
b + b^3)*d*f^3)*x)*sinh(d*x + c)^2 + 12*((a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*
e*f^2*x + a^2*b*d^2*e^2*f)*cosh(d*x + c) + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^
2*e*f^2*x + a^2*b*d^2*e^2*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqr
t((a^2 + b^2)/b^2) - b)/b + 1) - 12*((a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^
2*x + a^2*b*d^2*e^2*f)*cosh(d*x + c) + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*
f^2*x + a^2*b*d^2*e^2*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b + 1) - 4*((a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a
^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*cosh(d*x + c) + (a^2*b*d^3*e^3 - 3*a^...
```

3.228.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.228.7 Maxima [F]

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*e^3*(2*a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
- a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2*d) - 2*(d*x + c)*a/(b^2*d) +
e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d)) - 1/4*(a*d^4*f^3*x^4*e^c + 4*a*d^4
*e*f^2*x^3*e^c + 6*a*d^4*e^2*f*x^2*e^c - 2*(b*d^3*f^3*x^3*e^(2*c) + 3*(d^3
*e*f^2 - d^2*f^3)*b*x^2*e^(2*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b*
x*e^(2*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b*e^(2*c))*e^(d*x) - 2*(b*d^
3*f^3*x^3 + 3*(d^3*e*f^2 + d^2*f^3)*b*x^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2 + 2
*d*f^3)*b*x + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b)*e^(-d*x))*e^(-c)/(b^2*d
^4) + integrate(2*(a^2*f^3*x^3*e^c + 3*a^2*e*f^2*x^2*e^c + 3*a^2*e^2*f*x*e
^c)*e^(d*x)/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)
```

3.228.8 Giac [F]

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.229 $\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.229.1 Optimal result

Integrand size = 28, antiderivative size = 407

$$\begin{aligned}\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{a(e+fx)^3}{3b^2f}+\frac{2f^2\cosh(c+dx)}{bd^3}+\frac{(e+fx)^2\cosh(c+dx)}{bd}\\
&+\frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d}\\
&-\frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d}\\
&+\frac{2a^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d^2}\\
&-\frac{2a^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d^2}\\
&-\frac{2a^2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d^3}\\
&+\frac{2a^2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d^3}\\
&-\frac{2f(e+fx)\sinh(c+dx)}{bd^2}\end{aligned}$$

output
```
-1/3*a*(f*x+e)^3/b^2/f+2*f^2*cosh(d*x+c)/b/d^3+(f*x+e)^2*cosh(d*x+c)/b/d-2
*f*(f*x+e)*sinh(d*x+c)/b/d^2+a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))/b^2/d/(a^2+b^2)^(1/2)-a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/b^2/d/(a^2+b^2)^(1/2)+2*a^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/b^2/d^2/(a^2+b^2)^(1/2)-2*a^2*f*(f*x+e)*polylog(2,-b*exp
(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^2/(a^2+b^2)^(1/2)-2*a^2*f^2*polylog(3,-
b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^3/(a^2+b^2)^(1/2)+2*a^2*f^2*polylo
g(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^3/(a^2+b^2)^(1/2)
```

3.229.2 Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.11

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-ax(3e^2+3efx+f^2x^2)+\frac{3a^2\left(-2d^2e^2\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^2f^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2d^2ef\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(a*x*(3*e^2 + 3*e*f*x + f^2*x^2)) + (3*a^2*(-2*d^2*e^2*ArcTanh[(a + b*E^
(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^2*f^2*x
^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLo
g[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*f^2*PolyLog[3, (b*E^(c + d*x
))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^3) + (3*b*Cosh[d*x]*((2*f^2 + d^2*(e +
f*x)^2)*Cosh[c] - 2*d*f*(e + f*x)*Sinh[c]))/d^3 + (3*b*(-2*d*f*(e + f*x)*C
osh[c] + (2*f^2 + d^2*(e + f*x)^2)*Sinh[c])*Sinh[d*x])/d^3)/(3*b^2)
```

3.229.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.92, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6091, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 6091, 17, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \ 6091$$

$$\frac{\int (e+fx)^2\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)^2\sin(ic+idx)dx}{b}$$

$$\Big\downarrow \ 26$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)^2\sin(ic+idx)dx}{b}$$

$$\Big\downarrow \ 3777$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}$$

$$\Big\downarrow \ 3777$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \ 26$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{6091}$$

$$-\frac{a\left(\frac{\int (e+fx)^2dx}{b}-\frac{a\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{a\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{a\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{3803}$$

$$-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

3.229. $\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 25$$

$$-\frac{a\left(\frac{2a\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^3}{3bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 2694$$

$$-\frac{a\left(\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^3}{3bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^3}{3bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 3011$$

3.229. $\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\cdots\right.}{bd}\cdots\right.}{\cdots}\cdots\right)}{b}\cdots\right)}{b}\cdots$$

$$\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \; 2720$$

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right)}{b}\cdots\right)}{b}\cdots$$

$$\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

3.229. $\displaystyle\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{7143}$$

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\cdots\right.}{\cdots}}{b}\right.}{b}$$

$$\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

input
```
Int[((e + f*x)^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((a*((e + f*x)^3/(3*b*f) + (2*a*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)
*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2
])))/b))/b) - (I*((I*(e + f*x)^2*Cosh[c + d*x])/d - ((2*I)*f*(-((f*Cosh[c
+ d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d))/d))/b
```

3.229. $\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.229.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.229. $\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.229.4 Maple [F]

$$\int \frac{(fx+e)^2 \sinh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.229.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1689 vs. $2(373) = 746$.

Time = 0.30 (sec) , antiderivative size = 1689, normalized size of antiderivative = 4.15

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*(a^2*b + b^3)*d^2*f^2*x^2 + 3*(a^2*b + b^3)*d^2*e^2 + 6*(a^2*b + b^
3)*d*e*f + 6*(a^2*b + b^3)*f^2 + 3*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b
^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*(a^2*b + b^3)*f^2 + 2*((a^2*b + b^
3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c)^2 + 3*((a^2*b + b^3)*d^
2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*(a^2*b + b^3
)*f^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*sinh(d*x + c)^2
 + 12*((a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c) + (a^2*b*d*f^2*x + a^2*
b*d*e*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*s
inh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
 b)/b + 1) - 12*((a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c) + (a^2*b*d*f^
2*x + a^2*b*d*e*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b + 1) - 6*((a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2
)*cosh(d*x + c) + (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*((a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f +
a^2*b*c^2*f^2)*cosh(d*x + c) + (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^
2*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*si
nh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*((a^2*b*d^2*f^2*x^2 + 2
*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c) + (a^...
```

3.229.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.229.7 Maxima [F]

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*e^2*(2*a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
- a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2*d) - 2*(d*x + c)*a/(b^2*d) +
e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d)) - 1/6*(2*a*d^3*f^2*x^3*e^c + 6*a*d
^3*e*f*x^2*e^c - 3*(b*d^2*f^2*x^2*e^(2*c) + 2*(d^2*e*f - d*f^2)*b*x*e^(2*c
) - 2*(d*e*f - f^2)*b*e^(2*c))*e^(d*x) - 3*(b*d^2*f^2*x^2 + 2*(d^2*e*f + d
*f^2)*b*x + 2*(d*e*f + f^2)*b)*e^(-d*x))*e^(-c)/(b^2*d^3) + integrate(2*(a
^2*f^2*x^2*e^c + 2*a^2*e*f*x*e^c)*e^(d*x)/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e
^(d*x + c) - b^3), x)
```

3.229.8 Giac [F]

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.230 $\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.230.1 Optimal result

Integrand size = 26, antiderivative size = 264

$$\begin{aligned}\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = &-\frac{aex}{b^2}-\frac{afx^2}{2b^2}+\frac{(e+fx)\cosh(c+dx)}{bd}\\ &+\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d}\\ &-\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d}\\ &+\frac{a^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d^2}\\ &-\frac{a^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d^2}-\frac{f\sinh(c+dx)}{bd^2}\end{aligned}$$

output
```
-a*e*x/b^2-1/2*a*f*x^2/b^2+(f*x+e)*cosh(d*x+c)/b/d-f*sinh(d*x+c)/b/d^2+a^2
*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d/(a^2+b^2)^(1/2)-a^2*
(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d/(a^2+b^2)^(1/2)+a^2*f
*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^2/(a^2+b^2)^(1/2)-a^2*
f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^2/(a^2+b^2)^(1/2)
```

3.230.2 Mathematica [A] (verified)

Time = 2.75 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.99

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{a(c+dx)(cf-d(2e+fx))+2bd(e+fx)\cosh(c+dx)+\frac{2a^2\left(-2de\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2cf\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)\right.}{}}{}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(a*(c + d*x)*(c*f - d*(2*e + f*x)) + 2*b*d*(e + f*x)*Cosh[c + d*x] + (2*a^
2*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a
+ b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog
[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] - 2*b*f*Si
nh[c + d*x])/(2*b^2*d^2)
```

3.230.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.98, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {6091, 3042, 26, 3777, 3042, 3117, 6091, 17, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{6091}$$
$$\frac{\int (e+fx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)\sin(ic+idx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)\sin(ic+idx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{6091}$$

$$-\frac{a\left(\frac{\int (e+fx)dx}{b}-\frac{a\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{a\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{3803}$$

$$-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{2a\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{25}$$

3.230. $\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{2a\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^2}{2bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \; 2694$$

$$-\frac{a\left(\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \; 2715$$

3.230. $\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)\cdots}{bd^2}\cdots}{2\sqrt{a^2+b^2}}\cdots}{b}\cdots\right)}{b}\cdots$$

$$\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \; 2838$$

$$-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)\cdots}{2bf}\cdots\right)}{b}\cdots$$

$$\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

input
```
Int[((e + f*x)*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((a*((e + f*x)^2/(2*b*f) + (2*a*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2])]])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2])]])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/b))/b) - (I*((I
*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/b
```

3.230. $\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.230.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.230. $\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

3.230.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(242) = 484$.

Time = 1.89 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.93

method	result
risch	$-\frac{a f x^2}{2b^2} - \frac{aex}{b^2} + \frac{(dfx+de-f)\mathrm{e}^{dx+c}}{2b\,d^2} + \frac{(dfx+de+f)\mathrm{e}^{-dx-c}}{2b\,d^2} - \frac{2a^2 e \,\mathrm{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d\,b^2\sqrt{a^2+b^2}} + \frac{a^2 f \ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right.}{d\,b^2\sqrt{a^2+b^2}}$

input
```
int((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*f*x^2/b^2-a*e*x/b^2+1/2*(d*f*x+d*e-f)/b/d^2*exp(d*x+c)+1/2*(d*f*x+d
*e+f)/b/d^2*exp(-d*x-c)-2/d*a^2/b^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp
(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d*a^2/b^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x
+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d*a^2/b^2*f/(a^2+b^2)^(1/
2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*a^2/b^
2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/
2)))*c-1/d^2*a^2/b^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)
/(a+(a^2+b^2)^(1/2)))*c+1/d^2*a^2/b^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+
c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*a^2/b^2*f/(a^2+b^2)^(1/2
)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d^2*a^2/b^
2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

3.230.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 946 vs. $2(240) = 480$.

Time = 0.27 (sec) , antiderivative size = 946, normalized size of antiderivative = 3.58

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + ((a^2*b + b^3)*d*f*x + (a^2
*b + b^3)*d*e - (a^2*b + b^3)*f)*cosh(d*x + c)^2 + ((a^2*b + b^3)*d*f*x +
(a^2*b + b^3)*d*e - (a^2*b + b^3)*f)*sinh(d*x + c)^2 + 2*(a^2*b*f*cosh(d*x
 + c) + a^2*b*f*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c
) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2) - b)/b + 1) - 2*(a^2*b*f*cosh(d*x + c) + a^2*b*f*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*((a^2*b*d*e -
 a^2*b*c*f)*cosh(d*x + c) + (a^2*b*d*e - a^2*b*c*f)*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 +
 b^2)/b^2) + 2*a) + 2*((a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c) + (a^2*b*d*e
- a^2*b*c*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((a^2*b*d*f*x + a
^2*b*c*f)*cosh(d*x + c) + (a^2*b*d*f*x + a^2*b*c*f)*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((a^2*b*d*f*x + a^2*b
*c*f)*cosh(d*x + c) + (a^2*b*d*f*x + a^2*b*c*f)*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a^2*b + b^3)*f - ((a^3 + a
*b^2)*d^2*f*x^2 + 2*(a^3 + a*b^2)*d^2*e*x)*cosh(d*x + c) - ((a^3 + a*b^...
```

3.230.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.230.7 Maxima [F]

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\sinh{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(4*a^2*integrate(x*e^(d*x + c)/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x +
 c) - b^3), x) - (a*d^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*e^(d*x) - (b
*d*x + b)*e^(-d*x))*e^(-c)/(b^2*d^2))*f + 1/2*e*(2*a^2*log((b*e^(-d*x - c)
 - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2
+ b^2)*b^2*d) - 2*(d*x + c)*a/(b^2*d) + e^(d*x + c)/(b*d) + e^(-d*x - c)/(
b*d))
```

3.230.8 Giac [F]

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\sinh{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\sinh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh{(c+d\,x)}}\,dx$$

input
```
int((sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.231 $\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.231.1 Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{ax}{b^2} - \frac{2a^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d} + \frac{\cosh(c+dx)}{bd}$$

output
```
-a*x/b^2+cosh(d*x+c)/b/d-2*a^2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)
^(1/2))/b^2/d/(a^2+b^2)^(1/2)
```

3.231.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-a\left(c+dx-\frac{2a\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)+b\cosh(c+dx)}{b^2d}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(a*(c + d*x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/S
qrt[-a^2 - b^2])) + b*Cosh[c + d*x])/(b^2*d)
```

3.231.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 25, 3225, 26, 27, 3042, 26, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3225}$$

$$\frac{\cosh(c+dx)}{bd}-\frac{i\int -\frac{ia\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\cosh(c+dx)}{bd}-\frac{\int \frac{a\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\cosh(c+dx)}{bd}-\frac{a\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh(c+dx)}{bd}-\frac{a\int -\frac{i\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{ia\int \frac{\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{ia\left(\frac{ix}{b}-\frac{ia\int\frac{1}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cosh(c+dx)}{bd}+\frac{ia\left(\frac{ix}{b}-\frac{ia\int\frac{1}{a-ib\sin(ic+idx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3139$$

$$\frac{\cosh(c+dx)}{bd}+\frac{ia\left(\frac{ix}{b}-\frac{2a\int\frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\right)}{b}$$

$$\Big\downarrow \; 1083$$

$$\frac{\cosh(c+dx)}{bd}+\frac{ia\left(\frac{4a\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{bd}+\frac{ix}{b}\right)}{b}$$

$$\Big\downarrow \; 217$$

$$\frac{\cosh(c+dx)}{bd}+\frac{ia\left(\frac{ix}{b}-\frac{2ia\,\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}}\right)}{b}$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
(I*a*((I*x)/b - ((2*I)*a*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(
b*Sqrt[a^2 + b^2]*d)))/b + Cosh[c + d*x]/(b*d)
```

3.231.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.231. $\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.231.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.70

method	result
derivativedivides	$\frac{\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{2a^2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{b^2}}{d}$
default	$\frac{\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{2a^2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{b^2}}{d}$
risch	$-\frac{ax}{b^2}+\frac{\mathrm{e}^{dx+c}}{2bd}+\frac{\mathrm{e}^{-dx-c}}{2bd}+\frac{a^2\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,b^2}-\frac{a^2\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,b^2}$

input
```
int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b/(tanh(1/2*d*x+1/2*c)+1)-a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)+2*a^2/b^2
/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)
)-1/b/(tanh(1/2*d*x+1/2*c)-1)+a/b^2*ln(tanh(1/2*d*x+1/2*c)-1))
```

3.231.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(68) = 136.

Time = 0.26 (sec) , antiderivative size = 331, normalized size of antiderivative = 4.66

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{2\left(a^3+ab^2\right)dx\cosh\left(dx+c\right)-a^2b-b^3-\left(a^2b+b^3\right)\cosh\left(dx+c\right)^2-\left(a^2b+b^3\right)\sinh\left(dx+c\right)^2-2\left(a^2\,c\right.}{}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*(a^3 + a*b^2)*d*x*cosh(d*x + c) - a^2*b - b^3 - (a^2*b + b^3)*cosh
(d*x + c)^2 - (a^2*b + b^3)*sinh(d*x + c)^2 - 2*(a^2*cosh(d*x + c) + a^2*s
inh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)
^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(
d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*c
osh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c
) + a)*sinh(d*x + c) - b)) + 2*((a^3 + a*b^2)*d*x - (a^2*b + b^3)*cosh(d*x
 + c))*sinh(d*x + c))/((a^2*b^2 + b^4)*d*cosh(d*x + c) + (a^2*b^2 + b^4)*d
*sinh(d*x + c))
```

3.231.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1748 vs. $2(61) = 122$.

Time = 176.26 (sec) , antiderivative size = 1748, normalized size of antiderivative = 24.62

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Piecewise((zoo*x*sinh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (cosh(c + d*x)/
(b*d), Eq(a, 0)), (-b*d*x*tanh(c/2 + d*x/2)**3/(b**2*d*tanh(c/2 + d*x/2)**
2 - b**2*d + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 - b*d*sqrt(-b**2)*tanh(c
/2 + d*x/2)) + b*d*x*tanh(c/2 + d*x/2)/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2
*d + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 - b*d*sqrt(-b**2)*tanh(c/2 + d*x
/2)) + 2*b*tanh(c/2 + d*x/2)**2/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d + b*
d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)) -
4*b/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d + b*d*sqrt(-b**2)*tanh(c/2 + d*x
/2)**3 - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)) + d*x*sqrt(-b**2)*tanh(c/2 + d
*x/2)**2/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d + b*d*sqrt(-b**2)*tanh(c/2
+ d*x/2)**3 - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)) - d*x*sqrt(-b**2)/(b**2*d
*tanh(c/2 + d*x/2)**2 - b**2*d + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 - b*
d*sqrt(-b**2)*tanh(c/2 + d*x/2)) - 2*sqrt(-b**2)*tanh(c/2 + d*x/2)/(b**2*d
*tanh(c/2 + d*x/2)**2 - b**2*d + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 - b*
d*sqrt(-b**2)*tanh(c/2 + d*x/2)), Eq(a, -sqrt(-b**2))), (-b*d*x*tanh(c/2 +
 d*x/2)**3/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d - b*d*sqrt(-b**2)*tanh(c/
2 + d*x/2)**3 + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)) + b*d*x*tanh(c/2 + d*x/
2)/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d - b*d*sqrt(-b**2)*tanh(c/2 + d*x/
2)**3 + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)) + 2*b*tanh(c/2 + d*x/2)**2/(b**
2*d*tanh(c/2 + d*x/2)**2 - b**2*d - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**...
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.68

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a^2\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^2d} - \frac{(dx+c)a}{b^2d} + \frac{e^{(dx+c)}}{2\,bd} + \frac{e^{(-dx-c)}}{2\,bd}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2*d) - (d*x + c)*a/(b^2*d) + 1/2*e^(d*x +
c)/(b*d) + 1/2*e^(-d*x - c)/(b*d)
```

3.231.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.56

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,a^2\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2} - \frac{2\,(dx+c)a}{b^2} + \frac{e^{(dx+c)}}{b} + \frac{e^{(-dx-c)}}{b}}{2\,d}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*a^2*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d
*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) - 2*(d*x + c)*a/
b^2 + e^(d*x + c)/b + e^(-d*x - c)/b)/d
```

3.231.9 Mupad [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.34

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\mathrm{e}^{c+d\,x}}{2\,b\,d} + \frac{\mathrm{e}^{-c-d\,x}}{2\,b\,d} - \frac{a\,x}{b^2} - \frac{a^2\,\ln\left(-\frac{2\,a^2\,\mathrm{e}^{c+d\,x}}{b^3} - \frac{2\,a^2\,\left(b-a\,\mathrm{e}^{c+d\,x}\right)}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,d\,\sqrt{a^2+b^2}} + \frac{a^2\,\ln\left(\frac{2\,a^2\,\left(b-a\,\mathrm{e}^{c+d\,x}\right)}{b^3\,\sqrt{a^2+b^2}} - \frac{2\,a^2\,\mathrm{e}^{c+d\,x}}{b^3}\right)}{b^2\,d\,\sqrt{a^2+b^2}}$$

input
```
int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)),x)
```

output
```
exp(c + d*x)/(2*b*d) + exp(- c - d*x)/(2*b*d) - (a*x)/b^2 - (a^2*log(- (2*
a^2*exp(c + d*x))/b^3 - (2*a^2*(b - a*exp(c + d*x)))/(b^3*(a^2 + b^2)^(1/2
))))/(b^2*d*(a^2 + b^2)^(1/2)) + (a^2*log((2*a^2*(b - a*exp(c + d*x)))/(b^
3*(a^2 + b^2)^(1/2)) - (2*a^2*exp(c + d*x))/b^3))/(b^2*d*(a^2 + b^2)^(1/2)
)
```

3.231. $\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.232 $\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.232.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Int}\left(\frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.232.2 Mathematica [N/A]

Not integrable

Time = 9.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.232.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.232.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.232.4 Maple [N/A] (verified)

Not integrable

Time = 0.85 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(dx+c)^2}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.232.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.232.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.232.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 158, normalized size of antiderivative = 5.64

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2*a^2*integrate(-e^(d*x + c)/(b^3*f*x + b^3*e - (b^3*f*x*e^(2*c) + b^3*e*e
^(2*c))*e^(2*d*x) - 2*(a*b^2*f*x*e^c + a*b^2*e*e^c)*e^(d*x)), x) + 1/2*e^(
-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d*e/f)*exp
_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f)
```

3.232.8 Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.232.9 Mupad [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(sinh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(sinh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.233 $\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.233.1 Optimal result

Integrand size = 28, antiderivative size = 712

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{3ef^2x}{4bd^2}-\frac{3f^3x^2}{8bd^2}+\frac{a^2(e+fx)^4}{4b^3f}-\frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3}-\frac{a(e+fx)^3\cosh(c+dx)}{b^2d} - \frac{a^3(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d} + \frac{a^3(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d} - \frac{3a^3f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^2} + \frac{3a^3f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^2} + \frac{6a^3f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^3} - \frac{6a^3f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^3} - \frac{6a^3f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^4} + \frac{6a^3f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d^4} + \frac{6af^3\sinh(c+dx)}{b^2d^4}+\frac{3af(e+fx)^2\sinh(c+dx)}{b^2d^2} + \frac{3f^2(e+fx)\cosh(c+dx)\sinh(c+dx)}{4bd^3} + \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{3f^3\sinh^2(c+dx)}{8bd^4}-\frac{3f(e+fx)^2\sinh^2(c+dx)}{4bd^2}$$

output
```
-3/4*e*f^2*x/b/d^2-3/8*f^3*x^2/b/d^2+1/4*a^2*(f*x+e)^4/b^3/f-1/8*(f*x+e)^4
/b/f-6*a*f^2*(f*x+e)*cosh(d*x+c)/b^2/d^3-a*(f*x+e)^3*cosh(d*x+c)/b^2/d+6*a
*f^3*sinh(d*x+c)/b^2/d^4+3*a*f*(f*x+e)^2*sinh(d*x+c)/b^2/d^2+3/4*f^2*(f*x+
e)*cosh(d*x+c)*sinh(d*x+c)/b/d^3+1/2*(f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/b/d
-3/8*f^3*sinh(d*x+c)^2/b/d^4-3/4*f*(f*x+e)^2*sinh(d*x+c)^2/b/d^2-a^3*(f*x+
e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d/(a^2+b^2)^(1/2)+a^3*(f*x
+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d/(a^2+b^2)^(1/2)-3*a^3*f
*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2/(a^2+b^2)^
(1/2)+3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d
^2/(a^2+b^2)^(1/2)+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))/b^3/d^3/(a^2+b^2)^(1/2)-6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/
(a+(a^2+b^2)^(1/2)))/b^3/d^3/(a^2+b^2)^(1/2)-6*a^3*f^3*polylog(4,-b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^4/(a^2+b^2)^(1/2)+6*a^3*f^3*polylog(4,-b*e
xp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^4/(a^2+b^2)^(1/2)
```

3.233.2 Mathematica [A] (verified)

Time = 4.15 (sec) , antiderivative size = 1407, normalized size of antiderivative = 1.98

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{16a^2\sqrt{a^2+b^2}d^4e^3x - 8b^2\sqrt{a^2+b^2}d^4e^3x + 24a^2\sqrt{a^2+b^2}d^4e^2fx^2 - 12b^2\sqrt{a^2+b^2}d^4e^2fx^2 + 16a^2\sqrt{a^2+b^2}}{}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(16*a^2*Sqrt[a^2 + b^2]*d^4*e^3*x - 8*b^2*Sqrt[a^2 + b^2]*d^4*e^3*x + 24*a
^2*Sqrt[a^2 + b^2]*d^4*e^2*f*x^2 - 12*b^2*Sqrt[a^2 + b^2]*d^4*e^2*f*x^2 +
16*a^2*Sqrt[a^2 + b^2]*d^4*e*f^2*x^3 - 8*b^2*Sqrt[a^2 + b^2]*d^4*e*f^2*x^3
 + 4*a^2*Sqrt[a^2 + b^2]*d^4*f^3*x^4 - 2*b^2*Sqrt[a^2 + b^2]*d^4*f^3*x^4 +
 32*a^3*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 16*a*b*Sqrt
[a^2 + b^2]*d^3*e^3*Cosh[c + d*x] - 96*a*b*Sqrt[a^2 + b^2]*d*e*f^2*Cosh[c
+ d*x] - 48*a*b*Sqrt[a^2 + b^2]*d^3*e^2*f*x*Cosh[c + d*x] - 96*a*b*Sqrt[a^
2 + b^2]*d*f^3*x*Cosh[c + d*x] - 48*a*b*Sqrt[a^2 + b^2]*d^3*e*f^2*x^2*Cosh
[c + d*x] - 16*a*b*Sqrt[a^2 + b^2]*d^3*f^3*x^3*Cosh[c + d*x] - 6*b^2*Sqrt[
a^2 + b^2]*d^2*e^2*f*Cosh[2*(c + d*x)] - 3*b^2*Sqrt[a^2 + b^2]*f^3*Cosh[2*
(c + d*x)] - 12*b^2*Sqrt[a^2 + b^2]*d^2*e*f^2*x*Cosh[2*(c + d*x)] - 6*b^2*
Sqrt[a^2 + b^2]*d^2*f^3*x^2*Cosh[2*(c + d*x)] - 48*a^3*d^3*e^2*f*x*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 48*a^3*d^3*e*f^2*x^2*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 16*a^3*d^3*f^3*x^3*Log[1 + (b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2])] + 48*a^3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])] + 48*a^3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])] + 16*a^3*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])] - 48*a^3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a +
Sqrt[a^2 + b^2])] + 48*a^3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))] + 96*a^3*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a...
```

3.233.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{6091}$$
$$\frac{\int (e+fx)^3\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{a\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -(e+fx)^3\sin(ic+idx)^2dx}{b}$$
$$\Big\downarrow \text{25}$$
$$-\frac{a\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\int (e+fx)^3\sin(ic+idx)^2dx}{b}$$

$$\Big\downarrow \text{3792}$$

$$-\frac{a\int\frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3f^2\int-((e+fx)\sinh^2(c+dx))dx}{2d^2}+\frac{1}{2}\int(e+fx)^3dx+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{a\int\frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3f^2\int-((e+fx)\sinh^2(c+dx))dx}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\int\frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{3f^2\int(e+fx)\sinh^2(c+dx)dx}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int\frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{3f^2\int-((e+fx)\sin(ic+idx)^2)dx}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\int\frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3f^2\int(e+fx)\sin(ic+idx)^2dx}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}$$

$$\Big\downarrow \text{3791}$$

$$-\frac{a\int\frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3f^2\left(\frac{1}{2}\int(e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}$$

$$\Big\downarrow \text{17}$$

3.233. $\int\frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}$$

$$\Big\downarrow \text{6091}$$

$$-\frac{a\left(\frac{\int (e+fx)^3\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)^3\sin(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)^3\sin(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\cosh(c+dx)dx}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

3.233. $\displaystyle\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3777$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int-i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

3.233. $\int\frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}\right)}{b}\right)}{b}$$

3.233. $\displaystyle\int\frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\downarrow \text{3117}$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\downarrow \text{6091}$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^3dx}{b}-\frac{a\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\downarrow \text{17}$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{a\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}\right)}{b}$$

3.233. $\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{a\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 3803$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^4}{4bf}-\frac{2a\int-\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\left(\frac{2a\int\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^4}{4bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}\right)}{b}$$

3.233. $\displaystyle\int\frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{ 2694}$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^4}{4bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2i}{}\right.}{}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^4}{4bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(i(e+fx)\right.}{}\right.}{d}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{ 2620}$$

3.233. $\displaystyle\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{\frac{3f^2\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}+\frac{3f(e+fx)^2\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$\frac{a\left(a-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{b}\right.}{b}\right.}{b}$$

input
```
Int[((e + f*x)^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.233.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.233. $\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

3.233.4 Maple [F]

$$\int \frac{(fx+e)^3 \sinh (dx+c)^3}{a+b \sinh (dx+c)}dx$$

input
```
int((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.233.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5191 vs. $2(654) = 1308$.

Time = 0.36 (sec) , antiderivative size = 5191, normalized size of antiderivative = 7.29

$$\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

3.233.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.233.7 Maxima [F]

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*e^3*(8*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
 - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3*d) + (4*a*e^(-d*x - c) - b)*
e^(2*d*x + 2*c)/(b^2*d) - 4*(2*a^2 - b^2)*(d*x + c)/(b^3*d) + (4*a*e^(-d*x
 - c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/32*(4*(2*a^2*d^4*f^3*e^(2*c) - b^
2*d^4*f^3*e^(2*c))*x^4 + 16*(2*a^2*d^4*e*f^2*e^(2*c) - b^2*d^4*e*f^2*e^(2*
c))*x^3 + 24*(2*a^2*d^4*e^2*f*e^(2*c) - b^2*d^4*e^2*f*e^(2*c))*x^2 + (4*b^
2*d^3*f^3*x^3*e^(4*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*b^2*x^2*e^(4*c) + 6*(2*d
^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*b^2*x*e^(4*c) - 3*(2*d^2*e^2*f - 2*d*e*f^2
 + f^3)*b^2*e^(4*c))*e^(2*d*x) - 16*(a*b*d^3*f^3*x^3*e^(3*c) + 3*(d^3*e*f^
2 - d^2*f^3)*a*b*x^2*e^(3*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a*b*x
*e^(3*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b*e^(3*c))*e^(d*x) - 16*(a*
b*d^3*f^3*x^3*e^c + 3*(d^3*e*f^2 + d^2*f^3)*a*b*x^2*e^c + 3*(d^3*e^2*f + 2
*d^2*e*f^2 + 2*d*f^3)*a*b*x*e^c + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a*b*e^
c)*e^(-d*x) - (4*b^2*d^3*f^3*x^3 + 6*(2*d^3*e*f^2 + d^2*f^3)*b^2*x^2 + 6*(
2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*b^2*x + 3*(2*d^2*e^2*f + 2*d*e*f^2 + f^
3)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^4) - integrate(2*(a^3*f^3*x^3*e^c + 3*
a^3*e*f^2*x^2*e^c + 3*a^3*e^2*f*x*e^c)*e^(d*x)/(b^4*e^(2*d*x + 2*c) + 2*a*
b^3*e^(d*x + c) - b^4), x)
```

3.233.8 Giac [F]

$$\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)^3\,(e+f\,x)^3}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.234 $\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.234.1 Optimal result

Integrand size = 28, antiderivative size = 522

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{f^2x}{4bd^2}+\frac{a^2(e+fx)^3}{3b^3f}-\frac{(e+fx)^3}{6bf} -\frac{2af^2\cosh(c+dx)}{b^2d^3}-\frac{a(e+fx)^2\cosh(c+dx)}{b^2d} -\frac{a^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}d} +\frac{a^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}d} -\frac{2a^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}d^2} +\frac{2a^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}d^2} +\frac{2a^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}d^3} -\frac{2a^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}d^3} +\frac{2af(e+fx)\sinh(c+dx)}{b^2d^2}+\frac{f^2\cosh(c+dx)\sinh(c+dx)}{4bd^3} +\frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{2bd} -\frac{f(e+fx)\sinh^2(c+dx)}{2bd^2}$$

output
```
-1/4*f^2*x/b/d^2+1/3*a^2*(f*x+e)^3/b^3/f-1/6*(f*x+e)^3/b/f-2*a*f^2*cosh(d*
x+c)/b^2/d^3-a*(f*x+e)^2*cosh(d*x+c)/b^2/d+2*a*f*(f*x+e)*sinh(d*x+c)/b^2/d
^2+1/4*f^2*cosh(d*x+c)*sinh(d*x+c)/b/d^3+1/2*(f*x+e)^2*cosh(d*x+c)*sinh(d*
x+c)/b/d-1/2*f*(f*x+e)*sinh(d*x+c)^2/b/d^2-a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)
/(a-(a^2+b^2)^(1/2)))/b^3/d/(a^2+b^2)^(1/2)+a^3*(f*x+e)^2*ln(1+b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/b^3/d/(a^2+b^2)^(1/2)-2*a^3*f*(f*x+e)*polylog(2,-b*
exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2/(a^2+b^2)^(1/2)+2*a^3*f*(f*x+e)*po
lylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^2/(a^2+b^2)^(1/2)+2*a^3*f
^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^3/(a^2+b^2)^(1/2)-2*
a^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^3/(a^2+b^2)^(1/
2)
```

3.234.2 Mathematica [A] (verified)

Time = 4.10 (sec) , antiderivative size = 740, normalized size of antiderivative = 1.42

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{24a^2e^2x - 12b^2e^2x + 24a^2efx^2 - 12b^2efx^2 + 8a^2f^2x^3 - 4b^2f^2x^3 + \frac{48a^3e^2\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d} - \frac{24abe^2\cosh(c+}{d}}{}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(24*a^2*e^2*x - 12*b^2*e^2*x + 24*a^2*e*f*x^2 - 12*b^2*e*f*x^2 + 8*a^2*f^2
*x^3 - 4*b^2*f^2*x^3 + (48*a^3*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
b^2]])/(Sqrt[a^2 + b^2]*d) - (24*a*b*e^2*Cosh[c + d*x])/d - (48*a*b*f^2*Co
sh[c + d*x])/d^3 - (48*a*b*e*f*x*Cosh[c + d*x])/d - (24*a*b*f^2*x^2*Cosh[c
 + d*x])/d - (6*b^2*e*f*Cosh[2*(c + d*x)])/d^2 - (6*b^2*f^2*x*Cosh[2*(c +
d*x)])/d^2 - (48*a^3*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])
/(Sqrt[a^2 + b^2]*d) - (24*a^3*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2])])/(Sqrt[a^2 + b^2]*d) + (48*a^3*e*f*x*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*d) + (24*a^3*f^2*x^2*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*d) - (48*a^3*f*(e + f*
x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*d^
2) + (48*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
))])/(Sqrt[a^2 + b^2]*d^2) + (48*a^3*f^2*PolyLog[3, (b*E^(c + d*x))/(-a +
Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*d^3) - (48*a^3*f^2*PolyLog[3, -((b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*d^3) + (48*a*b*e*f*Sin
h[c + d*x])/d^2 + (48*a*b*f^2*x*Sinh[c + d*x])/d^2 + (6*b^2*e^2*Sinh[2*(c
+ d*x)])/d + (3*b^2*f^2*Sinh[2*(c + d*x)])/d^3 + (12*b^2*e*f*x*Sinh[2*(c +
 d*x)])/d + (6*b^2*f^2*x^2*Sinh[2*(c + d*x)])/d)/(24*b^3)
```

3.234.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \; 6091$$

$$\frac{\int (e+fx)^2\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -(e+fx)^2\sin(ic+idx)^2dx}{b}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\int (e+fx)^2\sin(ic+idx)^2dx}{b}$$

$$\Big\downarrow \textbf{3792}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{f^2\int -\sinh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^2dx+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}}{b}$$

$$\Big\downarrow \textbf{17}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{f^2\int -\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{f^2\int \sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{f^2\int -\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{f^2\int \sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}$$

$$\Big\downarrow \textbf{3115}$$

3.234. $\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}$$

$$\Big\downarrow \text{6091}$$

$$-\frac{a\left(\frac{\int (e+fx)^2\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)^2\sin(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)^2\sin(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{b}\right)}{b}$$

3.234. $\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

3.234. $\int\frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 3118$$

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 6091$$

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^2dx}{b}-\frac{a\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 17$$

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{a\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 3042$$

3.234. $\displaystyle\int\frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{a\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow 3803$$

$$-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^3}{3bf}-\frac{2a\int-\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\left(\frac{2a\int\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^3}{3bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow 2694$$

$$-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^3}{3bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-f\,\mathrm{co}\cdots\right)}{d}\cdots\right)}{b}\cdots\right)}{b}$$

3.234. $\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$\downarrow$ 27

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^3}{3bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right.}{b}\right.}{b}$$

$\downarrow$ 2620

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{b}\right.}{b}\right.}{b}$$

$\downarrow$ 3011

3.234. $\int\frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{-\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-$$

$$a\left(-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{bd}\right.}{}\right.}{b}\right.}{b}\right.$$

$\downarrow$ 2720

3.234. $\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-$$

$$a\left(-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log}{}\right.}{}\right.}{b}\right.}{b}\right.$$

input
```
Int[((e + f*x)^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.234. $\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.234.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.234. $\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.234. $\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

3.234.4 Maple [F]

$$\int \frac{(fx+e)^2 \sinh (dx+c)^3}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.234.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3247 vs. $2(478) = 956$.

Time = 0.32 (sec) , antiderivative size = 3247, normalized size of antiderivative = 6.22

$$\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/48*(6*(a^2*b^2 + b^4)*d^2*f^2*x^2 + 6*(a^2*b^2 + b^4)*d^2*e^2 - 3*(2*(a
^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^2*b^2 + b^4)*d^2*e^2 - 2*(a^2*b^2 + b^4)*
d*e*f + (a^2*b^2 + b^4)*f^2 + 2*(2*(a^2*b^2 + b^4)*d^2*e*f - (a^2*b^2 + b^
4)*d*f^2)*x)*cosh(d*x + c)^4 - 3*(2*(a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^2*b
^2 + b^4)*d^2*e^2 - 2*(a^2*b^2 + b^4)*d*e*f + (a^2*b^2 + b^4)*f^2 + 2*(2*(
a^2*b^2 + b^4)*d^2*e*f - (a^2*b^2 + b^4)*d*f^2)*x)*sinh(d*x + c)^4 + 6*(a^
2*b^2 + b^4)*d*e*f + 24*((a^3*b + a*b^3)*d^2*f^2*x^2 + (a^3*b + a*b^3)*d^2
*e^2 - 2*(a^3*b + a*b^3)*d*e*f + 2*(a^3*b + a*b^3)*f^2 + 2*((a^3*b + a*b^3
)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^3 + 12*(2*(a^3*b + a*b
^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e^2 - 4*(a^3*b + a*b^3)*d*e*f + 4*
(a^3*b + a*b^3)*f^2 + 4*((a^3*b + a*b^3)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*
x - (2*(a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^2*b^2 + b^4)*d^2*e^2 - 2*(a^2*b^
2 + b^4)*d*e*f + (a^2*b^2 + b^4)*f^2 + 2*(2*(a^2*b^2 + b^4)*d^2*e*f - (a^2
*b^2 + b^4)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2*b^2 + b^4)*f
^2 - 8*((2*a^4 + a^2*b^2 - b^4)*d^3*f^2*x^3 + 3*(2*a^4 + a^2*b^2 - b^4)*d^
3*e*f*x^2 + 3*(2*a^4 + a^2*b^2 - b^4)*d^3*e^2*x)*cosh(d*x + c)^2 - 2*(4*(2
*a^4 + a^2*b^2 - b^4)*d^3*f^2*x^3 + 12*(2*a^4 + a^2*b^2 - b^4)*d^3*e*f*x^2
 + 12*(2*a^4 + a^2*b^2 - b^4)*d^3*e^2*x + 9*(2*(a^2*b^2 + b^4)*d^2*f^2*x^2
 + 2*(a^2*b^2 + b^4)*d^2*e^2 - 2*(a^2*b^2 + b^4)*d*e*f + (a^2*b^2 + b^4)*f
^2 + 2*(2*(a^2*b^2 + b^4)*d^2*e*f - (a^2*b^2 + b^4)*d*f^2)*x)*cosh(d*x ...
```

3.234.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.234.7 Maxima [F]

$$\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*e^2*(8*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
 - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3*d) + (4*a*e^(-d*x - c) - b)*
e^(2*d*x + 2*c)/(b^2*d) - 4*(2*a^2 - b^2)*(d*x + c)/(b^3*d) + (4*a*e^(-d*x
 - c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/48*(8*(2*a^2*d^3*f^2*e^(2*c) - b^
2*d^3*f^2*e^(2*c))*x^3 + 24*(2*a^2*d^3*e*f*e^(2*c) - b^2*d^3*e*f*e^(2*c))*
x^2 + 3*(2*b^2*d^2*f^2*x^2*e^(4*c) + 2*(2*d^2*e*f - d*f^2)*b^2*x*e^(4*c) -
 (2*d*e*f - f^2)*b^2*e^(4*c))*e^(2*d*x) - 24*(a*b*d^2*f^2*x^2*e^(3*c) + 2*
(d^2*e*f - d*f^2)*a*b*x*e^(3*c) - 2*(d*e*f - f^2)*a*b*e^(3*c))*e^(d*x) - 2
4*(a*b*d^2*f^2*x^2*e^c + 2*(d^2*e*f + d*f^2)*a*b*x*e^c + 2*(d*e*f + f^2)*a
*b*e^c)*e^(-d*x) - 3*(2*b^2*d^2*f^2*x^2 + 2*(2*d^2*e*f + d*f^2)*b^2*x + (2
*d*e*f + f^2)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^3) - integrate(2*(a^3*f^2*x
^2*e^c + 2*a^3*e*f*x*e^c)*e^(d*x)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x +
c) - b^4), x)
```

3.234.8 Giac [F]

$$\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.235 $\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.235.1 Optimal result

Integrand size = 26, antiderivative size = 335

$$\begin{aligned}\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{a^2ex}{b^3}-\frac{ex}{2b}+\frac{a^2fx^2}{2b^3}-\frac{fx^2}{4b}-\frac{a(e+fx)\cosh(c+dx)}{b^2d}\\
&-\frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}d}\\
&+\frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}d}\\
&-\frac{a^3f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}d^2}\\
&+\frac{a^3f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}d^2}+\frac{af\sinh(c+dx)}{b^2d^2}\\
&+\frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd}-\frac{f\sinh^2(c+dx)}{4bd^2}\end{aligned}$$

output
```
a^2*e*x/b^3-1/2*e*x/b+1/2*a^2*f*x^2/b^3-1/4*f*x^2/b-a*(f*x+e)*cosh(d*x+c)/
b^2/d+a*f*sinh(d*x+c)/b^2/d^2+1/2*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b/d-1/4*
f*sinh(d*x+c)^2/b/d^2-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b
^3/d/(a^2+b^2)^(1/2)+a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^
3/d/(a^2+b^2)^(1/2)-a^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3
/d^2/(a^2+b^2)^(1/2)+a^3*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^
3/d^2/(a^2+b^2)^(1/2)
```

3.235.2 Mathematica [A] (verified)

Time = 5.51 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.91

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{2(2a^2-b^2)(c+dx)(cf-d(2e+fx))+8abd(e+fx)\cosh(c+dx)+b^2f\cosh(2(c+dx))+\frac{8a^3\left(-2de\,\mathrm{a}\right.}{}}{}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/8*(2*(2*a^2 - b^2)*(c + d*x)*(c*f - d*(2*e + f*x)) + 8*a*b*d*(e + f*x)*
Cosh[c + d*x] + b^2*f*Cosh[2*(c + d*x)] + (8*a^3*(-2*d*e*ArcTanh[(a + b*E^
(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
 b^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c
 + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E
^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] - 8*a*b*f*Sinh[c + d*x] - 2*b^2*d*(e
+ f*x)*Sinh[2*(c + d*x)])/(b^3*d^2)
```

3.235.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.97, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.808, Rules used = {6091, 3042, 25, 3791, 17, 6091, 3042, 26, 3777, 3042, 3117, 6091, 17, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6091}$$

$$\frac{\int (e+fx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -\left((e+fx)\sin(ic+idx)^2\right)dx}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\int (e+fx)\sin(ic+idx)^2dx}{b}$$

$$\Big\downarrow \text{3791}$$

$$-\frac{a\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{1}{2}\int (e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{a\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}$$

$$\Big\downarrow \text{6091}$$

$$-\frac{a\left(\frac{\int (e+fx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)\sin(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)\sin(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

3.235. $\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}\right)}{b}$$

$$\downarrow \text{3117}$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\downarrow \text{6091}$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)dx}{b}-\frac{a\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\downarrow \text{17}$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{a\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\downarrow \text{3042}$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{a\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\downarrow \text{3803}$$

3.235. $\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\left(\frac{(e+fx)^2}{2bf}-\frac{2a\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}}{}$$

$\Big\downarrow$ 25

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\left(\frac{2a\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b}+\frac{(e+fx)^2}{2bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 2694

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 27

3.235. $\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}\right)}{b}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)^2}{2bf}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 2715$$

3.235. $\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+\right.}{bd^2}}{2\sqrt{a^2+b^2}}\right.}{b}}{b}\right.}{b}$$

$$\downarrow \text{2838}$$

$$-\frac{\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{2a\left(\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b}+\frac{(e+fx)}{2bf}\right.}{b}\right.}{b}$$

input
```
Int[((e + f*x)*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

3.235. $\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
-(((e + f*x)^2/(4*f) - ((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (f*
Sinh[c + d*x]^2)/(4*d^2))/b) - (a*(-((a*((e + f*x)^2/(2*b*f) + (2*a*(-1/2*
(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*
PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 +
 b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*
d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2
*Sqrt[a^2 + b^2])))/b))/b) - (I*((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh
[c + d*x])/d^2))/b))/b
```

3.235.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

3.235. $\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6091
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sinh[
c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

3.235. $\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.235.4 Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.76

method	result
risch	$\frac{a^2 f\,x^2}{2b^3}-\frac{f\,x^2}{4b}+\frac{a^2 ex}{b^3}-\frac{ex}{2b}+\frac{(2dfx+2de-f)\,\mathrm{e}^{2dx+2c}}{16b\,d^2}-\frac{a(dfx+de-f)\,\mathrm{e}^{dx+c}}{2b^2d^2}-\frac{a(dfx+de+f)\,\mathrm{e}^{-dx-c}}{2b^2d^2}-\frac{(2dfx+2de+f)\mathrm{e}}{16b\,d^2}$

input
```
int((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*f*x^2/b^3-1/4*f*x^2/b+a^2*e*x/b^3-1/2*e*x/b+1/16*(2*d*f*x+2*d*e-f)
/b/d^2*exp(2*d*x+2*c)-1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp(d*x+c)-1/2*a*(d*f*x+
d*e+f)/b^2/d^2*exp(-d*x-c)-1/16*(2*d*f*x+2*d*e+f)/b/d^2*exp(-2*d*x-2*c)+2/
d*a^3/b^3*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/
2))-1/d*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a
+(a^2+b^2)^(1/2)))*x+1/d*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b
^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((-b
*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2*a^3/b^3*f/(a^
2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/
d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+
(a^2+b^2)^(1/2)))+1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2
+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/d^2*a^3/b^3*f*c/(a^2+b^2)^(1/2)*arct
anh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

3.235.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1727 vs. 2(303) = 606.

Time = 0.31 (sec) , antiderivative size = 1727, normalized size of antiderivative = 5.16

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Too large to display}$$

input
```
integrate((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*((2*(a^2*b^2 + b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d*e - (a^2*b^2 + b^4)*f
)*cosh(d*x + c)^4 + (2*(a^2*b^2 + b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d*e - (a^
2*b^2 + b^4)*f)*sinh(d*x + c)^4 - 2*(a^2*b^2 + b^4)*d*f*x - 8*((a^3*b + a*
b^3)*d*f*x + (a^3*b + a*b^3)*d*e - (a^3*b + a*b^3)*f)*cosh(d*x + c)^3 - 4*
(2*(a^3*b + a*b^3)*d*f*x + 2*(a^3*b + a*b^3)*d*e - 2*(a^3*b + a*b^3)*f - (
2*(a^2*b^2 + b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d*e - (a^2*b^2 + b^4)*f)*cosh(
d*x + c))*sinh(d*x + c)^3 - 2*(a^2*b^2 + b^4)*d*e + 4*((2*a^4 + a^2*b^2 -
b^4)*d^2*f*x^2 + 2*(2*a^4 + a^2*b^2 - b^4)*d^2*e*x)*cosh(d*x + c)^2 + 2*(2
*(2*a^4 + a^2*b^2 - b^4)*d^2*f*x^2 + 4*(2*a^4 + a^2*b^2 - b^4)*d^2*e*x + 3
*(2*(a^2*b^2 + b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d*e - (a^2*b^2 + b^4)*f)*cos
h(d*x + c)^2 - 12*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e - (a^3*b +
a*b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 16*(a^3*b*f*cosh(d*x + c)^2 + 2
*a^3*b*f*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh(d*x + c)^2)*sqrt((a^2
+ b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 16*(a^3*b*f*cosh(d*x
+ c)^2 + 2*a^3*b*f*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh(d*x + c)^2)*
sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 16*((a^3*b*
d*e - a^3*b*c*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)
*sinh(d*x + c) + (a^3*b*d*e - a^3*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b...
```

3.235.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.235.7 Maxima [F]

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/16*(32*a^3*integrate(x*e^(d*x + c)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*
x + c) - b^4), x) - (4*(2*a^2*d^2*e^(2*c) - b^2*d^2*e^(2*c))*x^2 + (2*b^2*
d*x*e^(4*c) - b^2*e^(4*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) - a*b*e^(3*c))*e
^(d*x) - 8*(a*b*d*x*e^c + a*b*e^c)*e^(-d*x) - (2*b^2*d*x + b^2)*e^(-2*d*x)
)*e^(-2*c)/(b^3*d^2))*f - 1/8*e*(8*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2
+ b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3*d) +
(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - 4*(2*a^2 - b^2)*(d*x + c)
/(b^3*d) + (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d))
```

3.235.8 Giac [F]

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\sinh(c+d\,x)^3\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.236 $\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.236.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\frac{(2a^2-b^2)\,x}{2b^3}+\frac{2a^3\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}d}-\frac{a\cosh(c+dx)}{b^2d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}$$

output
```
1/2*(2*a^2-b^2)*x/b^3-a*cosh(d*x+c)/b^2/d+1/2*cosh(d*x+c)*sinh(d*x+c)/b/d+
2*a^3*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/b^3/d/(a^2+b^2)^(
1/2)
```

3.236.2 Mathematica [A] (verified)

Time = 3.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\frac{-2(-2a^2+b^2)\,(c+dx)-\frac{8a^3\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-4ab\cosh(c+dx)+b^2\sinh(2(c+dx))}{4b^3d}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
(-2*(-2*a^2 + b^2)*(c + d*x) - (8*a^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqr
t[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c + d
*x)])/(4*b^3*d)
```

3.236.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 26, 3272, 3042, 3502, 26, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3272}$$

$$i\left(\frac{i\int \frac{2a\sinh^2(c+dx)+b\sinh(c+dx)+a}{a+b\sinh(c+dx)}dx}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\int \frac{-2a\sin(ic+idx)^2-ib\sin(ic+idx)+a}{a-ib\sin(ic+idx)}dx}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

$$\Big\downarrow \text{3502}$$

$$i\left(\frac{i\left(\frac{2a\cosh(c+dx)}{bd}+\frac{i\int -\frac{i\left(ab-\left(2a^2-b^2\right)\sinh(c+dx)\right)}{a+b\sinh(c+dx)}dx}{b}\right)}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

$$\downarrow 26$$

$$i\left(\frac{i\left(\frac{\int \frac{ab-\left(2a^2-b^2\right)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{2a\cosh(c+dx)}{bd}\right)}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

$$\downarrow 3042$$

$$i\left(\frac{i\left(\frac{2a\cosh(c+dx)}{bd}+\frac{\int \frac{ab+i\left(2a^2-b^2\right)\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{b}\right)}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

$$\downarrow 3214$$

$$i\left(\frac{i\left(\frac{\frac{2a^3\int \frac{1}{a+b\sinh(c+dx)}dx}{b}-\frac{x\left(2a^2-b^2\right)}{b}}{b}+\frac{2a\cosh(c+dx)}{bd}\right)}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

$$\downarrow 3042$$

$$i\left(\frac{i\left(\frac{2a\cosh(c+dx)}{bd}+\frac{-\frac{x\left(2a^2-b^2\right)}{b}+\frac{2a^3\int \frac{1}{a-ib\sin(ic+idx)}dx}{b}}{b}\right)}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

$$\downarrow 3139$$

$$i\left(\frac{i\left(\frac{2a\cosh(c+dx)}{bd}+\frac{-\frac{x\left(2a^2-b^2\right)}{b}-\frac{4ia^3\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}}{b}\right)}{2b}-\frac{i\sinh(c+dx)\cosh(c+d}{2bd}\right.$$

$$\downarrow 1083$$

3.236. $\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$i\left(\frac{i\left(\frac{2a\cosh(c+dx)}{bd}+\frac{-\frac{x(2a^2-b^2)}{b}+\frac{8ia^3\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{bd}}{b}\right)}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

$$\Big\downarrow \text{217}$$

$$i\left(\frac{i\left(\frac{\frac{4a^3\operatorname{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}}-\frac{x(2a^2-b^2)}{b}}{b}+\frac{2a\cosh(c+dx)}{bd}\right)}{2b}-\frac{i\sinh(c+dx)\cosh(c+dx)}{2bd}\right)$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
I*(((I/2)*((-(((2*a^2 - b^2)*x)/b) + (4*a^3*ArcTanh[Tanh[(c + d*x)/2]/(2*S
qrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d))/b + (2*a*Cosh[c + d*x])/(b*d)))/b
 - ((I/2)*Cosh[c + d*x]*Sinh[c + d*x])/(b*d))
```

3.236.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.236. $\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3272

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.236. $\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.236.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.79

method	result
derivativedivides	$\frac{\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{-b-2a}{2b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-2a^{2}+b^{2}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^{3}}-\frac{2a^{3}\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{b^{3}\sqrt{a^{2}+b^{2}}}-\frac{}{2b(\mathrm{ta}}}{d}$
default	$\frac{\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{-b-2a}{2b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-2a^{2}+b^{2}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^{3}}-\frac{2a^{3}\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{b^{3}\sqrt{a^{2}+b^{2}}}-\frac{}{2b(\mathrm{ta}}}{d}$
risch	$\frac{x a^{2}}{b^{3}}-\frac{x}{2b}+\frac{\mathrm{e}^{2dx+2c}}{8bd}-\frac{a\,\mathrm{e}^{dx+c}}{2b^{2}d}-\frac{a\,\mathrm{e}^{-dx-c}}{2b^{2}d}-\frac{\mathrm{e}^{-2dx-2c}}{8bd}+\frac{a^{3}\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{\sqrt{a^{2}+b^{2}}\,b}\right)}{\sqrt{a^{2}+b^{2}}\,d\,b^{3}}-\frac{a^{3}\ln\left(\mathrm{e}^{dx+c}\right.}{\sqrt{}}$

input
```
int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/2*(-b-2*a)/b^2/(tanh(1/2*d*x+1/2*c)
-1)+1/2/b^3*(-2*a^2+b^2)*ln(tanh(1/2*d*x+1/2*c)-1)-2*a^3/b^3/(a^2+b^2)^(1/
2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1/2/b/(tanh(
1/2*d*x+1/2*c)+1)^2-1/2*(-b+2*a)/b^2/(tanh(1/2*d*x+1/2*c)+1)+1/2*(2*a^2-b^
2)/b^3*ln(tanh(1/2*d*x+1/2*c)+1))
```

3.236.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(100) = 200$.

Time = 0.26 (sec) , antiderivative size = 601, normalized size of antiderivative = 5.62

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{4\left(2a^4+a^2b^2-b^4\right)dx\cosh\left(dx+c\right)^2+\left(a^2b^2+b^4\right)\cosh\left(dx+c\right)^4+\left(a^2b^2+b^4\right)\sinh\left(dx+c\right)^4-a^2b^2-b^4}{}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/8*(4*(2*a^4 + a^2*b^2 - b^4)*d*x*cosh(d*x + c)^2 + (a^2*b^2 + b^4)*cosh(
d*x + c)^4 + (a^2*b^2 + b^4)*sinh(d*x + c)^4 - a^2*b^2 - b^4 - 4*(a^3*b +
a*b^3)*cosh(d*x + c)^3 - 4*(a^3*b + a*b^3 - (a^2*b^2 + b^4)*cosh(d*x + c))
*sinh(d*x + c)^3 + 2*(2*(2*a^4 + a^2*b^2 - b^4)*d*x + 3*(a^2*b^2 + b^4)*co
sh(d*x + c)^2 - 6*(a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*(a^3*
cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x + c)^2)
*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*co
sh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*
sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^
2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d
*x + c) - b)) - 4*(a^3*b + a*b^3)*cosh(d*x + c) - 4*(a^3*b + a*b^3 - 2*(2*
a^4 + a^2*b^2 - b^4)*d*x*cosh(d*x + c) - (a^2*b^2 + b^4)*cosh(d*x + c)^3 +
 3*(a^3*b + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^2*b^3 + b^5)*d*cosh
(d*x + c)^2 + 2*(a^2*b^3 + b^5)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2*b^3 +
 b^5)*d*sinh(d*x + c)^2)
```

3.236.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.236.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.53

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a^3\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^3d} - \frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^2d} + \frac{(2\,a^2-b^2)(dx+c)}{2\,b^3d} - \frac{4\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}}{8\,b^2d}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3*d) - 1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*
x + 2*c)/(b^2*d) + 1/2*(2*a^2 - b^2)*(d*x + c)/(b^3*d) - 1/8*(4*a*e^(-d*x
- c) + b*e^(-2*d*x - 2*c))/(b^2*d)
```

3.236.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.41

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{8\,a^3\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^3}-\frac{4\,(2\,a^2-b^2)(dx+c)}{b^3}-\frac{be^{(2\,dx+2\,c)}-4\,ae^{(dx+c)}}{b^2}+\frac{(4\,abe^{(dx+c)}+b^2)e^{(-2\,dx-2\,c)}}{b^3}}{8\,d}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/8*(8*a^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(
d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3) - 4*(2*a^2 - b^
2)*(d*x + c)/b^3 - (b*e^(2*d*x + 2*c) - 4*a*e^(d*x + c))/b^2 + (4*a*b*e^(d
*x + c) + b^2)*e^(-2*d*x - 2*c)/b^3)/d
```

3.236.9 Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.98

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x\,(2\,a^2-b^2)}{2\,b^3}-\frac{\mathrm{e}^{-2\,c-2\,d\,x}}{8\,b\,d}+\frac{\mathrm{e}^{2\,c+2\,d\,x}}{8\,b\,d}-\frac{a\,\mathrm{e}^{-c-d\,x}}{2\,b^2\,d}-\frac{a\,\mathrm{e}^{c+d\,x}}{2\,b^2\,d}-\frac{a^3\,\ln\left(\frac{2\,a^3\,\mathrm{e}^{c+d\,x}}{b^4}-\frac{2\,a^3\,\left(b-a\,\mathrm{e}^{c+d\,x}\right)}{b^4\,\sqrt{a^2+b^2}}\right)}{b^3\,d\,\sqrt{a^2+b^2}}+\frac{a^3\,\ln\left(\frac{2\,a^3\,\mathrm{e}^{c+d\,x}}{b^4}+\frac{2\,a^3\,\left(b-a\,\mathrm{e}^{c+d\,x}\right)}{b^4\,\sqrt{a^2+b^2}}\right)}{b^3\,d\,\sqrt{a^2+b^2}}$$

input
```
int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)),x)
```

output
```
(x*(2*a^2 - b^2))/(2*b^3) - exp(- 2*c - 2*d*x)/(8*b*d) + exp(2*c + 2*d*x)/
(8*b*d) - (a*exp(- c - d*x))/(2*b^2*d) - (a*exp(c + d*x))/(2*b^2*d) - (a^3
*log((2*a^3*exp(c + d*x))/b^4 - (2*a^3*(b - a*exp(c + d*x)))/(b^4*(a^2 + b
^2)^(1/2))))/(b^3*d*(a^2 + b^2)^(1/2)) + (a^3*log((2*a^3*exp(c + d*x))/b^4
 + (2*a^3*(b - a*exp(c + d*x)))/(b^4*(a^2 + b^2)^(1/2))))/(b^3*d*(a^2 + b^
2)^(1/2))
```

3.236. $\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.237 $\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.237.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.237.2 Mathematica [N/A]

Not integrable

Time = 11.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.237.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.237.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.237.4 Maple [N/A] (verified)

Not integrable

Time = 0.66 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.237.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.237.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.237.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 236, normalized size of antiderivative = 8.43

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*a^3*integrate(-e^(d*x + c)/(b^4*f*x + b^4*e - (b^4*f*x*e^(2*c) + b^4*e*
e^(2*c))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*b^3*e*e^c)*e^(d*x)), x) - 1/4*e^
(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) - 1/2*a*e^(-c +
d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c - d*e/f)*exp_
integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f)*exp_integral
_e(1, -2*(f*x + e)*d/f)/(b*f) + 1/2*(2*a^2 - b^2)*log(f*x + e)/(b^3*f)
```

3.237.8 Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.237.9 Mupad [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)^3}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(sinh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(sinh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.238 $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.238.1 Optimal result

Integrand size = 26, antiderivative size = 605

$$\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2(e+fx)^3\operatorname{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}$$
$$+\frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}$$
$$-\frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}$$
$$+\frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}$$
$$-\frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d^2}$$
$$+\frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d^2}$$
$$+\frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}$$
$$-\frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3}$$
$$+\frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d^3}$$
$$-\frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d^3}$$
$$-\frac{6f^3\operatorname{PolyLog}\left(4,-e^{c+dx}\right)}{ad^4}+\frac{6f^3\operatorname{PolyLog}\left(4,e^{c+dx}\right)}{ad^4}$$
$$-\frac{6bf^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d^4}$$
$$+\frac{6bf^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d^4}$$

output
```
-2*(f*x+e)^3*arctanh(exp(d*x+c))/a/d-3*f*(f*x+e)^2*polylog(2,-exp(d*x+c))/
a/d^2+3*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a/d^2+6*f^2*(f*x+e)*polylog(3,-e
xp(d*x+c))/a/d^3-6*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a/d^3-6*f^3*polylog(4
,-exp(d*x+c))/a/d^4+6*f^3*polylog(4,exp(d*x+c))/a/d^4-b*(f*x+e)^3*ln(1+b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d/(a^2+b^2)^(1/2)+b*(f*x+e)^3*ln(1+b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d/(a^2+b^2)^(1/2)-3*b*f*(f*x+e)^2*polylog(2,
-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^2/(a^2+b^2)^(1/2)+3*b*f*(f*x+e)^2*p
olylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^2/(a^2+b^2)^(1/2)+6*b*f^2*
(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^3/(a^2+b^2)^(1/2)
-6*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^3/(a^2+b
^2)^(1/2)-6*b*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^4/(a^2+
b^2)^(1/2)+6*b*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^4/(a^2
+b^2)^(1/2)
```

3.238.2 Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.21

$$\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{d^3(e+fx)^3\log\left(1-e^{c+dx}\right)-d^3(e+fx)^3\log\left(1+e^{c+dx}\right)-3f\left(d^2(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)-2df(e\right.}{}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(d^3*(e + f*x)^3*Log[1 - E^(c + d*x)] - d^3*(e + f*x)^3*Log[1 + E^(c + d*x
)] - 3*f*(d^2*(e + f*x)^2*PolyLog[2, -E^(c + d*x)] - 2*d*f*(e + f*x)*PolyL
og[3, -E^(c + d*x)] + 2*f^2*PolyLog[4, -E^(c + d*x)]) + 3*f*(d^2*(e + f*x)
^2*PolyLog[2, E^(c + d*x)] - 2*d*f*(e + f*x)*PolyLog[3, E^(c + d*x)] + 2*f
^2*PolyLog[4, E^(c + d*x)]) + (b*(2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sq
rt[a^2 + b^2]] - 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])] - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^3
*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e^2*f*x*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-
a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))] + 6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a
^2 + b^2])] + 6*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 - 6*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*f^
3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*f^3*PolyLog[4
, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*f^3*PolyLog[4, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2])/(a*d^4)
```

3.238.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.45 (sec) , antiderivative size = 558, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {6109, 3042, 26, 3803, 25, 2694, 27, 2620, 3011, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)^3\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int i(e+fx)^3\csc(ic+idx)dx}{a}-\frac{b\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}-\frac{b\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a}$$

$$\Big\downarrow \text{ 3803}$$

$$-\frac{2b\int -\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2b\int \frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{ 2694}$$

$$\frac{2b\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2b\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}$$

$$\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{ 3011}$$

3.238. $\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+c}}{a-\sqrt{a^2}}\right.}{bd}\right.}{}\right.}{a}$$

$$\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \ 4670$$

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+c}}{a-\sqrt{a^2}}\right.}{bd}\right.}{}\right.}{a}$$

$$\frac{i\left(\frac{3if\int(e+fx)^2\log(1-e^{c+dx})dx}{d}-\frac{3if\int(e+fx)^2\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^3\text{arctanh}(e^{c+dx})}{d}\right)}{a}$$

$$\Big\downarrow \ 3011$$

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+c}}{a-\sqrt{a^2}}\right.}{bd}\right.}{}\right.}{a}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}(2,e^{c+dx})}{d}\right)}{d}\right.}{a}$$

$$\Big\downarrow \ 7163$$

3.238. $\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}}{bd}\right.}{2\sqrt{a^2+b^2}}\right.$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-e^{c+dx}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{d}-\frac{f\int\operatorname{P}}{}\right.}{d}\right.}{}\right.}{a}$$

$$\Big\downarrow \text{2720}$$

3.238. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-e^{c+dx}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,e\right.}{d}\right.}{}\right.}{}\right.}{a}$$

$$\Big\downarrow \; 7143$$

3.238. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right.}{2\sqrt{a^2+b^2}}\right.$$

$$\frac{i\left(\frac{2i(e+fx)^3\operatorname{arctanh}(e^{c+dx})}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}(3,-e^{c+dx})}{d}-\frac{f\operatorname{PolyLog}(4,-e^{c+dx})}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(2f\left((e+\right.\right.}{}\right.}{a}$$

input
```
Int[((e + f*x)^3*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.238. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(I*(((2*I)*(e + f*x)^3*ArcTanh[E^(c + d*x)])/d - ((3*I)*f*(-(((e + f*x)^2*
PolyLog[2, -E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, -E^(c + d*x)])/
d - (f*PolyLog[4, -E^(c + d*x)])/d^2))/d))/d + ((3*I)*f*(-(((e + f*x)^2*Po
lyLog[2, E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, E^(c + d*x)])/d -
(f*PolyLog[4, E^(c + d*x)])/d^2))/d))/d))/a + (2*b*(-1/2*(b*(((e + f*x)^3*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)
^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e +
f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[
4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/Sqrt[a^2 +
 b^2] + (b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(
b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2
))/d))/(b*d)))/(2*Sqrt[a^2 + b^2])))/a
```

3.238.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

3.238. $\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.238. $\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.238.4 Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.238.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1645 vs. $2(554) = 1108$.

Time = 0.29 (sec) , antiderivative size = 1645, normalized size of antiderivative = 2.72

$$\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(6*b^2*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*
b^2*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c
) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(a^2
 + b^2)*f^3*polylog(4, cosh(d*x + c) + sinh(d*x + c)) + 6*(a^2 + b^2)*f^3*
polylog(4, -cosh(d*x + c) - sinh(d*x + c)) + 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^
2*e*f^2*x + b^2*d^2*e^2*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) +
a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
) - b)/b + 1) - 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*sq
rt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^2*d^3*e^3
- 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*sqrt((a^2 + b^2)/b^
2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) +
 2*a) + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3
)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sq
rt((a^2 + b^2)/b^2) + 2*a) + (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^
2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*sqrt(
(a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^2*d^3*f^3*x^3 + 3*
b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d...
```

3.238.6 Sympy [F]

$$\int \frac{(e+fx)^3 \mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3\,\mathrm{csch}\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)**3*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.238.7 Maxima [F]

$$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \operatorname{csch}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^3*(b*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + s
qrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + log(e^(-d*x - c) + 1)/(a*d) - log
(e^(-d*x - c) - 1)/(a*d)) - 3*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x +
c)))*e^2*f/(a*d^2) + 3*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*e^
2*f/(a*d^2) - 3*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c))
- 2*polylog(3, -e^(d*x + c)))*e*f^2/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c)
+ 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*e*f^2/(a*d^3)
 - (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*p
olylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x
^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3,
 e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*f^3/(a*d^4) - integrate(2*(b*f^
3*x^3*e^c + 3*b*e*f^2*x^2*e^c + 3*b*e^2*f*x*e^c)*e^(d*x)/(a*b*e^(2*d*x + 2
*c) + 2*a^2*e^(d*x + c) - a*b), x)
```

3.238.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{{\left(e+f\,x\right)}^3}{\mathrm{sinh}\left(c+d\,x\right)\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^3/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.239 $\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.239.1 Optimal result

Integrand size = 26, antiderivative size = 433

$$\begin{aligned}\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{2(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{ad}-\frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}\\
&+\frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}\\
&-\frac{2f(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}\\
&+\frac{2f(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}\\
&-\frac{2bf(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}\\
&+\frac{2bf(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}\\
&+\frac{2f^2\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}-\frac{2f^2\,\text{PolyLog}\left(3,e^{c+dx}\right)}{ad^3}\\
&+\frac{2bf^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}\\
&-\frac{2bf^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^3}\end{aligned}$$

output
```
-2*(f*x+e)^2*arctanh(exp(d*x+c))/a/d-2*f*(f*x+e)*polylog(2,-exp(d*x+c))/a/
d^2+2*f*(f*x+e)*polylog(2,exp(d*x+c))/a/d^2+2*f^2*polylog(3,-exp(d*x+c))/a
/d^3-2*f^2*polylog(3,exp(d*x+c))/a/d^3-b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/a/d/(a^2+b^2)^(1/2)+b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+
b^2)^(1/2)))/a/d/(a^2+b^2)^(1/2)-2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/a/d^2/(a^2+b^2)^(1/2)+2*b*f*(f*x+e)*polylog(2,-b*exp(d*x
+c)/(a+(a^2+b^2)^(1/2)))/a/d^2/(a^2+b^2)^(1/2)+2*b*f^2*polylog(3,-b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/a/d^3/(a^2+b^2)^(1/2)-2*b*f^2*polylog(3,-b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^3/(a^2+b^2)^(1/2)
```

3.239.2 Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.05

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{d^2(e+fx)^2\log\left(1-e^{c+dx}\right)-d^2(e+fx)^2\log\left(1+e^{c+dx}\right)-2df(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)+2df(e+f}{}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(d^2*(e + f*x)^2*Log[1 - E^(c + d*x)] - d^2*(e + f*x)^2*Log[1 + E^(c + d*x
)] - 2*d*f*(e + f*x)*PolyLog[2, -E^(c + d*x)] + 2*d*f*(e + f*x)*PolyLog[2,
 E^(c + d*x)] + 2*f^2*PolyLog[3, -E^(c + d*x)] - 2*f^2*PolyLog[3, E^(c + d
*x)] + (b*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*d^2*
e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2
+ b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))] + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*f^2*P
olyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2])/(a*
d^3)
```

3.239.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.94, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {6109, 3042, 26, 3803, 25, 2694, 27, 2620, 3011, 2720, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)^2\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int i(e+fx)^2\csc(ic+idx)dx}{a}-\frac{b\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}-\frac{b\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a}$$

$$\Big\downarrow \text{3803}$$

$$-\frac{2b\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{2694}$$

$$\frac{2b\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}$$

$$\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{}}{a}$$

$$\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{2720}$$

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{b}{a-}\right.}{bd}\right.}{}}{a}$$

$$\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{4670}$$

3.239. $\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{b}{a-}\right.}{bd}\right.}{}\right.}{a}$$

$$\frac{i\left(\frac{2if\int(e+fx)\log(1-e^{c+dx})dx}{d}-\frac{2if\int(e+fx)\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^2\operatorname{arctanh}(e^{c+dx})}{d}\right)}{a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{b}{a-}\right.}{bd}\right.}{}\right.}{a}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}+\frac{2i(e+fx)^2\operatorname{arc}}{d}\right.}{a}$$

$$\Big\downarrow \text{2720}$$

3.239. $\int \frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{b}{a-}\right.}{bd}\right.}{}\right.}{a}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f}{}\right.}{}\right.}{a}$$

$$\frac{i\left(\frac{2i(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{2if\left(\frac{f\text{PolyLog}\left(3,-e^{c+dx}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\text{PolyLog}\left(3,e^{c+dx}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right.}{d}\right.}{d}\right.}{a}$$

input
```
Int[((e + f*x)^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.239. $\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(I*(((2*I)*(e + f*x)^2*ArcTanh[E^(c + d*x)])/d - ((2*I)*f*(-(((e + f*x)*Po
lyLog[2, -E^(c + d*x)])/d) + (f*PolyLog[3, -E^(c + d*x)])/d^2))/d + ((2*I)
*f*(-(((e + f*x)*PolyLog[2, E^(c + d*x)])/d) + (f*PolyLog[3, E^(c + d*x)])
/d^2))/d))/a + (2*b*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2]))])/d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/a
```

3.239.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

3.239. $\int \frac{(e+fx)^2 \text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.239. $\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.239.4 Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.239.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. $2(394) = 788$.

Time = 0.29 (sec) , antiderivative size = 1096, normalized size of antiderivative = 2.53

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*b^2*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x
+ c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*b
^2*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*(a^2
+ b^2)*f^2*polylog(3, cosh(d*x + c) + sinh(d*x + c)) + 2*(a^2 + b^2)*f^2*p
olylog(3, -cosh(d*x + c) - sinh(d*x + c)) - 2*(b^2*d*f^2*x + b^2*d*e*f)*sq
rt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^2*d*f^2*
x + b^2*d*e*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x +
 c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1
) + (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(
2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) -
 (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(2*b
*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b
^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*sqrt((a^2
+ b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^2*d^2*f^2*x^2 + 2*b^2*d
^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh
(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b) + 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*dil...
```

3.239.6 Sympy [F]

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.239.7 Maxima [F]

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\operatorname{csch}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^2*(b*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + s
qrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + log(e^(-d*x - c) + 1)/(a*d) - log
(e^(-d*x - c) - 1)/(a*d)) - 2*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x +
c)))*e*f/(a*d^2) + 2*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*e*f/
(a*d^2) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*po
lylog(3, -e^(d*x + c)))*f^2/(a*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d
*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^2/(a*d^3) - integrate
(2*(b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^(d*x)/(a*b*e^(2*d*x + 2*c) + 2*a^2*e^
(d*x + c) - a*b), x)
```

3.239.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{{\left(e+f\,x\right)}^2}{\mathrm{sinh}\left(c+d\,x\right)\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^2/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.240 $\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.240.1 Optimal result

Integrand size = 24, antiderivative size = 261

$$\begin{aligned}\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx =& -\frac{2(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} \\ &+ \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} \\ &- \frac{f\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} + \frac{f\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} \\ &- \frac{bf\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2} + \frac{bf\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d^2}\end{aligned}$$

output

```
-2*(f*x+e)*arctanh(exp(d*x+c))/a/d-f*polylog(2,-exp(d*x+c))/a/d^2+f*polylo
g(2,exp(d*x+c))/a/d^2-b*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d
/(a^2+b^2)^(1/2)+b*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d/(a^2
+b^2)^(1/2)-b*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^2/(a^2+b^
2)^(1/2)+b*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^2/(a^2+b^2)^
(1/2)
```

3.240.2 Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.02

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{d(e+fx)\left(\log\left(1-e^{c+dx}\right)-\log\left(1+e^{c+dx}\right)\right)-f\,\text{PolyLog}\left(2,-e^{c+dx}\right)+f\,\text{PolyLog}\left(2,e^{c+dx}\right)-\frac{b\left(-2de\,\text{ar}\right.}{}}{}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(d*(e + f*x)*(Log[1 - E^(c + d*x)] - Log[1 + E^(c + d*x)]) - f*PolyLog[2,
-E^(c + d*x)] + f*PolyLog[2, E^(c + d*x)] - (b*(-2*d*e*ArcTanh[(a + b*E^(c
 + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b
^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c +
 d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(
c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))]))/Sqrt[a^2 + b^2])/(a*d^2)
```

3.240.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {6109, 3042, 26, 3803, 25, 2694, 27, 2620, 2715, 2838, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6109$$

$$\frac{\int (e+fx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int i(e+fx)\csc(ic+idx)dx}{a}-\frac{b\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-\frac{b\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a}$$

$$\Big\downarrow \; 3803$$

$$-\frac{2b\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{2b\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \; 2694$$

$$\frac{2b\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \; 27$$

$$\frac{2b\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \; 2620$$

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \; 2715$$

$$\frac{2b\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}$$

3.240. $\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{2838}$$

$$\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{4670}$$

$$\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\left(\frac{if\int \log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{a}$$

3.240. $\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(I*(((2*I)*(e + f*x)*ArcTanh[E^(c + d*x)])/d + (I*f*PolyLog[2, -E^(c + d*x
)])/d^2 - (I*f*PolyLog[2, E^(c + d*x)])/d^2))/a + (2*b*(-1/2*(b*(((e + f*x
)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*((
(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyL
og[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b
^2])))/a
```

3.240.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.240. $\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

3.240.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(238) = 476$.

Time = 2.25 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.04

method	result
risch	$\frac{e\ln(\mathrm{e}^{dx+c}-1)}{ad}+\frac{2eb\,\operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{da\sqrt{a^2+b^2}}-\frac{e\ln(\mathrm{e}^{dx+c}+1)}{ad}-\frac{f\,\mathrm{dilog}(\mathrm{e}^{dx+c})}{d^2a}-\frac{f\ln(\mathrm{e}^{dx+c}+1)x}{ad}-\frac{f\,\mathrm{dilog}(\mathrm{e}^{dx+c}+1)}{d^2a}-$

input
```
int((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

3.240. $\int \frac{(e+fx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
1/a/d*e*ln(exp(d*x+c)-1)+2/d*e/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*
x+c)+2*a)/(a^2+b^2)^(1/2))-1/a/d*e*ln(exp(d*x+c)+1)-1/d^2*f/a*dilog(exp(d*
x+c))-1/a/d*f*ln(exp(d*x+c)+1)*x-1/d^2*f/a*dilog(exp(d*x+c)+1)-1/d*f/a*b/(
a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*
x+1/d*f/a*b/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^
2)^(1/2)))*x-1/d^2*f/a*b/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2*f/a*b/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(
a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2*f/a*b/(a^2+b^2)^(1/2)*dilog
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2*f/a*b/(a^2+
b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/a
/d^2*c*f*ln(exp(d*x+c)-1)-2/d^2*c*f/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*e
xp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

3.240.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(234) = 468$.

Time = 0.28 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.49

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{b^2 f\sqrt{\frac{a^2+b^2}{b^2}}\operatorname{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)-b^2 f\sqrt{\frac{a^2+b^2}{b^2}}\operatorname{Li}_2\left(\frac{a\cosh(dx+c)}{}\right.}{}$$

input
```
integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(b^2*f*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - b^2
*f*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (a^2 + b
^2)*f*dilog(cosh(d*x + c) + sinh(d*x + c)) + (a^2 + b^2)*f*dilog(-cosh(d*x
 + c) - sinh(d*x + c)) - (b^2*d*e - b^2*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*b
*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b
^2*d*e - b^2*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d
*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^2*d*f*x + b^2*c*f)*sqrt((a
^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^2*d*f*x + b^2*c*f)*s
qrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + ((a^2 + b^2)*d*f*
x + (a^2 + b^2)*d*e)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - ((a^2 + b^2)
*d*e - (a^2 + b^2)*c*f)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - ((a^2 + b
^2)*d*f*x + (a^2 + b^2)*c*f)*log(-cosh(d*x + c) - sinh(d*x + c) + 1))/((a^
3 + a*b^2)*d^2)
```

3.240.6 Sympy [F]

$$\int \frac{(e+fx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.240.7 Maxima [F]

$$\int \frac{(e+fx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\operatorname{csch}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e*(b*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqr
t(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + log(e^(-d*x - c) + 1)/(a*d) - log(e
^(-d*x - c) - 1)/(a*d)) + 2*f*integrate(2*x/((b*(e^(d*x + c) - e^(-d*x - c
)) + 2*a)*(e^(d*x + c) - e^(-d*x - c))), x)
```

3.240.8 Giac [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*csch(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\sinh\,(c+d\,x)\,\,(a+b\sinh\,(c+d\,x))}\,dx$$

input
```
int((e + f*x)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.241 $\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.241.1 Optimal result

Integrand size = 19, antiderivative size = 64

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx=-\frac{\operatorname{arctanh}(\cosh(c+dx))}{ad}+\frac{2b\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$

output
```
-arctanh(cosh(d*x+c))/a/d+2*b*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^
(1/2))/a/d/(a^2+b^2)^(1/2)
```

3.241.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.28

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx =\frac{-\frac{2b\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

input
```
Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
((-2*b*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2
] - Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)/2]])/(a*d)
```

3.241.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 26, 3226, 26, 3042, 26, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3226}$$

$$i\left(\frac{ib\int \frac{1}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i\text{csch}(c+dx)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{ib\int \frac{1}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int \text{csch}(c+dx)dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{ib\int \frac{1}{a-ib\sin(ic+idx)}dx}{a}-\frac{i\int i\csc(ic+idx)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{ib\int \frac{1}{a-ib\sin(ic+idx)}dx}{a}+\frac{\int \csc(ic+idx)dx}{a}\right)$$

$$\Big\downarrow \text{3139}$$

$$i\left(\frac{2b\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}+\frac{\int \csc(ic+idx)dx}{a}\right)$$

$$\Big\downarrow \text{1083}$$

$$i\left(\frac{\int \csc(ic+idx)dx}{a}-\frac{4b\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{ad}\right)$$

$$\Big\downarrow \text{217}$$

$$i\left(\frac{\int \csc(ic+idx)dx}{a}+\frac{2ib\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}\right)$$

$$\Big\downarrow \text{4257}$$

$$i\left(\frac{2ib\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}+\frac{i\text{arctanh}(\cosh(c+dx))}{ad}\right)$$

input
```
Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
I*((I*ArcTanh[Cosh[c + d*x]])/(a*d) + ((2*I)*b*ArcTanh[Tanh[(c + d*x)/2]/(
2*Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d))
```

3.241.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.241. $\int \frac{\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.241.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{-\frac{2b\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$	63
default	$\frac{-\frac{2b\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$	63
risch	$\frac{b\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,da}-\frac{b\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,da}-\frac{\ln\left(\mathrm{e}^{dx+c}+1\right)}{da}+\frac{\ln\left(\mathrm{e}^{dx+c}-1\right)}{da}$	152

input
```
int(csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2
+b^2)^(1/2))+1/a*ln(tanh(1/2*d*x+1/2*c)))
```

3.241.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(61) = 122$.

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.48

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\sqrt{a^2+b^2}b\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)+2\,\sqrt{a^2+b^2}(b\cosh(dx+c)+}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right.}{(}$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(sqrt(a^2 + b^2)*b*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*
cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) +
2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c
)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh
(d*x + c) - b)) - (a^2 + b^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a^
2 + b^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1))/((a^3 + a*b^2)*d)
```

3.241.6 Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.241.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.75

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{b\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}ad} - \frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-b*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) - log(e^(-d*x - c) + 1)/(a*d) + log(e^(-
d*x - c) - 1)/(a*d)
```

3.241.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.59

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{b\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a}+\frac{\log\left(e^{(dx+c)}+1\right)}{a}-\frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{a}}{d}$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(b*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
 + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(e^(d*x + c) + 1)/a
- log(abs(e^(d*x + c) - 1))/a)/d
```

3.241.9 Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 347, normalized size of antiderivative = 5.42

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\ln\left(32\,a-32\,a\,e^{d\,x}\,e^{c}\right)}{a\,d}-\frac{\ln\left(32\,a+32\,a\,e^{d\,x}\,e^{c}\right)}{a\,d}$$
$$+\frac{b\,\ln\left(128\,a^5\,e^{d\,x}\,e^{c}-64\,a^2\,b^3-64\,a^4\,b+32\,a\,b^3\,\sqrt{a^2+b^2}+64\,a^3\,b\,\sqrt{a^2+b^2}+160\,a^3\,b^2\,e^{d\,x}\,e^{c}-128\,a^4\,e\right.}{d\,a^3+d\,a\,b^2}$$
$$-\frac{b\,\ln\left(64\,a^4\,b+64\,a^2\,b^3-128\,a^5\,e^{d\,x}\,e^{c}+32\,a\,b^3\,\sqrt{a^2+b^2}+64\,a^3\,b\,\sqrt{a^2+b^2}-160\,a^3\,b^2\,e^{d\,x}\,e^{c}-128\,a^4\,e\right.}{d\,a^3+d\,a\,b^2}$$

input
```
int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
log(32*a - 32*a*exp(d*x)*exp(c))/(a*d) - log(32*a + 32*a*exp(d*x)*exp(c))/
(a*d) + (b*log(128*a^5*exp(d*x)*exp(c) - 64*a^2*b^3 - 64*a^4*b + 32*a*b^3*
(a^2 + b^2)^(1/2) + 64*a^3*b*(a^2 + b^2)^(1/2) + 160*a^3*b^2*exp(d*x)*exp(
c) - 128*a^4*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 32*a*b^4*exp(d*x)*exp(c)
- 96*a^2*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^3*d
+ a*b^2*d) - (b*log(64*a^4*b + 64*a^2*b^3 - 128*a^5*exp(d*x)*exp(c) + 32*a
*b^3*(a^2 + b^2)^(1/2) + 64*a^3*b*(a^2 + b^2)^(1/2) - 160*a^3*b^2*exp(d*x)
*exp(c) - 128*a^4*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) - 32*a*b^4*exp(d*x)*ex
p(c) - 96*a^2*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a
^3*d + a*b^2*d)
```

3.241. $\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.242 $\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.242.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.242.2 Mathematica [N/A]

Not integrable

Time = 4.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Csch[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Csch[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.242.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.242.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.242.4 Maple [N/A] (verified)

Not integrable

Time = 0.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.242.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(csch(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.242.6 Sympy [N/A]

Not integrable

Time = 5.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.242.7 Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(csch(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.242.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.242.9 Mupad [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.243 $\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.243.1 Optimal result

Integrand size = 28, antiderivative size = 745

$$\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^3}{ad}+\frac{2b(e+fx)^3\operatorname{arctanh}(e^{c+dx})}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d} - \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d} + \frac{3f(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad^2} + \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2} - \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2} + \frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^2} - \frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^2} + \frac{3f^2(e+fx)\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3} - \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{a^2d^3} + \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{a^2d^3} - \frac{6b^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^3} + \frac{6b^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^3} - \frac{3f^3\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^4}+\frac{6bf^3\operatorname{PolyLog}\left(4,-e^{c+dx}\right)}{a^2d^4} - \frac{6bf^3\operatorname{PolyLog}\left(4,e^{c+dx}\right)}{a^2d^4}+\frac{6b^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^4} - \frac{6b^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^4}$$

output
```
-(f*x+e)^3/a/d+2*b*(f*x+e)^3*arctanh(exp(d*x+c))/a^2/d-(f*x+e)^3*coth(d*x+
c)/a/d+3*f*(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a/d^2+3*b*f*(f*x+e)^2*polylog(2,
-exp(d*x+c))/a^2/d^2-3*b*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a^2/d^2+3*f^2*(
f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^3-6*b*f^2*(f*x+e)*polylog(3,-exp(d*x+
c))/a^2/d^3+6*b*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a^2/d^3-3/2*f^3*polylog(
3,exp(2*d*x+2*c))/a/d^4+6*b*f^3*polylog(4,-exp(d*x+c))/a^2/d^4-6*b*f^3*pol
ylog(4,exp(d*x+c))/a^2/d^4+b^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/a^2/d/(a^2+b^2)^(1/2)-b^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/a^2/d/(a^2+b^2)^(1/2)+3*b^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)-3*b^2*f*(f*x+e)^2*polylog(2,-b*e
xp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)-6*b^2*f^2*(f*x+e)*p
olylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^3/(a^2+b^2)^(1/2)+6*b^2*
f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^3/(a^2+b^2)
^(1/2)+6*b^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^4/(a^2
+b^2)^(1/2)-6*b^2*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^4
/(a^2+b^2)^(1/2)
```

3.243.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1493 vs. 2(745) = 1490.

Time = 8.54 (sec) , antiderivative size = 1493, normalized size of antiderivative = 2.00

$$\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-(d^3*e^2*(-1 + E^(2*c))*(b*d*e - 3*a*f)*x) + d^3*e^2*(-1 + E^(2*c))*(b
*d*e + 3*a*f)*x + 2*a*d^3*(e + f*x)^3 + 3*d^2*e*(-1 + E^(2*c))*f*(b*d*e -
2*a*f)*x*Log[1 - E^(-c - d*x)] + 3*d^2*(-1 + E^(2*c))*f^2*(b*d*e - a*f)*x^
2*Log[1 - E^(-c - d*x)] + b*d^3*(-1 + E^(2*c))*f^3*x^3*Log[1 - E^(-c - d*x
)] - 3*d^2*e*(-1 + E^(2*c))*f*(b*d*e + 2*a*f)*x*Log[1 + E^(-c - d*x)] - 3*
d^2*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*x^2*Log[1 + E^(-c - d*x)] - b*d^3*(-1
 + E^(2*c))*f^3*x^3*Log[1 + E^(-c - d*x)] + d^2*e^2*(-1 + E^(2*c))*(b*d*e
- 3*a*f)*Log[1 - E^(c + d*x)] - d^2*e^2*(-1 + E^(2*c))*(b*d*e + 3*a*f)*Log
[1 + E^(c + d*x)] + 3*d*e*(-1 + E^(2*c))*f*(b*d*e + 2*a*f)*PolyLog[2, -E^(
-c - d*x)] + 6*d*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*x*PolyLog[2, -E^(-c - d*
x)] + 3*b*d^2*(-1 + E^(2*c))*f^3*x^2*PolyLog[2, -E^(-c - d*x)] - 3*d*e*(-1
 + E^(2*c))*f*(b*d*e - 2*a*f)*PolyLog[2, E^(-c - d*x)] - 6*d*(-1 + E^(2*c)
)*f^2*(b*d*e - a*f)*x*PolyLog[2, E^(-c - d*x)] - 3*b*d^2*(-1 + E^(2*c))*f^
3*x^2*PolyLog[2, E^(-c - d*x)] + 6*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*PolyLo
g[3, -E^(-c - d*x)] + 6*b*d*(-1 + E^(2*c))*f^3*x*PolyLog[3, -E^(-c - d*x)]
 + 6*(-1 + E^(2*c))*f^2*(-(b*d*e) + a*f)*PolyLog[3, E^(-c - d*x)] - 6*b*d*
(-1 + E^(2*c))*f^3*x*PolyLog[3, E^(-c - d*x)] + 6*b*(-1 + E^(2*c))*f^3*Pol
yLog[4, -E^(-c - d*x)] - 6*b*(-1 + E^(2*c))*f^3*PolyLog[4, E^(-c - d*x)])/
(a^2*d^4*(-1 + E^(2*c)))) + (b^2*(-2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/S
qrt[a^2 + b^2]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +...
```

3.243.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 4.43 (sec) , antiderivative size = 715, normalized size of antiderivative = 0.96, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6109, 3042, 25, 4672, 26, 3042, 26, 4201, 2620, 3011, 2720, 6109, 3042, 26, 3803, 25, 2694, 27, 2620, 3011, 4670, 3011, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)^3\text{csch}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -(e+fx)^3\csc(ic+idx)^2dx}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int (e+fx)^3\csc(ic+idx)^2dx}{a}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\coth(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3f\int (e+fx)^2\coth(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int (e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \text{3011}$$

3.243. $\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int\frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-$$
$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}}{d}}{a}$$

$$\Big\downarrow \; 2720$$

$$-\frac{b\int\frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-$$
$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$\Big\downarrow \; 6109$$

$$-\frac{b\left(\frac{\int(e+fx)^3\text{csch}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-$$
$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\left(\frac{\int i(e+fx)^3\csc(ic+idx)dx}{a}-\frac{b\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-$$
$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\left(\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}-\frac{b\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-$$
$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

3.243. $\int\frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 3803$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+\cdots}\right)}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}-\frac{b\left(-\frac{2b\int-\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+\cdots}\right)}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}-\frac{b\left(\frac{2b\int\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-$$

$$\Big\downarrow \; 2694$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+\cdots}\right)}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}-\frac{b\left(\frac{2b\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+\cdots}\right)}{2d}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{a}-\frac{b\left(\frac{2b\left(\frac{b\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-$$

3.243. $\displaystyle\int\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$\downarrow$ 2620

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right.}{2\sqrt{a^2+b^2}}}{a}\right.}{a}$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+\cdots}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$\downarrow$ 3011

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c\cdots}}{a-\sqrt{\cdots}}\right.}{bd}\right.}{\cdots}}{a}\right.}{a}$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+\cdots}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$\downarrow$ 4670

3.243. $\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c}}{a-\sqrt{}}\right.}{bd}\right.}{}\right)}{a}}{}$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}$$

$\downarrow$ 3011

3.243. $\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c}\cdots}{a-\sqrt{\cdots}}\cdots\right)}{bd}\cdots\right.}{\cdots}\right)}{a}\cdots\right)}{\cdots}$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+\cdots}\right)}{2d}\cdots\right)}{d}\cdots\right)\cdots\right)}{d}\cdots}{a}$$

$$\Big\downarrow \; 7143$$

3.243. $\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c}\cdots}{a-\sqrt{\cdots}}\cdots\right)}{bd}\cdots\right.}{\cdots}\right.}{a}\cdots\right.}{\cdots}$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\,\text{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\downarrow \color{blue}{7163}$$

3.243. $\displaystyle\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+\cdots}}{a+\sqrt{a^2\cdots}}\right)}{d}\cdots\right)}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\,\text{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \ 2720$$

3.243. $\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{\frac{\coth(c+dx)(e+fx)^3}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\,\text{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$b\left(\frac{i\left(\frac{2i\text{arctanh}\left(e^{c+dx}\right)(e+fx)^3}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{d}-\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(3,-e^{c+dx}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\right.}{a}\right.$$

$$\downarrow 7143$$

3.243. $\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right.}{d}}{bd}}{2\sqrt{a^2+b^2}}}{\quad}\right.}{\quad}$$

$$\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\,\text{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

input
```
Int[((e + f*x)^3*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

3.243. $\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
-((((e + f*x)^3*Coth[c + d*x])/d + ((3*I)*f*(((-1/3*I)*(e + f*x)^3)/f + (2
*I)*(((e + f*x)^2*Log[1 + E^(2*c - I*Pi + 2*d*x)])/(2*d) - (f*(-1/2*((e +
f*x)*PolyLog[2, -E^(2*c - I*Pi + 2*d*x)])/d + (f*PolyLog[3, -E^(2*c - I*Pi
 + 2*d*x)])/(4*d^2)))/d)))/d)/a) - (b*((I*(((2*I)*(e + f*x)^3*ArcTanh[E^(c
 + d*x)])/d - ((3*I)*f*(-(((e + f*x)^2*PolyLog[2, -E^(c + d*x)])/d) + (2*f
*(((e + f*x)*PolyLog[3, -E^(c + d*x)])/d - (f*PolyLog[4, -E^(c + d*x)])/d^
2))/d))/d + ((3*I)*f*(-(((e + f*x)^2*PolyLog[2, E^(c + d*x)])/d) + (2*f*((
(e + f*x)*PolyLog[3, E^(c + d*x)])/d - (f*PolyLog[4, E^(c + d*x)])/d^2))/d
))/d))/a + (2*b*(-1/2*(b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/d^2))/d))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^3*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog
[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyL
og[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(2*Sqrt[a^2 + b^2]))
)/a))/a
```

3.243.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.243. $\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.243.4 Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.243.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6416 vs. 2(691) = 1382.

Time = 0.39 (sec) , antiderivative size = 6416, normalized size of antiderivative = 8.61

$$\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

3.243.6 Sympy [F]

$$\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**3*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*csch(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.243.7 Maxima [F]

$$\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\operatorname{csch}(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
e^3*(b^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a +
sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2*d) + b*log(e^(-d*x - c) + 1)/(a^2*d
) - b*log(e^(-d*x - c) - 1)/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d)) - 6*
e^2*f*x/(a*d) + 3*e^2*f*log(e^(d*x + c) + 1)/(a*d^2) + 3*e^2*f*log(e^(d*x
+ c) - 1)/(a*d^2) - 2*(f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x)/(a*d*e^(2*d*x +
2*c) - a*d) + (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)
) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b*f^3/(a^
2*d^4) - (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6
*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b*f^3/(a^2*d^4)
+ 3*(b*d*e^2*f + 2*a*e*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)
))/(a^2*d^3) - 3*(b*d*e^2*f - 2*a*e*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilo
g(e^(d*x + c)))/(a^2*d^3) + 3*(b*d*e*f^2 + a*f^3)*(d^2*x^2*log(e^(d*x + c)
 + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^2*d^4)
- 3*(b*d*e*f^2 - a*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*
x + c)) - 2*polylog(3, e^(d*x + c)))/(a^2*d^4) - 1/4*(b*d^4*f^3*x^4 + 4*(b
*d*e*f^2 + a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f + 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^
4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 - a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f -
 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) + integrate(2*(b^2*f^3*x^3*e^c + 3*b^2*e*
f^2*x^2*e^c + 3*b^2*e^2*f*x*e^c)*e^(d*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^
(d*x + c) - a^2*b), x)
```

3.243.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^3/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.244 $\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.244.1 Optimal result

Integrand size = 28, antiderivative size = 535

$$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^2}{ad}+\frac{2b(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad}+\frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d} - \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d} + \frac{2f(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad^2} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2} - \frac{2bf(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2} + \frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^2} - \frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^2} + \frac{f^2\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3} - \frac{2bf^2\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{a^2d^3} + \frac{2bf^2\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{a^2d^3} - \frac{2b^2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^3} + \frac{2b^2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d^3}$$

output
```
-(f*x+e)^2/a/d+2*b*(f*x+e)^2*arctanh(exp(d*x+c))/a^2/d-(f*x+e)^2*coth(d*x+
c)/a/d+2*f*(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d^2+2*b*f*(f*x+e)*polylog(2,-exp
(d*x+c))/a^2/d^2-2*b*f*(f*x+e)*polylog(2,exp(d*x+c))/a^2/d^2+f^2*polylog(2
,exp(2*d*x+2*c))/a/d^3-2*b*f^2*polylog(3,-exp(d*x+c))/a^2/d^3+2*b*f^2*poly
log(3,exp(d*x+c))/a^2/d^3+b^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/a^2/d/(a^2+b^2)^(1/2)-b^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/a^2/d/(a^2+b^2)^(1/2)+2*b^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)-2*b^2*f*(f*x+e)*polylog(2,-b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)-2*b^2*f^2*polylog(3,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^3/(a^2+b^2)^(1/2)+2*b^2*f^2*polylog(3
,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^3/(a^2+b^2)^(1/2)
```

3.244. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.244.2 Mathematica [A] (verified)

Time = 7.99 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.72

$$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{-d^2e(-1+e^{2c})(bde-2af)x+d^2e(-1+e^{2c})(bde+2af)x+2ad^2(e+fx)^2+2d(-1+e^{2c})f(bde-c}{}$$

$$+\frac{b^2\left(-2d^2e^2\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^2f^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2d^2efx\log}{}$$

$$+\frac{\operatorname{sech}\left(\frac{c}{2}\right)\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-e^2\sinh\left(\frac{dx}{2}\right)-2efx\sinh\left(\frac{dx}{2}\right)-f^2x^2\sinh\left(\frac{dx}{2}\right)\right)}{2ad}$$

$$+\frac{\operatorname{csch}\left(\frac{c}{2}\right)\operatorname{csch}\left(\frac{c}{2}+\frac{dx}{2}\right)\left(e^2\sinh\left(\frac{dx}{2}\right)+2efx\sinh\left(\frac{dx}{2}\right)+f^2x^2\sinh\left(\frac{dx}{2}\right)\right)}{2ad}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-(d^2*e*(-1 + E^(2*c))*(b*d*e - 2*a*f)*x) + d^2*e*(-1 + E^(2*c))*(b*d*e
 + 2*a*f)*x + 2*a*d^2*(e + f*x)^2 + 2*d*(-1 + E^(2*c))*f*(b*d*e - a*f)*x*L
og[1 - E^(-c - d*x)] + b*d^2*(-1 + E^(2*c))*f^2*x^2*Log[1 - E^(-c - d*x)]
- 2*d*(-1 + E^(2*c))*f*(b*d*e + a*f)*x*Log[1 + E^(-c - d*x)] - b*d^2*(-1 +
 E^(2*c))*f^2*x^2*Log[1 + E^(-c - d*x)] + d*e*(-1 + E^(2*c))*(b*d*e - 2*a*
f)*Log[1 - E^(c + d*x)] - d*e*(-1 + E^(2*c))*(b*d*e + 2*a*f)*Log[1 + E^(c
+ d*x)] + 2*(-1 + E^(2*c))*f*(b*d*e + a*f)*PolyLog[2, -E^(-c - d*x)] + 2*b
*d*(-1 + E^(2*c))*f^2*x*PolyLog[2, -E^(-c - d*x)] + 2*(-1 + E^(2*c))*f*(-(
b*d*e) + a*f)*PolyLog[2, E^(-c - d*x)] - 2*b*d*(-1 + E^(2*c))*f^2*x*PolyLo
g[2, E^(-c - d*x)] + 2*b*(-1 + E^(2*c))*f^2*PolyLog[3, -E^(-c - d*x)] - 2*
b*(-1 + E^(2*c))*f^2*PolyLog[3, E^(-c - d*x)])/(a^2*d^3*(-1 + E^(2*c)))) +
 (b^2*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f
*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b
*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b
^2])] - 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)] - 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*Poly
Log[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2*Sqrt[a^2 + b^2]*d^
3) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(e^2*Sinh[(d*x)/2]) - 2*e*f*x*Sin...
```

3.244.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.99 (sec) , antiderivative size = 523, normalized size of antiderivative = 0.98, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.893$, Rules used = {6109, 3042, 25, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838, 6109, 3042, 26, 3803, 25, 2694, 27, 2620, 3011, 2720, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)^2\text{csch}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -(e+fx)^2\csc(ic+idx)^2dx}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int (e+fx)^2\csc(ic+idx)^2dx}{a}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2if\int -i(e+fx)\coth(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int (e+fx)\coth(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int \log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{6109}$$

$$-\frac{b\left(\frac{\int (e+fx)^2\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(\frac{\int i(e+fx)^2\csc(ic+idx)dx}{a}-\frac{b\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

3.244. $\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\downarrow 26$$

$$-\frac{b\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-\frac{b\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\downarrow 3803$$

$$-\frac{b\left(-\frac{2b\int-\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\downarrow 25$$

$$-\frac{b\left(\frac{2b\int\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\downarrow 2694$$

$$-\frac{b\left(\frac{2b\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\downarrow 27$$

3.244. $\int\frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{2b\left(\frac{b\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}\right.}{a}$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \text{3011}$$

3.244. $\int\frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right.}{bd}\right.}{}\right)}{a}\right)}{a}$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \; 2720$$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(-\right.}{b}\right.}{}\right)}{a}\right)}{a}$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

3.244. $\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \ 4670$$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\cdots\right)}{b\cdots}\cdots\right.}{\cdots}\cdots\right)}{a}\cdots\right)}{\cdots}$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

$$\Big\downarrow \ 3011$$

3.244. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\cdots\right.}{b\cdots}\cdots\right.}{\cdots}}{a}\cdots\right.}{\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}}$$

$$\Big\downarrow \; 2720$$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\cdots\right.}{b\cdots}\cdots\right.}{\cdots}}{a}\cdots\right.}{\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}}$$

3.244. $\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 7143$$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\cdots\right)}{\cdots}\right)}{a}\cdots\right)}{\cdots}$$

$$\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}$$

input

```
Int[((e + f*x)^2*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output

```
-((((e + f*x)^2*Coth[c + d*x])/d + ((2*I)*f*(((-1/2*I)*(e + f*x)^2)/f + (2
*I)*(((e + f*x)*Log[1 + E^(2*c - I*Pi + 2*d*x)])/(2*d) + (f*PolyLog[2, -E^
(2*c - I*Pi + 2*d*x)])/(4*d^2))))/d)/a) - (b*((I*(((2*I)*(e + f*x)^2*ArcTa
nh[E^(c + d*x)])/d - ((2*I)*f*(-(((e + f*x)*PolyLog[2, -E^(c + d*x)])/d) +
 (f*PolyLog[3, -E^(c + d*x)])/d^2))/d + ((2*I)*f*(-(((e + f*x)*PolyLog[2,
E^(c + d*x)])/d) + (f*PolyLog[3, E^(c + d*x)])/d^2))/d))/a + (2*b*(-1/2*(b
*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (2*
f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) +
 (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/Sq
rt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/a))/a
```

3.244. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.244.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.244. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

3.244. $\int \frac{(e+fx)^2 \text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.244.4 Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.244.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3805 vs. 2(495) = 990.

Time = 0.33 (sec) , antiderivative size = 3805, normalized size of antiderivative = 7.11

$$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*(a^3 + a*b^2)*d^2*e^2 - 4*(a^3 + a*b^2)*c*d*e*f + 2*(a^3 + a*b^2)*c^2*
f^2 + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 +
a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)^2 + 4*((a^3 + a*b^2)
*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3
+ a*b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + 2*((a^3 + a*b^2)*d^2*f^2*x
^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c
^2*f^2)*sinh(d*x + c)^2 + 2*(b^3*d*f^2*x + b^3*d*e*f - (b^3*d*f^2*x + b^3*
d*e*f)*cosh(d*x + c)^2 - 2*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)*sinh(d*
x + c) - (b^3*d*f^2*x + b^3*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*
dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x +
 c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^3*d*f^2*x + b^3*d*e*f - (b^3
*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)^2 - 2*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d
*x + c)*sinh(d*x + c) - (b^3*d*f^2*x + b^3*d*e*f)*sinh(d*x + c)^2)*sqrt((a
^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*d^2*e^2 - 2*b
^3*c*d*e*f + b^3*c^2*f^2 - (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cos
h(d*x + c)^2 - 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)
*sinh(d*x + c) - (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sinh(d*x + c)
^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*
sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f...
```

3.244.6 Sympy [F]

$$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*csch(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.244.7 Maxima [F]

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{csch}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
e^2*(b^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a +
sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2*d) + b*log(e^(-d*x - c) + 1)/(a^2*d
) - b*log(e^(-d*x - c) - 1)/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d)) - 4*
e*f*x/(a*d) - 2*(f^2*x^2 + 2*e*f*x)/(a*d*e^(2*d*x + 2*c) - a*d) + 2*e*f*lo
g(e^(d*x + c) + 1)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/(a*d^2) + (d^2*x^2
*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x +
 c)))*b*f^2/(a^2*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*
x + c)) - 2*polylog(3, e^(d*x + c)))*b*f^2/(a^2*d^3) + 2*(b*d*e*f + a*f^2)
*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 2*(b*d*e*f -
 a*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) - 1/3*(
b*d^3*f^2*x^3 + 3*(b*d*e*f + a*f^2)*d^2*x^2)/(a^2*d^3) + 1/3*(b*d^3*f^2*x^
3 + 3*(b*d*e*f - a*f^2)*d^2*x^2)/(a^2*d^3) + integrate(2*(b^2*f^2*x^2*e^c
+ 2*b^2*e*f*x*e^c)*e^(d*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x + c) - a^
2*b), x)
```

3.244.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^2/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.245 $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.245.1 Optimal result

Integrand size = 26, antiderivative size = 306

$$\begin{aligned}\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{2b(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{a^2d}-\frac{(e+fx)\coth(c+dx)}{ad}\\
&\quad+\frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}\\
&\quad-\frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d}\\
&\quad+\frac{f\log(\sinh(c+dx))}{ad^2}+\frac{bf\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2}\\
&\quad-\frac{bf\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2}+\frac{b^2f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}\\
&\quad-\frac{b^2f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d^2}\end{aligned}$$

output
```
2*b*(f*x+e)*arctanh(exp(d*x+c))/a^2/d-(f*x+e)*coth(d*x+c)/a/d+f*ln(sinh(d*
x+c))/a/d^2+b*f*polylog(2,-exp(d*x+c))/a^2/d^2-b*f*polylog(2,exp(d*x+c))/a
^2/d^2+b^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d/(a^2+b^2)^
(1/2)-b^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d/(a^2+b^2)^(
1/2)+b^2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^
(1/2)-b^2*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)
^(1/2)
```

3.245.2 Mathematica [A] (verified)

Time = 5.91 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.14

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{ad(e+fx)\coth\left(\frac{1}{2}(c+dx)\right)-2\big(af(c+dx)+(af-bd(e+fx))\log\left(1-e^{-c-dx}\right)+(af+bd(e+fx)$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(a*d*(e + f*x)*Coth[(c + d*x)/2] - 2*(a*f*(c + d*x) + (a*f - b*d*(e +
 f*x))*Log[1 - E^(-c - d*x)] + (a*f + b*d*(e + f*x))*Log[1 + E^(-c - d*x)]
 - b*f*PolyLog[2, -E^(-c - d*x)] + b*f*PolyLog[2, E^(-c - d*x)]) - (2*b^2*
(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a +
b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] + a*d*(e + f
*x)*Tanh[(c + d*x)/2])/(a^2*d^2)
```

3.245.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.99, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.808$, Rules used = {6109, 3042, 25, 4672, 26, 3042, 26, 3956, 6109, 3042, 26, 3803, 25, 2694, 27, 2620, 2715, 2838, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)\text{csch}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -\left((e+fx)\csc(ic+idx)^2\right)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int (e+fx)\csc(ic+idx)^2dx}{a}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{if\int -i\coth(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int \coth(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}+\frac{if\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{6109}$$

$$-\frac{b\left(\frac{\int (e+fx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(\frac{\int i(e+fx)\csc(ic+idx)dx}{a}-\frac{b\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(\frac{i\int (e+fx)\csc(ic+idx)dx}{a}-\frac{b\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

3.245. $\displaystyle\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3803}$$

$$-\frac{b\left(-\frac{2b\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\left(\frac{2b\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{b\left(\frac{2b\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\left(\frac{2b\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)\ldots}{}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

3.245. $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d}{bd^2}\right.}{2\sqrt{a^2+b^2}}\right.}{a}\right.}{a}$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int(e+f}{}\right.}{a}$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\left(\frac{if\int 1}{}\right.}{}\right.}{a}$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$$\Big\downarrow \text{2715}$$

3.245. $\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\left(\frac{if\int e\right.}{}\right.}{a}$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

$\Big\downarrow$ 2838

$$-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\left(\frac{2i(e+\right.}{}\right.}{a}$$

$$\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}$$

input
```
Int[((e + f*x)*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((((e + f*x)*Coth[c + d*x])/d - (f*Log[(-I)*Sinh[c + d*x]])/d^2)/a) - (b*
((I*(((2*I)*(e + f*x)*ArcTanh[E^(c + d*x)])/d + (I*f*PolyLog[2, -E^(c + d*
x)])/d^2 - (I*f*PolyLog[2, E^(c + d*x)])/d^2))/a + (2*b*(-1/2*(b*(((e + f*
x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -
((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(
((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 +
b^2])))/a))/a
```

3.245. $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.245.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.245. $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

3.245.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(283) = 566$.

Time = 1.62 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.05

method	result
risch	$-\frac{2(fx+e)}{da\left(e^{2dx+2c}-1\right)}-\frac{be\ln\left(e^{dx+c}-1\right)}{a^2d}+\frac{be\ln\left(e^{dx+c}+1\right)}{a^2d}+\frac{bcf\ln\left(e^{dx+c}-1\right)}{a^2d^2}-\frac{2b^2e\,\operatorname{arctanh}\left(\frac{2b\,e^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{a^2d\sqrt{a^2+b^2}}+\frac{2b^2cf\,\operatorname{arctan}}{a^2d^2}$

input
```
int((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(f*x+e)/a/(exp(2*d*x+2*c)-1)-1/a^2/d*b*e*ln(exp(d*x+c)-1)+1/a^2/d*b*e
*ln(exp(d*x+c)+1)+1/a^2/d^2*b*c*f*ln(exp(d*x+c)-1)-2/a^2/d*b^2*e/(a^2+b^2)
^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/a^2/d^2*b^2*c*f
/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/a^2/d
^2*f*b*dilog(exp(d*x+c))+1/a^2/d^2*f*b*dilog(exp(d*x+c)+1)+1/a^2/d*f*b^2/(
a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*
x-1/a^2/d*f*b^2/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^
2+b^2)^(1/2)))*x+1/a^2/d*f*b*ln(exp(d*x+c)+1)*x+1/a^2/d^2*f*b^2/(a^2+b^2)^
(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/a^2/d
^2*f*b^2/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^
(1/2)))*c+1/a^2/d^2*f*b^2/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/a^2/d^2*f*b^2/(a^2+b^2)^(1/2)*dilog((b*exp
(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d^2/a*f*ln(exp(d*x+c)-1)
+1/a/d^2*f*ln(exp(d*x+c)+1)-2/a/d^2*f*ln(exp(d*x+c))
```

3.245.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1830 vs. $2(279) = 558$.

Time = 0.31 (sec) , antiderivative size = 1830, normalized size of antiderivative = 5.98

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*(a^3 + a*b^2)*d*e - 2*(a^3 + a*b^2)*c*f + 2*((a^3 + a*b^2)*d*f*x + (a^
3 + a*b^2)*c*f)*cosh(d*x + c)^2 + 4*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c
*f)*cosh(d*x + c)*sinh(d*x + c) + 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c
*f)*sinh(d*x + c)^2 - (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x + c)*sinh(
d*x + c) + b^3*f*sinh(d*x + c)^2 - b^3*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*c
osh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) + (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x +
 c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 - b^3*f)*sqrt((a^2 + b^2)/b^2)*d
ilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*d*e - b^3*c*f - (b^3*d*e - b^
3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)*sinh(d*x + c)
 - (b^3*d*e - b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh
(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d*
e - b^3*c*f - (b^3*d*e - b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*e - b^3*c*f)*
cosh(d*x + c)*sinh(d*x + c) - (b^3*d*e - b^3*c*f)*sinh(d*x + c)^2)*sqrt((a
^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 +
 b^2)/b^2) + 2*a) + (b^3*d*f*x + b^3*c*f - (b^3*d*f*x + b^3*c*f)*cosh(d*x
+ c)^2 - 2*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*f*x
+ b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) +
a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/...
```

3.245.6 Sympy [F]

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\,\text{csch}^2\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*csch(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.245.7 Maxima [F]

$$\int \frac{(e+fx)\mathrm{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\mathrm{csch}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
(4*b^2*integrate(1/2*x*e^(d*x + c)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x +
 c) - a^2*b), x) - 4*b*d*integrate(1/4*x/(a^2*d*e^(d*x + c) + a^2*d), x) -
 4*b*d*integrate(1/4*x/(a^2*d*e^(d*x + c) - a^2*d), x) - a*((d*x + c)/(a^2
*d^2) - log(e^(d*x + c) + 1)/(a^2*d^2)) - a*((d*x + c)/(a^2*d^2) - log(e^(
d*x + c) - 1)/(a^2*d^2)) - 2*x/(a*d*e^(2*d*x + 2*c) - a*d))*f + e*(b^2*log
((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b
^2)))/(sqrt(a^2 + b^2)*a^2*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^
(-d*x - c) - 1)/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d))
```

3.245.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\mathrm{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\mathrm{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\sinh\,(c+d\,x)^2\,\,(a+b\sinh\,(c+d\,x))}\,dx$$

input
```
int((e + f*x)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.246 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.246.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\operatorname{arctanh}(\cosh(c+dx))}{a^2d} - \frac{2b^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d} - \frac{\coth(c+dx)}{ad}$$

output
```
b*arctanh(cosh(d*x+c))/a^2/d-coth(d*x+c)/a/d-2*b^2*arctanh((b-a*tanh(1/2*d
*x+1/2*c))/(a^2+b^2)^(1/2))/a^2/d/(a^2+b^2)^(1/2)
```

3.246.2 Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\coth\left(\frac{1}{2}(c+dx)\right)+2b\left(-\frac{2b\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)}{2a^2d}$$

input
```
Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(a*Coth[(c + d*x)/2] + 2*b*((-2*b*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sq
rt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c +
 d*x)/2]]) + a*Tanh[(c + d*x)/2])/(a^2*d)
```

3.246.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 25, 3281, 27, 3042, 26, 3226, 26, 3042, 26, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3281}$$

$$-\frac{\int \frac{b\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\coth(c+dx)}{ad}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\coth(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{b\int \frac{i}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\int \frac{1}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx}{a}$$

$$\Big\downarrow \text{3226}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\left(\frac{ib\int\frac{1}{a+b\sinh(c+dx)}dx}{a}+\frac{\int-i\operatorname{csch}(c+dx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\left(\frac{ib\int\frac{1}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int\operatorname{csch}(c+dx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\left(\frac{ib\int\frac{1}{a-ib\sin(ic+idx)}dx}{a}-\frac{i\int i\csc(ic+idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\left(\frac{ib\int\frac{1}{a-ib\sin(ic+idx)}dx}{a}+\frac{\int\csc(ic+idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\left(\frac{2b\int\frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}+\frac{\int\csc(ic+idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\left(\frac{\int\csc(ic+idx)dx}{a}-\frac{4b\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{ad}\right)}{a}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\left(\frac{\int\csc(ic+idx)dx}{a}+\frac{2ib\,\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}\right)}{a}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{ib\left(\frac{2ib\,\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}+\frac{i\,\text{arctanh}(\cosh(c+dx))}{ad}\right)}{a}$$

input
```
Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

3.246. $\int\frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
((-I)*b*((I*ArcTanh[Cosh[c + d*x]])/(a*d) + ((2*I)*b*ArcTanh[Tanh[(c + d*x
)/2]/(2*Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d)))/a - Coth[c + d*x]/(a*d)
```

3.246.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.246. $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.246.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{2b^2\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{2b^2\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}}{d}$
risch	$-\frac{2}{da\left(e^{2dx+2c}-1\right)}+\frac{b\ln\left(e^{dx+c}+1\right)}{a^2d}-\frac{b\ln\left(e^{dx+c}-1\right)}{a^2d}+\frac{b^2\ln\left(e^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,a^2}-$

input
```
int(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)-1/2/a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(tanh(
1/2*d*x+1/2*c))+2/a^2*b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/
2*c)-2*b)/(a^2+b^2)^(1/2)))
```

3.246.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(77) = 154$.

Time = 0.28 (sec) , antiderivative size = 479, normalized size of antiderivative = 5.99

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2\,a^3+2\,ab^2-\left(b^2\cosh\left(dx+c\right)^2+2\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b^2\sinh\left(dx+c\right)^2-b^2\right)\sqrt{a^2+b^2}\,\mathrm{log}}{}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*a^3 + 2*a*b^2 - (b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x +
c) + b^2*sinh(d*x + c)^2 - b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 +
 b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x
 + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d
*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) +
 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + (a^2*b + b^3 - (a^2*b + b^3
)*cosh(d*x + c)^2 - 2*(a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c) - (a^2*b +
 b^3)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a^2*b + b
^3 - (a^2*b + b^3)*cosh(d*x + c)^2 - 2*(a^2*b + b^3)*cosh(d*x + c)*sinh(d*
x + c) - (a^2*b + b^3)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c)
- 1))/((a^4 + a^2*b^2)*d*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*d*cosh(d*x +
c)*sinh(d*x + c) + (a^4 + a^2*b^2)*d*sinh(d*x + c)^2 - (a^4 + a^2*b^2)*d)
```

3.246.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\operatorname{csch}^2\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.246.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^2\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^2d} + \frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d} - \frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d} + \frac{2}{(ae^{(-2\,dx-2\,c)}-a)d}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
b^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b
*log(e^(-d*x - c) - 1)/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d)
```

3.246.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.54

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{b^2\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^2} + \frac{b\log\left(e^{(dx+c)}+1\right)}{a^2} - \frac{b\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^2} - \frac{2}{a\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
(b^2*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c
) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) + b*log(e^(d*x + c) +
1)/a^2 - b*log(abs(e^(d*x + c) - 1))/a^2 - 2/(a*(e^(2*d*x + 2*c) - 1)))/d
```

3.246.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 360, normalized size of antiderivative = 4.50

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2}{a\,d-a\,d\,\mathrm{e}^{2\,c+2\,d\,x}}$$
$$+\frac{b^2\,\ln\left(128\,a^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-64\,a\,b^3-64\,a^3\,b-32\,b^3\,\sqrt{a^2+b^2}+32\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-64\,a^2\,b\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}\right.}{d\,a^4+d\,a^2\,b^2}$$
$$-\frac{b^2\,\ln\left(32\,b^3\,\sqrt{a^2+b^2}-64\,a\,b^3-64\,a^3\,b+128\,a^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+32\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+64\,a^2\,b\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}\right.}{d\,a^4+d\,a^2\,b^2}$$
$$-\frac{b\,\ln\left(32\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-32\right)}{a^2\,d}+\frac{b\,\ln\left(32\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+32\right)}{a^2\,d}$$

input
```
int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
2/(a*d - a*d*exp(2*c + 2*d*x)) + (b^2*log(128*a^4*exp(d*x)*exp(c) - 64*a*b
^3 - 64*a^3*b - 32*b^3*(a^2 + b^2)^(1/2) + 32*b^4*exp(d*x)*exp(c) - 64*a^2
*b*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(d*x)*exp(c) + 128*a^3*exp(d*x)*exp(
c)*(a^2 + b^2)^(1/2) + 96*a*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 +
b^2)^(1/2))/(a^4*d + a^2*b^2*d) - (b^2*log(32*b^3*(a^2 + b^2)^(1/2) - 64*a
*b^3 - 64*a^3*b + 128*a^4*exp(d*x)*exp(c) + 32*b^4*exp(d*x)*exp(c) + 64*a^
2*b*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(d*x)*exp(c) - 128*a^3*exp(d*x)*exp
(c)*(a^2 + b^2)^(1/2) - 96*a*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 +
 b^2)^(1/2))/(a^4*d + a^2*b^2*d) - (b*log(32*exp(d*x)*exp(c) - 32))/(a^2*d
) + (b*log(32*exp(d*x)*exp(c) + 32))/(a^2*d)
```

3.247 $\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.247.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.247.2 Mathematica [N/A]

Not integrable

Time = 92.95 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Csch[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Csch[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.247.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.247.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.247.4 Maple [N/A] (verified)

Not integrable

Time = 0.85 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.247.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(csch(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.247.6 Sympy [N/A]

Not integrable

Time = 6.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^2\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(csch(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.247.7 Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 304, normalized size of antiderivative = 10.86

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
4*b^2*integrate(-1/2*e^(d*x + c)/(a^2*b*f*x + a^2*b*e - (a^2*b*f*x*e^(2*c)
 + a^2*b*e*e^(2*c))*e^(2*d*x) - 2*(a^3*f*x*e^c + a^3*e*e^c)*e^(d*x)), x) +
 2/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*x)) - 4*int
egrate(-1/4*(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d
*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x
) - 4*integrate(1/4*(b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x
 + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(
d*x)), x)
```

3.247.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.247.9 Mupad [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.248 $\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.248.1 Optimal result

Integrand size = 28, antiderivative size = 1053

$$\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{ad^3}$$
$$+ \frac{(e+fx)^3\operatorname{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^3\operatorname{arctanh}\left(e^{c+dx}\right)}{a^3d}$$
$$+ \frac{b(e+fx)^3\coth(c+dx)}{a^2d} - \frac{3f(e+fx)^2\operatorname{csch}(c+dx)}{2ad^2}$$
$$- \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$
$$- \frac{b^3(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d}$$
$$+ \frac{b^3(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d}$$
$$- \frac{3bf(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{a^2d^2} - \frac{3f^3\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^4}$$
$$+ \frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{2ad^2}$$
$$- \frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2}$$
$$+ \frac{3f^3\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^4} - \frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{2ad^2}$$
$$+ \frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2}$$
$$- \frac{3b^3f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d^2}$$
$$+ \frac{3b^3f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d^2}$$
$$- \frac{3bf^2(e+fx)\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{a^2d^3}$$
$$- \frac{3f^2(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}$$
$$+ \frac{6b^2f^2(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{a^3d^3}$$
$$+ \frac{3f^2(e+fx)\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3}$$
$$- \frac{6b^2f^2(e+fx)\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{a^3d^3}$$
$$+ \frac{6b^3f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d^3}$$
$$- \frac{6b^3f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d^3}$$

output
```
-3*b^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^2/(a
^2+b^2)^(1/2)+3*b^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)
))/a^3/d^2/(a^2+b^2)^(1/2)+6*b^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/a^3/d^3/(a^2+b^2)^(1/2)-6*b^3*f^2*(f*x+e)*polylog(3,-b*exp
(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^3/(a^2+b^2)^(1/2)-3*b^2*f*(f*x+e)^2*pol
ylog(2,-exp(d*x+c))/a^3/d^2+3*b^2*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a^3/d^
2-3*b*f^2*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a^2/d^3+6*b^2*f^2*(f*x+e)*poly
log(3,-exp(d*x+c))/a^3/d^3-6*b^2*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a^3/d^3
-3*b*f*(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a^2/d^2-6*b^3*f^3*polylog(4,-b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^4/(a^2+b^2)^(1/2)+6*b^3*f^3*polylog(4,-b*
exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^4/(a^2+b^2)^(1/2)+3*f^3*polylog(4,-e
xp(d*x+c))/a/d^4-3*f^3*polylog(4,exp(d*x+c))/a/d^4-6*f^2*(f*x+e)*arctanh(e
xp(d*x+c))/a/d^3-3/2*f*(f*x+e)^2*csch(d*x+c)/a/d^2-1/2*(f*x+e)^3*coth(d*x+
c)*csch(d*x+c)/a/d+3/2*f*(f*x+e)^2*polylog(2,-exp(d*x+c))/a/d^2-3/2*f*(f*x
+e)^2*polylog(2,exp(d*x+c))/a/d^2-3*f^2*(f*x+e)*polylog(3,-exp(d*x+c))/a/d
^3+3*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a/d^3-b^3*(f*x+e)^3*ln(1+b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/a^3/d/(a^2+b^2)^(1/2)+b^3*(f*x+e)^3*ln(1+b*exp(d*x
+c)/(a+(a^2+b^2)^(1/2)))/a^3/d/(a^2+b^2)^(1/2)+b*(f*x+e)^3*coth(d*x+c)/a^2
/d+b*(f*x+e)^3/a^2/d-3*f^3*polylog(2,-exp(d*x+c))/a/d^4+3*f^3*polylog(2,ex
p(d*x+c))/a/d^4-2*b^2*(f*x+e)^3*arctanh(exp(d*x+c))/a^3/d+3/2*b*f^3*pol...
```

3.248.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2801 vs. $2(1053) = 2106$.

Time = 9.46 (sec) , antiderivative size = 2801, normalized size of antiderivative = 2.66

$$\int \frac{(e+fx)^3 \text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(12*a*b*d^3*e^2*E^(2*c)*f*x + 12*a*b*d^3*e*E^(2*c)*f^2*x^2 + 4*a*b*d^3*E^(
2*c)*f^3*x^3 - 2*a^2*d^3*e^3*ArcTanh[E^(c + d*x)] + 4*b^2*d^3*e^3*ArcTanh[
E^(c + d*x)] + 2*a^2*d^3*e^3*E^(2*c)*ArcTanh[E^(c + d*x)] - 4*b^2*d^3*e^3*
E^(2*c)*ArcTanh[E^(c + d*x)] + 12*a^2*d*e*f^2*ArcTanh[E^(c + d*x)] - 12*a^
2*d*e*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] + 3*a^2*d^3*e^2*f*x*Log[1 - E^(c +
d*x)] - 6*b^2*d^3*e^2*f*x*Log[1 - E^(c + d*x)] - 3*a^2*d^3*e^2*E^(2*c)*f*x
*Log[1 - E^(c + d*x)] + 6*b^2*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(c + d*x)] - 6
*a^2*d*f^3*x*Log[1 - E^(c + d*x)] + 6*a^2*d*E^(2*c)*f^3*x*Log[1 - E^(c + d
*x)] + 3*a^2*d^3*e*f^2*x^2*Log[1 - E^(c + d*x)] - 6*b^2*d^3*e*f^2*x^2*Log[
1 - E^(c + d*x)] - 3*a^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 6*b^
2*d^3*e*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + a^2*d^3*f^3*x^3*Log[1 - E^(
c + d*x)] - 2*b^2*d^3*f^3*x^3*Log[1 - E^(c + d*x)] - a^2*d^3*E^(2*c)*f^3*x
^3*Log[1 - E^(c + d*x)] + 2*b^2*d^3*E^(2*c)*f^3*x^3*Log[1 - E^(c + d*x)] -
 3*a^2*d^3*e^2*f*x*Log[1 + E^(c + d*x)] + 6*b^2*d^3*e^2*f*x*Log[1 + E^(c +
 d*x)] + 3*a^2*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - 6*b^2*d^3*e^2*E^
(2*c)*f*x*Log[1 + E^(c + d*x)] + 6*a^2*d*f^3*x*Log[1 + E^(c + d*x)] - 6*a^
2*d*E^(2*c)*f^3*x*Log[1 + E^(c + d*x)] - 3*a^2*d^3*e*f^2*x^2*Log[1 + E^(c
+ d*x)] + 6*b^2*d^3*e*f^2*x^2*Log[1 + E^(c + d*x)] + 3*a^2*d^3*e*E^(2*c)*f
^2*x^2*Log[1 + E^(c + d*x)] - 6*b^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E^(c + d
*x)] - a^2*d^3*f^3*x^3*Log[1 + E^(c + d*x)] + 2*b^2*d^3*f^3*x^3*Log[1 +...
```

3.248.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)^3\text{csch}^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^3\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int (e+fx)^3\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(-\frac{3f^2\int -i(e+fx)\text{csch}(c+dx)dx}{d^2}+\frac{1}{2}\int -i(e+fx)^3\text{csch}(c+dx)dx-\frac{3if(e+fx)^2\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)^3\coth(c+dx)\text{csch}(c+d}{2d}\right.}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{3if^2\int (e+fx)\text{csch}(c+dx)dx}{d^2}-\frac{1}{2}i\int (e+fx)^3\text{csch}(c+dx)dx-\frac{3if(e+fx)^2\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{3if^2\int i(e+fx)\csc(ic+idx)dx}{d^2}-\frac{1}{2}i\int i(e+fx)^3\csc(ic+idx)dx-\frac{3if(e+fx)^2\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(-\frac{3f^2\int (e+fx)\csc(ic+idx)dx}{d^2}+\frac{1}{2}\int (e+fx)^3\csc(ic+idx)dx-\frac{3if(e+fx)^2\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(-\frac{3f^2\left(\frac{if\int \log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(\frac{3if\int (e+fx)^2\log(1-e^{c+dx})dx}{d}-\frac{3if\int (e+fx)^2\log}{d}\right.\right.}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(-\frac{3f^2\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(\frac{3if\int (e+fx)^2\log(1-e^{c+dx})}{d}\right.\right.}{a}$$

3.248. $\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-$$

$$\frac{i\left(\frac{1}{2}\left(\frac{3if\int (e+fx)^2\log\left(1-e^{c+dx}\right)dx}{d}-\frac{3if\int (e+fx)^2\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^3\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-\frac{3f^2\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right.}{}\right.}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-$$

$$i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.\right.$$

$$\Big\downarrow \text{6109}$$

$$-\frac{b\left(\frac{\int (e+fx)^3\text{csch}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-$$

$$i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.\right.}{}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -(e+fx)^3\csc(ic+idx)^2dx}{a}\right)}{a}$$

$$\Big\downarrow \text{25}$$

3.248. $\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+d}\right.}{d}\right.}{d}\right.\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int(e+fx)^3\csc(ic+idx)^2dx}{a}\right)}{a}}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+d}\right.}{d}\right.}{d}\right.\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3if\int-i(e+fx)^2\coth(c+dx)dx}{d}}{a}\right)}{a}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3f\int(e+fx)^2\coth(c+dx)dx}{d}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right.}{d}\right.}{d}\right.\right.}{}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+d}\right.}{d}\right.}{d}\right.\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{3f\int-i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}\right)}{a}}$$

$$\Big\downarrow \text{26}$$

3.248. $\int\frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+c}\right.}{d}\right.}{d}\right.\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\int(e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}\right)}{a}}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+c}\right.}{d}\right.}{d}\right.\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}\right)}{a}}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+c}\right.}{d}\right.}{d}\right.\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}\right)}{a}}$$

$$\Big\downarrow \text{3011}$$

3.248. $\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+d}\right.}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\operatorname{PolyLog}\left(\right.}{2}\right.}{d}\right.\right.}{d}}{a}\right.}{a}$$

$$\Big\downarrow \; 2720$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+d}\right.}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2}}{4d^2}\right.}{d}\right.\right.}{d}}{a}\right.}{a}$$

$$\Big\downarrow \; 6109$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+d}\right.}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\left(\frac{\int(e+fx)^3\operatorname{csch}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLo}}{}\right.}{}\right.\right.}{}}{a}\right.}{a}$$

$$\Big\downarrow \; 3042$$

3.248. $\displaystyle\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\left(\frac{\int i(e+fx)^3\csc(ic+idx)dx}{a}-\frac{b\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(f\int e^{-2c-2dx+i\pi}\operatorname{PolyL}\cdots\right.}{\cdots}\right.\right.}{a\cdots}\right.}\cdots$$

$$\Big\downarrow \; 26$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\left(\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}-\frac{b\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(f\int e^{-2c-2dx+i\pi}\operatorname{PolyL}\cdots\right.}{\cdots}\right.\right.}{a\cdots}\right.}\cdots$$

$$\Big\downarrow \; 3803$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\left(-\frac{2b\int-\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(f\int e^{-2c-\cdots}\right.}{\cdots}\right.\right.}{\cdots}\right.}\cdots$$

$$\Big\downarrow \; 25$$

3.248. $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.\right.}{}$$

$$\frac{b\left(-\frac{b\left(\frac{2b\int\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx}\cdots}{}\right.}{}\right.\right.}{}\right.}{a}$$

$\downarrow$ 2694

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+f\cdots}{}\right.\right.}{}\right.}{a}$$

$$\frac{i\left(\frac{1}{2}\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.\right.}{}$$

$\downarrow$ 27

3.248. $\int\frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-\frac{(e+fx)^3\coth(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e\cdots\right.}{2d}\cdots\right.\right.}{\cdots}\cdots\right)}{i\left(\frac{1}{2}\left(\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\cdots\right.}{d}\cdots\right.\right.\right.}\cdots a$$

$$\Big\downarrow \text{2620}$$

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+\cdots\right.}{bd}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{a}\cdots\right.}{a}\cdots\right.}{i\left(\frac{1}{2}\left(\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\cdots\right.}{d}\cdots\right.\right.\right.}$$

input
```
Int[((e + f*x)^3*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

3.248. $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
$Aborted
```

3.248.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.248. $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.248. $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

3.248.4 Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.248.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18159 vs. $2(977) = 1954$.

Time = 0.58 (sec) , antiderivative size = 18159, normalized size of antiderivative = 17.25

$$\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

3.248.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.248.7 Maxima [F]

$$\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \operatorname{csch}(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*e^3*(2*b^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
 - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d) - 2*(a*e^(-d*x - c) + 2*b
*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2*b)/((2*a^2*e^(-2*d*x - 2*c) - a
^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 - 2*b^2)*log(e^(-d*x - c) + 1)/(a^3*d
) + (a^2 - 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d)) - (2*b*d*f^3*x^3 + 6*b*d*
e*f^2*x^2 + 6*b*d*e^2*f*x + (a*d*f^3*x^3*e^(3*c) + 3*a*e^2*f*e^(3*c) + 3*(
d*e*f^2 + f^3)*a*x^2*e^(3*c) + 3*(d*e^2*f + 2*e*f^2)*a*x*e^(3*c))*e^(3*d*x
) - 2*(b*d*f^3*x^3*e^(2*c) + 3*b*d*e*f^2*x^2*e^(2*c) + 3*b*d*e^2*f*x*e^(2*
c))*e^(2*d*x) + (a*d*f^3*x^3*e^c - 3*a*e^2*f*e^c + 3*(d*e*f^2 - f^3)*a*x^2
*e^c + 3*(d*e^2*f - 2*e*f^2)*a*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) -
2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) + 3*(b*d*e^2*f + a*e*f^2)*x/(a^2*d^2)
 + 3*(b*d*e^2*f - a*e*f^2)*x/(a^2*d^2) - 3*(b*d*e^2*f + a*e*f^2)*log(e^(d*
x + c) + 1)/(a^2*d^3) - 3*(b*d*e^2*f - a*e*f^2)*log(e^(d*x + c) - 1)/(a^2*
d^3) + 1/2*(d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) -
 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*(a^2*f^3 - 2
*b^2*f^3)/(a^3*d^4) - 1/2*(d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog
(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))
*(a^2*f^3 - 2*b^2*f^3)/(a^3*d^4) + 3/2*(a^2*d*e*f^2 - 2*b^2*d*e*f^2 - 2*a*
b*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polyl
og(3, -e^(d*x + c)))/(a^3*d^4) - 3/2*(a^2*d*e*f^2 - 2*b^2*d*e*f^2 + 2*a...
```

3.248.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\sinh\left(c+d\,x\right)^3\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^3/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.249 $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.249.1 Optimal result

Integrand size = 28, antiderivative size = 725

$$\begin{aligned}
\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{b(e+fx)^2}{a^2d}+\frac{(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{ad}\\
&-\frac{2b^2(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{a^3d}-\frac{f^2\operatorname{arctanh}(\cosh(c+dx))}{ad^3}\\
&+\frac{b(e+fx)^2\coth(c+dx)}{a^2d}-\frac{f(e+fx)\operatorname{csch}(c+dx)}{ad^2}\\
&-\frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}\\
&-\frac{b^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d}\\
&+\frac{b^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d}\\
&-\frac{2bf(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^2d^2}\\
&+\frac{f(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}\\
&-\frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2}\\
&-\frac{f(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}\\
&+\frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2}\\
&-\frac{2b^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d^2}\\
&+\frac{2b^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d^2}\\
&-\frac{bf^2\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{a^2d^3}-\frac{f^2\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}\\
&+\frac{2b^2f^2\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{a^3d^3}+\frac{f^2\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3}\\
&-\frac{2b^2f^2\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{a^3d^3}+\frac{2b^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d^3}\\
&-\frac{2b^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d^3}
\end{aligned}$$

output
```
b*(f*x+e)^2/a^2/d+(f*x+e)^2*arctanh(exp(d*x+c))/a/d-2*b^2*(f*x+e)^2*arctan
h(exp(d*x+c))/a^3/d-f^2*arctanh(cosh(d*x+c))/a/d^3+b*(f*x+e)^2*coth(d*x+c)
/a^2/d-f*(f*x+e)*csch(d*x+c)/a/d^2-1/2*(f*x+e)^2*coth(d*x+c)*csch(d*x+c)/a
/d-2*b*f*(f*x+e)*ln(1-exp(2*d*x+2*c))/a^2/d^2+f*(f*x+e)*polylog(2,-exp(d*x
+c))/a/d^2-2*b^2*f*(f*x+e)*polylog(2,-exp(d*x+c))/a^3/d^2-f*(f*x+e)*polylo
g(2,exp(d*x+c))/a/d^2+2*b^2*f*(f*x+e)*polylog(2,exp(d*x+c))/a^3/d^2-b*f^2*
polylog(2,exp(2*d*x+2*c))/a^2/d^3-f^2*polylog(3,-exp(d*x+c))/a/d^3+2*b^2*f
^2*polylog(3,-exp(d*x+c))/a^3/d^3+f^2*polylog(3,exp(d*x+c))/a/d^3-2*b^2*f^
2*polylog(3,exp(d*x+c))/a^3/d^3-b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/a^3/d/(a^2+b^2)^(1/2)+b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/a^3/d/(a^2+b^2)^(1/2)-2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/a^3/d^2/(a^2+b^2)^(1/2)+2*b^3*f*(f*x+e)*polylog(2,-b*
exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^2/(a^2+b^2)^(1/2)+2*b^3*f^2*polylog(
3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^3/(a^2+b^2)^(1/2)-2*b^3*f^2*pol
ylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^3/(a^2+b^2)^(1/2)
```

3.249.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1532 vs. $2(725) = 1450$.

Time = 8.24 (sec) , antiderivative size = 1532, normalized size of antiderivative = 2.11

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(8*a*b*d^2*e*E^(2*c)*f*x + 4*a*b*d^2*E^(2*c)*f^2*x^2 - 2*a^2*d^2*e^2*ArcTa
nh[E^(c + d*x)] + 4*b^2*d^2*e^2*ArcTanh[E^(c + d*x)] + 2*a^2*d^2*e^2*E^(2*
c)*ArcTanh[E^(c + d*x)] - 4*b^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] + 4*a
^2*f^2*ArcTanh[E^(c + d*x)] - 4*a^2*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] + 2*a
^2*d^2*e*f*x*Log[1 - E^(c + d*x)] - 4*b^2*d^2*e*f*x*Log[1 - E^(c + d*x)] -
 2*a^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 4*b^2*d^2*e*E^(2*c)*f*x*Lo
g[1 - E^(c + d*x)] + a^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)] - 2*b^2*d^2*f^2*
x^2*Log[1 - E^(c + d*x)] - a^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] +
2*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] - 2*a^2*d^2*e*f*x*Log[1 + E
^(c + d*x)] + 4*b^2*d^2*e*f*x*Log[1 + E^(c + d*x)] + 2*a^2*d^2*e*E^(2*c)*f
*x*Log[1 + E^(c + d*x)] - 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - a
^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)] + 2*b^2*d^2*f^2*x^2*Log[1 + E^(c + d*x
)] + a^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] - 2*b^2*d^2*E^(2*c)*f^2*
x^2*Log[1 + E^(c + d*x)] + 4*a*b*d*e*f*Log[1 - E^(2*(c + d*x))] - 4*a*b*d*
e*E^(2*c)*f*Log[1 - E^(2*(c + d*x))] + 4*a*b*d*f^2*x*Log[1 - E^(2*(c + d*x
))] - 4*a*b*d*E^(2*c)*f^2*x*Log[1 - E^(2*(c + d*x))] + 2*(a^2 - 2*b^2)*d*(
-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -E^(c + d*x)] - 2*(a^2 - 2*b^2)*d*(-1
 + E^(2*c))*f*(e + f*x)*PolyLog[2, E^(c + d*x)] + 2*a*b*f^2*PolyLog[2, E^(
2*(c + d*x))] - 2*a*b*E^(2*c)*f^2*PolyLog[2, E^(2*(c + d*x))] + 2*a^2*f^2*
PolyLog[3, -E^(c + d*x)] - 4*b^2*f^2*PolyLog[3, -E^(c + d*x)] - 2*a^2*E...
```

3.249.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \ 6109$$

$$\frac{\int (e+fx)^2\text{csch}^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^2\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \ 26$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int (e+fx)^2\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{ 4674}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(-\frac{f^2\int -i\text{csch}(c+dx)dx}{d^2}+\frac{1}{2}\int -i(e+fx)^2\text{csch}(c+dx)dx-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{if^2\int \text{csch}(c+dx)dx}{d^2}-\frac{1}{2}i\int (e+fx)^2\text{csch}(c+dx)dx-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{if^2\int i\csc(ic+idx)dx}{d^2}-\frac{1}{2}i\int i(e+fx)^2\csc(ic+idx)dx-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(-\frac{f^2\int \csc(ic+idx)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\csc(ic+idx)dx-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{ 4257}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\int (e+fx)^2\csc(ic+idx)dx-\frac{if^2\text{arctanh}(\cosh(c+dx))}{d^3}-\frac{if(e+fx)\text{csch}(c+dx)}{d^2}-\frac{i(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{ 4670}$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2if\int (e+fx)\log\left(1-e^{c+dx}\right)dx}{d}-\frac{2if\int (e+fx)\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-\frac{if^2\text{arctanh}(\cosh(c+dx))}{d^3}-if(\right.}{a}$$

$$\Big\downarrow \text{ 3011}$$

3.249. $\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}+\frac{2i(e+fx)^2}{}\right.\right.}{a}$$

$$\Big\downarrow \; 2720$$

$$-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2\right.}{d}\right.}{d}\right.\right.}{a}$$

$$\Big\downarrow \; 6109$$

$$-\frac{b\left(\frac{\int(e+fx)^2\text{csch}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2\right.}{d}\right.}{d}\right.\right.}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLo}}{d}\right.}{d}\right.\right.}{}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -(e+fx)^2\csc(ic+idx)^2dx}{a}\right)}{a}$$

$$\Big\downarrow \; 25$$

3.249. $\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int(e+fx)^2\csc(ic+idx)^2dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 4672}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2if\int-i(e+fx)\coth(c+dx)dx}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int(e+fx)\coth(c+dx)dx}{d}}{a}\right)}{a}-$$

$$\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}(2}{d}\right.}{d}\right.\right.}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int-i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{ 26}$$

3.249. $\int\frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLo}}{d}\right.}{d}\right.\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\int(e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}\right)}{a}}$$

$\downarrow$ 4201

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLo}}{d}\right.}{d}\right.\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}\right)}{a}}$$

$\downarrow$ 2620

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLo}}{d}\right.}{d}\right.\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}\right)}{a}}$$

$\downarrow$ 2715

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLo}}{d}\right.}{d}\right.\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\right.}{d}}{a}\right.}{a}}$$

3.249. $\int\frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{2838}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{6109}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\left(\frac{\int(e+fx)^2\operatorname{csch}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)\right.}{d}}{a}\right.}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLo}}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\left(\frac{\int i(e+fx)^2\csc(ic+idx)dx}{a}-\frac{b\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right.\right.}{d}}{a}\right.}{a}$$

$$\Big\downarrow \text{26}$$

3.249. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\left(\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}-\frac{b\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right.\right.}{d}}{a}\right.}{a}$$

$$\Big\downarrow \color{blue}{3803}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\left(-\frac{2b\int-\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+\right.}{2d}\right.\right.}{d}}{a}\right.}{a}$$

$$\Big\downarrow \color{blue}{25}$$

$$-\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}}{d}\right.}{d}\right.\right.}{b\left(-\frac{b\left(\frac{2b\int\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c-}\right.}{2d}\right.\right.}{d}}{a}\right.}{a}$$

$$\Big\downarrow \color{blue}{2694}$$

3.249. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\mathrm{Poly}\cdots}{\cdots}\right.\right.}{\cdots}\cdots\right)}{a}$$

$$\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2\cdots\right.}{d}\right.}{d}\cdots\right.\right.}{a\cdots}$$

$$\Big\downarrow \color{blue}{27}$$

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2\cdots}\right.}{4d^2}\cdots\right.\right.}{\cdots}\cdots\right)}{a}$$

$$\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2\cdots\right.}{d}\right.}{d}\cdots\right.\right.}{a\cdots}$$

$$\Big\downarrow \color{blue}{2620}$$

3.249. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)\cdots}{bd}\cdots}{2\sqrt{a^2+b^2}}\cdots\right)}{a}\cdots}{a}\cdots\right)}{a\cdots}$$

$$\frac{i\left(\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(\cdots\right)}{d}\cdots\right)}{d}\cdots\right)\cdots\right)}{a\cdots}$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.249.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.249. $\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.249. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

3.249. $\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.249.4 Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.249.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10341 vs. $2(676) = 1352$.

Time = 0.40 (sec) , antiderivative size = 10341, normalized size of antiderivative = 14.26

$$\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

3.249.6 Sympy [F]

$$\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*csch(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

3.249.7 Maxima [F]

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{csch}\,(dx+c)^3}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*e^2*(2*b^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
 - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d) - 2*(a*e^(-d*x - c) + 2*b
*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2*b)/((2*a^2*e^(-2*d*x - 2*c) - a
^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 - 2*b^2)*log(e^(-d*x - c) + 1)/(a^3*d
) + (a^2 - 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d)) - (2*b*d*f^2*x^2 + 4*b*d*
e*f*x + (a*d*f^2*x^2*e^(3*c) + 2*a*e*f*e^(3*c) + 2*(d*e*f + f^2)*a*x*e^(3*
c))*e^(3*d*x) - 2*(b*d*f^2*x^2*e^(2*c) + 2*b*d*e*f*x*e^(2*c))*e^(2*d*x) +
(a*d*f^2*x^2*e^c - 2*a*e*f*e^c + 2*(d*e*f - f^2)*a*x*e^c)*e^(d*x))/(a^2*d^
2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) + (2*b*d*e*f + a*
f^2)*x/(a^2*d^2) + (2*b*d*e*f - a*f^2)*x/(a^2*d^2) - (2*b*d*e*f + a*f^2)*l
og(e^(d*x + c) + 1)/(a^2*d^3) - (2*b*d*e*f - a*f^2)*log(e^(d*x + c) - 1)/(
a^2*d^3) + 1/2*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) -
 2*polylog(3, -e^(d*x + c)))*(a^2*f^2 - 2*b^2*f^2)/(a^3*d^3) - 1/2*(d^2*x^
2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x +
 c)))*(a^2*f^2 - 2*b^2*f^2)/(a^3*d^3) + (a^2*d*e*f - 2*b^2*d*e*f - 2*a*b*f
^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^3*d^3) - (a^2*d*e*
f - 2*b^2*d*e*f + 2*a*b*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c
)))/(a^3*d^3) + 1/6*((a^2*f^2 - 2*b^2*f^2)*d^3*x^3 + 3*(a^2*d*e*f - 2*b^2*
d*e*f + 2*a*b*f^2)*d^2*x^2)/(a^3*d^3) - 1/6*((a^2*f^2 - 2*b^2*f^2)*d^3*x^3
 + 3*(a^2*d*e*f - 2*b^2*d*e*f - 2*a*b*f^2)*d^2*x^2)/(a^3*d^3) - integra...
```

3.249.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\sinh\left(c+d\,x\right)^3\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^2/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.250 $\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.250.1 Optimal result

Integrand size = 26, antiderivative size = 420

$$\begin{aligned}
\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{ad}-\frac{2b^2(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{a^3d} \\
&+\frac{b(e+fx)\coth(c+dx)}{a^2d}-\frac{f\text{csch}(c+dx)}{2ad^2} \\
&-\frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2ad} \\
&-\frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}d} \\
&+\frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}d} \\
&-\frac{bf\log(\sinh(c+dx))}{a^2d^2}+\frac{f\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2ad^2} \\
&-\frac{b^2f\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2}-\frac{f\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2ad^2} \\
&+\frac{b^2f\,\text{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2}-\frac{b^3f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}d^2} \\
&+\frac{b^3f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}d^2}
\end{aligned}$$

output
```
(f*x+e)*arctanh(exp(d*x+c))/a/d-2*b^2*(f*x+e)*arctanh(exp(d*x+c))/a^3/d+b*
(f*x+e)*coth(d*x+c)/a^2/d-1/2*f*csch(d*x+c)/a/d^2-1/2*(f*x+e)*coth(d*x+c)*
csch(d*x+c)/a/d-b*f*ln(sinh(d*x+c))/a^2/d^2+1/2*f*polylog(2,-exp(d*x+c))/a
/d^2-b^2*f*polylog(2,-exp(d*x+c))/a^3/d^2-1/2*f*polylog(2,exp(d*x+c))/a/d^
2+b^2*f*polylog(2,exp(d*x+c))/a^3/d^2-b^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/a^3/d/(a^2+b^2)^(1/2)+b^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/a^3/d/(a^2+b^2)^(1/2)-b^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/a^3/d^2/(a^2+b^2)^(1/2)+b^3*f*polylog(2,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/a^3/d^2/(a^2+b^2)^(1/2)
```

3.250.2 Mathematica [A] (verified)

Time = 8.42 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.47

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\left(2bde\cosh\left(\frac{1}{2}(c+dx)\right)-af\cosh\left(\frac{1}{2}(c+dx)\right)-2bcf\cosh\left(\frac{1}{2}(c+dx)\right)+2bf(c+dx)\cosh\left(\frac{1}{2}(c+dx)\right)\right)\mathrm{c}}{4a^2d^2}$$
$$+\frac{(-de+cf-f(c+dx))\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$-\frac{2abf(c+dx)+(2abf+a^2(de+dfx)-2b^2(de+dfx))\log\left(1-e^{-c-dx}\right)+\left(2abf-a^2(de+dfx)+2b^2\right.}{2a^3d}$$
$$-\frac{b^3\left(-2de\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2cf\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(\right.\right.}{a^3\sqrt{a^2+b^2}d^2}$$
$$+\frac{(-de+cf-f(c+dx))\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(2bde\sinh\left(\frac{1}{2}(c+dx)\right)+af\sinh\left(\frac{1}{2}(c+dx)\right)-2bcf\sinh\left(\frac{1}{2}(c+dx)\right)+2bf(c+dx)\mathrm{si}\right.}{4a^2d^2}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*
x)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2)
+ ((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) - (2*a*b*f*
(c + d*x) + (2*a*b*f + a^2*(d*e + d*f*x) - 2*b^2*(d*e + d*f*x))*Log[1 - E^
(-c - d*x)] + (2*a*b*f - a^2*(d*e + d*f*x) + 2*b^2*(d*e + d*f*x))*Log[1 +
E^(-c - d*x)] + (a^2 - 2*b^2)*f*PolyLog[2, -E^(-c - d*x)] - (a^2 - 2*b^2)*
f*PolyLog[2, E^(-c - d*x)])/(2*a^3*d^2) - (b^3*(-2*d*e*ArcTanh[(a + b*E^(c
 + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b
^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c +
 d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(
c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))]))/(a^3*Sqrt[a^2 + b^2]*d^2) + ((-(d*e) + c*f - f*(c + d*x
))*Sech[(c + d*x)/2]^2)/(8*a*d^2) + (Sech[(c + d*x)/2]*(2*b*d*e*Sinh[(c +
d*x)/2] + a*f*Sinh[(c + d*x)/2] - 2*b*c*f*Sinh[(c + d*x)/2] + 2*b*f*(c + d
*x)*Sinh[(c + d*x)/2]))/(4*a^2*d^2)
```

3.250.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6109}$$

$$\frac{\int (e+fx)\text{csch}^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int (e+fx)\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{4673}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\int -i(e+fx)\text{csch}(c+dx)dx-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \textbf{26}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(-\frac{1}{2}i\int (e+fx)\text{csch}(c+dx)dx-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(-\frac{1}{2}i\int i(e+fx)\csc(ic+idx)dx-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \textbf{26}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\int (e+fx)\csc(ic+idx)dx-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \textbf{4670}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\left(\frac{if\int \log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \textbf{2715}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(\frac{1}{2}\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)}{}\right)}{a}$$

$$\Big\downarrow \textbf{2838}$$

3.250. $\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \; 6109$$

$$-\frac{b\left(\frac{\int(e+fx)\text{csch}^2(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int-\left((e+fx)\csc(ic+idx)^2\right)dx}{a}\right)}{a}-$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \; 25$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int(e+fx)\csc(ic+idx)^2dx}{a}\right)}{a}-$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \; 4672$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{if\int-i\coth(c+dx)dx}{d}}{a}\right)}{a}-$$
$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \; 26$$

3.250. $\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int \coth(c+dx)dx}{d}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}+\frac{if\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{6109}$$

$$-\frac{b\left(-\frac{b\left(\frac{\int (e+fx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{3042}$$

3.250. $\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\left(\frac{\int i(e+fx)\csc(ic+idx)dx}{a}-\frac{b\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\left(-\frac{b\left(\frac{i\int(e+fx)\csc(ic+idx)dx}{a}-\frac{b\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \; 3803$$

$$-\frac{b\left(-\frac{b\left(-\frac{2b\int-\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \; 25$$

$$-\frac{b\left(-\frac{b\left(\frac{2b\int\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a}+\frac{i\int(e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)}{a}-\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \; 2694$$

3.250. $\int\frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+d}{d^2}}{a}}{a}$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}}{a}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}}{a}\right)}{a}-$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}}{a}$$

$$\Big\downarrow \textbf{2620}$$

3.250. $\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int(e+fx}{}\right)}{a}}{a}$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\operatorname{arctanh}(e^{c+dx})}{d}+\frac{if\operatorname{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\operatorname{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\operatorname{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}\right.}{bd^2}\right.}{2\sqrt{a^2+b^2}}\right.}{a}\right.}{a}\right.}{a}$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\operatorname{arctanh}(e^{c+dx})}{d}+\frac{if\operatorname{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\operatorname{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\operatorname{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{2d}\right.}{a}$$

3.250. $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\downarrow \text{2838}$$

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\int(e+\cdots}{\cdots}\right)}{a}\cdots\right)}{a}$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\cdots\right.}{a}$$

$$\downarrow \text{4670}$$

3.250. $\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\left(\frac{if}{\cdots}\right.}{\cdots}\right)}{a}\cdots\right)}{a}$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\cdots\right.}{a}$$

$$\Big\downarrow \; 2715$$

$$-\frac{b\left(-\frac{b\left(\frac{2b\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{i\left(\frac{if}{\cdots}\right.}{\cdots}\right)}{a}\cdots\right)}{a}$$

$$\frac{i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{if\text{csch}(c+dx)}{2d^2}-\frac{i(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2d}\cdots\right.}{a}$$

3.250. $\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.250.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.250. $\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 6109
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Csch[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a +
 b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 IGtQ[n, 0]
```

3.250. $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.250.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(386) = 772$.

Time = 1.95 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.05

method	result
risch	$-\frac{adfx\,\mathrm{e}^{3dx+3c}+ade\,\mathrm{e}^{3dx+3c}-2bdfx\,\mathrm{e}^{2dx+2c}+adfx\,\mathrm{e}^{dx+c}+af\,\mathrm{e}^{3dx+3c}-2bde\,\mathrm{e}^{2dx+2c}+ade\,\mathrm{e}^{dx+c}+2bdfx-af\,\mathrm{e}^{dx+c}+2bed}{d^2a^2\left(\mathrm{e}^{2dx+2c}-1\right)^2}+\ f\ln(\mathrm{e}$

input
```
int((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(a*d*f*x*exp(3*d*x+3*c)+a*d*e*exp(3*d*x+3*c)-2*b*d*f*x*exp(2*d*x+2*c)+a*d
*f*x*exp(d*x+c)+a*f*exp(3*d*x+3*c)-2*b*d*e*exp(2*d*x+2*c)+a*d*e*exp(d*x+c)
+2*b*d*f*x-a*f*exp(d*x+c)+2*b*e*d)/d^2/a^2/(exp(2*d*x+2*c)-1)^2+1/2/a/d*f*
ln(exp(d*x+c)+1)*x-1/d/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^
2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+2/d^2/a^2*b*f*ln(exp(d*x+c))+1/d/a^3*b
^2*e*ln(exp(d*x+c)-1)-1/d/a^3*b^2*e*ln(exp(d*x+c)+1)-1/d^2/a^3*b^2*c*f*ln(
exp(d*x+c)-1)-1/d^2/a^2*b*f*ln(exp(d*x+c)-1)-1/d^2/a^2*b*f*ln(exp(d*x+c)+1
)+1/d/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^
2+b^2)^(1/2)))*x-2/d^2/a^3*b^3*c*f/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*
x+c)+2*a)/(a^2+b^2)^(1/2))+1/2/d^2*f/a*dilog(exp(d*x+c))+1/2/d^2*f/a*dilog
(exp(d*x+c)+1)+2/d/a^3*b^3*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2
*a)/(a^2+b^2)^(1/2))+1/d^2/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2
+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((
-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d/a^3*b^2*f*ln(
exp(d*x+c)+1)*x+1/2/a/d^2*c*f*ln(exp(d*x+c)-1)-1/d^2/a^3*b^3*f/(a^2+b^2)^(
1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2/a
^3*b^3*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^
2)^(1/2)))-1/d^2/a^3*b^2*f*dilog(exp(d*x+c))-1/d^2/a^3*b^2*f*dilog(exp(d*x
+c)+1)-1/2/a/d*e*ln(exp(d*x+c)-1)+1/2/a/d*e*ln(exp(d*x+c)+1)
```

3.250.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4720 vs. $2(380) = 760$.

Time = 0.35 (sec) , antiderivative size = 4720, normalized size of antiderivative = 11.24

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(4*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*c*f)*cosh(d*x + c)^4 + 4*(
(a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*c*f)*sinh(d*x + c)^4 - 2*((a^4 + a
^2*b^2)*d*f*x + (a^4 + a^2*b^2)*d*e + (a^4 + a^2*b^2)*f)*cosh(d*x + c)^3 -
 2*((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)*d*e + (a^4 + a^2*b^2)*f - 8*((
a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*c*f)*cosh(d*x + c))*sinh(d*x + c)^3
 - 4*(a^3*b + a*b^3)*d*e + 4*(a^3*b + a*b^3)*c*f - 4*((a^3*b + a*b^3)*d*f*
x - (a^3*b + a*b^3)*d*e + 2*(a^3*b + a*b^3)*c*f)*cosh(d*x + c)^2 - 2*(2*(a
^3*b + a*b^3)*d*f*x - 2*(a^3*b + a*b^3)*d*e + 4*(a^3*b + a*b^3)*c*f - 12*(
(a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*c*f)*cosh(d*x + c)^2 + 3*((a^4 + a
^2*b^2)*d*f*x + (a^4 + a^2*b^2)*d*e + (a^4 + a^2*b^2)*f)*cosh(d*x + c))*si
nh(d*x + c)^2 - 2*(b^4*f*cosh(d*x + c)^4 + 4*b^4*f*cosh(d*x + c)*sinh(d*x
+ c)^3 + b^4*f*sinh(d*x + c)^4 - 2*b^4*f*cosh(d*x + c)^2 + b^4*f + 2*(3*b^
4*f*cosh(d*x + c)^2 - b^4*f)*sinh(d*x + c)^2 + 4*(b^4*f*cosh(d*x + c)^3 -
b^4*f*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*
x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2) - b)/b + 1) + 2*(b^4*f*cosh(d*x + c)^4 + 4*b^4*f*cosh(d*x + c)*
sinh(d*x + c)^3 + b^4*f*sinh(d*x + c)^4 - 2*b^4*f*cosh(d*x + c)^2 + b^4*f
+ 2*(3*b^4*f*cosh(d*x + c)^2 - b^4*f)*sinh(d*x + c)^2 + 4*(b^4*f*cosh(d*x
+ c)^3 - b^4*f*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((
a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))...
```

3.250.6 Sympy [F]

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\operatorname{csch}^3{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*csch(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

3.250.7 Maxima [F]

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)^3}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(8*b^3*integrate(1/4*x*e^(d*x + c)/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x
+ c) - a^3*b), x) + 8*a^2*d*integrate(1/16*x/(a^3*d*e^(d*x + c) + a^3*d),
x) - 16*b^2*d*integrate(1/16*x/(a^3*d*e^(d*x + c) + a^3*d), x) + 8*a^2*d*i
ntegrate(1/16*x/(a^3*d*e^(d*x + c) - a^3*d), x) - 16*b^2*d*integrate(1/16*
x/(a^3*d*e^(d*x + c) - a^3*d), x) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x
+ c) + 1)/(a^3*d^2)) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^
3*d^2)) - (2*b*d*x*e^(2*d*x + 2*c) - 2*b*d*x - (a*d*x*e^(3*c) + a*e^(3*c))
*e^(3*d*x) - (a*d*x*e^c - a*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2
*d^2*e^(2*d*x + 2*c) + a^2*d^2))*f - 1/2*e*(2*b^3*log((b*e^(-d*x - c) - a
- sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2
)*a^3*d) - 2*(a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) -
 2*b)/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 - 2
*b^2)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2 - 2*b^2)*log(e^(-d*x - c) - 1)/
(a^3*d))
```

3.250.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{e+f\,x}{\sinh\left(c+d\,x\right)^3\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.251 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.251.1 Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{(a^2-2b^2)\operatorname{arctanh}(\cosh(c+dx))}{2a^3d} + \frac{2b^3\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

output
```
1/2*(a^2-2*b^2)*arctanh(cosh(d*x+c))/a^3/d+b*coth(d*x+c)/a^2/d-1/2*coth(d*
x+c)*csch(d*x+c)/a/d+2*b^3*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/
2))/a^3/d/(a^2+b^2)^(1/2)
```

3.251.2 Mathematica [A] (verified)

Time = 2.44 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.48

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{16b^3\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4ab\coth\left(\frac{1}{2}(c+dx)\right) + a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right) - 4(a^2-2b^2)\log\left(\cosh\left(\frac{1}{2}(c+\right.\right.}{8a^3d}$$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/8*((16*b^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^
2 - b^2] - 4*a*b*Coth[(c + d*x)/2] + a^2*Csch[(c + d*x)/2]^2 - 4*(a^2 - 2*
b^2)*Log[Cosh[(c + d*x)/2]] + 4*(a^2 - 2*b^2)*Log[Sinh[(c + d*x)/2]] + a^2
*Sech[(c + d*x)/2]^2 - 4*a*b*Tanh[(c + d*x)/2])/(a^3*d)
```

3.251.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.22, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 26, 3281, 26, 3042, 25, 3534, 25, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3281}$$

$$-i\left(\frac{\int -\frac{i\operatorname{csch}^2(c+dx)\left(b\sinh^2(c+dx)+a\sinh(c+dx)+2b\right)}{a+b\sinh(c+dx)}dx}{2a}-\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{i\int \frac{\operatorname{csch}^2(c+dx)\left(b\sinh^2(c+dx)+a\sinh(c+dx)+2b\right)}{a+b\sinh(c+dx)}dx}{2a}-\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{i\int -\frac{-b\sin(ic+idx)^2-ia\sin(ic+idx)+2b}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx}{2a}-\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(\frac{i\int \frac{-b\sin(ic+idx)^2-ia\sin(ic+idx)+2b}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{3534}$$

$$-i\left(\frac{i\left(\frac{\int -\frac{\text{csch}(c+dx)\left(a^2+b\sinh(c+dx)a-2b^2\right)}{a+b\sinh(c+dx)}dx}{a}+\frac{2b\coth(c+dx)}{ad}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{\int \frac{\text{csch}(c+dx)\left(a^2+b\sinh(c+dx)a-2b^2\right)}{a+b\sinh(c+dx)}dx}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{\int \frac{i\left(a^2-ib\sin(ic+idx)a-2b^2\right)}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\int \frac{a^2-ib\sin(ic+idx)a-2b^2}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{3480}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\left(\frac{\left(a^2-2b^2\right)\int -i\text{csch}(c+dx)dx}{a}-\frac{2ib^3\int \frac{1}{a+b\sinh(c+dx)}dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

3.251. $\int \frac{\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\left(-\frac{i\left(a^2-2b^2\right)\int \text{csch}(c+dx)dx}{a}-\frac{2ib^3\int \frac{1}{a+b\sinh(c+dx)}dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\left(-\frac{i\left(a^2-2b^2\right)\int i\csc(ic+idx)dx}{a}-\frac{2ib^3\int \frac{1}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\left(\frac{\left(a^2-2b^2\right)\int \csc(ic+idx)dx}{a}-\frac{2ib^3\int \frac{1}{a-ib\sin(ic+idx)}dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{3139}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\left(\frac{\left(a^2-2b^2\right)\int \csc(ic+idx)dx}{a}-\frac{4b^3\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\right)}{a}\right)}{2a}-\frac{i\coth(c+\cdots}{}\right.$$

$$\Big\downarrow \text{1083}$$

3.251. $\int \frac{\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\left(\frac{\left(a^2-2b^2\right)\int \csc(ic+idx)dx}{a}+\frac{8b^3\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{ad}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\mathrm{cs}}{2ad}\right.$$

$$\Big\downarrow \ 217$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\left(\frac{\left(a^2-2b^2\right)\int \csc(ic+idx)dx}{a}-\frac{4ib^3\operatorname{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \ 4257$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}-\frac{i\left(\frac{i\left(a^2-2b^2\right)\operatorname{arctanh}(\cosh(c+dx))}{ad}-\frac{4ib^3\operatorname{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}\right)$$

input
```
Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

3.251. $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(-I)*(((I/2)*(((-I)*((I*(a^2 - 2*b^2)*ArcTanh[Cosh[c + d*x]])/(a*d) - ((4*
I)*b^3*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*
d)))/a + (2*b*Coth[c + d*x])/(a*d)))/a - ((I/2)*Coth[c + d*x]*Csch[c + d*x
])/(a*d))
```

3.251.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

3.251. $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.251. $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.251.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^{2}}-\frac{2b^{3}\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{a^{3}\sqrt{a^{2}+b^{2}}}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{\left(-2a^{2}+4b^{2}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^{3}}+}{d}$
default	$\frac{\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^{2}}-\frac{2b^{3}\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{a^{3}\sqrt{a^{2}+b^{2}}}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{\left(-2a^{2}+4b^{2}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^{3}}+}{d}$
risch	$-\frac{a\,e^{dx+c}+e^{3dx+3c}a-2b\,e^{2dx+2c}+2b}{a^{2}d\left(e^{2dx+2c}-1\right)^{2}}-\frac{\ln\left(e^{dx+c}-1\right)}{2da}+\frac{\ln\left(e^{dx+c}-1\right)b^{2}}{d\,a^{3}}+\frac{\ln\left(e^{dx+c}+1\right)}{2da}-\frac{\ln\left(e^{dx+c}+1\right)b^{2}}{d\,a^{3}}+\frac{b^{3}}{}$

input
```
int(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4/a^2*(1/2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c))-2/a^3*b
^3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2))-1/8/a/tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(-2*a^2+4*b^2)*ln(tanh(1/2*d*x+1/2
*c))+1/2*b/a^2/tanh(1/2*d*x+1/2*c))
```

3.251.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1203 vs. 2(106) = 212.

Time = 0.30 (sec) , antiderivative size = 1203, normalized size of antiderivative = 10.65

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(4*a^3*b + 4*a*b^3 + 2*(a^4 + a^2*b^2)*cosh(d*x + c)^3 + 2*(a^4 + a^2
*b^2)*sinh(d*x + c)^3 - 4*(a^3*b + a*b^3)*cosh(d*x + c)^2 - 2*(2*a^3*b + 2
*a*b^3 - 3*(a^4 + a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b^3*cosh(d*
x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4 - 2*b
^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^2
 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b
^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) +
 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^
2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*
x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b))
 + 2*(a^4 + a^2*b^2)*cosh(d*x + c) - ((a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c
)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - a^2
*b^2 - 2*b^4)*sinh(d*x + c)^4 + a^4 - a^2*b^2 - 2*b^4 - 2*(a^4 - a^2*b^2 -
 2*b^4)*cosh(d*x + c)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 - a^2*b^2 - 2*
b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*cosh(d*
x + c)^3 - (a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(
d*x + c) + sinh(d*x + c) + 1) + ((a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^4 +
 4*(a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - a^2*b^2
- 2*b^4)*sinh(d*x + c)^4 + a^4 - a^2*b^2 - 2*b^4 - 2*(a^4 - a^2*b^2 - 2*b^
4)*cosh(d*x + c)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 - a^2*b^2 - 2*b^...
```

3.251.6 Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\operatorname{csch}^3{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

3.251.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.87

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b^3\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^3d} + \frac{ae^{(-dx-c)}+2\,be^{(-2\,dx-2\,c)}+ae^{(-3\,dx-3\,c)}-2\,b}{(2\,a^2e^{(-2\,dx-2\,c)}-a^2e^{(-4\,dx-4\,c)}-a^2)d} + \frac{(a^2-2\,b^2)\log\left(e^{(-dx-c)}+1\right)}{2\,a^3d} - \frac{(a^2-2\,b^2)\log\left(e^{(-dx-c)}-1\right)}{2\,a^3d}$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-b^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d) + (a*e^(-d*x - c) + 2*b*e^(-2*d*x -
2*c) + a*e^(-3*d*x - 3*c) - 2*b)/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x
- 4*c) - a^2)*d) + 1/2*(a^2 - 2*b^2)*log(e^(-d*x - c) + 1)/(a^3*d) - 1/2*(
a^2 - 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d)
```

3.251.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2\,b^3\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^3} - \frac{(a^2-2\,b^2)\log\left(e^{(dx+c)}+1\right)}{a^3} + \frac{(a^2-2\,b^2)\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^3} + \frac{2\,(ae^{(3\,dx+3\,c)}-2\,be^{(2\,dx+2\,c)}+ae^{(dx}}{a^2\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*b^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(
d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) - (a^2 - 2*b^2)
*log(e^(d*x + c) + 1)/a^3 + (a^2 - 2*b^2)*log(abs(e^(d*x + c) - 1))/a^3 +
2*(a*e^(3*d*x + 3*c) - 2*b*e^(2*d*x + 2*c) + a*e^(d*x + c) + 2*b)/(a^2*(e^
(2*d*x + 2*c) - 1)^2))/d
```

3.251.9 Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 776, normalized size of antiderivative = 6.87

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{e^{c+dx}}{a\,d-a\,d\,e^{2\,c+2\,d\,x}} - \frac{2\,e^{c+d\,x}}{a\,d-2\,a\,d\,e^{2\,c+2\,d\,x}+a\,d\,e^{4\,c+4\,d\,x}} - \frac{2\,b}{a^2\,d-a^2\,d\,e^{2\,c+2\,d\,x}}$$
$$- \frac{\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,e^{d\,x}\,e^c-24\,b^4\,e^{d\,x}\,e^c+20\,a^2\,b^2\,e^{d\,x}\,e^c\right)}{2\,a\,d}$$
$$+ \frac{\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2+4\,a^4\,e^{d\,x}\,e^c+24\,b^4\,e^{d\,x}\,e^c-20\,a^2\,b^2\,e^{d\,x}\,e^c\right)}{2\,a\,d}$$
$$- \frac{b^3\,\ln\left(16\,a^5\,b-48\,a\,b^5-24\,b^5\,\sqrt{a^2+b^2}-32\,a^3\,b^3-40\,a^2\,b^3\,\sqrt{a^2+b^2}-32\,a^6\,e^{d\,x}\,e^c+24\,b^6\,e^{d\,x}\,e^c+16\,\ldots}{}$$
$$+ \frac{b^3\,\ln\left(24\,b^5\,\sqrt{a^2+b^2}-48\,a\,b^5+16\,a^5\,b-32\,a^3\,b^3+40\,a^2\,b^3\,\sqrt{a^2+b^2}-32\,a^6\,e^{d\,x}\,e^c+24\,b^6\,e^{d\,x}\,e^c-16\,\ldots}{}$$
$$+ \frac{b^2\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,e^{d\,x}\,e^c-24\,b^4\,e^{d\,x}\,e^c+20\,a^2\,b^2\,e^{d\,x}\,e^c\right)}{a^3\,d}$$
$$- \frac{b^2\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2+4\,a^4\,e^{d\,x}\,e^c+24\,b^4\,e^{d\,x}\,e^c-20\,a^2\,b^2\,e^{d\,x}\,e^c\right)}{a^3\,d}$$

input
```
int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
exp(c + d*x)/(a*d - a*d*exp(2*c + 2*d*x)) - (2*exp(c + d*x))/(a*d - 2*a*d*
exp(2*c + 2*d*x) + a*d*exp(4*c + 4*d*x)) - (2*b)/(a^2*d - a^2*d*exp(2*c +
2*d*x)) - log(4*a^4 + 24*b^4 - 20*a^2*b^2 - 4*a^4*exp(d*x)*exp(c) - 24*b^4
*exp(d*x)*exp(c) + 20*a^2*b^2*exp(d*x)*exp(c))/(2*a*d) + log(4*a^4 + 24*b^
4 - 20*a^2*b^2 + 4*a^4*exp(d*x)*exp(c) + 24*b^4*exp(d*x)*exp(c) - 20*a^2*b
^2*exp(d*x)*exp(c))/(2*a*d) - (b^3*log(16*a^5*b - 48*a*b^5 - 24*b^5*(a^2 +
 b^2)^(1/2) - 32*a^3*b^3 - 40*a^2*b^3*(a^2 + b^2)^(1/2) - 32*a^6*exp(d*x)*
exp(c) + 24*b^6*exp(d*x)*exp(c) + 16*a^4*b*(a^2 + b^2)^(1/2) + 112*a^2*b^4
*exp(d*x)*exp(c) + 56*a^4*b^2*exp(d*x)*exp(c) - 32*a^5*exp(d*x)*exp(c)*(a^
2 + b^2)^(1/2) + 72*a*b^4*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 72*a^3*b^2*e
xp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^5*d + a^3*b^2*d) +
 (b^3*log(24*b^5*(a^2 + b^2)^(1/2) - 48*a*b^5 + 16*a^5*b - 32*a^3*b^3 + 40
*a^2*b^3*(a^2 + b^2)^(1/2) - 32*a^6*exp(d*x)*exp(c) + 24*b^6*exp(d*x)*exp(
c) - 16*a^4*b*(a^2 + b^2)^(1/2) + 112*a^2*b^4*exp(d*x)*exp(c) + 56*a^4*b^2
*exp(d*x)*exp(c) + 32*a^5*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) - 72*a*b^4*exp
(d*x)*exp(c)*(a^2 + b^2)^(1/2) - 72*a^3*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1
/2))*(a^2 + b^2)^(1/2))/(a^5*d + a^3*b^2*d) + (b^2*log(4*a^4 + 24*b^4 - 20
*a^2*b^2 - 4*a^4*exp(d*x)*exp(c) - 24*b^4*exp(d*x)*exp(c) + 20*a^2*b^2*exp
(d*x)*exp(c)))/(a^3*d) - (b^2*log(4*a^4 + 24*b^4 - 20*a^2*b^2 + 4*a^4*exp(
d*x)*exp(c) + 24*b^4*exp(d*x)*exp(c) - 20*a^2*b^2*exp(d*x)*exp(c)))/(a^...
```

3.251. $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.252 $\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.252.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.252.2 Mathematica [N/A]

Not integrable

Time = 77.77 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Csch[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Csch[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.252.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$
$$\Big\downarrow \; 6111$$
$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Csch[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.252.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.252.4 Maple [N/A] (verified)

Not integrable

Time = 0.57 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.252.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(csch(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.252.6 Sympy [N/A]

Not integrable

Time = 17.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(csch(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)**3/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.252.7 Maxima [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 757, normalized size of antiderivative = 27.04

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-8*b^3*integrate(-1/4*e^(d*x + c)/(a^3*b*f*x + a^3*b*e - (a^3*b*f*x*e^(2*c
) + a^3*b*e*e^(2*c))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e*e^c)*e^(d*x)), x)
- (2*b*d*f*x + 2*b*d*e + (a*d*f*x*e^(3*c) + (d*e - f)*a*e^(3*c))*e^(3*d*x)
 - 2*(b*d*f*x*e^(2*c) + b*d*e*e^(2*c))*e^(2*d*x) + (a*d*f*x*e^c + (d*e + f
)*a*e^c)*e^(d*x))/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*
d^2*f^2*x^2*e^(4*c) + 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e^(4*
d*x) - 2*(a^2*d^2*f^2*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e^2*
e^(2*c))*e^(2*d*x)) - 8*integrate(1/16*(2*b^2*d^2*e^2 + 2*a*b*d*e*f - (d^2
*e^2 - 2*f^2)*a^2 - (a^2*d^2*f^2 - 2*b^2*d^2*f^2)*x^2 - 2*(a^2*d^2*e*f - 2
*b^2*d^2*e*f - a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^
3*d^2*e^2*f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e
^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) - 8*integrate(-
1/16*(2*b^2*d^2*e^2 - 2*a*b*d*e*f - (d^2*e^2 - 2*f^2)*a^2 - (a^2*d^2*f^2 -
 2*b^2*d^2*f^2)*x^2 - 2*(a^2*d^2*e*f - 2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*
d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 + (a^3
*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d
^2*e^3*e^c)*e^(d*x)), x)
```

3.252.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.252.9 Mupad [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^3\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.252. $\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.253 $\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.253.1 Optimal result

Integrand size = 29, antiderivative size = 139

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i(e+fx)^4}{4af} - \frac{2i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad} - \frac{6if(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2} + \frac{12if^2(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3} - \frac{12if^3\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)}{ad^4}$$

output
```
1/4*I*(f*x+e)^4/a/f-2*I*(f*x+e)^3*ln(1+I*exp(d*x+c))/a/d-6*I*f*(f*x+e)^2*p
olylog(2,-I*exp(d*x+c))/a/d^2+12*I*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/a/
d^3-12*I*f^3*polylog(4,-I*exp(d*x+c))/a/d^4
```

3.253.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.85

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i\left(\frac{(e+fx)^4}{f} - \frac{8(e+fx)^3\log(1+ie^{c+dx})}{d} - \frac{24f\left(d^2(e+fx)^2\operatorname{PolyLog}(2,-ie^{c+dx}) - 2df(e+fx)\operatorname{PolyLog}(3,-ie^{c+dx}) + 2f^2\operatorname{PolyLog}(4,-ie^{c+dx})\right)}{d^4}\right)}{4a}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((I/4)*((e + f*x)^4/f - (8*(e + f*x)^3*Log[1 + I*E^(c + d*x)])/d - (24*f*(
d^2*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)] - 2*d*f*(e + f*x)*PolyLog[3,
(-I)*E^(c + d*x)] + 2*f^2*PolyLog[4, (-I)*E^(c + d*x)]))/d^4))/a
```

3.253.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {6093, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6093}$$

$$2\int \frac{e^{c+dx}(e+fx)^3}{ie^{c+dx}a+a}dx+\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{2620}$$

$$2\left(\frac{3if\int (e+fx)^2\log\left(1+ie^{c+dx}\right)dx}{ad}-\frac{i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{3011}$$

$$2\left(\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}(2,-ie^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-ie^{c+dx})}{d}\right)}{ad}-\frac{i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{7163}$$

$$2\left(\frac{3if\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\int \text{PolyLog}\left(3,-ie^{c+dx}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}(2,-ie^{c+dx})}{d}\right)}{ad}-\frac{i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}\right.$$

$$\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{2720}$$

$$2\left(\frac{3if\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(3,-ie^{c+dx}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}(2,-ie^{c+dx})}{d}\right)}{ad}-\frac{i(e+fx)^3\log\left(\right.}{ad}\right.$$

$$\frac{i(e+fx)^4}{4af}$$

$$\Big\downarrow \text{7143}$$

$$2\left(\frac{3if\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\,\text{PolyLog}\left(4,-ie^{c+dx}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}(2,-ie^{c+dx})}{d}\right)}{ad}-\frac{i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}\right)$$

$$\frac{i(e+fx)^4}{4af}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((I/4)*(e + f*x)^4)/(a*f) + 2*(((-I)*(e + f*x)^3*Log[1 + I*E^(c + d*x)])/(
a*d) + ((3*I)*f*(-(((e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/d) + (2*f*((
(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/d - (f*PolyLog[4, (-I)*E^(c + d*x)
])/d^2))/d))/(a*d))
```

3.253. $\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.253.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6093
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + Simp[2   Int[(e + f*x)^m*(E^(c + d*x)/(a + b*E^(c + d*x))), x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.253. $\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.253.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 646 vs. 2(124) = 248.

Time = 6.17 (sec) , antiderivative size = 647, normalized size of antiderivative = 4.65

method	result
risch	$\frac{if^3x^4}{4a} - \frac{ie^4}{4af} - \frac{ie^3x}{a} + \frac{if^2e\,x^3}{a} + \frac{3if\,e^2x^2}{2a} + \frac{6ie^2fc\ln(\mathrm{e}^{dx+c}-i)}{d^2a} - \frac{6ie^2fc\ln(\mathrm{e}^{dx+c})}{d^2a} - \frac{6if^2ex\,c^2}{d^2a} - \frac{6if^2e\ln(1+i\mathrm{e}^{dx+c}}{da}$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3/2*I/d^4/a*f^3*c^4+2*I/d/a*ln(exp(d*x+c))*e^3-2*I/d/a*ln(exp(d*x+c)-I)*e^
3+I/a*f^2*e*x^3-12*I*f^3*polylog(4,-I*exp(d*x+c))/a/d^4+3/2*I/a*f*e^2*x^2-
I/a*e^3*x-1/4*I/a/f*e^4+6*I/d^2/a*e^2*f*c*ln(exp(d*x+c)-I)-6*I/d^2/a*e^2*f
*c*ln(exp(d*x+c))-6*I/d^2/a*f^2*e*x*c^2-6*I/d/a*f^2*e*ln(1+I*exp(d*x+c))*x
^2+6*I/d^3/a*f^2*e*ln(1+I*exp(d*x+c))*c^2-12*I/d^2/a*f^2*e*polylog(2,-I*ex
p(d*x+c))*x+6*I/d/a*f*e^2*c*x-6*I/d/a*f*e^2*ln(1+I*exp(d*x+c))*x-6*I/d^2/a
*f*e^2*ln(1+I*exp(d*x+c))*c-6*I/d^3/a*e*f^2*c^2*ln(exp(d*x+c)-I)+6*I/d^3/a
*e*f^2*c^2*ln(exp(d*x+c))-2*I/d^4/a*f^3*ln(1+I*exp(d*x+c))*c^3-6*I/d^2/a*f
^3*polylog(2,-I*exp(d*x+c))*x^2+3*I/d^2/a*f*e^2*c^2-6*I/d^2/a*f*e^2*polylo
g(2,-I*exp(d*x+c))+2*I/d^4/a*f^3*c^3*ln(exp(d*x+c)-I)-2*I/d^4/a*f^3*c^3*ln
(exp(d*x+c))-4*I/d^3/a*f^2*e*c^3+12*I/d^3/a*f^3*polylog(3,-I*exp(d*x+c))*x
+2*I/d^3/a*f^3*c^3*x-2*I/d/a*f^3*ln(1+I*exp(d*x+c))*x^3+1/4*I/a*f^3*x^4+12
*I/d^3/a*f^2*e*polylog(3,-I*exp(d*x+c))
```

3.253.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(114) = 228.

Time = 0.25 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.15

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{i\,d^4f^3x^4 + 4i\,d^4ef^2x^3 + 6i\,d^4e^2fx^2 + 4i\,d^4e^3x + 8i\,cd^3e^3 - 12i\,c^2d^2e^2f + 8i\,c^3def^2 - 2i\,c^4f^3 - 48i\,f^3\mathrm{poly}}{}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(I*d^4*f^3*x^4 + 4*I*d^4*e*f^2*x^3 + 6*I*d^4*e^2*f*x^2 + 4*I*d^4*e^3*x
 + 8*I*c*d^3*e^3 - 12*I*c^2*d^2*e^2*f + 8*I*c^3*d*e*f^2 - 2*I*c^4*f^3 - 48
*I*f^3*polylog(4, -I*e^(d*x + c)) - 24*(I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x +
I*d^2*e^2*f)*dilog(-I*e^(d*x + c)) - 8*(I*d^3*e^3 - 3*I*c*d^2*e^2*f + 3*I*
c^2*d*e*f^2 - I*c^3*f^3)*log(e^(d*x + c) - I) - 8*(I*d^3*f^3*x^3 + 3*I*d^3
*e*f^2*x^2 + 3*I*d^3*e^2*f*x + 3*I*c*d^2*e^2*f - 3*I*c^2*d*e*f^2 + I*c^3*f
^3)*log(I*e^(d*x + c) + 1) - 48*(-I*d*f^3*x - I*d*e*f^2)*polylog(3, -I*e^(
d*x + c)))/(a*d^4)
```

3.253.6 Sympy [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^3\cosh(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{f^3x^3\cosh(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{3ef^2x^2\cosh(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{3e^2fx\cosh(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**3*cosh(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**3*x**
*3*cosh(c + d*x)/(sinh(c + d*x) - I), x) + Integral(3*e*f**2*x**2*cosh(c +
 d*x)/(sinh(c + d*x) - I), x) + Integral(3*e**2*f*x*cosh(c + d*x)/(sinh(c
+ d*x) - I), x))/a
```

3.253.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(114) = 228$.

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.90

$$\begin{aligned}\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{i\,e^3\log\left(i\,a\sinh\left(dx+c\right)+a\right)}{ad}\\ &- \frac{6i\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)\right)e^2f}{ad^2} - \frac{i\left(f^3x^4+4\,ef^2x^3+6\,e^2fx^2\right)}{4\,a}\\ &- \frac{6i\left(d^2x^2\log\left(i\,e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-2\,\mathrm{Li}_3\left(-i\,e^{(dx+c)}\right)\right)ef^2}{ad^3}\\ &- \frac{2i\left(d^3x^3\log\left(i\,e^{(dx+c)}+1\right)+3\,d^2x^2\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-6\,dx\mathrm{Li}_3\left(-i\,e^{(dx+c)}\right)+6\,\mathrm{Li}_4\left(-i\,e^{(dx+c)}\right)\right)f^3}{ad^4}\\ &+ \frac{i\,d^4f^3x^4+4i\,d^4ef^2x^3+6i\,d^4e^2fx^2}{2\,ad^4}\end{aligned}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-I*e^3*log(I*a*sinh(d*x + c) + a)/(a*d) - 6*I*(d*x*log(I*e^(d*x + c) + 1)
+ dilog(-I*e^(d*x + c)))*e^2*f/(a*d^2) - 1/4*I*(f^3*x^4 + 4*e*f^2*x^3 + 6*
e^2*f*x^2)/a - 6*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x
 + c)) - 2*polylog(3, -I*e^(d*x + c)))*e*f^2/(a*d^3) - 2*I*(d^3*x^3*log(I*
e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-I*e^(d*x + c)) - 6*d*x*polylog(3, -I*e
^(d*x + c)) + 6*polylog(4, -I*e^(d*x + c)))*f^3/(a*d^4) + 1/2*(I*d^4*f^3*x
^4 + 4*I*d^4*e*f^2*x^3 + 6*I*d^4*e^2*f*x^2)/(a*d^4)
```

3.253.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^3}{a+a\sinh(c+d\,x)\,1\mathrm{i}}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i), x)
```

3.254 $\int \frac{(e+fx)^2 \cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.254.1 Optimal result

Integrand size = 29, antiderivative size = 106

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{i(e+fx)^3}{3af}-\frac{2i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}-\frac{4if(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}+\frac{4if^2\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3}$$

output
```
1/3*I*(f*x+e)^3/a/f-2*I*(f*x+e)^2*ln(1+I*exp(d*x+c))/a/d-4*I*f*(f*x+e)*pol
ylog(2,-I*exp(d*x+c))/a/d^2+4*I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3
```

3.254.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i\left(d^2(e+fx)^2\left(d(e+fx)-6f\log\left(1+ie^{c+dx}\right)\right)-12df^2(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)+12f^3\,\mathrm{PolyLog}\left(3\right.\right.}{3ad^3f}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((I/3)*(d^2*(e + f*x)^2*(d*(e + f*x) - 6*f*Log[1 + I*E^(c + d*x)]) - 12*d*
f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)] + 12*f^3*PolyLog[3, (-I)*E^(c +
 d*x)]))/(a*d^3*f)
```

3.254.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {6093, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6093}$$

$$2\int \frac{e^{c+dx}(e+fx)^2}{ie^{c+dx}a+a}dx+\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{2620}$$

$$2\left(\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{ad}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{3011}$$

$$2\left(\frac{2if\left(\frac{f\int \text{PolyLog}(2,-ie^{c+dx})dx}{d}-\frac{(e+fx)\text{PolyLog}(2,-ie^{c+dx})}{d}\right)}{ad}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{2720}$$

$$2\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}(2,-ie^{c+dx})de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}(2,-ie^{c+dx})}{d}\right)}{ad}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^3}{3af}$$

$$\Big\downarrow \text{7143}$$

$$2\left(\frac{2if\left(\frac{f\,\mathrm{PolyLog}(3,-ie^{c+dx})}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}(2,-ie^{c+dx})}{d}\right)}{ad}-\frac{i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^3}{3af}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((I/3)*(e + f*x)^3)/(a*f) + 2*(((-I)*(e + f*x)^2*Log[1 + I*E^(c + d*x)])/(
a*d) + ((2*I)*f*(-(((e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d) + (f*PolyLo
g[3, (-I)*E^(c + d*x)])/d^2))/(a*d))
```

3.254.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6093
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + Simp[2   Int[(e + f*x)^m*(E^(c + d*x)/(a + b*E^(c + d*x))), x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.254.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 404 vs. 2(94) = 188.

Time = 2.90 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.82

method	result
risch	$-\frac{4if^2c^3}{3d^3a}+\frac{2if^2c^2\ln(\mathrm{e}^{dx+c})}{d^3a}-\frac{4iefc\ln(\mathrm{e}^{dx+c})}{d^2a}-\frac{2if^2c^2\ln(\mathrm{e}^{dx+c}-i)}{d^3a}+\frac{2ief\,c^2}{d^2a}+\frac{ife\,x^2}{a}-\frac{ie^3}{3af}-\frac{4ief\ln(1+i\mathrm{e}^{dx+c})c}{d^2a}$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-4/3*I/d^3/a*f^2*c^3+2*I/d^3/a*f^2*c^2*ln(exp(d*x+c))-4*I/d^2/a*e*f*c*ln(e
xp(d*x+c))-2*I/d^3/a*f^2*c^2*ln(exp(d*x+c)-I)+2*I/d^2/a*e*f*c^2+I/a*f*e*x^
2-1/3*I/a/f*e^3-4*I/d^2/a*e*f*ln(1+I*exp(d*x+c))*c+2*I/d^3/a*f^2*ln(1+I*ex
p(d*x+c))*c^2+2*I/d/a*ln(exp(d*x+c))*e^2+4*I/d/a*e*f*c*x-2*I/d^2/a*f^2*x*c
^2+4*I/d^2/a*e*f*c*ln(exp(d*x+c)-I)-4*I/d^2/a*e*f*polylog(2,-I*exp(d*x+c))
-I/a*e^2*x-4*I/d^2/a*f^2*polylog(2,-I*exp(d*x+c))*x+4*I*f^2*polylog(3,-I*e
xp(d*x+c))/a/d^3-2*I/d/a*ln(exp(d*x+c)-I)*e^2+1/3*I/a*f^2*x^3-2*I/d/a*f^2*
ln(1+I*exp(d*x+c))*x^2-4*I/d/a*e*f*ln(1+I*exp(d*x+c))*x
```

3.254.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(86) = 172.

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.74

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i\,d^3f^2x^3+3i\,d^3efx^2+3i\,d^3e^2x+6i\,cd^2e^2-6i\,c^2def+2i\,c^3f^2+12i\,f^2\mathrm{polylog}\left(3,-i\,e^{(dx+c)}\right)-12\,(i\,df^2x}{}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(I*d^3*f^2*x^3 + 3*I*d^3*e*f*x^2 + 3*I*d^3*e^2*x + 6*I*c*d^2*e^2 - 6*I
*c^2*d*e*f + 2*I*c^3*f^2 + 12*I*f^2*polylog(3, -I*e^(d*x + c)) - 12*(I*d*f
^2*x + I*d*e*f)*dilog(-I*e^(d*x + c)) - 6*(I*d^2*e^2 - 2*I*c*d*e*f + I*c^2
*f^2)*log(e^(d*x + c) - I) - 6*(I*d^2*f^2*x^2 + 2*I*d^2*e*f*x + 2*I*c*d*e*
f - I*c^2*f^2)*log(I*e^(d*x + c) + 1))/(a*d^3)
```

3.254.6 Sympy [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^2\cosh(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{f^2x^2\cosh(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{2efx\cosh(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**2*cosh(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**2*x**
*2*cosh(c + d*x)/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*cosh(c + d*x)/
(sinh(c + d*x) - I), x))/a
```

3.254.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.55

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\,e^2\log\left(i\,a\sinh\left(dx+c\right)+a\right)}{ad} - \frac{i\,f^2x^3+3i\,efx^2}{3\,a} - \frac{4i\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)\right)ef}{ad^2} - \frac{2i\left(d^2x^2\log\left(i\,e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-2\,\mathrm{Li}_3\left(-i\,e^{(dx+c)}\right)\right)f^2}{ad^3} - \frac{2\left(-i\,d^3f^2x^3-3i\,d^3efx^2\right)}{3\,ad^3}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-I*e^2*log(I*a*sinh(d*x + c) + a)/(a*d) - 1/3*(I*f^2*x^3 + 3*I*e*f*x^2)/a
- 4*I*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*e*f/(a*d^2) - 2
*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*polyl
og(3, -I*e^(d*x + c)))*f^2/(a*d^3) - 2/3*(-I*d^3*f^2*x^3 - 3*I*d^3*e*f*x^2
)/(a*d^3)
```

3.254.8 Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)\,(e+f\,x)^2}{a+a\sinh(c+d\,x)\,1\mathrm{i}}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i), x)
```

3.255 $\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.255.1 Optimal result

Integrand size = 27, antiderivative size = 73

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i(e+fx)^2}{2af} - \frac{2i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad} - \frac{2if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}$$

output
```
1/2*I*(f*x+e)^2/a/f-2*I*(f*x+e)*ln(1+I*exp(d*x+c))/a/d-2*I*f*polylog(2,-I*
exp(d*x+c))/a/d^2
```

3.255.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i\left(d(e+fx)\left(d(e+fx)-4f\log\left(1+ie^{c+dx}\right)\right)-4f^2\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)\right)}{2ad^2f}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((I/2)*(d*(e + f*x)*(d*(e + f*x) - 4*f*Log[1 + I*E^(c + d*x)]) - 4*f^2*Pol
yLog[2, (-I)*E^(c + d*x)]))/(a*d^2*f)
```

3.255.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6093, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6093}$$

$$2\int \frac{e^{c+dx}(e+fx)}{ie^{c+dx}a+a}dx+\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \text{2620}$$

$$2\left(\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{ad}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \text{2715}$$

$$2\left(\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{ad^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^2}{2af}$$

$$\Big\downarrow \text{2838}$$

$$2\left(-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}-\frac{i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad}\right)+\frac{i(e+fx)^2}{2af}$$

input
```
Int[((e + f*x)*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((I/2)*(e + f*x)^2)/(a*f) + 2*(((-I)*(e + f*x)*Log[1 + I*E^(c + d*x)])/(a*
d) - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2))
```

3.255.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6093
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + Simp[2   Int[(e + f*x)^m*(E^(c + d*x)/(a + b*E^(c + d*x))), x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]
```

3.255.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(64) = 128$.

Time = 1.83 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.58

method	result
risch	$\frac{if\,x^2}{2a} - \frac{iex}{a} - \frac{2i\ln\left(e^{dx+c}-i\right)e}{da} + \frac{2i\ln\left(e^{dx+c}\right)e}{da} + \frac{2ifcx}{da} + \frac{if\,c^2}{d^2a} - \frac{2if\ln\left(1+ie^{dx+c}\right)x}{da} - \frac{2if\ln\left(1+ie^{dx+c}\right)c}{d^2a} - 2if\,\mathrm{poly}$

input
```
int((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*f*x^2/a-I*e*x/a-2*I/d/a*ln(exp(d*x+c)-I)*e+2*I/d/a*ln(exp(d*x+c))*e+
2*I/d/a*f*c*x+I/d^2/a*f*c^2-2*I/d/a*f*ln(1+I*exp(d*x+c))*x-2*I/d^2/a*f*ln(
1+I*exp(d*x+c))*c-2*I*f*polylog(2,-I*exp(d*x+c))/a/d^2+2*I/d^2/a*c*f*ln(ex
p(d*x+c)-I)-2*I/d^2/a*c*f*ln(exp(d*x+c))
```

3.255.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{i\,d^2fx^2+2i\,d^2ex+4i\,cde-2i\,c^2f-4i\,f\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-4\left(i\,de-i\,cf\right)\log\left(e^{(dx+c)}-i\right)-4\left(i\,dfx+i\,cf\right.}{2\,ad^2}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(I*d^2*f*x^2 + 2*I*d^2*e*x + 4*I*c*d*e - 2*I*c^2*f - 4*I*f*dilog(-I*e^
(d*x + c)) - 4*(I*d*e - I*c*f)*log(e^(d*x + c) - I) - 4*(I*d*f*x + I*c*f)*
log(I*e^(d*x + c) + 1))/(a*d^2)
```

3.255.6 Sympy [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e\cosh(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{fx\cosh(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e*cosh(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f*x*cosh(c
 + d*x)/(sinh(c + d*x) - I), x))/a
```

3.255.7 Maxima [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*f*(-I*x^2/a + 4*integrate(x/(a*e^(d*x + c) - I*a), x)) - I*e*log(I*a*s
inh(d*x + c) + a)/(a*d)
```

3.255.8 Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,(e+f\,x)}{a+a\sinh(c+d\,x)\,1\mathrm{i}}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

output
```
int((cosh(c + d*x)*(e + f*x))/(a + a*sinh(c + d*x)*1i), x)
```

3.256 $\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.256.1 Optimal result

Integrand size = 22, antiderivative size = 23

$$\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\log(i-\sinh(c+dx))}{ad}$$

output
```
-I*ln(I-sinh(d*x+c))/a/d
```

3.256.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\log(i-\sinh(c+dx))}{ad}$$

input
```
Integrate[Cosh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-I)*Log[I - Sinh[c + d*x]])/(a*d)
```

3.256.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\cos(ic+idx)}{a+a\sin(ic+idx)}dx$$
$$\Big\downarrow \ 3146$$
$$-\frac{i\int \frac{1}{i\sinh(c+dx)a+a}d(ia\sinh(c+dx))}{ad}$$
$$\Big\downarrow \ 16$$
$$-\frac{i\log(a+ia\sinh(c+dx))}{ad}$$

input
```
Int[Cosh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-I)*Log[a + I*a*Sinh[c + d*x]])/(a*d)
```

3.256.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.256.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

method	result	size
risch	$\frac{ix}{a}+\frac{2ic}{ad}-\frac{2i\ln\left(\mathrm{e}^{dx+c}-i\right)}{ad}$	38
derivativedivides	$-\frac{i\ln\left(a^2\sinh(dx+c)^2+a^2\right)}{2da}+\frac{\arctan(\sinh(dx+c))}{ad}$	42
default	$-\frac{i\ln\left(a^2\sinh(dx+c)^2+a^2\right)}{2da}+\frac{\arctan(\sinh(dx+c))}{ad}$	42

input
```
int(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
I*x/a+2*I/a/d*c-2*I/a/d*ln(exp(d*x+c)-I)
```

3.256.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i\,dx-2i\,\log\left(e^{(dx+c)}-i\right)}{ad}$$

input
```
integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(I*d*x - 2*I*log(e^(d*x + c) - I))/(a*d)
```

3.256.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{ix}{a} - \frac{2i\log\left(e^{dx}-ie^{-c}\right)}{ad}$$

input
```
integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
I*x/a - 2*I*log(exp(d*x) - I*exp(-c))/(a*d)
```

3.256.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\,\log\left(i\,a\sinh\left(dx+c\right)+a\right)}{ad}$$

input
```
integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-I*log(I*a*sinh(d*x + c) + a)/(a*d)
```

3.256.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{-\frac{i\,(dx+c)}{a}+\frac{2i\,\log\left(e^{(dx+c)}-i\right)}{a}}{d}$$

input
```
integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(-I*(d*x + c)/a + 2*I*log(e^(d*x + c) - I)/a)/d
```

3.256.9 Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\ln\left(\sinh(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{a\,d}$$

input
```
int(cosh(c + d*x)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
-(log(sinh(c + d*x) - 1i)*1i)/(a*d)
```

3.257 $\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

3.257.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.257.2 Mathematica [N/A]

Not integrable

Time = 6.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Cosh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Cosh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

3.257.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Cosh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.257.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.257.4 Maple [N/A] (verified)

Not integrable

Time = 0.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\cosh\left(dx+c\right)}{\left(fx+e\right)\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.257.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((-I*e^(d*x + c) + 1)/(-I*a*f*x - I*a*e + (a*f*x + a*e)*e^(d*x + c
)), x)
```

3.257.6 Sympy [N/A]

Not integrable

Time = 4.97 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\cosh(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

input
```
integrate(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(cosh(c + d*x)/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) - I*f
*x), x)/a
```

3.257.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-I*log(f*x + e)/(a*f) + 2*integrate(1/(-I*a*f*x - I*a*e + (a*f*x*e^c + a*e
*e^c)*e^(d*x)), x)
```

3.257.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

3.257.9 Mupad [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)}{(e+f\,x)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(cosh(c + d*x)/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(cosh(c + d*x)/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.258 $\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

3.258.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.258.2 Mathematica [N/A]

Not integrable

Time = 26.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Cosh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Cosh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

3.258.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \ 6111$$
$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Cosh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.258.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.258.4 Maple [N/A] (verified)

Not integrable

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\cosh\left(dx+c\right)}{\left(fx+e\right)^{2}\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.258.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.21

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((-I*e^(d*x + c) + 1)/(-I*a*f^2*x^2 - 2*I*a*e*f*x - I*a*e^2 + (a*f
^2*x^2 + 2*a*e*f*x + a*e^2)*e^(d*x + c)), x)
```

3.258.6 Sympy [N/A]

Not integrable

Time = 22.51 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.45

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\cosh(c+dx)}{e^2\sinh(c+dx)-ie^2+2efx\sinh(c+dx)-2iefx+f^2x^2\sinh(c+dx)-if^2x^2}\,dx}{a}$$

input
```
integrate(cosh(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(cosh(c + d*x)/(e**2*sinh(c + d*x) - I*e**2 + 2*e*f*x*sinh(c +
d*x) - 2*I*e*f*x + f**2*x**2*sinh(c + d*x) - I*f**2*x**2), x)/a
```

3.258.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.59

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
I/(a*f^2*x + a*e*f) + 2*integrate(1/(-I*a*f^2*x^2 - 2*I*a*e*f*x - I*a*e^2
+ (a*f^2*x^2*e^c + 2*a*e*f*x*e^c + a*e^2*e^c)*e^(d*x)), x)
```

3.258.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(fx+e\right)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

3.258.9 Mupad [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)}{{\left(e+f\,x\right)}^2\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int(cosh(c + d*x)/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(cosh(c + d*x)/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.259 $\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.259.1 Optimal result

Integrand size = 31, antiderivative size = 108

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{6if^3\sinh(c+dx)}{ad^4} + \frac{3if(e+fx)^2\sinh(c+dx)}{ad^2}$$

output
```
1/4*(f*x+e)^4/a/f-6*I*f^2*(f*x+e)*cosh(d*x+c)/a/d^3-I*(f*x+e)^3*cosh(d*x+c
)/a/d+6*I*f^3*sinh(d*x+c)/a/d^4+3*I*f*(f*x+e)^2*sinh(d*x+c)/a/d^2
```

3.259.2 Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.98

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{d^4x(4e^3+6e^2fx+4ef^2x^2+f^3x^3) - 4id(e+fx)\left(6f^2+d^2(e+fx)^2\right)\cosh(c+dx) + 12if\left(2f^2+d^2(e+\right.}{4ad^4}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) - (4*I)*d*(e + f*x)*(6*
f^2 + d^2*(e + f*x)^2)*Cosh[c + d*x] + (12*I)*f*(2*f^2 + d^2*(e + f*x)^2)*
Sinh[c + d*x])/(4*a*d^4)
```

3.259.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {6097, 17, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6097}$$

$$\frac{\int (e+fx)^3dx}{a}-\frac{i\int (e+fx)^3\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(e+fx)^4}{4af}-\frac{i\int (e+fx)^3\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^4}{4af}-\frac{i\int -i(e+fx)^3\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{(e+fx)^4}{4af}-\frac{\int (e+fx)^3\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\cosh(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{3117}$$

$$\frac{(e+fx)^4}{4af}-\frac{\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(e + f*x)^4/(4*a*f) - ((I*(e + f*x)^3*Cosh[c + d*x])/d - ((3*I)*f*(((e + f
*x)^2*Sinh[c + d*x])/d + ((2*I)*f*((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Si
nh[c + d*x])/d^2))/d))/d)/a
```

3.259. $\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.259.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

3.259. $\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.259.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(102) = 204.

Time = 15.66 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.46

method	result
risch	$\frac{f^3x^4}{4a}+\frac{f^2e\,x^3}{a}+\frac{3f\,e^2x^2}{2a}+\frac{e^3x}{a}+\frac{e^4}{4af}-\frac{i(d^3x^3f^3+3d^3e\,f^2x^2+3d^3e^2f x-3d^2f^3x^2+e^3d^3-6d^2e\,f^2x-3d^2e^2f+}{2d^4a}$
derivativedivides	$-\frac{-3ic\,f^3\left((dx+c)^2\cosh(dx+c)-2(dx+c)\sinh(dx+c)+2\cosh(dx+c)\right)+3id^2e^2f((dx+c)\cosh(dx+c)-\sinh(dx+c))+3i}{}$
default	$-\frac{-3ic\,f^3\left((dx+c)^2\cosh(dx+c)-2(dx+c)\sinh(dx+c)+2\cosh(dx+c)\right)+3id^2e^2f((dx+c)\cosh(dx+c)-\sinh(dx+c))+3i}{}$

input
```
int((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*f^3*x^4+1/a*f^2*e*x^3+3/2/a*f*e^2*x^2+1/a*e^3*x+1/4/a/f*e^4-1/2*I*(d
^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x-3*d^2*f^3*x^2+d^3*e^3-6*d^2*e*f^2
*x-3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-6*f^3)/d^4/a*exp(d*x+c)-1/2*I*(d^3*f^3*
x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x+3*d^2*f^3*x^2+d^3*e^3+6*d^2*e*f^2*x+3*d^
2*e^2*f+6*d*f^3*x+6*d*e*f^2+6*f^3)/d^4/a*exp(-d*x-c)
```

3.259.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(98) = 196.

Time = 0.25 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.44

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{\left(-2i\,d^3f^3x^3-2i\,d^3e^3-6i\,d^2e^2f-12i\,def^2-12i\,f^3-6\left(i\,d^3ef^2+i\,d^2f^3\right)x^2-6\left(i\,d^3e^2f+2i\,d^2ef^2+2i\right.}{}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/4*(-2*I*d^3*f^3*x^3 - 2*I*d^3*e^3 - 6*I*d^2*e^2*f - 12*I*d*e*f^2 - 12*I*
f^3 - 6*(I*d^3*e*f^2 + I*d^2*f^3)*x^2 - 6*(I*d^3*e^2*f + 2*I*d^2*e*f^2 + 2
*I*d*f^3)*x - 2*(I*d^3*f^3*x^3 + I*d^3*e^3 - 3*I*d^2*e^2*f + 6*I*d*e*f^2 -
 6*I*f^3 + 3*(I*d^3*e*f^2 - I*d^2*f^3)*x^2 + 3*(I*d^3*e^2*f - 2*I*d^2*e*f^
2 + 2*I*d*f^3)*x)*e^(2*d*x + 2*c) + (d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4
*e^2*f*x^2 + 4*d^4*e^3*x)*e^(d*x + c))*e^(-d*x - c)/(a*d^4)
```

3.259.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 518, normalized size of antiderivative = 4.80

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \begin{cases} \frac{((-2iad^7e^3-6iad^7e^2fx-6iad^7ef^2x^2-2iad^7f^3x^3-6iad^6e^2f-12iad^6ef^2x-6iad^6f^3x^2-12iad^5ef^2-12iad^5f^3x-12iad^4f^3)e^{-dx}+(-2iad^7e^3}{} \\ \frac{x^4(-if^3e^{2c}+if^3)e^{-c}}{8a}+\frac{x^3(-ief^2e^{2c}+ief^2)e^{-c}}{2a}+\frac{x^2(-3ie^2fe^{2c}+3ie^2f)e^{-c}}{4a}+\frac{x(-ie^3e^{2c}+ie^3)e^{-c}}{2a} \end{cases}$$
$$+\frac{e^3x}{a}+\frac{3e^2fx^2}{2a}+\frac{ef^2x^3}{a}+\frac{f^3x^4}{4a}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise((((-2*I*a*d**7*e**3 - 6*I*a*d**7*e**2*f*x - 6*I*a*d**7*e*f**2*x*
*2 - 2*I*a*d**7*f**3*x**3 - 6*I*a*d**6*e**2*f - 12*I*a*d**6*e*f**2*x - 6*I
*a*d**6*f**3*x**2 - 12*I*a*d**5*e*f**2 - 12*I*a*d**5*f**3*x - 12*I*a*d**4*
f**3)*exp(-d*x) + (-2*I*a*d**7*e**3*exp(2*c) - 6*I*a*d**7*e**2*f*x*exp(2*c
) - 6*I*a*d**7*e*f**2*x**2*exp(2*c) - 2*I*a*d**7*f**3*x**3*exp(2*c) + 6*I*
a*d**6*e**2*f*exp(2*c) + 12*I*a*d**6*e*f**2*x*exp(2*c) + 6*I*a*d**6*f**3*x
**2*exp(2*c) - 12*I*a*d**5*e*f**2*exp(2*c) - 12*I*a*d**5*f**3*x*exp(2*c) +
 12*I*a*d**4*f**3*exp(2*c))*exp(d*x))*exp(-c)/(4*a**2*d**8), Ne(a**2*d**8*
exp(c), 0)), (x**4*(-I*f**3*exp(2*c) + I*f**3)*exp(-c)/(8*a) + x**3*(-I*e*
f**2*exp(2*c) + I*e*f**2)*exp(-c)/(2*a) + x**2*(-3*I*e**2*f*exp(2*c) + 3*I
*e**2*f)*exp(-c)/(4*a) + x*(-I*e**3*exp(2*c) + I*e**3)*exp(-c)/(2*a), True
)) + e**3*x/a + 3*e**2*f*x**2/(2*a) + e*f**2*x**3/a + f**3*x**4/(4*a)
```

3.259.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(98) = 196.

Time = 0.35 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.45

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{3}{2}e^2f\left(\frac{2\,xe^{(dx+c)}}{ade^{(dx+c)}-i\,ad} - \frac{i\,d^2x^2e^c + i\,dxe^c - \left(-i\,dxe^{(3\,c)} + i\,e^{(3\,c)}\right)e^{(2\,dx)} - \left(d^2x^2e^{(2\,c)} - 3\,dxe^{(2\,c)} + e^{(2\,c)}\right)e}{ad^2e^{(dx+2\,c)} - i\,ad^2e^c}\right.$$
$$+ \frac{1}{2}e^3\left(\frac{2\,(dx+c)}{ad} - \frac{i\,e^{(dx+c)}}{ad} - \frac{i\,e^{(-dx-c)}}{ad}\right)$$
$$+ \frac{\left(2\,d^3x^3e^c + 3\left(-i\,d^2x^2e^{(2\,c)} + 2i\,dxe^{(2\,c)} - 2i\,e^{(2\,c)}\right)e^{(dx)} + 3\left(-i\,d^2x^2 - 2i\,dx - 2i\right)e^{(-dx)}\right)ef^2e^{(-c)}}{2\,ad^3}$$
$$+ \frac{\left(d^4x^4e^c + 2\left(-i\,d^3x^3e^{(2\,c)} + 3i\,d^2x^2e^{(2\,c)} - 6i\,dxe^{(2\,c)} + 6i\,e^{(2\,c)}\right)e^{(dx)} + 2\left(-i\,d^3x^3 - 3i\,d^2x^2 - 6i\,dx - 6i\right)\right.}{4\,ad^4}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
3/2*e^2*f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) - (I*d^2*x^2*e^c + I*
d*x*e^c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d*
x*e^(2*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x +
 2*c) - I*a*d^2*e^c)) + 1/2*e^3*(2*(d*x + c)/(a*d) - I*e^(d*x + c)/(a*d) -
 I*e^(-d*x - c)/(a*d)) + 1/2*(2*d^3*x^3*e^c + 3*(-I*d^2*x^2*e^(2*c) + 2*I*
d*x*e^(2*c) - 2*I*e^(2*c))*e^(d*x) + 3*(-I*d^2*x^2 - 2*I*d*x - 2*I)*e^(-d*
x))*e*f^2*e^(-c)/(a*d^3) + 1/4*(d^4*x^4*e^c + 2*(-I*d^3*x^3*e^(2*c) + 3*I*
d^2*x^2*e^(2*c) - 6*I*d*x*e^(2*c) + 6*I*e^(2*c))*e^(d*x) + 2*(-I*d^3*x^3 -
 3*I*d^2*x^2 - 6*I*d*x - 6*I)*e^(-d*x))*f^3*e^(-c)/(a*d^4)
```

3.259.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(98) = 196.

Time = 0.29 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.29

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(d^4f^3x^4e^{(dx+c)} + 4\,d^4ef^2x^3e^{(dx+c)} - 2i\,d^3f^3x^3e^{(2\,dx+2\,c)} + 6\,d^4e^2fx^2e^{(dx+c)} - 2i\,d^3f^3x^3 - 6i\,d^3ef^2x^2e^{(2\,dx+2}\right.}{}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(d^4*f^3*x^4*e^(d*x + c) + 4*d^4*e*f^2*x^3*e^(d*x + c) - 2*I*d^3*f^3*x
^3*e^(2*d*x + 2*c) + 6*d^4*e^2*f*x^2*e^(d*x + c) - 2*I*d^3*f^3*x^3 - 6*I*d
^3*e*f^2*x^2*e^(2*d*x + 2*c) + 4*d^4*e^3*x*e^(d*x + c) - 6*I*d^3*e*f^2*x^2
 - 6*I*d^3*e^2*f*x*e^(2*d*x + 2*c) + 6*I*d^2*f^3*x^2*e^(2*d*x + 2*c) - 6*I
*d^3*e^2*f*x - 6*I*d^2*f^3*x^2 - 2*I*d^3*e^3*e^(2*d*x + 2*c) + 12*I*d^2*e*
f^2*x*e^(2*d*x + 2*c) - 2*I*d^3*e^3 - 12*I*d^2*e*f^2*x + 6*I*d^2*e^2*f*e^(
2*d*x + 2*c) - 12*I*d*f^3*x*e^(2*d*x + 2*c) - 6*I*d^2*e^2*f - 12*I*d*f^3*x
 - 12*I*d*e*f^2*e^(2*d*x + 2*c) - 12*I*d*e*f^2 + 12*I*f^3*e^(2*d*x + 2*c)
- 12*I*f^3)*e^(-d*x - c)/(a*d^4)
```

3.259.9 Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.49

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \mathrm{e}^{c+d\,x}\left(\frac{\left(-d^3\,e^3+3\,d^2\,e^2\,f-6\,d\,e\,f^2+6\,f^3\right)\,1\mathrm{i}}{2\,a\,d^4}-\frac{f^3\,x^3\,1\mathrm{i}}{2\,a\,d}+\frac{f^2\,x^2\,\left(f-d\,e\right)\,3\mathrm{i}}{2\,a\,d^2}-\frac{f\,x\,\left(d^2\,e^2-2\,d\,e\,f+2\,f^2\right)\,3\mathrm{i}}{2\,a\,d^3}\right)-\mathrm{e}^{-c-d\,x}\left(\frac{\left(d^3\,e^3+3\,d^2\,e^2\,f+6\,d\,e\,f^2+6\,f^3\right)\,1\mathrm{i}}{2\,a\,d^4}+\frac{f^3\,x^3\,1\mathrm{i}}{2\,a\,d}+\frac{f^2\,x^2\,\left(f+d\,e\right)\,3\mathrm{i}}{2\,a\,d^2}+\frac{f\,x\,\left(d^2\,e^2+2\,d\,e\,f+2\,f^2\right)\,3\mathrm{i}}{2\,a\,d^3}\right)+\frac{e^3\,x}{a}+\frac{f^3\,x^4}{4\,a}+\frac{3\,e^2\,f\,x^2}{2\,a}+\frac{e\,f^2\,x^3}{a}$$

input
```
int((cosh(c + d*x)^2*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
exp(c + d*x)*(((6*f^3 - d^3*e^3 + 3*d^2*e^2*f - 6*d*e*f^2)*1i)/(2*a*d^4) -
 (f^3*x^3*1i)/(2*a*d) + (f^2*x^2*(f - d*e)*3i)/(2*a*d^2) - (f*x*(2*f^2 + d
^2*e^2 - 2*d*e*f)*3i)/(2*a*d^3)) - exp(- c - d*x)*(((6*f^3 + d^3*e^3 + 3*d
^2*e^2*f + 6*d*e*f^2)*1i)/(2*a*d^4) + (f^3*x^3*1i)/(2*a*d) + (f^2*x^2*(f +
 d*e)*3i)/(2*a*d^2) + (f*x*(2*f^2 + d^2*e^2 + 2*d*e*f)*3i)/(2*a*d^3)) + (e
^3*x)/a + (f^3*x^4)/(4*a) + (3*e^2*f*x^2)/(2*a) + (e*f^2*x^3)/a
```

3.260 $\int \frac{(e+fx)^2 \cosh^2(c+dx)}{a+ia \sinh(c+dx)} dx$

3.260.1 Optimal result

Integrand size = 31, antiderivative size = 82

$$\int \frac{(e+fx)^2 \cosh^2(c+dx)}{a+ia \sinh(c+dx)} dx = \frac{(e+fx)^3}{3af} - \frac{2if^2 \cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2 \cosh(c+dx)}{ad} + \frac{2if(e+fx) \sinh(c+dx)}{ad^2}$$

output
```
1/3*(f*x+e)^3/a/f-2*I*f^2*cosh(d*x+c)/a/d^3-I*(f*x+e)^2*cosh(d*x+c)/a/d+2*
I*f*(f*x+e)*sinh(d*x+c)/a/d^2
```

3.260.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{(e+fx)^2 \cosh^2(c+dx)}{a+ia \sinh(c+dx)} dx = \frac{d^3x(3e^2+3efx+f^2x^2) - 3i(2f^2+d^2(e+fx)^2) \cosh(c+dx) + 6idf(e+fx) \sinh(c+dx)}{3ad^3}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2) - (3*I)*(2*f^2 + d^2*(e + f*x)^2)*Cosh[
c + d*x] + (6*I)*d*f*(e + f*x)*Sinh[c + d*x])/(3*a*d^3)
```

3.260.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {6097, 17, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6097}$$

$$\frac{\int (e+fx)^2dx}{a}-\frac{i\int (e+fx)^2\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{17}$$

$$\frac{(e+fx)^3}{3af}-\frac{i\int (e+fx)^2\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3}{3af}-\frac{i\int -i(e+fx)^2\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{(e+fx)^3}{3af}-\frac{\int (e+fx)^2\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(e+fx)^3}{3af}-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3}{3af}-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(e+fx)^3}{3af}-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{(e+fx)^3}{3af}-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \, 3042$$

$$\frac{(e+fx)^3}{3af}-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \, 26$$

$$\frac{(e+fx)^3}{3af}-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \, 3118$$

$$\frac{(e+fx)^3}{3af}-\frac{\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}}{a}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(e + f*x)^3/(3*a*f) - ((I*(e + f*x)^2*Cosh[c + d*x])/d - ((2*I)*f*(-((f*Co
sh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d))/d)/a
```

3.260.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.260. $\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

3.260.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(77) = 154$.

Time = 6.81 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.95

method	result
risch	$\frac{f^2x^3}{3a} + \frac{fe\,x^2}{a} + \frac{e^2x}{a} + \frac{e^3}{3af} - \frac{i(d^2x^2f^2+2d^2efx+d^2e^2-2xd\,f^2-2def+2f^2)\,\mathrm{e}^{dx+c}}{2d^3a} - \frac{i(d^2x^2f^2+2d^2efx+d^2e^2\cdots}{2\cdots}$
derivativedivides	$-\frac{ic^2f^2\cosh(dx+c)-2icdef\cosh(dx+c)-2ic\,f^2((dx+c)\cosh(dx+c)-\sinh(dx+c))+id^2e^2\cosh(dx+c)+2idef((dx+c)\,\cdots}{\cdots}$
default	$-\frac{ic^2f^2\cosh(dx+c)-2icdef\cosh(dx+c)-2ic\,f^2((dx+c)\cosh(dx+c)-\sinh(dx+c))+id^2e^2\cosh(dx+c)+2idef((dx+c)\,\cdots}{\cdots}$

input
```
int((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*f^2*x^3+1/a*f*e*x^2+1/a*e^2*x+1/3/a/f*e^3-1/2*I*(d^2*f^2*x^2+2*d^2*e
*f*x+d^2*e^2-2*d*f^2*x-2*d*e*f+2*f^2)/d^3/a*exp(d*x+c)-1/2*I*(d^2*f^2*x^2+
2*d^2*e*f*x+d^2*e^2+2*d*f^2*x+2*d*e*f+2*f^2)/d^3/a*exp(-d*x-c)
```

3.260.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(74) = 148$.

Time = 0.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.95

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(-3i\,d^2f^2x^2 - 3i\,d^2e^2 - 6i\,def - 6i\,f^2 - 6\,(i\,d^2ef + i\,df^2)x - 3\,(i\,d^2f^2x^2 + i\,d^2e^2 - 2i\,def + 2i\,f^2 + 2\,(i\,\cdots\right.}{6\,ad^3}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/6*(-3*I*d^2*f^2*x^2 - 3*I*d^2*e^2 - 6*I*d*e*f - 6*I*f^2 - 6*(I*d^2*e*f +
 I*d*f^2)*x - 3*(I*d^2*f^2*x^2 + I*d^2*e^2 - 2*I*d*e*f + 2*I*f^2 + 2*(I*d^
2*e*f - I*d*f^2)*x)*e^(2*d*x + 2*c) + 2*(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d
^3*e^2*x)*e^(d*x + c))*e^(-d*x - c)/(a*d^3)
```

3.260.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.88

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \begin{cases} \frac{((-2iad^5e^2-4iad^5efx-2iad^5f^2x^2-4iad^4ef-4iad^4f^2x-4iad^3f^2)e^{-dx}+(-2iad^5e^2e^{2c}-4iad^5efxe^{2c}-2iad^5f^2x^2e^{2c}+4iad^4efe^{2c}+4iad^4f^2}{4a^2d^6} \\ \frac{x^3(-if^2e^{2c}+if^2)e^{-c}}{6a}+\frac{x^2(-iefe^{2c}+ief)e^{-c}}{2a}+\frac{x(-ie^2e^{2c}+ie^2)e^{-c}}{2a} \end{cases}$$
$$+\frac{e^2x}{a}+\frac{efx^2}{a}+\frac{f^2x^3}{3a}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise((((-2*I*a*d**5*e**2 - 4*I*a*d**5*e*f*x - 2*I*a*d**5*f**2*x**2 -
4*I*a*d**4*e*f - 4*I*a*d**4*f**2*x - 4*I*a*d**3*f**2)*exp(-d*x) + (-2*I*a*
d**5*e**2*exp(2*c) - 4*I*a*d**5*e*f*x*exp(2*c) - 2*I*a*d**5*f**2*x**2*exp(
2*c) + 4*I*a*d**4*e*f*exp(2*c) + 4*I*a*d**4*f**2*x*exp(2*c) - 4*I*a*d**3*f
**2*exp(2*c))*exp(d*x))*exp(-c)/(4*a**2*d**6), Ne(a**2*d**6*exp(c), 0)), (
x**3*(-I*f**2*exp(2*c) + I*f**2)*exp(-c)/(6*a) + x**2*(-I*e*f*exp(2*c) + I
*e*f)*exp(-c)/(2*a) + x*(-I*e**2*exp(2*c) + I*e**2)*exp(-c)/(2*a), True))
 + e**2*x/a + e*f*x**2/a + f**2*x**3/(3*a)
```

3.260.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(74) = 148$.

Time = 0.31 (sec) , antiderivative size = 270, normalized size of antiderivative = 3.29

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= ef\left(\frac{2\,xe^{(dx+c)}}{ade^{(dx+c)}-i\,ad} - \frac{i\,d^2x^2e^c + i\,dxe^c - \left(-i\,dxe^{(3\,c)} + i\,e^{(3\,c)}\right)e^{(2\,dx)} - \left(d^2x^2e^{(2\,c)} - 3\,dxe^{(2\,c)} + e^{(2\,c)}\right)e^{(dx}}{ad^2e^{(dx+2\,c)} - i\,ad^2e^c}\right.$$
$$+ \frac{1}{2}\,e^2\left(\frac{2\,(dx+c)}{ad} - \frac{i\,e^{(dx+c)}}{ad} - \frac{i\,e^{(-dx-c)}}{ad}\right)$$
$$+ \frac{\left(2\,d^3x^3e^c + 3\left(-i\,d^2x^2e^{(2\,c)} + 2i\,dxe^{(2\,c)} - 2i\,e^{(2\,c)}\right)e^{(dx)} + 3\left(-i\,d^2x^2 - 2i\,dx - 2i\right)e^{(-dx)}\right)f^2e^{(-c)}}{6\,ad^3}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
e*f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) - (I*d^2*x^2*e^c + I*d*x*e^
c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d*x*e^(2
*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x + 2*c)
- I*a*d^2*e^c)) + 1/2*e^2*(2*(d*x + c)/(a*d) - I*e^(d*x + c)/(a*d) - I*e^(
-d*x - c)/(a*d)) + 1/6*(2*d^3*x^3*e^c + 3*(-I*d^2*x^2*e^(2*c) + 2*I*d*x*e^
(2*c) - 2*I*e^(2*c))*e^(d*x) + 3*(-I*d^2*x^2 - 2*I*d*x - 2*I)*e^(-d*x))*f^
2*e^(-c)/(a*d^3)
```

3.260.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(74) = 148$.

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.54

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(2\,d^3f^2x^3e^{(dx+c)} + 6\,d^3efx^2e^{(dx+c)} - 3i\,d^2f^2x^2e^{(2\,dx+2\,c)} + 6\,d^3e^2xe^{(dx+c)} - 3i\,d^2f^2x^2 - 6i\,d^2efxe^{(2\,dx+2\,c)} -\right.}{}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(2*d^3*f^2*x^3*e^(d*x + c) + 6*d^3*e*f*x^2*e^(d*x + c) - 3*I*d^2*f^2*x
^2*e^(2*d*x + 2*c) + 6*d^3*e^2*x*e^(d*x + c) - 3*I*d^2*f^2*x^2 - 6*I*d^2*e
*f*x*e^(2*d*x + 2*c) - 6*I*d^2*e*f*x - 3*I*d^2*e^2*e^(2*d*x + 2*c) + 6*I*d
*f^2*x*e^(2*d*x + 2*c) - 3*I*d^2*e^2 - 6*I*d*f^2*x + 6*I*d*e*f*e^(2*d*x +
2*c) - 6*I*d*e*f - 6*I*f^2*e^(2*d*x + 2*c) - 6*I*f^2)*e^(-d*x - c)/(a*d^3)
```

3.260.9 Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.04

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{e^2\,x}{a} - \mathrm{e}^{-c-d\,x}\left(\frac{\left(d^2\,e^2+2\,d\,e\,f+2\,f^2\right)\,1\mathrm{i}}{2\,a\,d^3}+\frac{f^2\,x^2\,1\mathrm{i}}{2\,a\,d}+\frac{f\,x\,\left(f+d\,e\right)\,1\mathrm{i}}{a\,d^2}\right) - \mathrm{e}^{c+d\,x}\left(\frac{\left(d^2\,e^2-2\,d\,e\,f+2\,f^2\right)\,1\mathrm{i}}{2\,a\,d^3}+\frac{f^2\,x^2\,1\mathrm{i}}{2\,a\,d}-\frac{f\,x\,\left(f-d\,e\right)\,1\mathrm{i}}{a\,d^2}\right)+\frac{f^2\,x^3}{3\,a}+\frac{e\,f\,x^2}{a}$$

input
```
int((cosh(c + d*x)^2*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
(e^2*x)/a - exp(- c - d*x)*(((2*f^2 + d^2*e^2 + 2*d*e*f)*1i)/(2*a*d^3) + (
f^2*x^2*1i)/(2*a*d) + (f*x*(f + d*e)*1i)/(a*d^2)) - exp(c + d*x)*(((2*f^2
+ d^2*e^2 - 2*d*e*f)*1i)/(2*a*d^3) + (f^2*x^2*1i)/(2*a*d) - (f*x*(f - d*e)
*1i)/(a*d^2)) + (f^2*x^3)/(3*a) + (e*f*x^2)/a
```

3.261 $\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.261.1 Optimal result

Integrand size = 29, antiderivative size = 56

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{ex}{a}+\frac{fx^2}{2a}-\frac{i(e+fx)\cosh(c+dx)}{ad}+\frac{if\sinh(c+dx)}{ad^2}$$

output
```
e*x/a+1/2*f*x^2/a-I*(f*x+e)*cosh(d*x+c)/a/d+I*f*sinh(d*x+c)/a/d^2
```

3.261.2 Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{(c+dx)(-2de+cf-dfx)+2id(e+fx)\cosh(c+dx)-2if\sinh(c+dx)}{2ad^2}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-1/2*((c + d*x)*(-2*d*e + c*f - d*f*x) + (2*I)*d*(e + f*x)*Cosh[c + d*x] -
 (2*I)*f*Sinh[c + d*x])/(a*d^2)
```

3.261.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {6097, 17, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6097}$$

$$\frac{\int (e+fx)dx}{a}-\frac{i\int (e+fx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{17}$$

$$\frac{(e+fx)^2}{2af}-\frac{i\int (e+fx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^2}{2af}-\frac{i\int -i(e+fx)\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{(e+fx)^2}{2af}-\frac{\int (e+fx)\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(e+fx)^2}{2af}-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^2}{2af}-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{a}$$

$$\Big\downarrow \text{3117}$$

$$\frac{(e+fx)^2}{2af}-\frac{\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}}{a}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

3.261. $\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

output
```
(e + f*x)^2/(2*a*f) - ((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])
/d^2)/a
```

3.261.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

3.261. $\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.261.4 Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25

method	result	size
risch	$\frac{f x^2}{2a}+\frac{ex}{a}-\frac{i(dfx+de-f)\mathrm{e}^{dx+c}}{2a\,d^2}-\frac{i(dfx+de+f)\mathrm{e}^{-dx-c}}{2a\,d^2}$	70
derivativedivides	$-\frac{-ifc\cosh(dx+c)+ide\cosh(dx+c)+if((dx+c)\cosh(dx+c)-\sinh(dx+c))+fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}}{d^2a}$	84
default	$-\frac{-ifc\cosh(dx+c)+ide\cosh(dx+c)+if((dx+c)\cosh(dx+c)-\sinh(dx+c))+fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}}{d^2a}$	84

input
```
int((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*f*x^2/a+e*x/a-1/2*I*(d*f*x+d*e-f)/a/d^2*exp(d*x+c)-1/2*I*(d*f*x+d*e+f)
/a/d^2*exp(-d*x-c)
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(-i\,dfx-i\,de+(-i\,dfx-i\,de+i\,f)e^{(2\,dx+2\,c)}+(d^2fx^2+2\,d^2ex)e^{(dx+c)}-i\,f\right)e^{(-dx-c)}}{2\,ad^2}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(-I*d*f*x - I*d*e + (-I*d*f*x - I*d*e + I*f)*e^(2*d*x + 2*c) + (d^2*f*
x^2 + 2*d^2*e*x)*e^(d*x + c) - I*f)*e^(-d*x - c)/(a*d^2)
```

3.261.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.98

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \begin{cases} \frac{((-2iad^3e-2iad^3fx-2iad^2f)e^{-dx}+(-2iad^3ee^{2c}-2iad^3fxe^{2c}+2iad^2fe^{2c})e^{dx})e^{-c}}{4a^2d^4} & \text{for } a^2d^4e^c \neq 0 \\ \frac{x^2(-ife^{2c}+if)e^{-c}}{4a} + \frac{x(-iee^{2c}+ie)e^{-c}}{2a} & \text{otherwise} \end{cases} + \frac{ex}{a} + \frac{fx^2}{2a}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise((((-2*I*a*d**3*e - 2*I*a*d**3*f*x - 2*I*a*d**2*f)*exp(-d*x) + (-
2*I*a*d**3*e*exp(2*c) - 2*I*a*d**3*f*x*exp(2*c) + 2*I*a*d**2*f*exp(2*c))*e
xp(d*x))*exp(-c)/(4*a**2*d**4), Ne(a**2*d**4*exp(c), 0)), (x**2*(-I*f*exp(
2*c) + I*f)*exp(-c)/(4*a) + x*(-I*e*exp(2*c) + I*e)*exp(-c)/(2*a), True))
+ e*x/a + f*x**2/(2*a)
```

3.261.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(53) = 106.

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.36

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{1}{2}f\left(\frac{2\,xe^{(dx+c)}}{ade^{(dx+c)}-i\,ad} - \frac{i\,d^2x^2e^c + i\,dxe^c - \left(-i\,dxe^{(3\,c)} + i\,e^{(3\,c)}\right)e^{(2\,dx)} - \left(d^2x^2e^{(2\,c)} - 3\,dxe^{(2\,c)} + e^{(2\,c)}\right)e^{(d}}{ad^2e^{(dx+2\,c)} - i\,ad^2e^c} + \frac{1}{2}e\left(\frac{2\,(dx+c)}{ad} - \frac{i\,e^{(dx+c)}}{ad} - \frac{i\,e^{(-dx-c)}}{ad}\right)$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) - (I*d^2*x^2*e^c + I*d*x*
e^c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d*x*e^
(2*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x + 2*c
) - I*a*d^2*e^c)) + 1/2*e*(2*(d*x + c)/(a*d) - I*e^(d*x + c)/(a*d) - I*e^(
-d*x - c)/(a*d))
```

3.261.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.71

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(d^2fx^2e^{(dx+c)}+2\,d^2exe^{(dx+c)}-i\,dfxe^{(2\,dx+2\,c)}-i\,dfx-i\,dee^{(2\,dx+2\,c)}-i\,de+i\,fe^{(2\,dx+2\,c)}-i\,f\right)e^{(-dx-c)}}{2\,ad^2}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(d^2*f*x^2*e^(d*x + c) + 2*d^2*e*x*e^(d*x + c) - I*d*f*x*e^(2*d*x + 2*
c) - I*d*f*x - I*d*e*e^(2*d*x + 2*c) - I*d*e + I*f*e^(2*d*x + 2*c) - I*f)*
e^(-d*x - c)/(a*d^2)
```

3.261.9 Mupad [B] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{f\,x^2}{2\,a}+\mathrm{e}^{c+d\,x}\left(\frac{(f-d\,e)\,1\mathrm{i}}{2\,a\,d^2}-\frac{f\,x\,1\mathrm{i}}{2\,a\,d}\right) - \mathrm{e}^{-c-d\,x}\left(\frac{(f+d\,e)\,1\mathrm{i}}{2\,a\,d^2}+\frac{f\,x\,1\mathrm{i}}{2\,a\,d}\right)+\frac{e\,x}{a}$$

input
```
int((cosh(c + d*x)^2*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

output
```
exp(c + d*x)*(((f - d*e)*1i)/(2*a*d^2) - (f*x*1i)/(2*a*d)) - exp(- c - d*x
)*(((f + d*e)*1i)/(2*a*d^2) + (f*x*1i)/(2*a*d)) + (f*x^2)/(2*a) + (e*x)/a
```

3.262 $\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.262.1 Optimal result

Integrand size = 24, antiderivative size = 22

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{x}{a}-\frac{i\cosh(c+dx)}{ad}$$

output
```
x/a-I*cosh(d*x+c)/a/d
```

3.262.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 139 vs. 2(22) = 44.

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 6.32

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\cosh^3(c+dx)\left(-2\arcsin\left(\frac{\sqrt{1-i\sinh(c+dx)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(c+dx)}+\sqrt{1+i\sinh(c+dx)}-i\sqrt{1+i\sinh(c+}\right.}{ad\sqrt{1+i\sinh(c+dx)}(-i+\sinh(c+dx))(i+\sinh(c+dx))^2}$$

input
```
Integrate[Cosh[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(Cosh[c + d*x]^3*(-2*ArcSin[Sqrt[1 - I*Sinh[c + d*x]]/Sqrt[2]]*Sqrt[1 - I*
Sinh[c + d*x]] + Sqrt[1 + I*Sinh[c + d*x]] - I*Sqrt[1 + I*Sinh[c + d*x]]*S
inh[c + d*x]))/(a*d*Sqrt[1 + I*Sinh[c + d*x]]*(-I + Sinh[c + d*x])*(I + Si
nh[c + d*x])^2)
```

3.262.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ic+idx)^2}{a+a\sin(ic+idx)}dx$$

$$\Big\downarrow \text{ 3161}$$

$$\frac{\int 1dx}{a}-\frac{i\cosh(c+dx)}{ad}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x}{a}-\frac{i\cosh(c+dx)}{ad}$$

input
```
Int[Cosh[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

output
```
x/a - (I*Cosh[c + d*x])/(a*d)
```

3.262.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

3.262.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

method	result	size
risch	$\frac{x}{a}-\frac{i e^{dx+c}}{2ad}-\frac{i e^{-dx-c}}{2ad}$	40
derivativedivides	$\frac{-\frac{i}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{2i}{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}$	70
default	$\frac{-\frac{i}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{2i}{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}$	70

input
```
int(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
x/a-1/2*I/a/d*exp(d*x+c)-1/2*I/a/d*exp(-d*x-c)
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{\left(2\,dxe^{(dx+c)}-i\,e^{(2\,dx+2\,c)}-i\right)e^{(-dx-c)}}{2\,ad}$$

input
```
integrate(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*d*x*e^(d*x + c) - I*e^(2*d*x + 2*c) - I)*e^(-d*x - c)/(a*d)
```

3.262.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.55

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \begin{cases} \frac{(-2iade^{2c}e^{dx}-2iade^{-dx})e^{-c}}{4a^2d^2} & \text{for } a^2d^2e^c \neq 0 \\ x\left(\frac{(-ie^{2c}+2e^c+i)e^{-c}}{2a}-\frac{1}{a}\right) & \text{otherwise} \end{cases} + \frac{x}{a}$$

input
```
integrate(cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise(((-2*I*a*d*exp(2*c)*exp(d*x) - 2*I*a*d*exp(-d*x))*exp(-c)/(4*a**
2*d**2), Ne(a**2*d**2*exp(c), 0)), (x*((-I*exp(2*c) + 2*exp(c) + I)*exp(-c
)/(2*a) - 1/a), True)) + x/a
```

3.262.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(20) = 40$.

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{dx+c}{ad} - \frac{i\,e^{(dx+c)}}{2\,ad} - \frac{i\,e^{(-dx-c)}}{2\,ad}$$

input
```
integrate(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
(d*x + c)/(a*d) - 1/2*I*e^(d*x + c)/(a*d) - 1/2*I*e^(-d*x - c)/(a*d)
```

3.262.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\frac{2\,(dx+c)}{a} - \frac{i\,e^{(dx+c)}}{a} - \frac{i\,e^{(-dx-c)}}{a}}{2\,d}$$

input
```
integrate(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)/a - I*e^(d*x + c)/a - I*e^(-d*x - c)/a)/d
```

3.262.9 Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{x}{a} - \frac{\frac{\mathrm{e}^{c+d\,x}\,1\mathrm{i}}{2} + \frac{\mathrm{e}^{-c-d\,x}\,1\mathrm{i}}{2}}{a\,d}$$

input
```
int(cosh(c + d*x)^2/(a + a*sinh(c + d*x)*1i),x)
```

output
```
x/a - ((exp(c + d*x)*1i)/2 + (exp(- c - d*x)*1i)/2)/(a*d)
```

3.263 $\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

3.263.1 Optimal result

Integrand size = 31, antiderivative size = 76

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\frac{\log(e+fx)}{af}-\frac{i\mathrm{Chi}\left(\frac{de}{f}+dx\right)\sinh\left(c-\frac{de}{f}\right)}{af}-\frac{i\cosh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(\frac{de}{f}+dx\right)}{af}$$

output
```
ln(f*x+e)/a/f-I*cosh(c-d*e/f)*Shi(d*e/f+d*x)/a/f-I*Chi(d*e/f+d*x)*sinh(c-d
*e/f)/a/f
```

3.263.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\frac{\log(e+fx)-i\mathrm{Chi}\left(d\left(\frac{e}{f}+x\right)\right)\sinh\left(c-\frac{de}{f}\right)-i\cosh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(d\left(\frac{e}{f}+x\right)\right)}{af}$$

input
```
Integrate[Cosh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
(Log[e + f*x] - I*CoshIntegral[d*(e/f + x)]*Sinh[c - (d*e)/f] - I*Cosh[c -
 (d*e)/f]*SinhIntegral[d*(e/f + x)])/(a*f)
```

3.263.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {6097, 16, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

$$\Big\downarrow \text{6097}$$

$$\frac{\int \frac{1}{e+fx}dx}{a}-\frac{i\int \frac{\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(e+fx)}{af}-\frac{i\int \frac{\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\log(e+fx)}{af}-\frac{i\int -\frac{i\sin(ic+idx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\log(e+fx)}{af}-\frac{\int \frac{\sin(ic+idx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3784}$$

$$\frac{\log(e+fx)}{af}-\frac{i\sinh\left(c-\frac{de}{f}\right)\int \frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx+\cosh\left(c-\frac{de}{f}\right)\int \frac{i\sinh\left(\frac{de}{f}+dx\right)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\log(e+fx)}{af}-\frac{i\sinh\left(c-\frac{de}{f}\right)\int \frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx+i\cosh\left(c-\frac{de}{f}\right)\int \frac{\sinh\left(\frac{de}{f}+dx\right)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\log(e+fx)}{af}-\frac{i\sinh\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx+i\cosh\left(c-\frac{de}{f}\right)\int -\frac{i\sin\left(\frac{ide}{f}+idx\right)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\log(e+fx)}{af}-\frac{i\sinh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx+\cosh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx\right)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\log(e+fx)}{af}-\frac{i\sinh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}$$

$$\Big\downarrow \text{3782}$$

$$\frac{\log(e+fx)}{af}-\frac{\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}$$

input
```
Int[Cosh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Log[e + f*x]/(a*f) - ((I*CoshIntegral[(d*e)/f + d*x]*Sinh[c - (d*e)/f])/f
+ (I*Cosh[c - (d*e)/f]*SinhIntegral[(d*e)/f + d*x])/f)/a
```

3.263.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

3.263. $\int\frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

3.263.4 Maple [A] (verified)

Time = 12.95 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.36

method	result	size
risch	$\frac{\ln(fx+e)}{af}+\frac{i\mathrm{e}^{\frac{cf-de}{f}}\operatorname{Ei}_1\left(-dx-c-\frac{-cf+de}{f}\right)}{2af}-\frac{i\mathrm{e}^{-\frac{cf-de}{f}}\operatorname{Ei}_1\left(dx+c-\frac{cf-de}{f}\right)}{2af}$	103

input
```
int(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
ln(f*x+e)/a/f+1/2*I/a/f*exp((c*f-d*e)/f)*Ei(1,-d*x-c-(-c*f+d*e)/f)-1/2*I/a
/f*exp(-(c*f-d*e)/f)*Ei(1,d*x+c-(c*f-d*e)/f)
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int\frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{i\operatorname{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(\frac{de-cf}{f}\right)}-i\operatorname{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(-\frac{de-cf}{f}\right)}+2\log\left(\frac{fx+e}{f}\right)}{2\,af}$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(I*Ei(-(d*f*x + d*e)/f)*e^((d*e - c*f)/f) - I*Ei((d*f*x + d*e)/f)*e^(-
(d*e - c*f)/f) + 2*log((f*x + e)/f))/(a*f)
```

3.263.6 Sympy [F]

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\cosh^2(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

input
```
integrate(cosh(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(cosh(c + d*x)**2/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) -
I*f*x), x)/a
```

3.263.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\,e^{\left(-c+\frac{de}{f}\right)}E_1\left(\frac{(fx+e)d}{f}\right)}{2\,af} + \frac{i\,e^{\left(c-\frac{de}{f}\right)}E_1\left(-\frac{(fx+e)d}{f}\right)}{2\,af} + \frac{\log(fx+e)}{af}$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*I*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(a*f) + 1/2*I*e^(c
- d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(a*f) + log(f*x + e)/(a*f)
```

3.263.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{\left(i\,\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(2\,c-\frac{de}{f}\right)} - i\,\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(\frac{de}{f}\right)} - 2\,e^{c}\log\left(i\,fx+i\,e\right)\right)e^{(-c)}}{2\,af}$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(I*Ei((d*f*x + d*e)/f)*e^(2*c - d*e/f) - I*Ei(-(d*f*x + d*e)/f)*e^(d*
e/f) - 2*e^c*log(I*f*x + I*e))*e^(-c)/(a*f)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^2}{(e+f\,x)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(cosh(c + d*x)^2/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(cosh(c + d*x)^2/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.264 $\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

3.264.1 Optimal result

Integrand size = 31, antiderivative size = 103

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{1}{af(e+fx)} - \frac{id\cosh\left(c-\frac{de}{f}\right)\mathrm{Chi}\left(\frac{de}{f}+dx\right)}{af^2} + \frac{i\sinh(c+dx)}{af(e+fx)} - \frac{id\sinh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(\frac{de}{f}+dx\right)}{af^2}$$

output
```
-1/a/f/(f*x+e)-I*d*Chi(d*e/f+d*x)*cosh(c-d*e/f)/a/f^2-I*d*Shi(d*e/f+d*x)*s
inh(c-d*e/f)/a/f^2+I*sinh(d*x+c)/a/f/(f*x+e)
```

3.264.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{i\left(d(e+fx)\cosh\left(c-\frac{de}{f}\right)\mathrm{Chi}\left(d\left(\frac{e}{f}+x\right)\right)-f(i+\sinh(c+dx))+d(e+fx)\sinh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(d\left(\frac{e}{f}\right.\right.\right.}{af^2(e+fx)}$$

input
```
Integrate[Cosh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
((-I)*(d*(e + f*x)*Cosh[c - (d*e)/f]*CoshIntegral[d*(e/f + x)] - f*(I + Si
nh[c + d*x]) + d*(e + f*x)*Sinh[c - (d*e)/f]*SinhIntegral[d*(e/f + x)]))/(
a*f^2*(e + f*x))
```

3.264.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {6097, 17, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

$$\Big\downarrow \text{ 6097}$$

$$\frac{\int \frac{1}{(e+fx)^2}dx}{a}-\frac{i\int \frac{\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{1}{af(e+fx)}-\frac{i\int \frac{\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{af(e+fx)}-\frac{i\int -\frac{i\sin(ic+idx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{1}{af(e+fx)}-\frac{\int \frac{\sin(ic+idx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \text{ 3778}$$

$$-\frac{1}{af(e+fx)}-\frac{\frac{id\int \frac{\cosh(c+dx)}{e+fx}dx}{f}-\frac{i\sinh(c+dx)}{f(e+fx)}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{af(e+fx)}-\frac{\frac{id\int \frac{\sin\left(ic+idx+\frac{\pi}{2}\right)}{e+fx}dx}{f}-\frac{i\sinh(c+dx)}{f(e+fx)}}{a}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{1}{af(e+fx)}-\frac{\frac{id\left(\cosh\left(c-\frac{de}{f}\right)\int\frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx-i\sinh\left(c-\frac{de}{f}\right)\int\frac{i\sinh\left(\frac{de}{f}+dx\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(c+dx)}{f(e+fx)}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{af(e+fx)}-\frac{\frac{id\left(\sinh\left(c-\frac{de}{f}\right)\int\frac{\sinh\left(\frac{de}{f}+dx\right)}{e+fx}dx+\cosh\left(c-\frac{de}{f}\right)\int\frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(c+dx)}{f(e+fx)}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{af(e+fx)}-\frac{\frac{id\left(\sinh\left(c-\frac{de}{f}\right)\int-\frac{i\sin\left(\frac{ide}{f}+idx\right)}{e+fx}dx+\cosh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(c+dx)}{f(e+fx)}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{af(e+fx)}-\frac{\frac{id\left(\cosh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx-i\sinh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(c+dx)}{f(e+fx)}}{a}$$

$$\Big\downarrow \text{3779}$$

$$-\frac{1}{af(e+fx)}-\frac{\frac{id\left(\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}+\cosh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(c+dx)}{f(e+fx)}}{a}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{1}{af(e+fx)}-\frac{\frac{id\left(\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}-\frac{i\sinh(c+dx)}{f(e+fx)}}{a}$$

input
```
Int[Cosh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
-(1/(a*f*(e + f*x))) - (((-I)*Sinh[c + d*x])/(f*(e + f*x)) + (I*d*((Cosh[c
 - (d*e)/f]*CoshIntegral[(d*e)/f + d*x])/f + (Sinh[c - (d*e)/f]*SinhIntegr
al[(d*e)/f + d*x])/f))/f)/a
```

3.264. $\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

3.264.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

3.264. $\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

3.264.4 Maple [A] (verified)

Time = 29.96 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.59

method	result	siz
risch	$-\frac{1}{af(fx+e)}+\frac{id\,\mathrm{e}^{dx+c}}{2a\,f^{2}\left(\frac{de}{f}+dx\right)}+\frac{id\,\mathrm{e}^{\frac{cf-de}{f}}\,\mathrm{Ei}_{1}\left(-dx-c-\frac{-cf+de}{f}\right)}{2a\,f^{2}}-\frac{id\,\mathrm{e}^{-dx-c}}{2af(dfx+de)}+\frac{id\,\mathrm{e}^{-\frac{cf-de}{f}}\,\mathrm{Ei}_{1}\left(dx+c-\frac{cf-de}{f}\right)}{2a\,f^{2}}$	16

input
```
int(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/a/f/(f*x+e)+1/2*I*d/a/f^2*exp(d*x+c)/(d*e/f+d*x)+1/2*I*d/a/f^2*exp((c*f
-d*e)/f)*Ei(1,-d*x-c-(-c*f+d*e)/f)-1/2*I/a*d*exp(-d*x-c)/f/(d*f*x+d*e)+1/2
*I/a*d/f^2*exp(-(c*f-d*e)/f)*Ei(1,d*x+c-(c*f-d*e)/f)
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.25

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$=\frac{\left(i\,fe^{(2\,dx+2\,c)}+\left((-i\,dfx-i\,de)\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(\frac{de-cf}{f}\right)}+(-i\,dfx-i\,de)\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(-\frac{de-cf}{f}\right)}-2\,f\right)e^{(dx+}\right.}{2\,(af^3x+aef^2)}$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/2*(I*f*e^(2*d*x + 2*c) + ((-I*d*f*x - I*d*e)*Ei(-(d*f*x + d*e)/f)*e^((d*
e - c*f)/f) + (-I*d*f*x - I*d*e)*Ei((d*f*x + d*e)/f)*e^(-(d*e - c*f)/f) -
2*f)*e^(d*x + c) - I*f)*e^(-d*x - c)/(a*f^3*x + a*e*f^2)
```

3.264.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.264.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{1}{af^2x+aef} - \frac{i\,e^{\left(-c+\frac{de}{f}\right)}E_2\left(\frac{(fx+e)d}{f}\right)}{2\,(fx+e)af} + \frac{i\,e^{\left(c-\frac{de}{f}\right)}E_2\left(-\frac{(fx+e)d}{f}\right)}{2\,(fx+e)af}$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-1/(a*f^2*x + a*e*f) - 1/2*I*e^(-c + d*e/f)*exp_integral_e(2, (f*x + e)*d/
f)/((f*x + e)*a*f) + 1/2*I*e^(c - d*e/f)*exp_integral_e(2, -(f*x + e)*d/f)
/((f*x + e)*a*f)
```

3.264.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(97) = 194$.

Time = 0.32 (sec) , antiderivative size = 572, normalized size of antiderivative = 5.55

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{\left(i\,(fx+e)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)d^2\mathrm{Ei}\left(-\frac{(fx+e)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf}{f}\right)e^{\left(\frac{de-cf}{f}\right)} + i\,d^3e\mathrm{Ei}\left(-\frac{(fx+e)\left(d-\frac{de}{fx+e}+\right.}{f}\right.\right.}{}$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(I*(f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))*d^2*Ei(-((f*x + e)*(
d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^((d*e - c*f)/f) + I*d
^3*e*Ei(-((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^
((d*e - c*f)/f) - I*c*d^2*f*Ei(-((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x +
 e)) + d*e - c*f)/f)*e^((d*e - c*f)/f) + I*(f*x + e)*(d - d*e/(f*x + e) +
c*f/(f*x + e))*d^2*Ei(((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e
 - c*f)/f)*e^(-(d*e - c*f)/f) + I*d^3*e*Ei(((f*x + e)*(d - d*e/(f*x + e) +
 c*f/(f*x + e)) + d*e - c*f)/f)*e^(-(d*e - c*f)/f) - I*c*d^2*f*Ei(((f*x +
e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^(-(d*e - c*f)/f)
- I*d^2*f*e^((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))/f) + I*d^2*f*e^
(-(f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))/f) + 2*d^2*f)*f^2/(((f*x +
 e)*a*(d - d*e/(f*x + e) + c*f/(f*x + e))*f^4 + a*d*e*f^4 - a*c*f^5)*d)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^2}{(e+f\,x)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(cosh(c + d*x)^2/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(cosh(c + d*x)^2/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.265 $\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.265.1 Optimal result

Integrand size = 31, antiderivative size = 231

$$\begin{aligned}\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = &-\frac{3if^3x}{8ad^3}-\frac{i(e+fx)^3}{4ad}-\frac{6f^3\cosh(c+dx)}{ad^4}\\&-\frac{3f(e+fx)^2\cosh(c+dx)}{ad^2}+\frac{6f^2(e+fx)\sinh(c+dx)}{ad^3}\\&+\frac{(e+fx)^3\sinh(c+dx)}{ad}+\frac{3if^3\cosh(c+dx)\sinh(c+dx)}{8ad^4}\\&+\frac{3if(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{4ad^2}\\&-\frac{3if^2(e+fx)\sinh^2(c+dx)}{4ad^3}-\frac{i(e+fx)^3\sinh^2(c+dx)}{2ad}\end{aligned}$$

output
```
-3/8*I*f^3*x/a/d^3-1/4*I*(f*x+e)^3/a/d-6*f^3*cosh(d*x+c)/a/d^4-3*f*(f*x+e)
^2*cosh(d*x+c)/a/d^2+6*f^2*(f*x+e)*sinh(d*x+c)/a/d^3+(f*x+e)^3*sinh(d*x+c)
/a/d+3/8*I*f^3*cosh(d*x+c)*sinh(d*x+c)/a/d^4+3/4*I*f*(f*x+e)^2*cosh(d*x+c)
*sinh(d*x+c)/a/d^2-3/4*I*f^2*(f*x+e)*sinh(d*x+c)^2/a/d^3-1/2*I*(f*x+e)^3*s
inh(d*x+c)^2/a/d
```

3.265.2 Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.58

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{-96f(2f^2+d^2(e+fx)^2)\cosh(c+dx)-4id(e+fx)\left(3f^2+2d^2(e+fx)^2\right)\cosh(2(c+dx))+4(8d(e+f}{32ad^4}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-96*f*(2*f^2 + d^2*(e + f*x)^2)*Cosh[c + d*x] - (4*I)*d*(e + f*x)*(3*f^2
+ 2*d^2*(e + f*x)^2)*Cosh[2*(c + d*x)] + 4*(8*d*(e + f*x)*(6*f^2 + d^2*(e
+ f*x)^2) + (3*I)*f*(f^2 + 2*d^2*(e + f*x)^2)*Cosh[c + d*x])*Sinh[c + d*x]
)/(32*a*d^4)
```

3.265.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.97, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.742$, Rules used = {6097, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 5969, 3042, 25, 3792, 17, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6097}$$

$$\frac{\int (e+fx)^3\cosh(c+dx)dx}{a}-\frac{i\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}-\frac{i\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\sinh(c+dx)dx}{d}}{a}-\frac{i\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int(e+fx)^2\sinh(c+dx)dx}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int-i(e+fx)^2\sin(ic+idx)dx}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\int(e+fx)^2\sin(ic+idx)dx}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

3.265. $\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \textbf{26}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \textbf{3118}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \textbf{5969}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{(e+fx)^3\sinh^2(c+dx)}{2d}-\frac{3f\int(e+fx)^2\sinh^2(c+dx)dx}{2d}\right)}{a}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{(e+fx)^3\sinh^2(c+dx)}{2d}-\frac{3f\int-(e+fx)^2\sin(ic+idx)^2dx}{2d}\right)}{a}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\int(e+fx)^2\sin(ic+idx)^2dx}{2d}\right)}{a}$$

3.265. $\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3792}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{3f\left(\frac{f^2\int -\sinh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^2dx+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{17}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{3f\left(\frac{f^2\int -\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{3f\left(-\frac{f^2\int \sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\left(-\frac{f^2\int -\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \text{25}$$

3.265. $\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\left(\frac{f^2\int\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \textbf{3115}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{3f\left(\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \textbf{24}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{a}-\frac{i\left(\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}\right)}{a}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(((e + f*x)^3*Sinh[c + d*x])/d + ((3*I)*f*((I*(e + f*x)^2*Cosh[c + d*x])/d
 - ((2*I)*f*(-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d))/d))/
d)/a - (I*(((e + f*x)^3*Sinh[c + d*x]^2)/(2*d) + (3*f*((e + f*x)^3/(6*f) -
 ((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (f*(e + f*x)*Sinh[c + d
*x]^2)/(2*d^2) + (f^2*(x/2 - (Cosh[c + d*x]*Sinh[c + d*x])/(2*d)))/(2*d^2)
))/(2*d)))/a
```

3.265. $\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.265.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.265. $\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

3.265.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(213) = 426$.

Time = 34.93 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{i\left(4d^3x^3f^3+12d^3e\,f^2x^2+12d^3e^2fx-6d^2f^3x^2+4e^3d^3-12d^2e\,f^2x-6d^2e^2f+6d\,f^3x+6e\,f^2d-3f^3\right)\mathrm{e}^{2dx+2c}}{32d^4a}+\frac{(d^3x^3 \cdots}{\cdots}$
derivativedivides	$-\frac{3ide\,f^2\left(\frac{(dx+c)^2\cosh(dx+c)^2}{2}-\frac{(dx+c)\cosh(dx+c)\sinh(dx+c)}{2}-\frac{(dx+c)^2}{4}+\frac{\cosh(dx+c)^2}{4}\right)-\frac{3ic\,d^2e^2f\cosh(dx+c)^2}{2}+3ic^2f^3 \left(\cdots\right.}{\cdots}$
default	$-\frac{3ide\,f^2\left(\frac{(dx+c)^2\cosh(dx+c)^2}{2}-\frac{(dx+c)\cosh(dx+c)\sinh(dx+c)}{2}-\frac{(dx+c)^2}{4}+\frac{\cosh(dx+c)^2}{4}\right)-\frac{3ic\,d^2e^2f\cosh(dx+c)^2}{2}+3ic^2f^3 \left(\cdots\right.}{\cdots}$

input
```
int((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/32*I*(4*d^3*f^3*x^3+12*d^3*e*f^2*x^2+12*d^3*e^2*f*x-6*d^2*f^3*x^2+4*d^3
*e^3-12*d^2*e*f^2*x-6*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-3*f^3)/d^4/a*exp(2*d*x
+2*c)+1/2*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x-3*d^2*f^3*x^2+d^3*e^3
-6*d^2*e*f^2*x-3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-6*f^3)/d^4/a*exp(d*x+c)-1/2
*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x+3*d^2*f^3*x^2+d^3*e^3+6*d^2*e*
f^2*x+3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+6*f^3)/d^4/a*exp(-d*x-c)-1/32*I*(4*d
^3*f^3*x^3+12*d^3*e*f^2*x^2+12*d^3*e^2*f*x+6*d^2*f^3*x^2+4*d^3*e^3+12*d^2*
e*f^2*x+6*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+3*f^3)/a/d^4*exp(-2*d*x-2*c)
```

3.265.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.75

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{(-4i\,d^3f^3x^3 - 4i\,d^3e^3 - 6i\,d^2e^2f - 6i\,def^2 - 3i\,f^3 - 6\,(2i\,d^3ef^2 + i\,d^2f^3)x^2 - 6\,(2i\,d^3e^2f + 2i\,d^2ef^2 + i\,c}{}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/32*(-4*I*d^3*f^3*x^3 - 4*I*d^3*e^3 - 6*I*d^2*e^2*f - 6*I*d*e*f^2 - 3*I*f
^3 - 6*(2*I*d^3*e*f^2 + I*d^2*f^3)*x^2 - 6*(2*I*d^3*e^2*f + 2*I*d^2*e*f^2
+ I*d*f^3)*x + (-4*I*d^3*f^3*x^3 - 4*I*d^3*e^3 + 6*I*d^2*e^2*f - 6*I*d*e*f
^2 + 3*I*f^3 - 6*(2*I*d^3*e*f^2 - I*d^2*f^3)*x^2 - 6*(2*I*d^3*e^2*f - 2*I*
d^2*e*f^2 + I*d*f^3)*x)*e^(4*d*x + 4*c) + 16*(d^3*f^3*x^3 + d^3*e^3 - 3*d^
2*e^2*f + 6*d*e*f^2 - 6*f^3 + 3*(d^3*e*f^2 - d^2*f^3)*x^2 + 3*(d^3*e^2*f -
 2*d^2*e*f^2 + 2*d*f^3)*x)*e^(3*d*x + 3*c) - 16*(d^3*f^3*x^3 + d^3*e^3 + 3
*d^2*e^2*f + 6*d*e*f^2 + 6*f^3 + 3*(d^3*e*f^2 + d^2*f^3)*x^2 + 3*(d^3*e^2*
f + 2*d^2*e*f^2 + 2*d*f^3)*x)*e^(d*x + c))*e^(-2*d*x - 2*c)/(a*d^4)
```

3.265.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1040 vs. $2(214) = 428$.

Time = 0.60 (sec) , antiderivative size = 1040, normalized size of antiderivative = 4.50

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \begin{cases} \frac{((-2048a^3d^{15}e^3e^{2c} - 6144a^3d^{15}e^2fxe^{2c} - 6144a^3d^{15}ef^2x^2e^{2c} - 2048a^3d^{15}f^3x^3e^{2c} - 6144a^3d^{14}e^2fe^{2c} - 12288a^3d^{14}ef^2xe^{2c} - 6144a^3d^{14}f^3x^2e}{} \\ \frac{x^4(-if^3e^{4c}+2f^3e^{3c}+2f^3e^c+if^3)e^{-2c}}{16a} + \frac{x^3(-ief^2e^{4c}+2ef^2e^{3c}+2ef^2e^c+ief^2)e^{-2c}}{4a} + \frac{x^2(-3ie^2fe^{4c}+6e^2fe^{3c}+6e^2fe^c+3ie^2f)e^{-2c}}{8a} + \end{cases}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise((((-2048*a**3*d**15*e**3*exp(2*c) - 6144*a**3*d**15*e**2*f*x*exp
(2*c) - 6144*a**3*d**15*e*f**2*x**2*exp(2*c) - 2048*a**3*d**15*f**3*x**3*e
xp(2*c) - 6144*a**3*d**14*e**2*f*exp(2*c) - 12288*a**3*d**14*e*f**2*x*exp(
2*c) - 6144*a**3*d**14*f**3*x**2*exp(2*c) - 12288*a**3*d**13*e*f**2*exp(2*
c) - 12288*a**3*d**13*f**3*x*exp(2*c) - 12288*a**3*d**12*f**3*exp(2*c))*ex
p(-d*x) + (2048*a**3*d**15*e**3*exp(4*c) + 6144*a**3*d**15*e**2*f*x*exp(4*
c) + 6144*a**3*d**15*e*f**2*x**2*exp(4*c) + 2048*a**3*d**15*f**3*x**3*exp(
4*c) - 6144*a**3*d**14*e**2*f*exp(4*c) - 12288*a**3*d**14*e*f**2*x*exp(4*c
) - 6144*a**3*d**14*f**3*x**2*exp(4*c) + 12288*a**3*d**13*e*f**2*exp(4*c)
+ 12288*a**3*d**13*f**3*x*exp(4*c) - 12288*a**3*d**12*f**3*exp(4*c))*exp(d
*x) + (-512*I*a**3*d**15*e**3*exp(c) - 1536*I*a**3*d**15*e**2*f*x*exp(c) -
 1536*I*a**3*d**15*e*f**2*x**2*exp(c) - 512*I*a**3*d**15*f**3*x**3*exp(c)
- 768*I*a**3*d**14*e**2*f*exp(c) - 1536*I*a**3*d**14*e*f**2*x*exp(c) - 768
*I*a**3*d**14*f**3*x**2*exp(c) - 768*I*a**3*d**13*e*f**2*exp(c) - 768*I*a*
*3*d**13*f**3*x*exp(c) - 384*I*a**3*d**12*f**3*exp(c))*exp(-2*d*x) + (-512
*I*a**3*d**15*e**3*exp(5*c) - 1536*I*a**3*d**15*e**2*f*x*exp(5*c) - 1536*I
*a**3*d**15*e*f**2*x**2*exp(5*c) - 512*I*a**3*d**15*f**3*x**3*exp(5*c) + 7
68*I*a**3*d**14*e**2*f*exp(5*c) + 1536*I*a**3*d**14*e*f**2*x*exp(5*c) + 76
8*I*a**3*d**14*f**3*x**2*exp(5*c) - 768*I*a**3*d**13*e*f**2*exp(5*c) - 768
*I*a**3*d**13*f**3*x*exp(5*c) + 384*I*a**3*d**12*f**3*exp(5*c))*exp(2*d...
```

3.265.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.265.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(207) = 414$.

Time = 0.29 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.68

$$\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx =$$
$$-\frac{\left(4i\,d^3f^3x^3e^{(4\,dx+4\,c)} - 16\,d^3f^3x^3e^{(3\,dx+3\,c)} + 16\,d^3f^3x^3e^{(dx+c)} + 4i\,d^3f^3x^3 + 12i\,d^3ef^2x^2e^{(4\,dx+4\,c)} - 48\,d^3\right.}{}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/32*(4*I*d^3*f^3*x^3*e^(4*d*x + 4*c) - 16*d^3*f^3*x^3*e^(3*d*x + 3*c) +
16*d^3*f^3*x^3*e^(d*x + c) + 4*I*d^3*f^3*x^3 + 12*I*d^3*e*f^2*x^2*e^(4*d*x
 + 4*c) - 48*d^3*e*f^2*x^2*e^(3*d*x + 3*c) + 48*d^3*e*f^2*x^2*e^(d*x + c)
+ 12*I*d^3*e*f^2*x^2 + 12*I*d^3*e^2*f*x*e^(4*d*x + 4*c) - 6*I*d^2*f^3*x^2*
e^(4*d*x + 4*c) - 48*d^3*e^2*f*x*e^(3*d*x + 3*c) + 48*d^2*f^3*x^2*e^(3*d*x
 + 3*c) + 48*d^3*e^2*f*x*e^(d*x + c) + 48*d^2*f^3*x^2*e^(d*x + c) + 12*I*d
^3*e^2*f*x + 6*I*d^2*f^3*x^2 + 4*I*d^3*e^3*e^(4*d*x + 4*c) - 12*I*d^2*e*f^
2*x*e^(4*d*x + 4*c) - 16*d^3*e^3*e^(3*d*x + 3*c) + 96*d^2*e*f^2*x*e^(3*d*x
 + 3*c) + 16*d^3*e^3*e^(d*x + c) + 96*d^2*e*f^2*x*e^(d*x + c) + 4*I*d^3*e^
3 + 12*I*d^2*e*f^2*x - 6*I*d^2*e^2*f*e^(4*d*x + 4*c) + 6*I*d*f^3*x*e^(4*d*
x + 4*c) + 48*d^2*e^2*f*e^(3*d*x + 3*c) - 96*d*f^3*x*e^(3*d*x + 3*c) + 48*
d^2*e^2*f*e^(d*x + c) + 96*d*f^3*x*e^(d*x + c) + 6*I*d^2*e^2*f + 6*I*d*f^3
*x + 6*I*d*e*f^2*e^(4*d*x + 4*c) - 96*d*e*f^2*e^(3*d*x + 3*c) + 96*d*e*f^2
*e^(d*x + c) + 6*I*d*e*f^2 - 3*I*f^3*e^(4*d*x + 4*c) + 96*f^3*e^(3*d*x + 3
*c) + 96*f^3*e^(d*x + c) + 3*I*f^3)*e^(-2*d*x - 2*c)/(a*d^4)
```

3.265.9 Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.94

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -e^{c+dx}\left(\frac{-d^3e^3+3d^2e^2f-6def^2+6f^3}{2ad^4}-\frac{f^3x^3}{2ad}+\frac{3f^2x^2(f-de)}{2ad^2}-\frac{3fx(d^2e^2-2def+2f^2)}{2ad^3}\right) - e^{-2c-2dx}\left(\frac{(4d^3e^3+6d^2e^2f+6def^2+3f^3)\,1i}{32ad^4}+\frac{f^3x^3\,1i}{8ad}+\frac{fx(2d^2e^2+2def+f^2)\,3i}{16ad^3}+\frac{f^2x^2(f+2de)\,3i}{16ad^2}\right) + e^{2c+2dx}\left(\frac{(-4d^3e^3+6d^2e^2f-6def^2+3f^3)\,1i}{32ad^4}-\frac{f^3x^3\,1i}{8ad}-\frac{fx(2d^2e^2-2def+f^2)\,3i}{16ad^3}+\frac{f^2x^2(f-2de)\,3i}{16ad^2}\right) - e^{-c-dx}\left(\frac{d^3e^3+3d^2e^2f+6def^2+6f^3}{2ad^4}+\frac{f^3x^3}{2ad}+\frac{3f^2x^2(f+de)}{2ad^2}+\frac{3fx(d^2e^2+2def+2f^2)}{2ad^3}\right)$$

input
```
int((cosh(c + d*x)^3*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
exp(2*c + 2*d*x)*(((3*f^3 - 4*d^3*e^3 + 6*d^2*e^2*f - 6*d*e*f^2)*1i)/(32*a
*d^4) - (f^3*x^3*1i)/(8*a*d) - (f*x*(f^2 + 2*d^2*e^2 - 2*d*e*f)*3i)/(16*a*
d^3) + (f^2*x^2*(f - 2*d*e)*3i)/(16*a*d^2)) - exp(- 2*c - 2*d*x)*(((3*f^3
+ 4*d^3*e^3 + 6*d^2*e^2*f + 6*d*e*f^2)*1i)/(32*a*d^4) + (f^3*x^3*1i)/(8*a*
d) + (f*x*(f^2 + 2*d^2*e^2 + 2*d*e*f)*3i)/(16*a*d^3) + (f^2*x^2*(f + 2*d*e
)*3i)/(16*a*d^2)) - exp(c + d*x)*((6*f^3 - d^3*e^3 + 3*d^2*e^2*f - 6*d*e*f
^2)/(2*a*d^4) - (f^3*x^3)/(2*a*d) + (3*f^2*x^2*(f - d*e))/(2*a*d^2) - (3*f
*x*(2*f^2 + d^2*e^2 - 2*d*e*f))/(2*a*d^3)) - exp(- c - d*x)*((6*f^3 + d^3*
e^3 + 3*d^2*e^2*f + 6*d*e*f^2)/(2*a*d^4) + (f^3*x^3)/(2*a*d) + (3*f^2*x^2*
(f + d*e))/(2*a*d^2) + (3*f*x*(2*f^2 + d^2*e^2 + 2*d*e*f))/(2*a*d^3))
```

3.266 $\int \frac{(e+fx)^2 \cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.266.1 Optimal result

Integrand size = 31, antiderivative size = 171

$$\begin{aligned}\int \frac{(e+fx)^2 \cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = &-\frac{iefx}{2ad}-\frac{if^2x^2}{4ad}-\frac{2f(e+fx)\cosh(c+dx)}{ad^2}\\&+\frac{2f^2\sinh(c+dx)}{ad^3}+\frac{(e+fx)^2\sinh(c+dx)}{ad}\\&+\frac{if(e+fx)\cosh(c+dx)\sinh(c+dx)}{2ad^2}\\&-\frac{if^2\sinh^2(c+dx)}{4ad^3}-\frac{i(e+fx)^2\sinh^2(c+dx)}{2ad}\end{aligned}$$

output
```
-1/2*I*e*f*x/a/d-1/4*I*f^2*x^2/a/d-2*f*(f*x+e)*cosh(d*x+c)/a/d^2+2*f^2*sin
h(d*x+c)/a/d^3+(f*x+e)^2*sinh(d*x+c)/a/d+1/2*I*f*(f*x+e)*cosh(d*x+c)*sinh(
d*x+c)/a/d^2-1/4*I*f^2*sinh(d*x+c)^2/a/d^3-1/2*I*(f*x+e)^2*sinh(d*x+c)^2/a
/d
```

3.266.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.58

$$\int \frac{(e+fx)^2 \cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{-32df(e+fx)\cosh(c+dx)-2i(f^2+2d^2(e+fx)^2)\cosh(2(c+dx))+8(2(2f^2+d^2(e+fx)^2)+idf(e-}{16ad^3}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-32*d*f*(e + f*x)*Cosh[c + d*x] - (2*I)*(f^2 + 2*d^2*(e + f*x)^2)*Cosh[2*
(c + d*x)] + 8*(2*(2*f^2 + d^2*(e + f*x)^2) + I*d*f*(e + f*x)*Cosh[c + d*x
])*Sinh[c + d*x])/(16*a*d^3)
```

3.266.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.90, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {6097, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 5969, 3042, 25, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6097}$$

$$\frac{\int (e+fx)^2\cosh(c+dx)dx}{a}-\frac{i\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}-\frac{i\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}}{a}-\frac{i\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}}{a}-\frac{i\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}}{a}-\frac{i\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}}{a}-\frac{i\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{a}-\frac{i\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{a}-\frac{i\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{5969}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{a}-\frac{i\left(\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int(e+fx)\sinh^2(c+dx)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{a}-\frac{i\left(\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int-\left((e+fx)\sin(ic+idx)^2\right)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{a}-\frac{i\left(\frac{(e+fx)^2\sinh^2(c+dx)}{2d}+\frac{f\int(e+fx)\sin(ic+idx)^2dx}{d}\right)}{a}$$

$$\Big\downarrow \text{3791}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{a}-\frac{i\left(\frac{f\left(\frac{1}{2}\int(e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{17}$$

3.266. $\displaystyle\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{a}-\frac{i\left(\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}\right)}{a}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(((e + f*x)^2*Sinh[c + d*x])/d + ((2*I)*f*((I*(e + f*x)*Cosh[c + d*x])/d -
 (I*f*Sinh[c + d*x])/d^2))/d)/a - (I*(((e + f*x)^2*Sinh[c + d*x]^2)/(2*d)
+ (f*((e + f*x)^2/(4*f) - ((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) +
(f*Sinh[c + d*x]^2)/(4*d^2)))/d))/a
```

3.266.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.266. $\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

3.266.4 Maple [A] (verified)

Time = 16.57 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{i(2d^2x^2f^2+4d^2efx+2d^2e^2-2xd\,f^2-2def+f^2)\,\mathrm{e}^{2dx+2c}}{16d^3a}+\frac{(d^2x^2f^2+2d^2efx+d^2e^2-2xd\,f^2-2def+2f^2)\,\mathrm{e}^{dx+c}}{2d^3a}-\frac{(d^2x^2f^2+2d}{}$

input
```
int((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/16*I*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2-2*d*f^2*x-2*d*e*f+f^2)/d^3/a*
exp(2*d*x+2*c)+1/2*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2-2*d*f^2*x-2*d*e*f+2*f^
2)/d^3/a*exp(d*x+c)-1/2*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2+2*d*f^2*x+2*d*e*f
+2*f^2)/d^3/a*exp(-d*x-c)-1/16*I*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2+2*d*
f^2*x+2*d*e*f+f^2)/a/d^3*exp(-2*d*x-2*c)
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.33

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{(-2i\,d^2f^2x^2-2i\,d^2e^2-2i\,def-i\,f^2-2\,(2i\,d^2ef+i\,df^2)x+(-2i\,d^2f^2x^2-2i\,d^2e^2+2i\,def-i\,f^2-2\,(}{}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/16*(-2*I*d^2*f^2*x^2 - 2*I*d^2*e^2 - 2*I*d*e*f - I*f^2 - 2*(2*I*d^2*e*f
+ I*d*f^2)*x + (-2*I*d^2*f^2*x^2 - 2*I*d^2*e^2 + 2*I*d*e*f - I*f^2 - 2*(2*
I*d^2*e*f - I*d*f^2)*x)*e^(4*d*x + 4*c) + 8*(d^2*f^2*x^2 + d^2*e^2 - 2*d*e
*f + 2*f^2 + 2*(d^2*e*f - d*f^2)*x)*e^(3*d*x + 3*c) - 8*(d^2*f^2*x^2 + d^2
*e^2 + 2*d*e*f + 2*f^2 + 2*(d^2*e*f + d*f^2)*x)*e^(d*x + c))*e^(-2*d*x - 2
*c)/(a*d^3)
```

3.266.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 631 vs. 2(150) = 300.

Time = 0.44 (sec) , antiderivative size = 631, normalized size of antiderivative = 3.69

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \begin{cases} \frac{((-512a^3d^{11}e^2e^{2c}-1024a^3d^{11}efxe^{2c}-512a^3d^{11}f^2x^2e^{2c}-1024a^3d^{10}efe^{2c}-1024a^3d^{10}f^2xe^{2c}-1024a^3d^9f^2e^{2c})e^{-dx}+(512a^3d^{11}e^2e^{4c}+1024}{} \\ \frac{x^3(-if^2e^{4c}+2f^2e^{3c}+2f^2e^c+if^2)e^{-2c}}{12a}+\frac{x^2(-iefe^{4c}+2efe^{3c}+2efe^c+ief)e^{-2c}}{4a}+\frac{x(-ie^2e^{4c}+2e^2e^{3c}+2e^2e^c+ie^2)e^{-2c}}{4a} \end{cases}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise((((-512*a**3*d**11*e**2*exp(2*c) - 1024*a**3*d**11*e*f*x*exp(2*c
) - 512*a**3*d**11*f**2*x**2*exp(2*c) - 1024*a**3*d**10*e*f*exp(2*c) - 102
4*a**3*d**10*f**2*x*exp(2*c) - 1024*a**3*d**9*f**2*exp(2*c))*exp(-d*x) + (
512*a**3*d**11*e**2*exp(4*c) + 1024*a**3*d**11*e*f*x*exp(4*c) + 512*a**3*d
**11*f**2*x**2*exp(4*c) - 1024*a**3*d**10*e*f*exp(4*c) - 1024*a**3*d**10*f
**2*x*exp(4*c) + 1024*a**3*d**9*f**2*exp(4*c))*exp(d*x) + (-128*I*a**3*d**
11*e**2*exp(c) - 256*I*a**3*d**11*e*f*x*exp(c) - 128*I*a**3*d**11*f**2*x**
2*exp(c) - 128*I*a**3*d**10*e*f*exp(c) - 128*I*a**3*d**10*f**2*x*exp(c) -
64*I*a**3*d**9*f**2*exp(c))*exp(-2*d*x) + (-128*I*a**3*d**11*e**2*exp(5*c)
 - 256*I*a**3*d**11*e*f*x*exp(5*c) - 128*I*a**3*d**11*f**2*x**2*exp(5*c) +
 128*I*a**3*d**10*e*f*exp(5*c) + 128*I*a**3*d**10*f**2*x*exp(5*c) - 64*I*a
**3*d**9*f**2*exp(5*c))*exp(2*d*x))*exp(-3*c)/(1024*a**4*d**12), Ne(a**4*d
**12*exp(3*c), 0)), (x**3*(-I*f**2*exp(4*c) + 2*f**2*exp(3*c) + 2*f**2*exp
(c) + I*f**2)*exp(-2*c)/(12*a) + x**2*(-I*e*f*exp(4*c) + 2*e*f*exp(3*c) +
2*e*f*exp(c) + I*e*f)*exp(-2*c)/(4*a) + x*(-I*e**2*exp(4*c) + 2*e**2*exp(3
*c) + 2*e**2*exp(c) + I*e**2)*exp(-2*c)/(4*a), True))
```

3.266.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.266.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(151) = 302$.

Time = 0.30 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.98

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\left(2i\,d^2f^2x^2e^{(4\,dx+4\,c)} - 8\,d^2f^2x^2e^{(3\,dx+3\,c)} + 8\,d^2f^2x^2e^{(dx+c)} + 2i\,d^2f^2x^2 + 4i\,d^2efxe^{(4\,dx+4\,c)} - 16\,d^2efxe^{(}\cdots}{\cdots}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/16*(2*I*d^2*f^2*x^2*e^(4*d*x + 4*c) - 8*d^2*f^2*x^2*e^(3*d*x + 3*c) + 8
*d^2*f^2*x^2*e^(d*x + c) + 2*I*d^2*f^2*x^2 + 4*I*d^2*e*f*x*e^(4*d*x + 4*c)
 - 16*d^2*e*f*x*e^(3*d*x + 3*c) + 16*d^2*e*f*x*e^(d*x + c) + 4*I*d^2*e*f*x
 + 2*I*d^2*e^2*e^(4*d*x + 4*c) - 2*I*d*f^2*x*e^(4*d*x + 4*c) - 8*d^2*e^2*e
^(3*d*x + 3*c) + 16*d*f^2*x*e^(3*d*x + 3*c) + 8*d^2*e^2*e^(d*x + c) + 16*d
*f^2*x*e^(d*x + c) + 2*I*d^2*e^2 + 2*I*d*f^2*x - 2*I*d*e*f*e^(4*d*x + 4*c)
 + 16*d*e*f*e^(3*d*x + 3*c) + 16*d*e*f*e^(d*x + c) + 2*I*d*e*f + I*f^2*e^(
4*d*x + 4*c) - 16*f^2*e^(3*d*x + 3*c) + 16*f^2*e^(d*x + c) + I*f^2)*e^(-2*
d*x - 2*c)/(a*d^3)
```

3.266.9 Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.58

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \mathrm{e}^{c+d\,x}\left(\frac{d^2\,e^2-2\,d\,e\,f+2\,f^2}{2\,a\,d^3}+\frac{f^2\,x^2}{2\,a\,d}-\frac{f\,x\,(f-d\,e)}{a\,d^2}\right) - \mathrm{e}^{-2\,c-2\,d\,x}\left(\frac{(2\,d^2\,e^2+2\,d\,e\,f+f^2)\ 1\mathrm{i}}{16\,a\,d^3}+\frac{f^2\,x^2\ 1\mathrm{i}}{8\,a\,d} + \frac{f\,x\,(f+2\,d\,e)\ 1\mathrm{i}}{8\,a\,d^2}\right) - \mathrm{e}^{2\,c+2\,d\,x}\left(\frac{(2\,d^2\,e^2-2\,d\,e\,f+f^2)\ 1\mathrm{i}}{16\,a\,d^3} + \frac{f^2\,x^2\ 1\mathrm{i}}{8\,a\,d}-\frac{f\,x\,(f-2\,d\,e)\ 1\mathrm{i}}{8\,a\,d^2}\right) - \mathrm{e}^{-c-d\,x}\left(\frac{d^2\,e^2+2\,d\,e\,f+2\,f^2}{2\,a\,d^3}+\frac{f^2\,x^2}{2\,a\,d}+\frac{f\,x\,(f+d\,e)}{a\,d^2}\right)$$

input
```
int((cosh(c + d*x)^3*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
```

output
```
exp(c + d*x)*((2*f^2 + d^2*e^2 - 2*d*e*f)/(2*a*d^3) + (f^2*x^2)/(2*a*d) -
(f*x*(f - d*e))/(a*d^2)) - exp(- 2*c - 2*d*x)*(((f^2 + 2*d^2*e^2 + 2*d*e*f
)*1i)/(16*a*d^3) + (f^2*x^2*1i)/(8*a*d) + (f*x*(f + 2*d*e)*1i)/(8*a*d^2))
- exp(2*c + 2*d*x)*(((f^2 + 2*d^2*e^2 - 2*d*e*f)*1i)/(16*a*d^3) + (f^2*x^2
*1i)/(8*a*d) - (f*x*(f - 2*d*e)*1i)/(8*a*d^2)) - exp(- c - d*x)*((2*f^2 +
d^2*e^2 + 2*d*e*f)/(2*a*d^3) + (f^2*x^2)/(2*a*d) + (f*x*(f + d*e))/(a*d^2)
)
```

3.267 $\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.267.1 Optimal result

Integrand size = 29, antiderivative size = 98

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{ifx}{4ad}-\frac{f\cosh(c+dx)}{ad^2}+\frac{(e+fx)\sinh(c+dx)}{ad}+\frac{if\cosh(c+dx)\sinh(c+dx)}{4ad^2}-\frac{i(e+fx)\sinh^2(c+dx)}{2ad}$$

output
```
-1/4*I*f*x/a/d-f*cosh(d*x+c)/a/d^2+(f*x+e)*sinh(d*x+c)/a/d+1/4*I*f*cosh(d*
x+c)*sinh(d*x+c)/a/d^2-1/2*I*(f*x+e)*sinh(d*x+c)^2/a/d
```

3.267.2 Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.61

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{if\cosh(c+dx)(4i+\sinh(c+dx))+d(e+fx)(-i\cosh(2(c+dx))+4\sinh(c+dx))}{4ad^2}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(I*f*Cosh[c + d*x]*(4*I + Sinh[c + d*x]) + d*(e + f*x)*((-I)*Cosh[2*(c + d
*x)] + 4*Sinh[c + d*x]))/(4*a*d^2)
```

3.267.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {6097, 3042, 3777, 26, 3042, 26, 3118, 5969, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6097}$$

$$\frac{\int (e+fx)\cosh(c+dx)dx}{a}-\frac{i\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}-\frac{i\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}}{a}-\frac{i\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}}{a}-\frac{i\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}}{a}-\frac{i\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}}{a}-\frac{i\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{a}-\frac{i\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}$$

$$\Big\downarrow \text{5969}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{a}-\frac{i\left(\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int \sinh^2(c+dx)dx}{2d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{a}-\frac{i\left(\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int-\sin(ic+idx)^2dx}{2d}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{a}-\frac{i\left(\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\int\sin(ic+idx)^2dx}{2d}\right)}{a}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{a}-\frac{i\left(\frac{f\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)\sinh^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{a}-\frac{i\left(\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}\right)}{a}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d)/a - (I*(((e + f*x
)*Sinh[c + d*x]^2)/(2*d) + (f*(x/2 - (Cosh[c + d*x]*Sinh[c + d*x])/(2*d)))
/(2*d)))/a
```

3.267.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

3.267.4 Maple [A] (verified)

Time = 6.80 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{i(2dfx+2de-f)\mathrm{e}^{2dx+2c}}{16a\,d^2}+\frac{(dfx+de-f)\mathrm{e}^{dx+c}}{2a\,d^2}-\frac{(dfx+de+f)\mathrm{e}^{-dx-c}}{2a\,d^2}-\frac{i(2dfx+2de+f)\mathrm{e}^{-2dx-2c}}{16a\,d^2}$
derivativedivides	$-\frac{-\frac{icf\cosh(dx+c)^2}{2}+\frac{ide\cosh(dx+c)^2}{2}+if\left(\frac{(dx+c)\cosh(dx+c)^2}{2}-\frac{\sinh(dx+c)\cosh(dx+c)}{4}-\frac{dx}{4}-\frac{c}{4}\right)+\sinh(dx+c)cf-\sinh(dx}{d^2a}$
default	$-\frac{-\frac{icf\cosh(dx+c)^2}{2}+\frac{ide\cosh(dx+c)^2}{2}+if\left(\frac{(dx+c)\cosh(dx+c)^2}{2}-\frac{\sinh(dx+c)\cosh(dx+c)}{4}-\frac{dx}{4}-\frac{c}{4}\right)+\sinh(dx+c)cf-\sinh(dx}{d^2a}$

input
```
int((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/16*I*(2*d*f*x+2*d*e-f)/a/d^2*exp(2*d*x+2*c)+1/2*(d*f*x+d*e-f)/a/d^2*exp
(d*x+c)-1/2*(d*f*x+d*e+f)/a/d^2*exp(-d*x-c)-1/16*I*(2*d*f*x+2*d*e+f)/a/d^2
*exp(-2*d*x-2*c)
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\left(-2i\,dfx - 2i\,de + (-2i\,dfx - 2i\,de + i\,f)e^{(4\,dx+4\,c)} + 8\,(dfx + de - f)e^{(3\,dx+3\,c)} - 8\,(dfx + de + f)e^{(dx+c)}\right.}{16\,ad^2}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*(-2*I*d*f*x - 2*I*d*e + (-2*I*d*f*x - 2*I*d*e + I*f)*e^(4*d*x + 4*c)
+ 8*(d*f*x + d*e - f)*e^(3*d*x + 3*c) - 8*(d*f*x + d*e + f)*e^(d*x + c) -
I*f)*e^(-2*d*x - 2*c)/(a*d^2)
```

3.267.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(82) = 164$.

Time = 0.30 (sec) , antiderivative size = 321, normalized size of antiderivative = 3.28

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \begin{cases} \frac{\left((-512a^3d^7ee^{2c}-512a^3d^7fxe^{2c}-512a^3d^6fe^{2c})e^{-dx}+(512a^3d^7ee^{4c}+512a^3d^7fxe^{4c}-512a^3d^6fe^{4c})e^{dx}+(-128ia^3d^7ee^{c}-128ia^3d^7fxe^{c}-64i\right.}{1024a^4d^8} \\ \frac{x^2(-ife^{4c}+2fe^{3c}+2fe^{c}+if)e^{-2c}}{8a}+\frac{x(-iee^{4c}+2ee^{3c}+2ee^{c}+ie)e^{-2c}}{4a} \end{cases}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise((((-512*a**3*d**7*e*exp(2*c) - 512*a**3*d**7*f*x*exp(2*c) - 512*
a**3*d**6*f*exp(2*c))*exp(-d*x) + (512*a**3*d**7*e*exp(4*c) + 512*a**3*d**
7*f*x*exp(4*c) - 512*a**3*d**6*f*exp(4*c))*exp(d*x) + (-128*I*a**3*d**7*e*
exp(c) - 128*I*a**3*d**7*f*x*exp(c) - 64*I*a**3*d**6*f*exp(c))*exp(-2*d*x)
 + (-128*I*a**3*d**7*e*exp(5*c) - 128*I*a**3*d**7*f*x*exp(5*c) + 64*I*a**3
*d**6*f*exp(5*c))*exp(2*d*x))*exp(-3*c)/(1024*a**4*d**8), Ne(a**4*d**8*exp
(3*c), 0)), (x**2*(-I*f*exp(4*c) + 2*f*exp(3*c) + 2*f*exp(c) + I*f)*exp(-2
*c)/(8*a) + x*(-I*e*exp(4*c) + 2*e*exp(3*c) + 2*e*exp(c) + I*e)*exp(-2*c)/
(4*a), True))
```

3.267.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.267.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.41

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\left(2i\,dfxe^{(4\,dx+4\,c)} - 8\,dfxe^{(3\,dx+3\,c)} + 8\,dfxe^{(dx+c)} + 2i\,dfx + 2i\,dee^{(4\,dx+4\,c)} - 8\,dee^{(3\,dx+3\,c)} + 8\,dee^{(dx+c)} - \cdots}{16\,ad^2}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/16*(2*I*d*f*x*e^(4*d*x + 4*c) - 8*d*f*x*e^(3*d*x + 3*c) + 8*d*f*x*e^(d*
x + c) + 2*I*d*f*x + 2*I*d*e*e^(4*d*x + 4*c) - 8*d*e*e^(3*d*x + 3*c) + 8*d
*e*e^(d*x + c) + 2*I*d*e - I*f*e^(4*d*x + 4*c) + 8*f*e^(3*d*x + 3*c) + 8*f
*e^(d*x + c) + I*f)*e^(-2*d*x - 2*c)/(a*d^2)
```

3.267.9 Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.47

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\mathrm{e}^{-c-d\,x}\left(\frac{f+d\,e}{2\,a\,d^2}+\frac{f\,x}{2\,a\,d}\right) - \mathrm{e}^{-2\,c-2\,d\,x}\left(\frac{(f+2\,d\,e)\ 1\mathrm{i}}{16\,a\,d^2}+\frac{f\,x\,1\mathrm{i}}{8\,a\,d}\right) + \mathrm{e}^{2\,c+2\,d\,x}\left(\frac{(f-2\,d\,e)\ 1\mathrm{i}}{16\,a\,d^2}-\frac{f\,x\,1\mathrm{i}}{8\,a\,d}\right) - \mathrm{e}^{c+d\,x}\left(\frac{f-d\,e}{2\,a\,d^2}-\frac{f\,x}{2\,a\,d}\right)$$

input
```
int((cosh(c + d*x)^3*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

output
```
exp(2*c + 2*d*x)*(((f - 2*d*e)*1i)/(16*a*d^2) - (f*x*1i)/(8*a*d)) - exp(-
2*c - 2*d*x)*(((f + 2*d*e)*1i)/(16*a*d^2) + (f*x*1i)/(8*a*d)) - exp(- c -
d*x)*((f + d*e)/(2*a*d^2) + (f*x)/(2*a*d)) - exp(c + d*x)*((f - d*e)/(2*a*
d^2) - (f*x)/(2*a*d))
```

3.268 $\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.268.1 Optimal result

Integrand size = 24, antiderivative size = 34

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{\sinh(c+dx)}{ad}-\frac{i\sinh^2(c+dx)}{2ad}$$

output
```
sinh(d*x+c)/a/d-1/2*I*sinh(d*x+c)^2/a/d
```

3.268.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{(2-i\sinh(c+dx))\sinh(c+dx)}{2ad}$$

input
```
Integrate[Cosh[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((2 - I*Sinh[c + d*x])*Sinh[c + d*x])/(2*a*d)
```

3.268.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^3}{a+a\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3146}$$

$$-\frac{i\int (a-ia\sinh(c+dx))d(ia\sinh(c+dx))}{a^3d}$$

$$\Big\downarrow \text{17}$$

$$\frac{i(a-ia\sinh(c+dx))^2}{2a^3d}$$

input
```
Int[Cosh[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((I/2)*(a - I*a*Sinh[c + d*x])^2)/(a^3*d)
```

3.268.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.268.4 Maple [A] (verified)

Time = 6.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{i\left(\frac{\sinh(dx+c)^{2}}{2}+i\sinh(dx+c)\right)}{ad}$	30
default	$-\frac{i\left(\frac{\sinh(dx+c)^{2}}{2}+i\sinh(dx+c)\right)}{ad}$	30
risch	$-\frac{i\mathrm{e}^{2dx+2c}}{8ad}+\frac{\mathrm{e}^{dx+c}}{2ad}-\frac{\mathrm{e}^{-dx-c}}{2ad}-\frac{i\mathrm{e}^{-2dx-2c}}{8ad}$	69

input
```
int(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-I/a/d*(1/2*sinh(d*x+c)^2+I*sinh(d*x+c))
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(-i\,e^{(4\,dx+4\,c)}+4\,e^{(3\,dx+3\,c)}-4\,e^{(dx+c)}-i\right)e^{(-2\,dx-2\,c)}}{8\,ad}$$

input
```
integrate(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/8*(-I*e^(4*d*x + 4*c) + 4*e^(3*d*x + 3*c) - 4*e^(d*x + c) - I)*e^(-2*d*x
 - 2*c)/(a*d)
```

3.268.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(24) = 48.

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.91

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \begin{cases} \frac{(-32ia^3d^3e^{5c}e^{2dx}+128a^3d^3e^{4c}e^{dx}-128a^3d^3e^{2c}e^{-dx}-32ia^3d^3e^ce^{-2dx})e^{-3c}}{256a^4d^4} & \text{for } a^4d^4e^{3c}\neq 0 \\ \frac{x(-ie^{4c}+2e^{3c}+2e^c+i)e^{-2c}}{4a} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
Piecewise(((-32*I*a**3*d**3*exp(5*c)*exp(2*d*x) + 128*a**3*d**3*exp(4*c)*e
xp(d*x) - 128*a**3*d**3*exp(2*c)*exp(-d*x) - 32*I*a**3*d**3*exp(c)*exp(-2*
d*x))*exp(-3*c)/(256*a**4*d**4), Ne(a**4*d**4*exp(3*c), 0)), (x*(-I*exp(4*
c) + 2*exp(3*c) + 2*exp(c) + I)*exp(-2*c)/(4*a), True))
```

3.268.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.76

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(4i\,e^{(-dx-c)}+1\right)e^{(2\,dx+2\,c)}}{8\,ad} - \frac{i\left(-4i\,e^{(-dx-c)}+e^{(-2\,dx-2\,c)}\right)}{8\,ad}$$

input
```
integrate(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*I*(4*I*e^(-d*x - c) + 1)*e^(2*d*x + 2*c)/(a*d) - 1/8*I*(-4*I*e^(-d*x
- c) + e^(-2*d*x - 2*c))/(a*d)
```

3.268.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\frac{(4\,e^{(dx+c)}+i)e^{(-2\,dx-2\,c)}}{a}+\frac{i\,ae^{(2\,dx+2\,c)}-4\,ae^{(dx+c)}}{a^2}}{8\,d}$$

input
```
integrate(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/8*((4*e^(d*x + c) + I)*e^(-2*d*x - 2*c)/a + (I*a*e^(2*d*x + 2*c) - 4*a*
e^(d*x + c))/a^2)/d
```

3.268.9 Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{4\sinh(c+d\,x)-\cosh(2\,c+2\,d\,x)\ 1\mathrm{i}}{4\,a\,d}$$

input
```
int(cosh(c + d*x)^3/(a + a*sinh(c + d*x)*1i),x)
```

output
```
(4*sinh(c + d*x) - cosh(2*c + 2*d*x)*1i)/(4*a*d)
```

3.269 $\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

3.269.1 Optimal result

Integrand size = 31, antiderivative size = 131

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{af} - \frac{i\text{Chi}\left(\frac{2de}{f}+2dx\right)\sinh\left(2c-\frac{2de}{f}\right)}{2af} + \frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{af} - \frac{i\cosh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\frac{2de}{f}+2dx\right)}{2af}$$

output
```
Chi(d*e/f+d*x)*cosh(c-d*e/f)/a/f-1/2*I*cosh(2*c-2*d*e/f)*Shi(2*d*e/f+2*d*x
)/a/f-1/2*I*Chi(2*d*e/f+2*d*x)*sinh(2*c-2*d*e/f)/a/f+Shi(d*e/f+d*x)*sinh(c
-d*e/f)/a/f
```

3.269.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$
$$= \frac{2\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(d\left(\frac{e}{f}+x\right)\right)-i\left(\text{Chi}\left(\frac{2d(e+fx)}{f}\right)\sinh\left(2c-\frac{2de}{f}\right)+2i\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(d\left(\frac{e}{f}+x\right)\right)+\right.}{2af}$$

input
```
Integrate[Cosh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
(2*Cosh[c - (d*e)/f]*CoshIntegral[d*(e/f + x)] - I*(CoshIntegral[(2*d*(e +
 f*x))/f]*Sinh[2*c - (2*d*e)/f] + (2*I)*Sinh[c - (d*e)/f]*SinhIntegral[d*(
e/f + x)] + Cosh[2*c - (2*d*e)/f]*SinhIntegral[(2*d*(e + f*x))/f]))/(2*a*f
)
```

3.269.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {6097, 3042, 3784, 26, 3042, 26, 3779, 3782, 5971, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \; 6097$$
$$\frac{\int \frac{\cosh(c+dx)}{e+fx}dx}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}dx}{a}$$
$$\Big\downarrow \; 3042$$
$$\frac{\int \frac{\sin\left(ic+idx+\frac{\pi}{2}\right)}{e+fx}dx}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}dx}{a}$$
$$\Big\downarrow \; 3784$$

$$\frac{\cosh\left(c-\frac{de}{f}\right)\int\frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx-i\sinh\left(c-\frac{de}{f}\right)\int\frac{i\sinh\left(\frac{de}{f}+dx\right)}{e+fx}dx}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sinh\left(c-\frac{de}{f}\right)\int\frac{\sinh\left(\frac{de}{f}+dx\right)}{e+fx}dx+\cosh\left(c-\frac{de}{f}\right)\int\frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh\left(c-\frac{de}{f}\right)\int-\frac{i\sin\left(\frac{ide}{f}+idx\right)}{e+fx}dx+\cosh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\cosh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx-i\sinh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx\right)}{e+fx}dx}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}+\cosh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{3782}$$

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}dx}{a}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\int\frac{\sinh(2c+2dx)}{2(e+fx)}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\int\frac{\sinh(2c+2dx)}{e+fx}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\int-\frac{i\sin(2ic+2idx)}{e+fx}dx}{2a}$$

$$\Big\downarrow \text{26}$$

3.269. $\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{\int\frac{\sin(2ic+2idx)}{e+fx}dx}{2a}$$

$$\Big\downarrow \; 3784$$

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\sinh\left(2c-\frac{2de}{f}\right)\int\frac{\cosh\left(\frac{2de}{f}+2dx\right)}{e+fx}dx+\cosh\left(2c-\frac{2de}{f}\right)\int\frac{i\sinh\left(\frac{2de}{f}+2dx\right)}{e+fx}dx}{2a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\sinh\left(2c-\frac{2de}{f}\right)\int\frac{\cosh\left(\frac{2de}{f}+2dx\right)}{e+fx}dx+i\cosh\left(2c-\frac{2de}{f}\right)\int\frac{\sinh\left(\frac{2de}{f}+2dx\right)}{e+fx}dx}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\sinh\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2ide}{f}+2idx+\frac{\pi}{2}\right)}{e+fx}dx+i\cosh\left(2c-\frac{2de}{f}\right)\int-\frac{i\sin\left(\frac{2ide}{f}+2idx\right)}{e+fx}dx}{2a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\sinh\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2ide}{f}+2idx+\frac{\pi}{2}\right)}{e+fx}dx+\cosh\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2ide}{f}+2idx\right)}{e+fx}dx}{2a}$$

$$\Big\downarrow \; 3779$$

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{i\sinh\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2ide}{f}+2idx+\frac{\pi}{2}\right)}{e+fx}dx+\frac{i\cosh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\frac{2de}{f}+2dx\right)}{f}}{2a}$$

$$\Big\downarrow \; 3782$$

3.269. $\int\frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

$$\frac{\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}}{a}-\frac{\frac{i\sinh\left(2c-\frac{2de}{f}\right)\text{Chi}\left(\frac{2de}{f}+2dx\right)}{f}+\frac{i\cosh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\frac{2de}{f}+2dx\right)}{f}}{2a}$$

input
```
Int[Cosh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
((Cosh[c - (d*e)/f]*CoshIntegral[(d*e)/f + d*x])/f + (Sinh[c - (d*e)/f]*Si
nhIntegral[(d*e)/f + d*x])/f)/a - ((I*CoshIntegral[(2*d*e)/f + 2*d*x]*Sinh
[2*c - (2*d*e)/f])/f + (I*Cosh[2*c - (2*d*e)/f]*SinhIntegral[(2*d*e)/f + 2
*d*x])/f)/(2*a)
```

3.269.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

3.269. $\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

3.269.4 Maple [A] (verified)

Time = 33.33 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{\mathrm{e}^{-\frac{cf-de}{f}} \mathrm{Ei}_{1}\left(dx+c-\frac{cf-de}{f}\right)}{2af}-\frac{\mathrm{e}^{\frac{cf-de}{f}} \mathrm{Ei}_{1}\left(-dx-c-\frac{-cf+de}{f}\right)}{2af}+\frac{i\mathrm{e}^{\frac{2cf-2de}{f}} \mathrm{Ei}_{1}\left(-2dx-2c-\frac{2(-cf+de)}{f}\right)}{4af}-\frac{i\mathrm{e}^{-\frac{2(cf-de)}{f}} \mathrm{Ei}}{}$

input
```
int(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/f*exp(-(c*f-d*e)/f)*Ei(1,d*x+c-(c*f-d*e)/f)-1/2/a/f*exp((c*f-d*e)/f
)*Ei(1,-d*x-c-(-c*f+d*e)/f)+1/4*I/a/f*exp(2*(c*f-d*e)/f)*Ei(1,-2*d*x-2*c-2
*(-c*f+d*e)/f)-1/4*I/a/f*exp(-2*(c*f-d*e)/f)*Ei(1,2*d*x+2*c-2*(c*f-d*e)/f)
```

3.269.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \frac{i\,\mathrm{Ei}\left(-\frac{2\,(dfx+de)}{f}\right)e^{\left(\frac{2\,(de-cf)}{f}\right)}+2\,\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(\frac{de-cf}{f}\right)}+2\,\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(-\frac{de-cf}{f}\right)}-i\,\mathrm{Ei}\left(\frac{2\,(dfx+de)}{f}\right)e^{\left(-\frac{2\,(de-c}{f}\right.}}{4\,af}$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(I*Ei(-2*(d*f*x + d*e)/f)*e^(2*(d*e - c*f)/f) + 2*Ei(-(d*f*x + d*e)/f)
*e^((d*e - c*f)/f) + 2*Ei((d*f*x + d*e)/f)*e^(-(d*e - c*f)/f) - I*Ei(2*(d*
f*x + d*e)/f)*e^(-2*(d*e - c*f)/f))/(a*f)
```

3.269.6 Sympy [F]

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\cosh^3(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

input
```
integrate(cosh(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(cosh(c + d*x)**3/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) -
I*f*x), x)/a
```

3.269.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.269.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.10

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx =$$
$$-\frac{\left(i\,\mathrm{Ei}\left(\frac{2\,(dfx+de)}{f}\right)e^{\left(4\,c-\frac{2\,de}{f}\right)}-2\,\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(3\,c-\frac{de}{f}\right)}-2\,\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(c+\frac{de}{f}\right)}-i\,\mathrm{Ei}\left(-\frac{2\,(dfx+de)}{f}\right)e^{\left(\frac{2\,de}{f}\right)}\right.}{4\,af}$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(I*Ei(2*(d*f*x + d*e)/f)*e^(4*c - 2*d*e/f) - 2*Ei((d*f*x + d*e)/f)*e^
(3*c - d*e/f) - 2*Ei(-(d*f*x + d*e)/f)*e^(c + d*e/f) - I*Ei(-2*(d*f*x + d*
e)/f)*e^(2*d*e/f) + 3*I*e^(2*c)*log(f*x + e) - 3*I*e^(2*c)*log(I*f*x + I*e
))*e^(-2*c)/(a*f)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^3}{(e+f\,x)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(cosh(c + d*x)^3/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(cosh(c + d*x)^3/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.270 $\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

3.270.1 Optimal result

Integrand size = 31, antiderivative size = 180

$$\begin{aligned}\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx &= -\frac{\cosh(c+dx)}{af(e+fx)}-\frac{id\cosh\left(2c-\frac{2de}{f}\right)\mathrm{Chi}\left(\frac{2de}{f}+2dx\right)}{af^2}\\&\quad+\frac{d\mathrm{Chi}\left(\frac{de}{f}+dx\right)\sinh\left(c-\frac{de}{f}\right)}{af^2}+\frac{i\sinh(2c+2dx)}{2af(e+fx)}\\&\quad+\frac{d\cosh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(\frac{de}{f}+dx\right)}{af^2}\\&\quad-\frac{id\sinh\left(2c-\frac{2de}{f}\right)\mathrm{Shi}\left(\frac{2de}{f}+2dx\right)}{af^2}\end{aligned}$$

output
```
-I*d*Chi(2*d*e/f+2*d*x)*cosh(2*c-2*d*e/f)/a/f^2-cosh(d*x+c)/a/f/(f*x+e)+d*
cosh(c-d*e/f)*Shi(d*e/f+d*x)/a/f^2-I*d*Shi(2*d*e/f+2*d*x)*sinh(2*c-2*d*e/f
)/a/f^2+d*Chi(d*e/f+d*x)*sinh(c-d*e/f)/a/f^2+1/2*I*sinh(2*d*x+2*c)/a/f/(f*
x+e)
```

3.270.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.18

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$= \frac{-2f\cosh(c+dx)-2id(e+fx)\cosh\left(2c-\frac{2de}{f}\right)\mathrm{Chi}\left(\frac{2d(e+fx)}{f}\right)+2d(e+fx)\mathrm{Chi}\left(d\left(\frac{e}{f}+x\right)\right)\sinh\left(c-\cdots\right.}{\cdots}$$

input
```
Integrate[Cosh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
(-2*f*Cosh[c + d*x] - (2*I)*d*(e + f*x)*Cosh[2*c - (2*d*e)/f]*CoshIntegral
[(2*d*(e + f*x))/f] + 2*d*(e + f*x)*CoshIntegral[d*(e/f + x)]*Sinh[c - (d*
e)/f] + I*f*Sinh[2*(c + d*x)] + 2*d*e*Cosh[c - (d*e)/f]*SinhIntegral[d*(e/
f + x)] + 2*d*f*x*Cosh[c - (d*e)/f]*SinhIntegral[d*(e/f + x)] - (2*I)*d*e*
Sinh[2*c - (2*d*e)/f]*SinhIntegral[(2*d*(e + f*x))/f] - (2*I)*d*f*x*Sinh[2
*c - (2*d*e)/f]*SinhIntegral[(2*d*(e + f*x))/f])/(2*a*f^2*(e + f*x))
```

3.270.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.774, Rules used = {6097, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782, 5971, 27, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \ 6097$$
$$\frac{\int \frac{\cosh(c+dx)}{(e+fx)^2}dx}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$
$$\Big\downarrow \ 3042$$
$$\frac{\int \frac{\sin\left(ic+idx+\frac{\pi}{2}\right)}{(e+fx)^2}dx}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$
$$\Big\downarrow \ 3778$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}+\frac{id\int -\frac{i\sinh(c+dx)}{e+fx}dx}{f}}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{d\int \frac{\sinh(c+dx)}{e+fx}dx}{f}-\frac{\cosh(c+dx)}{f(e+fx)}}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}+\frac{d\int -\frac{i\sin(ic+idx)}{e+fx}dx}{f}}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\int \frac{\sin(ic+idx)}{e+fx}dx}{f}}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \; 3784$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(i\sinh\left(c-\frac{de}{f}\right)\int \frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx+\cosh\left(c-\frac{de}{f}\right)\int \frac{i\sinh\left(\frac{de}{f}+dx\right)}{e+fx}dx\right)}{f}}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(i\sinh\left(c-\frac{de}{f}\right)\int \frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx+i\cosh\left(c-\frac{de}{f}\right)\int \frac{\sinh\left(\frac{de}{f}+dx\right)}{e+fx}dx\right)}{f}}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(i\sinh\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx+i\cosh\left(c-\frac{de}{f}\right)\int -\frac{i\sin\left(\frac{ide}{f}+idx\right)}{e+fx}dx\right)}{f}}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(i\sinh\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx+\cosh\left(c-\frac{de}{f}\right)\int \frac{\sin\left(\frac{ide}{f}+idx\right)}{e+fx}dx\right)}{f}}{a}-\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \; 3779$$

3.270. $\displaystyle\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(i\sinh\left(c-\frac{de}{f}\right)\int\frac{\sin\left(\frac{ide}{f}+idx+\frac{\pi}{2}\right)}{e+fx}dx+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \ 3782$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{i\int\frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}dx}{a}$$

$$\Big\downarrow \ 5971$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{i\int\frac{\sinh(2c+2dx)}{2(e+fx)^2}dx}{a}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{i\int\frac{\sinh(2c+2dx)}{(e+fx)^2}dx}{2a}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{i\int-\frac{i\sin(2ic+2idx)}{(e+fx)^2}dx}{2a}$$

$$\Big\downarrow \ 26$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\int\frac{\sin(2ic+2idx)}{(e+fx)^2}dx}{2a}$$

$$\Big\downarrow \ 3778$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\frac{2id\int\frac{\cosh(2c+2dx)}{e+fx}dx}{f}-\frac{i\sinh(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\frac{2id\int\frac{\sin\left(2ic+2idx+\frac{\pi}{2}\right)}{e+fx}dx}{f}-\frac{i\sinh(2c+2dx)}{f(e+fx)}}{2a}$$

3.270. $\displaystyle\int\frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

$$\Big\downarrow \; 3784$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\frac{2id\left(\cosh\left(2c-\frac{2de}{f}\right)\int\frac{\cosh\left(\frac{2de}{f}+2dx\right)}{e+fx}dx-i\sinh\left(2c-\frac{2de}{f}\right)\int\frac{i\sinh\left(\frac{2de}{f}+2dx\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\frac{2id\left(\sinh\left(2c-\frac{2de}{f}\right)\int\frac{\sinh\left(\frac{2de}{f}+2dx\right)}{e+fx}dx+\cosh\left(2c-\frac{2de}{f}\right)\int\frac{\cosh\left(\frac{2de}{f}+2dx\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\frac{2id\left(\sinh\left(2c-\frac{2de}{f}\right)\int-\frac{i\sin\left(\frac{2ide}{f}+2idx\right)}{e+fx}dx+\cosh\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2ide}{f}+2idx+\frac{\pi}{2}\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\frac{2id\left(\cosh\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2ide}{f}+2idx+\frac{\pi}{2}\right)}{e+fx}dx-i\sinh\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2ide}{f}+2idx\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \; 3779$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\frac{2id\left(\frac{\sinh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\frac{2de}{f}+2dx\right)}{f}+\cosh\left(2c-\frac{2de}{f}\right)\int\frac{\sin\left(\frac{2ide}{f}+2idx+\frac{\pi}{2}\right)}{e+fx}dx\right)}{f}-\frac{i\sinh(2c+2dx)}{f(e+fx)}}{2a}$$

$$\Big\downarrow \; 3782$$

3.270. $$\int\frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

$$\frac{-\frac{\cosh(c+dx)}{f(e+fx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{de}{f}\right)\mathrm{Chi}\left(\frac{de}{f}+dx\right)}{f}+\frac{i\cosh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(\frac{de}{f}+dx\right)}{f}\right)}{f}}{a}-\frac{\frac{2id\left(\frac{\cosh\left(2c-\frac{2de}{f}\right)\mathrm{Chi}\left(\frac{2de}{f}+2dx\right)}{f}+\frac{\sinh\left(2c-\frac{2de}{f}\right)\mathrm{Shi}\left(\frac{2de}{f}+2dx\right)}{f}\right)}{f}-\frac{i\sinh(2c+2dx)}{f(e+fx)}}{2a}$$

input
```
Int[Cosh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
(-(Cosh[c + d*x]/(f*(e + f*x))) - (I*d*((I*CoshIntegral[(d*e)/f + d*x]*Sin
h[c - (d*e)/f])/f + (I*Cosh[c - (d*e)/f]*SinhIntegral[(d*e)/f + d*x])/f))/
f)/a - (((-I)*Sinh[2*c + 2*d*x])/(f*(e + f*x)) + ((2*I)*d*((Cosh[2*c - (2*
d*e)/f]*CoshIntegral[(2*d*e)/f + 2*d*x])/f + (Sinh[2*c - (2*d*e)/f]*SinhIn
tegral[(2*d*e)/f + 2*d*x])/f))/f)/(2*a)
```

3.270.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

3.270. $\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6097
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Cosh[c
 + d*x]^(n - 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*
Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && E
qQ[a^2 + b^2, 0]
```

3.270.4 Maple [A] (verified)

Time = 117.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.66

method	result
risch	$-\frac{d\,\mathrm{e}^{-dx-c}}{2af(dfx+de)}+\frac{d\,\mathrm{e}^{-\frac{cf-de}{f}}\,\mathrm{Ei}_{1}\left(dx+c-\frac{cf-de}{f}\right)}{2a\,f^{2}}-\frac{d\,\mathrm{e}^{dx+c}}{2f^{2}a\left(\frac{de}{f}+dx\right)}-\frac{d\,\mathrm{e}^{\frac{cf-de}{f}}\,\mathrm{Ei}_{1}\left(-dx-c-\frac{-cf+de}{f}\right)}{2f^{2}a}+\frac{id\,\mathrm{e}^{2dx+2c}}{4a\,f^{2}\left(\frac{de}{f}+dx\right)}+$

input
```
int(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d/a*exp(-d*x-c)/f/(d*f*x+d*e)+1/2*d/a/f^2*exp(-(c*f-d*e)/f)*Ei(1,d*x+
c-(c*f-d*e)/f)-1/2/f^2*d/a*exp(d*x+c)/(d*e/f+d*x)-1/2/f^2*d/a*exp((c*f-d*e
)/f)*Ei(1,-d*x-c-(-c*f+d*e)/f)+1/4*I*d/a/f^2*exp(2*d*x+2*c)/(d*e/f+d*x)+1/
2*I*d/a/f^2*exp(2*(c*f-d*e)/f)*Ei(1,-2*d*x-2*c-2*(-c*f+d*e)/f)-1/4*I/a*d*e
xp(-2*d*x-2*c)/f/(d*f*x+d*e)+1/2*I/a*d/f^2*exp(-2*(c*f-d*e)/f)*Ei(1,2*d*x+
2*c-2*(c*f-d*e)/f)
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.26

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$= \frac{\left(i\,fe^{(4\,dx+4\,c)} - 2\,fe^{(3\,dx+3\,c)} - 2\left((i\,dfx+i\,de)\mathrm{Ei}\left(-\frac{2\,(dfx+de)}{f}\right)e^{\left(\frac{2\,(de-cf)}{f}\right)} + (dfx+de)\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(\frac{de-}{f}\right.}\right.\right.}{4}$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/4*(I*f*e^(4*d*x + 4*c) - 2*f*e^(3*d*x + 3*c) - 2*((I*d*f*x + I*d*e)*Ei(-
2*(d*f*x + d*e)/f)*e^(2*(d*e - c*f)/f) + (d*f*x + d*e)*Ei(-(d*f*x + d*e)/f
)*e^((d*e - c*f)/f) - (d*f*x + d*e)*Ei((d*f*x + d*e)/f)*e^(-(d*e - c*f)/f)
 + (I*d*f*x + I*d*e)*Ei(2*(d*f*x + d*e)/f)*e^(-2*(d*e - c*f)/f))*e^(2*d*x
+ 2*c) - 2*f*e^(d*x + c) - I*f)*e^(-2*d*x - 2*c)/(a*f^3*x + a*e*f^2)
```

3.270.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.270.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.270.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1080 vs. $2(172) = 344$.

Time = 0.35 (sec) , antiderivative size = 1080, normalized size of antiderivative = 6.00

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(2*I*(f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))*d^2*Ei(-2*((f*x +
e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^(2*(d*e - c*f)/f)
 + 2*I*d^3*e*Ei(-2*((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e -
c*f)/f)*e^(2*(d*e - c*f)/f) - 2*I*c*d^2*f*Ei(-2*((f*x + e)*(d - d*e/(f*x +
 e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^(2*(d*e - c*f)/f) + 2*(f*x + e)*(d
- d*e/(f*x + e) + c*f/(f*x + e))*d^2*Ei(-((f*x + e)*(d - d*e/(f*x + e) + c
*f/(f*x + e)) + d*e - c*f)/f)*e^((d*e - c*f)/f) + 2*d^3*e*Ei(-((f*x + e)*(
d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^((d*e - c*f)/f) - 2*c
*d^2*f*Ei(-((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*
e^((d*e - c*f)/f) - 2*(f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))*d^2*Ei
(((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^(-(d*e -
 c*f)/f) - 2*d^3*e*Ei(((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e
 - c*f)/f)*e^(-(d*e - c*f)/f) + 2*c*d^2*f*Ei(((f*x + e)*(d - d*e/(f*x + e)
 + c*f/(f*x + e)) + d*e - c*f)/f)*e^(-(d*e - c*f)/f) + 2*I*(f*x + e)*(d -
d*e/(f*x + e) + c*f/(f*x + e))*d^2*Ei(2*((f*x + e)*(d - d*e/(f*x + e) + c*
f/(f*x + e)) + d*e - c*f)/f)*e^(-2*(d*e - c*f)/f) + 2*I*d^3*e*Ei(2*((f*x +
 e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^(-2*(d*e - c*f)/
f) - 2*I*c*d^2*f*Ei(2*((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e
 - c*f)/f)*e^(-2*(d*e - c*f)/f) - I*d^2*f*e^(2*(f*x + e)*(d - d*e/(f*x + e
) + c*f/(f*x + e))/f) + 2*d^2*f*e^((f*x + e)*(d - d*e/(f*x + e) + c*f/(...
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^3}{(e+f\,x)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(cosh(c + d*x)^3/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(cosh(c + d*x)^3/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.271 $\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.271.1 Optimal result

Integrand size = 29, antiderivative size = 463

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{3if(e+fx)^2}{2ad^2} - \frac{6f^2(e+fx)\arctan\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\arctan\left(e^{c+dx}\right)}{ad} + \frac{3if^2(e+fx)\log\left(1+e^{2(c+dx)}\right)}{ad^3} + \frac{3if^3\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^4} - \frac{3if(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2ad^2} - \frac{3if^3\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{ad^4} + \frac{3if(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2ad^2} + \frac{3if^3\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2ad^4} + \frac{3if^2(e+fx)\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3} - \frac{3if^2(e+fx)\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{ad^3} - \frac{3if^3\,\text{PolyLog}\left(4,-ie^{c+dx}\right)}{ad^4} + \frac{3if^3\,\text{PolyLog}\left(4,ie^{c+dx}\right)}{ad^4} + \frac{3f(e+fx)^2\text{sech}(c+dx)}{2ad^2} + \frac{i(e+fx)^3\text{sech}^2(c+dx)}{2ad} - \frac{3if(e+fx)^2\tanh(c+dx)}{2ad^2} + \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{2ad}$$

output
```
3/2*I*f*(f*x+e)^2*polylog(2,I*exp(d*x+c))/a/d^2-6*f^2*(f*x+e)*arctan(exp(d
*x+c))/a/d^3+(f*x+e)^3*arctan(exp(d*x+c))/a/d+3*I*f^3*polylog(2,-I*exp(d*x
+c))/a/d^4+3/2*I*f^3*polylog(2,-exp(2*d*x+2*c))/a/d^4+1/2*I*(f*x+e)^3*sech
(d*x+c)^2/a/d-3*I*f^3*polylog(2,I*exp(d*x+c))/a/d^4-3/2*I*f*(f*x+e)^2*tanh
(d*x+c)/a/d^2+3*I*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/a/d^3-3*I*f^3*polyl
og(4,-I*exp(d*x+c))/a/d^4+3*I*f^3*polylog(4,I*exp(d*x+c))/a/d^4-3/2*I*f*(f
*x+e)^2/a/d^2+3*I*f^2*(f*x+e)*ln(1+exp(2*d*x+2*c))/a/d^3+3/2*f*(f*x+e)^2*s
ech(d*x+c)/a/d^2-3/2*I*f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/a/d^2-3*I*f^2*
(f*x+e)*polylog(3,I*exp(d*x+c))/a/d^3+1/2*(f*x+e)^3*sech(d*x+c)*tanh(d*x+c
)/a/d
```

3.271.2 Mathematica [A] (verified)

Time = 8.07 (sec) , antiderivative size = 828, normalized size of antiderivative = 1.79

$$\int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx =$$

$$-\frac{\frac{(e+fx)^4}{f}+\frac{4(1-ie^c)(e+fx)^3\log(1+ie^{-c-dx})}{d}+\frac{12i(i+e^c)f(d^2(e+fx)^2\operatorname{PolyLog}(2,-ie^{-c-dx})+2f(d(e+fx)\operatorname{PolyLog}(3,-ie^{-c-dx})+f\operatorname{P}}{d^4}}{8a\left(i+e^c\right)}$$

$$-\frac{-4d^2e(1+ie^c)\,f(d^2e^2-12f^2)\,x+\left(-12f^2+d^2(e+fx)^2\right)^2+12d(1+ie^c)\,f^2(d^2e^2-4f^2)\,x\log\left(1-ie\right.}{}$$

$$+\frac{x(4e^3+6e^2fx+4ef^2x^2+f^3x^3)}{8a\left(\cosh\left(\frac{c}{2}\right)-i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)}$$

$$+\frac{i(e+fx)^3}{2ad\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)+i\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}$$

$$-\frac{3i\left(e^2f\sinh\left(\frac{dx}{2}\right)+2ef^2x\sinh\left(\frac{dx}{2}\right)+f^3x^2\sinh\left(\frac{dx}{2}\right)\right)}{ad^2\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)+i\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

input
```
Integrate[((e + f*x)^3*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-1/8*((e + f*x)^4/f + (4*(1 - I*E^c)*(e + f*x)^3*Log[1 + I*E^(-c - d*x)])/
d + ((12*I)*(I + E^c)*f*(d^2*(e + f*x)^2*PolyLog[2, (-I)*E^(-c - d*x)] + 2
*f*(d*(e + f*x)*PolyLog[3, (-I)*E^(-c - d*x)] + f*PolyLog[4, (-I)*E^(-c -
d*x)])))/d^4)/(a*(I + E^c)) - (-4*d^2*e*(1 + I*E^c)*f*(d^2*e^2 - 12*f^2)*x
 + (-12*f^2 + d^2*(e + f*x)^2)^2 + 12*d*(1 + I*E^c)*f^2*(d^2*e^2 - 4*f^2)*
x*Log[1 - I*E^(-c - d*x)] + 12*d^3*e*(1 + I*E^c)*f^3*x^2*Log[1 - I*E^(-c -
 d*x)] + 4*d^3*(1 + I*E^c)*f^4*x^3*Log[1 - I*E^(-c - d*x)] + 4*d*e*(1 + I*
E^c)*f*(d^2*e^2 - 12*f^2)*Log[I - E^(c + d*x)] + 12*(1 + I*E^c)*f^2*(-(d^2
*e^2) + 4*f^2)*PolyLog[2, I*E^(-c - d*x)] - 24*d^2*e*(1 + I*E^c)*f^3*x*Pol
yLog[2, I*E^(-c - d*x)] - 12*d^2*(1 + I*E^c)*f^4*x^2*PolyLog[2, I*E^(-c -
d*x)] - 24*d*e*(1 + I*E^c)*f^3*PolyLog[3, I*E^(-c - d*x)] - 24*d*(1 + I*E^
c)*f^4*x*PolyLog[3, I*E^(-c - d*x)] - 24*(1 + I*E^c)*f^4*PolyLog[4, I*E^(-
c - d*x)])/(8*a*d^4*(-I + E^c)*f) + (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 +
f^3*x^3))/(8*a*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])) + ((I/
2)*(e + f*x)^3)/(a*d*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2])^2) - ((
3*I)*(e^2*f*Sinh[(d*x)/2] + 2*e*f^2*x*Sinh[(d*x)/2] + f^3*x^2*Sinh[(d*x)/2
]))/(a*d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (
d*x)/2]))
```

3.271.3 Rubi [A] (verified)

Time = 2.90 (sec) , antiderivative size = 442, normalized size of antiderivative = 0.95, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.724$, Rules used = {6105, 3042, 4674, 3042, 4668, 2715, 2838, 3011, 5974, 3042, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \textrm{6105}$$

$$\frac{\int (e+fx)^3\text{sech}^3(c+dx)dx}{a}-\frac{i\int (e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \textrm{3042}$$

$$\frac{\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{a}-\frac{i\int (e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \textrm{4674}$$

$$\frac{-\frac{3f^2\int (e+fx)\text{sech}(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^3\text{sech}(c+dx)dx+\frac{3f(e+fx)^2\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)^3\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\int (e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \textrm{3042}$$

$$\frac{-\frac{3f^2\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3f(e+fx)^2\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)^3\tanh(c+dx)\text{sech}(c+}{2d}}{a}$$

$$\frac{i\int (e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \textrm{4668}$$

$$\frac{-\frac{3f^2\left(-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(-\frac{3if\int (e+fx)^2\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{3if\int (e+fx)^2\log(1}{d}\right.}{a}$$

$$\frac{i\int (e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \textrm{2715}$$

$$-\frac{3f^2\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(-\frac{3if\int(e+fx)^2\log(1-ie^{c+dx})dx}{d}\right.$$
$$a$$

$$\frac{i\int(e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 2838$$

$$\frac{\frac{1}{2}\left(-\frac{3if\int(e+fx)^2\log(1-ie^{c+dx})dx}{d}+\frac{3if\int(e+fx)^2\log(1+ie^{c+dx})dx}{d}+\frac{2(e+fx)^3\arctan(e^{c+dx})}{d}\right)-\frac{3f^2\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{Poly}}{}\right.}{}}{a}$$

$$\frac{i\int(e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 3011$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\right.}{}$$

$$\frac{i\int(e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 5974$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\right.}{}$$

$$\frac{i\left(\frac{3f\int(e+fx)^2\text{sech}^2(c+dx)dx}{2d}-\frac{(e+fx)^3\text{sech}^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\right.}{}$$

$$\frac{i\left(-\frac{(e+fx)^3\text{sech}^2(c+dx)}{2d}+\frac{3f\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d}\right)}{a}$$

$$\Big\downarrow \; 4672$$

3.271. $\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\right.}{\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2if\int-i(e+fx)\tanh(c+dx)dx}{d}\right)}{2d}\right)}{a}}$$

$$\Big\downarrow \textbf{26}$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\right.}{\frac{i\left(\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int(e+fx)\tanh(c+dx)dx}{d}\right)}{2d}-\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{2d}\right)}{a}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\right.}{\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int-i(e+fx)\tan(ic+idx)dx}{d}\right)}{2d}\right)}{a}}$$

$$\Big\downarrow \textbf{26}$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\right.}{\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\int(e+fx)\tan(ic+idx)dx}{d}\right)}{2d}\right)}{a}}$$

$$\Big\downarrow \textbf{4201}$$

3.271. $\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\right.}{i\left(-\frac{(e+fx)^3\text{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\int\frac{e^{2(c+dx)}(e+fx)}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\right.}{i\left(-\frac{(e+fx)^3\text{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int\log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\right.}{i\left(-\frac{(e+fx)^3\text{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int e^{-2(c+dx)}\log\left(1+e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right.}{2d}\right.}{a}$$

$$\Big\downarrow \text{2838}$$

3.271. $\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^3\text{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \; 7163$$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\int \text{PolyLog}\left(3,-ie^{c+dx}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{d}-\frac{f\int}{}\right.}{d}\right.}{}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^3\text{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \; 2720$$

3.271. $\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{1}{2}\left(\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}(3,-ie^{c+dx})}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}(3,-ie^{c+dx})de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-ie^{c+dx})}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}(3,\cdots}{d}\right.}{}\right.}{}\right.}{}$$

$$\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}(2,-e^{2(c+dx)})}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \text{7143}$$

$$\frac{-\frac{3f^2\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{if\operatorname{PolyLog}(2,ie^{c+dx})}{d^2}\right)}{d^2}+\frac{1}{2}\left(\frac{2(e+fx)^3\arctan(e^{c+dx})}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}(3,-i\cdots}{d}\right.}{}\right.}{}\right.}{}$$

$$\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{2d}+\frac{3f\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}(2,-e^{2(c+dx)})}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}\right)}{a}$$

input
```
Int[((e + f*x)^3*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

3.271. $\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

output
```
((-3*f^2*((2*(e + f*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c
+ d*x)])/d^2 + (I*f*PolyLog[2, I*E^(c + d*x)])/d^2))/d^2 + ((2*(e + f*x)^3
*ArcTan[E^(c + d*x)])/d + ((3*I)*f*(-(((e + f*x)^2*PolyLog[2, (-I)*E^(c +
d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/d - (f*PolyLog[
4, (-I)*E^(c + d*x)])/d^2))/d))/d - ((3*I)*f*(-(((e + f*x)^2*PolyLog[2, I*
E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, I*E^(c + d*x)])/d - (f*Poly
Log[4, I*E^(c + d*x)])/d^2))/d))/d)/2 + (3*f*(e + f*x)^2*Sech[c + d*x])/(2
*d^2) + ((e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/a - (I*(-1/2*((e
+ f*x)^3*Sech[c + d*x]^2)/d + (3*f*(((2*I)*f*(((-1/2*I)*(e + f*x)^2)/f + (
2*I)*(((e + f*x)*Log[1 + E^(2*(c + d*x))])/(2*d) + (f*PolyLog[2, -E^(2*(c
+ d*x))])/(4*d^2))))/d + ((e + f*x)^2*Tanh[c + d*x])/d))/(2*d)))/a
```

3.271.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.271. $\int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

3.271. $\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.271.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1079 vs. 2(416) = 832.

Time = 22.22 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.33

method	result	size
risch	Expression too large to display	1080

input
```
int((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(d*f^3*x^3*exp(d*x+c)+3*d*e*f^2*x^2*exp(d*x+c)+3*d*e^2*f*x*exp(d*x+c)+d*e^
3*exp(d*x+c)+3*f^3*x^2*exp(d*x+c)-3*I*f^3*x^2+6*e*f^2*x*exp(d*x+c)-6*I*e*f
^2*x+3*e^2*f*exp(d*x+c)-3*I*e^2*f)/(exp(d*x+c)-I)^2/d^2/a-3*I*f^3*polylog(
4,-I*exp(d*x+c))/a/d^4-3/2*I/a/d*e*f^2*ln(1+I*exp(d*x+c))*x^2-3*I/a/d^2*e*
f^2*polylog(2,-I*exp(d*x+c))*x+3*I*f^3*polylog(4,I*exp(d*x+c))/a/d^4-3*I/a
/d^4*f^3*c*ln(1+exp(2*d*x+2*c))+6*I/a/d^4*f^3*c*ln(exp(d*x+c))+3/2*I/a/d^2
*e^2*f*polylog(2,I*exp(d*x+c))-3/2*I/a/d^2*e^2*f*polylog(2,-I*exp(d*x+c))+
1/2*I/a/d^4*f^3*ln(1-I*exp(d*x+c))*c^3-1/2*I/a/d^4*f^3*ln(1+I*exp(d*x+c))*
c^3+1/2*I/a/d*f^3*ln(1-I*exp(d*x+c))*x^3+3/2*I/a/d^2*f^3*polylog(2,I*exp(d
*x+c))*x^2-3*I/a/d^3*f^3*polylog(3,I*exp(d*x+c))*x-1/2*I/a/d*f^3*ln(1+I*ex
p(d*x+c))*x^3-3/2*I/a/d^2*f^3*polylog(2,-I*exp(d*x+c))*x^2+3*I/a/d^3*f^3*p
olylog(3,-I*exp(d*x+c))*x+1/a/d*e^3*arctan(exp(d*x+c))-3/a/d^2*e^2*f*c*arc
tan(exp(d*x+c))-3/2*I/a/d^3*e*f^2*ln(1-I*exp(d*x+c))*c^2+3/2*I/a/d^3*e*f^2
*ln(1+I*exp(d*x+c))*c^2+3/2*I/a/d*e^2*f*ln(1-I*exp(d*x+c))*x+3/2*I/a/d^2*e
^2*f*ln(1-I*exp(d*x+c))*c-3/2*I/a/d*e^2*f*ln(1+I*exp(d*x+c))*x-3/2*I/a/d^2
*e^2*f*ln(1+I*exp(d*x+c))*c+3/2*I/a/d*e*f^2*ln(1-I*exp(d*x+c))*x^2+3*I/a/d
^2*e*f^2*polylog(2,I*exp(d*x+c))*x+3/a/d^3*f^2*c^2*e*arctan(exp(d*x+c))+3*
I/a/d^3*e*f^2*polylog(3,-I*exp(d*x+c))+3*I/a/d^3*e*f^2*ln(1+exp(2*d*x+2*c)
)-6*I/a/d^3*e*f^2*ln(exp(d*x+c))-3*I/a/d^3*e*f^2*polylog(3,I*exp(d*x+c))-6
*I/a/d^3*f^3*c*x+6*I/a/d^3*f^3*ln(1+I*exp(d*x+c))*x+6*I/a/d^4*f^3*ln(1+...
```

3.271.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1461 vs. $2(390) = 780$.

Time = 0.26 (sec) , antiderivative size = 1461, normalized size of antiderivative = 3.16

$$\int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(-6*I*d^2*e^2*f + 12*I*c*d*e*f^2 - 6*I*c^2*f^3 - 3*(I*d^2*f^3*x^2 + 2*
I*d^2*e*f^2*x + I*d^2*e^2*f + (-I*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x - I*d^2*e^
2*f)*e^(2*d*x + 2*c) - 2*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*e^(d*x
+ c))*dilog(I*e^(d*x + c)) - 3*(-I*d^2*f^3*x^2 - 2*I*d^2*e*f^2*x - I*d^2*e
^2*f + 4*I*f^3 + (I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x + I*d^2*e^2*f - 4*I*f^3)
*e^(2*d*x + 2*c) + 2*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f - 4*f^3)*e^(
d*x + c))*dilog(-I*e^(d*x + c)) - 6*(I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x + 2*I
*c*d*e*f^2 - I*c^2*f^3)*e^(2*d*x + 2*c) + 2*(d^3*f^3*x^3 + d^3*e^3 + 3*d^2
*e^2*f - 12*c*d*e*f^2 + 6*c^2*f^3 + 3*(d^3*e*f^2 - d^2*f^3)*x^2 + 3*(d^3*e
^2*f - 2*d^2*e*f^2)*x)*e^(d*x + c) + (-I*d^3*e^3 + 3*I*c*d^2*e^2*f - 3*I*c
^2*d*e*f^2 + I*c^3*f^3 + (I*d^3*e^3 - 3*I*c*d^2*e^2*f + 3*I*c^2*d*e*f^2 -
I*c^3*f^3)*e^(2*d*x + 2*c) + 2*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 -
c^3*f^3)*e^(d*x + c))*log(e^(d*x + c) + I) + (I*d^3*e^3 - 3*I*c*d^2*e^2*f
- 3*(-I*c^2 + 4*I)*d*e*f^2 + (-I*c^3 + 12*I*c)*f^3 + (-I*d^3*e^3 + 3*I*c*d
^2*e^2*f - 3*(I*c^2 - 4*I)*d*e*f^2 + (I*c^3 - 12*I*c)*f^3)*e^(2*d*x + 2*c)
 - 2*(d^3*e^3 - 3*c*d^2*e^2*f + 3*(c^2 - 4)*d*e*f^2 - (c^3 - 12*c)*f^3)*e^
(d*x + c))*log(e^(d*x + c) - I) + (I*d^3*f^3*x^3 + 3*I*d^3*e*f^2*x^2 + 3*I
*c*d^2*e^2*f - 3*I*c^2*d*e*f^2 + (I*c^3 - 12*I*c)*f^3 - 3*(-I*d^3*e^2*f +
4*I*d*f^3)*x + (-I*d^3*f^3*x^3 - 3*I*d^3*e*f^2*x^2 - 3*I*c*d^2*e^2*f + 3*I
*c^2*d*e*f^2 + (-I*c^3 + 12*I*c)*f^3 - 3*(I*d^3*e^2*f - 4*I*d*f^3)*x)*e...
```

3.271.6 Sympy [F]

$$\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^3\operatorname{sech}(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{f^3x^3\operatorname{sech}(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{3ef^2x^2\operatorname{sech}(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{3e^2fx\operatorname{sech}(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**3*sech(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**3*x**
*3*sech(c + d*x)/(sinh(c + d*x) - I), x) + Integral(3*e*f**2*x**2*sech(c +
 d*x)/(sinh(c + d*x) - I), x) + Integral(3*e**2*f*x*sech(c + d*x)/(sinh(c
+ d*x) - I), x))/a
```

3.271.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.48

$$\int \frac{(e+fx)^3\mathrm{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx =$$
$$-\frac{1}{2}\,e^3\left(\frac{4\,e^{(-dx-c)}}{-2\,(-2i\,ae^{(-dx-c)}-ae^{(-2\,dx-2\,c)}+a)d}+\frac{i\,\log\left(e^{(-dx-c)}+i\right)}{ad}-\frac{i\,\log\left(i\,e^{(-dx-c)}+1\right)}{ad}\right)$$
$$+\frac{3i\,\left(dx\log\left(-i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(i\,e^{(dx+c)}\right)\right)e^2f}{2\,ad^2}-\frac{6i\,ef^2x}{ad^2}$$
$$+\frac{-3i\,f^3x^2-6i\,ef^2x-3i\,e^2f+(df^3x^3e^c+3\,e^2fe^c+3\,(def^2+f^3)x^2e^c+3\,(de^2f+2\,ef^2)xe^c)e^{(dx)}}{ad^2e^{(2\,dx+2\,c)}-2i\,ad^2e^{(dx+c)}-ad^2}$$
$$-\frac{3i\,\left(d^2x^2\log\left(i\,e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-2\,\mathrm{Li}_3(-i\,e^{(dx+c)})\right)ef^2}{2\,ad^3}$$
$$+\frac{3i\,\left(d^2x^2\log\left(-i\,e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(i\,e^{(dx+c)}\right)-2\,\mathrm{Li}_3(i\,e^{(dx+c)})\right)ef^2}{2\,ad^3}$$
$$+\frac{6i\,ef^2\log\left(i\,e^{(dx+c)}+1\right)}{ad^3}$$
$$-\frac{i\,\left(d^3x^3\log\left(i\,e^{(dx+c)}+1\right)+3\,d^2x^2\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-6\,dx\mathrm{Li}_3(-i\,e^{(dx+c)})+6\,\mathrm{Li}_4(-i\,e^{(dx+c)})\right)f^3}{2\,ad^4}$$
$$+\frac{i\,\left(d^3x^3\log\left(-i\,e^{(dx+c)}+1\right)+3\,d^2x^2\mathrm{Li}_2\left(i\,e^{(dx+c)}\right)-6\,dx\mathrm{Li}_3(i\,e^{(dx+c)})+6\,\mathrm{Li}_4(i\,e^{(dx+c)})\right)f^3}{2\,ad^4}$$
$$-\frac{3i\,(d^2e^2f-4\,f^3)\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)\right)}{2\,ad^4}$$
$$-\frac{i\,d^4f^3x^4+4i\,d^4ef^2x^3+6i\,d^4e^2fx^2}{8\,ad^4}+\frac{i\,d^4f^3x^4+4i\,d^4ef^2x^3-6\,(-i\,d^2e^2f+4i\,f^3)d^2x^2}{8\,ad^4}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*e^3*(4*e^(-d*x - c)/((4*I*a*e^(-d*x - c) + 2*a*e^(-2*d*x - 2*c) - 2*a
)*d) + I*log(e^(-d*x - c) + I)/(a*d) - I*log(I*e^(-d*x - c) + 1)/(a*d)) +
3/2*I*(d*x*log(-I*e^(d*x + c) + 1) + dilog(I*e^(d*x + c)))*e^2*f/(a*d^2) -
 6*I*e*f^2*x/(a*d^2) + (-3*I*f^3*x^2 - 6*I*e*f^2*x - 3*I*e^2*f + (d*f^3*x^
3*e^c + 3*e^2*f*e^c + 3*(d*e*f^2 + f^3)*x^2*e^c + 3*(d*e^2*f + 2*e*f^2)*x*
e^c)*e^(d*x))/(a*d^2*e^(2*d*x + 2*c) - 2*I*a*d^2*e^(d*x + c) - a*d^2) - 3/
2*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*poly
log(3, -I*e^(d*x + c)))*e*f^2/(a*d^3) + 3/2*I*(d^2*x^2*log(-I*e^(d*x + c)
+ 1) + 2*d*x*dilog(I*e^(d*x + c)) - 2*polylog(3, I*e^(d*x + c)))*e*f^2/(a*
d^3) + 6*I*e*f^2*log(I*e^(d*x + c) + 1)/(a*d^3) - 1/2*I*(d^3*x^3*log(I*e^(
d*x + c) + 1) + 3*d^2*x^2*dilog(-I*e^(d*x + c)) - 6*d*x*polylog(3, -I*e^(d
*x + c)) + 6*polylog(4, -I*e^(d*x + c)))*f^3/(a*d^4) + 1/2*I*(d^3*x^3*log(
-I*e^(d*x + c) + 1) + 3*d^2*x^2*dilog(I*e^(d*x + c)) - 6*d*x*polylog(3, I*
e^(d*x + c)) + 6*polylog(4, I*e^(d*x + c)))*f^3/(a*d^4) - 3/2*I*(d^2*e^2*f
 - 4*f^3)*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))/(a*d^4) - 1
/8*(I*d^4*f^3*x^4 + 4*I*d^4*e*f^2*x^3 + 6*I*d^4*e^2*f*x^2)/(a*d^4) + 1/8*(
I*d^4*f^3*x^4 + 4*I*d^4*e*f^2*x^3 - 6*(-I*d^2*e^2*f + 4*I*f^3)*d^2*x^2)/(a
*d^4)
```

3.271.8 Giac [F]

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\,\text{sech}\,(dx+c)}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sech(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(e+f\,x)^3}{\cosh\,(c+d\,x)\,\left(a+a\sinh\,(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.272 $\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.272.1 Optimal result

Integrand size = 29, antiderivative size = 268

$$
\begin{aligned}
\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{(e+fx)^2\arctan\left(e^{c+dx}\right)}{ad}-\frac{f^2\arctan(\sinh(c+dx))}{ad^3}\\
&+\frac{if^2\log(\cosh(c+dx))}{ad^3}-\frac{if(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}\\
&+\frac{if(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{ad^2}+\frac{if^2\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3}\\
&-\frac{if^2\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{ad^3}+\frac{f(e+fx)\text{sech}(c+dx)}{ad^2}\\
&+\frac{i(e+fx)^2\text{sech}^2(c+dx)}{2ad}-\frac{if(e+fx)\tanh(c+dx)}{ad^2}\\
&+\frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{2ad}
\end{aligned}
$$

output
```
(f*x+e)^2*arctan(exp(d*x+c))/a/d-f^2*arctan(sinh(d*x+c))/a/d^3+I*f^2*ln(co
sh(d*x+c))/a/d^3-I*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^2+I*f*(f*x+e)*po
lylog(2,I*exp(d*x+c))/a/d^2+I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3-I*f^2*pol
ylog(3,I*exp(d*x+c))/a/d^3+f*(f*x+e)*sech(d*x+c)/a/d^2+1/2*I*(f*x+e)^2*sec
h(d*x+c)^2/a/d-I*f*(f*x+e)*tanh(d*x+c)/a/d^2+1/2*(f*x+e)^2*sech(d*x+c)*tan
h(d*x+c)/a/d
```

3.272.2 Mathematica [A] (verified)

Time = 5.80 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.98

$$\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\frac{\frac{(e+fx)^3}{f}+\frac{3(1-ie^c)(e+fx)^2\log\left(1+ie^{-c-dx}\right)}{d}+\frac{6i(i+e^c)f\left(d(e+fx)\operatorname{PolyLog}\left(2,-ie^{-c-dx}\right)+f\operatorname{PolyLog}\left(3,-ie^{-c-dx}\right)\right)}{d^3}}{i+e^c}+\frac{3d^2e^2x-12f^2x-3(1+ie^c)(\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-1/6*(((e + f*x)^3/f + (3*(1 - I*E^c)*(e + f*x)^2*Log[1 + I*E^(-c - d*x)])
/d + ((6*I)*(I + E^c)*f*(d*(e + f*x)*PolyLog[2, (-I)*E^(-c - d*x)] + f*Pol
yLog[3, (-I)*E^(-c - d*x)]))/d^3)/(I + E^c) + (3*d^2*e^2*x - 12*f^2*x - 3*
(1 + I*E^c)*(d^2*e^2 - 4*f^2)*x + 3*d^2*e*f*x^2 + d^2*f^2*x^3 + 6*d*e*(1 +
 I*E^c)*f*x*Log[1 - I*E^(-c - d*x)] + 3*d*(1 + I*E^c)*f^2*x^2*Log[1 - I*E^
(-c - d*x)] + (3*(1 + I*E^c)*(d^2*e^2 - 4*f^2)*Log[I - E^(c + d*x)])/d - 6
*e*(1 + I*E^c)*f*PolyLog[2, I*E^(-c - d*x)] - 6*(1 + I*E^c)*f^2*x*PolyLog[
2, I*E^(-c - d*x)] - (6*(1 + I*E^c)*f^2*PolyLog[3, I*E^(-c - d*x)])/d)/(d^
2*(-I + E^c)) - x*(3*e^2 + 3*e*f*x + f^2*x^2)*Sech[c] - ((3*I)*(e + f*x)^2
)/(d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2) + ((12*I)*f*(e + f*x)*Si
nh[(d*x)/2])/(d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c
 + d*x)/2])))/a
```

3.272.3 Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.94, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.552, Rules used = {6105, 3042, 4674, 3042, 4257, 4668, 3011, 2720, 5974, 3042, 4672, 26, 3042, 26, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6105}$$

$$\frac{\int (e+fx)^2\operatorname{sech}^3(c+dx)dx}{a}-\frac{i\int (e+fx)^2\operatorname{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2 \csc\left(ic+idx+\frac{\pi}{2}\right)^3 dx}{a}-\frac{i\int (e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$\frac{-\frac{f^2\int \text{sech}(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\text{sech}(c+dx)dx+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\int (e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f^2\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\int (e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{1}{2}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\int (e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4668}$$

$$\frac{\frac{1}{2}\left(-\frac{2if\int (e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}\right)-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{f(e+fx)\text{se}}{d^2}}{a}-\frac{i\int (e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+\frac{2(e+fx)^2\text{arc}}{d}\right.}{a}-\frac{i\int (e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2720}$$

3.272. $\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\int(e+fx)^2\operatorname{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{5974}$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{f\int(e+fx)\operatorname{sech}^2(c+dx)dx}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}+\frac{f\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{d}\right)}{a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int-i\tanh(c+dx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

3.272. $\int\frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int\tanh(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int-i\tan(ic+idx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow 26$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int\tan(ic+idx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow 3956$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow 7143$$

3.272. $\int\frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{-\frac{f^2 \arctan(\sinh(c+dx))}{d^3} + \frac{1}{2}\left(\frac{2(e+fx)^2 \arctan\left(e^{c+dx}\right)}{d} + \frac{2if\left(\frac{f \operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2} - \frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d} - \frac{2if\left(\frac{f \operatorname{PolyLog}\left(3,ie^{c+\cdots}\right)}{d^2}\cdots\right.}{\cdots}\right.}{a}$$

$$\frac{i\left(\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d} - \frac{f\log(\cosh(c+dx))}{d^2}\right)}{d} - \frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}\right)}{a}$$

input
```
Int[((e + f*x)^2*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-((f^2*ArcTan[Sinh[c + d*x]])/d^3) + ((2*(e + f*x)^2*ArcTan[E^(c + d*x)])
/d + ((2*I)*f*(-(((e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d) + (f*PolyLog[
3, (-I)*E^(c + d*x)])/d^2))/d - ((2*I)*f*(-(((e + f*x)*PolyLog[2, I*E^(c +
 d*x)])/d) + (f*PolyLog[3, I*E^(c + d*x)])/d^2))/d)/2 + (f*(e + f*x)*Sech[
c + d*x])/d^2 + ((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/a - (I*(-
1/2*((e + f*x)^2*Sech[c + d*x]^2)/d + (f*(-((f*Log[Cosh[c + d*x]])/d^2) +
((e + f*x)*Tanh[c + d*x])/d))/d))/a
```

3.272.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

3.272. $\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.272.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(248) = 496.

Time = 9.47 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.05

method	result
risch	$\frac{d x^2 f^2 \mathrm{e}^{dx+c}+2 d e f x \,\mathrm{e}^{dx+c}+d e^2 \mathrm{e}^{dx+c}-2 i f^2 x+2 f^2 x \,\mathrm{e}^{dx+c}-2 i e f+2 e f \,\mathrm{e}^{dx+c}}{\left(\mathrm{e}^{dx+c}-i\right)^2 d^2 a}+\frac{i \ln\left(1-i \mathrm{e}^{dx+c}\right) e f x}{a d}-\frac{i \ln\left(1+i \mathrm{e}^{dx+c}\right) f^2 x^2}{2 a d}+$

input
```
int((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(d*x^2*f^2*exp(d*x+c)+2*d*e*f*x*exp(d*x+c)+d*e^2*exp(d*x+c)-2*I*f^2*x+2*f^
2*x*exp(d*x+c)-2*I*e*f+2*e*f*exp(d*x+c))/(exp(d*x+c)-I)^2/d^2/a+I/a/d*ln(1
-I*exp(d*x+c))*e*f*x-1/2*I/a/d*ln(1+I*exp(d*x+c))*f^2*x^2+1/2*I/a/d^3*ln(1
+I*exp(d*x+c))*c^2*f^2-2/a/d^2*c*e*f*arctan(exp(d*x+c))-2/a/d^3*f^2*arctan
(exp(d*x+c))+1/a/d*e^2*arctan(exp(d*x+c))+1/a/d^3*c^2*f^2*arctan(exp(d*x+c
))-I/a/d*ln(1+I*exp(d*x+c))*e*f*x-2*I/a/d^3*f^2*ln(exp(d*x+c))+1/2*I/a/d*l
n(1-I*exp(d*x+c))*f^2*x^2-I/a/d^2*e*f*polylog(2,-I*exp(d*x+c))+I/a/d^2*ln(
1-I*exp(d*x+c))*c*e*f+I/a/d^2*e*f*polylog(2,I*exp(d*x+c))-I/a/d^2*polylog(
2,-I*exp(d*x+c))*f^2*x-I/a/d^2*ln(1+I*exp(d*x+c))*c*e*f+I/a/d^2*polylog(2,
I*exp(d*x+c))*f^2*x+I/a/d^3*f^2*ln(1+exp(2*d*x+2*c))-1/2*I/a/d^3*ln(1-I*ex
p(d*x+c))*c^2*f^2+I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3-I*f^2*polylog(3,I*e
xp(d*x+c))/a/d^3
```

3.272.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 805 vs. $2(235) = 470$.

Time = 0.25 (sec) , antiderivative size = 805, normalized size of antiderivative = 3.00

$$\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{-4i\,def+4i\,cf^2-2\left(i\,df^2x+i\,def+(-i\,df^2x-i\,def)e^{(2\,dx+2\,c)}-2\,(df^2x+def)e^{(dx+c)}\right)\mathrm{Li}_2\left(i\,e^{(dx+c)}\right)-}{}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(-4*I*d*e*f + 4*I*c*f^2 - 2*(I*d*f^2*x + I*d*e*f + (-I*d*f^2*x - I*d*e
*f)*e^(2*d*x + 2*c) - 2*(d*f^2*x + d*e*f)*e^(d*x + c))*dilog(I*e^(d*x + c)
) - 2*(-I*d*f^2*x - I*d*e*f + (I*d*f^2*x + I*d*e*f)*e^(2*d*x + 2*c) + 2*(d
*f^2*x + d*e*f)*e^(d*x + c))*dilog(-I*e^(d*x + c)) - 4*(I*d*f^2*x + I*c*f^
2)*e^(2*d*x + 2*c) + 2*(d^2*f^2*x^2 + d^2*e^2 + 2*d*e*f - 4*c*f^2 + 2*(d^2
*e*f - d*f^2)*x)*e^(d*x + c) + (-I*d^2*e^2 + 2*I*c*d*e*f - I*c^2*f^2 + (I*
d^2*e^2 - 2*I*c*d*e*f + I*c^2*f^2)*e^(2*d*x + 2*c) + 2*(d^2*e^2 - 2*c*d*e*
f + c^2*f^2)*e^(d*x + c))*log(e^(d*x + c) + I) + (I*d^2*e^2 - 2*I*c*d*e*f
+ (I*c^2 - 4*I)*f^2 + (-I*d^2*e^2 + 2*I*c*d*e*f + (-I*c^2 + 4*I)*f^2)*e^(2
*d*x + 2*c) - 2*(d^2*e^2 - 2*c*d*e*f + (c^2 - 4)*f^2)*e^(d*x + c))*log(e^(
d*x + c) - I) + (I*d^2*f^2*x^2 + 2*I*d^2*e*f*x + 2*I*c*d*e*f - I*c^2*f^2 +
 (-I*d^2*f^2*x^2 - 2*I*d^2*e*f*x - 2*I*c*d*e*f + I*c^2*f^2)*e^(2*d*x + 2*c
) - 2*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(d*x + c))*log(I
*e^(d*x + c) + 1) + (-I*d^2*f^2*x^2 - 2*I*d^2*e*f*x - 2*I*c*d*e*f + I*c^2*
f^2 + (I*d^2*f^2*x^2 + 2*I*d^2*e*f*x + 2*I*c*d*e*f - I*c^2*f^2)*e^(2*d*x +
 2*c) + 2*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(d*x + c))*l
og(-I*e^(d*x + c) + 1) - 2*(I*f^2*e^(2*d*x + 2*c) + 2*f^2*e^(d*x + c) - I*
f^2)*polylog(3, I*e^(d*x + c)) - 2*(-I*f^2*e^(2*d*x + 2*c) - 2*f^2*e^(d*x
+ c) + I*f^2)*polylog(3, -I*e^(d*x + c)))/(a*d^3*e^(2*d*x + 2*c) - 2*I*a*d
^3*e^(d*x + c) - a*d^3)
```

3.272.6 Sympy [F]

$$\int \frac{(e+fx)^2\mathrm{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= -\frac{i\left(\int \frac{e^2\,\mathrm{sech}\,(c+dx)}{\sinh\,(c+dx)-i}\,dx + \int \frac{f^2x^2\,\mathrm{sech}\,(c+dx)}{\sinh\,(c+dx)-i}\,dx + \int \frac{2efx\,\mathrm{sech}\,(c+dx)}{\sinh\,(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**2*sech(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**2*x*
*2*sech(c + d*x)/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*sech(c + d*x)/
(sinh(c + d*x) - I), x))/a
```

3.272.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.44

$$\int \frac{(e+fx)^2\mathrm{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx =$$
$$-\frac{1}{2}\,e^2\left(\frac{4\,e^{(-dx-c)}}{-2\,(-2i\,ae^{(-dx-c)}-ae^{(-2\,dx-2\,c)}+a)d}+\frac{i\,\log\left(e^{(-dx-c)}+i\right)}{ad}-\frac{i\,\log\left(i\,e^{(-dx-c)}+1\right)}{ad}\right)$$
$$+\frac{-2i\,f^2x-2i\,ef+(df^2x^2e^c+2\,efe^c+2\,(def+f^2)xe^c)e^{(dx)}}{ad^2e^{(2\,dx+2\,c)}-2i\,ad^2e^{(dx+c)}-ad^2}$$
$$-\frac{i\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)\right)ef}{ad^2}$$
$$+\frac{i\left(dx\log\left(-i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(i\,e^{(dx+c)}\right)\right)ef}{ad^2}-\frac{2i\,f^2x}{ad^2}$$
$$-\frac{i\left(d^2x^2\log\left(i\,e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-2\,\mathrm{Li}_3(-i\,e^{(dx+c)})\right)f^2}{2\,ad^3}$$
$$+\frac{i\left(d^2x^2\log\left(-i\,e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(i\,e^{(dx+c)}\right)-2\,\mathrm{Li}_3(i\,e^{(dx+c)})\right)f^2}{2\,ad^3}$$
$$+\frac{2i\,f^2\log\left(i\,e^{(dx+c)}+1\right)}{ad^3}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*e^2*(4*e^(-d*x - c)/((4*I*a*e^(-d*x - c) + 2*a*e^(-2*d*x - 2*c) - 2*a
)*d) + I*log(e^(-d*x - c) + I)/(a*d) - I*log(I*e^(-d*x - c) + 1)/(a*d)) +
(-2*I*f^2*x - 2*I*e*f + (d*f^2*x^2*e^c + 2*e*f*e^c + 2*(d*e*f + f^2)*x*e^c
)*e^(d*x))/(a*d^2*e^(2*d*x + 2*c) - 2*I*a*d^2*e^(d*x + c) - a*d^2) - I*(d*
x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*e*f/(a*d^2) + I*(d*x*log
(-I*e^(d*x + c) + 1) + dilog(I*e^(d*x + c)))*e*f/(a*d^2) - 2*I*f^2*x/(a*d^
2) - 1/2*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) -
 2*polylog(3, -I*e^(d*x + c)))*f^2/(a*d^3) + 1/2*I*(d^2*x^2*log(-I*e^(d*x
+ c) + 1) + 2*d*x*dilog(I*e^(d*x + c)) - 2*polylog(3, I*e^(d*x + c)))*f^2/
(a*d^3) + 2*I*f^2*log(I*e^(d*x + c) + 1)/(a*d^3)
```

3.272.8 Giac [F]

$$\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}\,(dx+c)}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sech(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\cosh\,(c+d\,x)\,\left(a+a\sinh\,(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.273 $\int \frac{(e+fx)\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.273.1 Optimal result

Integrand size = 27, antiderivative size = 161

$$\begin{aligned}\int \frac{(e+fx)\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{(e+fx)\arctan\left(e^{c+dx}\right)}{ad} - \frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2ad^2} \\ &\quad + \frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2ad^2} + \frac{f\text{sech}(c+dx)}{2ad^2} \\ &\quad + \frac{i(e+fx)\text{sech}^2(c+dx)}{2ad} - \frac{if\tanh(c+dx)}{2ad^2} \\ &\quad + \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{2ad}\end{aligned}$$

output
```
(f*x+e)*arctan(exp(d*x+c))/a/d-1/2*I*f*polylog(2,-I*exp(d*x+c))/a/d^2+1/2*
I*f*polylog(2,I*exp(d*x+c))/a/d^2+1/2*f*sech(d*x+c)/a/d^2+1/2*I*(f*x+e)*se
ch(d*x+c)^2/a/d-1/2*I*f*tanh(d*x+c)/a/d^2+1/2*(f*x+e)*sech(d*x+c)*tanh(d*x
+c)/a/d
```

3.273.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 400 vs. 2(161) = 322.

Time = 2.24 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.48

$$\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx =$$
$$-\frac{-2id(e+fx)+(c+dx)(cf-d(2e+fx))\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2+(1-i)\left(\frac{1}{2}d^2fx^2\right.}{}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-1/4*((-2*I)*d*(e + f*x) + (c + d*x)*(c*f - d*(2*e + f*x))*(Cosh[(c + d*x)
/2] + I*Sinh[(c + d*x)/2])^2 + (1 - I)*((d^2*f*x^2)/2 + d*e*(c + d*x) - (1
 - I)*(d*e - c*f)*(c + d*x) + (1 - I)*f*(c + d*x)*Log[1 + I*E^(-c - d*x)]
+ (1 - I)*(d*e - c*f)*Log[I + E^(c + d*x)] - (1 - I)*f*PolyLog[2, (-I)*E^(
-c - d*x)])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 + (1 + I)*((d^2*f*
x^2)/2 + d*e*(c + d*x) - (1 + I)*(d*e - c*f)*(c + d*x) + (1 + I)*f*(c + d*
x)*Log[1 - I*E^(-c - d*x)] + (1 + I)*(d*e - c*f)*Log[I - E^(c + d*x)] - (1
 + I)*f*PolyLog[2, I*E^(-c - d*x)])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/
2])^2 - 4*f*Sinh[(c + d*x)/2]*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])
)/(d^2*(a + I*a*Sinh[c + d*x]))
```

3.273.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6105, 3042, 4673, 3042, 4668, 2715, 2838, 5974, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}dx$$
$$\Big\downarrow \text{6105}$$
$$\frac{\int (e+fx)\operatorname{sech}^3(c+dx)dx}{a}-\frac{i\int (e+fx)\operatorname{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$
$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{a}-\frac{i\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 4673}$$

$$\frac{\frac{1}{2}\int (e+fx)\text{sech}(c+dx)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{2}\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 4668}$$

$$\frac{\frac{1}{2}\left(-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{\frac{1}{2}\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+(e+fx)\tanh(}{a}$$
$$\frac{i\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 5974}$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{a}-\frac{i\left(\frac{f\int \text{sech}^2(c+dx)dx}{2d}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{ 3042}$$

3.273. $\int \frac{(e+fx)\text{sech}(c+dx)}{a+ia\sinh(c+dx)}dx$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\operatorname{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}}{a}-\frac{i\left(-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}+\frac{f\int\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d}\right)}{a}$$

$$\Big\downarrow \; 4254$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\operatorname{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}}{a}-\frac{i\left(-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}+\frac{if\int 1d(-i\tanh(c+dx))}{2d^2}\right)}{a}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\operatorname{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}}{a}-\frac{i\left(\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}\right)}{a}$$

input
```
Int[((e + f*x)*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-I)*(-1/2*((e + f*x)*Sech[c + d*x]^2)/d + (f*Tanh[c + d*x])/(2*d^2)))/a
+ (((2*(e + f*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)
])/d^2 + (I*f*PolyLog[2, I*E^(c + d*x)])/d^2)/2 + (f*Sech[c + d*x])/(2*d^2
) + ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/a
```

3.273.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.273. $\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

3.273. $\int \frac{(e+fx)\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.273.4 Maple [A] (verified)

Time = 5.38 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.35

method	result
risch	$\frac{d f x\,\mathrm{e}^{d x+c}+d e\,\mathrm{e}^{d x+c}+f\,\mathrm{e}^{d x+c}-i f}{\left(\mathrm{e}^{d x+c}-i\right)^{2} d^{2} a}+\frac{e \arctan\left(\mathrm{e}^{d x+c}\right)}{d a}+\frac{i f \ln\left(1-i \mathrm{e}^{d x+c}\right) x}{2 d a}+\frac{i f \ln\left(1-i \mathrm{e}^{d x+c}\right) c}{2 d^{2} a}+\frac{i f \,\mathrm{polylog}\left(2, i \mathrm{e}^{d x+c}\right)}{2 a\,d^{2}}-\frac{i f}{}$

input
```
int((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(d*f*x*exp(d*x+c)+d*e*exp(d*x+c)+f*exp(d*x+c)-I*f)/(exp(d*x+c)-I)^2/d^2/a+
1/d/a*e*arctan(exp(d*x+c))+1/2*I/d/a*f*ln(1-I*exp(d*x+c))*x+1/2*I/d^2/a*f*
ln(1-I*exp(d*x+c))*c+1/2*I*f*polylog(2,I*exp(d*x+c))/a/d^2-1/2*I/d/a*f*ln(
1+I*exp(d*x+c))*x-1/2*I/d^2/a*f*ln(1+I*exp(d*x+c))*c-1/2*I*f*polylog(2,-I*
exp(d*x+c))/a/d^2-1/d^2/a*f*c*arctan(exp(d*x+c))
```

3.273.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(135) = 270.

Time = 0.25 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.19

$$\int \frac{(e+fx)\mathrm{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(i\,fe^{(2\,dx+2\,c)}+2\,fe^{(dx+c)}-i\,f\right)\mathrm{Li}_2\left(i\,e^{(dx+c)}\right)+\left(-i\,fe^{(2\,dx+2\,c)}-2\,fe^{(dx+c)}+i\,f\right)\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)+2\,(dfx}{}$$

input
```
integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((I*f*e^(2*d*x + 2*c) + 2*f*e^(d*x + c) - I*f)*dilog(I*e^(d*x + c)) +
(-I*f*e^(2*d*x + 2*c) - 2*f*e^(d*x + c) + I*f)*dilog(-I*e^(d*x + c)) + 2*(
d*f*x + d*e + f)*e^(d*x + c) + (-I*d*e + I*c*f + (I*d*e - I*c*f)*e^(2*d*x
+ 2*c) + 2*(d*e - c*f)*e^(d*x + c))*log(e^(d*x + c) + I) + (I*d*e - I*c*f
+ (-I*d*e + I*c*f)*e^(2*d*x + 2*c) - 2*(d*e - c*f)*e^(d*x + c))*log(e^(d*x
 + c) - I) + (I*d*f*x + I*c*f + (-I*d*f*x - I*c*f)*e^(2*d*x + 2*c) - 2*(d*
f*x + c*f)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (-I*d*f*x - I*c*f + (I*d*
f*x + I*c*f)*e^(2*d*x + 2*c) + 2*(d*f*x + c*f)*e^(d*x + c))*log(-I*e^(d*x
+ c) + 1) - 2*I*f)/(a*d^2*e^(2*d*x + 2*c) - 2*I*a*d^2*e^(d*x + c) - a*d^2)
```

3.273.6 Sympy [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e\,\text{sech}\,(c+dx)}{\sinh\,(c+dx)-i}\,dx + \int \frac{fx\,\text{sech}\,(c+dx)}{\sinh\,(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e*sech(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f*x*sech(c
 + d*x)/(sinh(c + d*x) - I), x))/a
```

3.273.7 Maxima [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{sech}\,(dx+c)}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
f*(((d*x*e^c + e^c)*e^(d*x) - I)/(a*d^2*e^(2*d*x + 2*c) - 2*I*a*d^2*e^(d*x
 + c) - a*d^2) + 2*integrate(1/4*x/(a*e^(d*x + c) + I*a), x) + 2*integrate
(1/4*x/(a*e^(d*x + c) - I*a), x)) - 1/2*e*(4*e^(-d*x - c)/((4*I*a*e^(-d*x
- c) + 2*a*e^(-2*d*x - 2*c) - 2*a)*d) + I*log(e^(-d*x - c) + I)/(a*d) - I*
log(I*e^(-d*x - c) + 1)/(a*d))
```

3.273.8 Giac [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{sech}\,(dx+c)}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sech(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\left(c+d\,x\right)\,\left(a+a\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.274 $\int \frac{\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.274.1 Optimal result

Integrand size = 22, antiderivative size = 42

$$\int \frac{\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\arctan(\sinh(c+dx))}{2ad} + \frac{i}{2d(a+ia\sinh(c+dx))}$$

output
```
1/2*arctan(sinh(d*x+c))/a/d+1/2*I/d/(a+I*a*sinh(d*x+c))
```

3.274.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\arctan(\sinh(c+dx)) + \frac{1}{-i+\sinh(c+dx)}}{2ad}$$

input
```
Integrate[Sech[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(ArcTan[Sinh[c + d*x]] + (-I + Sinh[c + d*x])^(-1))/(2*a*d)
```

3.274.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)(a+a\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3146}$$

$$-\frac{ia\int \frac{1}{(a-ia\sinh(c+dx))(i\sinh(c+dx)a+a)^2}d(ia\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia\int \left(\frac{1}{2(\sinh^2(c+dx)a^2+a^2)a}+\frac{1}{2(i\sinh(c+dx)a+a)^2a}\right)d(ia\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia\left(\frac{i\arctan(\sinh(c+dx))}{2a^2}-\frac{1}{2a(a+ia\sinh(c+dx))}\right)}{d}$$

input
```
Int[Sech[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-I)*a*(((I/2)*ArcTan[Sinh[c + d*x]])/a^2 - 1/(2*a*(a + I*a*Sinh[c + d*x]
))))/d
```

3.274.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.274.4 Maple [A] (verified)

Time = 3.74 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.52

method	result	size
risch	$\frac{e^{dx+c}}{\left(e^{dx+c}-i\right)^{2} d a}-\frac{i \ln \left(e^{dx+c}-i\right)}{2 a d}+\frac{i \ln \left(e^{dx+c}+i\right)}{2 a d}$	64
derivativedivides	$\frac{\frac{i \ln \left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)+i\right)}{2}-\frac{i}{\left(-i+\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{i \ln \left(-i+\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{2}-\frac{1}{-i+\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)}}{a d}$	75
default	$\frac{\frac{i \ln \left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)+i\right)}{2}-\frac{i}{\left(-i+\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{i \ln \left(-i+\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{2}-\frac{1}{-i+\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)}}{a d}$	75

input
```
int(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
exp(d*x+c)/(exp(d*x+c)-I)^2/d/a-1/2*I/a/d*ln(exp(d*x+c)-I)+1/2*I/a/d*ln(ex
p(d*x+c)+I)
```

3.274.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(34) = 68.

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.43

$$\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\left(i\,e^{(2\,dx+2\,c)}+2\,e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}+i\right)+\left(-i\,e^{(2\,dx+2\,c)}-2\,e^{(dx+c)}+i\right)\log\left(e^{(dx+c)}-i\right)+2\,e^{(dx+c)}}{2\left(ade^{(2\,dx+2\,c)}-2i\,ade^{(dx+c)}-ad\right)}$$

input
```
integrate(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((I*e^(2*d*x + 2*c) + 2*e^(d*x + c) - I)*log(e^(d*x + c) + I) + (-I*e^
(2*d*x + 2*c) - 2*e^(d*x + c) + I)*log(e^(d*x + c) - I) + 2*e^(d*x + c))/(
a*d*e^(2*d*x + 2*c) - 2*I*a*d*e^(d*x + c) - a*d)
```

3.274.6 Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int\frac{\operatorname{sech}(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

input
```
integrate(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)/(sinh(c + d*x) - I), x)/a
```

3.274.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(34) = 68.

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.07

$$\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{2\,e^{(-dx-c)}}{-2\left(-2i\,ae^{(-dx-c)}-ae^{(-2\,dx-2\,c)}+a\right)d} - \frac{i\,\log\left(e^{(-dx-c)}+i\right)}{2\,ad} + \frac{i\,\log\left(i\,e^{(-dx-c)}+1\right)}{2\,ad}$$

input
```
integrate(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*e^(-d*x - c)/((4*I*a*e^(-d*x - c) + 2*a*e^(-2*d*x - 2*c) - 2*a)*d) - 1/
2*I*log(e^(-d*x - c) + I)/(a*d) + 1/2*I*log(I*e^(-d*x - c) + 1)/(a*d)
```

3.274.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.43

$$\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{-\frac{i\log\left(e^{(dx+c)}-e^{(-dx-c)}+2i\right)}{a}+\frac{i\log\left(e^{(dx+c)}-e^{(-dx-c)}-2i\right)}{a}+\frac{-i\,e^{(dx+c)}+i\,e^{(-dx-c)}-6}{a\left(e^{(dx+c)}-e^{(-dx-c)}-2i\right)}}{4\,d}$$

input
```
integrate(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(-I*log(e^(d*x + c) - e^(-d*x - c) + 2*I)/a + I*log(e^(d*x + c) - e^(
-d*x - c) - 2*I)/a + (-I*e^(d*x + c) + I*e^(-d*x - c) - 6)/(a*(e^(d*x + c)
 - e^(-d*x - c) - 2*I)))/d
```

3.274.9 Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.76

$$\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\sqrt{a^2\,d^2}}{a\,d}\right)}{\sqrt{a^2\,d^2}}+\frac{1}{a\,d\,\left(e^{c+d\,x}-\mathrm{i}\right)}-\frac{1\mathrm{i}}{a\,d\,{\left(1+e^{c+d\,x}\,1\mathrm{i}\right)}^2}$$

input
```
int(1/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
atan((exp(d*x)*exp(c)*(a^2*d^2)^(1/2))/(a*d))/(a^2*d^2)^(1/2) + 1/(a*d*(ex
p(c + d*x) - 1i)) - 1i/(a*d*(exp(c + d*x)*1i + 1)^2)
```

3.275 $\int \frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

3.275.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.275.2 Mathematica [N/A]

Not integrable

Time = 84.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Sech[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Sech[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

3.275.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))} \, dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.275.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.275.4 Maple [N/A] (verified)

Not integrable

Time = 0.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{sech}\left(d x +c \right)}{\left(f x +e \right) \left(a +i a \sinh \left(d x +c \right)\right)}d x$$

input
```
int(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.275.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 391, normalized size of antiderivative = 13.48

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-((d*f*x + d*e - f)*e^(d*x + c) - (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e
^2 - (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^(2*d*x + 2*c) + 2*(I*a*
d^2*f^2*x^2 + 2*I*a*d^2*e*f*x + I*a*d^2*e^2)*e^(d*x + c))*integral((-2*I*f
^2 + (d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 - 2*f^2)*e^(d*x + c))/(a*d^2*f^3
*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3 +
3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(2*d*x + 2*c)), x) + I*
f)/(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2 - (a*d^2*f^2*x^2 + 2*a*d^2*e
*f*x + a*d^2*e^2)*e^(2*d*x + 2*c) + 2*(I*a*d^2*f^2*x^2 + 2*I*a*d^2*e*f*x +
 I*a*d^2*e^2)*e^(d*x + c))
```

3.275.6 Sympy [N/A]

Not integrable

Time = 4.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\operatorname{sech}(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

input
```
integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) - I*f
*x), x)/a
```

3.275.7 Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 327, normalized size of antiderivative = 11.28

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-((d*f*x*e^c + (d*e - f)*e^c)*e^(d*x) + I*f)/(a*d^2*f^2*x^2 + 2*a*d^2*e*f*
x + a*d^2*e^2 - (a*d^2*f^2*x^2*e^(2*c) + 2*a*d^2*e*f*x*e^(2*c) + a*d^2*e^2
*e^(2*c))*e^(2*d*x) + 2*(I*a*d^2*f^2*x^2*e^c + 2*I*a*d^2*e*f*x*e^c + I*a*d
^2*e^2*e^c)*e^(d*x)) + 2*integrate((d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 -
4*f^2)/(-4*I*a*d^2*f^3*x^3 - 12*I*a*d^2*e*f^2*x^2 - 12*I*a*d^2*e^2*f*x - 4
*I*a*d^2*e^3 + 4*(a*d^2*f^3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c + 3*a*d^2*e^2*
f*x*e^c + a*d^2*e^3*e^c)*e^(d*x)), x) + 2*integrate(1/(4*I*a*f*x + 4*I*a*e
 + 4*(a*f*x*e^c + a*e*e^c)*e^(d*x)), x)
```

3.275.8 Giac [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sech(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

3.275.9 Mupad [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(cosh(c + d*x)*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.276 $\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx$

3.276.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx = \operatorname{Int}\left(\frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}, x\right)$$

output
```
Unintegrable(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.276.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} \, dx = \$\text{Aborted}$$

input
```
Integrate[Sech[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.276.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \; 6111$$
$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.276.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.276.4 Maple [N/A] (verified)

Not integrable

Time = 1.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{sech}\left(dx+c\right)}{\left(fx+e\right)^{2}\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.276.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 511, normalized size of antiderivative = 17.62

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-((d*f*x + d*e - 2*f)*e^(d*x + c) - (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3
*a*d^2*e^2*f*x + a*d^2*e^3 - (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*
e^2*f*x + a*d^2*e^3)*e^(2*d*x + 2*c) + 2*(I*a*d^2*f^3*x^3 + 3*I*a*d^2*e*f^
2*x^2 + 3*I*a*d^2*e^2*f*x + I*a*d^2*e^3)*e^(d*x + c))*integral((-6*I*f^2 +
 (d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 - 6*f^2)*e^(d*x + c))/(a*d^2*f^4*x^4
 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4 +
 (a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*
x + a*d^2*e^4)*e^(2*d*x + 2*c)), x) + 2*I*f)/(a*d^2*f^3*x^3 + 3*a*d^2*e*f^
2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 - (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 +
 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(2*d*x + 2*c) + 2*(I*a*d^2*f^3*x^3 + 3*I*a
*d^2*e*f^2*x^2 + 3*I*a*d^2*e^2*f*x + I*a*d^2*e^3)*e^(d*x + c))
```

3.276.6 Sympy [N/A]

Not integrable

Time = 30.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.45

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\operatorname{sech}(c+dx)}{e^2\sinh(c+dx)-ie^2+2efx\sinh(c+dx)-2iefx+f^2x^2\sinh(c+dx)-if^2x^2}\,dx}{a}$$

input
```
integrate(sech(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)/(e**2*sinh(c + d*x) - I*e**2 + 2*e*f*x*sinh(c +
d*x) - 2*I*e*f*x + f**2*x**2*sinh(c + d*x) - I*f**2*x**2), x)/a
```

3.276.7 Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 436, normalized size of antiderivative = 15.03

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)}{\left(fx+e\right)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-((d*f*x*e^c + (d*e - 2*f)*e^c)*e^(d*x) + 2*I*f)/(a*d^2*f^3*x^3 + 3*a*d^2*
e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 - (a*d^2*f^3*x^3*e^(2*c) + 3*a*d^2
*e*f^2*x^2*e^(2*c) + 3*a*d^2*e^2*f*x*e^(2*c) + a*d^2*e^3*e^(2*c))*e^(2*d*x
) + 2*(I*a*d^2*f^3*x^3*e^c + 3*I*a*d^2*e*f^2*x^2*e^c + 3*I*a*d^2*e^2*f*x*e
^c + I*a*d^2*e^3*e^c)*e^(d*x)) + 2*integrate((d^2*f^2*x^2 + 2*d^2*e*f*x +
d^2*e^2 - 12*f^2)/(-4*I*a*d^2*f^4*x^4 - 16*I*a*d^2*e*f^3*x^3 - 24*I*a*d^2*
e^2*f^2*x^2 - 16*I*a*d^2*e^3*f*x - 4*I*a*d^2*e^4 + 4*(a*d^2*f^4*x^4*e^c +
4*a*d^2*e*f^3*x^3*e^c + 6*a*d^2*e^2*f^2*x^2*e^c + 4*a*d^2*e^3*f*x*e^c + a*
d^2*e^4*e^c)*e^(d*x)), x) + 2*integrate(1/(4*I*a*f^2*x^2 + 8*I*a*e*f*x + 4
*I*a*e^2 + 4*(a*f^2*x^2*e^c + 2*a*e*f*x*e^c + a*e^2*e^c)*e^(d*x)), x)
```

3.276.8 Giac [N/A]

Not integrable

Time = 32.95 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)}{\left(fx+e\right)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sech(d*x + c)/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

3.276.9 Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\cosh(c+d\,x)\,\left(e+f\,x\right)^2\,\left(a+a\sinh(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(cosh(c + d*x)*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.277 $\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.277.1 Optimal result

Integrand size = 31, antiderivative size = 450

$$
\begin{aligned}
\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{2(e+fx)^3}{3ad}-\frac{if(e+fx)^2\arctan\left(e^{c+dx}\right)}{ad^2} \\
&\quad+\frac{if^3\arctan(\sinh(c+dx))}{ad^4}-\frac{2f(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{ad^2} \\
&\quad+\frac{f^3\log(\cosh(c+dx))}{ad^4}-\frac{f^2(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3} \\
&\quad+\frac{f^2(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{ad^3} \\
&\quad-\frac{2f^2(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{ad^3} \\
&\quad+\frac{f^3\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4}-\frac{f^3\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{ad^4} \\
&\quad+\frac{f^3\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{ad^4}-\frac{if^2(e+fx)\operatorname{sech}(c+dx)}{ad^3} \\
&\quad+\frac{f(e+fx)^2\operatorname{sech}^2(c+dx)}{2ad^2}+\frac{i(e+fx)^3\operatorname{sech}^3(c+dx)}{3ad} \\
&\quad-\frac{f^2(e+fx)\tanh(c+dx)}{ad^3}+\frac{2(e+fx)^3\tanh(c+dx)}{3ad} \\
&\quad-\frac{if(e+fx)^2\operatorname{sech}(c+dx)\tanh(c+dx)}{2ad^2} \\
&\quad+\frac{(e+fx)^3\operatorname{sech}^2(c+dx)\tanh(c+dx)}{3ad}
\end{aligned}
$$

output
```
2/3*(f*x+e)^3/a/d-1/2*I*f*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/a/d^2-I*f*(f*x
+e)^2*arctan(exp(d*x+c))/a/d^2-2*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/a/d^2+f^
3*ln(cosh(d*x+c))/a/d^4-f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^3+f^2*(f*
x+e)*polylog(2,I*exp(d*x+c))/a/d^3-2*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c)
)/a/d^3+f^3*polylog(3,-I*exp(d*x+c))/a/d^4-f^3*polylog(3,I*exp(d*x+c))/a/d
^4+f^3*polylog(3,-exp(2*d*x+2*c))/a/d^4+1/3*I*(f*x+e)^3*sech(d*x+c)^3/a/d+
1/2*f*(f*x+e)^2*sech(d*x+c)^2/a/d^2-I*f^2*(f*x+e)*sech(d*x+c)/a/d^3-f^2*(f
*x+e)*tanh(d*x+c)/a/d^3+2/3*(f*x+e)^3*tanh(d*x+c)/a/d+I*f^3*arctan(sinh(d*
x+c))/a/d^4+1/3*(f*x+e)^3*sech(d*x+c)^2*tanh(d*x+c)/a/d
```

3.277.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1078 vs. $2(450) = 900$.

Time = 8.48 (sec) , antiderivative size = 1078, normalized size of antiderivative = 2.40

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx =$$

$$-\frac{if\left(\frac{(e+fx)^3}{f}+\frac{3(1-ie^c)(e+fx)^2\log(1+ie^{-c-dx})}{d}+\frac{6i(i+e^c)f\left(d(e+fx)\,\text{PolyLog}(2,-ie^{-c-dx})+f\,\text{PolyLog}(3,-ie^{-c-dx})\right)}{d^3}\right)}{2ad\,(i+e^c)}$$

$$+\frac{if\Big(15d^2e^2x-12f^2x-3(1+ie^c)\left(5d^2e^2-4f^2\right)x+15d^2efx^2+5d^2f^2x^3+30de(1+ie^c)\,fx\log\left(1-ie\right.}{}$$

$$+\frac{e^3\sinh\left(\frac{dx}{2}\right)+3e^2fx\sinh\left(\frac{dx}{2}\right)+3ef^2x^2\sinh\left(\frac{dx}{2}\right)+f^3x^3\sinh\left(\frac{dx}{2}\right)}{2ad\left(\cosh\left(\frac{c}{2}\right)-i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)-i\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

$$+\frac{e^3\sinh\left(\frac{dx}{2}\right)+3e^2fx\sinh\left(\frac{dx}{2}\right)+3ef^2x^2\sinh\left(\frac{dx}{2}\right)+f^3x^3\sinh\left(\frac{dx}{2}\right)}{3ad\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)+i\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3}$$

$$+\frac{ide^3\cosh\left(\frac{c}{2}\right)+3e^2f\cosh\left(\frac{c}{2}\right)+3ide^2fx\cosh\left(\frac{c}{2}\right)+6ef^2x\cosh\left(\frac{c}{2}\right)+3idef^2x^2\cosh\left(\frac{c}{2}\right)+3f^3x^2\cosh\left(\right.}{6ad^2\left(\cosh\right.}$$

$$+\frac{5d^2e^3\sinh\left(\frac{dx}{2}\right)-12ef^2\sinh\left(\frac{dx}{2}\right)+15d^2e^2fx\sinh\left(\frac{dx}{2}\right)-12f^3x\sinh\left(\frac{dx}{2}\right)+15d^2ef^2x^2\sinh\left(\frac{dx}{2}\right)+5d^2}{6ad^3\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)+i\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

input
```
Integrate[((e + f*x)^3*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-1/2*I)*f*((e + f*x)^3/f + (3*(1 - I*E^c)*(e + f*x)^2*Log[1 + I*E^(-c -
d*x)])/d + ((6*I)*(I + E^c)*f*(d*(e + f*x)*PolyLog[2, (-I)*E^(-c - d*x)] +
 f*PolyLog[3, (-I)*E^(-c - d*x)]))/d^3))/(a*d*(I + E^c)) + ((I/6)*f*(15*d^
2*e^2*x - 12*f^2*x - 3*(1 + I*E^c)*(5*d^2*e^2 - 4*f^2)*x + 15*d^2*e*f*x^2
+ 5*d^2*f^2*x^3 + 30*d*e*(1 + I*E^c)*f*x*Log[1 - I*E^(-c - d*x)] + 15*d*(1
 + I*E^c)*f^2*x^2*Log[1 - I*E^(-c - d*x)] + (3*(1 + I*E^c)*(5*d^2*e^2 - 4*
f^2)*Log[I - E^(c + d*x)])/d - 30*e*(1 + I*E^c)*f*PolyLog[2, I*E^(-c - d*x
)] - 30*(1 + I*E^c)*f^2*x*PolyLog[2, I*E^(-c - d*x)] - (30*(1 + I*E^c)*f^2
*PolyLog[3, I*E^(-c - d*x)])/d))/(a*d^3*(-I + E^c)) + (e^3*Sinh[(d*x)/2] +
 3*e^2*f*x*Sinh[(d*x)/2] + 3*e*f^2*x^2*Sinh[(d*x)/2] + f^3*x^3*Sinh[(d*x)/
2])/(2*a*d*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] - I*Sinh[c/2 + (
d*x)/2])) + (e^3*Sinh[(d*x)/2] + 3*e^2*f*x*Sinh[(d*x)/2] + 3*e*f^2*x^2*Sin
h[(d*x)/2] + f^3*x^3*Sinh[(d*x)/2])/(3*a*d*(Cosh[c/2] + I*Sinh[c/2])*(Cosh
[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2])^3) + (I*d*e^3*Cosh[c/2] + 3*e^2*f
*Cosh[c/2] + (3*I)*d*e^2*f*x*Cosh[c/2] + 6*e*f^2*x*Cosh[c/2] + (3*I)*d*e*f
^2*x^2*Cosh[c/2] + 3*f^3*x^2*Cosh[c/2] + I*d*f^3*x^3*Cosh[c/2] + d*e^3*Sin
h[c/2] + (3*I)*e^2*f*Sinh[c/2] + 3*d*e^2*f*x*Sinh[c/2] + (6*I)*e*f^2*x*Sin
h[c/2] + 3*d*e*f^2*x^2*Sinh[c/2] + (3*I)*f^3*x^2*Sinh[c/2] + d*f^3*x^3*Sin
h[c/2])/(6*a*d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c
/2 + (d*x)/2])^2) + (5*d^2*e^3*Sinh[(d*x)/2] - 12*e*f^2*Sinh[(d*x)/2] +...
```

3.277.3 Rubi [A] (verified)

Time = 2.82 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.98, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.710, Rules used = {6105, 3042, 4674, 3042, 4672, 26, 3042, 26, 3956, 4201, 2620, 3011, 2720, 5974, 3042, 4674, 3042, 4257, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6105}$$

$$\frac{\int (e+fx)^3\text{sech}^4(c+dx)dx}{a}-\frac{i\int (e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^4dx}{a}-\frac{i\int (e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 4674$$

$$\frac{-\frac{f^2\int (e+fx)\text{sech}^2(c+dx)dx}{d^2}+\frac{2}{3}\int (e+fx)^3\text{sech}^2(c+dx)dx+\frac{f(e+fx)^2\text{sech}^2(c+dx)}{2d^2}+\frac{(e+fx)^3\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f^2\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{d^2}+\frac{2}{3}\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx+\frac{f(e+fx)^2\text{sech}^2(c+dx)}{2d^2}+\frac{(e+fx)^3\tanh(c+dx)\text{sech}^2(c}{3d}}{a}\ \frac{i\int (e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 4672$$

$$\frac{-\frac{f^2\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int -i\tanh(c+dx)dx}{d}\right)}{d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\tanh(c+dx)dx}{d}\right)+\frac{f(e+fx)^2\text{sech}^2(c+dx)}{2d^2}+}{a}\ \frac{i\int (e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{f^2\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int \tanh(c+dx)dx}{d}\right)}{d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3f\int (e+fx)^2\tanh(c+dx)dx}{d}\right)+\frac{f(e+fx)^2\text{sech}^2(c+dx)}{2d^2}+\frac{(e+fx}{}}{a}\ \frac{i\int (e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f^2\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int -i\tan(ic+idx)dx}{d}\right)}{d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\tan(ic+idx)dx}{d}\right)+\frac{f(e+fx)^2\text{sech}^2(c+dx)}{2d^2}+}{a}\ \frac{i\int (e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{f^2\left(\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int \tan(ic+idx)dx}{d}\right)}{d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\int (e+fx)^2\tan(ic+idx)dx}{d}\right)+\frac{f(e+fx)^2\text{sech}^2(c+dx)}{2d^2}+\frac{(e+}{}}{a}\ \frac{i\int (e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 3956$$

3.277. $\displaystyle\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\int(e+fx)^2\tan(ic+idx)dx}{d}\right)-\frac{f^2\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d^2}+\frac{f(e+fx)^2\text{sech}^2(c+dx)}{2d^2}+\frac{(e+fx)}{}}{a}$$

$$\frac{i\int(e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4201}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\int\frac{e^{2(c+dx)}(e+fx)^2}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}\right)-\frac{f^2\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d^2}+\frac{f(e+fx)^2\text{sech}^2(c+d}{2d^2}}{a}$$

$$\frac{i\int(e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2(c+dx)}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}\right)-\frac{f^2\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(c}{}\right)}{d^2}}{a}$$

$$\frac{i\int(e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int\text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}\right)}{a}$$

$$\frac{i\int(e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2720}$$

3.277. $\int\frac{(e+fx)^3\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right.\right.}{d}\right.}{a}$$

$$\frac{i\int(e+fx)^3\operatorname{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{5974}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right.\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{f\int(e+fx)^2\operatorname{sech}^3(c+dx)dx}{d}-\frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{3d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right.\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{3d}+\frac{f\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{d}\right)}{a}$$

$$\Big\downarrow \text{4674}$$

3.277. $\int\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right.\right.}{d}\right.}{a}$$

$$\frac{i\left(\frac{f\left(-\frac{f^2\int \operatorname{sech}(c+dx)dx}{d^2}+\frac{1}{2}\int(e+fx)^2\operatorname{sech}(c+dx)dx+\frac{f(e+fx)\operatorname{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}\right)}{d}-\frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{3d}\right)}{a}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right.\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{3d}+\frac{f\left(-\frac{f^2\int\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f(e+fx)\operatorname{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}\right.}{d}\right.}{a}$$

$$\Big\downarrow \color{blue}{4257}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right.\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{3d}+\frac{f\left(\frac{1}{2}\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{f(e+fx)\operatorname{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}\right)}{d}\right.}{a}$$

$$\Big\downarrow \color{blue}{4668}$$

3.277. $\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right.\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{3d}+\frac{f\left(\frac{1}{2}\left(-\frac{2if\int(e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int(e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}\right)-\frac{f^2\arctan(\sinh(c+\right.}{d^3}}{d}\right.}{a}$$

$$\Big\downarrow \textbf{3011}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right.\right.}{d}\right.}{a}$$

$$\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{3d}+\frac{f\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyL}}{}\right.}{d}\right.\right.}{}\right.}{a}$$

$$\Big\downarrow \textbf{2720}$$

3.277. $\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right.\right.}{d}\right.}{i\left(-\frac{(e+fx)^3\text{sech}^3(c+dx)}{3d}+\frac{f\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+}\right.}{d^2}\right.}{}\right.\right.}{}\right.}$$

$$a$$

$$\Big\downarrow \text{7143}$$

$$\frac{-\frac{f^2\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+}{}\right.}{d}\right.\right.}{d}\right.}{i\left(-\frac{(e+fx)^3\text{sech}^3(c+dx)}{3d}+\frac{f\left(-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{1}{2}\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}\right.\right.}{}\right.}$$

$$a$$

input
```
Int[((e + f*x)^3*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

3.277. $\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

output
```
((f*(e + f*x)^2*Sech[c + d*x]^2)/(2*d^2) + ((e + f*x)^3*Sech[c + d*x]^2*Ta
nh[c + d*x])/(3*d) - (f^2*(-((f*Log[Cosh[c + d*x]])/d^2) + ((e + f*x)*Tanh
[c + d*x])/d))/d^2 + (2*(((3*I)*f*(((-1/3*I)*(e + f*x)^3)/f + (2*I)*(((e +
 f*x)^2*Log[1 + E^(2*(c + d*x))])/(2*d) - (f*(-1/2*((e + f*x)*PolyLog[2, -
E^(2*(c + d*x))])/d + (f*PolyLog[3, -E^(2*(c + d*x))])/(4*d^2)))/d)))/d +
((e + f*x)^3*Tanh[c + d*x])/d))/3)/a - (I*(-1/3*((e + f*x)^3*Sech[c + d*x]
^3)/d + (f*(-((f^2*ArcTan[Sinh[c + d*x]])/d^3) + ((2*(e + f*x)^2*ArcTan[E^
(c + d*x)])/d + ((2*I)*f*(-(((e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d) +
(f*PolyLog[3, (-I)*E^(c + d*x)])/d^2))/d - ((2*I)*f*(-(((e + f*x)*PolyLog[
2, I*E^(c + d*x)])/d) + (f*PolyLog[3, I*E^(c + d*x)])/d^2))/d)/2 + (f*(e +
 f*x)*Sech[c + d*x])/d^2 + ((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
))/d))/a
```

3.277.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.277. $\int \frac{(e+fx)^3 \text{sech}^2(c+dx)}{a+ia\sinh(c+dx)} \, dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

3.277. $\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.277.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1020 vs. $2(424) = 848$.

Time = 89.80 (sec) , antiderivative size = 1021, normalized size of antiderivative = 2.27

method	result
risch	$-\frac{3f^3\ln\left(1-i\mathrm{e}^{dx+c}\right)x^2}{2a\,d^2}+\frac{4f^2e\,c^2}{a\,d^3}+\frac{4f^2e\,x^2}{ad}+\frac{4f^3x^3}{3ad}+\frac{f^3\ln\left(1+\mathrm{e}^{2dx+2c}\right)}{a\,d^4}-\frac{2f^3\ln\left(\mathrm{e}^{dx+c}\right)}{a\,d^4}+\frac{i\left(-4id^2e^3+8d^2f^3x^3\mathrm{e}^{dx+c}\right.}{}$

input
```
int((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3*f^3*polylog(3,I*exp(d*x+c))/a/d^4+5*f^3*polylog(3,-I*exp(d*x+c))/a/d^4-3
/2/a/d^2*f^3*ln(1-I*exp(d*x+c))*x^2-3/a/d^3*f^3*polylog(2,I*exp(d*x+c))*x+
3/2/a/d^4*f^3*ln(1-I*exp(d*x+c))*c^2+4/a/d^3*f^2*e*c^2+4/a/d*f^2*e*x^2+2*I
/a/d^3*f^2*c*e*arctan(exp(d*x+c))+4/3/a/d*f^3*x^3+1/a/d^4*f^3*ln(1+exp(2*d
*x+2*c))-2/a/d^4*f^3*ln(exp(d*x+c))+1/3*I*(-4*I*d^2*e^3+8*d^2*f^3*x^3*exp(
d*x+c)-3*d*f^3*x^2*exp(3*d*x+3*c)+6*I*e*f^2+24*d^2*e*f^2*x^2*exp(d*x+c)-6*
d*e*f^2*x*exp(3*d*x+3*c)-12*I*d^2*e^2*f*x+6*I*f^3*x*exp(2*d*x+2*c)+24*d^2*
e^2*f*x*exp(d*x+c)-3*d*e^2*f*exp(3*d*x+3*c)-3*d*f^3*x^2*exp(d*x+c)-6*f^3*x
*exp(3*d*x+3*c)-4*I*d^2*x^3*f^3+6*I*e*f^2*exp(2*d*x+2*c)+8*d^2*e^3*exp(d*x
+c)-6*d*e*f^2*x*exp(d*x+c)-6*e*f^2*exp(3*d*x+3*c)+6*I*f^3*x-3*d*e^2*f*exp(
d*x+c)-6*f^3*x*exp(d*x+c)-12*I*d^2*e*f^2*x^2-6*e*f^2*exp(d*x+c))/(exp(d*x+
c)+I)/(exp(d*x+c)-I)^3/d^3/a-3/a/d^3*f^2*e*polylog(2,I*exp(d*x+c))+2*I/a/d
^4*f^3*arctan(exp(d*x+c))-3/a/d^2*f^2*e*ln(1-I*exp(d*x+c))*x-3/a/d^3*f^2*e
*ln(1-I*exp(d*x+c))*c+8/a/d^2*f^2*e*c*x-I/a/d^4*f^3*c^2*arctan(exp(d*x+c))
-I/a/d^2*f*e^2*arctan(exp(d*x+c))-8/a/d^3*f^2*e*c*ln(exp(d*x+c))-5/d^3/a*f
^2*e*polylog(2,-I*exp(d*x+c))-2/d^4/a*f^3*c^2*ln(1+exp(2*d*x+2*c))+4/d^4/a
*c^2*f^3*ln(exp(d*x+c))-2/d^2/a*e^2*f*ln(1+exp(2*d*x+2*c))+4/d^2/a*e^2*f*l
n(exp(d*x+c))+5/2/d^4/a*f^3*ln(1+I*exp(d*x+c))*c^2-4/d^3/a*f^3*x*c^2-5/2/d
^2/a*f^3*ln(1+I*exp(d*x+c))*x^2-5/d^3/a*f^3*polylog(2,-I*exp(d*x+c))*x-8/3
/d^4/a*f^3*c^3-5/d^3/a*f^2*e*ln(1+I*exp(d*x+c))*c-5/d^2/a*f^2*e*ln(1+I*...
```

3.277.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1405 vs. 2(412) = 824.

Time = 0.27 (sec) , antiderivative size = 1405, normalized size of antiderivative = 3.12

$$\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/6*(8*d^3*e^3 - 24*c*d^2*e^2*f + 12*(2*c^2 - 1)*d*e*f^2 - 4*(2*c^3 - 3*c)
*f^3 + 18*(d*f^3*x + d*e*f^2 - (d*f^3*x + d*e*f^2)*e^(4*d*x + 4*c) - 2*(-I
*d*f^3*x - I*d*e*f^2)*e^(3*d*x + 3*c) - 2*(-I*d*f^3*x - I*d*e*f^2)*e^(d*x
+ c))*dilog(I*e^(d*x + c)) + 30*(d*f^3*x + d*e*f^2 - (d*f^3*x + d*e*f^2)*e
^(4*d*x + 4*c) - 2*(-I*d*f^3*x - I*d*e*f^2)*e^(3*d*x + 3*c) - 2*(-I*d*f^3*
x - I*d*e*f^2)*e^(d*x + c))*dilog(-I*e^(d*x + c)) + 4*(2*d^3*f^3*x^3 + 6*d
^3*e*f^2*x^2 + 6*c*d^2*e^2*f - 6*c^2*d*e*f^2 + (2*c^3 - 3*c)*f^3 + 3*(2*d^
3*e^2*f - d*f^3)*x)*e^(4*d*x + 4*c) - 2*(8*I*d^3*f^3*x^3 + 3*(8*I*c + I)*d
^2*e^2*f + 6*(-4*I*c^2 + I)*d*e*f^2 + 4*(2*I*c^3 - 3*I*c)*f^3 + 3*(8*I*d^3
*e*f^2 + I*d^2*f^3)*x^2 + 6*(4*I*d^3*e^2*f + I*d^2*e*f^2 - I*d*f^3)*x)*e^(
3*d*x + 3*c) - 12*(d*f^3*x + d*e*f^2)*e^(2*d*x + 2*c) - 2*(3*I*d^2*f^3*x^2
 - 8*I*d^3*e^3 + 3*(8*I*c + I)*d^2*e^2*f + 6*(-4*I*c^2 + I)*d*e*f^2 + 4*(2
*I*c^3 - 3*I*c)*f^3 + 6*(I*d^2*e*f^2 - I*d*f^3)*x)*e^(d*x + c) + 9*(d^2*e^
2*f - 2*c*d*e*f^2 + c^2*f^3 - (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*e^(4*d*x
 + 4*c) - 2*(-I*d^2*e^2*f + 2*I*c*d*e*f^2 - I*c^2*f^3)*e^(3*d*x + 3*c) - 2
*(-I*d^2*e^2*f + 2*I*c*d*e*f^2 - I*c^2*f^3)*e^(d*x + c))*log(e^(d*x + c) +
 I) + 3*(5*d^2*e^2*f - 10*c*d*e*f^2 + (5*c^2 - 4)*f^3 - (5*d^2*e^2*f - 10*
c*d*e*f^2 + (5*c^2 - 4)*f^3)*e^(4*d*x + 4*c) - 2*(-5*I*d^2*e^2*f + 10*I*c*
d*e*f^2 + (-5*I*c^2 + 4*I)*f^3)*e^(3*d*x + 3*c) - 2*(-5*I*d^2*e^2*f + 10*I
*c*d*e*f^2 + (-5*I*c^2 + 4*I)*f^3)*e^(d*x + c))*log(e^(d*x + c) - I) + ...
```

3.277.6 Sympy [F]

$$\int \frac{(e+fx)^3 \operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= -\frac{i\left(\int \frac{e^3\operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{f^3x^3\operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{3ef^2x^2\operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{3e^2fx\operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**3*sech(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**3*sech(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(f**3
*x**3*sech(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(3*e*f**2*x**2*se
ch(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(3*e**2*f*x*sech(c + d*x)
**2/(sinh(c + d*x) - I), x))/a
```

3.277.7 Maxima [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.62

$$\int \frac{(e+fx)^3\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{1}{2}e^2f\left(\frac{24\left(4i\,dxe^{(4\,dx+4\,c)}+\left(8\,dxe^{(3\,c)}+e^{(3\,c)}\right)e^{(3\,dx)}+e^{(dx+c)}\right)}{12i\,ad^2e^{(4\,dx+4\,c)}+24\,ad^2e^{(3\,dx+3\,c)}+24\,ad^2e^{(dx+c)}-12i\,ad^2}-\frac{3\,\log\left(\left(e^{(dx+c)}+i\right)e^{(-c)}\right)}{ad^2}-\frac{5\,\log\left(-\right.}{}\right.$$
$$+\frac{4}{3}e^3\left(\frac{2\,e^{(-dx-c)}}{\left(2\,ae^{(-dx-c)}+2\,ae^{(-3\,dx-3\,c)}-i\,ae^{(-4\,dx-4\,c)}+i\,a\right)d}+\frac{i}{\left(2\,ae^{(-dx-c)}+2\,ae^{(-3\,dx-3\,c)}-i\,ae^{(-4\,dx-4\,c)}\right.}\right.$$
$$+\frac{4i\,d^2f^3x^3+12i\,d^2ef^2x^2-6i\,f^3x-6i\,ef^2+3\left(df^3x^2e^{(3\,c)}+2\,ef^2e^{(3\,c)}+2\left(def^2+f^3\right)xe^{(3\,c)}\right)e^{(3\,dx)}-6}{3i\,ad^3e^{(4\,dx+4\,c)}+6\,ad^3e^{(3\,dx}}$$
$$-\frac{5\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)\right)ef^2}{ad^3}$$
$$-\frac{3\left(dx\log\left(-i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(i\,e^{(dx+c)}\right)\right)ef^2}{ad^3}-\frac{2\,f^3x}{ad^3}$$
$$-\frac{5\left(d^2x^2\log\left(i\,e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-2\,\mathrm{Li}_3\left(-i\,e^{(dx+c)}\right)\right)f^3}{2\,ad^4}$$
$$-\frac{3\left(d^2x^2\log\left(-i\,e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(i\,e^{(dx+c)}\right)-2\,\mathrm{Li}_3\left(i\,e^{(dx+c)}\right)\right)f^3}{2\,ad^4}$$
$$+\frac{2\,f^3\log\left(e^{(dx+c)}-i\right)}{ad^4}+\frac{4\left(d^3f^3x^3+3\,d^3ef^2x^2\right)}{3\,ad^4}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
1/2*e^2*f*(24*(4*I*d*x*e^(4*d*x + 4*c) + (8*d*x*e^(3*c) + e^(3*c))*e^(3*d*
x) + e^(d*x + c))/(12*I*a*d^2*e^(4*d*x + 4*c) + 24*a*d^2*e^(3*d*x + 3*c) +
 24*a*d^2*e^(d*x + c) - 12*I*a*d^2) - 3*log((e^(d*x + c) + I)*e^(-c))/(a*d
^2) - 5*log(-I*(I*e^(d*x + c) + 1)*e^(-c))/(a*d^2)) + 4/3*e^3*(2*e^(-d*x -
 c)/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x - 4*c) + I*a
)*d) + I/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x - 4*c)
+ I*a)*d)) + (4*I*d^2*f^3*x^3 + 12*I*d^2*e*f^2*x^2 - 6*I*f^3*x - 6*I*e*f^2
 + 3*(d*f^3*x^2*e^(3*c) + 2*e*f^2*e^(3*c) + 2*(d*e*f^2 + f^3)*x*e^(3*c))*e
^(3*d*x) - 6*(I*f^3*x*e^(2*c) + I*e*f^2*e^(2*c))*e^(2*d*x) - (8*d^2*f^3*x^
3*e^c - 6*e*f^2*e^c + 3*(8*d^2*e*f^2 - d*f^3)*x^2*e^c - 6*(d*e*f^2 + f^3)*
x*e^c)*e^(d*x))/(3*I*a*d^3*e^(4*d*x + 4*c) + 6*a*d^3*e^(3*d*x + 3*c) + 6*a
*d^3*e^(d*x + c) - 3*I*a*d^3) - 5*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e
^(d*x + c)))*e*f^2/(a*d^3) - 3*(d*x*log(-I*e^(d*x + c) + 1) + dilog(I*e^(d
*x + c)))*e*f^2/(a*d^3) - 2*f^3*x/(a*d^3) - 5/2*(d^2*x^2*log(I*e^(d*x + c)
 + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a
*d^4) - 3/2*(d^2*x^2*log(-I*e^(d*x + c) + 1) + 2*d*x*dilog(I*e^(d*x + c))
- 2*polylog(3, I*e^(d*x + c)))*f^3/(a*d^4) + 2*f^3*log(e^(d*x + c) - I)/(a
*d^4) + 4/3*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2)/(a*d^4)
```

3.277.8 Giac [F]

$$\int \frac{(e+fx)^3\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\,\mathrm{sech}\,(dx+c)^2}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sech(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{(e+f\,x)^3}{\cosh\,(c+d\,x)^2\,\left(a+a\sinh\,(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.278 $\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.278.1 Optimal result

Integrand size = 31, antiderivative size = 325

$$\begin{aligned}\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{2(e+fx)^2}{3ad}-\frac{2if(e+fx)\arctan\left(e^{c+dx}\right)}{3ad^2}\\ &-\frac{4f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{3ad^2}\\ &-\frac{f^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{3ad^3}+\frac{f^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{3ad^3}\\ &-\frac{2f^2\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{3ad^3}-\frac{if^2\text{sech}(c+dx)}{3ad^3}\\ &+\frac{f(e+fx)\text{sech}^2(c+dx)}{3ad^2}+\frac{i(e+fx)^2\text{sech}^3(c+dx)}{3ad}\\ &-\frac{f^2\tanh(c+dx)}{3ad^3}+\frac{2(e+fx)^2\tanh(c+dx)}{3ad}\\ &-\frac{if(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{3ad^2}\\ &+\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{3ad}\end{aligned}$$

output
```
2/3*(f*x+e)^2/a/d-2/3*I*f*(f*x+e)*arctan(exp(d*x+c))/a/d^2-4/3*f*(f*x+e)*l
n(1+exp(2*d*x+2*c))/a/d^2-1/3*f^2*polylog(2,-I*exp(d*x+c))/a/d^3+1/3*f^2*p
olylog(2,I*exp(d*x+c))/a/d^3-2/3*f^2*polylog(2,-exp(2*d*x+2*c))/a/d^3-1/3*
I*f^2*sech(d*x+c)/a/d^3+1/3*f*(f*x+e)*sech(d*x+c)^2/a/d^2+1/3*I*(f*x+e)^2*
sech(d*x+c)^3/a/d-1/3*f^2*tanh(d*x+c)/a/d^3+2/3*(f*x+e)^2*tanh(d*x+c)/a/d-
1/3*I*f*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/a/d^2+1/3*(f*x+e)^2*sech(d*x+c)^2*
tanh(d*x+c)/a/d
```

3.278.2 Mathematica [A] (verified)

Time = 4.61 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.77

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{\frac{10id(e+fx)\left(d(e+fx)+2(1+ie^c)f\log\left(1-ie^{-c-dx}\right)\right)}{-i+e^c}+\frac{6\left(d(e+fx)\left(d(e+fx)+2(1-ie^c)f\log\left(1+ie^{-c-dx}\right)\right)+2i(i+e^c)f^2\,\text{PolyLog}\left(2,-ie^{-c-dx}\right)\right)}{-1+ie^c}}{}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(((10*I)*d*(e + f*x)*(d*(e + f*x) + 2*(1 + I*E^c)*f*Log[1 - I*E^(-c - d*x)
]))/(-I + E^c) + (6*(d*(e + f*x)*(d*(e + f*x) + 2*(1 - I*E^c)*f*Log[1 + I*
E^(-c - d*x)]) + (2*I)*(I + E^c)*f^2*PolyLog[2, (-I)*E^(-c - d*x)]))/(-1 +
 I*E^c) + 20*f^2*PolyLog[2, I*E^(-c - d*x)] + ((-2*I)*f^2*Cosh[c] + 2*d*f*
(e + f*x)*Cosh[d*x] - (2*I)*d^2*e^2*Cosh[c + d*x] + (4*I)*f^2*Cosh[c + d*x
] - (4*I)*d^2*e*f*x*Cosh[c + d*x] - (2*I)*d^2*f^2*x^2*Cosh[c + d*x] + 2*d*
e*f*Cosh[2*c + d*x] + 2*d*f^2*x*Cosh[2*c + d*x] + (4*I)*d^2*e^2*Cosh[c + 2
*d*x] - (2*I)*f^2*Cosh[c + 2*d*x] + (8*I)*d^2*e*f*x*Cosh[c + 2*d*x] + (4*I
)*d^2*f^2*x^2*Cosh[c + 2*d*x] + 8*d^2*e^2*Sinh[d*x] - 2*f^2*Sinh[d*x] + 16
*d^2*e*f*x*Sinh[d*x] + 8*d^2*f^2*x^2*Sinh[d*x] + d^2*e^2*Sinh[2*(c + d*x)]
 - 2*f^2*Sinh[2*(c + d*x)] + 2*d^2*e*f*x*Sinh[2*(c + d*x)] + d^2*f^2*x^2*S
inh[2*(c + d*x)] + 2*f^2*Sinh[2*c + d*x])/((Cosh[c/2] - I*Sinh[c/2])*(Cosh
[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2])*(Cosh[(c +
d*x)/2] + I*Sinh[(c + d*x)/2])^3))/(12*a*d^3)
```

3.278.3 Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.95, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.677$, Rules used = {6105, 3042, 4674, 3042, 4254, 24, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838, 5974, 3042, 4673, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$\Big\downarrow \; 6105$$

$$\frac{\int (e+fx)^2\text{sech}^4(c+dx)dx}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^4dx}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$\frac{-\frac{f^2\int \text{sech}^2(c+dx)dx}{3d^2}+\frac{2}{3}\int (e+fx)^2\text{sech}^2(c+dx)dx+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f^2\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{3d^2}+\frac{2}{3}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4254}$$

$$\frac{-\frac{if^2\int 1d(-i\tanh(c+dx))}{3d^3}+\frac{2}{3}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{2}{3}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2if\int -i(e+fx)\tanh(c+dx)dx}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

3.278. $\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}dx$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int(e+fx)\tanh(c+dx)dx}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int(e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int-i(e+fx)\tan(ic+idx)dx}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int(e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\int(e+fx)\tan(ic+idx)dx}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int(e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4201}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\int\frac{e^{2(c+dx)}(e+fx)}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int(e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int\log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\cdots}{a}-\frac{i\int(e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2715}$$

3.278. $\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int e^{-2(c+dx)}\log\left(1+e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+f}{a}$$

$$\frac{i\int(e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2838}$$

$$\frac{-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)+\frac{f(e+fx)\text{sech}^2}{3d^2}}{a}$$

$$\frac{i\int(e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{5974}$$

$$\frac{-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)+\frac{f(e+fx)\text{sech}^2}{3d^2}}{a}$$

$$\frac{i\left(\frac{2f\int(e+fx)\text{sech}^3(c+dx)dx}{3d}-\frac{(e+fx)^2\text{sech}^3(c+dx)}{3d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)+\frac{f(e+fx)\text{sech}^2}{3d^2}}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\text{sech}^3(c+dx)}{3d}+\frac{2f\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d}\right)}{a}$$

$$\Big\downarrow \text{4673}$$

3.278. $\displaystyle\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)+\frac{f(e+fx)\mathrm{sech}^2}{3d^2}}{a}$$

$$\frac{i\left(\frac{2f\left(\frac{1}{2}\int(e+fx)\mathrm{sech}(c+dx)dx+\frac{f\mathrm{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}\right)}{3d}-\frac{(e+fx)^2\mathrm{sech}^3(c+dx)}{3d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)+\frac{f(e+fx)\mathrm{sech}^2}{3d^2}}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\mathrm{sech}^3(c+dx)}{3d}+\frac{2f\left(\frac{1}{2}\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f\mathrm{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}\right)}{3d}\right)}{a}$$

$$\Big\downarrow \text{4668}$$

$$\frac{-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)+\frac{f(e+fx)\mathrm{sech}^2}{3d^2}}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\mathrm{sech}^3(c+dx)}{3d}+\frac{2f\left(\frac{1}{2}\left(-\frac{if\int\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\mathrm{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)}{2d}\right.}{3d}}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)+\frac{f(e+fx)\mathrm{sech}^2}{3d^2}}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\mathrm{sech}^3(c+dx)}{3d}+\frac{2f\left(\frac{1}{2}\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\mathrm{sech}}{2d}\right.}{3d}}{a}$$

$$\Big\downarrow \text{2838}$$

3.278. $\displaystyle\int \frac{(e+fx)^2\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)+\frac{f(e+fx)\mathrm{sech}^2}{3d^2}}{a}$$

$$\frac{i\left(-\frac{(e+fx)^2\mathrm{sech}^3(c+dx)}{3d}+\frac{2f\left(\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\mathrm{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx}{2d}\right.}{3d}\right.}{a}$$

input
```
Int[((e + f*x)^2*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((f*(e + f*x)*Sech[c + d*x]^2)/(3*d^2) - (f^2*Tanh[c + d*x])/(3*d^3) + ((e
 + f*x)^2*Sech[c + d*x]^2*Tanh[c + d*x])/(3*d) + (2*(((2*I)*f*(((-1/2*I)*(
e + f*x)^2)/f + (2*I)*(((e + f*x)*Log[1 + E^(2*(c + d*x))])/(2*d) + (f*Pol
yLog[2, -E^(2*(c + d*x))])/(4*d^2))))/d + ((e + f*x)^2*Tanh[c + d*x])/d))/
3)/a - (I*(-1/3*((e + f*x)^2*Sech[c + d*x]^3)/d + (2*f*(((2*(e + f*x)*ArcT
an[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*f*PolyLog
[2, I*E^(c + d*x)])/d^2)/2 + (f*Sech[c + d*x])/(2*d^2) + ((e + f*x)*Sech[c
 + d*x]*Tanh[c + d*x])/(2*d)))/(3*d)))/a
```

3.278.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

3.278. $\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

3.278.4 Maple [A] (verified)

Time = 24.63 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.57

method	result
risch	$\frac{2i\left(-2id^2x^2f^2+4d^2x^2f^2\mathrm{e}^{dx+c}-d\,f^2x\,\mathrm{e}^{3dx+3c}-4id^2efx+8d^2efx\,\mathrm{e}^{dx+c}-def\,\mathrm{e}^{3dx+3c}-2id^2e^2+if^2\mathrm{e}^{2dx+2c}+4d^2e^2\mathrm{e}^{dx+c}-\mathrm{e}^{dx+c}d\,f^2\right.}{3\left(\mathrm{e}^{dx+c}+i\right)\left(\mathrm{e}^{dx+c}-i\right)^3d^3a}$

input
```
int((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*(-2*I*d^2*x^2*f^2+4*d^2*x^2*f^2*exp(d*x+c)-d*f^2*x*exp(3*d*x+3*c)-4*
I*d^2*e*f*x+8*d^2*e*f*x*exp(d*x+c)-d*e*f*exp(3*d*x+3*c)-2*I*d^2*e^2+I*f^2*
exp(2*d*x+2*c)+4*d^2*e^2*exp(d*x+c)-exp(d*x+c)*d*f^2*x-f^2*exp(3*d*x+3*c)-
exp(d*x+c)*d*e*f+I*f^2-f^2*exp(d*x+c))/(exp(d*x+c)+I)/(exp(d*x+c)-I)^3/d^3
/a-5/3*f^2*polylog(2,-I*exp(d*x+c))/a/d^3+4/3/a/d^3*f^2*c^2+2/3*I/a/d^3*f^
2*c*arctan(exp(d*x+c))+8/3/a/d^2*f*ln(exp(d*x+c))*e-5/3/a/d^3*f^2*ln(1+I*e
xp(d*x+c))*c+4/3/d^3/a*f^2*c*ln(1+exp(2*d*x+2*c))-8/3/a/d^3*f^2*c*ln(exp(d
*x+c))-f^2*polylog(2,I*exp(d*x+c))/a/d^3-1/a/d^2*f^2*ln(1-I*exp(d*x+c))*x-
1/a/d^3*f^2*ln(1-I*exp(d*x+c))*c-4/3/a/d^2*f*e*ln(1+exp(2*d*x+2*c))+4/3*f^
2*x^2/a/d-5/3/a/d^2*f^2*ln(1+I*exp(d*x+c))*x+8/3/a/d^2*f^2*c*x-2/3*I/a/d^2
*f*e*arctan(exp(d*x+c))
```

3.278.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(281) = 562$.

Time = 0.25 (sec) , antiderivative size = 714, normalized size of antiderivative = 2.20

$$\int \frac{(e+fx)^2\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{4\,d^2e^2-8\,cdef+2\,(2\,c^2-1)f^2-2\,f^2e^{(2\,dx+2\,c)}-3\,\left(f^2e^{(4\,dx+4\,c)}-2i\,f^2e^{(3\,dx+3\,c)}-2i\,f^2e^{(dx+c)}-f^2\right)\mathrm{Li}_2(}{}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/3*(4*d^2*e^2 - 8*c*d*e*f + 2*(2*c^2 - 1)*f^2 - 2*f^2*e^(2*d*x + 2*c) - 3
*(f^2*e^(4*d*x + 4*c) - 2*I*f^2*e^(3*d*x + 3*c) - 2*I*f^2*e^(d*x + c) - f^
2)*dilog(I*e^(d*x + c)) - 5*(f^2*e^(4*d*x + 4*c) - 2*I*f^2*e^(3*d*x + 3*c)
 - 2*I*f^2*e^(d*x + c) - f^2)*dilog(-I*e^(d*x + c)) + 4*(d^2*f^2*x^2 + 2*d
^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(4*d*x + 4*c) - 2*(4*I*d^2*f^2*x^2 + (8*
I*c + I)*d*e*f + (-4*I*c^2 + I)*f^2 + (8*I*d^2*e*f + I*d*f^2)*x)*e^(3*d*x
+ 3*c) - 2*(-4*I*d^2*e^2 + (8*I*c + I)*d*e*f + I*d*f^2*x + (-4*I*c^2 + I)*
f^2)*e^(d*x + c) + 3*(d*e*f - c*f^2 - (d*e*f - c*f^2)*e^(4*d*x + 4*c) - 2*
(-I*d*e*f + I*c*f^2)*e^(3*d*x + 3*c) - 2*(-I*d*e*f + I*c*f^2)*e^(d*x + c))
*log(e^(d*x + c) + I) + 5*(d*e*f - c*f^2 - (d*e*f - c*f^2)*e^(4*d*x + 4*c)
 - 2*(-I*d*e*f + I*c*f^2)*e^(3*d*x + 3*c) - 2*(-I*d*e*f + I*c*f^2)*e^(d*x
+ c))*log(e^(d*x + c) - I) + 5*(d*f^2*x + c*f^2 - (d*f^2*x + c*f^2)*e^(4*d
*x + 4*c) - 2*(-I*d*f^2*x - I*c*f^2)*e^(3*d*x + 3*c) - 2*(-I*d*f^2*x - I*c
*f^2)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + 3*(d*f^2*x + c*f^2 - (d*f^2*x
+ c*f^2)*e^(4*d*x + 4*c) - 2*(-I*d*f^2*x - I*c*f^2)*e^(3*d*x + 3*c) - 2*(-
I*d*f^2*x - I*c*f^2)*e^(d*x + c))*log(-I*e^(d*x + c) + 1))/(a*d^3*e^(4*d*x
 + 4*c) - 2*I*a*d^3*e^(3*d*x + 3*c) - 2*I*a*d^3*e^(d*x + c) - a*d^3)
```

3.278.6 Sympy [F]

$$\int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^2 \operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{f^2x^2 \operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{2efx \operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**2*sech(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**2*sech(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(f**2
*x**2*sech(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*sech(c +
 d*x)**2/(sinh(c + d*x) - I), x))/a
```

3.278.7 Maxima [F]

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}\,(dx+c)^2}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
4*f^2*((2*I*d^2*x^2 + (d*x*e^(3*c) + e^(3*c))*e^(3*d*x) - (4*d^2*x^2*e^c -
 d*x*e^c - e^c)*e^(d*x) - I*e^(2*d*x + 2*c) - I)/(6*I*a*d^3*e^(4*d*x + 4*c
) + 12*a*d^3*e^(3*d*x + 3*c) + 12*a*d^3*e^(d*x + c) - 6*I*a*d^3) + I*integ
rate(1/4*x/(a*d*e^(d*x + c) + I*a*d), x) - 5*I*integrate(1/12*x/(a*d*e^(d*
x + c) - I*a*d), x)) + 1/3*e*f*(24*(4*I*d*x*e^(4*d*x + 4*c) + (8*d*x*e^(3*
c) + e^(3*c))*e^(3*d*x) + e^(d*x + c))/(12*I*a*d^2*e^(4*d*x + 4*c) + 24*a*
d^2*e^(3*d*x + 3*c) + 24*a*d^2*e^(d*x + c) - 12*I*a*d^2) - 3*log((e^(d*x +
 c) + I)*e^(-c))/(a*d^2) - 5*log(-I*(I*e^(d*x + c) + 1)*e^(-c))/(a*d^2)) +
 4/3*e^2*(2*e^(-d*x - c)/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e
^(-4*d*x - 4*c) + I*a)*d) + I/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) -
I*a*e^(-4*d*x - 4*c) + I*a)*d))
```

3.278.8 Giac [F]

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}\,(dx+c)^2}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sech(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{{\left(e+f\,x\right)}^2}{{\mathrm{cosh}\left(c+d\,x\right)}^2\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

input
```
int((e + f*x)^2/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.279 $\int \frac{(e+fx)\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.279.1 Optimal result

Integrand size = 29, antiderivative size = 158

$$\int \frac{(e+fx)\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{if\arctan(\sinh(c+dx))}{6ad^2} - \frac{2f\log(\cosh(c+dx))}{3ad^2} + \frac{f\mathrm{sech}^2(c+dx)}{6ad^2} + \frac{i(e+fx)\mathrm{sech}^3(c+dx)}{3ad} + \frac{2(e+fx)\tanh(c+dx)}{3ad} - \frac{if\mathrm{sech}(c+dx)\tanh(c+dx)}{6ad^2} + \frac{(e+fx)\mathrm{sech}^2(c+dx)\tanh(c+dx)}{3ad}$$

output
```
-1/6*I*f*arctan(sinh(d*x+c))/a/d^2-2/3*f*ln(cosh(d*x+c))/a/d^2+1/6*f*sech(
d*x+c)^2/a/d^2+1/3*I*(f*x+e)*sech(d*x+c)^3/a/d+2/3*(f*x+e)*tanh(d*x+c)/a/d
-1/6*I*f*sech(d*x+c)*tanh(d*x+c)/a/d^2+1/3*(f*x+e)*sech(d*x+c)^2*tanh(d*x+
c)/a/d
```

3.279.2 Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.23

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{2d(e+fx)(\cosh(2(c+dx))-2i\sinh(c+dx))+\cosh(c+dx)\left(-de-if+cf-2f\arctan\left(\tanh\left(\frac{1}{2}(c+\right.\right.\right.}{6ad^2\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)\right)(\text{co}}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(2*d*(e + f*x)*(Cosh[2*(c + d*x)] - (2*I)*Sinh[c + d*x]) + Cosh[c + d*x]*(
-(d*e) - I*f + c*f - 2*f*ArcTan[Tanh[(c + d*x)/2]] + (4*I)*f*Log[Cosh[c +
d*x]] - I*(d*e - c*f + 2*f*ArcTan[Tanh[(c + d*x)/2]] - (4*I)*f*Log[Cosh[c
+ d*x]])*Sinh[c + d*x]))/(6*a*d^2*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2]
)*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(-I + Sinh[c + d*x]))
```

3.279.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {6105, 3042, 4673, 3042, 4672, 26, 3042, 26, 3956, 5974, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{6105}$$
$$\frac{\int (e+fx)\text{sech}^4(c+dx)dx}{a}-\frac{i\int (e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^4dx}{a}-\frac{i\int (e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$
$$\Big\downarrow \text{4673}$$

$$\frac{\frac{2}{3}\int (e+fx)\text{sech}^2(c+dx)dx+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2}{3}\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int -i\tanh(c+dx)dx}{d}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int \tanh(c+dx)dx}{d}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int -i\tan(ic+idx)dx}{d}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int \tan(ic+idx)dx}{d}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\int (e+fx)\text{sech}^3(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{5974}$$

3.279. $\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\left(\frac{f\int\text{sech}^3(c+dx)dx}{3d}-\frac{(e+fx)\text{sech}^3(c+dx)}{3d}\right)}{a}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\left(-\frac{(e+fx)\text{sech}^3(c+dx)}{3d}+\frac{f\int\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d}\right)}{a}$$

$$\Big\downarrow \textbf{4255}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\left(\frac{f\left(\frac{1}{2}\int\text{sech}(c+dx)dx+\frac{\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{3d}-\frac{(e+fx)\text{sech}^3(c+dx)}{3d}\right)}{a}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\left(-\frac{(e+fx)\text{sech}^3(c+dx)}{3d}+\frac{f\left(\frac{\tanh(c+dx)\text{sech}(c+dx)}{2d}+\frac{1}{2}\int\csc\left(ic+idx+\frac{\pi}{2}\right)dx\right)}{3d}\right)}{a}$$

$$\Big\downarrow \textbf{4257}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)+\frac{f\text{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3d}}{a}-\frac{i\left(\frac{f\left(\frac{\arctan(\sinh(c+dx))}{2d}+\frac{\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{3d}-\frac{(e+fx)\text{sech}^3(c+dx)}{3d}\right)}{a}$$

input
```
Int[((e + f*x)*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

3.279. $\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

output
```
((f*Sech[c + d*x]^2)/(6*d^2) + ((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(
3*d) + (2*(-((f*Log[Cosh[c + d*x]])/d^2) + ((e + f*x)*Tanh[c + d*x])/d))/3
)/a - (I*(-1/3*((e + f*x)*Sech[c + d*x]^3)/d + (f*(ArcTan[Sinh[c + d*x]]/(
2*d) + (Sech[c + d*x]*Tanh[c + d*x])/(2*d)))/(3*d)))/a
```

3.279.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

3.279. $\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

3.279.4 Maple [A] (verified)

Time = 11.56 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{4fx}{3ad}+\frac{4fc}{3a\,d^{2}}-\frac{i\left(-8dfx\,{\mathrm e}^{dx+c}+f\,{\mathrm e}^{3dx+3c}-8de\,{\mathrm e}^{dx+c}+f\,{\mathrm e}^{dx+c}+4idfx+4ide\right)}{3\left({\mathrm e}^{dx+c}+i\right)\left({\mathrm e}^{dx+c}-i\right)^{3}d^{2}a}-\frac{f\ln\left({\mathrm e}^{dx+c}+i\right)}{2a\,d^{2}}-\frac{5f\ln\left({\mathrm e}^{dx+c}-i\right)}{6a\,d^{2}}$	143

input
```
int((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
4/3*f*x/a/d+4/3*f/a/d^2*c-1/3*I*(-8*d*f*x*exp(d*x+c)+f*exp(3*d*x+3*c)-8*d*
e*exp(d*x+c)+f*exp(d*x+c)+4*I*d*f*x+4*I*d*e)/(exp(d*x+c)+I)/(exp(d*x+c)-I)
^3/d^2/a-1/2*f/a/d^2*ln(exp(d*x+c)+I)-5/6*f/a/d^2*ln(exp(d*x+c)-I)
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.27

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$=\frac{8\,dfxe^{(4\,dx+4\,c)}+8\,de-2\,(8i\,dfx+i\,f)e^{(3\,dx+3\,c)}-2\,(-8i\,de+i\,f)e^{(dx+c)}-3\,\left(fe^{(4\,dx+4\,c)}-2i\,fe^{(3\,dx+3\,c)}-\right.}{6\,\left(ad^2e^{(4\,dx+4\,c)}-2i\,ad^2e^{(3\,dx+3}\right.}$$

input
```
integrate((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(8*d*f*x*e^(4*d*x + 4*c) + 8*d*e - 2*(8*I*d*f*x + I*f)*e^(3*d*x + 3*c)
 - 2*(-8*I*d*e + I*f)*e^(d*x + c) - 3*(f*e^(4*d*x + 4*c) - 2*I*f*e^(3*d*x
+ 3*c) - 2*I*f*e^(d*x + c) - f)*log(e^(d*x + c) + I) - 5*(f*e^(4*d*x + 4*c
) - 2*I*f*e^(3*d*x + 3*c) - 2*I*f*e^(d*x + c) - f)*log(e^(d*x + c) - I))/(
a*d^2*e^(4*d*x + 4*c) - 2*I*a*d^2*e^(3*d*x + 3*c) - 2*I*a*d^2*e^(d*x + c)
- a*d^2)
```

3.279.6 Sympy [F]

$$\int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e\operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{fx\operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)*sech(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e*sech(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(f*x*sec
h(c + d*x)**2/(sinh(c + d*x) - I), x))/a
```

3.279.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.59

$$\int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{1}{6}f\left(\frac{24\left(4i\,dxe^{(4\,dx+4\,c)} + \left(8\,dxe^{(3\,c)} + e^{(3\,c)}\right)e^{(3\,dx)} + e^{(dx+c)}\right)}{12i\,ad^2e^{(4\,dx+4\,c)} + 24\,ad^2e^{(3\,dx+3\,c)} + 24\,ad^2e^{(dx+c)} - 12i\,ad^2} - \frac{3\log\left(\left(e^{(dx+c)}+i\right)e^{(-c)}\right)}{ad^2} - \frac{5\log\left(-i\right.}{}\right.$$
$$+\frac{4}{3}e\left(\frac{2\,e^{(-dx-c)}}{\left(2\,ae^{(-dx-c)} + 2\,ae^{(-3\,dx-3\,c)} - i\,ae^{(-4\,dx-4\,c)} + i\,a\right)d} + \frac{i}{\left(2\,ae^{(-dx-c)} + 2\,ae^{(-3\,dx-3\,c)} - i\,ae^{(-4\,dx-4\,c)} +\right.}\right.$$

input
```
integrate((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*f*(24*(4*I*d*x*e^(4*d*x + 4*c) + (8*d*x*e^(3*c) + e^(3*c))*e^(3*d*x) +
 e^(d*x + c))/(12*I*a*d^2*e^(4*d*x + 4*c) + 24*a*d^2*e^(3*d*x + 3*c) + 24*
a*d^2*e^(d*x + c) - 12*I*a*d^2) - 3*log((e^(d*x + c) + I)*e^(-c))/(a*d^2)
- 5*log(-I*(I*e^(d*x + c) + 1)*e^(-c))/(a*d^2)) + 4/3*e*(2*e^(-d*x - c)/((
2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x - 4*c) + I*a)*d) +
 I/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x - 4*c) + I*a)
*d))
```

3.279.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.65

$$\int \frac{(e+fx)\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{8\,dfxe^{(4\,dx+4\,c)} - 16i\,dfxe^{(3\,dx+3\,c)} + 16i\,dee^{(dx+c)} - 3\,fe^{(4\,dx+4\,c)}\log\left(e^{(dx+c)}+i\right) + 6i\,fe^{(3\,dx+3\,c)}\log\left(e^{(dx+c}\right.}{}$$

input
```
integrate((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(8*d*f*x*e^(4*d*x + 4*c) - 16*I*d*f*x*e^(3*d*x + 3*c) + 16*I*d*e*e^(d*
x + c) - 3*f*e^(4*d*x + 4*c)*log(e^(d*x + c) + I) + 6*I*f*e^(3*d*x + 3*c)*
log(e^(d*x + c) + I) + 6*I*f*e^(d*x + c)*log(e^(d*x + c) + I) - 5*f*e^(4*d
*x + 4*c)*log(e^(d*x + c) - I) + 10*I*f*e^(3*d*x + 3*c)*log(e^(d*x + c) -
I) + 10*I*f*e^(d*x + c)*log(e^(d*x + c) - I) + 8*d*e - 2*I*f*e^(3*d*x + 3*
c) - 2*I*f*e^(d*x + c) + 3*f*log(e^(d*x + c) + I) + 5*f*log(e^(d*x + c) -
I))/(a*d^2*e^(4*d*x + 4*c) - 2*I*a*d^2*e^(3*d*x + 3*c) - 2*I*a*d^2*e^(d*x
+ c) - a*d^2)
```

3.279.9 Mupad [B] (verification not implemented)

Time = 3.46 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.30

$$\begin{aligned}\int \frac{(e+fx)\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{4\,f\,x}{3\,a\,d} - \frac{f+3\,d\,e+3\,d\,f\,x}{3\,a\,d^2\,\left(1-\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{c+d\,x}\,2\mathrm{i}\right)} \\ &\quad - \frac{5\,f\,\ln\left(f+f\,\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}{6\,a\,d^2} \\ &\quad - \frac{(e+f\,x)\,2\mathrm{i}}{3\,a\,d\,\left(3\,\mathrm{e}^{c+d\,x}+\mathrm{e}^{2\,c+2\,d\,x}\,3\mathrm{i}-\mathrm{e}^{3\,c+3\,d\,x}-\mathrm{i}\right)} \\ &\quad - \frac{(e+f\,x)\,1\mathrm{i}}{2\,a\,d\,\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)} - \frac{f\,\ln\left(-1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}{2\,a\,d^2} \\ &\quad + \frac{(3\,d\,e-2\,f+3\,d\,f\,x)\,1\mathrm{i}}{6\,a\,d^2\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}\end{aligned}$$

input
```
int((e + f*x)/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
(4*f*x)/(3*a*d) - (f + 3*d*e + 3*d*f*x)/(3*a*d^2*(exp(c + d*x)*2i - exp(2*
c + 2*d*x) + 1)) - (5*f*log(f + f*exp(c + d*x)*1i))/(6*a*d^2) - ((e + f*x)
*2i)/(3*a*d*(3*exp(c + d*x) + exp(2*c + 2*d*x)*3i - exp(3*c + 3*d*x) - 1i)
) - ((e + f*x)*1i)/(2*a*d*(exp(c + d*x) + 1i)) - (f*log(exp(c + d*x)*1i -
1))/(2*a*d^2) + ((3*d*e - 2*f + 3*d*f*x)*1i)/(6*a*d^2*(exp(c + d*x) - 1i))
```

3.279. $\int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.280 $\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.280.1 Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{i\operatorname{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}+\frac{2\tanh(c+dx)}{3ad}$$

output
```
1/3*I*sech(d*x+c)/d/(a+I*a*sinh(d*x+c))+2/3*tanh(d*x+c)/a/d
```

3.280.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{\operatorname{sech}(c+dx)(\cosh(2(c+dx))-2i\sinh(c+dx))}{3ad(-i+\sinh(c+dx))}$$

input
```
Integrate[Sech[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(Sech[c + d*x]*(Cosh[2*(c + d*x)] - (2*I)*Sinh[c + d*x]))/(3*a*d*(-I + Sin
h[c + d*x]))
```

3.280.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3042, 3151, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)^2(a+a\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3151}$$

$$\frac{2\int \operatorname{sech}^2(c+dx)dx}{3a}+\frac{i\operatorname{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{3a}+\frac{i\operatorname{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}$$

$$\Big\downarrow \text{4254}$$

$$\frac{2i\int 1d(-i\tanh(c+dx))}{3ad}+\frac{i\operatorname{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}$$

$$\Big\downarrow \text{24}$$

$$\frac{2\tanh(c+dx)}{3ad}+\frac{i\operatorname{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}$$

input
```
Int[Sech[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((I/3)*Sech[c + d*x])/(d*(a + I*a*Sinh[c + d*x])) + (2*Tanh[c + d*x])/(3*a
*d)
```

3.280.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.280.4 Maple [A] (verified)

Time = 10.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{4i(2\,\mathrm{e}^{dx+c}-i)}{3(\mathrm{e}^{dx+c}-i)^3(\mathrm{e}^{dx+c}+i)ad}$	43
derivativedivides	$\frac{\frac{2}{4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}-\frac{2}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{3}{2\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{ad}$	75
default	$\frac{\frac{2}{4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}-\frac{2}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{3}{2\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{ad}$	75

input
```
int(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
4/3*I*(2*exp(d*x+c)-I)/(exp(d*x+c)-I)^3/(exp(d*x+c)+I)/a/d
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{4\left(-2i\,e^{(dx+c)}-1\right)}{3\left(ade^{(4\,dx+4\,c)}-2i\,ade^{(3\,dx+3\,c)}-2i\,ade^{(dx+c)}-ad\right)}$$

input
```
integrate(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-4/3*(-2*I*e^(d*x + c) - 1)/(a*d*e^(4*d*x + 4*c) - 2*I*a*d*e^(3*d*x + 3*c)
 - 2*I*a*d*e^(d*x + c) - a*d)
```

3.280.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int \frac{\operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

input
```
integrate(sech(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)**2/(sinh(c + d*x) - I), x)/a
```

3.280.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(39) = 78.

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.21

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{8\,e^{(-dx-c)}}{3\,(2\,ae^{(-dx-c)}+2\,ae^{(-3\,dx-3\,c)}-i\,ae^{(-4\,dx-4\,c)}+i\,a)d} + \frac{4i}{3\,(2\,ae^{(-dx-c)}+2\,ae^{(-3\,dx-3\,c)}-i\,ae^{(-4\,dx-4\,c)}+i\,a)d}$$

input
```
integrate(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
8/3*e^(-d*x - c)/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x
 - 4*c) + I*a)*d) + 4/3*I/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*
e^(-4*d*x - 4*c) + I*a)*d)
```

3.280.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\frac{3}{a\left(i\,e^{(dx+c)}-1\right)} - \frac{-3i\,e^{(2\,dx+2\,c)}-12\,e^{(dx+c)}+5i}{a\left(e^{(dx+c)}-i\right)^3}}{6\,d}$$

input
```
integrate(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3/(a*(I*e^(d*x + c) - 1)) - (-3*I*e^(2*d*x + 2*c) - 12*e^(d*x + c) +
5*I)/(a*(e^(d*x + c) - I)^3))/d
```

3.280.9 Mupad [B] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{4\left(1+\mathrm{e}^{c+d\,x}\,2\mathrm{i}\right)\,\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)^2}{3\,a\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)^3}$$

input
```
int(1/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
(4*(exp(c + d*x)*2i + 1)*(exp(c + d*x) + 1i)^2)/(3*a*d*(exp(2*c + 2*d*x) +
 1)^3)
```

3.281 $\int \frac{\mathrm{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

3.281.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\mathrm{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \mathrm{Int}\left(\frac{\mathrm{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.281.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{\mathrm{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \$\mathrm{Aborted}$$

input
```
Integrate[Sech[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.281.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

$$\Big\downarrow \text{6111}$$

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.281.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.281.4 Maple [N/A] (verified)

Not integrable

Time = 1.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.281.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 768, normalized size of antiderivative = 24.77

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)^2}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3*(4*d^2*f^2*x^2 + 8*d^2*e*f*x + 4*d^2*e^2 - 2*f^2*e^(2*d*x + 2*c) - 2*
f^2 + (I*d*f^2*x + I*d*e*f - 2*I*f^2)*e^(3*d*x + 3*c) + (8*I*d^2*f^2*x^2 +
 8*I*d^2*e^2 + I*d*e*f - 2*I*f^2 + (16*I*d^2*e*f + I*d*f^2)*x)*e^(d*x + c)
 - 3*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3 - (a
*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*e^(4*d*x +
 4*c) + 2*(I*a*d^3*f^3*x^3 + 3*I*a*d^3*e*f^2*x^2 + 3*I*a*d^3*e^2*f*x + I*a
*d^3*e^3)*e^(3*d*x + 3*c) + 2*(I*a*d^3*f^3*x^3 + 3*I*a*d^3*e*f^2*x^2 + 3*I
*a*d^3*e^2*f*x + I*a*d^3*e^3)*e^(d*x + c))*integral(-1/3*(4*d^2*f^3*x^2 +
8*d^2*e*f^2*x + 4*d^2*e^2*f - 6*f^3 - (I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x + I
*d^2*e^2*f - 6*I*f^3)*e^(d*x + c))/(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*
a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4 + (a*d^3*f^4*x^4 + 4*a*d^3
*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*e^(2*d*x +
 2*c)), x))/(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e
^3 - (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*e^(
4*d*x + 4*c) + 2*(I*a*d^3*f^3*x^3 + 3*I*a*d^3*e*f^2*x^2 + 3*I*a*d^3*e^2*f*
x + I*a*d^3*e^3)*e^(3*d*x + 3*c) + 2*(I*a*d^3*f^3*x^3 + 3*I*a*d^3*e*f^2*x^
2 + 3*I*a*d^3*e^2*f*x + I*a*d^3*e^3)*e^(d*x + c))
```

3.281.6 Sympy [N/A]

Not integrable

Time = 4.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\operatorname{sech}^2(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

input
```
integrate(sech(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)**2/(e*sinh(c + d*x) - I*e + f*x**sinh(c + d*x) -
I*f*x), x)/a
```

3.281.7 Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 632, normalized size of antiderivative = 20.39

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\,(dx+c)^2}{(fx+e)(i\,a\sinh\,(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-4*I*f*integrate(1/(8*I*a*d*f^2*x^2 + 16*I*a*d*e*f*x + 8*I*a*d*e^2 + 8*(a*
d*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 1/3*(4*d^2*f
^2*x^2 + 8*d^2*e*f*x + 4*d^2*e^2 - 2*f^2*e^(2*d*x + 2*c) - 2*f^2 + (I*d*f^
2*x*e^(3*c) + (I*d*e*f - 2*I*f^2)*e^(3*c))*e^(3*d*x) + (8*I*d^2*f^2*x^2*e^
c + (16*I*d^2*e*f + I*d*f^2)*x*e^c + (8*I*d^2*e^2 + I*d*e*f - 2*I*f^2)*e^c
)*e^(d*x))/(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^
3 - (a*d^3*f^3*x^3*e^(4*c) + 3*a*d^3*e*f^2*x^2*e^(4*c) + 3*a*d^3*e^2*f*x*e
^(4*c) + a*d^3*e^3*e^(4*c))*e^(4*d*x) + 2*(I*a*d^3*f^3*x^3*e^(3*c) + 3*I*a
*d^3*e*f^2*x^2*e^(3*c) + 3*I*a*d^3*e^2*f*x*e^(3*c) + I*a*d^3*e^3*e^(3*c))*
e^(3*d*x) + 2*(I*a*d^3*f^3*x^3*e^c + 3*I*a*d^3*e*f^2*x^2*e^c + 3*I*a*d^3*e
^2*f*x*e^c + I*a*d^3*e^3*e^c)*e^(d*x)) - 4*integrate(1/24*(5*d^2*f^3*x^2 +
 10*d^2*e*f^2*x + 5*d^2*e^2*f - 12*f^3)/(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3
 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4 - (-I*a*d^3*f^4*x^4*e
^c - 4*I*a*d^3*e*f^3*x^3*e^c - 6*I*a*d^3*e^2*f^2*x^2*e^c - 4*I*a*d^3*e^3*f
*x*e^c - I*a*d^3*e^4*e^c)*e^(d*x)), x)
```

3.281.8 Giac [N/A]

Not integrable

Time = 53.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\,(dx+c)^2}{(fx+e)(i\,a\sinh\,(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sech(d*x + c)^2/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

3.281.9 Mupad [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(cosh(c + d*x)^2*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.282 $\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

3.282.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.282.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \$\text{Aborted}$$

input
```
Integrate[Sech[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.282.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\text{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.282.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.282.4 Maple [N/A] (verified)

Not integrable

Time = 0.89 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}\left(dx+c\right)^{2}}{\left(fx+e\right)^{2}\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.282.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 919, normalized size of antiderivative = 29.65

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^2}{\left(fx+e\right)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-1/3*(4*d^2*f^2*x^2 + 8*d^2*e*f*x + 4*d^2*e^2 - 6*f^2*e^(2*d*x + 2*c) - 6*
f^2 - 2*(-I*d*f^2*x - I*d*e*f + 3*I*f^2)*e^(3*d*x + 3*c) - 2*(-4*I*d^2*f^2
*x^2 - 4*I*d^2*e^2 - I*d*e*f + 3*I*f^2 + (-8*I*d^2*e*f - I*d*f^2)*x)*e^(d*
x + c) - 3*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*
d^3*e^3*f*x + a*d^3*e^4 - (a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2
*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*e^(4*d*x + 4*c) + 2*(I*a*d^3*f^4*x
^4 + 4*I*a*d^3*e*f^3*x^3 + 6*I*a*d^3*e^2*f^2*x^2 + 4*I*a*d^3*e^3*f*x + I*a
*d^3*e^4)*e^(3*d*x + 3*c) + 2*(I*a*d^3*f^4*x^4 + 4*I*a*d^3*e*f^3*x^3 + 6*I
*a*d^3*e^2*f^2*x^2 + 4*I*a*d^3*e^3*f*x + I*a*d^3*e^4)*e^(d*x + c))*integra
l(-2/3*(4*d^2*f^3*x^2 + 8*d^2*e*f^2*x + 4*d^2*e^2*f - 12*f^3 + (-I*d^2*f^3
*x^2 - 2*I*d^2*e*f^2*x - I*d^2*e^2*f + 12*I*f^3)*e^(d*x + c))/(a*d^3*f^5*x
^5 + 5*a*d^3*e*f^4*x^4 + 10*a*d^3*e^2*f^3*x^3 + 10*a*d^3*e^3*f^2*x^2 + 5*a
*d^3*e^4*f*x + a*d^3*e^5 + (a*d^3*f^5*x^5 + 5*a*d^3*e*f^4*x^4 + 10*a*d^3*e
^2*f^3*x^3 + 10*a*d^3*e^3*f^2*x^2 + 5*a*d^3*e^4*f*x + a*d^3*e^5)*e^(2*d*x
+ 2*c)), x))/(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*
a*d^3*e^3*f*x + a*d^3*e^4 - (a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e
^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*e^(4*d*x + 4*c) + 2*(I*a*d^3*f^4
*x^4 + 4*I*a*d^3*e*f^3*x^3 + 6*I*a*d^3*e^2*f^2*x^2 + 4*I*a*d^3*e^3*f*x + I
*a*d^3*e^4)*e^(3*d*x + 3*c) + 2*(I*a*d^3*f^4*x^4 + 4*I*a*d^3*e*f^3*x^3 + 6
*I*a*d^3*e^2*f^2*x^2 + 4*I*a*d^3*e^3*f*x + I*a*d^3*e^4)*e^(d*x + c))
```

3.282.6 Sympy [N/A]

Not integrable

Time = 31.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.35

$$\begin{aligned}&\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx\\&=-\frac{i\int \frac{\operatorname{sech}^2\left(c+dx\right)}{e^2\sinh\left(c+dx\right)-ie^2+2efx\sinh\left(c+dx\right)-2iefx+f^2x^2\sinh\left(c+dx\right)-if^2x^2}\,dx}{a}\end{aligned}$$

input
```
integrate(sech(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)**2/(e**2*sinh(c + d*x) - I*e**2 + 2*e*f*x*sinh(c
 + d*x) - 2*I*e*f*x + f**2*x**2*sinh(c + d*x) - I*f**2*x**2), x)/a
```

3.282.7 Maxima [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 762, normalized size of antiderivative = 24.58

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)^2}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-4*I*f*integrate(1/(4*I*a*d*f^3*x^3 + 12*I*a*d*e*f^2*x^2 + 12*I*a*d*e^2*f*
x + 4*I*a*d*e^3 + 4*(a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x
*e^c + a*d*e^3*e^c)*e^(d*x)), x) - 2/3*(2*d^2*f^2*x^2 + 4*d^2*e*f*x + 2*d^
2*e^2 - 3*f^2*e^(2*d*x + 2*c) - 3*f^2 + (I*d*f^2*x*e^(3*c) + (I*d*e*f - 3*
I*f^2)*e^(3*c))*e^(3*d*x) + (4*I*d^2*f^2*x^2*e^c + (8*I*d^2*e*f + I*d*f^2)
*x*e^c + (4*I*d^2*e^2 + I*d*e*f - 3*I*f^2)*e^c)*e^(d*x))/(a*d^3*f^4*x^4 +
4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4 - (a
*d^3*f^4*x^4*e^(4*c) + 4*a*d^3*e*f^3*x^3*e^(4*c) + 6*a*d^3*e^2*f^2*x^2*e^(
4*c) + 4*a*d^3*e^3*f*x*e^(4*c) + a*d^3*e^4*e^(4*c))*e^(4*d*x) + 2*(I*a*d^3
*f^4*x^4*e^(3*c) + 4*I*a*d^3*e*f^3*x^3*e^(3*c) + 6*I*a*d^3*e^2*f^2*x^2*e^(
3*c) + 4*I*a*d^3*e^3*f*x*e^(3*c) + I*a*d^3*e^4*e^(3*c))*e^(3*d*x) + 2*(I*a
*d^3*f^4*x^4*e^c + 4*I*a*d^3*e*f^3*x^3*e^c + 6*I*a*d^3*e^2*f^2*x^2*e^c + 4
*I*a*d^3*e^3*f*x*e^c + I*a*d^3*e^4*e^c)*e^(d*x)) - 4*integrate(1/12*(5*d^2
*f^3*x^2 + 10*d^2*e*f^2*x + 5*d^2*e^2*f - 24*f^3)/(a*d^3*f^5*x^5 + 5*a*d^3
*e*f^4*x^4 + 10*a*d^3*e^2*f^3*x^3 + 10*a*d^3*e^3*f^2*x^2 + 5*a*d^3*e^4*f*x
 + a*d^3*e^5 - (-I*a*d^3*f^5*x^5*e^c - 5*I*a*d^3*e*f^4*x^4*e^c - 10*I*a*d^
3*e^2*f^3*x^3*e^c - 10*I*a*d^3*e^3*f^2*x^2*e^c - 5*I*a*d^3*e^4*f*x*e^c - I
*a*d^3*e^5*e^c)*e^(d*x)), x)
```

3.282.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.282.9 Mupad [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^2\,\left(e+f\,x\right)^2\,\left(a+a\,\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(cosh(c + d*x)^2*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.283 $\int \frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.283.1 Optimal result

Integrand size = 31, antiderivative size = 667

$$
\begin{aligned}
\int \frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = & -\frac{if(e+fx)^2}{2ad^2}-\frac{5f^2(e+fx)\arctan\left(e^{c+dx}\right)}{ad^3} \\
& +\frac{3(e+fx)^3\arctan\left(e^{c+dx}\right)}{4ad}+\frac{if^2(e+fx)\log\left(1+e^{2(c+dx)}\right)}{ad^3} \\
& +\frac{5if^3\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{2ad^4} \\
& -\frac{9if(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{8ad^2} \\
& -\frac{5if^3\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{2ad^4} \\
& +\frac{9if(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{8ad^2} \\
& +\frac{if^3\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2ad^4} \\
& +\frac{9if^2(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{4ad^3} \\
& -\frac{9if^2(e+fx)\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{4ad^3} \\
& -\frac{9if^3\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)}{4ad^4} \\
& +\frac{9if^3\operatorname{PolyLog}\left(4,ie^{c+dx}\right)}{4ad^4}-\frac{f^3\operatorname{sech}(c+dx)}{4ad^4} \\
& +\frac{9f(e+fx)^2\operatorname{sech}(c+dx)}{8ad^2}-\frac{if^2(e+fx)\operatorname{sech}^2(c+dx)}{4ad^3} \\
& +\frac{f(e+fx)^2\operatorname{sech}^3(c+dx)}{4ad^2}+\frac{i(e+fx)^3\operatorname{sech}^4(c+dx)}{4ad} \\
& +\frac{if^3\tanh(c+dx)}{4ad^4}-\frac{if(e+fx)^2\tanh(c+dx)}{2ad^2} \\
& -\frac{f^2(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{4ad^3} \\
& +\frac{3(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)}{8ad} \\
& -\frac{if(e+fx)^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4ad^2} \\
& +\frac{(e+fx)^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4ad}
\end{aligned}
$$

output
```
1/4*I*(f*x+e)^3*sech(d*x+c)^4/a/d-5*f^2*(f*x+e)*arctan(exp(d*x+c))/a/d^3+3
/4*(f*x+e)^3*arctan(exp(d*x+c))/a/d+1/4*I*f^3*tanh(d*x+c)/a/d^4+9/8*I*f*(f
*x+e)^2*polylog(2,I*exp(d*x+c))/a/d^2+5/2*I*f^3*polylog(2,-I*exp(d*x+c))/a
/d^4+1/2*I*f^3*polylog(2,-exp(2*d*x+2*c))/a/d^4-5/2*I*f^3*polylog(2,I*exp(
d*x+c))/a/d^4-1/2*I*f*(f*x+e)^2*tanh(d*x+c)/a/d^2+9/4*I*f^2*(f*x+e)*polylo
g(3,-I*exp(d*x+c))/a/d^3-9/4*I*f^3*polylog(4,-I*exp(d*x+c))/a/d^4-1/4*I*f^
2*(f*x+e)*sech(d*x+c)^2/a/d^3+9/4*I*f^3*polylog(4,I*exp(d*x+c))/a/d^4-1/4*
f^3*sech(d*x+c)/a/d^4+9/8*f*(f*x+e)^2*sech(d*x+c)/a/d^2-1/2*I*f*(f*x+e)^2/
a/d^2+1/4*f*(f*x+e)^2*sech(d*x+c)^3/a/d^2+I*f^2*(f*x+e)*ln(1+exp(2*d*x+2*c
))/a/d^3-1/4*I*f*(f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/a/d^2-9/8*I*f*(f*x+e)
^2*polylog(2,-I*exp(d*x+c))/a/d^2-1/4*f^2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/
a/d^3+3/8*(f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/a/d-9/4*I*f^2*(f*x+e)*polylog(
3,I*exp(d*x+c))/a/d^3+1/4*(f*x+e)^3*sech(d*x+c)^3*tanh(d*x+c)/a/d
```

3.283.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2008 vs. 2(667) = 1334.

Time = 9.27 (sec) , antiderivative size = 2008, normalized size of antiderivative = 3.01

$$\int \frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(-3*E^c*((d^2*e^3*x)/E^c - (4*e*f^2*x)/E^c - (e*(1 - I*E^c)*(d^2*e^2 - 4*f
^2)*x)/E^c + (3*d^2*e^2*f*x^2)/(2*E^c) - (2*f^3*x^2)/E^c + (d^2*e*f^2*x^3)
/E^c + (d^2*f^3*x^4)/(4*E^c) + ((1 - I*E^c)*f*(3*d^2*e^2 - 4*f^2)*x*Log[1
+ I*E^(-c - d*x)])/(d*E^c) + (3*d*e*(1 - I*E^c)*f^2*x^2*Log[1 + I*E^(-c -
d*x)])/E^c + (d*(1 - I*E^c)*f^3*x^3*Log[1 + I*E^(-c - d*x)])/E^c + (e*(1 -
 I*E^c)*(d^2*e^2 - 4*f^2)*Log[I + E^(c + d*x)])/(d*E^c) - ((1 - I*E^c)*f*(
3*d^2*e^2 - 4*f^2)*PolyLog[2, (-I)*E^(-c - d*x)])/(d^2*E^c) - (6*e*(1 - I*
E^c)*f^2*x*PolyLog[2, (-I)*E^(-c - d*x)])/E^c - (3*(1 - I*E^c)*f^3*x^2*Pol
yLog[2, (-I)*E^(-c - d*x)])/E^c - (6*e*(1 - I*E^c)*f^2*PolyLog[3, (-I)*E^(
-c - d*x)])/(d*E^c) - (6*(1 - I*E^c)*f^3*x*PolyLog[3, (-I)*E^(-c - d*x)])/
(d*E^c) - (6*(1 - I*E^c)*f^3*PolyLog[4, (-I)*E^(-c - d*x)])/(d^2*E^c)))/(8
*a*d^2*(I + E^c)) - (-12*d^2*e*(1 + I*E^c)*f*(3*d^2*e^2 - 28*f^2)*x + (28*
f^2 - 3*d^2*(e + f*x)^2)^2 + 12*d*(1 + I*E^c)*f^2*(9*d^2*e^2 - 28*f^2)*x*L
og[1 - I*E^(-c - d*x)] + 108*d^3*e*(1 + I*E^c)*f^3*x^2*Log[1 - I*E^(-c - d
*x)] + 36*d^3*(1 + I*E^c)*f^4*x^3*Log[1 - I*E^(-c - d*x)] + 12*d*e*(1 + I*
E^c)*f*(3*d^2*e^2 - 28*f^2)*Log[I - E^(c + d*x)] + 12*(1 + I*E^c)*f^2*(-9*
d^2*e^2 + 28*f^2)*PolyLog[2, I*E^(-c - d*x)] - 216*d^2*e*(1 + I*E^c)*f^3*x
*PolyLog[2, I*E^(-c - d*x)] - 108*d^2*(1 + I*E^c)*f^4*x^2*PolyLog[2, I*E^(
-c - d*x)] - 216*d*e*(1 + I*E^c)*f^3*PolyLog[3, I*E^(-c - d*x)] - 216*d*(1
 + I*E^c)*f^4*x*PolyLog[3, I*E^(-c - d*x)] - 216*(1 + I*E^c)*f^4*PolyLo...
```

3.283.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6105}$$

$$\frac{\int (e+fx)^3\text{sech}^5(c+dx)dx}{a}-\frac{i\int (e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^5dx}{a}-\frac{i\int (e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$\frac{-\frac{f^2\int (e+fx)\text{sech}^3(c+dx)dx}{2d^2}+\frac{3}{4}\int (e+fx)^3\text{sech}^3(c+dx)dx+\frac{f(e+fx)^2\text{sech}^3(c+dx)}{4d^2}+\frac{(e+fx)^3\tanh(c+dx)\text{sech}^3(c+dx)}{4d}}{a}-$$

$$\frac{i\int (e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f^2\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{2d^2}+\frac{3}{4}\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{f(e+fx)^2\text{sech}^3(c+dx)}{4d^2}+\frac{(e+fx)^3\tanh(c+dx)\text{sech}^3(c}{4d}}{a}$$

$$\frac{i\int (e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4673}$$

$$\frac{-\frac{f^2\left(\frac{1}{2}\int (e+fx)\text{sech}(c+dx)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{2d^2}+\frac{3}{4}\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{f(e+fx)^2}{}}{a}$$

$$\frac{i\int (e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f^2\left(\frac{1}{2}\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{2d^2}+\frac{3}{4}\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{f(e+f}{}}{a}$$

$$\frac{i\int (e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4668}$$

$$\frac{-\frac{f^2\left(\frac{1}{2}\left(-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{2d^2}+\frac{3}{4}\int (e+}{a}$$

$$\frac{i\int (e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-\frac{f^2\left(\frac{1}{2}\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}}{2d}}{2d^2}}{a}$$

$$\frac{i\int (e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2838}$$

3.283. $\int \frac{(e+fx)^3\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{3}{4}\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx-\frac{f^2\left(\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\operatorname{sech}(c+dx)}{2d^2}+}{2d^2}}{a}$$

$$\frac{i\int (e+fx)^3\operatorname{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$\frac{\frac{3}{4}\left(-\frac{3f^2\int (e+fx)\operatorname{sech}(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^3\operatorname{sech}(c+dx)dx+\frac{3f(e+fx)^2\operatorname{sech}(c+dx)}{2d^2}+\frac{(e+fx)^3\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}\right)-}{}$$

$$\frac{i\int (e+fx)^3\operatorname{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\left(-\frac{3f^2\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3f(e+fx)^2\operatorname{sech}(c+dx)}{2d^2}+\frac{(e+fx)^3\tanh(c+dx)\operatorname{sech}(}{2d}\right.}{}$$

$$\frac{i\int (e+fx)^3\operatorname{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4668}$$

$$\frac{\frac{3}{4}\left(-\frac{3f^2\left(-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(-\frac{3if\int (e+fx)^2\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{3if\int (e+fx)^2\log}{}\right.\right.}{}$$

$$\frac{i\int (e+fx)^3\operatorname{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{3}{4}\left(-\frac{3f^2\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)}{d^2}+\frac{1}{2}\left(-\frac{3if\int (e+fx)^2\log\left(1-ie^{c+}\right.}{d}\right.\right.}{}$$

$$\frac{i\int (e+fx)^3\operatorname{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2838}$$

3.283. $\int \frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\frac{3if\int(e+fx)^2\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{3if\int(e+fx)^2\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^3\arctan\left(e^{c+dx}\right)}{d}\right)-\frac{3f^2\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if}{}\right.}{}\right.}{}$$

$$\frac{i\int(e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\int(e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{5974}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\left(\frac{3f\int(e+fx)^2\text{sech}^4(c+dx)dx}{4d}-\frac{(e+fx)^3\text{sech}^4(c+dx)}{4d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\left(-\frac{(e+fx)^3\text{sech}^4(c+dx)}{4d}+\frac{3f\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^4dx}{4d}\right)}{a}$$

$$\Big\downarrow \text{4674}$$

3.283. $\int \frac{(e+fx)^3\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right.}{d}\right.}{d}\right.\right.$$

$$\frac{i\left(\frac{3f\left(-\frac{f^2\int\operatorname{sech}^2(c+dx)dx}{3d^2}+\frac{2}{3}\int(e+fx)^2\operatorname{sech}^2(c+dx)dx+\frac{f(e+fx)\operatorname{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{4d}-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}\right.}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right.}{d}\right.}{d}\right.\right.$$

$$\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}+\frac{3f\left(-\frac{f^2\int\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{3d^2}+\frac{2}{3}\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx+\frac{f(e+fx)\operatorname{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}^2}{3d}\right.}{4d}\right.}{a}$$

$$\Big\downarrow \ 4254$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right.}{d}\right.}{d}\right.\right.$$

$$\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}+\frac{3f\left(-\frac{if^2\int 1d(-i\tanh(c+dx))}{3d^3}+\frac{2}{3}\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx+\frac{f(e+fx)\operatorname{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}^2}{3d}\right.}{4d}\right.}{a}$$

$$\Big\downarrow \ 24$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right.}{d}\right.}{d}\right.\right.$$

$$\frac{i\left(-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}+\frac{3f\left(\frac{2}{3}\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\operatorname{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{4d}\right)}{a}$$

3.283. $\displaystyle\int\frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$\downarrow$ 4672

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}+\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2if\int -i(e+fx)\tanh(c+dx)dx}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\operatorname{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c\right.}{4d}\right.}{a}$$

$\downarrow$ 26

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.\right.}{i\left(\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int(e+fx)\tanh(c+dx)dx}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\operatorname{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{4d}-\frac{(e+fx)^3\operatorname{s}}{}\right.}{a}$$

$\downarrow$ 3042

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}+\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int -i(e+fx)\tan(ic+idx)dx}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\operatorname{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c\right.}{4d}\right.}{a}$$

$\downarrow$ 26

3.283. $\displaystyle\int \frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+d}\right.}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\left(-\frac{(e+fx)^3\text{sech}^4(c+dx)}{4d}+\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\int(e+fx)\tan(ic+idx)dx}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e+fx)^2\tanh(c+d}{3d}\right.}{4d}\right.}{a}$$

$\downarrow$ 4201

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+d}\right.}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\left(-\frac{(e+fx)^3\text{sech}^4(c+dx)}{4d}+\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\int\frac{e^{2(c+dx)}(e+fx)}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}\right)-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3d^2}+\frac{(e}{}\right.}{4d}\right.}{a}$$

$\downarrow$ 2620

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+d}\right.}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\left(-\frac{(e+fx)^3\text{sech}^4(c+dx)}{4d}+\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int\log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)-\frac{f^2\tanh(c+}{3d^3}\right.}{4d}\right.}{a}$$

$\downarrow$ 2715

3.283. $\int \frac{(e+fx)^3\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}+\frac{3f\left(\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int e^{-2(c+dx)}\log\left(1+e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)-\frac{i(e+fx)}{2f}\right.}{d}\right.\right.}{4d}\right.}$$

$$a$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right.}{d}\right.}{d}\right.\right.}{i\left(-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}+\frac{3f\left(-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx}{2f}\right.}{d}\right.\right.}{4d}\right.}$$

$$a$$

$$\Big\downarrow \text{7163}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{d}\right.}{d}\right.}{}\right.\right.}{i\left(-\frac{(e+fx)^3\operatorname{sech}^4(c+dx)}{4d}+\frac{3f\left(-\frac{f^2\tanh(c+dx)}{3d^3}+\frac{2}{3}\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx}{2f}\right.}{d}\right.\right.}{4d}\right.}$$

$$a$$

3.283. $\int \frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^3*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.283.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.283. $\int \frac{(e+fx)^3\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

3.283. $\int \frac{(e+fx)^3\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.283.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1908 vs. $2(589) = 1178$.

Time = 188.52 (sec) , antiderivative size = 1909, normalized size of antiderivative = 2.86

method	result	size
risch	Expression too large to display	1909

input
```
int((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-9/4*I*f^3*polylog(4,-I*exp(d*x+c))/a/d^4+9/8*I/a/d*e*f^2*ln(1-I*exp(d*x+c
))*x^2+9/4*I/a/d^2*e*f^2*polylog(2,I*exp(d*x+c))*x+9/4*I*f^3*polylog(4,I*e
xp(d*x+c))/a/d^4+9/8*I/a/d^2*e^2*f*ln(1-I*exp(d*x+c))*c-9/8*I/a/d*e^2*f*ln
(1+I*exp(d*x+c))*x-9/8*I/a/d^2*e^2*f*ln(1+I*exp(d*x+c))*c-9/4*I/a/d^2*e*f^
2*polylog(2,-I*exp(d*x+c))*x-9/8*I/a/d*e*f^2*ln(1+I*exp(d*x+c))*x^2+1/4*(-
8*I*d^2*e*f^2*x-2*d*f^3*x*exp(d*x+c)-2*d*e*f^2*exp(d*x+c)+2*d^3*e^3*exp(3*
d*x+3*c)+3*d^3*e^3*exp(5*d*x+5*c)+2*I*f^3*exp(4*d*x+4*c)+4*I*f^3*exp(2*d*x
+2*c)-2*d^2*e*f^2*x*exp(d*x+c)+9*d^3*e*f^2*x^2*exp(d*x+c)-18*I*d^3*e*f^2*x
^2*exp(4*d*x+4*c)+2*I*f^3+3*d^3*f^3*x^3*exp(5*d*x+5*c)-2*d*f^3*x*exp(5*d*x
+5*c)-2*d*e*f^2*exp(5*d*x+5*c)+2*d^3*f^3*x^3*exp(3*d*x+3*c)+9*d^2*f^3*x^2*
exp(5*d*x+5*c)+9*d^2*e^2*f*exp(5*d*x+5*c)-18*I*d^3*e^2*f*x*exp(4*d*x+4*c)-
18*I*d^2*e^2*f*exp(4*d*x+4*c)-4*I*d^2*e^2*f-4*I*d^2*f^3*x^2+6*I*d^3*f^3*x^
3*exp(2*d*x+2*c)-18*I*d^2*f^3*x^2*exp(4*d*x+4*c)+18*d^2*e*f^2*x*exp(5*d*x+
5*c)+6*d^3*e*f^2*x^2*exp(3*d*x+3*c)+6*d^3*e^2*f*x*exp(3*d*x+3*c)+9*d^3*e*f
^2*x^2*exp(5*d*x+5*c)+9*d^3*e^2*f*x*exp(5*d*x+5*c)+8*d^2*f^3*x^2*exp(3*d*x
+3*c)+18*I*d^3*e*f^2*x^2*exp(2*d*x+2*c)-36*I*d^2*e*f^2*x*exp(4*d*x+4*c)-44
*I*d^2*e*f^2*x*exp(2*d*x+2*c)-22*I*d^2*f^3*x^2*exp(2*d*x+2*c)-22*I*d^2*e^2
*f*exp(2*d*x+2*c)-6*I*d^3*f^3*x^3*exp(4*d*x+4*c)+18*I*d^3*e^2*f*x*exp(2*d*
x+2*c)+9*d^3*e^2*f*x*exp(d*x+c)-6*I*d^3*e^3*exp(4*d*x+4*c)+6*I*d^3*e^3*exp
(2*d*x+2*c)-2*f^3*exp(d*x+c)-4*f^3*exp(3*d*x+3*c)-2*f^3*exp(5*d*x+5*c)+...
```

3.283.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3854 vs. $2(560) = 1120$.

Time = 0.29 (sec) , antiderivative size = 3854, normalized size of antiderivative = 5.78

$$\int \frac{(e+fx)^3\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/8*(-8*I*d^2*e^2*f + 16*I*c*d*e*f^2 - 4*(2*I*c^2 - I)*f^3 - 3*(3*I*d^2*f^
3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^2*f - 4*I*f^3 + (-3*I*d^2*f^3*x^2 - 6*
I*d^2*e*f^2*x - 3*I*d^2*e^2*f + 4*I*f^3)*e^(6*d*x + 6*c) - 2*(3*d^2*f^3*x^
2 + 6*d^2*e*f^2*x + 3*d^2*e^2*f - 4*f^3)*e^(5*d*x + 5*c) + (-3*I*d^2*f^3*x
^2 - 6*I*d^2*e*f^2*x - 3*I*d^2*e^2*f + 4*I*f^3)*e^(4*d*x + 4*c) - 4*(3*d^2
*f^3*x^2 + 6*d^2*e*f^2*x + 3*d^2*e^2*f - 4*f^3)*e^(3*d*x + 3*c) + (3*I*d^2
*f^3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^2*f - 4*I*f^3)*e^(2*d*x + 2*c) - 2*
(3*d^2*f^3*x^2 + 6*d^2*e*f^2*x + 3*d^2*e^2*f - 4*f^3)*e^(d*x + c))*dilog(I
*e^(d*x + c)) + (9*I*d^2*f^3*x^2 + 18*I*d^2*e*f^2*x + 9*I*d^2*e^2*f - 28*I
*f^3 + (-9*I*d^2*f^3*x^2 - 18*I*d^2*e*f^2*x - 9*I*d^2*e^2*f + 28*I*f^3)*e^
(6*d*x + 6*c) - 2*(9*d^2*f^3*x^2 + 18*d^2*e*f^2*x + 9*d^2*e^2*f - 28*f^3)*
e^(5*d*x + 5*c) + (-9*I*d^2*f^3*x^2 - 18*I*d^2*e*f^2*x - 9*I*d^2*e^2*f + 2
8*I*f^3)*e^(4*d*x + 4*c) - 4*(9*d^2*f^3*x^2 + 18*d^2*e*f^2*x + 9*d^2*e^2*f
 - 28*f^3)*e^(3*d*x + 3*c) + (9*I*d^2*f^3*x^2 + 18*I*d^2*e*f^2*x + 9*I*d^2
*e^2*f - 28*I*f^3)*e^(2*d*x + 2*c) - 2*(9*d^2*f^3*x^2 + 18*d^2*e*f^2*x + 9
*d^2*e^2*f - 28*f^3)*e^(d*x + c))*dilog(-I*e^(d*x + c)) - 8*(I*d^2*f^3*x^2
 + 2*I*d^2*e*f^2*x + 2*I*c*d*e*f^2 - I*c^2*f^3)*e^(6*d*x + 6*c) + 2*(3*d^3
*f^3*x^3 + 3*d^3*e^3 + 9*d^2*e^2*f - 2*(8*c + 1)*d*e*f^2 + 2*(4*c^2 - 1)*f
^3 + (9*d^3*e*f^2 + d^2*f^3)*x^2 + (9*d^3*e^2*f + 2*d^2*e*f^2 - 2*d*f^3)*x
)*e^(5*d*x + 5*c) - 4*(3*I*d^3*f^3*x^3 + 3*I*d^3*e^3 + 9*I*d^2*e^2*f + ...
```

3.283.6 Sympy [F]

$$\int \frac{(e+fx)^3 \operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= -\frac{i\left(\int \frac{e^3\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{f^3x^3\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{3ef^2x^2\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{3e^2fx\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**3*sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**3*sech(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(f**3
*x**3*sech(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(3*e*f**2*x**2*se
ch(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(3*e**2*f*x*sech(c + d*x)
**3/(sinh(c + d*x) - I), x))/a
```

3.283.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3 \mathrm{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.283.8 Giac [F]

$$\int \frac{(e+fx)^3 \mathrm{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\mathrm{sech}\,(dx+c)^3}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sech(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \mathrm{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\cosh\,(c+d\,x)^3\,\left(a+a\sinh\,(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

3.284 $\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.284.1 Optimal result

Integrand size = 31, antiderivative size = 423

$$\begin{aligned}\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = {} & \frac{3(e+fx)^2\arctan\left(e^{c+dx}\right)}{4ad}-\frac{5f^2\arctan(\sinh(c+dx))}{6ad^3} \\ & +\frac{if^2\log(\cosh(c+dx))}{3ad^3}-\frac{3if(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{4ad^2} \\ & +\frac{3if(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{4ad^2} \\ & +\frac{3if^2\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{4ad^3}-\frac{3if^2\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{4ad^3} \\ & +\frac{3f(e+fx)\text{sech}(c+dx)}{4ad^2}-\frac{if^2\text{sech}^2(c+dx)}{12ad^3} \\ & +\frac{f(e+fx)\text{sech}^3(c+dx)}{6ad^2}+\frac{i(e+fx)^2\text{sech}^4(c+dx)}{4ad} \\ & -\frac{if(e+fx)\tanh(c+dx)}{3ad^2}-\frac{f^2\text{sech}(c+dx)\tanh(c+dx)}{12ad^3} \\ & +\frac{3(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{8ad} \\ & -\frac{if(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{6ad^2} \\ & +\frac{(e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)}{4ad}\end{aligned}$$

output
```
3/4*(f*x+e)^2*arctan(exp(d*x+c))/a/d-5/6*f^2*arctan(sinh(d*x+c))/a/d^3-1/1
2*I*f^2*sech(d*x+c)^2/a/d^3+3/4*I*f*(f*x+e)*polylog(2,I*exp(d*x+c))/a/d^2+
1/3*I*f^2*ln(cosh(d*x+c))/a/d^3+3/4*I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3-1
/3*I*f*(f*x+e)*tanh(d*x+c)/a/d^2+3/4*f*(f*x+e)*sech(d*x+c)/a/d^2-3/4*I*f^2
*polylog(3,I*exp(d*x+c))/a/d^3+1/6*f*(f*x+e)*sech(d*x+c)^3/a/d^2+1/4*I*(f*
x+e)^2*sech(d*x+c)^4/a/d-3/4*I*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^2-1/
12*f^2*sech(d*x+c)*tanh(d*x+c)/a/d^3+3/8*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)
/a/d-1/6*I*f*(f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/a/d^2+1/4*(f*x+e)^2*sech(d*
x+c)^3*tanh(d*x+c)/a/d
```

3.284.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1363 vs. $2(423) = 846$.

Time = 8.18 (sec) , antiderivative size = 1363, normalized size of antiderivative = 3.22

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
-1/8*(E^c*((3*d^2*e^2*x)/E^c - (4*f^2*x)/E^c - ((1 - I*E^c)*(3*d^2*e^2 - 4
*f^2)*x)/E^c + (3*d^2*e*f*x^2)/E^c + (d^2*f^2*x^3)/E^c + (6*d*e*(1 - I*E^c
)*f*x*Log[1 + I*E^(-c - d*x)])/E^c + (3*d*(1 - I*E^c)*f^2*x^2*Log[1 + I*E^
(-c - d*x)])/E^c + ((1 - I*E^c)*(3*d^2*e^2 - 4*f^2)*Log[I + E^(c + d*x)])/
(d*E^c) - (6*e*(1 - I*E^c)*f*PolyLog[2, (-I)*E^(-c - d*x)])/E^c - (6*(1 -
I*E^c)*f^2*x*PolyLog[2, (-I)*E^(-c - d*x)])/E^c - (6*(1 - I*E^c)*f^2*PolyL
og[3, (-I)*E^(-c - d*x)])/(d*E^c)))/(a*d^2*(I + E^c)) - (9*d^2*e^2*x - 28*
f^2*x - (1 + I*E^c)*(9*d^2*e^2 - 28*f^2)*x + 9*d^2*e*f*x^2 + 3*d^2*f^2*x^3
 + 18*d*e*(1 + I*E^c)*f*x*Log[1 - I*E^(-c - d*x)] + 9*d*(1 + I*E^c)*f^2*x^
2*Log[1 - I*E^(-c - d*x)] + ((1 + I*E^c)*(9*d^2*e^2 - 28*f^2)*Log[I - E^(c
 + d*x)])/d - 18*e*(1 + I*E^c)*f*PolyLog[2, I*E^(-c - d*x)] - 18*(1 + I*E^
c)*f^2*x*PolyLog[2, I*E^(-c - d*x)] - (18*(1 + I*E^c)*f^2*PolyLog[3, I*E^(
-c - d*x)])/d)/(24*a*d^2*(-I + E^c)) + ((3*e^2*x*Cosh[c])/(4*a) + (3*e^2*x
*Sinh[c])/(4*a))/(1 + Cosh[2*c] + Sinh[2*c]) + ((3*e*f*x^2*Cosh[c])/(4*a)
+ (3*e*f*x^2*Sinh[c])/(4*a))/(1 + Cosh[2*c] + Sinh[2*c]) + ((f^2*x^3*Cosh[
c])/(4*a) + (f^2*x^3*Sinh[c])/(4*a))/(1 + Cosh[2*c] + Sinh[2*c]) - ((I/8)*
(e^2 + 2*e*f*x + f^2*x^2))/(a*d*(Cosh[c/2 + (d*x)/2] - I*Sinh[c/2 + (d*x)/
2])^2) + ((I/2)*(e*f*Sinh[(d*x)/2] + f^2*x*Sinh[(d*x)/2]))/(a*d^2*(Cosh[c/
2] - I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] - I*Sinh[c/2 + (d*x)/2])) + ((I/8)*
(e^2 + 2*e*f*x + f^2*x^2))/(a*d*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*...
```

3.284.3 Rubi [A] (verified)

Time = 2.83 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.95, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.742$, Rules used = {6105, 3042, 4674, 3042, 4255, 3042, 4257, 4674, 3042, 4257, 4668, 3011, 2720, 5974, 3042, 4673, 3042, 4672, 26, 3042, 26, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}dx$$
$$\Big\downarrow \text{6105}$$
$$\frac{\int (e+fx)^2\text{sech}^5(c+dx)dx}{a}-\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^5dx}{a}-\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$\frac{-\frac{f^2\int \operatorname{sech}^3(c+dx)dx}{6d^2}+\frac{3}{4}\int (e+fx)^2\operatorname{sech}^3(c+dx)dx+\frac{f(e+fx)\operatorname{sech}^3(c+dx)}{6d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}}{a}-\frac{i\int (e+fx)^2\operatorname{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f^2\int \csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{6d^2}+\frac{3}{4}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{f(e+fx)\operatorname{sech}^3(c+dx)}{6d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}}{a}-\frac{i\int (e+fx)^2\operatorname{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4255}$$

$$\frac{-\frac{f^2\left(\frac{1}{2}\int \operatorname{sech}(c+dx)dx+\frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}\right)}{6d^2}+\frac{3}{4}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{f(e+fx)\operatorname{sech}^3(c+dx)}{6d^2}+\frac{(e+fx)^2\tan}{}}{a}\frac{i\int (e+fx)^2\operatorname{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f^2\left(\frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}+\frac{1}{2}\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx\right)}{6d^2}+\frac{3}{4}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{f(e+fx)\operatorname{sech}^3(c+dx)}{6d^2}+\frac{(e+fx)^2}{}}{a}\frac{i\int (e+fx)^2\operatorname{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{3}{4}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx-\frac{f^2\left(\frac{\arctan(\sinh(c+dx))}{2d}+\frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}\right)}{6d^2}+\frac{f(e+fx)\operatorname{sech}^3(c+dx)}{6d^2}+\frac{(e+fx)^2\tanh(c-}{}}{a}\frac{i\int (e+fx)^2\operatorname{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4674}$$

$$\frac{\frac{3}{4}\left(-\frac{f^2\int \operatorname{sech}(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\operatorname{sech}(c+dx)dx+\frac{f(e+fx)\operatorname{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}\right)-\frac{f^2\left(\arctan\right.}{}}{a}\frac{i\int (e+fx)^2\operatorname{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

3.284. $\int \frac{(e+fx)^2\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3}{4}\left(-\frac{f^2\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)-}{a}$$

$$\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 4257$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)-\frac{f}{}}{a}$$

$$\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 4668$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\frac{2if\int (e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}\right)-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{f(e+fx}{}}{}$$

$$\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 3011$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+\frac{2(e+fx)}{}\right.\right.}{}$$

$$\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 2720$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}(}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \; 5974$$

3.284. $\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\left(\frac{f\int(e+fx)\operatorname{sech}^4(c+dx)dx}{2d}-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}\right)}{a}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\left(-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}+\frac{f\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^4dx}{2d}\right)}{a}$$

$$\Big\downarrow \color{blue}{4673}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\left(\frac{f\left(\frac{2}{3}\int(e+fx)\operatorname{sech}^2(c+dx)dx+\frac{f\operatorname{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{2d}-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}\right)}{a}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\left(-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}+\frac{f\left(\frac{2}{3}\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx+\frac{f\operatorname{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \color{blue}{4672}$$

3.284. $\quad \int \frac{(e+fx)^2\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\left(-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}+\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int -i\tanh(c+dx)dx}{d}\right)+\frac{f\operatorname{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\left(\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int\tanh(c+dx)dx}{d}\right)+\frac{f\operatorname{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{2d}-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\left(-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}+\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int -i\tan(ic+idx)dx}{d}\right)+\frac{f\operatorname{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}(}{d}\right.}{d}\right.\right.}{}$$

$$\frac{i\left(-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}+\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int\tan(ic+idx)dx}{d}\right)+\frac{f\operatorname{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{2d}\right)}{a}$$

3.284. $\displaystyle\int \frac{(e+fx)^2\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(\right.}{d}\right.}{d}\right.\right.}{i\left(\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)+\frac{f\operatorname{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{2d}-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}\right)}$$

$$a$$

$$\Big\downarrow \text{7143}$$

$$\frac{-\frac{f^2\left(\frac{\arctan(\sinh(c+dx))}{2d}+\frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}\right)}{6d^2}+\frac{3}{4}\left(-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{1}{2}\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+}\right.}{d^2}\right.}{}\right.\right.}{i\left(\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)+\frac{f\operatorname{sech}^2(c+dx)}{6d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}\right)}{2d}-\frac{(e+fx)^2\operatorname{sech}^4(c+dx)}{4d}\right)}$$

$$a$$

input
```
Int[((e + f*x)^2*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((f*(e + f*x)*Sech[c + d*x]^3)/(6*d^2) + ((e + f*x)^2*Sech[c + d*x]^3*Tanh
[c + d*x])/(4*d) - (f^2*(ArcTan[Sinh[c + d*x]]/(2*d) + (Sech[c + d*x]*Tanh
[c + d*x])/(2*d)))/(6*d^2) + (3*(-((f^2*ArcTan[Sinh[c + d*x]])/d^3) + ((2*
(e + f*x)^2*ArcTan[E^(c + d*x)])/d + ((2*I)*f*(-(((e + f*x)*PolyLog[2, (-I
)*E^(c + d*x)])/d) + (f*PolyLog[3, (-I)*E^(c + d*x)])/d^2))/d - ((2*I)*f*(
-(((e + f*x)*PolyLog[2, I*E^(c + d*x)])/d) + (f*PolyLog[3, I*E^(c + d*x)])
/d^2))/d)/2 + (f*(e + f*x)*Sech[c + d*x])/d^2 + ((e + f*x)^2*Sech[c + d*x]
*Tanh[c + d*x])/(2*d)))/4)/a - (I*(-1/4*((e + f*x)^2*Sech[c + d*x]^4)/d +
(f*((f*Sech[c + d*x]^2)/(6*d^2) + ((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x]
)/(3*d) + (2*(-((f*Log[Cosh[c + d*x]])/d^2) + ((e + f*x)*Tanh[c + d*x])/d)
)/3))/(2*d)))/a
```

3.284. $\int \frac{(e+fx)^2\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.284.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.284. $\int \frac{(e+fx)^2\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

3.284. $\int \frac{(e+fx)^2 \operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.284.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 960 vs. $2(373) = 746$.

Time = 87.45 (sec) , antiderivative size = 961, normalized size of antiderivative = 2.27

method	result
risch	$\frac{-4f^2 e^{3dx+3c}+9d^2e^2 e^{5dx+5c}+18d^2 efx\, e^{dx+c}-2f^2 e^{dx+c}+9d^2x^2f^2 e^{dx+c}+9d^2e^2 e^{dx+c}+16d\, f^2x\, e^{3dx+3c}+18d^2efx\, e^{5dx+5c}+6d^2e^2 e}{}$

input
```
int((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-4*f^2*exp(3*d*x+3*c)+36*I*d^2*e*f*x*exp(2*d*x+2*c)-36*I*d^2*e*f*x*e
xp(4*d*x+4*c)+9*d^2*e^2*exp(5*d*x+5*c)+18*d^2*e*f*x*exp(d*x+c)-2*f^2*exp(d
*x+c)+9*d^2*x^2*f^2*exp(d*x+c)+9*d^2*e^2*exp(d*x+c)+18*I*d^2*f^2*x^2*exp(2
*d*x+2*c)-36*I*d*f^2*x*exp(4*d*x+4*c)+16*d*f^2*x*exp(3*d*x+3*c)+18*d^2*e*f
*x*exp(5*d*x+5*c)+6*d^2*e^2*exp(3*d*x+3*c)-2*f^2*exp(5*d*x+5*c)-2*exp(d*x+
c)*d*f^2*x-2*exp(d*x+c)*d*e*f-36*I*d*e*f*exp(4*d*x+4*c)+18*I*d^2*e^2*exp(2
*d*x+2*c)+6*d^2*f^2*x^2*exp(3*d*x+3*c)-44*I*d*e*f*exp(2*d*x+2*c)-18*I*d^2*
f^2*x^2*exp(4*d*x+4*c)+9*d^2*f^2*x^2*exp(5*d*x+5*c)+18*d*f^2*x*exp(5*d*x+5
*c)+18*d*e*f*exp(5*d*x+5*c)-18*I*d^2*e^2*exp(4*d*x+4*c)-8*I*d*e*f-8*I*d*f^
2*x+12*d^2*e*f*x*exp(3*d*x+3*c)-44*I*d*f^2*x*exp(2*d*x+2*c)+16*d*e*f*exp(3
*d*x+3*c))/(exp(d*x+c)+I)^2/(exp(d*x+c)-I)^4/d^3/a+3/4*I/a/d^2*polylog(2,I
*exp(d*x+c))*f^2*x+3/4*I/a/d*ln(1-I*exp(d*x+c))*e*f*x+3/4/a/d*e^2*arctan(e
xp(d*x+c))+3/4/a/d^3*c^2*f^2*arctan(exp(d*x+c))-3/4*I/a/d*ln(1+I*exp(d*x+c
))*e*f*x+3/8*I/a/d*ln(1-I*exp(d*x+c))*f^2*x^2-3/8*I/a/d*ln(1+I*exp(d*x+c))
*f^2*x^2+3/8*I/a/d^3*ln(1+I*exp(d*x+c))*c^2*f^2-3/2/a/d^2*c*e*f*arctan(exp
(d*x+c))-3/4*I/a/d^2*ln(1+I*exp(d*x+c))*c*e*f-3/4*I/a/d^2*polylog(2,-I*exp
(d*x+c))*f^2*x+3/4*I/a/d^2*e*f*polylog(2,I*exp(d*x+c))-2/3*I/a/d^3*f^2*ln(
exp(d*x+c))+1/3*I/a/d^3*f^2*ln(1+exp(2*d*x+2*c))-3/4*I*f^2*polylog(3,I*exp
(d*x+c))/a/d^3-3/4*I/a/d^2*e*f*polylog(2,-I*exp(d*x+c))+3/4*I/a/d^2*ln(1-I
*exp(d*x+c))*c*e*f-3/8*I/a/d^3*ln(1-I*exp(d*x+c))*c^2*f^2+3/4*I*f^2*pol...
```

3.284.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2066 vs. $2(358) = 716$.

Time = 0.29 (sec) , antiderivative size = 2066, normalized size of antiderivative = 4.88

$$\int \frac{(e+fx)^2 \operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/24*(-16*I*d*e*f + 16*I*c*f^2 - 18*(I*d*f^2*x + I*d*e*f + (-I*d*f^2*x - I
*d*e*f)*e^(6*d*x + 6*c) - 2*(d*f^2*x + d*e*f)*e^(5*d*x + 5*c) + (-I*d*f^2*
x - I*d*e*f)*e^(4*d*x + 4*c) - 4*(d*f^2*x + d*e*f)*e^(3*d*x + 3*c) + (I*d*
f^2*x + I*d*e*f)*e^(2*d*x + 2*c) - 2*(d*f^2*x + d*e*f)*e^(d*x + c))*dilog(
I*e^(d*x + c)) - 18*(-I*d*f^2*x - I*d*e*f + (I*d*f^2*x + I*d*e*f)*e^(6*d*x
 + 6*c) + 2*(d*f^2*x + d*e*f)*e^(5*d*x + 5*c) + (I*d*f^2*x + I*d*e*f)*e^(4
*d*x + 4*c) + 4*(d*f^2*x + d*e*f)*e^(3*d*x + 3*c) + (-I*d*f^2*x - I*d*e*f)
*e^(2*d*x + 2*c) + 2*(d*f^2*x + d*e*f)*e^(d*x + c))*dilog(-I*e^(d*x + c))
- 16*(I*d*f^2*x + I*c*f^2)*e^(6*d*x + 6*c) + 2*(9*d^2*f^2*x^2 + 9*d^2*e^2
+ 18*d*e*f - 2*(8*c + 1)*f^2 + 2*(9*d^2*e*f + d*f^2)*x)*e^(5*d*x + 5*c) -
4*(9*I*d^2*f^2*x^2 + 9*I*d^2*e^2 + 18*I*d*e*f + 4*I*c*f^2 + 2*(9*I*d^2*e*f
 + 11*I*d*f^2)*x)*e^(4*d*x + 4*c) + 4*(3*d^2*f^2*x^2 + 3*d^2*e^2 + 8*d*e*f
 - 2*(8*c + 1)*f^2 + 2*(3*d^2*e*f - 4*d*f^2)*x)*e^(3*d*x + 3*c) - 4*(-9*I*
d^2*f^2*x^2 - 9*I*d^2*e^2 + 22*I*d*e*f - 4*I*c*f^2 + 18*(-I*d^2*e*f + I*d*
f^2)*x)*e^(2*d*x + 2*c) + 2*(9*d^2*f^2*x^2 + 9*d^2*e^2 - 2*d*e*f - 2*(8*c
+ 1)*f^2 + 18*(d^2*e*f - d*f^2)*x)*e^(d*x + c) - 3*(3*I*d^2*e^2 - 6*I*c*d*
e*f + (3*I*c^2 - 4*I)*f^2 + (-3*I*d^2*e^2 + 6*I*c*d*e*f + (-3*I*c^2 + 4*I)
*f^2)*e^(6*d*x + 6*c) - 2*(3*d^2*e^2 - 6*c*d*e*f + (3*c^2 - 4)*f^2)*e^(5*d
*x + 5*c) + (-3*I*d^2*e^2 + 6*I*c*d*e*f + (-3*I*c^2 + 4*I)*f^2)*e^(4*d*x +
 4*c) - 4*(3*d^2*e^2 - 6*c*d*e*f + (3*c^2 - 4)*f^2)*e^(3*d*x + 3*c) + (...
```

3.284.6 Sympy [F]

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e^2\,\text{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{f^2x^2\,\text{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{2efx\,\text{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)**2*sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e**2*sech(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(f**2
*x**2*sech(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*sech(c +
 d*x)**3/(sinh(c + d*x) - I), x))/a
```

3.284.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.284.8 Giac [F]

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}\,(dx+c)^3}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sech(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\int \frac{{\left(e+f\,x\right)}^2}{{\mathrm{cosh}\left(c+d\,x\right)}^3\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

input
```
int((e + f*x)^2/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

3.285 $\int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.285.1 Optimal result

Integrand size = 29, antiderivative size = 233

$$\begin{aligned}\int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{3(e+fx)\arctan\left(e^{c+dx}\right)}{4ad}-\frac{3if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{8ad^2}\\&\quad+\frac{3if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{8ad^2}+\frac{3f\operatorname{sech}(c+dx)}{8ad^2}\\&\quad+\frac{f\operatorname{sech}^3(c+dx)}{12ad^2}+\frac{i(e+fx)\operatorname{sech}^4(c+dx)}{4ad}\\&\quad-\frac{if\tanh(c+dx)}{4ad^2}+\frac{3(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{8ad}\\&\quad+\frac{(e+fx)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4ad}+\frac{if\tanh^3(c+dx)}{12ad^2}\end{aligned}$$

output
```
3/4*(f*x+e)*arctan(exp(d*x+c))/a/d-3/8*I*f*polylog(2,-I*exp(d*x+c))/a/d^2+
3/8*I*f*polylog(2,I*exp(d*x+c))/a/d^2+3/8*f*sech(d*x+c)/a/d^2+1/12*f*sech(
d*x+c)^3/a/d^2+1/4*I*(f*x+e)*sech(d*x+c)^4/a/d-1/4*I*f*tanh(d*x+c)/a/d^2+3
/8*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/a/d+1/4*(f*x+e)*sech(d*x+c)^3*tanh(d*x+
c)/a/d+1/12*I*f*tanh(d*x+c)^3/a/d^2
```

3.285.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 617 vs. 2(233) = 466.

Time = 4.02 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.65

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{2(f+6id(e+fx)) + \frac{6id(e+fx)}{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2} - 9(c+dx)(cf-d(2e+fx))\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sin\right.}{}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(2*(f + (6*I)*d*(e + f*x)) + ((6*I)*d*(e + f*x))/(Cosh[(c + d*x)/2] + I*Si
nh[(c + d*x)/2])^2 - 9*(c + d*x)*(c*f - d*(2*e + f*x))*(Cosh[(c + d*x)/2]
+ I*Sinh[(c + d*x)/2])^2 - (9 - 9*I)*((d^2*f*x^2)/2 + d*e*(c + d*x) - (1 -
 I)*(d*e - c*f)*(c + d*x) + (1 - I)*f*(c + d*x)*Log[1 + I*E^(-c - d*x)] +
(1 - I)*(d*e - c*f)*Log[I + E^(c + d*x)] - (1 - I)*f*PolyLog[2, (-I)*E^(-c
 - d*x)])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 - (9 + 9*I)*((d^2*f*
x^2)/2 + d*e*(c + d*x) - (1 + I)*(d*e - c*f)*(c + d*x) + (1 + I)*f*(c + d*
x)*Log[1 - I*E^(-c - d*x)] + (1 + I)*(d*e - c*f)*Log[I - E^(c + d*x)] - (1
 + I)*f*PolyLog[2, I*E^(-c - d*x)])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/
2])^2 - ((6*I)*d*(e + f*x)*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2)/(C
osh[(c + d*x)/2] - I*Sinh[(c + d*x)/2])^2 - ((4*I)*f*Sinh[(c + d*x)/2])/(C
osh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]) + ((12*I)*f*(Cosh[(c + d*x)/2] + I
*Sinh[(c + d*x)/2])^2*Sinh[(c + d*x)/2])/(Cosh[(c + d*x)/2] - I*Sinh[(c +
d*x)/2]) + 28*f*Sinh[(c + d*x)/2]*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)
/2]))/(48*d^2*(a + I*a*Sinh[c + d*x]))
```

3.285.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {6105, 3042, 4673, 3042, 4673, 3042, 4668, 2715, 2838, 5974, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6105}$$

$$\frac{\int (e+fx)\text{sech}^5(c+dx)dx}{a}-\frac{i\int (e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^5dx}{a}-\frac{i\int (e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\frac{3}{4}\int (e+fx)\text{sech}^3(c+dx)dx+\frac{f\text{sech}^3(c+dx)}{12d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^3(c+dx)}{4d}}{a}-\frac{i\int (e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{f\text{sech}^3(c+dx)}{12d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^3(c+dx)}{4d}}{a}-\frac{i\int (e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int (e+fx)\text{sech}(c+dx)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)+\frac{f\text{sech}^3(c+dx)}{12d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}^3(}{4d}}{a}$$
$$\frac{i\int (e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)+\frac{f\text{sech}^3(c+dx)}{12d^2}+\frac{(e+fx)\tanh(c+dx}{4d}}{a}$$
$$\frac{i\int (e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{4668}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)}{a}$$
$$\frac{i\int (e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

3.285. $\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\text{ta}}{}\right.}{a}$$

$$\frac{i\int(e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)-}{a}$$

$$\frac{i\int(e+fx)\text{sech}^4(c+dx)\tanh(c+dx)dx}{a}$$

$$\Big\downarrow \text{5974}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)-}{a}$$

$$\frac{i\left(\frac{f\int\text{sech}^4(c+dx)dx}{4d}-\frac{(e+fx)\text{sech}^4(c+dx)}{4d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)-}{a}$$

$$\frac{i\left(-\frac{(e+fx)\text{sech}^4(c+dx)}{4d}+\frac{f\int\csc\left(ic+idx+\frac{\pi}{2}\right)^4dx}{4d}\right)}{a}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)-}{a}$$

$$\frac{i\left(-\frac{(e+fx)\text{sech}^4(c+dx)}{4d}+\frac{if\int\left(1-\tanh^2(c+dx)\right)d(-i\tanh(c+dx))}{4d^2}\right)}{a}$$

$$\Big\downarrow \text{2009}$$

3.285. $\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{2(e+fx)\arctan(e^{c+dx})}{d}-\frac{if\,\mathrm{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{if\,\mathrm{PolyLog}(2,ie^{c+dx})}{d^2}\right)+\frac{f\mathrm{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}\right)-}{a}$$

$$\frac{i\left(-\frac{(e+fx)\mathrm{sech}^4(c+dx)}{4d}+\frac{if\left(\frac{1}{3}i\tanh^3(c+dx)-i\tanh(c+dx)\right)}{4d^2}\right)}{a}$$

input
```
Int[((e + f*x)*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((f*Sech[c + d*x]^3)/(12*d^2) + ((e + f*x)*Sech[c + d*x]^3*Tanh[c + d*x])/
(4*d) + (3*(((2*(e + f*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^
(c + d*x)])/d^2 + (I*f*PolyLog[2, I*E^(c + d*x)])/d^2)/2 + (f*Sech[c + d*x
])/(2*d^2) + ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d)))/4)/a - (I*(-1
/4*((e + f*x)*Sech[c + d*x]^4)/d + ((I/4)*f*((-I)*Tanh[c + d*x] + (I/3)*Ta
nh[c + d*x]^3))/d^2))/a
```

3.285.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.285. $\int \frac{(e+fx)\mathrm{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6105
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Sech[
c + d*x]^(n + 2), x], x] + Simp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)
*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] &&
EqQ[a^2 + b^2, 0]
```

3.285.4 Maple [A] (verified)

Time = 25.57 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.70

method	result
risch	$\frac{9dfx\,e^{5dx+5c}+6de\,e^{3dx+3c}-18ide\,e^{4dx+4c}+9de\,e^{5dx+5c}-18idfx\,e^{4dx+4c}-22if\,e^{2dx+2c}+9de\,e^{dx+c}-18if\,e^{4dx+4c}-4if+6dfx\,e^{3dx+3}}{12\left(e^{dx+c}+i\right)^{2}\left(e^{dx+c}-i\right)^{4}d^{2}a}$

input
```
int((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/12*(9*d*f*x*exp(5*d*x+5*c)+6*d*e*exp(3*d*x+3*c)-18*I*d*e*exp(4*d*x+4*c)+
9*d*e*exp(5*d*x+5*c)-18*I*d*f*x*exp(4*d*x+4*c)-22*I*f*exp(2*d*x+2*c)+9*d*e
*exp(d*x+c)-18*I*f*exp(4*d*x+4*c)-4*I*f+6*d*f*x*exp(3*d*x+3*c)+18*I*d*e*ex
p(2*d*x+2*c)+18*I*d*f*x*exp(2*d*x+2*c)+9*d*f*x*exp(d*x+c)+9*f*exp(5*d*x+5*
c)+8*f*exp(3*d*x+3*c)-f*exp(d*x+c))/(exp(d*x+c)+I)^2/(exp(d*x+c)-I)^4/d^2/
a+3/4/d/a*e*arctan(exp(d*x+c))+3/8*I/d/a*f*ln(1-I*exp(d*x+c))*x+3/8*I/d^2/
a*f*ln(1-I*exp(d*x+c))*c+3/8*I*f*polylog(2,I*exp(d*x+c))/a/d^2-3/8*I/d/a*f
*ln(1+I*exp(d*x+c))*x-3/8*I/d^2/a*f*ln(1+I*exp(d*x+c))*c-3/8*I*f*polylog(2
,-I*exp(d*x+c))/a/d^2-3/4/d^2/a*f*c*arctan(exp(d*x+c))
```

3.285.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 917 vs. $2(197) = 394$.

Time = 0.27 (sec) , antiderivative size = 917, normalized size of antiderivative = 3.94

$$\int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/24*(9*(-I*f*e^(6*d*x + 6*c) - 2*f*e^(5*d*x + 5*c) - I*f*e^(4*d*x + 4*c)
 - 4*f*e^(3*d*x + 3*c) + I*f*e^(2*d*x + 2*c) - 2*f*e^(d*x + c) + I*f)*dilo
g(I*e^(d*x + c)) + 9*(I*f*e^(6*d*x + 6*c) + 2*f*e^(5*d*x + 5*c) + I*f*e^(4
*d*x + 4*c) + 4*f*e^(3*d*x + 3*c) - I*f*e^(2*d*x + 2*c) + 2*f*e^(d*x + c)
- I*f)*dilog(-I*e^(d*x + c)) - 18*(d*f*x + d*e + f)*e^(5*d*x + 5*c) + 36*(
I*d*f*x + I*d*e + I*f)*e^(4*d*x + 4*c) - 4*(3*d*f*x + 3*d*e + 4*f)*e^(3*d*
x + 3*c) + 4*(-9*I*d*f*x - 9*I*d*e + 11*I*f)*e^(2*d*x + 2*c) - 2*(9*d*f*x
+ 9*d*e - f)*e^(d*x + c) + 9*(I*d*e - I*c*f + (-I*d*e + I*c*f)*e^(6*d*x +
6*c) - 2*(d*e - c*f)*e^(5*d*x + 5*c) + (-I*d*e + I*c*f)*e^(4*d*x + 4*c) -
4*(d*e - c*f)*e^(3*d*x + 3*c) + (I*d*e - I*c*f)*e^(2*d*x + 2*c) - 2*(d*e -
 c*f)*e^(d*x + c))*log(e^(d*x + c) + I) + 9*(-I*d*e + I*c*f + (I*d*e - I*c
*f)*e^(6*d*x + 6*c) + 2*(d*e - c*f)*e^(5*d*x + 5*c) + (I*d*e - I*c*f)*e^(4
*d*x + 4*c) + 4*(d*e - c*f)*e^(3*d*x + 3*c) + (-I*d*e + I*c*f)*e^(2*d*x +
2*c) + 2*(d*e - c*f)*e^(d*x + c))*log(e^(d*x + c) - I) + 9*(-I*d*f*x - I*c
*f + (I*d*f*x + I*c*f)*e^(6*d*x + 6*c) + 2*(d*f*x + c*f)*e^(5*d*x + 5*c) +
 (I*d*f*x + I*c*f)*e^(4*d*x + 4*c) + 4*(d*f*x + c*f)*e^(3*d*x + 3*c) + (-I
*d*f*x - I*c*f)*e^(2*d*x + 2*c) + 2*(d*f*x + c*f)*e^(d*x + c))*log(I*e^(d*
x + c) + 1) + 9*(I*d*f*x + I*c*f + (-I*d*f*x - I*c*f)*e^(6*d*x + 6*c) - 2*
(d*f*x + c*f)*e^(5*d*x + 5*c) + (-I*d*f*x - I*c*f)*e^(4*d*x + 4*c) - 4*(d*
f*x + c*f)*e^(3*d*x + 3*c) + (I*d*f*x + I*c*f)*e^(2*d*x + 2*c) - 2*(d*f...
```

3.285.6 Sympy [F]

$$\int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\left(\int \frac{e\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{fx\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate((f*x+e)*sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*(Integral(e*sech(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(f*x*sec
h(c + d*x)**3/(sinh(c + d*x) - I), x))/a
```

3.285.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.285.8 Giac [F]

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{sech}\,(dx+c)^3}{i\,a\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sech(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\,(c+d\,x)^3\,\left(a+a\sinh\,(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

3.286 $\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

3.286.1 Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{3\arctan(\sinh(c+dx))}{8ad} - \frac{i}{8d(a-ia\sinh(c+dx))} + \frac{ia}{8d(a+ia\sinh(c+dx))^2} + \frac{i}{4d(a+ia\sinh(c+dx))}$$

output
```
3/8*arctan(sinh(d*x+c))/a/d-1/8*I/d/(a-I*a*sinh(d*x+c))+1/8*I*a/d/(a+I*a*s
inh(d*x+c))^2+1/4*I/d/(a+I*a*sinh(d*x+c))
```

3.286.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\operatorname{sech}^2(c+dx)\left(2-3i\arctan(\sinh(c+dx))+3(-i+\arctan(\sinh(c+dx)))\sinh(c+dx)+(3-3i\arctan\right.}{8ad(-i+\sinh(c+dx))}$$

input
```
Integrate[Sech[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

output
```
(Sech[c + d*x]^2*(2 - (3*I)*ArcTan[Sinh[c + d*x]] + 3*(-I + ArcTan[Sinh[c
+ d*x]])*Sinh[c + d*x] + (3 - (3*I)*ArcTan[Sinh[c + d*x]])*Sinh[c + d*x]^2
 + 3*ArcTan[Sinh[c + d*x]]*Sinh[c + d*x]^3))/(8*a*d*(-I + Sinh[c + d*x]))
```

3.286.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)^3(a+a\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3146}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\sinh(c+dx))^2(i\sinh(c+dx)a+a)^3}d(ia\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^3\int \left(\frac{1}{8a^3(a-ia\sinh(c+dx))^2}+\frac{1}{4a^3(i\sinh(c+dx)a+a)^2}+\frac{1}{4a^2(i\sinh(c+dx)a+a)^3}+\frac{3}{8a^3(\sinh^2(c+dx)a^2+a^2)}\right)d(ia\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^3\left(\frac{3i\arctan(\sinh(c+dx))}{8a^4}+\frac{1}{8a^3(a-ia\sinh(c+dx))}-\frac{1}{4a^3(a+ia\sinh(c+dx))}-\frac{1}{8a^2(a+ia\sinh(c+dx))^2}\right)}{d}$$

input
```
Int[Sech[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

output
```
((-I)*a^3*((((3*I)/8)*ArcTan[Sinh[c + d*x]])/a^4 + 1/(8*a^3*(a - I*a*Sinh[
c + d*x])) - 1/(8*a^2*(a + I*a*Sinh[c + d*x])^2) - 1/(4*a^3*(a + I*a*Sinh[
c + d*x]))))/d
```

3.286.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.286.4 Maple [A] (verified)

Time = 24.69 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.37

method	result
risch	$\frac{6i\,\mathrm{e}^{2dx+2c}-6i\,\mathrm{e}^{4dx+4c}+3\,\mathrm{e}^{dx+c}+2\,\mathrm{e}^{3dx+3c}+3\,\mathrm{e}^{5dx+5c}}{4\left(\mathrm{e}^{dx+c}+i\right)^{2}\left(\mathrm{e}^{dx+c}-i\right)^{4}da}+\frac{3i\ln\left(\mathrm{e}^{dx+c}+i\right)}{8ad}-\frac{3i\ln\left(\mathrm{e}^{dx+c}-i\right)}{8ad}$
derivativedivides	$\frac{\frac{i}{4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}+\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}{8}-\frac{1}{4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{i}{2\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{3i\ln\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{}{2(-i}}{ad}$
default	$\frac{\frac{i}{4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}+\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}{8}-\frac{1}{4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{i}{2\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{3i\ln\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{}{2(-i}}{ad}$

input
```
int(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(6*I*exp(2*d*x+2*c)-6*I*exp(4*d*x+4*c)+3*exp(d*x+c)+2*exp(3*d*x+3*c)+3
*exp(5*d*x+5*c))/(exp(d*x+c)+I)^2/(exp(d*x+c)-I)^4/d/a+3/8*I/a/d*ln(exp(d*
x+c)+I)-3/8*I/a/d*ln(exp(d*x+c)-I)
```

3.286.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(71) = 142$.

Time = 0.25 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.15

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{3\left(-i\,e^{(6\,dx+6\,c)}-2\,e^{(5\,dx+5\,c)}-i\,e^{(4\,dx+4\,c)}-4\,e^{(3\,dx+3\,c)}+i\,e^{(2\,dx+2\,c)}-2\,e^{(dx+c)}+i\right)\log\left(e^{(dx+c)}+i\right)+3}{8\left(ade^{(6\,dx+6\,c)}-2\right.}$$

input
```
integrate(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/8*(3*(-I*e^(6*d*x + 6*c) - 2*e^(5*d*x + 5*c) - I*e^(4*d*x + 4*c) - 4*e^
(3*d*x + 3*c) + I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + I)*log(e^(d*x + c) + I
) + 3*(I*e^(6*d*x + 6*c) + 2*e^(5*d*x + 5*c) + I*e^(4*d*x + 4*c) + 4*e^(3*
d*x + 3*c) - I*e^(2*d*x + 2*c) + 2*e^(d*x + c) - I)*log(e^(d*x + c) - I) -
 6*e^(5*d*x + 5*c) + 12*I*e^(4*d*x + 4*c) - 4*e^(3*d*x + 3*c) - 12*I*e^(2*
d*x + 2*c) - 6*e^(d*x + c))/(a*d*e^(6*d*x + 6*c) - 2*I*a*d*e^(5*d*x + 5*c)
 + a*d*e^(4*d*x + 4*c) - 4*I*a*d*e^(3*d*x + 3*c) - a*d*e^(2*d*x + 2*c) - 2
*I*a*d*e^(d*x + c) - a*d)
```

3.286.6 Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int \frac{\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

input
```
integrate(sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)**3/(sinh(c + d*x) - I), x)/a
```

3.286.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.286.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(71) = 142$.

Time = 0.35 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.90

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{-\frac{6i\log\left(e^{(dx+c)}-e^{(-dx-c)}+2i\right)}{a} + \frac{6i\log\left(e^{(dx+c)}-e^{(-dx-c)}-2i\right)}{a} - \frac{2\left(3e^{(dx+c)}-3e^{(-dx-c)}+10i\right)}{a\left(ie^{(dx+c)}-ie^{(-dx-c)}-2\right)} + \frac{-9i\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-52e^{(d}}{a\left(e^{(dx+c)}-e^{(-dx-}\right.}}{32d}$$

input
```
integrate(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/32*(-6*I*log(e^(d*x + c) - e^(-d*x - c) + 2*I)/a + 6*I*log(e^(d*x + c)
- e^(-d*x - c) - 2*I)/a - 2*(3*e^(d*x + c) - 3*e^(-d*x - c) + 10*I)/(a*(I*
e^(d*x + c) - I*e^(-d*x - c) - 2)) + (-9*I*(e^(d*x + c) - e^(-d*x - c))^2
- 52*e^(d*x + c) + 52*e^(-d*x - c) + 84*I)/(a*(e^(d*x + c) - e^(-d*x - c)
- 2*I)^2))/d
```

3.286.9 Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.51

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx=\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{a^2\,d^2}}{a\,d}\right)}{4\,\sqrt{a^2\,d^2}}+\frac{1}{2\,a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}+\frac{1}{4\,a\,d\,\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)}-\frac{1\mathrm{i}}{4\,a\,d\,{\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)}^2}-\frac{1\mathrm{i}}{a\,d\,{\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}^3}+\frac{1\mathrm{i}}{2\,a\,d\,{\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}^4}$$

input
```
int(1/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
(3*atan((exp(d*x)*exp(c)*(a^2*d^2)^(1/2))/(a*d)))/(4*(a^2*d^2)^(1/2)) + 1/
(2*a*d*(exp(c + d*x) - 1i)) + 1/(4*a*d*(exp(c + d*x) + 1i)) - 1i/(4*a*d*(e
xp(c + d*x) + 1i)^2) - 1i/(a*d*(exp(c + d*x)*1i + 1)^3) + 1i/(2*a*d*(exp(c
 + d*x)*1i + 1)^4)
```

3.287 $\int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

3.287.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\text{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.287.2 Mathematica [N/A]

Not integrable

Time = 51.85 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

input
```
Integrate[Sech[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
Integrate[Sech[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

3.287.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

$$\Big\downarrow \ 6111$$

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.287.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.287.4 Maple [N/A] (verified)

Not integrable

Time = 0.78 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}\left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+ia\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

3.287.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 1783, normalized size of antiderivative = 57.52

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^3}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/12*(4*I*d^2*f^3*x^2 + 8*I*d^2*e*f^2*x + 4*I*d^2*e^2*f - 6*I*f^3 + (9*d^
3*f^3*x^3 + 9*d^3*e^3 - 9*d^2*e^2*f - 2*d*e*f^2 + 6*f^3 + 9*(3*d^3*e*f^2 -
 d^2*f^3)*x^2 + (27*d^3*e^2*f - 18*d^2*e*f^2 - 2*d*f^3)*x)*e^(5*d*x + 5*c)
 - 6*(3*I*d^3*f^3*x^3 + 3*I*d^3*e^3 - 3*I*d^2*e^2*f + I*f^3 + 3*(3*I*d^3*e
*f^2 - I*d^2*f^3)*x^2 + 3*(3*I*d^3*e^2*f - 2*I*d^2*e*f^2)*x)*e^(4*d*x + 4*
c) + 2*(3*d^3*f^3*x^3 + 3*d^3*e^3 - 4*d^2*e^2*f - 2*d*e*f^2 + 6*f^3 + (9*d
^3*e*f^2 - 4*d^2*f^3)*x^2 + (9*d^3*e^2*f - 8*d^2*e*f^2 - 2*d*f^3)*x)*e^(3*
d*x + 3*c) - 2*(-9*I*d^3*f^3*x^3 - 9*I*d^3*e^3 - 11*I*d^2*e^2*f + 6*I*f^3
+ (-27*I*d^3*e*f^2 - 11*I*d^2*f^3)*x^2 + (-27*I*d^3*e^2*f - 22*I*d^2*e*f^2
)*x)*e^(2*d*x + 2*c) + (9*d^3*f^3*x^3 + 9*d^3*e^3 + d^2*e^2*f - 2*d*e*f^2
+ 6*f^3 + (27*d^3*e*f^2 + d^2*f^3)*x^2 + (27*d^3*e^2*f + 2*d^2*e*f^2 - 2*d
*f^3)*x)*e^(d*x + c) - 12*(a*d^4*f^4*x^4 + 4*a*d^4*e*f^3*x^3 + 6*a*d^4*e^2
*f^2*x^2 + 4*a*d^4*e^3*f*x + a*d^4*e^4 - (a*d^4*f^4*x^4 + 4*a*d^4*e*f^3*x^
3 + 6*a*d^4*e^2*f^2*x^2 + 4*a*d^4*e^3*f*x + a*d^4*e^4)*e^(6*d*x + 6*c) + 2
*(I*a*d^4*f^4*x^4 + 4*I*a*d^4*e*f^3*x^3 + 6*I*a*d^4*e^2*f^2*x^2 + 4*I*a*d^
4*e^3*f*x + I*a*d^4*e^4)*e^(5*d*x + 5*c) - (a*d^4*f^4*x^4 + 4*a*d^4*e*f^3*
x^3 + 6*a*d^4*e^2*f^2*x^2 + 4*a*d^4*e^3*f*x + a*d^4*e^4)*e^(4*d*x + 4*c) +
 4*(I*a*d^4*f^4*x^4 + 4*I*a*d^4*e*f^3*x^3 + 6*I*a*d^4*e^2*f^2*x^2 + 4*I*a*
d^4*e^3*f*x + I*a*d^4*e^4)*e^(3*d*x + 3*c) + (a*d^4*f^4*x^4 + 4*a*d^4*e*f^
3*x^3 + 6*a*d^4*e^2*f^2*x^2 + 4*a*d^4*e^3*f*x + a*d^4*e^4)*e^(2*d*x + 2...
```

3.287.6 Sympy [N/A]

Not integrable

Time = 8.77 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=-\frac{i\int \frac{\operatorname{sech}^3(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

input
```
integrate(sech(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)**3/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) -
I*f*x), x)/a
```

3.287.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.287.8 Giac [N/A]

Not integrable

Time = 150.91 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^3}{(fx+e)(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sech(d*x + c)^3/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

3.287.9 Mupad [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\,\left(e+f\,x\right)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(cosh(c + d*x)^3*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.288 $\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

3.288.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.288.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \$\text{Aborted}$$

input
```
Integrate[Sech[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.288.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

$$\Big\downarrow \; 6111$$

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.288.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.288.4 Maple [F(-1)]

Timed out.

hanged

input
```
int(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

3.288.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 2026, normalized size of antiderivative = 65.35

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^3}{(fx+e)^2(i\,a\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-1/12*(8*I*d^2*f^3*x^2 + 16*I*d^2*e*f^2*x + 8*I*d^2*e^2*f - 24*I*f^3 + 3*(
3*d^3*f^3*x^3 + 3*d^3*e^3 - 6*d^2*e^2*f - 2*d*e*f^2 + 8*f^3 + 3*(3*d^3*e*f
^2 - 2*d^2*f^3)*x^2 + (9*d^3*e^2*f - 12*d^2*e*f^2 - 2*d*f^3)*x)*e^(5*d*x +
 5*c) - 6*(3*I*d^3*f^3*x^3 + 3*I*d^3*e^3 - 6*I*d^2*e^2*f + 4*I*f^3 + 3*(3*
I*d^3*e*f^2 - 2*I*d^2*f^3)*x^2 + 3*(3*I*d^3*e^2*f - 4*I*d^2*e*f^2)*x)*e^(4
*d*x + 4*c) + 2*(3*d^3*f^3*x^3 + 3*d^3*e^3 - 8*d^2*e^2*f - 6*d*e*f^2 + 24*
f^3 + (9*d^3*e*f^2 - 8*d^2*f^3)*x^2 + (9*d^3*e^2*f - 16*d^2*e*f^2 - 6*d*f^
3)*x)*e^(3*d*x + 3*c) - 2*(-9*I*d^3*f^3*x^3 - 9*I*d^3*e^3 - 22*I*d^2*e^2*f
 + 24*I*f^3 + (-27*I*d^3*e*f^2 - 22*I*d^2*f^3)*x^2 + (-27*I*d^3*e^2*f - 44
*I*d^2*e*f^2)*x)*e^(2*d*x + 2*c) + (9*d^3*f^3*x^3 + 9*d^3*e^3 + 2*d^2*e^2*
f - 6*d*e*f^2 + 24*f^3 + (27*d^3*e*f^2 + 2*d^2*f^3)*x^2 + (27*d^3*e^2*f +
4*d^2*e*f^2 - 6*d*f^3)*x)*e^(d*x + c) - 12*(a*d^4*f^5*x^5 + 5*a*d^4*e*f^4*
x^4 + 10*a*d^4*e^2*f^3*x^3 + 10*a*d^4*e^3*f^2*x^2 + 5*a*d^4*e^4*f*x + a*d^
4*e^5 - (a*d^4*f^5*x^5 + 5*a*d^4*e*f^4*x^4 + 10*a*d^4*e^2*f^3*x^3 + 10*a*d
^4*e^3*f^2*x^2 + 5*a*d^4*e^4*f*x + a*d^4*e^5)*e^(6*d*x + 6*c) + 2*(I*a*d^4
*f^5*x^5 + 5*I*a*d^4*e*f^4*x^4 + 10*I*a*d^4*e^2*f^3*x^3 + 10*I*a*d^4*e^3*f
^2*x^2 + 5*I*a*d^4*e^4*f*x + I*a*d^4*e^5)*e^(5*d*x + 5*c) - (a*d^4*f^5*x^5
 + 5*a*d^4*e*f^4*x^4 + 10*a*d^4*e^2*f^3*x^3 + 10*a*d^4*e^3*f^2*x^2 + 5*a*d
^4*e^4*f*x + a*d^4*e^5)*e^(4*d*x + 4*c) + 4*(I*a*d^4*f^5*x^5 + 5*I*a*d^4*e
*f^4*x^4 + 10*I*a*d^4*e^2*f^3*x^3 + 10*I*a*d^4*e^3*f^2*x^2 + 5*I*a*d^4*...
```

3.288.6 Sympy [N/A]

Not integrable

Time = 31.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.35

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\operatorname{sech}^3(c+dx)}{e^2\sinh(c+dx)-ie^2+2efx\sinh(c+dx)-2iefx+f^2x^2\sinh(c+dx)-if^2x^2}\,dx}{a}$$

input
```
integrate(sech(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

output
```
-I*Integral(sech(c + d*x)**3/(e**2*sinh(c + d*x) - I*e**2 + 2*e*f*x*sinh(c
 + d*x) - 2*I*e*f*x + f**2*x**2*sinh(c + d*x) - I*f**2*x**2), x)/a
```

3.288.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.288.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.288.9 Mupad [N/A]

Not integrable

Time = 2.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$
$$=\int \frac{1}{\cosh\left(c+d\,x\right)^3\,\left(e+f\,x\right)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

output
```
int(1/(cosh(c + d*x)^3*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.289 $\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.289.1 Optimal result

Integrand size = 26, antiderivative size = 356

$$\begin{aligned}\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = &-\frac{(e+fx)^4}{4bf}+\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd}\\&+\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd}\\&+\frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}\\&+\frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}\\&-\frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3}\\&-\frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^3}\\&+\frac{6f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4}\\&+\frac{6f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^4}\end{aligned}$$

output
```
-1/4*(f*x+e)^4/b/f+(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d+(f
*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d+3*f*(f*x+e)^2*polylog(2
,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d^2+3*f*(f*x+e)^2*polylog(2,-b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/b/d^2-6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/b/d^3-6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/b/d^3+6*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d^4+6*
f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d^4
```

3.289.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.92

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{-\frac{(e+fx)^4}{f} + \frac{4(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d} + \frac{4(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d} + \frac{12f\left(d^2(e+fx)^2\operatorname{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)-2df(e+fx)\,\mathrm{P}\right.}{}}{}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-((e + f*x)^4/f) + (4*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/d + (4*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/d + (12*f*(d^2*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b
^2])] - 2*d*f*(e + f*x)*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 + 2*f^2*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]))/d^4 + (12*f*
(d^2*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*
d*f*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2
*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/d^4)/(4*b)
```

3.289.3 Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6095, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \ 6095$$

$$\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \ 2620$$

$$-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \ 3011$$

$$-\frac{3f\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \ 7163$$

$$-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \ 2720$$

3.289. $\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^4}{4bf}$$

$$\Big\downarrow \; 7143$$

$$-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^4}{4bf}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
))])/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/d^2))/d))/(b*d)
```

3.289. $\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.289.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.289. $\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.289.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh (dx+c)}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.289.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(328) = 656$.

Time = 0.27 (sec) , antiderivative size = 882, normalized size of antiderivative = 2.48

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{d^4f^3x^4+4\,d^4ef^2x^3+6\,d^4e^2fx^2+4\,d^4e^3x-24\,f^3\mathrm{polylog}\left(4,\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+}{b}\right.}{}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2 + 4*d^4*e^3*x - 24*f
^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 24*f^3*polylog(4, (a*cosh(d*x + c
) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2))/b) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*dilog((a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*dilog
((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 4*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e
*f^2 - c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2
+ b^2)/b^2) + 2*a) - 4*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)
*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) - 4*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3
*c^2*d*e*f^2 + c^3*f^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 4*(d^3*f^3*x^3
 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f
^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 24*(d*f^3*x + d*e*f^2)*polylog(3,
(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2))/b) + 24*(d*f^3*x + d*e*f^2)*polylog(3, (a*cosh(d*...
```

3.289.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.289.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
e^3*log(b*sinh(d*x + c) + a)/(b*d) + 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*
x^2)/b - integrate(-2*(b*f^3*x^3 + 3*b*e*f^2*x^2 + 3*b*e^2*f*x - (a*f^3*x^
3*e^c + 3*a*e*f^2*x^2*e^c + 3*a*e^2*f*x*e^c)*e^(d*x))/(b^2*e^(2*d*x + 2*c)
 + 2*a*b*e^(d*x + c) - b^2), x)
```

3.289.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.290 $\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.290.1 Optimal result

Integrand size = 26, antiderivative size = 264

$$\begin{aligned}\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{(e+fx)^3}{3bf}+\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd}\\ &\quad+\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd}\\ &\quad+\frac{2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}\\ &\quad+\frac{2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}\\ &\quad-\frac{2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3}\\ &\quad-\frac{2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^3}\end{aligned}$$

output
```
-1/3*(f*x+e)^3/b/f+(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d+(f
*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d+2*f*(f*x+e)*polylog(2,-
b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d^2+2*f*(f*x+e)*polylog(2,-b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/b/d^2-2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/b/d^3-2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d^3
```

3.290.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.92

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-\frac{(e+fx)^3}{f}+\frac{3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}+\frac{3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}+\frac{6f\left(d(e+fx)\operatorname{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)-f\operatorname{PolyLog}\left(3,\frac{\ }{-}\right.\right.}{d^3}}{3b}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-((e + f*x)^3/f) + (3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/d + (3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/d + (6*f*(d*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 - f*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]))/d^3 + (6*f*(d*(e
 + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - f*PolyLog[3
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/d^3)/(3*b)
```

3.290.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \ 6095$$
$$\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}$$
$$\Big\downarrow \ 2620$$
$$-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^3}{3bf}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^3}{3bf}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^3}{3bf}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{2f\left(\frac{f\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^3}{3bf}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.290. $\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/d^2))/(b*d)
```

3.290.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.290. $\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.290.4 Maple [F]

$$\int \frac{(fx+e)^2 \cosh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.290.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(242) = 484$.

Time = 0.24 (sec) , antiderivative size = 609, normalized size of antiderivative = 2.31

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{d^3f^2x^3+3\,d^3efx^2+3\,d^3e^2x+6\,f^2\mathrm{polylog}\left(3,\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}}{b}\right)+6}{}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x + 6*f^2*polylog(3, (a*cosh
(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2))/b) + 6*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(d*f^2*x
 + d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 6*(d*f^2*x + d*e*f)*dil
og((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(d^2*e^2 - 2*c*d*e*f + c^2*f^2)*lo
g(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
 - 3*(d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x
+ c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2
*c*d*e*f - c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(d^2*f^2*x^2 + 2
*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
- (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b))/(b*d^
3)
```

3.290.6 Sympy [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*cosh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.290.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
e^2*log(b*sinh(d*x + c) + a)/(b*d) + 1/3*(f^2*x^3 + 3*e*f*x^2)/b - integra
te(-2*(b*f^2*x^2 + 2*b*e*f*x - (a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x))/(b
^2*e^(2*d*x + 2*c) + 2*a*b*e^(d*x + c) - b^2), x)
```

3.290.8 Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.291 $\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.291.1 Optimal result

Integrand size = 24, antiderivative size = 170

$$\begin{aligned}\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = &-\frac{(e+fx)^2}{2bf}+\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd}\\&+\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd}\\&+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}\end{aligned}$$

output
```
-1/2*(f*x+e)^2/b/f+(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d+(f*x
+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d+f*polylog(2,-b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))/b/d^2+f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b
/d^2
```

3.291.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.92

$$\begin{aligned}&\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx\\&=\frac{-d(e+fx)\left(de+dfx-2f\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2f\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)+2f^2\,\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)}{2bd^2f}\end{aligned}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(d*(e + f*x)*(d*e + d*f*x - 2*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2])] - 2*f*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])) + 2*f^2*Poly
Log[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[2, -((b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2]))])/(2*b*d^2*f)
```

3.291.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6095}$$

$$\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}$$

$$\Big\downarrow \text{2838}$$

$$\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}$$

input
```
Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2
) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)
```

3.291.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

3.291.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(156) = 312$.

Time = 1.47 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.42

method	result
risch	$-\frac{f x^2}{2b}+\frac{ex}{b}-\frac{2e\ln\left(\mathrm{e}^{dx+c}\right)}{db}+\frac{e\ln\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)}{db}-\frac{f c^2}{d^2 b}+\frac{f\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)c}{d^2 b}+\frac{f\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+}}{-a+\sqrt{a^2+b^2}}\right.}{d^2 b}$

input
```
int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*f*x^2/b+e*x/b-2/d/b*e*ln(exp(d*x+c))+1/d/b*e*ln(b*exp(2*d*x+2*c)+2*a*
exp(d*x+c)-b)-1/d^2/b*f*c^2+1/d^2/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/
(a+(a^2+b^2)^(1/2)))*c+1/d^2/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+
(a^2+b^2)^(1/2)))*c+2/d^2/b*c*f*ln(exp(d*x+c))-1/d^2/b*c*f*ln(b*exp(2*d*x+
2*c)+2*a*exp(d*x+c)-b)+1/d/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))*x+1/d/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))*x+1/d^2/b*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))+1/d^2/b*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/
2)))-2/d/b*c*f*x
```

3.291.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(154) = 308$.

Time = 0.26 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.24

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{d^2 f x^2 + 2\,d^2 e x - 2\,f\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right) - 2\,f\mathrm{Li}_2\left(\frac{a\cosh(dx+}{}\right.}{}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(d^2*f*x^2 + 2*d^2*e*x - 2*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
 + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) -
 2*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(d*e - c*f)*log(2*b*cosh(d
*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(d*e -
c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2)
 + 2*a) - 2*(d*f*x + c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(d*f*x + c
*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b))/(b*d^2)
```

3.291.6 Sympy [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\cosh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*cosh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.291.7 Maxima [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*f*(x^2/b - integrate(4*(a*x*e^(d*x + c) - b*x)/(b^2*e^(2*d*x + 2*c) +
2*a*b*e^(d*x + c) - b^2), x)) + e*log(b*sinh(d*x + c) + a)/(b*d)
```

3.291.8 Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.292 $\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.292.1 Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx=\frac{\log(a+b\sinh(c+dx))}{bd}$$

output
```
ln(a+b*sinh(d*x+c))/b/d
```

3.292.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx=\frac{\log(a+b\sinh(c+dx))}{bd}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
Log[a + b*Sinh[c + d*x]]/(b*d)
```

3.292.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{\int \frac{1}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(a+b\sinh(c+dx))}{bd}$$

input
```
Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
Log[a + b*Sinh[c + d*x]]/(b*d)
```

3.292.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.292.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\ln(a+b\sinh(dx+c))}{bd}$	19
default	$\frac{\ln(a+b\sinh(dx+c))}{bd}$	19
risch	$-\frac{x}{b}-\frac{2c}{bd}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{bd}$	48

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*sinh(d*x+c))/b/d
```

3.292.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(18) = 36.

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{dx-\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{bd}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(d*x - log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))))/(b*d
)
```

3.292.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.

Time = 0.82 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \begin{cases} \frac{x\cosh(c)}{a} & \text{for } b=0 \wedge d=0 \\ \frac{\sinh(c+dx)}{ad} & \text{for } b=0 \\ \frac{x\cosh(c)}{a+b\sinh(c)} & \text{for } d=0 \\ \frac{\log\left(\frac{a}{b}+\sinh(c+dx)\right)}{bd} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Piecewise((x*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)/(a*d), Eq(b,
0)), (x*cosh(c)/(a + b*sinh(c)), Eq(d, 0)), (log(a/b + sinh(c + d*x))/(b*d
), True))
```

3.292.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\log\left(b\sinh(dx+c)+a\right)}{bd}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
log(b*sinh(d*x + c) + a)/(b*d)
```

3.292.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.83

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{bd}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(b*d)
```

3.292.9 Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\ln\left(a+b\sinh(c+d\,x)\right)}{b\,d}$$

input
```
int(cosh(c + d*x)/(a + b*sinh(c + d*x)),x)
```

output
```
log(a + b*sinh(c + d*x))/(b*d)
```

3.293 $\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.293.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.293.2 Mathematica [N/A]

Not integrable

Time = 5.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Cosh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Cosh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.293.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Cosh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.293.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.293.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.293.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(cosh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.293.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.293.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.58

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
log(f*x + e)/(b*f) - 1/2*integrate(-4*(a*e^(d*x + c) - b)/(b^2*f*x + b^2*e
 - (b^2*f*x*e^(2*c) + b^2*e*e^(2*c))*e^(2*d*x) - 2*(a*b*f*x*e^c + a*b*e*e^
c)*e^(d*x)), x)
```

3.293.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.293.9 Mupad [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

input
```
int(cosh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(cosh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.294 $\int \frac{(e+fx)^3 \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.294.1 Optimal result

Integrand size = 28, antiderivative size = 527

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a(e+fx)^4}{4b^2f}+\frac{6f^2(e+fx)\cosh(c+dx)}{bd^3}+\frac{(e+fx)^3\cosh(c+dx)}{bd}+\frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{3\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{3\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{6\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3}+\frac{6\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^3}+\frac{6\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^4}-\frac{6\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^4}-\frac{6f^3\sinh(c+dx)}{bd^4}-\frac{3f(e+fx)^2\sinh(c+dx)}{bd^2}$$

output
```
-1/4*a*(f*x+e)^4/b^2/f+6*f^2*(f*x+e)*cosh(d*x+c)/b/d^3+(f*x+e)^3*cosh(d*x+
c)/b/d-6*f^3*sinh(d*x+c)/b/d^4-3*f*(f*x+e)^2*sinh(d*x+c)/b/d^2+(f*x+e)^3*l
n(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d-(f*x+e)^3*ln(1
+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d+3*f*(f*x+e)^2*pol
ylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d^2-3*f*(f*x
+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d^2
-6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2
)/b^2/d^3+6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+
b^2)^(1/2)/b^2/d^3+6*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2
+b^2)^(1/2)/b^2/d^4-6*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^
2+b^2)^(1/2)/b^2/d^4
```

3.294.2 Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 933, normalized size of antiderivative = 1.77

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{4ad^4e^3x+6ad^4e^2fx^2+4ad^4ef^2x^3+ad^4f^3x^4+8\sqrt{a^2+b^2}d^3e^3\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-4bd^3e^3\cosh(c+dx}{}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/4*(4*a*d^4*e^3*x + 6*a*d^4*e^2*f*x^2 + 4*a*d^4*e*f^2*x^3 + a*d^4*f^3*x^
4 + 8*Sqrt[a^2 + b^2]*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]]
 - 4*b*d^3*e^3*Cosh[c + d*x] - 24*b*d*e*f^2*Cosh[c + d*x] - 12*b*d^3*e^2*f
*x*Cosh[c + d*x] - 24*b*d*f^3*x*Cosh[c + d*x] - 12*b*d^3*e*f^2*x^2*Cosh[c
+ d*x] - 4*b*d^3*f^3*x^3*Cosh[c + d*x] - 12*Sqrt[a^2 + b^2]*d^3*e^2*f*x*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 12*Sqrt[a^2 + b^2]*d^3*e*f^
2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 4*Sqrt[a^2 + b^2]*d
^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 12*Sqrt[a^2 +
b^2]*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 12*Sqrt[
a^2 + b^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] +
4*Sqrt[a^2 + b^2]*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)] - 12*Sqrt[a^2 + b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a +
 Sqrt[a^2 + b^2])] + 12*Sqrt[a^2 + b^2]*d^2*f*(e + f*x)^2*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 24*Sqrt[a^2 + b^2]*d*e*f^2*PolyLog[
3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 24*Sqrt[a^2 + b^2]*d*f^3*x*Po
lyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 24*Sqrt[a^2 + b^2]*d*e*
f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 24*Sqrt[a^2 + b
^2]*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 24*Sqrt
[a^2 + b^2]*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 24*Sq
rt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] ...
```

3.294.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.53 (sec) , antiderivative size = 501, normalized size of antiderivative = 0.95, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6099, 17, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 3803, 25, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6099}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^3dx}{b^2}+\frac{\int (e+fx)^3\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)^3\sinh(c+dx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int -i(e+fx)^3\sin(ic+idx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int (e+fx)^3\sin(ic+idx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{ 26}$$

3.294. $\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}$$

$$\Big\downarrow \text{ 3117}$$

3.294. $\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{3803}$$

$$\frac{2\left(a^2+b^2\right)\int-\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\left(a^2+b^2\right)\int\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{2\left(a^2+b^2\right)\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{27}$$

3.294. $\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$-\frac{2(a^2+b^2)\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+}}{a-\sqrt{}}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{b^2}$$

$$\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left((e+fx\right.}{}\right.}{b^2}$$

$$\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{7163}$$

3.294. $\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2\cdots\right.}{d}\cdots}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right.}{\cdots}$$

$$\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}$$

$$\downarrow 2720$$

3.294. $\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx}{}}{bd}\right)}{2\sqrt{a^2+b^2}}}{\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}}$$

$$\Big\downarrow \; 7143$$

3.294. $\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{}{a}\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/4*(a*(e + f*x)^4)/(b^2*f) - (2*(a^2 + b^2)*(-1/2*(b*(((e + f*x)^3*Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*Po
lyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*
PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/Sqrt[a^2 + b^2]
 + (b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d)
- (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/d)
)/(b*d)))/(2*Sqrt[a^2 + b^2])))/b^2 - (I*((I*(e + f*x)^3*Cosh[c + d*x])/d
- ((3*I)*f*(((e + f*x)^2*Sinh[c + d*x])/d + ((2*I)*f*((I*(e + f*x)*Cosh[c
+ d*x])/d - (I*f*Sinh[c + d*x])/d^2))/d))/d))/b
```

3.294. $\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.294.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.294. $\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.294. $\int \frac{(e+fx)^3 \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.294.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.294.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2020 vs. $2(483) = 966$.

Time = 0.29 (sec) , antiderivative size = 2020, normalized size of antiderivative = 3.83

$$\int \frac{(e+fx)^3 \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*b*d^3*f^3*x^3 + 2*b*d^3*e^3 + 6*b*d^2*e^2*f + 12*b*d*e*f^2 + 12*b*f
^3 + 6*(b*d^3*e*f^2 + b*d^2*f^3)*x^2 + 2*(b*d^3*f^3*x^3 + b*d^3*e^3 - 3*b*
d^2*e^2*f + 6*b*d*e*f^2 - 6*b*f^3 + 3*(b*d^3*e*f^2 - b*d^2*f^3)*x^2 + 3*(b
*d^3*e^2*f - 2*b*d^2*e*f^2 + 2*b*d*f^3)*x)*cosh(d*x + c)^2 + 2*(b*d^3*f^3*
x^3 + b*d^3*e^3 - 3*b*d^2*e^2*f + 6*b*d*e*f^2 - 6*b*f^3 + 3*(b*d^3*e*f^2 -
 b*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*b*d^2*e*f^2 + 2*b*d*f^3)*x)*sinh(d*x
+ c)^2 + 12*((b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)
 + (b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 12*((b*d^2*f^3*x^2
+ 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c) + (b*d^2*f^3*x^2 + 2*b*d^2*
e*f^2*x + b*d^2*e^2*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b + 1) - 4*((b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f
^2 - b*c^3*f^3)*cosh(d*x + c) + (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e
*f^2 - b*c^3*f^3)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*((b*d^3*e^3
- 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*cosh(d*x + c) + (b*d^3*e^
3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2...
```

3.294.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.294.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*e^3*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) - e^(-d*x - c)/(b*d) -
 2*sqrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x -
 c) - a + sqrt(a^2 + b^2)))/(b^2*d)) - 1/4*(a*d^4*f^3*x^4*e^c + 4*a*d^4*e*
f^2*x^3*e^c + 6*a*d^4*e^2*f*x^2*e^c - 2*(b*d^3*f^3*x^3*e^(2*c) + 3*(d^3*e*
f^2 - d^2*f^3)*b*x^2*e^(2*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b*x*e
^(2*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b*e^(2*c))*e^(d*x) - 2*(b*d^3*f
^3*x^3 + 3*(d^3*e*f^2 + d^2*f^3)*b*x^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*
f^3)*b*x + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b)*e^(-d*x))*e^(-c)/(b^2*d^4)
 + integrate(2*((a^2*f^3*e^c + b^2*f^3*e^c)*x^3 + 3*(a^2*e*f^2*e^c + b^2*e
*f^2*e^c)*x^2 + 3*(a^2*e^2*f*e^c + b^2*e^2*f*e^c)*x)*e^(d*x)/(b^3*e^(2*d*x
 + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)
```

3.294.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{{\cosh(c+d\,x)}^2\,{(e+f\,x)}^3}{a+b\,\sinh{(c+d\,x)}}\,dx$$

input
```
int((cosh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.295 $\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.295.1 Optimal result

Integrand size = 28, antiderivative size = 389

$$\begin{aligned}\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{a(e+fx)^3}{3b^2f}+\frac{2f^2\cosh(c+dx)}{bd^3}+\frac{(e+fx)^2\cosh(c+dx)}{bd}\\ &+\frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d}\\ &-\frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d}\\ &+\frac{2\sqrt{a^2+b^2}f(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}\\ &-\frac{2\sqrt{a^2+b^2}f(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2}\\ &-\frac{2\sqrt{a^2+b^2}f^2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3}\\ &+\frac{2\sqrt{a^2+b^2}f^2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^3}\\ &-\frac{2f(e+fx)\sinh(c+dx)}{bd^2}\end{aligned}$$

output
```
-1/3*a*(f*x+e)^3/b^2/f+2*f^2*cosh(d*x+c)/b/d^3+(f*x+e)^2*cosh(d*x+c)/b/d-2
*f*(f*x+e)*sinh(d*x+c)/b/d^2+(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2
)))*(a^2+b^2)^(1/2)/b^2/d-(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))
*(a^2+b^2)^(1/2)/b^2/d+2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))*(a^2+b^2)^(1/2)/b^2/d^2-2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+
b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d^2-2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d^3+2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d^3
```

3.295.2 Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.15

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{ad^3x(3e^2+3efx+f^2x^2)+3\sqrt{a^2+b^2}\left(2d^2e^2\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-d^2f^2x}{}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/3*(a*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2) + 3*Sqrt[a^2 + b^2]*(2*d^2*e^2*A
rcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2])] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*
(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e +
f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2*PolyLog[
3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*f^2*PolyLog[3, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))]) - 3*b*Cosh[d*x]*((2*f^2 + d^2*(e + f*x)^2)*
Cosh[c] - 2*d*f*(e + f*x)*Sinh[c]) + 3*b*(2*d*f*(e + f*x)*Cosh[c] - (2*f^2
 + d^2*(e + f*x)^2)*Sinh[c])*Sinh[d*x])/(b^2*d^3)
```

3.295.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.97, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.679$, Rules used = {6099, 17, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \; 6099$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^2dx}{b^2}+\frac{\int (e+fx)^2\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \; 17$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)^2\sinh(c+dx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int -i(e+fx)^2\sin(ic+idx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \; 26$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int (e+fx)^2\sin(ic+idx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \; 3777$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \; 3777$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}$$

$$\Big\downarrow \text{3118}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{3803}$$

$$\frac{2(a^2+b^2)\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{25}$$

3.295. $\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{2\left(a^2+b^2\right)\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{2\left(a^2+b^2\right)\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\left(a^2+b^2\right)\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}\cdots}{a-\sqrt{a^2\cdots}}\cdots\right.}{bd}\cdots}{2\sqrt{a^2+b^2}}\cdots\right.}{b^2}$$
$$\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

3.295. $\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\cdots\right.}{bd}\cdots\right.}{\cdots}\cdots\right)}{b^2}\cdots$$

$$\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \color{blue}{2720}$$

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\cdots\right.}{\cdots}\cdots\right)}{b^2}\cdots$$

$$\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

$$\Big\downarrow \color{blue}{7143}$$

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+\cdots}}{a-\sqrt{a\cdots}}\cdots\right.}{bd}\cdots\right.}{\cdots}\cdots\right)}{b^2}\cdots$$

$$\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}$$

3.295. $\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/3*(a*(e + f*x)^3)/(b^2*f) - (2*(a^2 + b^2)*(-1/2*(b*(((e + f*x)^2*Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*Poly
Log[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(
((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*
(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (
f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(2*S
qrt[a^2 + b^2])))/b^2 - (I*((I*(e + f*x)^2*Cosh[c + d*x])/d - ((2*I)*f*(-(
(f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d))/d))/b
```

3.295.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.295. $\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.295.4 Maple [F]

$$\int \frac{(fx+e)^2 \cosh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.295.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1313 vs. $2(355) = 710$.

Time = 0.27 (sec) , antiderivative size = 1313, normalized size of antiderivative = 3.38

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*b*d^2*f^2*x^2 + 3*b*d^2*e^2 + 6*b*d*e*f + 6*b*f^2 + 3*(b*d^2*f^2*x^
2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f - b*d*f^2)*x)*cosh(d*x
+ c)^2 + 3*(b*d^2*f^2*x^2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f
 - b*d*f^2)*x)*sinh(d*x + c)^2 + 12*((b*d*f^2*x + b*d*e*f)*cosh(d*x + c) +
 (b*d*f^2*x + b*d*e*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b + 1) - 12*((b*d*f^2*x + b*d*e*f)*cosh(d*x + c) + (b*d*
f^2*x + b*d*e*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2) - b)/b + 1) - 6*((b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c
) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2)
 + 2*a) + 6*((b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c) + (b*d^2*
e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*
b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6
*((b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*cosh(d*x + c)
+ (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 6*((b*d^2*f^2*x
^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*cosh(d*x + c) + (b*d^2*f^...
```

3.295.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.295.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*e^2*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) - e^(-d*x - c)/(b*d) -
 2*sqrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x -
 c) - a + sqrt(a^2 + b^2)))/(b^2*d)) - 1/6*(2*a*d^3*f^2*x^3*e^c + 6*a*d^3*
e*f*x^2*e^c - 3*(b*d^2*f^2*x^2*e^(2*c) + 2*(d^2*e*f - d*f^2)*b*x*e^(2*c) -
 2*(d*e*f - f^2)*b*e^(2*c))*e^(d*x) - 3*(b*d^2*f^2*x^2 + 2*(d^2*e*f + d*f^
2)*b*x + 2*(d*e*f + f^2)*b)*e^(-d*x))*e^(-c)/(b^2*d^3) + integrate(2*((a^2
*f^2*e^c + b^2*f^2*e^c)*x^2 + 2*(a^2*e*f*e^c + b^2*e*f*e^c)*x)*e^(d*x)/(b^
3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)
```

3.295.8 Giac [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.296 $\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.296.1 Optimal result

Integrand size = 26, antiderivative size = 252

$$\begin{aligned}\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{aex}{b^2}-\frac{afx^2}{2b^2}+\frac{(e+fx)\cosh(c+dx)}{bd} \\ &\quad+\frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} \\ &\quad-\frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d} \\ &\quad+\frac{\sqrt{a^2+b^2}f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2} \\ &\quad-\frac{\sqrt{a^2+b^2}f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{f\sinh(c+dx)}{bd^2}\end{aligned}$$

output
```
-a*e*x/b^2-1/2*a*f*x^2/b^2+(f*x+e)*cosh(d*x+c)/b/d-f*sinh(d*x+c)/b/d^2+(f*
x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d-(f*x+e)*
ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d+f*polylog(2,-
b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d^2-f*polylog(2,-b*e
xp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d^2
```

3.296.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.02

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{a(c+dx)(cf-d(2e+fx))+2bd(e+fx)\cosh(c+dx)+2\sqrt{a^2+b^2}\left(-2de\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2cf\operatorname{arct}\right.}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(a*(c + d*x)*(c*f - d*(2*e + f*x)) + 2*b*d*(e + f*x)*Cosh[c + d*x] + 2*Sqr
t[a^2 + b^2]*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*
ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
- f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 2*b*f*Sinh[c +
 d*x])/(2*b^2*d^2)
```

3.296.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {6099, 17, 3042, 26, 3777, 3042, 3117, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{ 6099}$$
$$\frac{(a^2+b^2)\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)dx}{b^2}+\frac{\int (e+fx)\sinh(c+dx)dx}{b}$$
$$\Big\downarrow \text{ 17}$$
$$\frac{(a^2+b^2)\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)\sinh(c+dx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a^2+b^2)\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int -i(e+fx)\sin(ic+idx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}$$

$$\Big\downarrow \; 26$$

$$\frac{(a^2+b^2)\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int (e+fx)\sin(ic+idx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}$$

$$\Big\downarrow \; 3777$$

$$\frac{(a^2+b^2)\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^2}{2b^2f}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a^2+b^2)\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^2}{2b^2f}$$

$$\Big\downarrow \; 3117$$

$$\frac{(a^2+b^2)\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \; 3803$$

$$\frac{2(a^2+b^2)\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{2(a^2+b^2)\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \; 2694$$

$$-\frac{2(a^2+b^2)\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \; 27$$

3.296. $\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{2(a^2+b^2)\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

3.296. $\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
-1/2*(a*(e + f*x)^2)/(b^2*f) - (2*(a^2 + b^2)*(-1/2*(b*(((e + f*x)*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/b^
2 - (I*((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/b
```

3.296.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.296. $\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

3.296.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(230) = 460$.

Time = 2.43 (sec) , antiderivative size = 901, normalized size of antiderivative = 3.58

method	result
risch	$-\frac{af\,x^2}{2b^2}-\frac{aex}{b^2}+\frac{(dfx+de-f)\,\mathrm{e}^{dx+c}}{2b\,d^2}+\frac{(dfx+de+f)\,\mathrm{e}^{-dx-c}}{2b\,d^2}-\frac{2a^2e\,\operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d\,b^2\sqrt{a^2+b^2}}-\frac{2e\,\operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$

input
```
int((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*f*x^2/b^2-a*e*x/b^2+1/2*(d*f*x+d*e-f)/b/d^2*exp(d*x+c)+1/2*(d*f*x+d
*e+f)/b/d^2*exp(-d*x-c)-2/d/b^2*a^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp
(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d
*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d/b^2*a^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c
)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d/b^2*a^2*f/(a^2+b^2)^(1/2)
*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2/b^2*a^2*
f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)
))*c-1/d^2/b^2*a^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(
a+(a^2+b^2)^(1/2)))*c+1/d^2/b^2*a^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2/b^2*a^2*f/(a^2+b^2)^(1/2)*
dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d*f/(a^2+b^2
)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d*f
/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*
x+1/d^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^
2)^(1/2)))*c-1/d^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(
a+(a^2+b^2)^(1/2)))*c+1/d^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^
2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x
+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d^2/b^2*a^2*f*c/(a^2+b^2)^(1
/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2*f*c/(a^2+b^2)^
(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

3.296.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 742 vs. $2(228) = 456$.

Time = 0.27 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.94

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{bdfx + bde + (bdfx + bde - bf)\cosh(dx+c)^2 + (bdfx + bde - bf)\sinh(dx+c)^2 + 2\left(bf\cosh(dx+c) + \right.}{}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(b*d*f*x + b*d*e + (b*d*f*x + b*d*e - b*f)*cosh(d*x + c)^2 + (b*d*f*x
+ b*d*e - b*f)*sinh(d*x + c)^2 + 2*(b*f*cosh(d*x + c) + b*f*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b*f*cos
h(d*x + c) + b*f*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2) - b)/b + 1) - 2*((b*d*e - b*c*f)*cosh(d*x + c) + (b*d*e - b*c*f)*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((b*d*e - b*c*f)*cosh(d*x + c) +
 (b*d*e - b*c*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c
) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((b*d*f*x + b
*c*f)*cosh(d*x + c) + (b*d*f*x + b*c*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((b*d*f*x + b*c*f)*cosh(d*x + c)
+ (b*d*f*x + b*c*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b) + b*f - (a*d^2*f*x^2 + 2*a*d^2*e*x)*cosh(d*x + c) - (a*d^
2*f*x^2 + 2*a*d^2*e*x - 2*(b*d*f*x + b*d*e - b*f)*cosh(d*x + c))*sinh(d*x
+ c))/(b^2*d^2*cosh(d*x + c) + b^2*d^2*sinh(d*x + c))
```

3.296.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.296.7 Maxima [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(4*(a^2*e^c + b^2*e^c)*integrate(x*e^(d*x)/(b^3*e^(2*d*x + 2*c) + 2*a*
b^2*e^(d*x + c) - b^3), x) - (a*d^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*
e^(d*x) - (b*d*x + b)*e^(-d*x))*e^(-c)/(b^2*d^2))*f - 1/2*e*(2*(d*x + c)*a
/(b^2*d) - e^(d*x + c)/(b*d) - e^(-d*x - c)/(b*d) - 2*sqrt(a^2 + b^2)*log(
(b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^
2)))/(b^2*d))
```

3.296.8 Giac [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\cosh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.297 $\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.297.1 Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=-\frac{ax}{b^2}-\frac{2\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\cosh(c+dx)}{bd}$$

output
```
-a*x/b^2+cosh(d*x+c)/b/d-2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/
2))*(a^2+b^2)^(1/2)/b^2/d
```

3.297.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 458, normalized size of antiderivative = 6.74

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{\cosh(c+dx)\left(-2\sqrt{a-ib}\sqrt{a+ib}\,\text{arctanh}\left(\frac{\sqrt{-\frac{b(i+\sinh(c+dx))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(c+dx))}{a+ib}}}\right)\sqrt{1+i\sinh(c+dx)}+2(a-ib)\,\text{arctanh}\left(\cdots\right.\right.}{\sqrt{\cdots}}$$

input
```
Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
(Cosh[c + d*x]*(-2*Sqrt[a - I*b]*Sqrt[a + I*b]*ArcTanh[Sqrt[-((b*(I + Sinh
[c + d*x]))/(a - I*b))]/Sqrt[-((b*(-I + Sinh[c + d*x]))/(a + I*b))]]*Sqrt[
1 + I*Sinh[c + d*x]] + 2*(a - I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + S
inh[c + d*x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[c + d*x]))
/(a + I*b))])]*Sqrt[1 + I*Sinh[c + d*x]] + Sqrt[a + I*b]*Sqrt[-((b*(-I + S
inh[c + d*x]))/(a + I*b))]*(-2*(-1)^(3/4)*Sqrt[b]*ArcSin[((-1)^(1/4)*Sqrt[
a - I*b]*Sqrt[-((b*(I + Sinh[c + d*x]))/(a - I*b))])/(Sqrt[2]*Sqrt[b])] +
Sqrt[a - I*b]*Sqrt[1 + I*Sinh[c + d*x]]*Sqrt[-((b*(I + Sinh[c + d*x]))/(a
- I*b))])))/(Sqrt[a - I*b]*Sqrt[a + I*b]*b*d*Sqrt[1 + I*Sinh[c + d*x]]*Sqr
t[-((b*(-I + Sinh[c + d*x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[c + d*x]))/(a
 - I*b))])
```

3.297.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3174, 26, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3174}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{i\int -\frac{i(b-a\sinh(c+dx))}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{b-a\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\cosh(c+dx)}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{\int \frac{b+ia\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{(a^2+b^2)\int \frac{1}{a+b\sinh(c+dx)}dx}{b}-\frac{ax}{b}}{b}+\frac{\cosh(c+dx)}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{-\frac{ax}{b}+\frac{(a^2+b^2)\int \frac{1}{a-ib\sin(ic+idx)}dx}{b}}{b}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{-\frac{ax}{b}-\frac{2i(a^2+b^2)\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}}{b}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\cosh(c+dx)}{bd}+\frac{-\frac{ax}{b}+\frac{4i(a^2+b^2)\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{bd}}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{ax}{b}}{b}+\frac{\cosh(c+dx)}{bd}$$

input
```
Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
(-((a*x)/b) + (2*Sqrt[a^2 + b^2]*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b
^2])])/(b*d))/b + Cosh[c + d*x]/(b*d)
```

3.297.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.297.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.79

method	result
risch	$-\frac{ax}{b^2}+\frac{e^{dx+c}}{2bd}+\frac{e^{-dx-c}}{2bd}+\frac{\sqrt{a^2+b^2}\,\ln\left(e^{dx+c}-\frac{-a+\sqrt{a^2+b^2}}{b}\right)}{d\,b^2}-\frac{\sqrt{a^2+b^2}\,\ln\left(e^{dx+c}+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{d\,b^2}$
derivativedivides	$\frac{\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{d}{}\right.\right.}{b^2}}{d}$
default	$\frac{\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{d}{}\right.\right.}{b^2}}{d}$

input
```
int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-a*x/b^2+1/2/b/d*exp(d*x+c)+1/2/b/d*exp(-d*x-c)+(a^2+b^2)^(1/2)/d/b^2*ln(e
xp(d*x+c)-(-a+(a^2+b^2)^(1/2))/b)-(a^2+b^2)^(1/2)/d/b^2*ln(exp(d*x+c)+(a+(
a^2+b^2)^(1/2))/b)
```

3.297.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(65) = 130$.

Time = 0.25 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.81

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{2\,adx\cosh\left(dx+c\right)-b\cosh\left(dx+c\right)^2-b\sinh\left(dx+c\right)^2-2\,\sqrt{a^2+b^2}(\cosh\left(dx+c\right)+\sinh\left(dx+c\right))\,\mathrm{lo}}{}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*a*d*x*cosh(d*x + c) - b*cosh(d*x + c)^2 - b*sinh(d*x + c)^2 - 2*sq
rt(a^2 + b^2)*(cosh(d*x + c) + sinh(d*x + c))*log((b^2*cosh(d*x + c)^2 + b
^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x +
 c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x
 + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2
*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a*d*x - b*cosh(d*x + c))*s
inh(d*x + c) - b)/(b^2*d*cosh(d*x + c) + b^2*d*sinh(d*x + c))
```

3.297.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(58) = 116$.

Time = 125.60 (sec) , antiderivative size = 503, normalized size of antiderivative = 7.40

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}x\cosh^2(c)}{\sinh(c)} \\ \frac{\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-d} - \frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-d} - \frac{2}{d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-d}}{b} \\ \frac{-\frac{x\sinh^2(c+dx)}{2}+\frac{x\cosh^2(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{a} \\ \frac{x\cosh^2(c)}{a+b\sinh(c)} \\ -\frac{adx\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{b^2d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d} + \frac{adx}{b^2d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d} - \frac{2b}{b^2d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d} - \frac{\sqrt{a^2+b^2}\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)\tanh^2\left(\right.}{b^2d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d} \end{cases}$$

input
```
integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Piecewise((zoo*x*cosh(c)**2/sinh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((lo
g(tanh(c/2 + d*x/2))*tanh(c/2 + d*x/2)**2/(d*tanh(c/2 + d*x/2)**2 - d) - l
og(tanh(c/2 + d*x/2))/(d*tanh(c/2 + d*x/2)**2 - d) - 2/(d*tanh(c/2 + d*x/2
)**2 - d))/b, Eq(a, 0)), ((-x*sinh(c + d*x)**2/2 + x*cosh(c + d*x)**2/2 +
sinh(c + d*x)*cosh(c + d*x)/(2*d))/a, Eq(b, 0)), (x*cosh(c)**2/(a + b*sinh
(c)), Eq(d, 0)), (-a*d*x*tanh(c/2 + d*x/2)**2/(b**2*d*tanh(c/2 + d*x/2)**2
 - b**2*d) + a*d*x/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d) - 2*b/(b**2*d*ta
nh(c/2 + d*x/2)**2 - b**2*d) - sqrt(a**2 + b**2)*log(tanh(c/2 + d*x/2) - b
/a - sqrt(a**2 + b**2)/a)*tanh(c/2 + d*x/2)**2/(b**2*d*tanh(c/2 + d*x/2)**
2 - b**2*d) + sqrt(a**2 + b**2)*log(tanh(c/2 + d*x/2) - b/a - sqrt(a**2 +
b**2)/a)/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d) + sqrt(a**2 + b**2)*log(ta
nh(c/2 + d*x/2) - b/a + sqrt(a**2 + b**2)/a)*tanh(c/2 + d*x/2)**2/(b**2*d*
tanh(c/2 + d*x/2)**2 - b**2*d) - sqrt(a**2 + b**2)*log(tanh(c/2 + d*x/2) -
 b/a + sqrt(a**2 + b**2)/a)/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d), True))
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.71

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{(dx+c)a}{b^2d} + \frac{e^{(dx+c)}}{2\,bd} + \frac{e^{(-dx-c)}}{2\,bd} + \frac{\sqrt{a^2+b^2}\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{b^2d}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) + 1/2*e^(-d*x - c)/(b*d) + sq
rt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) -
 a + sqrt(a^2 + b^2)))/(b^2*d)
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.62

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2\,(dx+c)a}{b^2} - \frac{e^{(dx+c)}}{b} - \frac{e^{(-dx-c)}}{b} - \frac{2\,\sqrt{a^2+b^2}\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{b^2}}{2\,d}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)*a/b^2 - e^(d*x + c)/b - e^(-d*x - c)/b - 2*sqrt(a^2 + b^
2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
+ 2*a + 2*sqrt(a^2 + b^2)))/b^2)/d
```

3.297.9 Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.78

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\mathrm{e}^{c+d\,x}}{2\,b\,d} - \frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-b^4\,d^2}}{b^2\,d\,\sqrt{a^2+b^2}} + \frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-b^4\,d^2}}{b\,d\,\sqrt{a^2+b^2}}\right)\,\sqrt{a^2+b^2}}{\sqrt{-b^4\,d^2}} + \frac{\mathrm{e}^{-c-d\,x}}{2\,b\,d} - \frac{a\,x}{b^2}$$

input
```
int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)),x)
```

output
```
exp(c + d*x)/(2*b*d) - (2*atan((a*(-b^4*d^2)^(1/2))/(b^2*d*(a^2 + b^2)^(1/
2)) + (exp(d*x)*exp(c)*(-b^4*d^2)^(1/2))/(b*d*(a^2 + b^2)^(1/2)))*(a^2 + b
^2)^(1/2))/(-b^4*d^2)^(1/2) + exp(- c - d*x)/(2*b*d) - (a*x)/b^2
```

3.297. $\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.298 $\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.298.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.298.2 Mathematica [N/A]

Not integrable

Time = 9.77 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Cosh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Cosh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.298.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{6111\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Cosh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.298.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.298.4 Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.298.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(cosh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.298.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.298.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 166, normalized size of antiderivative = 5.93

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2*(a^2*e^c + b^2*e^c)*integrate(-e^(d*x)/(b^3*f*x + b^3*e - (b^3*f*x*e^(2*
c) + b^3*e*e^(2*c))*e^(2*d*x) - 2*(a*b^2*f*x*e^c + a*b^2*e*e^c)*e^(d*x)),
x) + 1/2*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c
- d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f)
```

3.298.8 Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.298.9 Mupad [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(cosh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(cosh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.299 $\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.299.1 Optimal result

Integrand size = 28, antiderivative size = 642

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{3f^3x}{8bd^3}+\frac{(e+fx)^3}{4bd}-\frac{(a^2+b^2)(e+fx)^4}{4b^3f}+\frac{6af^3\cosh(c+dx)}{b^2d^4}+\frac{3af(e+fx)^2\cosh(c+dx)}{b^2d^2}+\frac{(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{3(a^2+b^2)f(e+fx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{3(a^2+b^2)f(e+fx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}-\frac{6(a^2+b^2)f^2(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{6(a^2+b^2)f^2(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3}+\frac{6(a^2+b^2)f^3\,\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^4}+\frac{6(a^2+b^2)f^3\,\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^4}-\frac{6af^2(e+fx)\sinh(c+dx)}{b^2d^3}-\frac{a(e+fx)^3\sinh(c+dx)}{b^2d}-\frac{3f^3\cosh(c+dx)\sinh(c+dx)}{8bd^4}-\frac{3f(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{4bd^2}+\frac{3f^2(e+fx)\sinh^2(c+dx)}{4bd^3}+\frac{(e+fx)^3\sinh^2(c+dx)}{2bd}$$

output
```
3/8*f^3*x/b/d^3+1/4*(f*x+e)^3/b/d-1/4*(a^2+b^2)*(f*x+e)^4/b^3/f+6*a*f^3*co
sh(d*x+c)/b^2/d^4+3*a*f*(f*x+e)^2*cosh(d*x+c)/b^2/d^2+(a^2+b^2)*(f*x+e)^3*
ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+(a^2+b^2)*(f*x+e)^3*ln(1+b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d+3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2+3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-
b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^2-6*(a^2+b^2)*f^2*(f*x+e)*polylog(
3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^3-6*(a^2+b^2)*f^2*(f*x+e)*polyl
og(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^3+6*(a^2+b^2)*f^3*polylog(4,
-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^4+6*(a^2+b^2)*f^3*polylog(4,-b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^4-6*a*f^2*(f*x+e)*sinh(d*x+c)/b^2/d^3-
a*(f*x+e)^3*sinh(d*x+c)/b^2/d-3/8*f^3*cosh(d*x+c)*sinh(d*x+c)/b/d^4-3/4*f*
(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b/d^2+3/4*f^2*(f*x+e)*sinh(d*x+c)^2/b/d^
3+1/2*(f*x+e)^3*sinh(d*x+c)^2/b/d
```

3.299.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1977 vs. $2(642) = 1284$.

Time = 10.71 (sec) , antiderivative size = 1977, normalized size of antiderivative = 3.08

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(32*(a^2 + b^2)*e^3*x*Coth[c] + 48*(a^2 + b^2)*e^2*f*x^2*Coth[c] + 32*(a^2
 + b^2)*e*f^2*x^3*Coth[c] + 8*(a^2 + b^2)*f^3*x^4*Coth[c] - (16*(a^2 + b^2
)*(4*e^3*E^(2*c)*x + 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)*f
^3*x^4 + (4*a*Sqrt[a^2 + b^2]*e^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b
^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (4*a*Sqrt[-a^2 - b^2]*e^3*ArcTanh[(a + b*
E^(c + d*x))/Sqrt[a^2 + b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) - (2*e^3*E^(2*c)*L
og[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))])/d + (2*e^3*Log[2*a*E^(c + d*x
) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2
*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 +
(b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f
^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d +
 (2*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])]
)/d - (2*E^(2*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^
2)*E^(2*c)])])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2
 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^
c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x)
)/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (
b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[
1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2...
```

3.299.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.55 (sec) , antiderivative size = 591, normalized size of antiderivative = 0.92, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.036, Rules used = {6099, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 5969, 3042, 25, 3792, 17, 25, 3042, 25, 3115, 24, 6095, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6099}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^3\cosh(c+dx)dx}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \ 3777$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int (e+fx)^2\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\int (e+fx)^2\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \ 3777$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \ 3042$$

3.299. $\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \; 26$$

3.299. $\displaystyle\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \color{blue}{3118}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \color{blue}{5969}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}-\frac{3f\int (e+fx)^2\sinh^2(c+dx)dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}-\frac{3f\int -(e+fx)^2\sin(ic+idx)^2dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \color{blue}{25}$$

3.299. $\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\int (e+fx)^2\sin(ic+idx)^2dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \ 3792$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{3f\left(\frac{f^2\int -\sinh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^2dx+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \ 17$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{3f\left(\frac{f^2\int -\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{3f\left(-\frac{f^2\int \sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

3.299. $\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\left(-\frac{f^2\int -\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}}{}$$

$$\Big\downarrow \text{25}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\left(\frac{f^2\int \sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{3f\left(\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{24}$$

3.299. $\displaystyle\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{6095}$$

$$\frac{\frac{(a^2+b^2)\left(\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{(a^2+b^2)\left(-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log}{}\right.}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{3011}$$

3.299. $\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\left(a^2+b^2\right)\left(-\frac{3f\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\right.}{bd}\right.}{b^2}$$

$$\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{7163}$$

$$\frac{\left(a^2+b^2\right)\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+}{}\right.}{}\right.}{}\right.}{}$$

$$\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{2720}$$

3.299. $\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{(a^2+b^2)\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\right.}{}$$

$$\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \; 7143$$

$$\frac{(a^2+b^2)\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(2f\left(\frac{(e+fx)}{}\right.\right.}{}\right.}{}$$

$$\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

3.299. $\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
((a^2 + b^2)*(-1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/b^2 - (a*(((e + f*x)^3*Sinh[c
 + d*x])/d + ((3*I)*f*((I*(e + f*x)^2*Cosh[c + d*x])/d - ((2*I)*f*(-((f*Co
sh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d))/d))/d))/b^2 + (((e + f*x
)^3*Sinh[c + d*x]^2)/(2*d) + (3*f*((e + f*x)^3/(6*f) - ((e + f*x)^2*Cosh[c
 + d*x]*Sinh[c + d*x])/(2*d) + (f*(e + f*x)*Sinh[c + d*x]^2)/(2*d^2) + (f^
2*(x/2 - (Cosh[c + d*x]*Sinh[c + d*x])/(2*d)))/(2*d^2)))/(2*d))/b
```

3.299.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.299. $\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.299. $\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+b\sinh(c+dx)} \, dx$

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.299.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh (dx+c)^3}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.299.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4371 vs. $2(602) = 1204$.

Time = 0.31 (sec) , antiderivative size = 4371, normalized size of antiderivative = 6.81

$$\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/32*(4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 + 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2
+ 3*b^2*f^3 + (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2
*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^
3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c)^4 + (4*b^2*d^3*f^3
*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*
b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 +
b^2*d*f^3)*x)*sinh(d*x + c)^4 - 16*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*b*
d^2*e^2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^
2 + 3*(a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c)^3 -
 4*(4*a*b*d^3*f^3*x^3 + 4*a*b*d^3*e^3 - 12*a*b*d^2*e^2*f + 24*a*b*d*e*f^2
- 24*a*b*f^3 + 12*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 12*(a*b*d^3*e^2*f -
2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x - (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*
b^2*d^2*e^2*f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f
^3)*x^2 + 6*(2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x +
c))*sinh(d*x + c)^3 + 6*(2*b^2*d^3*e*f^2 + b^2*d^2*f^3)*x^2 - 8*((a^2 + b^
2)*d^4*f^3*x^4 + 4*(a^2 + b^2)*d^4*e*f^2*x^3 + 6*(a^2 + b^2)*d^4*e^2*f*x^2
 + 4*(a^2 + b^2)*d^4*e^3*x + 8*(a^2 + b^2)*c*d^3*e^3 - 12*(a^2 + b^2)*c^2*
d^2*e^2*f + 8*(a^2 + b^2)*c^3*d*e*f^2 - 2*(a^2 + b^2)*c^4*f^3)*cosh(d*x +
c)^2 - 2*(4*(a^2 + b^2)*d^4*f^3*x^4 + 16*(a^2 + b^2)*d^4*e*f^2*x^3 + 24*(a
^2 + b^2)*d^4*e^2*f*x^2 + 16*(a^2 + b^2)*d^4*e^3*x + 32*(a^2 + b^2)*c*d...
```

3.299.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.299.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*e^3*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - 8*(a^2 + b^2)*(
d*x + c)/(b^3*d) - (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) - 8*(a^
2 + b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d)) + 1/32*(
8*(a^2*d^4*f^3*e^(2*c) + b^2*d^4*f^3*e^(2*c))*x^4 + 32*(a^2*d^4*e*f^2*e^(2
*c) + b^2*d^4*e*f^2*e^(2*c))*x^3 + 48*(a^2*d^4*e^2*f*e^(2*c) + b^2*d^4*e^2
*f*e^(2*c))*x^2 + (4*b^2*d^3*f^3*x^3*e^(4*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*b
^2*x^2*e^(4*c) + 6*(2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*b^2*x*e^(4*c) - 3*(
2*d^2*e^2*f - 2*d*e*f^2 + f^3)*b^2*e^(4*c))*e^(2*d*x) - 16*(a*b*d^3*f^3*x^
3*e^(3*c) + 3*(d^3*e*f^2 - d^2*f^3)*a*b*x^2*e^(3*c) + 3*(d^3*e^2*f - 2*d^2
*e*f^2 + 2*d*f^3)*a*b*x*e^(3*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b*e^
(3*c))*e^(d*x) + 16*(a*b*d^3*f^3*x^3*e^c + 3*(d^3*e*f^2 + d^2*f^3)*a*b*x^2
*e^c + 3*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*a*b*x*e^c + 3*(d^2*e^2*f + 2*
d*e*f^2 + 2*f^3)*a*b*e^c)*e^(-d*x) + (4*b^2*d^3*f^3*x^3 + 6*(2*d^3*e*f^2 +
 d^2*f^3)*b^2*x^2 + 6*(2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*b^2*x + 3*(2*d^2
*e^2*f + 2*d*e*f^2 + f^3)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^4) - integrate(
-2*((a^2*b*f^3 + b^3*f^3)*x^3 + 3*(a^2*b*e*f^2 + b^3*e*f^2)*x^2 + 3*(a^2*b
*e^2*f + b^3*e^2*f)*x - ((a^3*f^3*e^c + a*b^2*f^3*e^c)*x^3 + 3*(a^3*e*f^2*
e^c + a*b^2*e*f^2*e^c)*x^2 + 3*(a^3*e^2*f*e^c + a*b^2*e^2*f*e^c)*x)*e^(d*x
))/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

3.299.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{{\mathrm{cosh}\left(c+d\,x\right)}^3\,{\left(e+f\,x\right)}^3}{a+b\,\mathrm{sinh}\left(c+d\,x\right)}\,dx$$

input

```
int((cosh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output

```
int((cosh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.300 $\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.300.1 Optimal result

Integrand size = 28, antiderivative size = 477

$$\begin{aligned}\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{efx}{2bd}+\frac{f^2x^2}{4bd}-\frac{(a^2+b^2)(e+fx)^3}{3b^3f}\\
&+\frac{2af(e+fx)\cosh(c+dx)}{b^2d^2}\\
&+\frac{(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}\\
&+\frac{(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d}\\
&+\frac{2(a^2+b^2)f(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}\\
&+\frac{2(a^2+b^2)f(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}\\
&-\frac{2(a^2+b^2)f^2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}\\
&-\frac{2(a^2+b^2)f^2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3}\\
&-\frac{2af^2\sinh(c+dx)}{b^2d^3}-\frac{a(e+fx)^2\sinh(c+dx)}{b^2d}\\
&-\frac{f(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd^2}\\
&+\frac{f^2\sinh^2(c+dx)}{4bd^3}+\frac{(e+fx)^2\sinh^2(c+dx)}{2bd}\end{aligned}$$

output
```
1/2*e*f*x/b/d+1/4*f^2*x^2/b/d-1/3*(a^2+b^2)*(f*x+e)^3/b^3/f+2*a*f*(f*x+e)*
cosh(d*x+c)/b^2/d^2+(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/b^3/d+(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/
d+2*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d
^2+2*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/
d^2-2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^3-2
*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^3-2*a*f^
2*sinh(d*x+c)/b^2/d^3-a*(f*x+e)^2*sinh(d*x+c)/b^2/d-1/2*f*(f*x+e)*cosh(d*x
+c)*sinh(d*x+c)/b/d^2+1/4*f^2*sinh(d*x+c)^2/b/d^3+1/2*(f*x+e)^2*sinh(d*x+c
)^2/b/d
```

3.300.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1253 vs. $2(477) = 954$.

Time = 9.31 (sec) , antiderivative size = 1253, normalized size of antiderivative = 2.63

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{8(a^2+b^2)\,x(3e^2+3efx+f^2x^2)\coth(c) - \frac{8(a^2+b^2)\left(6e^2e^{2c}x+6ee^{2c}fx^2+2e^{2c}f^2x^3+\frac{6a\sqrt{a^2+b^2}e^2\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-\left(a^2+b^2\right)^2}d}+\frac{6a\sqrt{-(a \cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(8*(a^2 + b^2)*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Coth[c] - (8*(a^2 + b^2)*(6*e
^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2
]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*
d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt
[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTa
nh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqr
t[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]]
)/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c +
d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))
])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c
)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*
E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/
(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2
*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b
*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x
^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*
(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -...
```

3.300.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.34 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.90, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.679$, Rules used = {6099, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 5969, 3042, 25, 3791, 17, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \textcolor{blue}{6099}$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^2\cosh(c+dx)dx}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

3.300. $\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 3117$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 5969$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int (e+fx)\sinh^2(c+dx)dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int -\left((e+fx)\sin(ic+idx)^2\right)dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 25$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}+\frac{f\int (e+fx)\sin(ic+idx)^2dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 3791$$

$$\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{f\left(\frac{1}{2}\int (e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 17$$

3.300. $\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{6095}$$

$$\frac{\left(a^2+b^2\right)\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\left(a^2+b^2\right)\left(-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\right.}{b}\right.}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\left(a^2+b^2\right)\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(\right.}{d}\right.}{bd}\right.}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}$$

3.300. $\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{2720}$$

$$\frac{(a^2+b^2)\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\cdots}{d^2}\cdots\right.}{\cdots}\cdots\right.}{b^2}\cdots$$

$$\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{7143}$$

$$\frac{(a^2+b^2)\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{\cdots}}{a+\sqrt{\cdots}}\cdots\right)}{d}\cdots\right.}{bd}\cdots\right.}{b^2}\cdots$$

$$\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
((a^2 + b^2)*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/d^2))/(b*d)))/b^2 - (a*(((e + f*x)^2*Sinh[c + d*x])/d + (
(2*I)*f*((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/d))/b^2
 + (((e + f*x)^2*Sinh[c + d*x]^2)/(2*d) + (f*((e + f*x)^2/(4*f) - ((e + f*
x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (f*Sinh[c + d*x]^2)/(4*d^2)))/d)/b
```

3.300. $\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.300.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.300. $\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.300. $\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.300.4 Maple [F]

$$\int \frac{(fx+e)^2 \cosh (dx+c)^3}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.300.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2726 vs. $2(445) = 890$.

Time = 0.30 (sec) , antiderivative size = 2726, normalized size of antiderivative = 5.71

$$\int \frac{(e+fx)^2 \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/48*(6*b^2*d^2*f^2*x^2 + 6*b^2*d^2*e^2 + 6*b^2*d*e*f + 3*(2*b^2*d^2*f^2*x
^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)
*x)*cosh(d*x + c)^4 + 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f +
 b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*sinh(d*x + c)^4 + 3*b^2*f^2 -
24*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e
*f - a*b*d*f^2)*x)*cosh(d*x + c)^3 - 12*(2*a*b*d^2*f^2*x^2 + 2*a*b*d^2*e^2
 - 4*a*b*d*e*f + 4*a*b*f^2 + 4*(a*b*d^2*e*f - a*b*d*f^2)*x - (2*b^2*d^2*f^
2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f
^2)*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 16*((a^2 + b^2)*d^3*f^2*x^3 + 3*(a
^2 + b^2)*d^3*e*f*x^2 + 3*(a^2 + b^2)*d^3*e^2*x + 6*(a^2 + b^2)*c*d^2*e^2
- 6*(a^2 + b^2)*c^2*d*e*f + 2*(a^2 + b^2)*c^3*f^2)*cosh(d*x + c)^2 - 2*(8*
(a^2 + b^2)*d^3*f^2*x^3 + 24*(a^2 + b^2)*d^3*e*f*x^2 + 24*(a^2 + b^2)*d^3*
e^2*x + 48*(a^2 + b^2)*c*d^2*e^2 - 48*(a^2 + b^2)*c^2*d*e*f + 16*(a^2 + b^
2)*c^3*f^2 - 9*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2
+ 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*cosh(d*x + c)^2 + 36*(a*b*d^2*f^2*x^2 +
 a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*co
sh(d*x + c))*sinh(d*x + c)^2 + 6*(2*b^2*d^2*e*f + b^2*d*f^2)*x + 24*(a*b*d
^2*f^2*x^2 + a*b*d^2*e^2 + 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f + a*b*
d*f^2)*x)*cosh(d*x + c) + 96*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*co
sh(d*x + c)^2 + 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + ...
```

3.300.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.300.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*e^2*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - 8*(a^2 + b^2)*(
d*x + c)/(b^3*d) - (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) - 8*(a^
2 + b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d)) + 1/48*(
16*(a^2*d^3*f^2*e^(2*c) + b^2*d^3*f^2*e^(2*c))*x^3 + 48*(a^2*d^3*e*f*e^(2*
c) + b^2*d^3*e*f*e^(2*c))*x^2 + 3*(2*b^2*d^2*f^2*x^2*e^(4*c) + 2*(2*d^2*e*
f - d*f^2)*b^2*x*e^(4*c) - (2*d*e*f - f^2)*b^2*e^(4*c))*e^(2*d*x) - 24*(a*
b*d^2*f^2*x^2*e^(3*c) + 2*(d^2*e*f - d*f^2)*a*b*x*e^(3*c) - 2*(d*e*f - f^2
)*a*b*e^(3*c))*e^(d*x) + 24*(a*b*d^2*f^2*x^2*e^c + 2*(d^2*e*f + d*f^2)*a*b
*x*e^c + 2*(d*e*f + f^2)*a*b*e^c)*e^(-d*x) + 3*(2*b^2*d^2*f^2*x^2 + 2*(2*d
^2*e*f + d*f^2)*b^2*x + (2*d*e*f + f^2)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^3
) - integrate(-2*((a^2*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*e*f + b^3*e*f)*x -
((a^3*f^2*e^c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e*f*e^c)*x)*e^
(d*x))/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

3.300.8 Giac [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.301 $\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.301.1 Optimal result

Integrand size = 26, antiderivative size = 298

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{fx}{4bd} - \frac{(a^2+b^2)(e+fx)^2}{2b^3f} + \frac{af\cosh(c+dx)}{b^2d^2} + \frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d} + \frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d} + \frac{(a^2+b^2)f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2} + \frac{(a^2+b^2)f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2} - \frac{a(e+fx)\sinh(c+dx)}{b^2d} - \frac{f\cosh(c+dx)\sinh(c+dx)}{4bd^2} + \frac{(e+fx)\sinh^2(c+dx)}{2bd}$$

output
```
1/4*f*x/b/d-1/2*(a^2+b^2)*(f*x+e)^2/b^3/f+a*f*cosh(d*x+c)/b^2/d^2+(a^2+b^2
)*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+(a^2+b^2)*(f*x+e)*l
n(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d+(a^2+b^2)*f*polylog(2,-b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2+(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/b^3/d^2-a*(f*x+e)*sinh(d*x+c)/b^2/d-1/4*f*cosh(d*x+c)*si
nh(d*x+c)/b/d^2+1/2*(f*x+e)*sinh(d*x+c)^2/b/d
```

3.301.2 Mathematica [A] (verified)

Time = 3.57 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.40

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{8abf\cosh(c+dx)+2b^2d(e+fx)\cosh(2(c+dx))+4(a^2+b^2)\left(-2de(c+dx)+2cf(c+dx)-f(c+dx\right.}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(8*a*b*f*Cosh[c + d*x] + 2*b^2*d*(e + f*x)*Cosh[2*(c + d*x)] + 4*(a^2 + b^
2)*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b
^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2]
 - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^
2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(
c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
]))]) - 8*a*b*d*(e + f*x)*Sinh[c + d*x] - b^2*f*Sinh[2*(c + d*x)])/(8*b^3*
d^2)
```

3.301.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.92, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {6099, 3042, 3777, 26, 3042, 26, 3118, 5969, 3042, 25, 3115, 24, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6099}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)\cosh(c+dx)dx}{b^2}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}$$

$$\Big\downarrow \text{5969}$$

3.301. $\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int \sinh^2(c+dx)dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int -\sin(ic+idx)^2dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\int \sin(ic+idx)^2dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{f\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{\left(a^2+b^2\right)\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}$$

$$\Big\downarrow \text{6095}$$

$$\frac{\left(a^2+b^2\right)\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}$$

$$\Big\downarrow \text{2620}$$

3.301. $\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{(a^2+b^2)\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\right.}{b^2}$$

$$\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}$$

$$\Big\downarrow \; 2715$$

$$\frac{(a^2+b^2)\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)}{}\right.}{b^2}$$

$$\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}$$

$$\Big\downarrow \; 2838$$

$$\frac{(a^2+b^2)\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\right.}{b^2}$$

$$\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
((a^2 + b^2)*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
/(b*d^2)))/b^2 - (a*(-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/
d))/b^2 + (((e + f*x)*Sinh[c + d*x]^2)/(2*d) + (f*(x/2 - (Cosh[c + d*x]*Si
nh[c + d*x])/(2*d)))/(2*d))/b
```

3.301. $\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.301.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

3.301. $\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

3.301.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(278) = 556$.

Time = 4.12 (sec) , antiderivative size = 975, normalized size of antiderivative = 3.27

method	result
risch	$-\frac{a^2 f c^2}{d^2 b^3}+\frac{a^2 f \operatorname{dilog}\left(\frac{-b\,e^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)}{d^2 b^3}+\frac{a^2 f \operatorname{dilog}\left(\frac{b\,e^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)}{d^2 b^3}-\frac{2a^2 e \ln\left(e^{dx+c}\right)}{d\,b^3}+\frac{a^2 e \ln\left(b\,e^{2dx+2c}+2a\,e^{d}\right.}{d\,b^3}$

input
```
int((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/d^2/b^3*a^2*f*c^2+1/d^2/b^3*a^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-
a)/(-a+(a^2+b^2)^(1/2)))+1/d^2/b^3*a^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/
2)+a)/(a+(a^2+b^2)^(1/2)))-2/d/b^3*a^2*e*ln(exp(d*x+c))+1/d/b^3*a^2*e*ln(b
*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d/b^3*a^2*f*c*x-1/2*a^2*f*x^2/b^3+1/16
*(2*d*f*x+2*d*e-f)/b/d^2*exp(2*d*x+2*c)-1/d^2/b*f*c^2-2/d/b*e*ln(exp(d*x+c
))+1/d/b*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d^2/b*f*dilog((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2/b*f*dilog((b*exp(d*x+
c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp
(d*x+c)-1/d^2/b^3*c*a^2*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/2*f*x^2/
b+1/d^2/b^3*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)
))*c+1/d^2/b^3*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))*c+2/d^2/b^3*c*a^2*f*ln(exp(d*x+c))+1/d/b^3*a^2*f*ln((-b*exp(d*x+c)+(a^
2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d/b^3*a^2*f*ln((b*exp(d*x+c)+(a^
2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+e*x/b-1/d^2/b*c*f*ln(b*exp(2*d*x+2*
c)+2*a*exp(d*x+c)-b)+1/d/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b
^2)^(1/2)))*x+1/d/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(
1/2)))*x+1/d^2/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))*c+2/d^2/b*c*f*ln(exp(d*x+c))-2/d/b*c*f*x+1/d^2/b*f*ln((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/16*(2*d*f*x+2*d*e+f)/b/d^2*exp(-
2*d*x-2*c)+a^2*e*x/b^3+1/2*a*(d*f*x+d*e+f)/b^2/d^2*exp(-d*x-c)
```

3.301.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. $2(276) = 552$.

Time = 0.26 (sec) , antiderivative size = 1416, normalized size of antiderivative = 4.75

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*(2*b^2*d*f*x + (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^4 + (2
*b^2*d*f*x + 2*b^2*d*e - b^2*f)*sinh(d*x + c)^4 + 2*b^2*d*e - 8*(a*b*d*f*x
 + a*b*d*e - a*b*f)*cosh(d*x + c)^3 - 4*(2*a*b*d*f*x + 2*a*b*d*e - 2*a*b*f
 - (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*
f - 8*((a^2 + b^2)*d^2*f*x^2 + 2*(a^2 + b^2)*d^2*e*x + 4*(a^2 + b^2)*c*d*e
 - 2*(a^2 + b^2)*c^2*f)*cosh(d*x + c)^2 - 2*(4*(a^2 + b^2)*d^2*f*x^2 + 8*(
a^2 + b^2)*d^2*e*x + 16*(a^2 + b^2)*c*d*e - 8*(a^2 + b^2)*c^2*f - 3*(2*b^2
*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^2 + 12*(a*b*d*f*x + a*b*d*e - a*
b*f)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*(a*b*d*f*x + a*b*d*e + a*b*f)*cosh
(d*x + c) + 16*((a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(a^2 + b^2)*f*cosh(d*x +
 c)*sinh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c)
+ a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b + 1) + 16*((a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(a^2 + b^2)*f*cosh
(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c)^2)*dilog((a*cosh(d*x
 + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) + 16*(((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x +
 c)^2 + 2*((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)
+ ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*(((a^2 + b^
2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d*e - (a^2 +...
```

3.301.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.301.7 Maxima [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\cosh{(dx+c)}^3}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*e*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - 8*(a^2 + b^2)*(d*
x + c)/(b^3*d) - (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) - 8*(a^2
+ b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d)) + 1/16*f*(
(8*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*x^2 + (2*b^2*d*x*e^(4*c) - b^2*e^(4
*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) - a*b*e^(3*c))*e^(d*x) + 8*(a*b*d*x*e^
c + a*b*e^c)*e^(-d*x) + (2*b^2*d*x + b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^2) -
 2*integrate(16*((a^3*e^c + a*b^2*e^c)*x*e^(d*x) - (a^2*b + b^3)*x)/(b^4*e
^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x))
```

3.301.8 Giac [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\cosh{(dx+c)}^3}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.301.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{{\cosh(c+d\,x)}^3\,(e+f\,x)}{a+b\,\sinh{(c+d\,x)}}\,dx$$

input
```
int((cosh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.302 $\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.302.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{b^3d}-\frac{a\sinh(c+dx)}{b^2d}+\frac{\sinh^2(c+dx)}{2bd}$$

output
```
(a^2+b^2)*ln(a+b*sinh(d*x+c))/b^3/d-a*sinh(d*x+c)/b^2/d+1/2*sinh(d*x+c)^2/
b/d
```

3.302.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{-((a^2+b^2)\log(a+b\sinh(c+dx)))+ab\sinh(c+dx)-\frac{1}{2}b^2\sinh^2(c+dx)}{b^3d}$$

input
```
Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-((a^2 + b^2)*Log[a + b*Sinh[c + d*x]]) + a*b*Sinh[c + d*x] - (b^2*Sinh
[c + d*x]^2)/2)/(b^3*d))
```

3.302.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3147, 25, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$
$$\Big\downarrow \text{3147}$$
$$-\frac{\int -\frac{\sinh^2(c+dx)b^2+b^2}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{\sinh^2(c+dx)b^2+b^2}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$
$$\Big\downarrow \text{476}$$
$$\frac{\int \left(-a+b\sinh(c+dx)+\frac{a^2+b^2}{a+b\sinh(c+dx)}\right)d(b\sinh(c+dx))}{b^3d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{-\left(a^2+b^2\right)\log(a+b\sinh(c+dx))+ab\sinh(c+dx)-\frac{1}{2}b^2\sinh^2(c+dx)}{b^3d}$$

input
```
Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-((a^2 + b^2)*Log[a + b*Sinh[c + d*x]]) + a*b*Sinh[c + d*x] - (b^2*Sinh
[c + d*x]^2)/2)/(b^3*d))
```

3.302.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.302.4 Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{-\frac{\sinh(dx+c)^2 b}{2}+a\sinh(dx+c)}{b^2}+\frac{(a^2+b^2)\ln(a+b\sinh(dx+c))}{b^3}}{d}$
default	$\frac{-\frac{-\frac{\sinh(dx+c)^2 b}{2}+a\sinh(dx+c)}{b^2}+\frac{(a^2+b^2)\ln(a+b\sinh(dx+c))}{b^3}}{d}$
risch	$-\frac{x a^2}{b^3}-\frac{x}{b}+\frac{e^{2dx+2c}}{8bd}-\frac{a\,e^{dx+c}}{2b^2d}+\frac{a\,e^{-dx-c}}{2b^2d}+\frac{e^{-2dx-2c}}{8bd}-\frac{2a^2c}{b^3d}-\frac{2c}{bd}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)a^2}{b^3d}+$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^2*(-1/2*sinh(d*x+c)^2*b+a*sinh(d*x+c))+(a^2+b^2)/b^3*ln(a+b*sinh
(d*x+c)))
```

3.302.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(57) = 114.

Time = 0.24 (sec) , antiderivative size = 327, normalized size of antiderivative = 5.54

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{b^2\cosh(dx+c)^4 + b^2\sinh(dx+c)^4 - 8(a^2+b^2)dx\cosh(dx+c)^2 - 4ab\cosh(dx+c)^3 + 4(b^2\cosh(dx-}{}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/8*(b^2*cosh(d*x + c)^4 + b^2*sinh(d*x + c)^4 - 8*(a^2 + b^2)*d*x*cosh(d*
x + c)^2 - 4*a*b*cosh(d*x + c)^3 + 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x +
c)^3 + 4*a*b*cosh(d*x + c) + 2*(3*b^2*cosh(d*x + c)^2 - 4*(a^2 + b^2)*d*x
- 6*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + b^2 + 8*((a^2 + b^2)*cosh(d*x + c
)^2 + 2*(a^2 + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*sinh(d*x + c
)^2)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(b^2
*cosh(d*x + c)^3 - 4*(a^2 + b^2)*d*x*cosh(d*x + c) - 3*a*b*cosh(d*x + c)^2
 + a*b)*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 + 2*b^3*d*cosh(d*x + c)*sinh
(d*x + c) + b^3*d*sinh(d*x + c)^2)
```

3.302.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.302.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(57) = 114$.

Time = 0.23 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.15

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^2d} + \frac{(a^2+b^2)(dx+c)}{b^3d} + \frac{4\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}}{8\,b^2d} + \frac{(a^2+b^2)\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{b^3d}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) + (a^2 + b^2)*(d*x + c
)/(b^3*d) + 1/8*(4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) + (a^2 + b
^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d)
```

3.302.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.56

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-4\,a\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{b^2}+\frac{8\,(a^2+b^2)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^3}}{8\,d}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/8*((b*(e^(d*x + c) - e^(-d*x - c))^2 - 4*a*(e^(d*x + c) - e^(-d*x - c)))
/b^2 + 8*(a^2 + b^2)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/b^3)/d
```

3.302.9 Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.03

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\mathrm{e}^{-2\,c-2\,d\,x}}{8\,b\,d} - \frac{x\,(a^2+b^2)}{b^3} + \frac{\mathrm{e}^{2\,c+2\,d\,x}}{8\,b\,d} + \frac{\ln\left(2\,a\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c} - b + b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)\,(a^2+b^2)}{b^3\,d} + \frac{a\,\mathrm{e}^{-c-d\,x}}{2\,b^2\,d} - \frac{a\,\mathrm{e}^{c+d\,x}}{2\,b^2\,d}$$

input
```
int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)),x)
```

output
```
exp(- 2*c - 2*d*x)/(8*b*d) - (x*(a^2 + b^2))/b^3 + exp(2*c + 2*d*x)/(8*b*d
) + (log(2*a*exp(d*x)*exp(c) - b + b*exp(2*c)*exp(2*d*x))*(a^2 + b^2))/(b^
3*d) + (a*exp(- c - d*x))/(2*b^2*d) - (a*exp(c + d*x))/(2*b^2*d)
```

3.303 $\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.303.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.303.2 Mathematica [N/A]

Not integrable

Time = 31.58 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Cosh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Cosh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.303.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Cosh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.303.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.303.4 Maple [N/A] (verified)

Not integrable

Time = 0.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.303.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh{(dx+c)}^3}{(fx+e)(b\sinh{(dx+c)}+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(cosh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.303.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.303.7 Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 251, normalized size of antiderivative = 8.96

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh{(dx+c)}^3}{(fx+e)(b\sinh{(dx+c)}+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) + 1/2*a*e^
(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c - d*e/f
)*exp_integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f)*exp_in
tegral_e(1, -2*(f*x + e)*d/f)/(b*f) + (a^2 + b^2)*log(f*x + e)/(b^3*f) - 1
/8*integrate(16*(a^2*b + b^3 - (a^3*e^c + a*b^2*e^c)*e^(d*x))/(b^4*f*x + b
^4*e - (b^4*f*x*e^(2*c) + b^4*e*e^(2*c))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*
b^3*e*e^c)*e^(d*x)), x)
```

3.303.8 Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh{(dx+c)}^3}{(fx+e)(b\sinh{(dx+c)}+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.303.9 Mupad [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^3}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(cosh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(cosh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.304 $\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.304.1 Optimal result

Integrand size = 26, antiderivative size = 786

$$\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2a(e+fx)^3\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d}$$
$$+ \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d}$$
$$- \frac{b(e+fx)^3\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)\,d}$$
$$- \frac{3iaf(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^2}$$
$$+ \frac{3iaf(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^2}$$
$$+ \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^2}$$
$$+ \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^2}$$
$$- \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2\,(a^2+b^2)\,d^2}$$
$$+ \frac{6iaf^2(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{(a^2+b^2)\,d^3}$$
$$- \frac{6iaf^2(e+fx)\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{(a^2+b^2)\,d^3}$$
$$- \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^3}$$
$$- \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^3}$$
$$+ \frac{3bf^2(e+fx)\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2\,(a^2+b^2)\,d^3}$$
$$- \frac{6iaf^3\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)}{(a^2+b^2)\,d^4} + \frac{6iaf^3\operatorname{PolyLog}\left(4,ie^{c+dx}\right)}{(a^2+b^2)\,d^4}$$
$$+ \frac{6bf^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^4}$$
$$+ \frac{6bf^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^4}$$
$$- \frac{3bf^3\operatorname{PolyLog}\left(4,-e^{2(c+dx)}\right)}{4\,(a^2+b^2)\,d^4}$$

output
```
2*a*(f*x+e)^3*arctan(exp(d*x+c))/(a^2+b^2)/d-b*(f*x+e)^3*ln(1+exp(2*d*x+2*
c))/(a^2+b^2)/d+b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^
2)/d+b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d-6*I*a*
f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/(a^2+b^2)/d^3-3*I*a*f*(f*x+e)^2*polylo
g(2,-I*exp(d*x+c))/(a^2+b^2)/d^2-3/2*b*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*
c))/(a^2+b^2)/d^2+3*b*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/(a^2+b^2)/d^2+3*b*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/(a^2+b^2)/d^2+6*I*a*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/(a^2+b^2)/d
^3+6*I*a*f^3*polylog(4,I*exp(d*x+c))/(a^2+b^2)/d^4+3/2*b*f^2*(f*x+e)*polyl
og(3,-exp(2*d*x+2*c))/(a^2+b^2)/d^3-6*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c
)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^3-6*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^3+3*I*a*f*(f*x+e)^2*polylog(2,I*exp(d*
x+c))/(a^2+b^2)/d^2-6*I*a*f^3*polylog(4,-I*exp(d*x+c))/(a^2+b^2)/d^4-3/4*b
*f^3*polylog(4,-exp(2*d*x+2*c))/(a^2+b^2)/d^4+6*b*f^3*polylog(4,-b*exp(d*x
+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^4+6*b*f^3*polylog(4,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/(a^2+b^2)/d^4
```

3.304.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3078 vs. $2(786) = 1572$.

Time = 11.35 (sec) , antiderivative size = 3078, normalized size of antiderivative = 3.92

$$\int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(8*b*d^4*e^3*E^(2*c)*x + 12*b*d^4*e^2*E^(2*c)*f*x^2 + 8*b*d^4*e*E^(2*c)*f^
2*x^3 + 2*b*d^4*E^(2*c)*f^3*x^4 + 8*a*d^3*e^3*ArcTan[E^(c + d*x)] + 8*a*d^
3*e^3*E^(2*c)*ArcTan[E^(c + d*x)] + (12*I)*a*d^3*e^2*f*x*Log[1 - I*E^(c +
d*x)] + (12*I)*a*d^3*e^2*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (12*I)*a*d^3
*e*f^2*x^2*Log[1 - I*E^(c + d*x)] + (12*I)*a*d^3*e*E^(2*c)*f^2*x^2*Log[1 -
 I*E^(c + d*x)] + (4*I)*a*d^3*f^3*x^3*Log[1 - I*E^(c + d*x)] + (4*I)*a*d^3
*E^(2*c)*f^3*x^3*Log[1 - I*E^(c + d*x)] - (12*I)*a*d^3*e^2*f*x*Log[1 + I*E
^(c + d*x)] - (12*I)*a*d^3*e^2*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (12*I)
*a*d^3*e*f^2*x^2*Log[1 + I*E^(c + d*x)] - (12*I)*a*d^3*e*E^(2*c)*f^2*x^2*L
og[1 + I*E^(c + d*x)] - (4*I)*a*d^3*f^3*x^3*Log[1 + I*E^(c + d*x)] - (4*I)
*a*d^3*E^(2*c)*f^3*x^3*Log[1 + I*E^(c + d*x)] - 4*b*d^3*e^3*Log[1 + E^(2*(
c + d*x))] - 4*b*d^3*e^3*E^(2*c)*Log[1 + E^(2*(c + d*x))] - 12*b*d^3*e^2*f
*x*Log[1 + E^(2*(c + d*x))] - 12*b*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(2*(c + d
*x))] - 12*b*d^3*e*f^2*x^2*Log[1 + E^(2*(c + d*x))] - 12*b*d^3*e*E^(2*c)*f
^2*x^2*Log[1 + E^(2*(c + d*x))] - 4*b*d^3*f^3*x^3*Log[1 + E^(2*(c + d*x))]
 - 4*b*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(2*(c + d*x))] - (12*I)*a*d^2*(1 + E^
(2*c))*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)] + (12*I)*a*d^2*(1 + E^(2
*c))*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)] - 6*b*d^2*e^2*f*PolyLog[2, -E
^(2*(c + d*x))] - 6*b*d^2*e^2*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))] - 12*
b*d^2*e*f^2*x*PolyLog[2, -E^(2*(c + d*x))] - 12*b*d^2*e*E^(2*c)*f^2*x*P...
```

3.304.3 Rubi [A] (verified)

Time = 2.93 (sec) , antiderivative size = 690, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {6107, 6095, 2620, 3011, 7163, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6107}$$

$$\frac{b^2\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{6095}$$

$$\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$\Big\downarrow$ 2620

$$\frac{b^2\left(-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}\cdots}\right.}{bd}\cdots\right.}{a^2+b^2}$$
$$\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$\Big\downarrow$ 3011

$$\frac{b^2\left(-\frac{3f\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{P}\cdots}{\cdots}\right.}{bd}\cdots\right.}{a^2+b^2}$$
$$\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$\Big\downarrow$ 7163

$$b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLo}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\cdots\right.$$
$$\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$\Big\downarrow$ 2720

3.304. $\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.$$

$$\frac{\int(e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \; 7143$$

$$\frac{\int(e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+$$

$$b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3\cdots\right.}{d}\right.}{\cdots}\right.}{\cdots}\cdots\right.$$

$$\Big\downarrow \; 7293$$

$$\frac{\int\left(a(e+fx)^3\text{sech}(c+dx)-b(e+fx)^3\tanh(c+dx)\right)dx}{a^2+b^2}+$$

$$b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3\cdots\right.}{d}\right.}{\cdots}\right.}{\cdots}\cdots\right.$$

$$\Big\downarrow \; 2009$$

3.304. $\displaystyle\int\frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}(3}{d}\right.}{}\right.}{}\right.$$

$$\frac{2a(e+fx)^3\arctan(e^{c+dx})}{d}-\frac{6iaf^3\operatorname{PolyLog}(4,-ie^{c+dx})}{d^4}+\frac{6iaf^3\operatorname{PolyLog}(4,ie^{c+dx})}{d^4}+\frac{6iaf^2(e+fx)\operatorname{PolyLog}(3,-ie^{c+dx})}{d^3}-\frac{6iaf^2(e+fx)\operatorname{P}}{}$$

input
```
Int[((e + f*x)^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^2*(-1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^4)/(4*f)
+ (2*a*(e + f*x)^3*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^3*Log[1 + E^(2*(c
 + d*x))])/d - ((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/d^2 +
((3*I)*a*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/d^2 - (3*b*f*(e + f*x)^2
*PolyLog[2, -E^(2*(c + d*x))])/(2*d^2) + ((6*I)*a*f^2*(e + f*x)*PolyLog[3,
 (-I)*E^(c + d*x)])/d^3 - ((6*I)*a*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)]
)/d^3 + (3*b*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*d^3) - ((6*I)*
a*f^3*PolyLog[4, (-I)*E^(c + d*x)])/d^4 + ((6*I)*a*f^3*PolyLog[4, I*E^(c +
 d*x)])/d^4 - (3*b*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*d^4))/(a^2 + b^2)
```

3.304. $\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.304.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

3.304. $\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.304.4 Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.304.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1718 vs. 2(720) = 1440.

Time = 0.29 (sec) , antiderivative size = 1718, normalized size of antiderivative = 2.19

$$\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(6*b*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*b*f^3*polylog(4, (a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2))/b) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*dilog
((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b
*d^2*e^2*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(-I*a*d^2*f^3*x^2 +
 b*d^2*f^3*x^2 - 2*I*a*d^2*e*f^2*x + 2*b*d^2*e*f^2*x - I*a*d^2*e^2*f + b*d
^2*e^2*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - 3*(I*a*d^2*f^3*x^2 +
b*d^2*f^3*x^2 + 2*I*a*d^2*e*f^2*x + 2*b*d^2*e*f^2*x + I*a*d^2*e^2*f + b*d^
2*e^2*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (b*d^3*e^3 - 3*b*c*d^
2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*
x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*d^3*e^3 - 3*b*c*d^2*e^2*f +
 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) -
2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*
b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*log(-(a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b) + (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*
f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*log(-(a*cosh(d*x +...
```

3.304.6 Sympy [F]

$$\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^3\operatorname{sech}\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate((f*x+e)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.304.7 Maxima [F]

$$\int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \operatorname{sech}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^3*(2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - b*log(-2*a*e^(-d*x - c) +
 b*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a
^2 + b^2)*d)) + integrate(4*f^3*x^3/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a
)*(e^(d*x + c) + e^(-d*x - c))) + 12*e*f^2*x^2/((b*(e^(d*x + c) - e^(-d*x
- c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))) + 12*e^2*f*x/((b*(e^(d*x + c) -
 e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))), x)
```

3.304.8 Giac [F]

$$\int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \operatorname{sech}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\cosh(c+d\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.305 $\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.305.1 Optimal result

Integrand size = 26, antiderivative size = 558

$$\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2a(e+fx)^2\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d} - \frac{b(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)\,d} - \frac{2iaf(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{2iaf(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^2} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^2} - \frac{bf(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{(a^2+b^2)\,d^2} + \frac{2iaf^2\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{(a^2+b^2)\,d^3} - \frac{2iaf^2\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{(a^2+b^2)\,d^3} - \frac{2bf^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^3} - \frac{2bf^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^3} + \frac{bf^2\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2\,(a^2+b^2)\,d^3}$$

output
```
2*a*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)/d-b*(f*x+e)^2*ln(1+exp(2*d*x+2*
c))/(a^2+b^2)/d+b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^
2)/d+b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d-2*I*a*
f*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^2+2*I*a*f*(f*x+e)*polylog(2
,I*exp(d*x+c))/(a^2+b^2)/d^2-b*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/(a^2+b
^2)/d^2+2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^
2)/d^2+2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2
)/d^2+2*I*a*f^2*polylog(3,-I*exp(d*x+c))/(a^2+b^2)/d^3-2*I*a*f^2*polylog(3
,I*exp(d*x+c))/(a^2+b^2)/d^3+1/2*b*f^2*polylog(3,-exp(2*d*x+2*c))/(a^2+b^2
)/d^3-2*b*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^3-2
*b*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^3
```

3.305.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1639 vs. $2(558) = 1116$.

Time = 10.64 (sec) , antiderivative size = 1639, normalized size of antiderivative = 2.94

$$\int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x - 12*b*d^3*e*f*x^2
- 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] + 6*b*d
^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*a*d*e*(1
+ E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - Poly
Log[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*b*d*e*(1 + E^(2*
c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x)
)]) + (6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*
Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLo
g[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c
 + d*x)]) + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*
x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))
]))/(6*(a^2 + b^2)*d^3*(1 + E^(2*c))) - (b*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*
f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c +
d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)
^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)
^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt
[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*
c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) +
(3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)
*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 + ...
```

3.305.3 Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 501, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6107, 6095, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6107}$$

$$\frac{b^2\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{6095}$$

$$\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{2620}$$

$$\frac{b^2\left(-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right.}{bd}\right.}{a^2+b^2}$$

$$\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{3011}$$

$$\frac{b^2\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+}}{a+\sqrt{a^2}}\right.}{d}\right.}{bd}\right.}{a^2+b^2}$$

$$\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{2720}$$

$$\frac{b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\right.}{bd}\right.}{a^2+b^2}$$

$$\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+$$

$$\frac{b^2\left(-\frac{2f\left(\frac{f\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right.}{a^2+b^2}$$

$$\Big\downarrow \text{7293}$$

3.305. $\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\int \left(a(e+fx)^2\text{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+$$

$$\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right.}{a^2+b^2}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right.}{a^2+b^2}$$

$$\frac{\frac{2a(e+fx)^2\arctan(e^{c+dx})}{d}+\frac{2iaf^2\,\text{PolyLog}(3,-ie^{c+dx})}{d^3}-\frac{2iaf^2\,\text{PolyLog}(3,ie^{c+dx})}{d^3}-\frac{2iaf(e+fx)\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{2iaf(e+fx)\,\text{Poly}}{d^2}}{a^2+b^2}$$

input
```
Int[((e + f*x)^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^2*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/d^2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3)/(3*f) + (2*a*(e + f*
x)^2*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/d -
 ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((2*I)*a*f*(e +
f*x)*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*(e + f*x)*PolyLog[2, -E^(2*(c +
 d*x))])/d^2 + ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/d^3 - ((2*I)*a*f
^2*PolyLog[3, I*E^(c + d*x)])/d^3 + (b*f^2*PolyLog[3, -E^(2*(c + d*x))])/(
2*d^3))/(a^2 + b^2)
```

3.305. $\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.305.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

3.305. $\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.305.4 Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.305.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1094 vs. 2(512) = 1024.

Time = 0.28 (sec) , antiderivative size = 1094, normalized size of antiderivative = 1.96

$$\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*b*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*b*f^2*polylog(3, (a*cosh
(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2))/b) - 2*(b*d*f^2*x + b*d*e*f)*dilog((a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b + 1) - 2*(b*d*f^2*x + b*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c
) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
+ 2*(-I*a*d*f^2*x + b*d*f^2*x - I*a*d*e*f + b*d*e*f)*dilog(I*cosh(d*x + c)
 + I*sinh(d*x + c)) + 2*(I*a*d*f^2*x + b*d*f^2*x + I*a*d*e*f + b*d*e*f)*di
log(-I*cosh(d*x + c) - I*sinh(d*x + c)) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2
*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2
) + 2*a) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(2*b*cosh(d*x + c) + 2
*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b*d^2*f^2*x^2 + 2*b
*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) -
(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*log(-(a*cosh(d*x
 + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b) - (I*a*d^2*e^2 - b*d^2*e^2 - 2*I*a*c*d*e*f + 2*b*c*d*e*f
 + I*a*c^2*f^2 - b*c^2*f^2)*log(cosh(d*x + c) + sinh(d*x + c) + I) - (-I*a
*d^2*e^2 - b*d^2*e^2 + 2*I*a*c*d*e*f + 2*b*c*d*e*f - I*a*c^2*f^2 - b*c^...
```

3.305.6 Sympy [F]

$$\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.305.7 Maxima [F]

$$\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^2*(2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - b*log(-2*a*e^(-d*x - c) +
 b*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a
^2 + b^2)*d)) + integrate(4*f^2*x^2/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a
)*(e^(d*x + c) + e^(-d*x - c))) + 8*e*f*x/((b*(e^(d*x + c) - e^(-d*x - c))
 + 2*a)*(e^(d*x + c) + e^(-d*x - c))), x)
```

3.305.8 Giac [F]

$$\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\cosh\,(c+d\,x)\,\,(a+b\sinh\,(c+d\,x))}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.306 $\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.306.1 Optimal result

Integrand size = 24, antiderivative size = 334

$$\begin{aligned}\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{2a(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d} \\ &\quad + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d} \\ &\quad - \frac{b(e+fx)\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)\,d} - \frac{iaf\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^2} \\ &\quad + \frac{iaf\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{bf\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^2} \\ &\quad + \frac{bf\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d^2} - \frac{bf\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2\,(a^2+b^2)\,d^2}\end{aligned}$$

output
```
2*a*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)/d-b*(f*x+e)*ln(1+exp(2*d*x+2*c))/
(a^2+b^2)/d+b*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d+b
*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d-I*a*f*polylog(
2,-I*exp(d*x+c))/(a^2+b^2)/d^2+I*a*f*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2
-1/2*b*f*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^2+b*f*polylog(2,-b*exp(d*x
+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2+b*f*polylog(2,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/(a^2+b^2)/d^2
```

3.306.2 Mathematica [A] (verified)

Time = 2.79 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.56

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{4ade\arctan\left(e^{c+dx}\right)-4acf\arctan\left(e^{c+dx}\right)+\frac{4ab\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4ab\sqrt{-(a^2+b^2)^2}de\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)}{(-a^2-b^2)^{3/2}}}{}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(4*a*d*e*ArcTan[E^(c + d*x)] - 4*a*c*f*ArcTan[E^(c + d*x)] + (4*a*b*Sqrt[a
^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b
^2)^2] - (4*a*b*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[
a^2 + b^2]])/(-a^2 - b^2)^(3/2) + (2*I)*a*f*(c + d*x)*Log[1 - I*E^(c + d*x
)] - (2*I)*a*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + 2*b*f*(c + d*x)*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*b*f*(c + d*x)*Log[1 + (b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2])] - 2*b*d*e*Log[1 + E^(2*(c + d*x))] + 2*b*c*f
*Log[1 + E^(2*(c + d*x))] - 2*b*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] - 2*b
*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*b*d*e*Log[2*a*E^(c +
 d*x) + b*(-1 + E^(2*(c + d*x)))] - (2*I)*a*f*PolyLog[2, (-I)*E^(c + d*x)]
 + (2*I)*a*f*PolyLog[2, I*E^(c + d*x)] + 2*b*f*PolyLog[2, (b*E^(c + d*x))/
(-a + Sqrt[a^2 + b^2])] + 2*b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2]))] - b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^2)
```

3.306.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \; 6107$$

$$\frac{b^2\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \; 6095$$

$$\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \; 2620$$

$$\frac{b^2\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \; 2715$$

$$\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\cdots\right)}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \; 2838$$

$$\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 7293$$

$$\frac{\int (a(e+fx)\text{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 2009$$

3.306. $\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right.}{a^2+b^2}$$

$$\frac{\frac{2a(e+fx)\arctan(e^{c+dx})}{d}-\frac{iaf\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{iaf\,\text{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{bf\,\text{PolyLog}(2,-e^{2(c+dx)})}{2d^2}-\frac{b(e+fx)\log(e^{2(c+dx)}+1)}{d}+}{a^2+b^2}$$

input
```
Int[((e + f*x)*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^2*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)
))/(a^2 + b^2) + ((b*(e + f*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x
)])/d - (b*(e + f*x)*Log[1 + E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*
E^(c + d*x)])/d^2 + (I*a*f*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2
, -E^(2*(c + d*x))])/(2*d^2))/(a^2 + b^2)
```

3.306.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.306. $\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.306.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(313) = 626$.

Time = 3.25 (sec) , antiderivative size = 954, normalized size of antiderivative = 2.86

method	result
risch	$-\frac{2eb\ln\left(1+\mathrm{e}^{2dx+2c}\right)}{d(2a^2+2b^2)}+\frac{4ea\arctan\left(\mathrm{e}^{dx+c}\right)}{d(2a^2+2b^2)}+\frac{2eb\ln\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)}{d(2a^2+2b^2)}+\frac{2fb\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{d(2a^2+2b^2)}+\frac{2fb\ln\left(\frac{-b}{}\right.}{d}$

input
```
int((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d*e/(2*a^2+2*b^2)*b*ln(1+exp(2*d*x+2*c))+4/d*e/(2*a^2+2*b^2)*a*arctan(e
xp(d*x+c))+2/d*e*b/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d
*f*b/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))*x+2/d^2*f*b/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^
2+b^2)^(1/2)))*c+2/d*f*b/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)
/(a+(a^2+b^2)^(1/2)))*x+2/d^2*f*b/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)
^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d^2*f*b/(2*a^2+2*b^2)*dilog((-b*exp(d*x
+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+2/d^2*f*b/(2*a^2+2*b^2)*dilog
((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/d*f/(2*a^2+2*b^2)
*ln(1+I*exp(d*x+c))*b*x-2/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*b*c-2*I/d
^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))*a-2*I/d^2*f/(2*a^2+2*b^2)*ln(1+I*
exp(d*x+c))*a*c-2/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*b*x-2/d^2*f/(2*a^2+
2*b^2)*ln(1-I*exp(d*x+c))*b*c+2*I/d^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a
*c-2*I/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*x-2/d^2*f/(2*a^2+2*b^2)*dilo
g(1+I*exp(d*x+c))*b+2*I/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*x-2/d^2*f/(
2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))*b+2*I/d^2*f/(2*a^2+2*b^2)*dilog(1-I*exp
(d*x+c))*a+2/d^2*c*f/(2*a^2+2*b^2)*b*ln(1+exp(2*d*x+2*c))-4/d^2*c*f/(2*a^2
+2*b^2)*a*arctan(exp(d*x+c))-2/d^2*c*f*b/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)
+2*a*exp(d*x+c)-b)
```

3.306.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.76

$$\int \frac{(e+fx)\mathrm{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{bf\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)+bf\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx}{b}\right.}{}$$

input
```
integrate((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(b*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + b*f*dilog((a*cosh(d*x + c) +
 a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b + 1) + (I*a*f - b*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) +
(-I*a*f - b*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (b*d*e - b*c*f)
*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) + (b*d*e - b*c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt
((a^2 + b^2)/b^2) + 2*a) + (b*d*f*x + b*c*f)*log(-(a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b) + (b*d*f*x + b*c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (I*a*d*e - b*
d*e - I*a*c*f + b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) + I) + (-I*a*d*e
- b*d*e + I*a*c*f + b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) - I) + (-I*a*
d*f*x - b*d*f*x - I*a*c*f - b*c*f)*log(I*cosh(d*x + c) + I*sinh(d*x + c) +
 1) + (I*a*d*f*x - b*d*f*x + I*a*c*f - b*c*f)*log(-I*cosh(d*x + c) - I*sin
h(d*x + c) + 1))/((a^2 + b^2)*d^2)
```

3.306.6 Sympy [F]

$$\int \frac{(e+fx)\mathrm{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\,\mathrm{sech}\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.306.7 Maxima [F]

$$\int \frac{(e+fx)\mathrm{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\mathrm{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e*(2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - b*log(-2*a*e^(-d*x - c) + b
*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2
 + b^2)*d)) + 2*f*integrate(2*x/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e
^(d*x + c) + e^(-d*x - c))), x)
```

3.306.8 Giac [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\,(c+d\,x)\,\,(a+b\sinh\,(c+d\,x))}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.307 $\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.307.1 Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a\arctan(\sinh(c+dx))}{(a^2+b^2)\,d} - \frac{b\log(\cosh(c+dx))}{(a^2+b^2)\,d} + \frac{b\log(a+b\sinh(c+dx))}{(a^2+b^2)\,d}$$

output
```
a*arctan(sinh(d*x+c))/(a^2+b^2)/d-b*ln(cosh(d*x+c))/(a^2+b^2)/d+b*ln(a+b*s
inh(d*x+c))/(a^2+b^2)/d
```

3.307.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.65

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b\left(\left(-a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\sinh(c+dx)\right)-2\sqrt{-b^2}\log(a+b\sinh(c+dx))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-\right.\right.}{2\sqrt{-b^2}\,(a^2+b^2)\,d}$$

input
```
Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(b*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[c + d*x]] - 2*Sqrt[-b^2
]*Log[a + b*Sinh[c + d*x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh[c +
d*x]]))/(Sqrt[-b^2]*(a^2 + b^2)*d)
```

3.307.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 3147, 25, 479, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3147}$$

$$-\frac{b\int -\frac{1}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int \frac{1}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{479}$$

$$-\frac{b\left(-\frac{\int \frac{a-b\sinh(c+dx)}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))}{a^2+b^2}-\frac{\log(a+b\sinh(c+dx))}{a^2+b^2}\right)}{d}$$

$$\Big\downarrow \text{452}$$

$$-\frac{b\left(-\frac{a\int \frac{1}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))-\int \frac{b\sinh(c+dx)}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))}{a^2+b^2}-\frac{\log(a+b\sinh(c+dx))}{a^2+b^2}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$-\frac{b\left(-\frac{\frac{a\arctan(\sinh(c+dx))}{b}-\int \frac{b\sinh(c+dx)}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))}{a^2+b^2}-\frac{\log(a+b\sinh(c+dx))}{a^2+b^2}\right)}{d}$$

$$\Big\downarrow \text{240}$$

$$-\frac{b\left(-\frac{\frac{a\arctan(\sinh(c+dx))}{b}-\frac{1}{2}\log\left(b^2\sinh^2(c+dx)+b^2\right)}{a^2+b^2}-\frac{\log(a+b\sinh(c+dx))}{a^2+b^2}\right)}{d}$$

input
```
Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
-((b*(-(Log[a + b*Sinh[c + d*x]]/(a^2 + b^2)) - ((a*ArcTan[Sinh[c + d*x]])
/b - Log[b^2 + b^2*Sinh[c + d*x]^2]/2)/(a^2 + b^2)))/d)
```

3.307.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.307.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{\frac{-b\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+2a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2+b^2}+\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^2+b^2}}{d}$
default	$\frac{\frac{-b\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+2a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2+b^2}+\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^2+b^2}}{d}$
risch	$\frac{2b\,d^2x}{a^2d^2+b^2d^2}+\frac{2bdc}{a^2d^2+b^2d^2}-\frac{2bx}{a^2+b^2}-\frac{2bc}{d(a^2+b^2)}+\frac{i\ln(\mathrm{e}^{dx+c}+i)a}{(a^2+b^2)d}-\frac{\ln(\mathrm{e}^{dx+c}+i)b}{(a^2+b^2)d}-\frac{i\ln(\mathrm{e}^{dx+c}-i)a}{(a^2+b^2)d}-\frac{\ln(\mathrm{e}$

input
```
int(sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/(a^2+b^2)*(-1/2*b*ln(1+tanh(1/2*d*x+1/2*c)^2)+a*arctan(tanh(1/2*d*x
+1/2*c)))+b/(a^2+b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a
))
```

3.307.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{2\,a\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)+b\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)-b\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^2+b^2)d}$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*a*arctan(cosh(d*x + c) + sinh(d*x + c)) + b*log(2*(b*sinh(d*x + c) + a)
/(cosh(d*x + c) - sinh(d*x + c))) - b*log(2*cosh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))))/((a^2 + b^2)*d)
```

3.307.6 Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.307.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2\,a\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d} + \frac{b\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^2+b^2)d} - \frac{b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d}$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(-2*a*e^(-d*x - c) + b*e^
(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) - b*log(e^(-2*d*x - 2*c) + 1)/((a^2 +
b^2)*d)
```

3.307.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.75

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,b^2\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^2b+b^3} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)a}{a^2+b^2} - \frac{b\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2}}{2\,d}$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*b^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^2*b + b^3) +
(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*a/(a^2 + b^2) - b*
log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2))/d
```

3.307.9 Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.87

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\ln\left(2a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-4b^3-a^2\,b+4b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+a^2\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+8\,a\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{d\,a^2+d\,b^2} - \frac{\ln\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)}{b\,d+a\,d\,1\mathrm{i}} - \frac{\ln\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)\,1\mathrm{i}}{a\,d+b\,d\,1\mathrm{i}}$$

input
```
int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
(b*log(2*a^3*exp(d*x)*exp(c) - 4*b^3 - a^2*b + 4*b^3*exp(2*c)*exp(2*d*x) +
 a^2*b*exp(2*c)*exp(2*d*x) + 8*a*b^2*exp(d*x)*exp(c)))/(a^2*d + b^2*d) - (
log(exp(c + d*x)*1i + 1)*1i)/(a*d + b*d*1i) - log(exp(c + d*x) + 1i)/(a*d*
1i + b*d)
```

3.308 $\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.308.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.308.2 Mathematica [N/A]

Not integrable

Time = 12.59 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Sech[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Sech[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.308.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.308.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.308.4 Maple [N/A] (verified)

Not integrable

Time = 0.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.308.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.308.6 Sympy [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.308.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.308.8 Giac [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.308.9 Mupad [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.309 $\int \frac{(e+fx)^3 \text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.309.1 Optimal result

Integrand size = 28, antiderivative size = 780

$$\begin{aligned}\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{a(e+fx)^3}{(a^2+b^2)\,d}-\frac{6bf(e+fx)^2\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2}\\
&\quad+\frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}\\
&\quad-\frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}\\
&\quad-\frac{3af(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)\,d^2}\\
&\quad+\frac{6ibf^2(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^3}\\
&\quad-\frac{6ibf^2(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^3}\\
&\quad+\frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2}\\
&\quad-\frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2}\\
&\quad-\frac{3af^2(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{(a^2+b^2)\,d^3}\\
&\quad-\frac{6ibf^3\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{(a^2+b^2)\,d^4}+\frac{6ibf^3\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{(a^2+b^2)\,d^4}\\
&\quad-\frac{6b^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^3}\\
&\quad+\frac{6b^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^3}\\
&\quad+\frac{3af^3\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2\,(a^2+b^2)\,d^4}\\
&\quad+\frac{6b^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^4}\\
&\quad-\frac{6b^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^4}\\
&\quad+\frac{b(e+fx)^3\operatorname{sech}(c+dx)}{(a^2+b^2)\,d}+\frac{a(e+fx)^3\tanh(c+dx)}{(a^2+b^2)\,d}\end{aligned}$$

output
```
a*(f*x+e)^3/(a^2+b^2)/d-6*b*f*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)/d^2-3
*a*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d^2+b^2*(f*x+e)^3*ln(1+b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-b^2*(f*x+e)^3*ln(1+b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-6*I*b*f^2*(f*x+e)*polylog(2,I*
exp(d*x+c))/(a^2+b^2)/d^3+6*I*b*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+
b^2)/d^3-3*a*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^3+3*b^2*f*
(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2
-3*b^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^
(3/2)/d^2-6*I*b*f^3*polylog(3,-I*exp(d*x+c))/(a^2+b^2)/d^4+6*I*b*f^3*polyl
og(3,I*exp(d*x+c))/(a^2+b^2)/d^4+3/2*a*f^3*polylog(3,-exp(2*d*x+2*c))/(a^2
+b^2)/d^4-6*b^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(
a^2+b^2)^(3/2)/d^3+6*b^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))/(a^2+b^2)^(3/2)/d^3+6*b^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/(a^2+b^2)^(3/2)/d^4-6*b^2*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/(a^2+b^2)^(3/2)/d^4+b*(f*x+e)^3*sech(d*x+c)/(a^2+b^2)/d+a*(f*x+e
)^3*tanh(d*x+c)/(a^2+b^2)/d
```

3.309.2 Mathematica [A] (verified)

Time = 8.02 (sec) , antiderivative size = 1072, normalized size of antiderivative = 1.37

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= -\frac{f\left(-12ad^3e^2e^{2c}x+12ad^3e^2\left(1+e^{2c}\right)x+12ad^3efx^2+4ad^3f^2x^3+12bd^2e^2\left(1+e^{2c}\right)\arctan\left(e^{c+dx}\right)-6ad^2e^2\left(1+e^{2c}\right)\left(2dx-\log\left(1+e^{2(c+dx)}\right)\right)+12ib\cdots\right.}{\cdots}$$

input
```
Integrate[((e + f*x)^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-((f*(-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e
*f*x^2 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)]
- 6*a*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*
d*e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)])
 - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1
+ E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c
 + d*x))]) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d
^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x
*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3,
 I*E^(c + d*x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*
(c + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c
+ d*x))]))))/((a^2 + b^2)*(1 + E^(2*c)))) + (2*b^2*(-2*d^3*e^3*ArcTanh[(a +
 b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]
- 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^3*e*f
^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2,
(b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*e*f^2*PolyLog[3, (b*E^(c...
```

3.309.3 Rubi [A] (verified)

Time = 3.22 (sec) , antiderivative size = 654, normalized size of antiderivative = 0.84, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 7163, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6107}$$

$$\frac{b^2\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3803}$$

$$\frac{2b^2\int -\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int \frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}$$

$$\Big\downarrow \ 2694$$

$$\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$$\Big\downarrow \ 2620$$

$$\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{a^2+b^2}$$

$$\Big\downarrow \ 3011$$

$$\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c-}}{a-\sqrt{a}}\right.}{bd}\right.}{}\right.}{a^2+b^2}$$

3.309. $\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{7163}$$

$$\frac{\int (e+fx)^3 \text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2} - 2b^2 \left(\frac{b\left(\frac{(e+fx)^3 \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd} - \frac{3f\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d} - \frac{f\int \text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d} - \frac{(e+fx)^2 \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}}{bd} \right.}{2\sqrt{a^2+b^2}} \right.$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int (e+fx)^3 \text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2} - 2b^2 \left(\frac{b\left(\frac{(e+fx)^3 \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd} - \frac{3f\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d} - \frac{f\int e^{-c-dx}\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d} - \frac{(e+fx)^2 \text{PolyLog}\left(2\right.}{d}}{bd} \right.}{2\sqrt{a^2+b^2}} \right.$$

$$\Big\downarrow \text{7143}$$

3.309. $\int \frac{(e+fx)^3 \text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\int (e+fx)^3 \text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2} - \frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd} - \frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d} - \frac{f\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d} - \frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}$$

$\downarrow$ 7293

$$\frac{\int \left(a(e+fx)^3\text{sech}^2(c+dx) - b(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2} - \frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd} - \frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d} - \frac{f\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d} - \frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a}$$

$\downarrow$ 2009

3.309. $\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\cdots\right)}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}$$

$$\frac{3af^3\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2d^4}-\frac{3af^2(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{d^3}-\frac{3af(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{d^2}+\frac{a(e+fx)^3\tanh(c+dx)}{d}+\frac{a(e+fx)^3}{d}-\cdots$$

input
```
Int[((e + f*x)^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-2*b^2*(-1/2*(b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))
])/d^2))/d))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^3*Log[1 + (b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(a^2 +
 b^2) + ((a*(e + f*x)^3)/d - (6*b*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/d^2 -
 (3*a*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/d^2 + ((6*I)*b*f^2*(e + f*x)
*PolyLog[2, (-I)*E^(c + d*x)])/d^3 - ((6*I)*b*f^2*(e + f*x)*PolyLog[2, I*E
^(c + d*x)])/d^3 - (3*a*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/d^3 -
((6*I)*b*f^3*PolyLog[3, (-I)*E^(c + d*x)])/d^4 + ((6*I)*b*f^3*PolyLog[3, I
*E^(c + d*x)])/d^4 + (3*a*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*d^4) + (b*(
e + f*x)^3*Sech[c + d*x])/d + (a*(e + f*x)^3*Tanh[c + d*x])/d)/(a^2 + b^2)
```

3.309. $\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.309.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.309. $\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.309.4 Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.309.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6367 vs. 2(712) = 1424.

Time = 0.38 (sec) , antiderivative size = 6367, normalized size of antiderivative = 8.16

$$\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

3.309.6 Sympy [F]

$$\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.309.7 Maxima [F]

$$\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\operatorname{sech}(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
3*a*e^2*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2
+ b^2)*d^2)) - 6*b*f^3*integrate(x^2*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) +
b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 6*a*f^3*integrate(x^2/(a^2*d*
e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 12*b*e*f^2*
integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a
^2*d + b^2*d), x) + 12*a*e*f^2*integrate(x/(a^2*d*e^(2*d*x + 2*c) + b^2*d*
e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + e^3*(b^2*log((b*e^(-d*x - c) - a -
sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2
)*d) + 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*
d)) - 6*b*e^2*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) - 2*(a*f^3*x^3 + 3*a
*e*f^2*x^2 + 3*a*e^2*f*x - (b*f^3*x^3*e^c + 3*b*e*f^2*x^2*e^c + 3*b*e^2*f*
x*e^c)*e^(d*x))/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)
) + integrate(-2*(b^2*f^3*x^3*e^c + 3*b^2*e*f^2*x^2*e^c + 3*b^2*e^2*f*x*e^
c)*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(2*c))*e^(2*d*x) - 2*(a^3
*e^c + a*b^2*e^c)*e^(d*x)), x)
```

3.309.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\cosh(c+d\,x)^2\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.310 $\int \frac{(e+fx)^2 \text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.310.1 Optimal result

Integrand size = 28, antiderivative size = 548

$$
\begin{aligned}
\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = {} & \frac{a(e+fx)^2}{(a^2+b^2)\,d} - \frac{4bf(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} \\
& + \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} \\
& - \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} \\
& - \frac{2af(e+fx)\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)\,d^2} \\
& + \frac{2ibf^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^3} - \frac{2ibf^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^3} \\
& + \frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} \\
& - \frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} \\
& - \frac{af^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{(a^2+b^2)\,d^3} \\
& - \frac{2b^2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^3} \\
& + \frac{2b^2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^3} \\
& + \frac{b(e+fx)^2\operatorname{sech}(c+dx)}{(a^2+b^2)\,d} + \frac{a(e+fx)^2\tanh(c+dx)}{(a^2+b^2)\,d}
\end{aligned}
$$

output
```
a*(f*x+e)^2/(a^2+b^2)/d-4*b*f*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)/d^2-2*a
*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d^2+b^2*(f*x+e)^2*ln(1+b*exp(d*x
+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-b^2*(f*x+e)^2*ln(1+b*exp(d*x+c)
/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+2*I*b*f^2*polylog(2,-I*exp(d*x+c))
/(a^2+b^2)/d^3-2*I*b*f^2*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^3-a*f^2*polyl
og(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^3+2*b^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c
)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-2*b^2*f*(f*x+e)*polylog(2,-b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-2*b^2*f^2*polylog(3,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^3+2*b^2*f^2*polylog(3,-b*
exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^3+b*(f*x+e)^2*sech(d*x+c
)/(a^2+b^2)/d+a*(f*x+e)^2*tanh(d*x+c)/(a^2+b^2)/d
```

3.310.2 Mathematica [A] (verified)

Time = 4.33 (sec) , antiderivative size = 633, normalized size of antiderivative = 1.16

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{\frac{f\left(4ad^2ee^{2c}x-4ad^2e(1+e^{2c})x+2ad^2e^{2c}fx^2-2ad^2(1+e^{2c})fx^2-4bde(1+e^{2c})\arctan(e^{c+dx})+2ade(1+e^{2c})\left(2dx-\log\left(1+e^{2(c+dx)}\right)\right)+2ib(1+e^{2c})\right.}{(a^2}}{}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
((f*(4*a*d^2*e*E^(2*c)*x - 4*a*d^2*e*(1 + E^(2*c))*x + 2*a*d^2*E^(2*c)*f*x
^2 - 2*a*d^2*(1 + E^(2*c))*f*x^2 - 4*b*d*e*(1 + E^(2*c))*ArcTan[E^(c + d*x
)] + 2*a*d*e*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (2*I)*b*(1
 + E^(2*c))*f*(d*x*(-Log[1 - I*E^(c + d*x)] + Log[1 + I*E^(c + d*x)]) + Po
lyLog[2, (-I)*E^(c + d*x)] - PolyLog[2, I*E^(c + d*x)]) + a*(1 + E^(2*c))*
f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]))
)/((a^2 + b^2)*(1 + E^(2*c))) - (b^2*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x)
)/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d^2
*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*E
^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a +
Sqrt[a^2 + b^2])] - 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
]))]))/(a^2 + b^2)^(3/2) + (d^2*(e + f*x)^2*Sech[c + d*x]*(b + a*Sech[c]*S
inh[d*x]))/(a^2 + b^2))/d^3
```

3.310.3 Rubi [A] (verified)

Time = 2.43 (sec) , antiderivative size = 469, normalized size of antiderivative = 0.86, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6107}$$

$$\frac{b^2\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3803}$$

$$\frac{2b^2\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}$$

$$\Big\downarrow \text{2694}$$

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$\Big\downarrow$ 2620

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$\Big\downarrow$ 3011

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\cdots\right.}{}\right.}{a^2+b^2}$$

$\Big\downarrow$ 2720

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\cdots\right.}{\cdots}\right.}{}\right.}{a^2+b^2}$$

$\Big\downarrow$ 7143

3.310. $\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\cdots\right.}{\cdots}\right)}{a^2+b^2}$$

$\downarrow$ 7293

$$\frac{\int \left(a(e+fx)^2\text{sech}^2(c+dx)-b(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\cdots\right.}{\cdots}\right)}{a^2+b^2}$$

$\downarrow$ 2009

$$-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\cdots\right.}{\cdots}\right)}{a^2+b^2}$$

$$\frac{-\frac{af^2\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{d^3}-\frac{2af(e+fx)\log\left(e^{2(c+dx)}+1\right)}{d^2}+\frac{a(e+fx)^2\tanh(c+dx)}{d}+\frac{a(e+fx)^2}{d}-\frac{4bf(e+fx)\arctan\left(e^{c+dx}\right)}{d^2}+\frac{2ibf^2\,\text{Po}\cdots}{\cdots}}{a^2+b^2}$$

input
```
Int[((e + f*x)^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

3.310. $\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(-2*b^2*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + ((a*(e
 + f*x)^2)/d - (4*b*f*(e + f*x)*ArcTan[E^(c + d*x)])/d^2 - (2*a*f*(e + f*x
)*Log[1 + E^(2*(c + d*x))])/d^2 + ((2*I)*b*f^2*PolyLog[2, (-I)*E^(c + d*x)
])/d^3 - ((2*I)*b*f^2*PolyLog[2, I*E^(c + d*x)])/d^3 - (a*f^2*PolyLog[2, -
E^(2*(c + d*x))])/d^3 + (b*(e + f*x)^2*Sech[c + d*x])/d + (a*(e + f*x)^2*T
anh[c + d*x])/d)/(a^2 + b^2)
```

3.310.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.310. $\int \frac{(e+fx)^2 \text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.310.4 Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.310.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3582 vs. $2(502) = 1004$.

Time = 0.34 (sec) , antiderivative size = 3582, normalized size of antiderivative = 6.54

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*(a^3 + a*b^2)*d^2*e^2 - 4*(a^3 + a*b^2)*c*d*e*f + 2*(a^3 + a*b^2)*c^2*
f^2 - 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 +
a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)^2 - 2*((a^3 + a*b^2)
*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3
+ a*b^2)*c^2*f^2)*sinh(d*x + c)^2 - 2*(b^3*d*f^2*x + b^3*d*e*f + (b^3*d*f^
2*x + b^3*d*e*f)*cosh(d*x + c)^2 + 2*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x +
c)*sinh(d*x + c) + (b^3*d*f^2*x + b^3*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 +
b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^3*d*f^2*x + b^3*d*
e*f + (b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)^2 + 2*(b^3*d*f^2*x + b^3*d*e
*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d*f^2*x + b^3*d*e*f)*sinh(d*x + c)^
2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^3*d^2
*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^
2*f^2)*cosh(d*x + c)^2 + 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cos
h(d*x + c)*sinh(d*x + c) + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sin
h(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
 c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^
3*c^2*f^2 + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)^2 +
2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + ...
```

3.310.6 Sympy [F]

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\,\text{sech}^2\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.310.7 Maxima [F]

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2*a*e*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 +
b^2)*d^2)) - 4*b*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*
d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 4*a*f^2*integrate(x/(a^2*d*e^(2*d
*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + e^2*(b^2*log((b*e
^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))
/((a^2 + b^2)^(3/2)*d) + 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*
e^(-2*d*x - 2*c))*d)) - 4*b*e*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) - 2*
(a*f^2*x^2 + 2*a*e*f*x - (b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^(d*x))/(a^2*d +
 b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) + integrate(-2*(b^2*f^
2*x^2*e^c + 2*b^2*e*f*x*e^c)*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e
^(2*c))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x)
```

3.310.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.311 $\int \frac{(e+fx)\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.311.1 Optimal result

Integrand size = 26, antiderivative size = 295

$$
\begin{aligned}
\int \frac{(e+fx)\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = &-\frac{bf\arctan(\sinh(c+dx))}{(a^2+b^2)\,d^2}+\frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} \\
&-\frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} \\
&-\frac{af\log(\cosh(c+dx))}{(a^2+b^2)\,d^2}+\frac{b^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} \\
&-\frac{b^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} \\
&+\frac{b(e+fx)\mathrm{sech}(c+dx)}{(a^2+b^2)\,d}+\frac{a(e+fx)\tanh(c+dx)}{(a^2+b^2)\,d}
\end{aligned}
$$

output
```
-b*f*arctan(sinh(d*x+c))/(a^2+b^2)/d^2-a*f*ln(cosh(d*x+c))/(a^2+b^2)/d^2+b
^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-b^2*(f
*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+b^2*f*polyl
og(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-b^2*f*polylog(
2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2+b*(f*x+e)*sech(d*
x+c)/(a^2+b^2)/d+a*(f*x+e)*tanh(d*x+c)/(a^2+b^2)/d
```

3.311.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.13

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\frac{2f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ia-b}+\frac{2if\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a-ib}-\frac{f\log(\cosh(c+dx))}{a-ib}-\frac{f\log(\cosh(c+dx))}{a+ib}+\frac{2b^2\left(-2de\,\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)\right.}{}}{}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
((2*f*ArcTan[Tanh[(c + d*x)/2]])/(I*a - b) + ((2*I)*f*ArcTan[Tanh[(c + d*x
)/2]])/(a - I*b) - (f*Log[Cosh[c + d*x]])/(a - I*b) - (f*Log[Cosh[c + d*x]
])/(a + I*b) + (2*b^2*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]]
 + 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2
+ b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2 +
 b^2)^(3/2) + (2*d*(e + f*x)*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2 + b
^2))/(2*d^2)
```

3.311.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {6107, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \ 6107$$
$$\frac{b^2\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$
$$\Big\downarrow \ 3042$$

$$\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3803}$$

$$\frac{2b^2\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}$$

$$\Big\downarrow \text{2694}$$

$$\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

3.311. $\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \color{blue}{2838}$$

$$\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$$\Big\downarrow \color{blue}{7293}$$

$$\frac{\int \left(a(e+fx)\text{sech}^2(c+dx)-b(e+fx)\text{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

$$\Big\downarrow \color{blue}{2009}$$

$$\frac{-\frac{af\log(\cosh(c+dx))}{d^2}+\frac{a(e+fx)\tanh(c+dx)}{d}-\frac{bf\arctan(\sinh(c+dx))}{d^2}+\frac{b(e+fx)\text{sech}(c+dx)}{d}}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}$$

input
```
Int[((e + f*x)*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-2*b^2*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2
)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + (-((b*f*ArcTan[Sinh[c + d*
x]])/d^2) - (a*f*Log[Cosh[c + d*x]])/d^2 + (b*(e + f*x)*Sech[c + d*x])/d +
 (a*(e + f*x)*Tanh[c + d*x])/d)/(a^2 + b^2)
```

3.311. $\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.311.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.311. $\int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.311.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1927 vs. $2(275) = 550$.

Time = 8.74 (sec) , antiderivative size = 1928, normalized size of antiderivative = 6.54

method	result	size
risch	Expression too large to display	1928

input
```
int((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/(a^2+b^2)^(3/2)/d^2*b^2*c*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))*a^2+2/(a^2+b^2)^(3/2)/d*b^2*f/(2*a^2+2*b^2)*ln((-b*exp
(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2*x-2/(a^2+b^2)^(3/2)/d
*b^2*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))*a^2*x+2/(a^2+b^2)^(3/2)/d^2*b^2*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2
+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2*c-2/(a^2+b^2)^(3/2)/d^2*b^2*f/(2*
a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2*c+
2/(a^2+b^2)^(5/2)/d^2*a^2*b^2*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)
^(1/2))-4/(a^2+b^2)^(1/2)/d^2*a^2*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x
+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(3/2)/d^2*b^4*f/(2*a^2+2*b^2)*arctan
h(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)/d^2*b^2*f/(2*a^2+2
*b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/(a^2+b^2)^(1/2)/d^2*b^2*f/
(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/(a^2+b^2
)/d^2*b^2*f/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c))-2/(a^2+b^2)^(3/2)/d*b^4*e
/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^
2)^(3/2)/d^2*b^4*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(
-a+(a^2+b^2)^(1/2)))-2/(a^2+b^2)^(3/2)/d^2*b^4*f/(2*a^2+2*b^2)*dilog((b*ex
p(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/(a^2+b^2)^(1/2)/d*b^2*e
/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-4/(a^2+b^
2)/d^2*a^2*b*f/(2*a^2+2*b^2)*arctan(exp(d*x+c))-2/(a^2+b^2)^(3/2)/d^2*b...
```

3.311.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1296 vs. $2(273) = 546$.

Time = 0.31 (sec) , antiderivative size = 1296, normalized size of antiderivative = 4.39

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*(a^3 + a*b^2)*d*f*x*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*d*f*x*sinh(d*x +
c)^2 - 2*(a^3 + a*b^2)*d*e + (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x + c
)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 + b^3*f)*sqrt((a^2 + b^2)/b^2)*dil
og((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cos
h(d*x + c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 + b^3*f)*sqrt((a^2 + b^2)
/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*d*e - b^3*c*f + (b^3*d
*e - b^3*c*f)*cosh(d*x + c)^2 + 2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)*sinh(d
*x + c) + (b^3*d*e - b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2
*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
(b^3*d*e - b^3*c*f + (b^3*d*e - b^3*c*f)*cosh(d*x + c)^2 + 2*(b^3*d*e - b^
3*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d*e - b^3*c*f)*sinh(d*x + c)^2)*
sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt
((a^2 + b^2)/b^2) + 2*a) + (b^3*d*f*x + b^3*c*f + (b^3*d*f*x + b^3*c*f)*co
sh(d*x + c)^2 + 2*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3
*d*f*x + b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x
+ c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b) - (b^3*d*f*x + b^3*c*f + (b^3*d*f*x + b^3*c*f)*cosh(d*x +
 c)^2 + 2*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d*f*...
```

3.311.6 Sympy [F]

$$\int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)\operatorname{sech}^2\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.311.7 Maxima [F]

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{sech}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
(4*b^2*integrate(-1/2*x*e^(d*x + c)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^
(2*c))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x) + 2*(b*x*e^(d*x +
c) - a*x)/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) + 2*
a*x/((a^2 + b^2)*d) - 2*b*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) - a*log(e^
(2*d*x + 2*c) + 1)/((a^2 + b^2)*d^2))*f + e*(b^2*log((b*e^(-d*x - c) - a -
 sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/
2)*d) + 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))
*d))
```

3.311.8 Giac [F]

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{sech}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*sech(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\,(c+d\,x)^2\,\,(a+b\sinh\,(c+d\,x))}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.312 $\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.312.1 Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=-\frac{2b^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}+\frac{\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{(a^2+b^2)\,d}$$

output
```
-2*b^2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/
d+sech(d*x+c)*(b+a*sinh(d*x+c))/(a^2+b^2)/d
```

3.312.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2b^2\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)+b\sqrt{-a^2-b^2}\operatorname{sech}(c+dx)+a\sqrt{-a^2-b^2}\tanh(c+dx)}{(-a^2-b^2)^{3/2}\,d}$$

input
```
Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
-((2*b^2*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] + b*Sqrt[-a^2
- b^2]*Sech[c + d*x] + a*Sqrt[-a^2 - b^2]*Tanh[c + d*x])/((-a^2 - b^2)^(3/
2)*d))
```

3.312.3 Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3175, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3175}$$

$$\frac{\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{d\left(a^2+b^2\right)}-\frac{\int -\frac{b^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^2\int \frac{1}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{d\left(a^2+b^2\right)}+\frac{b^2\int \frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{d\left(a^2+b^2\right)}-\frac{2ib^2\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{d\left(a^2+b^2\right)}+\frac{4ib^2\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{2b^2\operatorname{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}+\frac{\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{d\left(a^2+b^2\right)}$$

input
```
Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
(2*b^2*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*
d) + (Sech[c + d*x]*(b + a*Sinh[c + d*x]))/((a^2 + b^2)*d)
```

3.312.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

3.312.4 Maple [A] (verified)

Time = 7.61 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{\frac{2b^2 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(-a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}{d}$
default	$\frac{\frac{2b^2 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(-a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}{d}$
risch	$-\frac{2\left(-b\,\mathrm{e}^{dx+c}+a\right)}{d\left(a^2+b^2\right)\left(1+\mathrm{e}^{2dx+2c}\right)}+\frac{b^2\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}-\frac{b^2\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}$

input
```
int(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b^2/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+
b^2)^(1/2))-2/(a^2+b^2)*(-a*tanh(1/2*d*x+1/2*c)-b)/(1+tanh(1/2*d*x+1/2*c)^
2))
```

3.312.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(74) = 148$.

Time = 0.25 (sec) , antiderivative size = 353, normalized size of antiderivative = 4.58

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{2\,a^3+2\,ab^2-\left(b^2\cosh\left(dx+c\right)^2+2\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b^2\sinh\left(dx+c\right)^2+b^2\right)\sqrt{a^2+b^2}\log}{\left(a^4+2\,a^2b^2+b^4\right)d\cosh\left(dx+c\right)^2+2\,(a^4}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*a^3 + 2*a*b^2 - (b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x +
c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 +
 b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x
 + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d
*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) +
 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(a^2*b + b^3)*cosh(d*x +
c) - 2*(a^2*b + b^3)*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x +
c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*
a^2*b^2 + b^4)*d*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)
```

3.312.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.312.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.49

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^2\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}d} + \frac{2\left(be^{(-dx-c)}+a\right)}{(a^2+b^2+(a^2+b^2)e^{(-2\,dx-2\,c)})d}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
b^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/((a^2 + b^2)^(3/2)*d) + 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 +
(a^2 + b^2)*e^(-2*d*x - 2*c))*d)
```

3.312.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{b^2 \log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2\,(be^{(dx+c)}-a)}{(a^2+b^2)(e^{(2\,dx+2\,c)}+1)}}{d}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
(b^2*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c
) + 2*a + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*e^(d*x + c) - a)/((
a^2 + b^2)*(e^(2*d*x + 2*c) + 1)))/d
```

3.312.9 Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 413, normalized size of antiderivative = 5.36

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2\,a}{d\,(a^2+b^2)}-\frac{2\,b\,\mathrm{e}^{c+d\,x}}{d\,(a^2+b^2)}}{\mathrm{e}^{2\,c+2\,d\,x}+1} - \frac{2\,\mathrm{atan}\left(\left(\frac{b^3\,\sqrt{-a^6\,d^2-3\,a^4\,b^2\,d^2-3\,a^2\,b^4\,d^2-b^6\,d^2}}{2}+\frac{a^2\,b\,\sqrt{-a^6\,d^2-3\,a^4\,b^2\,d^2-3\,a^2\,b^4\,d^2-b^6\,d^2}}{2}\right)\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\left(\frac{2}{d\,\sqrt{b^4}\,(a^2+b^2)^2}+\cdots\right.\right.\right.}{\sqrt{-a^6\,d^2-3\,a^4\,b^2\cdots}}$$

input
```
int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
- ((2*a)/(d*(a^2 + b^2)) - (2*b*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(2*c +
2*d*x) + 1) - (2*atan(((b^3*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b
^2*d^2)^(1/2))/2 + (a^2*b*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2
*d^2)^(1/2))/2)*(exp(d*x)*exp(c)*(2/(d*(b^4)^(1/2)*(a^2 + b^2)^2) + (2*a*(
a^3*d*(b^4)^(1/2) + a*b^2*d*(b^4)^(1/2)))/(b^4*(-d^2*(a^2 + b^2)^3)^(1/2)*
(a^2 + b^2)*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2*d^2)^(1/2)))
- (2*a*(b^3*d*(b^4)^(1/2) + a^2*b*d*(b^4)^(1/2)))/(b^4*(-d^2*(a^2 + b^2)^3
)^(1/2)*(a^2 + b^2)*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2*d^2)^
(1/2))))*(b^4)^(1/2))/(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2*d^2
)^(1/2)
```

3.313 $\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.313.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.313.2 Mathematica [N/A]

Not integrable

Time = 65.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Sech[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Sech[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.313.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$
$$\Big\downarrow \; 6111$$
$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.313.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.313.4 Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.313.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.313.6 Sympy [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}^2(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sech(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.313.7 Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 391, normalized size of antiderivative = 13.96

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
4*b^2*integrate(-1/2*e^(d*x + c)/(a^2*b*e + b^3*e + (a^2*b*f + b^3*f)*x -
(a^2*b*e*e^(2*c) + b^3*e*e^(2*c) + (a^2*b*f*e^(2*c) + b^3*f*e^(2*c))*x)*e^
(2*d*x) - 2*(a^3*e*e^c + a*b^2*e*e^c + (a^3*f*e^c + a*b^2*f*e^c)*x)*e^(d*x
)), x) + 2*(b*e^(d*x + c) - a)/(a^2*d*e + b^2*d*e + (a^2*d*f + b^2*d*f)*x
+ (a^2*d*e*e^(2*c) + b^2*d*e*e^(2*c) + (a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))
*x)*e^(2*d*x)) + 4*integrate(1/2*(b*f*e^(d*x + c) - a*f)/(a^2*d*e^2 + b^2*
d*e^2 + (a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*e*f + b^2*d*e*f)*x + (a^2*d
*e^2*e^(2*c) + b^2*d*e^2*e^(2*c) + (a^2*d*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))
*x^2 + 2*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x)*e^(2*d*x)), x)
```

3.313.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.313.9 Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.314 $\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.314.1 Optimal result

Integrand size = 28, antiderivative size = 928

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2ab^2(e+fx)^2\arctan\left(e^{c+dx}\right)}{(a^2+b^2)^2d} + \frac{a(e+fx)^2\arctan\left(e^{c+dx}\right)}{(a^2+b^2)d} - \frac{af^2\arctan(\sinh(c+dx))}{(a^2+b^2)d^3} + \frac{b^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2d} + \frac{b^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2d} - \frac{b^3(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)^2d} + \frac{bf^2\log(\cosh(c+dx))}{(a^2+b^2)d^3} - \frac{2iab^2f(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)^2d^2} - \frac{iaf(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)d^2} + \frac{2iab^2f(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)^2d^2} + \frac{iaf(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)d^2} + \frac{2b^3f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2d^2} + \frac{2b^3f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2d^2} - \frac{b^3f(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{(a^2+b^2)^2d^2} + \frac{2iab^2f^2\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{(a^2+b^2)^2d^3} + \frac{iaf^2\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{(a^2+b^2)d^3} - \frac{2iab^2f^2\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{(a^2+b^2)^2d^3} - \frac{iaf^2\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{(a^2+b^2)d^3} - \frac{2b^3f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2d^3} - \frac{2b^3f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2d^3} + \frac{b^3f^2\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2(a^2+b^2)^2d^3} + \frac{af(e+fx)\text{sech}(c+dx)}{(a^2+b^2)d^2} + \frac{b(e+fx)^2\text{sech}^2(c+dx)}{2(a^2+b^2)d} - \frac{bf(e+fx)\tanh(c+dx)}{(a^2+b^2)d^2}$$

output
```
-2*I*a*b^2*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b^2)^2/d^2+a*(f*x+e)^2*
arctan(exp(d*x+c))/(a^2+b^2)/d-I*a*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2
+b^2)/d^2-2*I*a*b^2*f^2*polylog(3,I*exp(d*x+c))/(a^2+b^2)^2/d^3+1/2*b^3*f^
2*polylog(3,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^3+1/2*b*(f*x+e)^2*sech(d*x+c)^2
/(a^2+b^2)/d-2*b^3*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b
^2)^2/d^3-2*b^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)
^2/d^3-a*f^2*arctan(sinh(d*x+c))/(a^2+b^2)/d^3-b^3*f*(f*x+e)*polylog(2,-ex
p(2*d*x+2*c))/(a^2+b^2)^2/d^2+I*a*f^2*polylog(3,-I*exp(d*x+c))/(a^2+b^2)/d
^3+a*f*(f*x+e)*sech(d*x+c)/(a^2+b^2)/d^2-b*f*(f*x+e)*tanh(d*x+c)/(a^2+b^2)
/d^2-b^3*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/(a^2+b^2)^2/d+b*f^2*ln(cosh(d*x+c)
)/(a^2+b^2)/d^3+b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+
b^2)^2/d+b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/
d+2*I*a*b^2*f^2*polylog(3,-I*exp(d*x+c))/(a^2+b^2)^2/d^3+I*a*f*(f*x+e)*pol
ylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2+2*I*a*b^2*f*(f*x+e)*polylog(2,I*exp(d*x
+c))/(a^2+b^2)^2/d^2+2*a*b^2*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)^2/d+1/
2*a*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a^2+b^2)/d+2*b^3*f*(f*x+e)*polylog(
2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2+2*b^3*f*(f*x+e)*polyl
og(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-I*a*f^2*polylog(3,
I*exp(d*x+c))/(a^2+b^2)/d^3
```

3.314.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3368 vs. 2(928) = 1856.

Time = 11.82 (sec) , antiderivative size = 3368, normalized size of antiderivative = 3.63

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(12*b^3*d^3*e^2*E^(2*c)*x - 12*a^2*b*d*E^(2*c)*f^2*x - 12*b^3*d*E^(2*c)*f^
2*x + 12*b^3*d^3*e*E^(2*c)*f*x^2 + 4*b^3*d^3*E^(2*c)*f^2*x^3 + 6*a^3*d^2*e
^2*ArcTan[E^(c + d*x)] + 18*a*b^2*d^2*e^2*ArcTan[E^(c + d*x)] + 6*a^3*d^2*
e^2*E^(2*c)*ArcTan[E^(c + d*x)] + 18*a*b^2*d^2*e^2*E^(2*c)*ArcTan[E^(c + d
*x)] - 12*a^3*f^2*ArcTan[E^(c + d*x)] - 12*a*b^2*f^2*ArcTan[E^(c + d*x)] -
 12*a^3*E^(2*c)*f^2*ArcTan[E^(c + d*x)] - 12*a*b^2*E^(2*c)*f^2*ArcTan[E^(c
 + d*x)] + (6*I)*a^3*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (18*I)*a*b^2*d^2*e
*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*a^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c +
 d*x)] + (18*I)*a*b^2*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (3*I)*a^3
*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] + (9*I)*a*b^2*d^2*f^2*x^2*Log[1 - I*E^
(c + d*x)] + (3*I)*a^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (9*I)*
a*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (6*I)*a^3*d^2*e*f*x*Log
[1 + I*E^(c + d*x)] - (18*I)*a*b^2*d^2*e*f*x*Log[1 + I*E^(c + d*x)] - (6*I
)*a^3*d^2*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (18*I)*a*b^2*d^2*e*E^(2*c
)*f*x*Log[1 + I*E^(c + d*x)] - (3*I)*a^3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)
] - (9*I)*a*b^2*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*a^3*d^2*E^(2*c)
*f^2*x^2*Log[1 + I*E^(c + d*x)] - (9*I)*a*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 +
I*E^(c + d*x)] - 6*b^3*d^2*e^2*Log[1 + E^(2*(c + d*x))] - 6*b^3*d^2*e^2*E^
(2*c)*Log[1 + E^(2*(c + d*x))] + 6*a^2*b*f^2*Log[1 + E^(2*(c + d*x))] + 6*
b^3*f^2*Log[1 + E^(2*(c + d*x))] + 6*a^2*b*E^(2*c)*f^2*Log[1 + E^(2*(c ...
```

3.314.3 Rubi [A] (verified)

Time = 3.26 (sec) , antiderivative size = 764, normalized size of antiderivative = 0.82, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {6107, 6107, 6095, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6107}$$

$$\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{6107}$$

$$\frac{\int (e+fx)^2 \text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 6095}$$

$$\frac{\int (e+fx)^2 \text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{\int (e+fx)^2 \text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx}{3bf}\right.}{a^2+b^2}\right.}{a^2+b^2}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+c}}{a+\sqrt{a^2}}\right.}{d}\right.}{bd}\right.}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{\int (e+fx)^2 \text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 2720}$$

3.314. $\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\right.}{bd}\right.}{a^2+b^2}\right.$$

$$\frac{\int(e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{7143}$$

$$b^2\left(\frac{\int(e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}(3,-}{d^2}\right.}{}\right.}{a^2+b^2}\right.$$

$$\frac{\int(e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \text{7293}$$

$$b^2\left(\frac{\int\left(a(e+fx)^2\text{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{Pol}}{}\right.}{}\right.}{a^2+b^2}\right.$$

$$\frac{\int\left(a(e+fx)^2\text{sech}^3(c+dx)-b(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}$$

$$\Big\downarrow \text{2009}$$

3.314. $\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right.}{a^2+b^2}\right.$$

$$-\frac{af^2\arctan(\sinh(c+dx))}{d^3}+\frac{a(e+fx)^2\arctan(e^{c+dx})}{d}+\frac{iaf^2\,\text{PolyLog}(3,-ie^{c+dx})}{d^3}-\frac{iaf^2\,\text{PolyLog}(3,ie^{c+dx})}{d^3}-\frac{iaf(e+fx)\,\text{PolyLog}(2,-ie^{c}}{d^2}$$

input
```
Int[((e + f*x)^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^2*((b^2*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))])/d^2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3)/(3*f) + (2*a*(
e + f*x)^2*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))
])/d - ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((2*I)*a*f
*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*(e + f*x)*PolyLog[2, -E^(
2*(c + d*x))])/d^2 + ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/d^3 - ((2*
I)*a*f^2*PolyLog[3, I*E^(c + d*x)])/d^3 + (b*f^2*PolyLog[3, -E^(2*(c + d*x
))])/(2*d^3))/(a^2 + b^2)))/(a^2 + b^2) + ((a*(e + f*x)^2*ArcTan[E^(c + d*
x)])/d - (a*f^2*ArcTan[Sinh[c + d*x]])/d^3 + (b*f^2*Log[Cosh[c + d*x]])/d^
3 - (I*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*a*f*(e + f*x)*
PolyLog[2, I*E^(c + d*x)])/d^2 + (I*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/d^
3 - (I*a*f^2*PolyLog[3, I*E^(c + d*x)])/d^3 + (a*f*(e + f*x)*Sech[c + d*x]
)/d^2 + (b*(e + f*x)^2*Sech[c + d*x]^2)/(2*d) - (b*f*(e + f*x)*Tanh[c + d*
x])/d^2 + (a*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/(a^2 + b^2)
```

3.314. $\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.314.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

3.314. $\int \frac{(e+fx)^2\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.314.4 Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.314.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10642 vs. $2(855) = 1710$.

Time = 0.44 (sec) , antiderivative size = 10642, normalized size of antiderivative = 11.47

$$\int \frac{(e+fx)^2\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

3.314.6 Sympy [F]

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\,\text{sech}^3\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sech(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

3.314.7 Maxima [F]

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}\,(dx+c)^3}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
a^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2
*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 +
 b^4*d^2), x) + 3*a*b^2*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*
x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d
^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*b^3*d^2*f^2*integrate(x^2/(a^4*d^2*e
^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) +
 a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^3*d^2*e*f*integrate(x*e^(d*x
 + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e
^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 6*a*b^2*d^2*e*f*
integrate(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x
+ 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x)
+ 4*b^3*d^2*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*
d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2),
 x) - a^2*b*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x
+ 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) - b^3*f^2*(2*(d*x + c)/((a^4 +
2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*
d^3)) + (b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2
*b^2 + b^4)*d) - b^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d)
 - (a^3 + 3*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (a*e
^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + ...
```

3.314.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)^3\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.315 $\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.315.1 Optimal result

Integrand size = 26, antiderivative size = 560

$$
\begin{aligned}
\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{2ab^2(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} + \frac{a(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d} \\
&+ \frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} \\
&+ \frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} \\
&- \frac{b^3(e+fx)\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)^2 d} - \frac{iab^2 f\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)^2 d^2} \\
&- \frac{iaf\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2\,(a^2+b^2)\,d^2} + \frac{iab^2 f\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)^2 d^2} \\
&+ \frac{iaf\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2\,(a^2+b^2)\,d^2} + \frac{b^3 f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d^2} \\
&+ \frac{b^3 f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d^2} - \frac{b^3 f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2\,(a^2+b^2)^2 d^2} \\
&+ \frac{af\,\text{sech}(c+dx)}{2\,(a^2+b^2)\,d^2} + \frac{b(e+fx)\text{sech}^2(c+dx)}{2\,(a^2+b^2)\,d} \\
&- \frac{bf\tanh(c+dx)}{2\,(a^2+b^2)\,d^2} + \frac{a(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{2\,(a^2+b^2)\,d}
\end{aligned}
$$

output
```
2*a*b^2*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)^2/d+a*(f*x+e)*arctan(exp(d*x+
c))/(a^2+b^2)/d-b^3*(f*x+e)*ln(1+exp(2*d*x+2*c))/(a^2+b^2)^2/d+b^3*(f*x+e)
*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d+b^3*(f*x+e)*ln(1+b*e
xp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-1/2*I*a*f*polylog(2,-I*exp(d*
x+c))/(a^2+b^2)/d^2+1/2*I*a*f*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2+I*a*b^
2*f*polylog(2,I*exp(d*x+c))/(a^2+b^2)^2/d^2-I*a*b^2*f*polylog(2,-I*exp(d*x
+c))/(a^2+b^2)^2/d^2-1/2*b^3*f*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2+
b^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2+b^3*f*p
olylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2+1/2*a*f*sech(d
*x+c)/(a^2+b^2)/d^2+1/2*b*(f*x+e)*sech(d*x+c)^2/(a^2+b^2)/d-1/2*b*f*tanh(d
*x+c)/(a^2+b^2)/d^2+1/2*a*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a^2+b^2)/d
```

3.315.2 Mathematica [A] (verified)

Time = 8.53 (sec) , antiderivative size = 832, normalized size of antiderivative = 1.49

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{b^3\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\,\text{arctanh}\left(\frac{a+be^{c+}}{\sqrt{a^2+b}}\right.}{(-a^2-b^2)^{3/2}}\right.}{}$$
$$-\frac{-2b^3de(c+dx)+2b^3cf(c+dx)-b^3f(c+dx)^2-2a^3de\arctan\left(e^{c+dx}\right)-6ab^2de\arctan\left(e^{c+dx}\right)+2a^3c}{}$$
$$+\frac{\text{sech}(c+dx)(af-bf\sinh(c+dx))}{2(a^2+b^2)d^2}$$
$$+\frac{\text{sech}^2(c+dx)(bde-bcf+bf(c+dx)+ade\sinh(c+dx)-acf\sinh(c+dx)+af(c+dx)\sinh(c+dx}{2(a^2+b^2)d^2}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^3*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 +
 b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^
2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E
^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-
a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))]))/(2*(a^2 + b^2)^2*d^2) - (-2*b^3*d*e*(c + d*x) + 2*b^3*c*f*(c + d*
x) - b^3*f*(c + d*x)^2 - 2*a^3*d*e*ArcTan[E^(c + d*x)] - 6*a*b^2*d*e*ArcTa
n[E^(c + d*x)] + 2*a^3*c*f*ArcTan[E^(c + d*x)] + 6*a*b^2*c*f*ArcTan[E^(c +
 d*x)] - I*a^3*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - (3*I)*a*b^2*f*(c + d*x
)*Log[1 - I*E^(c + d*x)] + I*a^3*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + (3*I
)*a*b^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + 2*b^3*d*e*Log[1 + E^(2*(c + d
*x))] - 2*b^3*c*f*Log[1 + E^(2*(c + d*x))] + 2*b^3*f*(c + d*x)*Log[1 + E^(
2*(c + d*x))] + I*a*(a^2 + 3*b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] - I*a*(a^
2 + 3*b^2)*f*PolyLog[2, I*E^(c + d*x)] + b^3*f*PolyLog[2, -E^(2*(c + d*x))
])/(2*(a^2 + b^2)^2*d^2) + (Sech[c + d*x]*(a*f - b*f*Sinh[c + d*x]))/(2*(a
^2 + b^2)*d^2) + (Sech[c + d*x]^2*(b*d*e - b*c*f + b*f*(c + d*x) + a*d*e*S
inh[c + d*x] - a*c*f*Sinh[c + d*x] + a*f*(c + d*x)*Sinh[c + d*x]))/(2*(...
```

3.315.3 Rubi [A] (verified)

Time = 1.94 (sec) , antiderivative size = 485, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6107, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{6107}$$
$$\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}$$
$$\Big\downarrow \text{6107}$$

$$\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \textbf{6095}$$

$$\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \textbf{2620}$$

$$\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+$$

$$\frac{b^2\left(\frac{b^2\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+fx)}{}\right.}{a^2+b^2}$$

$$\Big\downarrow \textbf{2715}$$

$$\frac{b^2\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right.}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \textbf{2838}$$

$$\frac{b^2\left(\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+f}{}\right.}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}$$

$$\Big\downarrow \textbf{7293}$$

3.315. $\displaystyle \int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{b^2\left(\frac{\int (a(e+fx)\text{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{\int \left(a(e+fx)\text{sech}^3(c+dx)-b(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\left(\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{2a(e+fx}{}\right.}{a^2+b^2}$$

$$\frac{\frac{a(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{iaf\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\frac{iaf\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2d^2}+\frac{af\text{sech}(c+dx)}{2d^2}+\frac{a(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}-\frac{bf\,t}{}}{a^2+b^2}$$

input
```
Int[((e + f*x)*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^2*((b^2*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
b*d^2)))/(a^2 + b^2) + ((b*(e + f*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c
 + d*x)])/d - (b*(e + f*x)*Log[1 + E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2,
 (-I)*E^(c + d*x)])/d^2 + (I*a*f*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*Pol
yLog[2, -E^(2*(c + d*x))])/(2*d^2))/(a^2 + b^2)))/(a^2 + b^2) + ((a*(e + f
*x)*ArcTan[E^(c + d*x)])/d - ((I/2)*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2
+ ((I/2)*a*f*PolyLog[2, I*E^(c + d*x)])/d^2 + (a*f*Sech[c + d*x])/(2*d^2)
+ (b*(e + f*x)*Sech[c + d*x]^2)/(2*d) - (b*f*Tanh[c + d*x])/(2*d^2) + (a*(
e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/(a^2 + b^2)
```

3.315. $\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.315.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.315. $\int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.315.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2050 vs. $2(520) = 1040$.

Time = 20.31 (sec) , antiderivative size = 2051, normalized size of antiderivative = 3.66

method	result	size
risch	Expression too large to display	2051

input
```
int((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-I/d/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x-I/d^2/(a^2+b^2)*a^
3*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c+3*I/d^2/(a^2+b^2)*a*b^2*f/(2*a^2+2*
b^2)*dilog(1-I*exp(d*x+c))-3*I/d^2/(a^2+b^2)*a*b^2*f/(2*a^2+2*b^2)*dilog(1
+I*exp(d*x+c))+I/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c+I/
d/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x-1/d/(a^2+b^2)^(3/2)*e
*a*b^3/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d
/(a^2+b^2)^(3/2)*e*a^3*b/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))+1/d/(a^2+b^2)^(1/2)*e*a*b/(2*a^2+2*b^2)*arctanh(1/2*(2*b*ex
p(d*x+c)+2*a)/(a^2+b^2)^(1/2))-6/d^2/(a^2+b^2)*c*a*b^2*f/(2*a^2+2*b^2)*arc
tan(exp(d*x+c))-2/d^2/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c-2
/d/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x-2/d^2/(a^2+b^2)*b^3*
f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c+6/d/(a^2+b^2)*e*a*b^2/(2*a^2+2*b^2)*a
rctan(exp(d*x+c))+I/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c)
)-2/d^2/(a^2+b^2)*c*a^3*f/(2*a^2+2*b^2)*arctan(exp(d*x+c))+1/d^2/(a^2+b^2)
^(3/2)*c*a*b^3*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^
(1/2))+1/d^2/(a^2+b^2)^(3/2)*c*a^3*b*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(
d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2/(a^2+b^2)^(1/2)*c*a*b*f/(2*a^2+2*b^2)*a
rctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d/(a^2+b^2)*a^3*e/(2*a^
2+2*b^2)*arctan(exp(d*x+c))-2/d/(a^2+b^2)*b^3*e/(2*a^2+2*b^2)*ln(1+exp(2*d
*x+2*c))+2/d/(a^2+b^2)*b^3*e/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(...
```

3.315.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4729 vs. 2(505) = 1010.

Time = 0.34 (sec) , antiderivative size = 4729, normalized size of antiderivative = 8.44

$$\int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*cosh(d*
x + c)^3 + 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*s
inh(d*x + c)^3 + 2*(2*(a^2*b + b^3)*d*f*x + 2*(a^2*b + b^3)*d*e + (a^2*b +
 b^3)*f)*cosh(d*x + c)^2 + 2*(2*(a^2*b + b^3)*d*f*x + 2*(a^2*b + b^3)*d*e
+ (a^2*b + b^3)*f + 3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*
b^2)*f)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a^2*b + b^3)*f - 2*((a^3 + a*b
^2)*d*f*x + (a^3 + a*b^2)*d*e - (a^3 + a*b^2)*f)*cosh(d*x + c) + 2*(b^3*f*
cosh(d*x + c)^4 + 4*b^3*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*f*sinh(d*x +
 c)^4 + 2*b^3*f*cosh(d*x + c)^2 + b^3*f + 2*(3*b^3*f*cosh(d*x + c)^2 + b^3
*f)*sinh(d*x + c)^2 + 4*(b^3*f*cosh(d*x + c)^3 + b^3*f*cosh(d*x + c))*sinh
(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^3*f*cosh(d*x + c)
^4 + 4*b^3*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*f*sinh(d*x + c)^4 + 2*b^3
*f*cosh(d*x + c)^2 + b^3*f + 2*(3*b^3*f*cosh(d*x + c)^2 + b^3*f)*sinh(d*x
+ c)^2 + 4*(b^3*f*cosh(d*x + c)^3 + b^3*f*cosh(d*x + c))*sinh(d*x + c))*di
log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - ((2*b^3*f - I*(a^3 + 3*a*b^2)*f)*co
sh(d*x + c)^4 + 4*(2*b^3*f - I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)*sinh(d*x +
 c)^3 + (2*b^3*f - I*(a^3 + 3*a*b^2)*f)*sinh(d*x + c)^4 + 2*b^3*f + 2*(2*b
^3*f - I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^2 + 2*(2*b^3*f + 3*(2*b^3*f -...
```

3.315.6 Sympy [F]

$$\int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*sech(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

3.315.7 Maxima [F]

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{sech}\,(dx+c)^3}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
(b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b
^4)*d) - b^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3
+ 3*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (a*e^(-d*x -
 c) + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^
2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e + f*(((a*d*x*e^(
3*c) + a*e^(3*c))*e^(3*d*x) + (2*b*d*x*e^(2*c) + b*e^(2*c))*e^(2*d*x) - (a
*d*x*e^c - a*e^c)*e^(d*x) + b)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^2
*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x))
 - 8*integrate(-1/4*(a*b^3*x*e^(d*x + c) - b^4*x)/(a^4*b + 2*a^2*b^3 + b^5
 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^
c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) + 8*integrate(1/8*(2*b^3*x + (
a^3*e^c + 3*a*b^2*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) +
2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x))
```

3.315.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\mathrm{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\left(c+d\,x\right)^3\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.316 $\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.316.1 Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a(a^2+3b^2)\arctan(\sinh(c+dx))}{2(a^2+b^2)^2 d} - \frac{b^3\log(\cosh(c+dx))}{(a^2+b^2)^2 d} + \frac{b^3\log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d} + \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)d}$$

output
```
1/2*a*(a^2+3*b^2)*arctan(sinh(d*x+c))/(a^2+b^2)^2/d-b^3*ln(cosh(d*x+c))/(a
^2+b^2)^2/d+b^3*ln(a+b*sinh(d*x+c))/(a^2+b^2)^2/d+1/2*sech(d*x+c)^2*(b+a*s
inh(d*x+c))/(a^2+b^2)/d
```

3.316.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a(a^2+b^2)\arctan(\sinh(c+dx)) - b^2((ia+b)\log(i-\sinh(c+dx)) + (-ia+b)\log(i+\sinh(c+dx)) - 2}{2(a^2+b^2)^2 d}$$

input
```
Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
(a*(a^2 + b^2)*ArcTan[Sinh[c + d*x]] - b^2*((I*a + b)*Log[I - Sinh[c + d*x
]] + ((-I)*a + b)*Log[I + Sinh[c + d*x]] - 2*b*Log[a + b*Sinh[c + d*x]]) +
 b*(a^2 + b^2)*Sech[c + d*x]^2 + a*(a^2 + b^2)*Sech[c + d*x]*Tanh[c + d*x]
)/(2*(a^2 + b^2)^2*d)
```

3.316.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3147, 496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{b^3\int \frac{1}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{496}$$

$$\frac{b^3\left(\frac{ab\sinh(c+dx)+b^2}{2b^2(a^2+b^2)\left(b^2\sinh^2(c+dx)+b^2\right)}-\frac{\int -\frac{a^2+b\sinh(c+dx)a+2b^2}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^3\left(\frac{\int \frac{a^2+b\sinh(c+dx)a+2b^2}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2b^2(a^2+b^2)}+\frac{ab\sinh(c+dx)+b^2}{2b^2(a^2+b^2)\left(b^2\sinh^2(c+dx)+b^2\right)}\right)}{d}$$

$$\Big\downarrow \text{657}$$

$$\frac{b^3\left(\frac{\int \left(\frac{2b^2}{\left(a^2+b^2\right)(a+b\sinh(c+dx))}+\frac{a^3+3b^2a-2b^3\sinh(c+dx)}{\left(a^2+b^2\right)\left(\sinh^2(c+dx)b^2+b^2\right)}\right)d(b\sinh(c+dx))}{2b^2(a^2+b^2)}+\frac{ab\sinh(c+dx)+b^2}{2b^2(a^2+b^2)\left(b^2\sinh^2(c+dx)+b^2\right)}\right)}{d}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{b^3\left(\frac{\frac{a(a^2+3b^2)\arctan(\sinh(c+dx))}{b(a^2+b^2)}-\frac{b^2\log(b^2\sinh^2(c+dx)+b^2)}{a^2+b^2}+\frac{2b^2\log(a+b\sinh(c+dx))}{a^2+b^2}}{2b^2(a^2+b^2)}+\frac{ab\sinh(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

input
```
Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^3*(((a*(a^2 + 3*b^2)*ArcTan[Sinh[c + d*x]])/(b*(a^2 + b^2)) + (2*b^2*Lo
g[a + b*Sinh[c + d*x]])/(a^2 + b^2) - (b^2*Log[b^2 + b^2*Sinh[c + d*x]^2])
/(a^2 + b^2))/(2*b^2*(a^2 + b^2)) + (b^2 + a*b*Sinh[c + d*x])/(2*b^2*(a^2
+ b^2)*(b^2 + b^2*Sinh[c + d*x]^2))))/d
```

3.316.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.316. $\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.316.4 Maple [A] (verified)

Time = 19.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.72

method	result
derivativedivides	$\frac{\frac{b^3 \ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^4+2a^2b^2+b^4}+\frac{\frac{2\left(\left(-\frac{1}{2}a^3-\frac{1}{2}a\,b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^2b-b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tan}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}}{a^4+2a^2b^2}}{d}$
default	$\frac{\frac{b^3 \ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^4+2a^2b^2+b^4}+\frac{\frac{2\left(\left(-\frac{1}{2}a^3-\frac{1}{2}a\,b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^2b-b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tan}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}}{a^4+2a^2b^2}}{d}$
risch	$\frac{2b^3d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2b^3dc}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2b^3x}{a^4+2a^2b^2+b^4}-\frac{2b^3c}{d(a^4+2a^2b^2+b^4)}+\frac{e^{dx+c}(a\,e^{2dx+2c}+2b\,e^{dx+c}}{d(a^2+b^2)(1+e^{2dx+2c})^2}$

input
```
int(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3/(a^4+2*a^2*b^2+b^4)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1
/2*c)-a)+2/(a^4+2*a^2*b^2+b^4)*(((-1/2*a^3-1/2*a*b^2)*tanh(1/2*d*x+1/2*c)^
3+(-a^2*b-b^3)*tanh(1/2*d*x+1/2*c)^2+(1/2*a^3+1/2*a*b^2)*tanh(1/2*d*x+1/2*
c))/(1+tanh(1/2*d*x+1/2*c)^2)^2-1/2*b^3*ln(1+tanh(1/2*d*x+1/2*c)^2)+1/2*(a
^3+3*a*b^2)*arctan(tanh(1/2*d*x+1/2*c))))
```

3.316.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. 2(115) = 230.

Time = 0.26 (sec) , antiderivative size = 893, normalized size of antiderivative = 7.50

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
((a^3 + a*b^2)*cosh(d*x + c)^3 + (a^3 + a*b^2)*sinh(d*x + c)^3 + 2*(a^2*b
+ b^3)*cosh(d*x + c)^2 + (2*a^2*b + 2*b^3 + 3*(a^3 + a*b^2)*cosh(d*x + c))
*sinh(d*x + c)^2 + ((a^3 + 3*a*b^2)*cosh(d*x + c)^4 + 4*(a^3 + 3*a*b^2)*co
sh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a*b^2)*sinh(d*x + c)^4 + a^3 + 3*a*
b^2 + 2*(a^3 + 3*a*b^2)*cosh(d*x + c)^2 + 2*(a^3 + 3*a*b^2 + 3*(a^3 + 3*a*
b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3 + 3*a*b^2)*cosh(d*x + c)^3
 + (a^3 + 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + si
nh(d*x + c)) - (a^3 + a*b^2)*cosh(d*x + c) + (b^3*cosh(d*x + c)^4 + 4*b^3*
cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^
2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*
x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(
cosh(d*x + c) - sinh(d*x + c))) - (b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x +
c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 + 2
*(3*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 +
b^3*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sin
h(d*x + c))) - (a^3 + a*b^2 - 3*(a^3 + a*b^2)*cosh(d*x + c)^2 - 4*(a^2*b +
 b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x +
c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 +
2*a^2*b^2 + b^4)*d*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x
+ c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + (a^4 + 2*a^2*...
```

3.316.6 Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sech(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

3.316.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.82

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^4+2\,a^2b^2+b^4)d} - \frac{b^3\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d} - \frac{(a^3+3\,ab^2)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d} + \frac{ae^{(-dx-c)}+2\,be^{(-2\,dx-2\,c)}-ae^{(-3\,dx-3\,c)}}{(a^2+b^2+2\,(a^2+b^2)e^{(-2\,dx-2\,c)}+(a^2+b^2)e^{(-4\,dx-4\,c)})d}$$

input
```
integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^
4)*d) - b^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3 +
 3*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (a*e^(-d*x -
c) + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2
)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d)
```

3.316.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(115) = 230$.

Time = 0.33 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.37

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{4\,b^4\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5} - \frac{2\,b^3\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^4+2\,a^2b^2+b^4} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(a^3+3\,ab^2)}{a^4+2\,a^2b^2+b^4} + \frac{2\left(b\right.}{}}{4\,d}$$

input
```
integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(4*b^4*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b + 2*a^2*b
^3 + b^5) - 2*b^3*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a^2*b^2
 + b^4) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a^3 + 3
*a*b^2)/(a^4 + 2*a^2*b^2 + b^4) + 2*(b^3*(e^(d*x + c) - e^(-d*x - c))^2 +
2*a^3*(e^(d*x + c) - e^(-d*x - c)) + 2*a*b^2*(e^(d*x + c) - e^(-d*x - c))
+ 4*a^2*b + 8*b^3)/((a^4 + 2*a^2*b^2 + b^4)*((e^(d*x + c) - e^(-d*x - c))^
2 + 4)))/d
```

3.316.9 Mupad [B] (verification not implemented)

Time = 3.11 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.20

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,(a^2\,b+b^3)}{d\,(a^2+b^2)^2}+\frac{\mathrm{e}^{c+d\,x}\,(a^3+a\,b^2)}{d\,(a^2+b^2)^2}}{\mathrm{e}^{2\,c+2\,d\,x}+1}-\frac{\frac{2\,b}{d\,(a^2+b^2)}+\frac{2\,a\,\mathrm{e}^{c+d\,x}}{d\,(a^2+b^2)}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}$$
$$-\frac{\ln\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)\,(2\,b+a\,1\mathrm{i})}{2\,(-d\,a^2+2\mathrm{i}\,d\,a\,b+d\,b^2)}-\frac{\ln\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)\,(a+b\,2\mathrm{i})}{2\,(-1\mathrm{i}\,d\,a^2+2\,d\,a\,b+1\mathrm{i}\,d\,b^2)}$$
$$+\frac{b^3\,\ln\left(2\,a^7\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-16\,b^7-9\,a^2\,b^5-6\,a^4\,b^3-a^6\,b+16\,b^7\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+a^6\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+18\,a^3\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+12\,a^5\right.}{d\,a^4+2\,d\,a^2\,b^2+d\,b^4}$$

input
```
int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
((2*(a^2*b + b^3))/(d*(a^2 + b^2)^2) + (exp(c + d*x)*(a*b^2 + a^3))/(d*(a^
2 + b^2)^2))/(exp(2*c + 2*d*x) + 1) - ((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c
+ d*x))/(d*(a^2 + b^2)))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (lo
g(exp(c + d*x) + 1i)*(a*1i + 2*b))/(2*(b^2*d - a^2*d + a*b*d*2i)) - (log(e
xp(c + d*x)*1i + 1)*(a + b*2i))/(2*(b^2*d*1i - a^2*d*1i + 2*a*b*d)) + (b^3
*log(2*a^7*exp(d*x)*exp(c) - 16*b^7 - 9*a^2*b^5 - 6*a^4*b^3 - a^6*b + 16*b
^7*exp(2*c)*exp(2*d*x) + a^6*b*exp(2*c)*exp(2*d*x) + 18*a^3*b^4*exp(d*x)*e
xp(c) + 12*a^5*b^2*exp(d*x)*exp(c) + 9*a^2*b^5*exp(2*c)*exp(2*d*x) + 6*a^4
*b^3*exp(2*c)*exp(2*d*x) + 32*a*b^6*exp(d*x)*exp(c)))/(a^4*d + b^4*d + 2*a
^2*b^2*d)
```

3.317 $\int \frac{\mathrm{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.317.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\mathrm{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \mathrm{Int}\left(\frac{\mathrm{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.317.2 Mathematica [N/A]

Not integrable

Time = 65.56 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\mathrm{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\mathrm{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Sech[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Sech[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.317.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Sech[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.317.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.317.4 Maple [N/A] (verified)

Not integrable

Time = 0.87 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}\left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.317.5 Fricas [N/A]

Not integrable

Time = 3.89 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sech(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.317.6 Sympy [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}^3(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sech(c + d*x)**3/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.317.7 Maxima [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 1100, normalized size of antiderivative = 39.29

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(b*f - (a*d*f*x*e^(3*c) + (d*e - f)*a*e^(3*c))*e^(3*d*x) - (2*b*d*f*x*e^(
2*c) + (2*d*e - f)*b*e^(2*c))*e^(2*d*x) + (a*d*f*x*e^c + (d*e + f)*a*e^c)*
e^(d*x))/(a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*
(a^2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^(4*c) + b^2*d^2*e^2*e^(4*c)
 + (a^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*e*f*e^(4*c
) + b^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + 2*(a^2*d^2*e^2*e^(2*c) + b^2*d^2*e
^2*e^(2*c) + (a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*
e*f*e^(2*c) + b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) + 8*integrate(1/8*(2*b^3*
d^2*f^2*x^2 + 4*b^3*d^2*e*f*x - 2*a^2*b*f^2 + 2*(d^2*e^2 - f^2)*b^3 + ((d^
2*e^2 - 2*f^2)*a^3*e^c + (3*d^2*e^2 - 2*f^2)*a*b^2*e^c + (a^3*d^2*f^2*e^c
+ 3*a*b^2*d^2*f^2*e^c)*x^2 + 2*(a^3*d^2*e*f*e^c + 3*a*b^2*d^2*e*f*e^c)*x)*
e^(d*x))/(a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2
*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f
^2 + b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2
*e^2*f)*x + (a^4*d^2*e^3*e^(2*c) + 2*a^2*b^2*d^2*e^3*e^(2*c) + b^4*d^2*e^3
*e^(2*c) + (a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*
e^(2*c))*x^3 + 3*(a^4*d^2*e*f^2*e^(2*c) + 2*a^2*b^2*d^2*e*f^2*e^(2*c) + b^
4*d^2*e*f^2*e^(2*c))*x^2 + 3*(a^4*d^2*e^2*f*e^(2*c) + 2*a^2*b^2*d^2*e^2*f*
e^(2*c) + b^4*d^2*e^2*f*e^(2*c))*x)*e^(2*d*x)), x) - 8*integrate(-1/4*(a*b
^3*e^(d*x + c) - b^4)/(a^4*b*e + 2*a^2*b^3*e + b^5*e + (a^4*b*f + 2*a^2...
```

3.317.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.317.9 Mupad [N/A]

Not integrable

Time = 3.75 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^3\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.318 $\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.318.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \mathrm{Int}\left(\frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)},x\right)$$

output
```
Unintegrable(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.318.2 Mathematica [N/A]

Not integrable

Time = 10.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
Integrate[(x^m*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
Integrate[(x^m*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]), x]
```

3.318.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$
$$\Big\downarrow \, 6111$$
$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

input
```
Int[(x^m*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.318.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.318.4 Maple [N/A] (verified)

Not integrable

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \cosh\left(dx+c\right)^3}{a+b\sinh\left(dx+c\right)}dx$$

input
```
int(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.318.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(x^m*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.318.6 Sympy [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(x**m*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(x**m*cosh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

3.318.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(x^m*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.318.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh{(dx+c)}^3}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^m*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.318.9 Mupad [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh(c+d\,x)^3}{a+b\sinh{(c+d\,x)}}\,dx$$

input
```
int((x^m*cosh(c + d*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((x^m*cosh(c + d*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.319 $\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.319.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \mathrm{Int}\left(\frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)},x\right)$$

output
```
Unintegrable(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.319.2 Mathematica [N/A]

Not integrable

Time = 8.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
Integrate[(x^m*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
Integrate[(x^m*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]), x]
```

3.319.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)} dx$$
$$\Big\downarrow \, 6111$$
$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)} dx$$

input
```
Int[(x^m*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.319.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.319.4 Maple [N/A] (verified)

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \cosh(dx+c)^2}{a+b\sinh(dx+c)} dx$$

input
```
int(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.319.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh\left(dx+c\right)^2}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(x^m*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.319.6 Sympy [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh^2\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(x**m*cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(x**m*cosh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.319.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh\left(dx+c\right)^2}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(x^m*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.319.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^m*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.319.9 Mupad [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh(c+d\,x)^2}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((x^m*cosh(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((x^m*cosh(c + d*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.320 $\int \frac{x^m \cosh(c+dx)}{a+b \sinh(c+dx)} \, dx$

3.320.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \cosh(c+dx)}{a+b \sinh(c+dx)} \, dx = \text{Int}\left(\frac{x^m \cosh(c+dx)}{a+b \sinh(c+dx)}, x\right)$$

output
```
Unintegrable(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.320.2 Mathematica [N/A]

Not integrable

Time = 2.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \cosh(c+dx)}{a+b \sinh(c+dx)} \, dx = \int \frac{x^m \cosh(c+dx)}{a+b \sinh(c+dx)} \, dx$$

input
```
Integrate[(x^m*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
Integrate[(x^m*Cosh[c + d*x])/(a + b*Sinh[c + d*x]), x]
```

3.320.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)} \, dx$$
$$\Big\downarrow \; 6111$$
$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)} dx$$

input
```
Int[(x^m*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.320.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.320.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \cosh (dx+c)}{a+b \sinh (dx+c)}dx$$

input
```
int(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.320.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(x^m*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.320.6 Sympy [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(x**m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(x**m*cosh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.320.7 Maxima [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 205, normalized size of antiderivative = 9.32

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
x*e^(2*d*x + m*log(x) + 2*c)/(b*(m + 1)*e^(2*d*x + 2*c) + 2*a*(m + 1)*e^(d
*x + c) - b*(m + 1)) - 1/2*integrate(2*(2*a*d*x*e^(3*d*x + 3*c) - 2*a*(m +
 1)*e^(d*x + c) + b*(m + 1) - (2*b*d*x*e^(2*c) + b*(m + 1)*e^(2*c))*e^(2*d
*x))*x^m/(b^2*(m + 1)*e^(4*d*x + 4*c) + 4*a*b*(m + 1)*e^(3*d*x + 3*c) - 4*
a*b*(m + 1)*e^(d*x + c) + b^2*(m + 1) + 2*(2*a^2*(m + 1)*e^(2*c) - b^2*(m
+ 1)*e^(2*c))*e^(2*d*x)), x)
```

3.320.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^m*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.320.9 Mupad [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh(c+d\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((x^m*cosh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((x^m*cosh(c + d*x))/(a + b*sinh(c + d*x)), x)
```

3.321 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

3.321.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = -\frac{2f\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2} - \frac{e+fx}{bd(a+b\sinh(c+dx))}$$

output
```
(-f*x-e)/b/d/(a+b*sinh(d*x+c))-2*f*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+
b^2)^(1/2))/b/d^2/(a^2+b^2)^(1/2)
```

3.321.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \frac{\frac{2f\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - \frac{d(e+fx)}{a+b\sinh(c+dx)}}{bd^2}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
((2*f*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
 - (d*(e + f*x))/(a + b*Sinh[c + d*x]))/(b*d^2)
```

3.321.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5987, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$$

$$\Big\downarrow \text{5987}$$

$$\frac{f\int \frac{1}{a+b\sinh(c+dx)}dx}{bd}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{e+fx}{bd(a+b\sinh(c+dx))}+\frac{f\int \frac{1}{a-ib\sin(ic+idx)}dx}{bd}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{e+fx}{bd(a+b\sinh(c+dx))}-\frac{2if\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd^2}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{e+fx}{bd(a+b\sinh(c+dx))}+\frac{4if\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{bd^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{2f\,\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

input
```
Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(2*f*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d^
2) - (e + f*x)/(b*d*(a + b*Sinh[c + d*x]))
```

3.321.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.321.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(71) = 142$.

Time = 3.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.22

method	result	size
risch	$-\frac{2(fx+e)\,\mathrm{e}^{dx+c}}{bd\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)}+\frac{f\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d^2b}-\frac{f\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d^2b}$	164

input
```
int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(f*x+e)/b/d*exp(d*x+c)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/(a^2+b^2)^
(1/2)*f/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(1/2)-a^2-b^2)/(a^2+b^2)^(1/2)/b)
-1/(a^2+b^2)^(1/2)*f/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(1/2)+a^2+b^2)/(a^2+
b^2)^(1/2)/b)
```

3.321.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(71) = 142$.

Time = 0.24 (sec) , antiderivative size = 411, normalized size of antiderivative = 5.55

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$$
$$= \frac{\left(bf\cosh\left(dx+c\right)^2+bf\sinh\left(dx+c\right)^2+2\,af\cosh\left(dx+c\right)-bf+2\left(bf\cosh\left(dx+c\right)+af\right)\sinh\left(dx+c\right)\right.}{(a^2b^2+b^4)d^2\cosh\left(dx+c\right)^2+}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
((b*f*cosh(d*x + c)^2 + b*f*sinh(d*x + c)^2 + 2*a*f*cosh(d*x + c) - b*f +
2*(b*f*cosh(d*x + c) + a*f)*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d
*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b
^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c)
 + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh
(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*((a^2 + b^2)*d
*f*x + (a^2 + b^2)*d*e)*cosh(d*x + c) - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)
*d*e)*sinh(d*x + c))/((a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + (a^2*b^2 + b^4
)*d^2*sinh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d^2*cosh(d*x + c) - (a^2*b^2 + b
^4)*d^2 + 2*((a^2*b^2 + b^4)*d^2*cosh(d*x + c) + (a^3*b + a*b^3)*d^2)*sinh
(d*x + c))
```

3.321.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

output
```
Timed out
```

3.321.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(71) = 142$.

Time = 0.36 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.12

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$$
$$= -f\left(\frac{2\,xe^{(dx+c)}}{b^2de^{(2\,dx+2\,c)}+2\,abde^{(dx+c)}-b^2d}-\frac{\log\left(\frac{be^{(dx+c)}+a-\sqrt{a^2+b^2}}{be^{(dx+c)}+a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}bd^2}\right)$$
$$-\frac{2\,ee^{(-dx-c)}}{(2\,abe^{(-dx-c)}-b^2e^{(-2\,dx-2\,c)}+b^2)d}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d)
- log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2
+ b^2)))/(sqrt(a^2 + b^2)*b*d^2)) - 2*e*e^(-d*x - c)/((2*a*b*e^(-d*x - c)
- b^2*e^(-2*d*x - 2*c) + b^2)*d)
```

3.321.8 Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

3.321.9 Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.69

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \frac{f\,\ln\left(\frac{2\,f\,\left(b-a\,{\mathrm{e}}^{c+d\,x}\right)}{b^2\,d\,\sqrt{a^2+b^2}}-\frac{2\,f\,{\mathrm{e}}^{c+d\,x}}{b^2\,d}\right)}{b\,d^2\,\sqrt{a^2+b^2}} - \frac{f\,\ln\left(-\frac{2\,f\,{\mathrm{e}}^{c+d\,x}}{b^2\,d}-\frac{2\,f\,\left(b-a\,{\mathrm{e}}^{c+d\,x}\right)}{b^2\,d\,\sqrt{a^2+b^2}}\right)}{b\,d^2\,\sqrt{a^2+b^2}} - \frac{2\,{\mathrm{e}}^{c+d\,x}\,\left(a^2\,e+b^2\,e+a^2\,f\,x+b^2\,f\,x\right)}{d\,\left(a^2\,b+b^3\right)\,\left(2\,a\,{\mathrm{e}}^{c+d\,x}-b+b\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}$$

input
```
int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^2,x)
```

output
```
(f*log((2*f*(b - a*exp(c + d*x)))/(b^2*d*(a^2 + b^2)^(1/2)) - (2*f*exp(c +
 d*x))/(b^2*d)))/(b*d^2*(a^2 + b^2)^(1/2)) - (f*log(- (2*f*exp(c + d*x))/(
b^2*d) - (2*f*(b - a*exp(c + d*x)))/(b^2*d*(a^2 + b^2)^(1/2))))/(b*d^2*(a^
2 + b^2)^(1/2)) - (2*exp(c + d*x)*(a^2*e + b^2*e + a^2*f*x + b^2*f*x))/(d*
(a^2*b + b^3)*(2*a*exp(c + d*x) - b + b*exp(2*c + 2*d*x)))
```

3.322 $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

3.322.1 Optimal result

Integrand size = 26, antiderivative size = 234

$$\begin{aligned}\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx &= \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} \\ &\quad-\frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} \\ &\quad+\frac{2f^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} \\ &\quad-\frac{2f^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}\end{aligned}$$

output
```
-(f*x+e)^2/b/d/(a+b*sinh(d*x+c))+2*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/b/d^2/(a^2+b^2)^(1/2)-2*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/b/d^2/(a^2+b^2)^(1/2)+2*f^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))/b/d^3/(a^2+b^2)^(1/2)-2*f^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/b/d^3/(a^2+b^2)^(1/2)
```

3.322.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.86

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx =$$
$$-\frac{2f\left(d\left(2e\,\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-fx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+fx\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)-f\,\text{PolyLog}\left(2,\frac{be^{c+}}{-a+\sqrt{a}}\right.}{b\sqrt{a^2+b^2}d^3}$$
$$-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(-2*f*(d*(2*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f*x*Log[1 + (
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*x*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])]) - f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
 f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*Sqrt[a^2 + b^
2]*d^3) - (e + f*x)^2/(b*d*(a + b*Sinh[c + d*x]))
```

3.322.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5987, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$$
$$\Big\downarrow \text{5987}$$
$$\frac{2f\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}+\frac{2f\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{bd}$$
$$\Big\downarrow \text{3803}$$

$$\frac{4f\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \; 25$$

$$-\frac{4f\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \; 2694$$

$$-\frac{4f\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \; 27$$

$$-\frac{4f\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \; 2620$$

$$-\frac{4f\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \; 2715$$

$$-\frac{4f\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2}\right)}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \; 2838$$

3.322. $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

$$-\frac{4f\left(\frac{b\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(-4*f*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])
/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2))
)/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/
(b*d^2)))/(2*Sqrt[a^2 + b^2])))/(b*d) - (e + f*x)^2/(b*d*(a + b*Sinh[c + d
*x]))
```

3.322.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.322.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(214) = 428$.

Time = 3.24 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.10

method	result
risch	$-\frac{2(x^2f^2+2efx+e^2)\mathrm{e}^{dx+c}}{bd\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)}-\frac{4fe\,\operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d^2b\sqrt{a^2+b^2}}+\frac{2f^2\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{d^2b\sqrt{a^2+b^2}}-\frac{2f^2\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right.}{d^2b\sqrt{a^2+b^2}}$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(f^2*x^2+2*e*f*x+e^2)/b/d*exp(d*x+c)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b
)-4/d^2/b*f*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))+2/d^2/b*f^2/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a
+(a^2+b^2)^(1/2)))*x-2/d^2/b*f^2/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2
)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+2/d^3/b*f^2/(a^2+b^2)^(1/2)*ln((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/d^3/b*f^2/(a^2+b^2)^(1/
2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d^3/b*f^2/
(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))-2/d^3/b*f^2/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+
(a^2+b^2)^(1/2)))+4/d^3/b*f^2*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c
)+2*a)/(a^2+b^2)^(1/2))
```

3.322.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1378 vs. $2(212) = 424$.

Time = 0.27 (sec) , antiderivative size = 1378, normalized size of antiderivative = 5.89

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
2*((b^2*f^2*cosh(d*x + c)^2 + b^2*f^2*sinh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x
 + c) - b^2*f^2 + 2*(b^2*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^2*f^2*cosh(d*
x + c)^2 + b^2*f^2*sinh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x + c) - b^2*f^2 + 2
*(b^2*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*di
log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^2*d*e*f - b^2*c*f^2 - (b^2*d*e*f
 - b^2*c*f^2)*cosh(d*x + c)^2 - (b^2*d*e*f - b^2*c*f^2)*sinh(d*x + c)^2 -
2*(a*b*d*e*f - a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*e*f - a*b*c*f^2 + (b^2*
d*e*f - b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log
(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
- (b^2*d*e*f - b^2*c*f^2 - (b^2*d*e*f - b^2*c*f^2)*cosh(d*x + c)^2 - (b^2*
d*e*f - b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*e*f - a*b*c*f^2)*cosh(d*x +
c) - 2*(a*b*d*e*f - a*b*c*f^2 + (b^2*d*e*f - b^2*c*f^2)*cosh(d*x + c))*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c
) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d*f^2*x + b^2*c*f^2 - (b^2*d*f
^2*x + b^2*c*f^2)*cosh(d*x + c)^2 - (b^2*d*f^2*x + b^2*c*f^2)*sinh(d*x + c
)^2 - 2*(a*b*d*f^2*x + a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*f^2*x + a*b*c*f
^2 + (b^2*d*f^2*x + b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 ...
```

3.322.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

output
```
Timed out
```

3.322.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-2*(x^2*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d)
- 2*integrate(x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) -
 b^2*d), x))*f^2 - 2*e*f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d
*e^(d*x + c) - b^2*d) - log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*
x + c) + a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d^2)) - 2*e^2*e^(-d*x -
c)/((2*a*b*e^(-d*x - c) - b^2*e^(-2*d*x - 2*c) + b^2)*d)
```

3.322.8 Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)\,(e+f\,x)^2}{(a+b\sinh(c+d\,x))^2}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^2,x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^2, x)
```

3.323 $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

3.323.1 Optimal result

Integrand size = 26, antiderivative size = 348

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} + \frac{6f^2(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} - \frac{6f^2(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} - \frac{6f^3\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^4} + \frac{6f^3\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^4} - \frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

output
```
-(f*x+e)^3/b/d/(a+b*sinh(d*x+c))+3*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/b/d^2/(a^2+b^2)^(1/2)-3*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+
b^2)^(1/2)))/b/d^2/(a^2+b^2)^(1/2)+6*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))/b/d^3/(a^2+b^2)^(1/2)-6*f^2*(f*x+e)*polylog(2,-b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/b/d^3/(a^2+b^2)^(1/2)-6*f^3*polylog(3,-b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/b/d^4/(a^2+b^2)^(1/2)+6*f^3*polylog(3,-b*exp(d*x
+c)/(a+(a^2+b^2)^(1/2)))/b/d^4/(a^2+b^2)^(1/2)
```

3.323.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.06

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$$
$$= \frac{3f\left(-2d^2e^2\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^2f^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2d^2efx\log\left(1\right.\right.}{}$$
$$-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(3*f*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*
x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])] - 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
] - 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyL
og[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*Sqrt[a^2 + b^2]*d^4)
- (e + f*x)^3/(b*d*(a + b*Sinh[c + d*x]))
```

3.323.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5987, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$$
$$\Big\downarrow \text{5987}$$
$$\frac{3f\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}+\frac{3f\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{bd}$$

$$\Big\downarrow \ \color{blue}{3803}$$

$$\frac{6f\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \ \color{blue}{25}$$

$$-\frac{6f\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \ \color{blue}{2694}$$

$$-\frac{6f\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \ \color{blue}{27}$$

$$-\frac{6f\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \ \color{blue}{2620}$$

$$-\frac{6f\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{bd}$$

$$\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \ \color{blue}{3011}$$

3.323. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

$$-\frac{6f\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+\right.}{bd}\right.}{}\right.}{bd}$$

$$\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{6f\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\right.}{}\right.}{}\right.}{bd}$$

$$\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{6f\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\right.}{}\right.}{bd}$$

$$\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

3.323. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

output
```
(-6*f*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^
2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(b*d) - (e + f*x)^3/(b
*d*(a + b*Sinh[c + d*x]))
```

3.323.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.323. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.323.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)}{(a+b\sinh(dx+c))^2}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

3.323.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2420 vs. $2(316) = 632$.

Time = 0.29 (sec) , antiderivative size = 2420, normalized size of antiderivative = 6.95

$$\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(6*(b^2*d*f^3*x + b^2*d*e*f^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)
^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^3*x + a*b*d*
e*f^2)*cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*e*f^2 + (b^2*d*f^3*x + b^2*d
*e*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b + 1) - 6*(b^2*d*f^3*x + b^2*d*e*f^2 - (b^2*d*f^3*x + b
^2*d*e*f^2)*cosh(d*x + c)^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*sinh(d*x + c)^2
 - 2*(a*b*d*f^3*x + a*b*d*e*f^2)*cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*e*f
^2 + (b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(b^2*d^2*e^2*f - 2*b
^2*c*d*e*f^2 + b^2*c^2*f^3 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^
3)*cosh(d*x + c)^2 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*sinh(
d*x + c)^2 - 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3)*cosh(d*x +
c) - 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (b^2*d^2*e^2*f - 2
*b^2*c*d*e*f^2 + b^2*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) + 3*(b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3 - (b^2*d^2*e
^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*cosh(d*x + c)^2 - (b^2*d^2*e^2*f - 2
*b^2*c*d*e*f^2 + b^2*c^2*f^3)*sinh(d*x + c)^2 - 2*(a*b*d^2*e^2*f - 2*a*...
```

3.323.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

output
```
Timed out
```

3.323.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-3*e^2*f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b
^2*d) - log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqr
t(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d^2)) - 2*e^3*e^(-d*x - c)/((2*a*b*e^(-d
*x - c) - b^2*e^(-2*d*x - 2*c) + b^2)*d) - 2*(f^3*x^3*e^c + 3*e*f^2*x^2*e^
c)*e^(d*x)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d) + integra
te(6*(f^3*x^2*e^c + 2*e*f^2*x*e^c)*e^(d*x)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*
d*e^(d*x + c) - b^2*d), x)
```

3.323.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)\,(e+f\,x)^3}{(a+b\sinh(c+d\,x))^2}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^2,x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^2, x)
```

3.324 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

3.324.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx=-\frac{2f\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

output
```
(-f*x-e)/b/d/(a+b*sinh(d*x+c))-2*f*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+
b^2)^(1/2))/b/d^2/(a^2+b^2)^(1/2)
```

3.324.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx=\frac{\frac{2f\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\frac{d(e+fx)}{a+b\sinh(c+dx)}}{bd^2}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
((2*f*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
 - (d*(e + f*x))/(a + b*Sinh[c + d*x]))/(b*d^2)
```

3.324.3 Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5987, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}dx$$

$$\Big\downarrow \text{5987}$$

$$\frac{f\int \frac{1}{a+b\sinh(c+dx)}dx}{bd}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{e+fx}{bd(a+b\sinh(c+dx))}+\frac{f\int \frac{1}{a-ib\sin(ic+idx)}dx}{bd}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{e+fx}{bd(a+b\sinh(c+dx))}-\frac{2if\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd^2}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{e+fx}{bd(a+b\sinh(c+dx))}+\frac{4if\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{bd^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{2f\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

input
```
Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(2*f*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d^
2) - (e + f*x)/(b*d*(a + b*Sinh[c + d*x]))
```

3.324.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.324.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(71) = 142$.

Time = 2.96 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.22

method	result	size
risch	$-\frac{2(fx+e)\,\mathrm{e}^{dx+c}}{bd\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)}+\frac{f\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d^2b}-\frac{f\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d^2b}$	164

input
```
int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(f*x+e)/b/d*exp(d*x+c)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/(a^2+b^2)^
(1/2)*f/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(1/2)-a^2-b^2)/(a^2+b^2)^(1/2)/b)
-1/(a^2+b^2)^(1/2)*f/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(1/2)+a^2+b^2)/(a^2+
b^2)^(1/2)/b)
```

3.324.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(71) = 142$.

Time = 0.25 (sec) , antiderivative size = 411, normalized size of antiderivative = 5.55

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \frac{\left(bf\cosh\left(dx+c\right)^2+bf\sinh\left(dx+c\right)^2+2\,af\cosh\left(dx+c\right)-bf+2\left(bf\cosh\left(dx+c\right)+af\right)\sinh\left(dx+c\right)}{(a^2b^2+b^4)d^2\cosh\left(dx+c\right)^2+}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
((b*f*cosh(d*x + c)^2 + b*f*sinh(d*x + c)^2 + 2*a*f*cosh(d*x + c) - b*f +
2*(b*f*cosh(d*x + c) + a*f)*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d
*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b
^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c)
 + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh
(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*((a^2 + b^2)*d
*f*x + (a^2 + b^2)*d*e)*cosh(d*x + c) - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)
*d*e)*sinh(d*x + c))/((a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + (a^2*b^2 + b^4
)*d^2*sinh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d^2*cosh(d*x + c) - (a^2*b^2 + b
^4)*d^2 + 2*((a^2*b^2 + b^4)*d^2*cosh(d*x + c) + (a^3*b + a*b^3)*d^2)*sinh
(d*x + c))
```

3.324.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

output
```
Timed out
```

3.324.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(71) = 142$.

Time = 0.35 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.12

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = -f\left(\frac{2\,xe^{(dx+c)}}{b^2de^{(2\,dx+2\,c)}+2\,abde^{(dx+c)}-b^2d}-\frac{\log\left(\frac{be^{(dx+c)}+a-\sqrt{a^2+b^2}}{be^{(dx+c)}+a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}bd^2}\right) - \frac{2\,ee^{(-dx-c)}}{(2\,abe^{(-dx-c)}-b^2e^{(-2\,dx-2\,c)}+b^2)d}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d)
- log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2
+ b^2)))/(sqrt(a^2 + b^2)*b*d^2)) - 2*e*e^(-d*x - c)/((2*a*b*e^(-d*x - c)
- b^2*e^(-2*d*x - 2*c) + b^2)*d)
```

3.324.8 Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

3.324.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.69

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \frac{f\,\ln\left(\frac{2\,f\,\left(b-a\,e^{c+d\,x}\right)}{b^2\,d\,\sqrt{a^2+b^2}}-\frac{2\,f\,e^{c+d\,x}}{b^2\,d}\right)}{b\,d^2\,\sqrt{a^2+b^2}} - \frac{f\,\ln\left(-\frac{2\,f\,e^{c+d\,x}}{b^2\,d}-\frac{2\,f\,\left(b-a\,e^{c+d\,x}\right)}{b^2\,d\,\sqrt{a^2+b^2}}\right)}{b\,d^2\,\sqrt{a^2+b^2}} - \frac{2\,e^{c+d\,x}\,\left(a^2\,e+b^2\,e+a^2\,f\,x+b^2\,f\,x\right)}{d\,\left(a^2\,b+b^3\right)\,\left(2\,a\,e^{c+d\,x}-b+b\,e^{2\,c+2\,d\,x}\right)}$$

input
```
int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^2,x)
```

output
```
(f*log((2*f*(b - a*exp(c + d*x)))/(b^2*d*(a^2 + b^2)^(1/2)) - (2*f*exp(c +
 d*x))/(b^2*d)))/(b*d^2*(a^2 + b^2)^(1/2)) - (f*log(- (2*f*exp(c + d*x))/(
b^2*d) - (2*f*(b - a*exp(c + d*x)))/(b^2*d*(a^2 + b^2)^(1/2))))/(b*d^2*(a^
2 + b^2)^(1/2)) - (2*exp(c + d*x)*(a^2*e + b^2*e + a^2*f*x + b^2*f*x))/(d*
(a^2*b + b^3)*(2*a*exp(c + d*x) - b + b*exp(2*c + 2*d*x)))
```

3.325 $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

3.325.1 Optimal result

Integrand size = 26, antiderivative size = 234

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} + \frac{2f^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} - \frac{2f^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} - \frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

output
```
-(f*x+e)^2/b/d/(a+b*sinh(d*x+c))+2*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/b/d^2/(a^2+b^2)^(1/2)-2*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/b/d^2/(a^2+b^2)^(1/2)+2*f^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))/b/d^3/(a^2+b^2)^(1/2)-2*f^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/b/d^3/(a^2+b^2)^(1/2)
```

3.325.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.86

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = -\frac{2f\left(d\left(2e\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-fx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+fx\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)-f\,\text{PolyLog}\left(2,\frac{be^{c+}}{-a+\sqrt{a}}\right.}{b\sqrt{a^2+b^2}d^3} - \frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(-2*f*(d*(2*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f*x*Log[1 + (
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*x*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])]) - f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
 f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*Sqrt[a^2 + b^
2]*d^3) - (e + f*x)^2/(b*d*(a + b*Sinh[c + d*x]))
```

3.325.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5987, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}dx$$

$$\Big\downarrow \text{5987}$$

$$\frac{2f\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{bd} - \frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))} + \frac{2f\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{bd}$$

$$\Big\downarrow \text{3803}$$

$$\frac{4f\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \ 25$$

$$-\frac{4f\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \ 2694$$

$$-\frac{4f\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \ 27$$

$$-\frac{4f\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \ 2620$$

$$-\frac{4f\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \ 2715$$

$$-\frac{4f\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2}\right.}{2\sqrt{a^2+b^2}}\right.}{bd}$$

$$\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \ 2838$$

3.325. $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

$$-\frac{4f\left(\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(-4*f*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])
/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2))
)/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/
(b*d^2)))/(2*Sqrt[a^2 + b^2])))/(b*d) - (e + f*x)^2/(b*d*(a + b*Sinh[c + d
*x]))
```

3.325.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.325.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(214) = 428$.

Time = 3.22 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.10

method	result
risch	$-\frac{2(x^2f^2+2efx+e^2)\mathrm{e}^{dx+c}}{bd\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)}-\frac{4fe\ \operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d^2b\sqrt{a^2+b^2}}+\frac{2f^2\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{d^2b\sqrt{a^2+b^2}}-\frac{2f^2\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right.}{d^2b\sqrt{a^2+b^2}}$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(f^2*x^2+2*e*f*x+e^2)/b/d*exp(d*x+c)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b
)-4/d^2/b*f*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))+2/d^2/b*f^2/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a
+(a^2+b^2)^(1/2)))*x-2/d^2/b*f^2/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2
)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+2/d^3/b*f^2/(a^2+b^2)^(1/2)*ln((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/d^3/b*f^2/(a^2+b^2)^(1/
2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d^3/b*f^2/
(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))-2/d^3/b*f^2/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+
(a^2+b^2)^(1/2)))+4/d^3/b*f^2*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c
)+2*a)/(a^2+b^2)^(1/2))
```

3.325.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1378 vs. $2(212) = 424$.

Time = 0.27 (sec) , antiderivative size = 1378, normalized size of antiderivative = 5.89

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
2*((b^2*f^2*cosh(d*x + c)^2 + b^2*f^2*sinh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x
 + c) - b^2*f^2 + 2*(b^2*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^2*f^2*cosh(d*
x + c)^2 + b^2*f^2*sinh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x + c) - b^2*f^2 + 2
*(b^2*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*di
log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^2*d*e*f - b^2*c*f^2 - (b^2*d*e*f
 - b^2*c*f^2)*cosh(d*x + c)^2 - (b^2*d*e*f - b^2*c*f^2)*sinh(d*x + c)^2 -
2*(a*b*d*e*f - a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*e*f - a*b*c*f^2 + (b^2*
d*e*f - b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log
(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
- (b^2*d*e*f - b^2*c*f^2 - (b^2*d*e*f - b^2*c*f^2)*cosh(d*x + c)^2 - (b^2*
d*e*f - b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*e*f - a*b*c*f^2)*cosh(d*x +
c) - 2*(a*b*d*e*f - a*b*c*f^2 + (b^2*d*e*f - b^2*c*f^2)*cosh(d*x + c))*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c
) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d*f^2*x + b^2*c*f^2 - (b^2*d*f
^2*x + b^2*c*f^2)*cosh(d*x + c)^2 - (b^2*d*f^2*x + b^2*c*f^2)*sinh(d*x + c
)^2 - 2*(a*b*d*f^2*x + a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*f^2*x + a*b*c*f
^2 + (b^2*d*f^2*x + b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 ...
```

3.325.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

output
```
Timed out
```

3.325.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-2*(x^2*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d)
- 2*integrate(x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) -
 b^2*d), x))*f^2 - 2*e*f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d
*e^(d*x + c) - b^2*d) - log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*
x + c) + a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d^2)) - 2*e^2*e^(-d*x -
c)/((2*a*b*e^(-d*x - c) - b^2*e^(-2*d*x - 2*c) + b^2)*d)
```

3.325.8 Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)\,(e+f\,x)^2}{(a+b\sinh(c+d\,x))^2}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^2,x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^2, x)
```

3.326 $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

3.326.1 Optimal result

Integrand size = 26, antiderivative size = 348

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} + \frac{6f^2(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} - \frac{6f^2(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} - \frac{6f^3\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^4} + \frac{6f^3\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^4} - \frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

output
```
-(f*x+e)^3/b/d/(a+b*sinh(d*x+c))+3*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/b/d^2/(a^2+b^2)^(1/2)-3*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+
b^2)^(1/2)))/b/d^2/(a^2+b^2)^(1/2)+6*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))/b/d^3/(a^2+b^2)^(1/2)-6*f^2*(f*x+e)*polylog(2,-b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/b/d^3/(a^2+b^2)^(1/2)-6*f^3*polylog(3,-b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/b/d^4/(a^2+b^2)^(1/2)+6*f^3*polylog(3,-b*exp(d*x
+c)/(a+(a^2+b^2)^(1/2)))/b/d^4/(a^2+b^2)^(1/2)
```

3.326.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.06

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$$
$$= \frac{3f\left(-2d^2e^2\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^2f^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2d^2efx\log\left(1\right.\right.}{}$$
$$-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

output
```
(3*f*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*
x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])] - 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
] - 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyL
og[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*Sqrt[a^2 + b^2]*d^4)
- (e + f*x)^3/(b*d*(a + b*Sinh[c + d*x]))
```

3.326.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5987, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$$
$$\Big\downarrow \text{ 5987}$$
$$\frac{3f\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$
$$\Big\downarrow \text{ 3042}$$

$$-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}+\frac{3f\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{bd}$$

$$\Big\downarrow \text{3803}$$

$$\frac{6f\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{6f\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{6f\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{6f\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{6f\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{bd}$$

$$\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{3011}$$

3.326. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

$$-\frac{6f\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+\right.}{bd}\right.}{}\right.}{bd}$$

$$\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{6f\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\right.}{}\right.}{}\right.}{bd}$$

$$\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{6f\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\right.}{}\right.}{bd}$$

$$\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

3.326. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

output
```
(-6*f*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^
2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(b*d) - (e + f*x)^3/(b
*d*(a + b*Sinh[c + d*x]))
```

3.326.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.326. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.326.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh (dx+c)}{(a+b\sinh (dx+c))^2}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

3.326.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2420 vs. $2(316) = 632$.

Time = 0.29 (sec) , antiderivative size = 2420, normalized size of antiderivative = 6.95

$$\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(6*(b^2*d*f^3*x + b^2*d*e*f^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)
^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^3*x + a*b*d*
e*f^2)*cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*e*f^2 + (b^2*d*f^3*x + b^2*d
*e*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b + 1) - 6*(b^2*d*f^3*x + b^2*d*e*f^2 - (b^2*d*f^3*x + b
^2*d*e*f^2)*cosh(d*x + c)^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*sinh(d*x + c)^2
- 2*(a*b*d*f^3*x + a*b*d*e*f^2)*cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*e*f
^2 + (b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(b^2*d^2*e^2*f - 2*b
^2*c*d*e*f^2 + b^2*c^2*f^3 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^
3)*cosh(d*x + c)^2 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*sinh(
d*x + c)^2 - 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3)*cosh(d*x +
c) - 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (b^2*d^2*e^2*f - 2
*b^2*c*d*e*f^2 + b^2*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) + 3*(b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3 - (b^2*d^2*e
^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*cosh(d*x + c)^2 - (b^2*d^2*e^2*f - 2
*b^2*c*d*e*f^2 + b^2*c^2*f^3)*sinh(d*x + c)^2 - 2*(a*b*d^2*e^2*f - 2*a*...
```

3.326.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

output
```
Timed out
```

3.326.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)^3\cosh{(dx+c)}}{(b\sinh{(dx+c)}+a)^2}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-3*e^2*f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b
^2*d) - log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqr
t(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d^2)) - 2*e^3*e^(-d*x - c)/((2*a*b*e^(-d
*x - c) - b^2*e^(-2*d*x - 2*c) + b^2)*d) - 2*(f^3*x^3*e^c + 3*e*f^2*x^2*e^
c)*e^(d*x)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d) + integra
te(6*(f^3*x^2*e^c + 2*e*f^2*x*e^c)*e^(d*x)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*
d*e^(d*x + c) - b^2*d), x)
```

3.326.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{(fx+e)^3\cosh{(dx+c)}}{(b\sinh{(dx+c)}+a)^2}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^3}{\left(a+b\sinh\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^2,x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^2, x)
```

3.327 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

3.327.1 Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{af\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2} - \frac{e+fx}{2bd(a+b\sinh(c+dx))^2} - \frac{f\cosh(c+dx)}{2\,(a^2+b^2)\,d^2(a+b\sinh(c+dx))}$$

output
```
-a*f*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(3/2)/
d^2+1/2*(-f*x-e)/b/d/(a+b*sinh(d*x+c))^2-1/2*f*cosh(d*x+c)/(a^2+b^2)/d^2/(
a+b*sinh(d*x+c))
```

3.327.2 Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{\frac{f\cosh(c+dx)}{(a^2+b^2)(a+b\sinh(c+dx))} + \frac{\frac{2af\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}} + \frac{d(e+fx)}{(a+b\sinh(c+dx))^2}}{b}}{2d^2}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*((f*Cosh[c + d*x])/((a^2 + b^2)*(a + b*Sinh[c + d*x])) + ((2*a*f*ArcT
an[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) + (d*(e
 + f*x))/(a + b*Sinh[c + d*x])^2)/b)/d^2
```

3.327.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5987, 3042, 3143, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}dx$$

$$\Big\downarrow \text{5987}$$

$$\frac{f\int \frac{1}{(a+b\sinh(c+dx))^2}dx}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\int \frac{1}{(a-ib\sin(ic+idx))^2}dx}{2bd}$$

$$\Big\downarrow \text{3143}$$

$$\frac{f\left(-\frac{\int -\frac{a}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f\left(\frac{\int \frac{a}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\left(\frac{a\int \frac{1}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}+\frac{a\int\frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{2bd}$$

$$\Big\downarrow \; 3139$$

$$-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}-\frac{2ia\int\frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d(a^2+b^2)}\right)}{2bd}$$

$$\Big\downarrow \; 1083$$

$$-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}+\frac{4ia\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{d(a^2+b^2)}\right)}{2bd}$$

$$\Big\downarrow \; 217$$

$$\frac{f\left(\frac{2a\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

input
```
Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*(e + f*x)/(b*d*(a + b*Sinh[c + d*x])^2) + (f*((2*a*ArcTanh[Tanh[(c +
d*x)/2]/(2*Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (b*Cosh[c + d*x])/((
a^2 + b^2)*d*(a + b*Sinh[c + d*x]))))/(2*b*d)
```

3.327.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.327.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(106) = 212$.

Time = 15.05 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.75

method	result
risch	$-\frac{2\,\mathrm{e}^{2dx+2c} a^{2} d f x+2 b^{2} d f x\,\mathrm{e}^{2dx+2c}+2\,\mathrm{e}^{2dx+2c} a^{2} d e-a b f\,\mathrm{e}^{3dx+3c}+2 b^{2} d e\,\mathrm{e}^{2dx+2c}-2 a^{2} f\,\mathrm{e}^{2dx+2c}+b^{2} f\,\mathrm{e}^{2dx+2c}+3 a f\,\mathrm{e}^{dx+c} b-f\,b^{2}}{b\,d^{2}\left(a^{2}+b^{2}\right)\left(b\,\mathrm{e}^{2dx+2c}+2 a\,\mathrm{e}^{dx+c}-b\right)^{2}}+$

input
```
int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(2*exp(2*d*x+2*c)*a^2*d*f*x+2*b^2*d*f*x*exp(2*d*x+2*c)+2*exp(2*d*x+2*
c)*a^2*d*e-a*b*f*exp(3*d*x+3*c)+2*b^2*d*e*exp(2*d*x+2*c)-2*a^2*f*exp(2*d*x
+2*c)+b^2*f*exp(2*d*x+2*c)+3*a*f*exp(d*x+c)*b-f*b^2)/d^2/(a^2+b^2)/(b*exp(
2*d*x+2*c)+2*a*exp(d*x+c)-b)^2+1/2/(a^2+b^2)^(3/2)*f*a/d^2/b*ln(exp(d*x+c)
+(a*(a^2+b^2)^(3/2)-a^4-2*a^2*b^2-b^4)/b/(a^2+b^2)^(3/2))-1/2/(a^2+b^2)^(3
/2)*f*a/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(3/2)+a^4+2*a^2*b^2+b^4)/b/(a^2+b
^2)^(3/2))
```

3.327.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1230 vs. $2(105) = 210$.

Time = 0.27 (sec) , antiderivative size = 1230, normalized size of antiderivative = 10.98

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(2*(a^3*b + a*b^3)*f*cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*f*sinh(d*x +
c)^3 - 6*(a^3*b + a*b^3)*f*cosh(d*x + c) - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*
f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*e - (2*a^4 + a^2*b^2 - b^4)*f)*cosh(d*x
+ c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*
e - 3*(a^3*b + a*b^3)*f*cosh(d*x + c) - (2*a^4 + a^2*b^2 - b^4)*f)*sinh(d*
x + c)^2 + (a*b^2*f*cosh(d*x + c)^4 + a*b^2*f*sinh(d*x + c)^4 + 4*a^2*b*f*
cosh(d*x + c)^3 - 4*a^2*b*f*cosh(d*x + c) + a*b^2*f + 2*(2*a^3 - a*b^2)*f*
cosh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c) + a^2*b*f)*sinh(d*x + c)^3 + 2*
(3*a*b^2*f*cosh(d*x + c)^2 + 6*a^2*b*f*cosh(d*x + c) + (2*a^3 - a*b^2)*f)*
sinh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c)^3 + 3*a^2*b*f*cosh(d*x + c)^2 -
 a^2*b*f + (2*a^3 - a*b^2)*f*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)
*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*
a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*
(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x +
 c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) +
2*(a^2*b^2 + b^4)*f + 2*(3*(a^3*b + a*b^3)*f*cosh(d*x + c)^2 - 3*(a^3*b +
a*b^3)*f - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*
d*e - (2*a^4 + a^2*b^2 - b^4)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b^3 +
 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c)^4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*si
nh(d*x + c)^4 + 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cosh(d*x + c)^3 + 2...
```

3.327.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

3.327.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(105) = 210$.

Time = 0.43 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.69

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$$
$$= \frac{1}{2}f\left(\frac{2\left(abe^{(3\,dx+3\,c)} - 3\,abe^{(dx+c)} + b^2 + \left(2\,a^2e^{(2\,c)} - b^2e^{(2\,c)} - 2\,(a^2de^{(\right.\right.}{a^2b^3d^2 + b^5d^2 + (a^2b^3d^2e^{(4\,c)} + b^5d^2e^{(4\,c)})e^{(4\,dx)} + 4\,(a^3b^2d^2e^{(3\,c)} + ab^4d^2e^{(3\,c)})e^{(3\,dx)} + 2\,(2\,a^4bd^2e^{(2\,c)} +}\right.$$
$$- \frac{2\,ee^{(-2\,dx-2\,c)}}{(4\,ab^2e^{(-dx-c)} - 4\,ab^2e^{(-3\,dx-3\,c)} + b^3e^{(-4\,dx-4\,c)} + b^3 + 2\,(2\,a^2b - b^3)e^{(-2\,dx-2\,c)})d}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(d*x + c) + b^2 + (2*a^2*e^(2*c) -
 b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e^(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^
2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b
^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d*x) + 2*(2*a^4*b*d^2*e^(2*c) + a
^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2*d*x) - 4*(a^3*b^2*d^2*e^c + a*b
^4*d^2*e^c)*e^(d*x)) + a*log((b*e^(d*x + 2*c) + a*e^c - sqrt(a^2 + b^2)*e^
c)/(b*e^(d*x + 2*c) + a*e^c + sqrt(a^2 + b^2)*e^c))/((a^2*b + b^3)*sqrt(a^
2 + b^2)*d^2)) - 2*e*e^(-2*d*x - 2*c)/((4*a*b^2*e^(-d*x - c) - 4*a*b^2*e^(
-3*d*x - 3*c) + b^3*e^(-4*d*x - 4*c) + b^3 + 2*(2*a^2*b - b^3)*e^(-2*d*x -
 2*c))*d)
```

3.327.8 Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{(fx+e)\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx=\int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)}{\left(a+b\sinh\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^3,x)
```

output
```
int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^3, x)
```

3.328 $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

3.328.1 Optimal result

Integrand size = 26, antiderivative size = 306

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^2} - \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^2} + \frac{f^2\log(a+b\sinh(c+dx))}{b\left(a^2+b^2\right)d^3} + \frac{af^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^3} - \frac{af^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^3} - \frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} - \frac{f(e+fx)\cosh(c+dx)}{\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}$$

output
```
f^2*ln(a+b*sinh(d*x+c))/b/(a^2+b^2)/d^3+a*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2-a*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a
^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+a*f^2*polylog(2,-b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-a*f^2*polylog(2,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-1/2*(f*x+e)^2/b/d/(a+b*sinh(d*x+c))^2-
f*(f*x+e)*cosh(d*x+c)/(a^2+b^2)/d^2/(a+b*sinh(d*x+c))
```

3.328.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 623 vs. $2(306) = 612$.

Time = 10.28 (sec) , antiderivative size = 623, normalized size of antiderivative = 2.04

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \frac{f^2x\coth(c)}{b(a^2+b^2)d^2}$$
$$+ \frac{2e^cf\left(-e^cfx+e^{-c}(-1+e^{2c})fx-\frac{aee^{-c}(-1+e^{2c})\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{ae^{-c}(-1+e^{2c})f\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}+\frac{1}{2}\right.}{}$$
$$-\frac{f^2x\cosh(c)\operatorname{csch}\left(\frac{c}{2}\right)\operatorname{sech}\left(\frac{c}{2}\right)}{2b(a^2+b^2)d^2}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$
$$+\frac{\operatorname{csch}\left(\frac{c}{2}\right)\operatorname{sech}\left(\frac{c}{2}\right)(aef\cosh(c)+af^2x\cosh(c)+bef\sinh(dx)+bf^2x\sinh(dx))}{2b(a^2+b^2)d^2(a+b\sinh(c+dx))}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
(f^2*x*Coth[c])/(b*(a^2 + b^2)*d^2) + (2*E^c*f*(-(E^c*f*x) + ((-1 + E^(2*c
))*f*x)/E^c - (a*e*(-1 + E^(2*c))*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b
^2]])/(Sqrt[a^2 + b^2]*E^c) + (a*(-1 + E^(2*c))*f*ArcTanh[(a + b*E^(c + d*
x))/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d*E^c) + ((-1 + E^(2*c))*f*(-2*x +
(2*a*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-a^2 - b^2]*d) +
Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d))/(2*E^c) + (a*(-1 + E^(
2*c))*f*(d*x*(Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]
)] - Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]) + Pol
yLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - PolyLo
g[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(2*d*Sqrt
[(a^2 + b^2)*E^(2*c)])))/(b*(a^2 + b^2)*d^2*(-1 + E^(2*c))) - (f^2*x*Cosh[
c]*Csch[c/2]*Sech[c/2])/(2*b*(a^2 + b^2)*d^2) - (e + f*x)^2/(2*b*d*(a + b*
Sinh[c + d*x])^2) + (Csch[c/2]*Sech[c/2]*(a*e*f*Cosh[c] + a*f^2*x*Cosh[c]
+ b*e*f*Sinh[d*x] + b*f^2*x*Sinh[d*x]))/(2*b*(a^2 + b^2)*d^2*(a + b*Sinh[c
 + d*x]))
```

3.328.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5987, 3042, 3805, 3042, 3147, 16, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}dx$$

$$\Big\downarrow \text{5987}$$

$$\frac{f\int \frac{e+fx}{(a+b\sinh(c+dx))^2}dx}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\int \frac{e+fx}{(a-ib\sin(ic+idx))^2}dx}{bd}$$

$$\Big\downarrow \text{3805}$$

$$\frac{f\left(\frac{a\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{bf\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{bf\int \frac{\cos(ic+idx)}{a-ib\sin(ic+idx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}$$

$$\Big\downarrow \text{3147}$$

$$-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(\frac{f\int \frac{1}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{d^2(a^2+b^2)}+\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}$$

$$\Big\downarrow \text{16}$$

$$-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}$$

$$\Big\downarrow \text{3803}$$

$$\frac{f\left(\frac{2a\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \ 25$$

$$\frac{f\left(-\frac{2a\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \ 2694$$

$$\frac{f\left(-\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{f\left(-\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \ 2620$$

3.328. $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

$$\frac{f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a+b}{d^2(}\right.}{bd}$$

$$\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 2715$$

$$\frac{f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2}\right.}{2\sqrt{a^2+b^2}}}{a^2+b^2}\right.}{bd}$$

$$\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 2838$$

$$\frac{f\left(-\frac{2a\left(\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a}{a}\right.}{bd}$$

$$\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

3.328. $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

output
```
-1/2*(e + f*x)^2/(b*d*(a + b*Sinh[c + d*x])^2) + (f*((f*Log[a + b*Sinh[c +
 d*x]])/((a^2 + b^2)*d^2) - (2*a*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) - (
b*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x]))))/(b*d)
```

3.328.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.328. $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.328. $\int \frac{(e+fx)^2 \cosh(c+dx)}{(a+b\sinh(c+dx))^3} \, dx$

3.328.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(284) = 568$.

Time = 16.25 (sec) , antiderivative size = 805, normalized size of antiderivative = 2.63

method	result
risch	$-\frac{2\left(a^{2} d\,f^{2} x^{2} {\mathrm e}^{2 d x +2 c}+b^{2} d\,f^{2} x^{2} {\mathrm e}^{2 d x +2 c}+2 a^{2} d e f x\,{\mathrm e}^{2 d x +2 c}-a b\,f^{2} x\,{\mathrm e}^{3 d x +3 c}+2 b^{2} d e f x\,{\mathrm e}^{2 d x +2 c}+a^{2} d\,e^{2} {\mathrm e}^{2 d x +2 c}-2 a^{2} f^{2} x\,{\mathrm e}^{2 d x +2 c}-ab}{b\,d^{2} \left(b\,{\mathrm e}^{2 d x +2 c}+2 a\,{\mathrm e}^{d x +c}}$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/b*(a^2*d*f^2*x^2*exp(2*d*x+2*c)+b^2*d*f^2*x^2*exp(2*d*x+2*c)+2*a^2*d*e*
f*x*exp(2*d*x+2*c)-a*b*f^2*x*exp(3*d*x+3*c)+2*b^2*d*e*f*x*exp(2*d*x+2*c)+a
^2*d*e^2*exp(2*d*x+2*c)-2*a^2*f^2*x*exp(2*d*x+2*c)-a*b*e*f*exp(3*d*x+3*c)+
b^2*d*e^2*exp(2*d*x+2*c)+b^2*f^2*x*exp(2*d*x+2*c)-2*a^2*e*f*exp(2*d*x+2*c)
+3*a*b*f^2*x*exp(d*x+c)+b^2*e*f*exp(2*d*x+2*c)+3*a*b*e*f*exp(d*x+c)-b^2*f^
2*x-b^2*e*f)/d^2/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)^2/(a^2+b^2)-2/(a^2+b^
2)/d^3/b*f^2*ln(exp(d*x+c))+1/(a^2+b^2)/d^3/b*f^2*ln(b*exp(2*d*x+2*c)+2*a*
exp(d*x+c)-b)-2/(a^2+b^2)^(3/2)/d^2/b*f*a*e*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(3/2)/d^2/b*f^2*a*ln((-b*exp(d*x+c)+(a^2+b
^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(3/2)/d^2/b*f^2*a*ln((b*e
xp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(3/2)/d^3/
b*f^2*a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/(a^
2+b^2)^(3/2)/d^3/b*f^2*a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^
(1/2)))*c+1/(a^2+b^2)^(3/2)/d^3/b*f^2*a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))-1/(a^2+b^2)^(3/2)/d^3/b*f^2*a*dilog((b*exp(d*x
+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/(a^2+b^2)^(3/2)/d^3/b*f^2*a*
c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

3.328.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5233 vs. $2(282) = 564$.

Time = 0.33 (sec) , antiderivative size = 5233, normalized size of antiderivative = 17.10

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.328.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

3.328.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
(2*a*d*integrate(x*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2
*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b^2*
d^2 - b^4*d^2), x) + b*(a*log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(
d*x + c) + a + sqrt(a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)*d^3) - 2
*(d*x + c)/((a^2*b^2 + b^4)*d^3) + log(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c)
 - b)/((a^2*b^2 + b^4)*d^3)) + 2*(a*b*x*e^(3*d*x + 3*c) - 3*a*b*x*e^(d*x +
 c) + b^2*x - ((a^2*d*e^(2*c) + b^2*d*e^(2*c))*x^2 - (2*a^2*e^(2*c) - b^2*
e^(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5
*d^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3
*d*x) + 2*(2*a^4*b*d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^
(2*d*x) - 4*(a^3*b^2*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) - a*log((b*e^(d*x +
 c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2 + b^2)))/((a^2*b
+ b^3)*sqrt(a^2 + b^2)*d^3))*f^2 + e*f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(
d*x + c) + b^2 + (2*a^2*e^(2*c) - b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e
^(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*
d^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*
d*x) + 2*(2*a^4*b*d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(
2*d*x) - 4*(a^3*b^2*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) + a*log((b*e^(d*x +
2*c) + a*e^c - sqrt(a^2 + b^2)*e^c)/(b*e^(d*x + 2*c) + a*e^c + sqrt(a^2 +
b^2)*e^c))/((a^2*b + b^3)*sqrt(a^2 + b^2)*d^2)) - 2*e^2*e^(-2*d*x - 2*c...
```

3.328.8 Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^2}{\left(a+b\sinh\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^3,x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^3, x)
```

3.329 $\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

3.329.1 Optimal result

Integrand size = 26, antiderivative size = 631

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} + \frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b(a^2+b^2)^{3/2}d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} - \frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2b(a^2+b^2)^{3/2}d^2} + \frac{3f^3\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^4} + \frac{3af^2(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^3} + \frac{3f^3\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^4} - \frac{3af^2(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^3} - \frac{3af^3\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^4} + \frac{3af^3\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^4} - \frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2} - \frac{3f(e+fx)^2\cosh(c+dx)}{2(a^2+b^2)d^2(a+b\sinh(c+dx))}$$

output
```
-3/2*f*(f*x+e)^2/b/(a^2+b^2)/d^2+3*f^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/b/(a^2+b^2)/d^3+3/2*a*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+3*f^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/b/(a^2+b^2)/d^3-3/2*a*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+3*f^3*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/b/(a^2+b^2)/d^4+3*a*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3+3*f^3*polylog(2,-b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/b/(a^2+b^2)/d^4-3*a*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-3*a*f^3*polylog(3,-b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^4+3*a*f^3*polylog(3,-b*exp(d*x+c)/(a+(
a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^4-1/2*(f*x+e)^3/b/d/(a+b*sinh(d*x+c))
^2-3/2*f*(f*x+e)^2*cosh(d*x+c)/(a^2+b^2)/d^2/(a+b*sinh(d*x+c))
```

3.329.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5753 vs. $2(631) = 1262$.

Time = 13.70 (sec) , antiderivative size = 5753, normalized size of antiderivative = 9.12

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
Result too large to show
```

3.329.3 Rubi [A] (verified)

Time = 2.88 (sec) , antiderivative size = 561, normalized size of antiderivative = 0.89, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5987, 3042, 3805, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 6095, 2620, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}dx$$

$$\Big\downarrow \text{5987}$$

$$\frac{3f\int \frac{(e+fx)^2}{(a+b\sinh(c+dx))^2}dx}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}+\frac{3f\int \frac{(e+fx)^2}{(a-ib\sin(ic+idx))^2}dx}{2bd}$$

$$\Big\downarrow \text{ 3805}$$

$$\frac{3f\left(\frac{a\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}+\frac{3f\left(\frac{a\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}$$

$$\Big\downarrow \text{ 3803}$$

$$\frac{3f\left(\frac{2a\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{3f\left(-\frac{2a\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{ 2694}$$

3.329. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

$$\frac{3f\left(-\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{3f\left(-\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 2620$$

$$\frac{3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{2bd}$$

$$-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 3011$$

3.329. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\cdots\right.}{bd}\cdots\right.}{\cdots}}{a^2+b^2}\cdots\right)}{2b\cdots}$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 2720$$

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\cdots}{\cdots}\right.}{\cdots}}{a^2+b^2}\cdots\right)}{\cdots}$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 6095$$

3.329. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log}{}\right.}{}\right)}{a^2+b^2}\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 2620$$

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log}{}\right.}{}\right)}{a^2+b^2}\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 2715$$

3.329. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log}{\cdots}\right.}{\cdots}\right.}{a^2+b^2}\cdots\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\downarrow$ 2838

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log}{\cdots}\right.}{\cdots}\right.}{a^2+b^2}\cdots\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$\downarrow$ 7143

3.329. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

$$3f\left(\frac{2bf\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{d(a^2+b^2)} - \frac{2a(b \cdots}{\cdots}\right)$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*(e + f*x)^3/(b*d*(a + b*Sinh[c + d*x])^2) + (3*f*((2*b*f*(-1/2*(e + f
*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) +
(f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/((a^2 + b^2)*d
) - (2*a*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])
/d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) - (b*(e
 + f*x)^2*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x]))))/(2*b*d)
```

3.329. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

3.329.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.329. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.329. $\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^3} \, dx$

3.329.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh (dx+c)}{(a+b\sinh (dx+c))^3}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

3.329.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11757 vs. $2(575) = 1150$.

Time = 0.43 (sec) , antiderivative size = 11757, normalized size of antiderivative = 18.63

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.329.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

3.329.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
3*a*d*f^3*integrate(x^2*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2
*e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2
*b^2*d^2 - b^4*d^2), x) + 6*a*d*e*f^2*integrate(x*e^(d*x + c)/(a^2*b^2*d^2
*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a
*b^3*d^2*e^(d*x + c) - a^2*b^2*d^2 - b^4*d^2), x) + 3*b*e*f^2*(a*log((b*e^
(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2 + b^2)))/((
a^2*b^2 + b^4)*sqrt(a^2 + b^2)*d^3) - 2*(d*x + c)/((a^2*b^2 + b^4)*d^3) +
log(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b)/((a^2*b^2 + b^4)*d^3)) - 6*a*
f^3*integrate(x*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*
x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b^2*d^2
 - b^4*d^2), x) + 6*b*f^3*integrate(x/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d
^2*e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a
^2*b^2*d^2 - b^4*d^2), x) + 3/2*e^2*f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(d
*x + c) + b^2 + (2*a^2*e^(2*c) - b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e^
(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d
^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d
*x) + 2*(2*a^4*b*d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2
*d*x) - 4*(a^3*b^2*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) + a*log((b*e^(d*x + 2
*c) + a*e^c - sqrt(a^2 + b^2)*e^c)/(b*e^(d*x + 2*c) + a*e^c + sqrt(a^2 + b
^2)*e^c))/((a^2*b + b^3)*sqrt(a^2 + b^2)*d^2)) - 2*e^3*e^(-2*d*x - 2*c)...
```

3.329.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^3}{\left(a+b\sinh\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^3,x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^3, x)
```

3.330 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

3.330.1 Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{af\,\mathrm{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2} - \frac{e+fx}{2bd(a+b\sinh(c+dx))^2} - \frac{f\cosh(c+dx)}{2\,(a^2+b^2)\,d^2(a+b\sinh(c+dx))}$$

output
```
-a*f*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(3/2)/
d^2+1/2*(-f*x-e)/b/d/(a+b*sinh(d*x+c))^2-1/2*f*cosh(d*x+c)/(a^2+b^2)/d^2/(
a+b*sinh(d*x+c))
```

3.330.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{\frac{f\cosh(c+dx)}{(a^2+b^2)(a+b\sinh(c+dx))} + \frac{\frac{2af\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}} + \frac{d(e+fx)}{(a+b\sinh(c+dx))^2}}{b}}{2d^2}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*((f*Cosh[c + d*x])/((a^2 + b^2)*(a + b*Sinh[c + d*x])) + ((2*a*f*ArcT
an[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) + (d*(e
 + f*x))/(a + b*Sinh[c + d*x])^2)/b)/d^2
```

3.330.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5987, 3042, 3143, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$$

$$\Big\downarrow \text{5987}$$

$$\frac{f\int \frac{1}{(a+b\sinh(c+dx))^2}dx}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\int \frac{1}{(a-ib\sin(ic+idx))^2}dx}{2bd}$$

$$\Big\downarrow \text{3143}$$

$$\frac{f\left(-\frac{\int -\frac{a}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f\left(\frac{\int \frac{a}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\left(\frac{a\int \frac{1}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}+\frac{a\int\frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{2bd}$$

$$\Big\downarrow \text{ 3139}$$

$$-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}-\frac{2ia\int\frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d(a^2+b^2)}\right)}{2bd}$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}+\frac{4ia\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{d(a^2+b^2)}\right)}{2bd}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{f\left(\frac{2a\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

input
```
Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*(e + f*x)/(b*d*(a + b*Sinh[c + d*x])^2) + (f*((2*a*ArcTanh[Tanh[(c +
d*x)/2]/(2*Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (b*Cosh[c + d*x])/((
a^2 + b^2)*d*(a + b*Sinh[c + d*x]))))/(2*b*d)
```

3.330.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.330.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(106) = 212$.

Time = 14.55 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.75

method	result
risch	$-\frac{2\,{\mathrm e}^{2 d x +2 c} a^{2} d f x +2 b^{2} d f x \,{\mathrm e}^{2 d x +2 c}+2\,{\mathrm e}^{2 d x +2 c} a^{2} d e -a b f \,{\mathrm e}^{3 d x +3 c}+2 b^{2} d e \,{\mathrm e}^{2 d x +2 c}-2 a^{2} f \,{\mathrm e}^{2 d x +2 c}+b^{2} f \,{\mathrm e}^{2 d x +2 c}+3 a f \,{\mathrm e}^{d x +c} b -f \,b^{2}}{b \,d^{2} \left(a^{2}+b^{2}\right) \left(b \,{\mathrm e}^{2 d x +2 c}+2 a \,{\mathrm e}^{d x +c}-b \right)^{2}}+$

input
```
int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(2*exp(2*d*x+2*c)*a^2*d*f*x+2*b^2*d*f*x*exp(2*d*x+2*c)+2*exp(2*d*x+2*
c)*a^2*d*e-a*b*f*exp(3*d*x+3*c)+2*b^2*d*e*exp(2*d*x+2*c)-2*a^2*f*exp(2*d*x
+2*c)+b^2*f*exp(2*d*x+2*c)+3*a*f*exp(d*x+c)*b-f*b^2)/d^2/(a^2+b^2)/(b*exp(
2*d*x+2*c)+2*a*exp(d*x+c)-b)^2+1/2/(a^2+b^2)^(3/2)*f*a/d^2/b*ln(exp(d*x+c)
+(a*(a^2+b^2)^(3/2)-a^4-2*a^2*b^2-b^4)/b/(a^2+b^2)^(3/2))-1/2/(a^2+b^2)^(3
/2)*f*a/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(3/2)+a^4+2*a^2*b^2+b^4)/b/(a^2+b
^2)^(3/2))
```

3.330.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1230 vs. $2(105) = 210$.

Time = 0.26 (sec) , antiderivative size = 1230, normalized size of antiderivative = 10.98

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(2*(a^3*b + a*b^3)*f*cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*f*sinh(d*x +
c)^3 - 6*(a^3*b + a*b^3)*f*cosh(d*x + c) - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*
f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*e - (2*a^4 + a^2*b^2 - b^4)*f)*cosh(d*x
+ c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*
e - 3*(a^3*b + a*b^3)*f*cosh(d*x + c) - (2*a^4 + a^2*b^2 - b^4)*f)*sinh(d*
x + c)^2 + (a*b^2*f*cosh(d*x + c)^4 + a*b^2*f*sinh(d*x + c)^4 + 4*a^2*b*f*
cosh(d*x + c)^3 - 4*a^2*b*f*cosh(d*x + c) + a*b^2*f + 2*(2*a^3 - a*b^2)*f*
cosh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c) + a^2*b*f)*sinh(d*x + c)^3 + 2*
(3*a*b^2*f*cosh(d*x + c)^2 + 6*a^2*b*f*cosh(d*x + c) + (2*a^3 - a*b^2)*f)*
sinh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c)^3 + 3*a^2*b*f*cosh(d*x + c)^2 -
 a^2*b*f + (2*a^3 - a*b^2)*f*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)
*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*
a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*
(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x +
 c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) +
2*(a^2*b^2 + b^4)*f + 2*(3*(a^3*b + a*b^3)*f*cosh(d*x + c)^2 - 3*(a^3*b +
a*b^3)*f - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*
d*e - (2*a^4 + a^2*b^2 - b^4)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b^3 +
 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c)^4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*si
nh(d*x + c)^4 + 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cosh(d*x + c)^3 + 2...
```

3.330.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

3.330.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(105) = 210$.

Time = 0.43 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.69

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$$
$$= \frac{1}{2}f\left(\frac{2\left(abe^{(3\,dx+3\,c)} - 3\,abe^{(dx+c)} + b^2 + \left(2\,a^2e^{(2\,c)} - b^2e^{(2\,c)} - 2\,(a^2de^{(}\right.\right.}{a^2b^3d^2 + b^5d^2 + (a^2b^3d^2e^{(4\,c)} + b^5d^2e^{(4\,c)})e^{(4\,dx)} + 4\,(a^3b^2d^2e^{(3\,c)} + ab^4d^2e^{(3\,c)})e^{(3\,dx)} + 2\,(2\,a^4bd^2e^{(2\,c)} +}\right.$$
$$- \frac{2\,ee^{(-2\,dx-2\,c)}}{(4\,ab^2e^{(-dx-c)} - 4\,ab^2e^{(-3\,dx-3\,c)} + b^3e^{(-4\,dx-4\,c)} + b^3 + 2\,(2\,a^2b - b^3)e^{(-2\,dx-2\,c)})d}$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(d*x + c) + b^2 + (2*a^2*e^(2*c) -
 b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e^(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^
2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b
^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d*x) + 2*(2*a^4*b*d^2*e^(2*c) + a
^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2*d*x) - 4*(a^3*b^2*d^2*e^c + a*b
^4*d^2*e^c)*e^(d*x)) + a*log((b*e^(d*x + 2*c) + a*e^c - sqrt(a^2 + b^2)*e^
c)/(b*e^(d*x + 2*c) + a*e^c + sqrt(a^2 + b^2)*e^c))/((a^2*b + b^3)*sqrt(a^
2 + b^2)*d^2)) - 2*e*e^(-2*d*x - 2*c)/((4*a*b^2*e^(-d*x - c) - 4*a*b^2*e^(
-3*d*x - 3*c) + b^3*e^(-4*d*x - 4*c) + b^3 + 2*(2*a^2*b - b^3)*e^(-2*d*x -
 2*c))*d)
```

3.330.8 Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{(fx+e)\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{\cosh(c+d\,x)\,(e+f\,x)}{(a+b\sinh(c+d\,x))^3}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^3,x)
```

output
```
int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^3, x)
```

3.331 $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

3.331.1 Optimal result

Integrand size = 26, antiderivative size = 306

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2} - \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2} + \frac{f^2\log(a+b\sinh(c+dx))}{b\,(a^2+b^2)\,d^3} + \frac{af^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^3} - \frac{af^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^3} - \frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} - \frac{f(e+fx)\cosh(c+dx)}{(a^2+b^2)\,d^2(a+b\sinh(c+dx))}$$

output
```
f^2*ln(a+b*sinh(d*x+c))/b/(a^2+b^2)/d^3+a*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2-a*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a
^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+a*f^2*polylog(2,-b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-a*f^2*polylog(2,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-1/2*(f*x+e)^2/b/d/(a+b*sinh(d*x+c))^2-
f*(f*x+e)*cosh(d*x+c)/(a^2+b^2)/d^2/(a+b*sinh(d*x+c))
```

3.331.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 623 vs. $2(306) = 612$.

Time = 6.68 (sec) , antiderivative size = 623, normalized size of antiderivative = 2.04

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \frac{f^2x\coth(c)}{b(a^2+b^2)d^2}$$

$$+ \frac{2e^cf\left(-e^cfx+e^{-c}(-1+e^{2c})fx-\frac{aee^{-c}(-1+e^{2c})\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{ae^{-c}(-1+e^{2c})f\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}+\cdots\right.}{\cdots}$$

$$-\frac{f^2x\cosh(c)\operatorname{csch}\left(\frac{c}{2}\right)\operatorname{sech}\left(\frac{c}{2}\right)}{2b(a^2+b^2)d^2}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$+\frac{\operatorname{csch}\left(\frac{c}{2}\right)\operatorname{sech}\left(\frac{c}{2}\right)(aef\cosh(c)+af^2x\cosh(c)+bef\sinh(dx)+bf^2x\sinh(dx))}{2b(a^2+b^2)d^2(a+b\sinh(c+dx))}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
(f^2*x*Coth[c])/(b*(a^2 + b^2)*d^2) + (2*E^c*f*(-(E^c*f*x) + ((-1 + E^(2*c
))*f*x)/E^c - (a*e*(-1 + E^(2*c))*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b
^2]])/(Sqrt[a^2 + b^2]*E^c) + (a*(-1 + E^(2*c))*f*ArcTanh[(a + b*E^(c + d*
x))/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d*E^c) + ((-1 + E^(2*c))*f*(-2*x +
(2*a*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-a^2 - b^2]*d) +
Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d))/(2*E^c) + (a*(-1 + E^(
2*c))*f*(d*x*(Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]
)] - Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]) + Pol
yLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - PolyLo
g[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(2*d*Sqrt
[(a^2 + b^2)*E^(2*c)])))/(b*(a^2 + b^2)*d^2*(-1 + E^(2*c))) - (f^2*x*Cosh[
c]*Csch[c/2]*Sech[c/2])/(2*b*(a^2 + b^2)*d^2) - (e + f*x)^2/(2*b*d*(a + b*
Sinh[c + d*x])^2) + (Csch[c/2]*Sech[c/2]*(a*e*f*Cosh[c] + a*f^2*x*Cosh[c]
+ b*e*f*Sinh[d*x] + b*f^2*x*Sinh[d*x]))/(2*b*(a^2 + b^2)*d^2*(a + b*Sinh[c
 + d*x]))
```

3.331.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5987, 3042, 3805, 3042, 3147, 16, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}dx$$

$$\Big\downarrow \ 5987$$

$$\frac{f\int \frac{e+fx}{(a+b\sinh(c+dx))^2}dx}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\int \frac{e+fx}{(a-ib\sin(ic+idx))^2}dx}{bd}$$

$$\Big\downarrow \ 3805$$

$$\frac{f\left(\frac{a\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{bf\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{bf\int \frac{\cos(ic+idx)}{a-ib\sin(ic+idx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}$$

$$\Big\downarrow \ 3147$$

$$-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(\frac{f\int \frac{1}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{d^2(a^2+b^2)}+\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}$$

$$\Big\downarrow \ 16$$

$$-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\left(\frac{a\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}$$

$$\Big\downarrow \ 3803$$

$$\frac{f\left(\frac{2a\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{f\left(-\frac{2a\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 2694$$

$$\frac{f\left(-\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{f\left(-\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a+b\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{b(e+fx)\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{bd}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 2620$$

3.331. $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

$$\frac{f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a+b\cdots}{d^2(\cdots}\cdots\right.}{bd}\cdots$$

$$\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 2715$$

$$\frac{f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)\cdots}{bd^2}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{a^2+b^2}\cdots\right.}{bd}\cdots$$

$$\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 2838$$

$$\frac{f\left(-\frac{2a\left(\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{f\log(a\cdots}{d\cdots}\cdots\right.}{bd}\cdots$$

$$\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

3.331. $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

output
```
-1/2*(e + f*x)^2/(b*d*(a + b*Sinh[c + d*x])^2) + (f*((f*Log[a + b*Sinh[c +
 d*x]])/((a^2 + b^2)*d^2) - (2*a*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) - (
b*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x]))))/(b*d)
```

3.331.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.331. $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.331. $\int \frac{(e+fx)^2 \cosh(c+dx)}{(a+b\sinh(c+dx))^3} \, dx$

3.331.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(284) = 568$.

Time = 15.88 (sec) , antiderivative size = 805, normalized size of antiderivative = 2.63

method	result
risch	$-\frac{2\left(a^2 d\, f^2 x^2 \mathrm{e}^{2dx+2c}+b^2 d\, f^2 x^2 \mathrm{e}^{2dx+2c}+2a^2 defx\, \mathrm{e}^{2dx+2c}-ab\, f^2 x\, \mathrm{e}^{3dx+3c}+2b^2 defx\, \mathrm{e}^{2dx+2c}+a^2 d\, e^2 \mathrm{e}^{2dx+2c}-2a^2 f^2 x\, \mathrm{e}^{2dx+2c}-ab\right.}{b\, d^2\left(b\, \mathrm{e}^{2dx+2c}+2a\, \mathrm{e}^{dx+c}\right.}$

input
```
int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/b*(a^2*d*f^2*x^2*exp(2*d*x+2*c)+b^2*d*f^2*x^2*exp(2*d*x+2*c)+2*a^2*d*e*
f*x*exp(2*d*x+2*c)-a*b*f^2*x*exp(3*d*x+3*c)+2*b^2*d*e*f*x*exp(2*d*x+2*c)+a
^2*d*e^2*exp(2*d*x+2*c)-2*a^2*f^2*x*exp(2*d*x+2*c)-a*b*e*f*exp(3*d*x+3*c)+
b^2*d*e^2*exp(2*d*x+2*c)+b^2*f^2*x*exp(2*d*x+2*c)-2*a^2*e*f*exp(2*d*x+2*c)
+3*a*b*f^2*x*exp(d*x+c)+b^2*e*f*exp(2*d*x+2*c)+3*a*b*e*f*exp(d*x+c)-b^2*f^
2*x-b^2*e*f)/d^2/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)^2/(a^2+b^2)-2/(a^2+b^
2)/d^3/b*f^2*ln(exp(d*x+c))+1/(a^2+b^2)/d^3/b*f^2*ln(b*exp(2*d*x+2*c)+2*a*
exp(d*x+c)-b)-2/(a^2+b^2)^(3/2)/d^2/b*f*a*e*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(3/2)/d^2/b*f^2*a*ln((-b*exp(d*x+c)+(a^2+b
^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(3/2)/d^2/b*f^2*a*ln((b*e
xp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(3/2)/d^3/
b*f^2*a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/(a^
2+b^2)^(3/2)/d^3/b*f^2*a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^
(1/2)))*c+1/(a^2+b^2)^(3/2)/d^3/b*f^2*a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))-1/(a^2+b^2)^(3/2)/d^3/b*f^2*a*dilog((b*exp(d*x
+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/(a^2+b^2)^(3/2)/d^3/b*f^2*a*
c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

3.331.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5233 vs. $2(282) = 564$.

Time = 0.34 (sec) , antiderivative size = 5233, normalized size of antiderivative = 17.10

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.331.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

3.331.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
(2*a*d*integrate(x*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2
*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b^2*
d^2 - b^4*d^2), x) + b*(a*log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(
d*x + c) + a + sqrt(a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)*d^3) - 2
*(d*x + c)/((a^2*b^2 + b^4)*d^3) + log(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c)
 - b)/((a^2*b^2 + b^4)*d^3)) + 2*(a*b*x*e^(3*d*x + 3*c) - 3*a*b*x*e^(d*x +
 c) + b^2*x - ((a^2*d*e^(2*c) + b^2*d*e^(2*c))*x^2 - (2*a^2*e^(2*c) - b^2*
e^(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5
*d^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3
*d*x) + 2*(2*a^4*b*d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^
(2*d*x) - 4*(a^3*b^2*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) - a*log((b*e^(d*x +
 c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2 + b^2)))/((a^2*b
+ b^3)*sqrt(a^2 + b^2)*d^3))*f^2 + e*f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(
d*x + c) + b^2 + (2*a^2*e^(2*c) - b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e
^(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*
d^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*
d*x) + 2*(2*a^4*b*d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(
2*d*x) - 4*(a^3*b^2*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) + a*log((b*e^(d*x +
2*c) + a*e^c - sqrt(a^2 + b^2)*e^c)/(b*e^(d*x + 2*c) + a*e^c + sqrt(a^2 +
b^2)*e^c))/((a^2*b + b^3)*sqrt(a^2 + b^2)*d^2)) - 2*e^2*e^(-2*d*x - 2*c...
```

3.331.8 Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx=\int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^2}{\left(a+b\sinh\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^3,x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^3, x)
```

3.332 $\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

3.332.1 Optimal result

Integrand size = 26, antiderivative size = 631

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{3f(e+fx)^2}{2b\,(a^2+b^2)\,d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)\,d^3} + \frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b\,(a^2+b^2)^{3/2}\,d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)\,d^3} - \frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2b\,(a^2+b^2)^{3/2}\,d^2} + \frac{3f^3\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)\,d^4} + \frac{3af^2(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^3} + \frac{3f^3\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)\,d^4} - \frac{3af^2(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^3} - \frac{3af^3\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^4} + \frac{3af^3\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^4} - \frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2} - \frac{3f(e+fx)^2\cosh(c+dx)}{2\,(a^2+b^2)\,d^2(a+b\sinh(c+dx))}$$

output
```
-3/2*f*(f*x+e)^2/b/(a^2+b^2)/d^2+3*f^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/b/(a^2+b^2)/d^3+3/2*a*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+3*f^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/b/(a^2+b^2)/d^3-3/2*a*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+3*f^3*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/b/(a^2+b^2)/d^4+3*a*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3+3*f^3*polylog(2,-b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/b/(a^2+b^2)/d^4-3*a*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-3*a*f^3*polylog(3,-b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^4+3*a*f^3*polylog(3,-b*exp(d*x+c)/(a+(
a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^4-1/2*(f*x+e)^3/b/d/(a+b*sinh(d*x+c))
^2-3/2*f*(f*x+e)^2*cosh(d*x+c)/(a^2+b^2)/d^2/(a+b*sinh(d*x+c))
```

3.332.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5753 vs. $2(631) = 1262$.

Time = 6.91 (sec) , antiderivative size = 5753, normalized size of antiderivative = 9.12

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
Result too large to show
```

3.332.3 Rubi [A] (verified)

Time = 2.89 (sec) , antiderivative size = 561, normalized size of antiderivative = 0.89, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5987, 3042, 3805, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 6095, 2620, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}dx$$

$$\Big\downarrow \text{5987}$$

$$\frac{3f\int \frac{(e+fx)^2}{(a+b\sinh(c+dx))^2}dx}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}+\frac{3f\int \frac{(e+fx)^2}{(a-ib\sin(ic+idx))^2}dx}{2bd}$$

$$\Big\downarrow \text{3805}$$

$$\frac{3f\left(\frac{a\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}+\frac{3f\left(\frac{a\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}$$

$$\Big\downarrow \text{3803}$$

$$\frac{3f\left(\frac{2a\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{3f\left(-\frac{2a\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{2694}$$

3.332. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

$$\frac{3f\left(-\frac{2a\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{3f\left(-\frac{2a\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{2bf\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{d(a^2+b^2)}-\frac{b(e+fx)^2\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}\right)}{2bd}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 2620$$

$$\frac{3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{a^2+b^2}\right.}{2bd}$$

$$-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 3011$$

3.332. $\displaystyle\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{bd}\right.}{}\right.}{a^2+b^2}\right.$$

$$2b$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 2720$$

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log}{}\right.}{}\right.}{a^2+b^2}\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 6095$$

3.332. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log}{}\right.}{}}{a^2+b^2}\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \color{blue}{2620}$$

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log}{}\right.}{}}{a^2+b^2}\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \color{blue}{2715}$$

3.332. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\cdots}{\cdots}\right.}{\cdots}\right)\cdots}{a^2+b^2\cdots}\right.\cdots}{\cdots}$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{ 2838}$$

$$3f\left(-\frac{2a\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\cdots}{\cdots}\right.}{\cdots}\right)\cdots}{a^2+b^2\cdots}\right.\cdots}{\cdots}$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{ 7143}$$

3.332. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

$$3f\left(\frac{2bf\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{d(a^2+b^2)}-\frac{2a\left(\cdots\right.}{\cdots}\right.$$

$$\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

output
```
-1/2*(e + f*x)^3/(b*d*(a + b*Sinh[c + d*x])^2) + (3*f*((2*b*f*(-1/2*(e + f
*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) +
(f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/((a^2 + b^2)*d
) - (2*a*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])
/d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) - (b*(e
 + f*x)^2*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x]))))/(2*b*d)
```

3.332. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

3.332.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.332. $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 5987
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.332. $\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

3.332.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)}{(a+b\sinh(dx+c))^3}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

3.332.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11757 vs. $2(575) = 1150$.

Time = 0.41 (sec) , antiderivative size = 11757, normalized size of antiderivative = 18.63

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.332.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

3.332.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
3*a*d*f^3*integrate(x^2*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2
*e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2
*b^2*d^2 - b^4*d^2), x) + 6*a*d*e*f^2*integrate(x*e^(d*x + c)/(a^2*b^2*d^2
*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a
*b^3*d^2*e^(d*x + c) - a^2*b^2*d^2 - b^4*d^2), x) + 3*b*e*f^2*(a*log((b*e^
(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2 + b^2)))/((
a^2*b^2 + b^4)*sqrt(a^2 + b^2)*d^3) - 2*(d*x + c)/((a^2*b^2 + b^4)*d^3) +
log(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b)/((a^2*b^2 + b^4)*d^3)) - 6*a*
f^3*integrate(x*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*
x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b^2*d^2
 - b^4*d^2), x) + 6*b*f^3*integrate(x/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d
^2*e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a
^2*b^2*d^2 - b^4*d^2), x) + 3/2*e^2*f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(d
*x + c) + b^2 + (2*a^2*e^(2*c) - b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e^
(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d
^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d
*x) + 2*(2*a^4*b*d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2
*d*x) - 4*(a^3*b^2*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) + a*log((b*e^(d*x + 2
*c) + a*e^c - sqrt(a^2 + b^2)*e^c)/(b*e^(d*x + 2*c) + a*e^c + sqrt(a^2 + b
^2)*e^c))/((a^2*b + b^3)*sqrt(a^2 + b^2)*d^2)) - 2*e^3*e^(-2*d*x - 2*c)...
```

3.332.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^3}{\left(a+b\sinh\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^3,x)
```

output
```
int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^3, x)
```

3.333 $\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.333.1 Optimal result

Integrand size = 32, antiderivative size = 448

$$\begin{aligned}
&\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^3\cosh(c+dx)}{bd^4} - \frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} \\
&\quad - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d} \\
&\quad - \frac{3af(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2} - \frac{3af(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2} \\
&\quad + \frac{6af^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3} + \frac{6af^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^3} \\
&\quad - \frac{6af^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^4} - \frac{6af^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^4} \\
&\quad + \frac{6f^2(e+fx)\sinh(c+dx)}{bd^3} + \frac{(e+fx)^3\sinh(c+dx)}{bd}
\end{aligned}$$

output
```
1/4*a*(f*x+e)^4/b^2/f-6*f^3*cosh(d*x+c)/b/d^4-3*f*(f*x+e)^2*cosh(d*x+c)/b/
d^2-a*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d-a*(f*x+e)^3*l
n(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d-3*a*f*(f*x+e)^2*polylog(2,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^2-3*a*f*(f*x+e)^2*polylog(2,-b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^2+6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/b^2/d^3+6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a
^2+b^2)^(1/2)))/b^2/d^3-6*a*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)
))/b^2/d^4-6*a*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^4+6*
f^2*(f*x+e)*sinh(d*x+c)/b/d^3+(f*x+e)^3*sinh(d*x+c)/b/d
```

3.333.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2809 vs. $2(448) = 896$.

Time = 9.73 (sec) , antiderivative size = 2809, normalized size of antiderivative = 6.27

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
((a*(4*e^3*E^(2*c)*x + 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)
*f^3*x^4 + (4*a*Sqrt[a^2 + b^2]*e^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 -
 b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (4*a*Sqrt[-a^2 - b^2]*e^3*ArcTanh[(a +
b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) - (2*e^3*E^(2*c)
*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))])/d + (2*e^3*Log[2*a*E^(c + d
*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/
(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^
(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1
+ (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)
*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d
 + (2*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]
)])/d - (2*E^(2*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
b^2)*E^(2*c)])])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a
^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*
E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*
x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 +
 (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Lo
g[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*
c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])
/d - (6*(-1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*...
```

3.333.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.28 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.01, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.594$, Rules used = {6113, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 6095, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)^3\cosh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\sinh(c+dx)dx}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int (e+fx)^2\sinh(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\sin(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\int (e+fx)^2\sin(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{d}}{b}$$

3.333. $\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{6095}$$

$$-\frac{a\left(\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{a\left(-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}}\right.}{bd}\right.}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{3011}$$

3.333. $\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(-\frac{3f\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-(e+fx)^2 \cdots\right.}{bd}\cdots\right.}{b}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{7163}$$

$$-\frac{a\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyL}\cdots}{}\right.}{}\right.}{}\cdots\right.}{}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{a\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\cdots\right.}{}\right.}{}\cdots\right.}{}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{7143}$$

3.333. $\displaystyle\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((a*(-1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/b) + (((e + f*x)^3*Sinh[c + d*x])/d +
 ((3*I)*f*((I*(e + f*x)^2*Cosh[c + d*x])/d - ((2*I)*f*(-((f*Cosh[c + d*x])
/d^2) + ((e + f*x)*Sinh[c + d*x])/d))/d))/d)/b
```

3.333.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.333. $\int \frac{(e+fx)^3 \cosh(c+dx) \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.333.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh (dx+c) \sinh (dx+c)}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.333.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1976 vs. $2(420) = 840$.

Time = 0.29 (sec) , antiderivative size = 1976, normalized size of antiderivative = 4.41

$$\int \frac{(e+fx)^3 \cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-1/4*(2*b*d^3*f^3*x^3 + 2*b*d^3*e^3 + 6*b*d^2*e^2*f + 12*b*d*e*f^2 + 12*b*
f^3 + 6*(b*d^3*e*f^2 + b*d^2*f^3)*x^2 - 2*(b*d^3*f^3*x^3 + b*d^3*e^3 - 3*b
*d^2*e^2*f + 6*b*d*e*f^2 - 6*b*f^3 + 3*(b*d^3*e*f^2 - b*d^2*f^3)*x^2 + 3*(
b*d^3*e^2*f - 2*b*d^2*e*f^2 + 2*b*d*f^3)*x)*cosh(d*x + c)^2 - 2*(b*d^3*f^3
*x^3 + b*d^3*e^3 - 3*b*d^2*e^2*f + 6*b*d*e*f^2 - 6*b*f^3 + 3*(b*d^3*e*f^2
- b*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*b*d^2*e*f^2 + 2*b*d*f^3)*x)*sinh(d*x
 + c)^2 + 6*(b*d^3*e^2*f + 2*b*d^2*e*f^2 + 2*b*d*f^3)*x - (a*d^4*f^3*x^4 +
 4*a*d^4*e*f^2*x^3 + 6*a*d^4*e^2*f*x^2 + 4*a*d^4*e^3*x + 8*a*c*d^3*e^3 - 1
2*a*c^2*d^2*e^2*f + 8*a*c^3*d*e*f^2 - 2*a*c^4*f^3)*cosh(d*x + c) + 12*((a*
d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*cosh(d*x + c) + (a*d^2*f^3*x^
2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) +
 a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b + 1) + 12*((a*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*cosh(
d*x + c) + (a*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*sinh(d*x + c))*
dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
 c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 4*((a*d^3*e^3 - 3*a*c*d^2*e^2*f +
 3*a*c^2*d*e*f^2 - a*c^3*f^3)*cosh(d*x + c) + (a*d^3*e^3 - 3*a*c*d^2*e^2*f
 + 3*a*c^2*d*e*f^2 - a*c^3*f^3)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b
*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*((a*d^3*e^3 - 3*a*c*
d^2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3)*cosh(d*x + c) + (a*d^3*e^3 - 3...
```

3.333.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.333.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/2*e^3*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) +
 2*a*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^2*d)) - 1/4*(a*d^4
*f^3*x^4*e^c + 4*a*d^4*e*f^2*x^3*e^c + 6*a*d^4*e^2*f*x^2*e^c - 2*(b*d^3*f^
3*x^3*e^(2*c) + 3*(d^3*e*f^2 - d^2*f^3)*b*x^2*e^(2*c) + 3*(d^3*e^2*f - 2*d
^2*e*f^2 + 2*d*f^3)*b*x*e^(2*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b*e^(2
*c))*e^(d*x) + 2*(b*d^3*f^3*x^3 + 3*(d^3*e*f^2 + d^2*f^3)*b*x^2 + 3*(d^3*e
^2*f + 2*d^2*e*f^2 + 2*d*f^3)*b*x + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b)*e
^(-d*x))*e^(-c)/(b^2*d^4) + integrate(-2*(a*b*f^3*x^3 + 3*a*b*e*f^2*x^2 +
3*a*b*e^2*f*x - (a^2*f^3*x^3*e^c + 3*a^2*e*f^2*x^2*e^c + 3*a^2*e^2*f*x*e^c
)*e^(d*x))/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)
```

3.333.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x
)
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.334 $\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.334.1 Optimal result

Integrand size = 32, antiderivative size = 330

$$\begin{aligned}\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{a(e+fx)^3}{3b^2f}-\frac{2f(e+fx)\cosh(c+dx)}{bd^2}\\
&-\frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d}\\
&-\frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d}\\
&-\frac{2af(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}\\
&-\frac{2af(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2}\\
&+\frac{2af^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3}\\
&+\frac{2af^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^3}\\
&+\frac{2f^2\sinh(c+dx)}{bd^3}+\frac{(e+fx)^2\sinh(c+dx)}{bd}\end{aligned}$$

output
```
1/3*a*(f*x+e)^3/b^2/f-2*f*(f*x+e)*cosh(d*x+c)/b/d^2-a*(f*x+e)^2*ln(1+b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d-a*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/b^2/d-2*a*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)
))/b^2/d^2-2*a*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/
d^2+2*a*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^3+2*a*f^2*p
olylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^3+2*f^2*sinh(d*x+c)/b/d^
3+(f*x+e)^2*sinh(d*x+c)/b/d
```

3.334.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1301 vs. $2(330) = 660$.

Time = 9.48 (sec) , antiderivative size = 1301, normalized size of antiderivative = 3.94

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{1}{2}\left(\frac{2a\left(6e^2e^{2c}x+6ee^{2c}fx^2+2e^{2c}f^2x^3+\frac{6a\sqrt{a^2+b^2}e^2\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}d}+\frac{6a\sqrt{-(a^2+b^2)^2}e^2e^{2c}\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{(a^2+b^2)^{3/2}d}-\frac{6a}{}\right.}{}\right.$$

$$-\frac{ax(3e^2+3efx+f^2x^2)\cosh(c)\operatorname{csch}\left(\frac{c}{2}\right)\operatorname{sech}\left(\frac{c}{2}\right)}{3b^2}$$

$$+\frac{2\cosh(dx)\left(-2def\cosh(c)-2df^2x\cosh(c)+d^2e^2\sinh(c)+2f^2\sinh(c)+2d^2efx\sinh(c)+d^2f^2x^2\sinh\right.}{bd^3}$$

$$+\frac{2(d^2e^2\cosh(c)+2f^2\cosh(c)+2d^2efx\cosh(c)+d^2f^2x^2\cosh(c)-2def\sinh(c)-2df^2x\sinh(c))\sinh(d}{bd^3}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
((2*a*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt
[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2
+ b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c +
d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]
*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d)
+ (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a
^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 +
E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(
c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b
^2)*E^(2*c)])]])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sq
rt[(a^2 + b^2)*E^(2*c)]])]/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c
 - Sqrt[(a^2 + b^2)*E^(2*c)])]])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c +
 d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])]])/d + (6*e*f*x*Log[1 + (b*E^(2*
c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]])/d - (6*e*E^(2*c)*f*x*Log[1
 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]])/d + (3*f^2*x^2*
Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]])/d - (3*E^(
2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]])
])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c
 - Sqrt[(a^2 + b^2)*E^(2*c)]))]])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyL
og[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]])/d^2 - ...
```

3.334.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6113, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 \sinh(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)^2\cosh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$$\Big\downarrow \text{6095}$$

$$-\frac{a\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$$\Big\downarrow \text{2620}$$

3.334. $\displaystyle\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2+}}\right.}{bd}\right.}{b}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{a\left(-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c}}{a+\sqrt{}}\right.}{d}\right.}{bd}\right.}{b}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{a\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\right.}{bd}\right.}{b}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{a\left(-\frac{2f\left(\frac{f\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right.}{b}$$

$$\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.334. $\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
-((a*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/d^2))/(b*d)))/b) + (((e + f*x)^2*Sinh[c + d*x])/d + ((2*I)*f*((I*
(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/d)/b
```

3.334.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.334.4 Maple [F]

$$\int \frac{(fx+e)^2 \cosh(dx+c)\sinh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.334.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. $2(308) = 616$.

Time = 0.29 (sec) , antiderivative size = 1265, normalized size of antiderivative = 3.83

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-1/6*(3*b*d^2*f^2*x^2 + 3*b*d^2*e^2 + 6*b*d*e*f + 6*b*f^2 - 3*(b*d^2*f^2*x
^2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f - b*d*f^2)*x)*cosh(d*x
 + c)^2 - 3*(b*d^2*f^2*x^2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*
f - b*d*f^2)*x)*sinh(d*x + c)^2 + 6*(b*d^2*e*f + b*d*f^2)*x - 2*(a*d^3*f^2
*x^3 + 3*a*d^3*e*f*x^2 + 3*a*d^3*e^2*x + 6*a*c*d^2*e^2 - 6*a*c^2*d*e*f + 2
*a*c^3*f^2)*cosh(d*x + c) + 12*((a*d*f^2*x + a*d*e*f)*cosh(d*x + c) + (a*d
*f^2*x + a*d*e*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
12*((a*d*f^2*x + a*d*e*f)*cosh(d*x + c) + (a*d*f^2*x + a*d*e*f)*sinh(d*x +
 c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 6*((a*d^2*e^2 - 2*a*c*d*e*f
+ a*c^2*f^2)*cosh(d*x + c) + (a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*sinh(d*
x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) + 6*((a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*cosh(d*x + c) + (a*d
^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2
*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*((a*d^2*f^2*x^2 +
2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*cosh(d*x + c) + (a*d^2*f^2*x^2 +
2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*sinh(d*x + c))*log(-(a*cosh(d*x +
 c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b) + 6*((a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c...
```

3.334.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.334.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh(dx+c)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/2*e^2*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) +
 2*a*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^2*d)) - 1/6*(2*a*d
^3*f^2*x^3*e^c + 6*a*d^3*e*f*x^2*e^c - 3*(b*d^2*f^2*x^2*e^(2*c) + 2*(d^2*e
*f - d*f^2)*b*x*e^(2*c) - 2*(d*e*f - f^2)*b*e^(2*c))*e^(d*x) + 3*(b*d^2*f^
2*x^2 + 2*(d^2*e*f + d*f^2)*b*x + 2*(d*e*f + f^2)*b)*e^(-d*x))*e^(-c)/(b^2
*d^3) + integrate(-2*(a*b*f^2*x^2 + 2*a*b*e*f*x - (a^2*f^2*x^2*e^c + 2*a^2
*e*f*x*e^c)*e^(d*x))/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)
```

3.334.8 Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh(dx+c)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x
)
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.335 $\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.335.1 Optimal result

Integrand size = 30, antiderivative size = 212

$$\begin{aligned}\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = {} & \frac{a(e+fx)^2}{2b^2f} - \frac{f\cosh(c+dx)}{bd^2} \\ & - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} \\ & - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d} \\ & - \frac{af\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2} \\ & - \frac{af\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2} \\ & + \frac{(e+fx)\sinh(c+dx)}{bd}\end{aligned}$$

output
```
1/2*a*(f*x+e)^2/b^2/f-f*cosh(d*x+c)/b/d^2-a*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))/b^2/d-a*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^
2/d-a*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^2-a*f*polylog(2
,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^2+(f*x+e)*sinh(d*x+c)/b/d
```

3.335.2 Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.78

$$\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-2bf\cosh(c+dx) - a\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}} - \frac{4a\sqrt{-(a^2}{}\right.}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-2*b*f*Cosh[c + d*x] - a*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x
)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]
])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(
c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x
))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[
2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))]) + 2*b*d*(e + f*x)*Sinh[c + d*x])/(2*b^2*d^2)
```

3.335.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {6113, 3042, 3777, 26, 3042, 26, 3118, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{6113}$$
$$\frac{\int (e+fx)\cosh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6095}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-$$

$$\frac{a\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-$$

$$\frac{a\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+}}{\sqrt{a^2+}}\right.}{bd}\right.}{b}$$

3.335. $\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{b}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((a*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)
))/b) + (-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d)/b
```

3.335.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.335. $\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.335.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(198) = 396$.

Time = 2.55 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.28

method	result
risch	$\frac{af\,x^2}{2b^2} - \frac{aex}{b^2} + \frac{(dfx+de-f)\mathrm{e}^{dx+c}}{2b\,d^2} - \frac{(dfx+de+f)\mathrm{e}^{-dx-c}}{2b\,d^2} + \frac{2ae\ln\left(\mathrm{e}^{dx+c}\right)}{d\,b^2} + \frac{af\,c^2}{d^2b^2} - \frac{af\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)c}{d^2b^2} -$

input
```
int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERB
OSE)
```

output
```
1/2*a*f*x^2/b^2-a*e*x/b^2+1/2*(d*f*x+d*e-f)/b/d^2*exp(d*x+c)-1/2*(d*f*x+d*
e+f)/b/d^2*exp(-d*x-c)+2/d*a/b^2*e*ln(exp(d*x+c))+1/d^2*a/b^2*f*c^2-1/d^2*
a/b^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2
*a/b^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d*a/
b^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d*a/b
^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a/b^
2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*a/
b^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/d^2*a/
b^2*c*f*ln(exp(d*x+c))+1/d^2*a/b^2*c*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-
b)+2/d*a/b^2*c*f*x-1/d*a/b^2*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)
```

3.335.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(196) = 392$.

Time = 0.26 (sec) , antiderivative size = 692, normalized size of antiderivative = 3.26

$$\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{bdfx + bde - (bdfx + bde - bf)\cosh(dx+c)^2 - (bdfx + bde - bf)\sinh(dx+c)^2 + bf - (ad^2fx^2 + 2}{}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
-1/2*(b*d*f*x + b*d*e - (b*d*f*x + b*d*e - b*f)*cosh(d*x + c)^2 - (b*d*f*x
 + b*d*e - b*f)*sinh(d*x + c)^2 + b*f - (a*d^2*f*x^2 + 2*a*d^2*e*x + 4*a*c
*d*e - 2*a*c^2*f)*cosh(d*x + c) + 2*(a*f*cosh(d*x + c) + a*f*sinh(d*x + c)
)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a*f*cosh(d*x + c) + a*f*sinh
(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*((a*d*e - a*c*f)*cos
h(d*x + c) + (a*d*e - a*c*f)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*si
nh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((a*d*e - a*c*f)*cosh(d
*x + c) + (a*d*e - a*c*f)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(
d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((a*d*f*x + a*c*f)*cosh(d*
x + c) + (a*d*f*x + a*c*f)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d
*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
) + 2*((a*d*f*x + a*c*f)*cosh(d*x + c) + (a*d*f*x + a*c*f)*sinh(d*x + c))*
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a*d^2*f*x^2 + 2*a*d^2*e*x + 4*a*c*d*e
 - 2*a*c^2*f + 2*(b*d*f*x + b*d*e - b*f)*cosh(d*x + c))*sinh(d*x + c))/(b^
2*d^2*cosh(d*x + c) + b^2*d^2*sinh(d*x + c))
```

3.335.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.335.7 Maxima [F]

$$\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
-1/2*e*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) + 2
*a*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^2*d)) - 1/4*f*(2*(a*
d^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*e^(d*x) + (b*d*x + b)*e^(-d*x))*
e^(-c)/(b^2*d^2) - integrate(8*(a^2*x*e^(d*x + c) - a*b*x)/(b^3*e^(2*d*x +
 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x))
```

3.335.8 Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.336 $\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.336.1 Optimal result

Integrand size = 25, antiderivative size = 34

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\log(a+b\sinh(c+dx))}{b^2d}+\frac{\sinh(c+dx)}{bd}$$

output
```
-a*ln(a+b*sinh(d*x+c))/b^2/d+sinh(d*x+c)/b/d
```

3.336.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{a\log(a+b\sinh(c+dx))}{b^2}-\frac{\sinh(c+dx)}{b}}{d}$$

input
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-(((a*Log[a + b*Sinh[c + d*x]])/b^2 - Sinh[c + d*x]/b)/d)
```

3.336.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 26, 3312, 26, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)\cos(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ic+idx)\sin(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3312}$$

$$-\frac{i\int \frac{i\sinh(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b\sinh(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^2d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(1-\frac{a}{a+b\sinh(c+dx)}\right)d(b\sinh(c+dx))}{b^2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\sinh(c+dx)-a\log(a+b\sinh(c+dx))}{b^2d}$$

input
```
Int[(Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(a*Log[a + b*Sinh[c + d*x]]) + b*Sinh[c + d*x])/(b^2*d)
```

3.336.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

3.336.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{\frac{\sinh(dx+c)}{b}-\frac{a\ln(a+b\sinh(dx+c))}{b^2}}{d}$	33
default	$\frac{\frac{\sinh(dx+c)}{b}-\frac{a\ln(a+b\sinh(dx+c))}{b^2}}{d}$	33
risch	$\frac{ax}{b^2}+\frac{e^{dx+c}}{2bd}-\frac{e^{-dx-c}}{2bd}+\frac{2ac}{b^2d}-\frac{a\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{b^2d}$	82

3.336. $\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
int(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b*sinh(d*x+c)-a/b^2*ln(a+b*sinh(d*x+c)))
```

3.336.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(34) = 68.

Time = 0.25 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.88

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2\,adx\cosh\left(dx+c\right)+b\cosh\left(dx+c\right)^{2}+b\sinh\left(dx+c\right)^{2}-2\left(a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)\right)\log\left(\frac{2\,(b\cdots}{\cosh(d\cdots}\right.}{2\left(b^{2}d\cosh\left(dx+c\right)+b^{2}d\sinh\left(dx+c\right)\right)}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*a*d*x*cosh(d*x + c) + b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 - 2*(a*
cosh(d*x + c) + a*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c
) - sinh(d*x + c))) + 2*(a*d*x + b*cosh(d*x + c))*sinh(d*x + c) - b)/(b^2*
d*cosh(d*x + c) + b^2*d*sinh(d*x + c))
```

3.336.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(27) = 54.

Time = 0.87 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \begin{cases} \frac{x\sinh\left(c\right)\cosh\left(c\right)}{a} & \text{for}\: b=0 \wedge d=0 \\ \frac{\cosh^{2}\left(c+dx\right)}{2ad} & \text{for}\: b=0 \\ \frac{x\sinh\left(c\right)\cosh\left(c\right)}{a+b\sinh\left(c\right)} & \text{for}\: d=0 \\ -\frac{a\log\left(\frac{a}{b}+\sinh\left(c+dx\right)\right)}{b^{2}d}+\frac{\sinh\left(c+dx\right)}{bd} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Piecewise((x*sinh(c)*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (cosh(c + d*x)**2/(2
*a*d), Eq(b, 0)), (x*sinh(c)*cosh(c)/(a + b*sinh(c)), Eq(d, 0)), (-a*log(a
/b + sinh(c + d*x))/(b**2*d) + sinh(c + d*x)/(b*d), True))
```

3.336.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(34) = 68$.

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.44

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(dx+c)a}{b^2d} + \frac{e^{(dx+c)}}{2\,bd} - \frac{e^{(-dx-c)}}{2\,bd} - \frac{a\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{b^2d}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) - 1/2*e^(-d*x - c)/(b*d) - a*
log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^2*d)
```

3.336.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.76

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{e^{(dx+c)}-e^{(-dx-c)}}{b} - \frac{2\,a\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^2}}{2\,d}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*((e^(d*x + c) - e^(-d*x - c))/b - 2*a*log(abs(b*(e^(d*x + c) - e^(-d*x
 - c)) + 2*a))/b^2)/d
```

3.336.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\,\ln\left(a+b\sinh(c+d\,x)\right)-b\sinh(c+d\,x)}{b^2\,d}$$

input
```
int((cosh(c + d*x)*sinh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

output
```
-(a*log(a + b*sinh(c + d*x)) - b*sinh(c + d*x))/(b^2*d)
```

3.337 $\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.337.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.337.2 Mathematica [N/A]

Not integrable

Time = 21.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

output
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

3.337.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{6125\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

$$\Big\downarrow \text{6125}$$

$$\int \frac{\sinh(c+dx)\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.337.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.337.4 Maple [N/A] (verified)

Not integrable

Time = 0.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(dx+c\right)\sinh\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.337.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
integral(cosh(d*x + c)*sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
 + c)), x)
```

3.337.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.337.7 Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 164, normalized size of antiderivative = 5.12

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
-1/2*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d*
e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f) + 1/
4*integrate(-8*(a^2*e^(d*x + c) - a*b)/(b^3*f*x + b^3*e - (b^3*f*x*e^(2*c)
 + b^3*e*e^(2*c))*e^(2*d*x) - 2*(a*b^2*f*x*e^c + a*b^2*e*e^c)*e^(d*x)), x)
```

3.337.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
integrate(cosh(d*x + c)*sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x
)
```

3.337.9 Mupad [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.338 $\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.338.1 Optimal result

Integrand size = 34, antiderivative size = 696

$$\begin{aligned}
&\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{3ef^2x}{4bd^2}+\frac{3f^3x^2}{8bd^2}+\frac{a^2(e+fx)^4}{4b^3f}+\frac{(e+fx)^4}{8bf}-\frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3} \\
&\quad-\frac{a(e+fx)^3\cosh(c+dx)}{b^2d}-\frac{3f^3\cosh^2(c+dx)}{8bd^4}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4bd^2} \\
&\quad-\frac{a\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{a\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d} \\
&\quad-\frac{3a\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2} \\
&\quad+\frac{3a\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2} \\
&\quad+\frac{6a\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3} \\
&\quad-\frac{6a\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3} \\
&\quad-\frac{6a\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^4}+\frac{6a\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^4} \\
&\quad+\frac{6af^3\sinh(c+dx)}{b^2d^4}+\frac{3af(e+fx)^2\sinh(c+dx)}{b^2d^2} \\
&\quad+\frac{3f^2(e+fx)\cosh(c+dx)\sinh(c+dx)}{4bd^3}+\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{2bd}
\end{aligned}$$

output
```
3/4*e*f^2*x/b/d^2+3/8*f^3*x^2/b/d^2+1/4*a^2*(f*x+e)^4/b^3/f+1/8*(f*x+e)^4/
b/f-6*a*f^2*(f*x+e)*cosh(d*x+c)/b^2/d^3-a*(f*x+e)^3*cosh(d*x+c)/b^2/d-3/8*
f^3*cosh(d*x+c)^2/b/d^4-3/4*f*(f*x+e)^2*cosh(d*x+c)^2/b/d^2+6*a*f^3*sinh(d
*x+c)/b^2/d^4+3*a*f*(f*x+e)^2*sinh(d*x+c)/b^2/d^2+3/4*f^2*(f*x+e)*cosh(d*x
+c)*sinh(d*x+c)/b/d^3+1/2*(f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/b/d-a*(f*x+e)^
3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d+a*(f*x+e)^3
*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d-3*a*f*(f*x+e
)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d^2+3
*a*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2
)/b^3/d^2+6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^
2+b^2)^(1/2)/b^3/d^3-6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))*(a^2+b^2)^(1/2)/b^3/d^3-6*a*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d^4+6*a*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d^4
```

3.338.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1994 vs. $2(696) = 1392$.

Time = 9.61 (sec) , antiderivative size = 1994, normalized size of antiderivative = 2.86

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(e^3*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(
Sqrt[-a^2 - b^2]*d)))/(4*b) + (3*e^2*f*(x^2 - (2*a*(d*x*(Log[1 + (b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2])] - Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])]) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - PolyLog[2, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^2)))/(8*b) +
(e*f^2*(x^3 - (3*a*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]
 - d^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[
2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^
2 + b^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqr
t[a^2 + b^2]*d^3)))/(4*b) + (f^3*(x^4 - (4*a*(d^3*x^3*Log[1 + (b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2])] - d^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])] + 3*d^2*x^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] -
 3*d^2*x^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*x*Po
lyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*x*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*PolyLog[4, (b*E^(c + d*x))/(-a + S
qrt[a^2 + b^2])] - 6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
)/(Sqrt[a^2 + b^2]*d^4)))/(16*b) + (e*f^2*(2*(4*a^2 + b^2)*x^3 - (6*a*(4*a
^2 + 3*b^2)*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*
x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (...
```

3.338.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.63 (sec) , antiderivative size = 641, normalized size of antiderivative = 0.92, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.853$, Rules used = {6113, 3042, 3792, 17, 3042, 3791, 17, 6099, 17, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 3803, 25, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)^3\cosh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}$$

$\big\downarrow$ 3792

$$\frac{\frac{3f^2\int (e+fx)\cosh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^3dx-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\big\downarrow$ 17

$$\frac{\frac{3f^2\int (e+fx)\cosh^2(c+dx)dx}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\big\downarrow$ 3042

$$-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{3f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}$$

$\big\downarrow$ 3791

$$\frac{\frac{3f^2\left(\frac{1}{2}\int (e+fx)dx-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\big\downarrow$ 17

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\big\downarrow$ 6099

3.338. $\displaystyle\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^3dx}{b^2}+\frac{\int(e+fx)^3\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)^3\sinh(c+dx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int-i(e+fx)^3\sin(ic+idx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int(e+fx)^3\sin(ic+idx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int(e+fx)^2\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

3.338. $\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int-i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

3.338. $\displaystyle\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b}-\frac{a(e+f}{4b^2f}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

3.338. $\displaystyle\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}\right)}{b}-a(\right.}{b}$$

$$\Big\downarrow \ 3117$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 3803$$

3.338. $\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$\frac{a\left(\frac{2\left(a^2+b^2\right)\int -\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+\dots}{d^2}\dots\right)}{d}\dots\right)}{d}\dots\right)}{b}\dots\right)}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$\frac{a\left(-\frac{2\left(a^2+b^2\right)\int \frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c\dots}{d^2}\dots\right)}{d}\dots\right)}{d}\dots\right)}{b}\dots\right)}{b}$$

$$\Big\downarrow \; 2694$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+\dots}{d}\dots\right)}{\dots}\right)}{\dots}\dots\right)}{b}$$

$$\Big\downarrow \; 27$$

3.338. $\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if(\cdots}{\cdots}\right.}{\cdots}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+\cdots}}\right.\cdots}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{b^2}\right.}{b}$$

$$\Big\downarrow \text{3011}$$

3.338. $\displaystyle\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\,\mathrm{lo}\cdots}{}\right.}{}\right.}{b^2}\right.$$

$$\Big\downarrow \; 7163$$

3.338. $\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}-}{b}$$

$$a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,\cdots}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{\cdots}\right)$$

↓ 2720

3.338. $\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)}{}\cdots}{bd}\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{}\cdots\right)$$

$\downarrow$ 7143

3.338. $\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-$$

$$a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{b}{a+}\right.}{d}}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.338. $\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
((e + f*x)^4/(8*f) - (3*f*(e + f*x)^2*Cosh[c + d*x]^2)/(4*d^2) + ((e + f*x
)^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (3*f^2*((e + f*x)^2/(4*f) - (f*Co
sh[c + d*x]^2)/(4*d^2) + ((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)))/(
2*d^2))/b - (a*(-1/4*(a*(e + f*x)^4)/(b^2*f) - (2*(a^2 + b^2)*(-1/2*(b*(((
e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-
(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (
2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d -
(f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))
/Sqrt[a^2 + b^2] + (b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))])/d^2))/d))/(b*d)))/(2*Sqrt[a^2 + b^2])))/b^2 - (I*((I*(e + f*x)^3*C
osh[c + d*x])/d - ((3*I)*f*(((e + f*x)^2*Sinh[c + d*x])/d + ((2*I)*f*((I*(
e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/d))/d))/b))/b
```

3.338.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.338. $\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.338. $\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.338. $\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} \, dx$

3.338.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)^2 \sinh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.338.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3847 vs. $2(638) = 1276$.

Time = 0.34 (sec) , antiderivative size = 3847, normalized size of antiderivative = 5.53

$$\int \frac{(e+fx)^3 \cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
-1/32*(4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 + 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2
 + 3*b^2*f^3 - (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^
2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d
^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c)^4 - (4*b^2*d^3*f^
3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2
*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 +
 b^2*d*f^3)*x)*sinh(d*x + c)^4 + 16*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*b
*d^2*e^2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x
^2 + 3*(a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c)^3
+ 4*(4*a*b*d^3*f^3*x^3 + 4*a*b*d^3*e^3 - 12*a*b*d^2*e^2*f + 24*a*b*d*e*f^2
 - 24*a*b*f^3 + 12*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 12*(a*b*d^3*e^2*f -
 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x - (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6
*b^2*d^2*e^2*f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*
f^3)*x^2 + 6*(2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x +
 c))*sinh(d*x + c)^3 + 6*(2*b^2*d^3*e*f^2 + b^2*d^2*f^3)*x^2 - 4*((2*a^2 +
 b^2)*d^4*f^3*x^4 + 4*(2*a^2 + b^2)*d^4*e*f^2*x^3 + 6*(2*a^2 + b^2)*d^4*e^
2*f*x^2 + 4*(2*a^2 + b^2)*d^4*e^3*x)*cosh(d*x + c)^2 - 2*(2*(2*a^2 + b^2)*
d^4*f^3*x^4 + 8*(2*a^2 + b^2)*d^4*e*f^2*x^3 + 12*(2*a^2 + b^2)*d^4*e^2*f*x
^2 + 8*(2*a^2 + b^2)*d^4*e^3*x + 3*(4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*
b^2*d^2*e^2*f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^...
```

3.338.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.338.7 Maxima [F]

$$\int \frac{(e+fx)^3 \cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \cosh(dx+c)^2 \sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/8*e^3*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) + 8*sqrt(a^2 + b^
2)*a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/(b^3*d) - 4*(2*a^2 + b^2)*(d*x + c)/(b^3*d) + (4*a*e^(-d*x -
 c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/32*(4*(2*a^2*d^4*f^3*e^(2*c) + b^2*
d^4*f^3*e^(2*c))*x^4 + 16*(2*a^2*d^4*e*f^2*e^(2*c) + b^2*d^4*e*f^2*e^(2*c)
)*x^3 + 24*(2*a^2*d^4*e^2*f*e^(2*c) + b^2*d^4*e^2*f*e^(2*c))*x^2 + (4*b^2*
d^3*f^3*x^3*e^(4*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*b^2*x^2*e^(4*c) + 6*(2*d^3
*e^2*f - 2*d^2*e*f^2 + d*f^3)*b^2*x*e^(4*c) - 3*(2*d^2*e^2*f - 2*d*e*f^2 +
 f^3)*b^2*e^(4*c))*e^(2*d*x) - 16*(a*b*d^3*f^3*x^3*e^(3*c) + 3*(d^3*e*f^2
- d^2*f^3)*a*b*x^2*e^(3*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a*b*x*e
^(3*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b*e^(3*c))*e^(d*x) - 16*(a*b*
d^3*f^3*x^3*e^c + 3*(d^3*e*f^2 + d^2*f^3)*a*b*x^2*e^c + 3*(d^3*e^2*f + 2*d
^2*e*f^2 + 2*d*f^3)*a*b*x*e^c + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a*b*e^c)
*e^(-d*x) - (4*b^2*d^3*f^3*x^3 + 6*(2*d^3*e*f^2 + d^2*f^3)*b^2*x^2 + 6*(2*
d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*b^2*x + 3*(2*d^2*e^2*f + 2*d*e*f^2 + f^3)
*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^4) - integrate(2*((a^3*f^3*e^c + a*b^2*f
^3*e^c)*x^3 + 3*(a^3*e*f^2*e^c + a*b^2*e*f^2*e^c)*x^2 + 3*(a^3*e^2*f*e^c +
 a*b^2*e^2*f*e^c)*x)*e^(d*x)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) -
b^4), x)
```

3.338.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a),
 x)
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{{\mathrm{cosh}\left(c+d\,x\right)}^2\,\mathrm{sinh}\left(c+d\,x\right)\,{\left(e+f\,x\right)}^3}{a+b\,\mathrm{sinh}\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.339 $\int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$

3.339.1 Optimal result

Integrand size = 34, antiderivative size = 510

$$
\begin{aligned}
&\int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx \\
&= \frac{f^2 x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3 f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2 \cosh(c+dx)}{b^2 d^3} \\
&\quad - \frac{a(e+fx)^2 \cosh(c+dx)}{b^2 d} - \frac{f(e+fx) \cosh^2(c+dx)}{2bd^2} \\
&\quad - \frac{a\sqrt{a^2+b^2}(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d} + \frac{a\sqrt{a^2+b^2}(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3 d} \\
&\quad - \frac{2a\sqrt{a^2+b^2} f(e+fx) \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^2} \\
&\quad + \frac{2a\sqrt{a^2+b^2} f(e+fx) \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3 d^2} \\
&\quad + \frac{2a\sqrt{a^2+b^2} f^2 \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^3} \\
&\quad - \frac{2a\sqrt{a^2+b^2} f^2 \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3 d^3} + \frac{2af(e+fx) \sinh(c+dx)}{b^2 d^2} \\
&\quad + \frac{f^2 \cosh(c+dx) \sinh(c+dx)}{4bd^3} + \frac{(e+fx)^2 \cosh(c+dx) \sinh(c+dx)}{2bd}
\end{aligned}
$$

output
```
1/4*f^2*x/b/d^2+1/3*a^2*(f*x+e)^3/b^3/f+1/6*(f*x+e)^3/b/f-2*a*f^2*cosh(d*x
+c)/b^2/d^3-a*(f*x+e)^2*cosh(d*x+c)/b^2/d-1/2*f*(f*x+e)*cosh(d*x+c)^2/b/d^
2+2*a*f*(f*x+e)*sinh(d*x+c)/b^2/d^2+1/4*f^2*cosh(d*x+c)*sinh(d*x+c)/b/d^3+
1/2*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b/d-a*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a
-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d+a*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d-2*a*f*(f*x+e)*polylog(2,-b*exp(d*x
+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d^2+2*a*f*(f*x+e)*polylog(2,-
b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d^2+2*a*f^2*polylog(
3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d^3-2*a*f^2*polyl
og(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d^3
```

3.339.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1188 vs. $2(510) = 1020$.

Time = 4.45 (sec) , antiderivative size = 1188, normalized size of antiderivative = 2.33

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{6b^2e^2\left(\frac{c}{d}+x-\frac{2a\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,d}\right)+6b^2ef\left(x^2-\frac{2a\left(dx\left(\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)+\text{PolyLog}\left(\right.\right.}{\sqrt{a^2+b^2}\,d^2}\right.}{}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(6*b^2*e^2*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^
2]])/(Sqrt[-a^2 - b^2]*d)) + 6*b^2*e*f*(x^2 - (2*a*(d*x*(Log[1 + (b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2])] - Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])]) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - PolyLog[2, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^2)) + 2*b^2*f
^2*(x^3 - (3*a*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d
^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (
b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 +
b^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^
2 + b^2]*d^3)) + f^2*(2*(4*a^2 + b^2)*x^3 - (6*a*(4*a^2 + 3*b^2)*(d^2*x^2*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*x^2*Log[1 + (b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*E^(c + d*x))/(-a + Sqr
t[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*PolyLog[3, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^3) - (24*a*b*
Cosh[d*x]*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c]))/d^3 + (3*b^2*Cosh[2*d*x
]*(-2*d*x*Cosh[2*c] + (1 + 2*d^2*x^2)*Sinh[2*c]))/d^3 - (24*a*b*(-2*d*x*Co
sh[c] + (2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*((1 + 2*d^2*x^2)*Co
sh[2*c] - 2*d*x*Sinh[2*c])*Sinh[2*d*x])/d^3) + (6*e^2*((4*a^2 + b^2)*(c...
```

3.339.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.74 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.95, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {6113, 3042, 3792, 17, 3042, 3115, 24, 6099, 17, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)^2\cosh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}$$

$$\Big\downarrow \text{ 3792}$$

$$\frac{\frac{f^2\int \cosh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int (e+fx)^2dx-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{\frac{f^2\int \cosh^2(c+dx)dx}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{f^2\int \sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{f^2\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 6099}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^2dx}{b^2}+\frac{\int (e+fx)^2\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 17}$$

3.339. $\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)^2\sinh(c+dx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int-i(e+fx)^2\sin(ic+idx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$$\Big\downarrow \color{blue}{26}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int(e+fx)^2\sin(ic+idx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$$\Big\downarrow \color{blue}{3777}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$$\Big\downarrow \color{blue}{3777}$$

3.339. $\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{b}$$

$$\Big\downarrow \ 3118$$

3.339. $\displaystyle\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3803}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{2(a^2+b^2)\int-\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\int\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{2694}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+d}{d}\right.}{d}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{27}$$

3.339. $\displaystyle \int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-$$

$$\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(}{d^2}\right.}{d}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-$$

$$\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{b^2}\right.}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-$$

$$a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+}}{a-\sqrt{a}}\right.}{bd}\right.}{}\right.}{b^2}\right.$$

3.339. $\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{2720}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx\cdots}{\cdots}\right.\cdots}{\cdots}\cdots\right)}{b^2}\cdots\right)}{\cdots}$$

$$\Big\downarrow \text{7143}$$

$$\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b\cdots}}\cdots\right.}{bd}\cdots\right.}{\cdots}\cdots\right)}{b^2}\cdots\right)}{\cdots}$$

3.339. $\displaystyle\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((e + f*x)^3/(6*f) - (f*(e + f*x)*Cosh[c + d*x]^2)/(2*d^2) + ((e + f*x)^2*
Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (f^2*(x/2 + (Cosh[c + d*x]*Sinh[c + d
*x])/(2*d)))/(2*d^2))/b - (a*(-1/3*(a*(e + f*x)^3)/(b^2*f) - (2*(a^2 + b^2
)*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/
(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/b^2 - (I*((I*(e + f*x)^2*C
osh[c + d*x])/d - ((2*I)*f*(-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c +
 d*x])/d))/d))/b))/b
```

3.339.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.339. $\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.339. $\int \frac{(e+fx)^2 \cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.339.4 Maple [F]

$$\int \frac{(fx+e)^2 \cosh(dx+c)^2 \sinh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.339.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2410 vs. $2(466) = 932$.

Time = 0.31 (sec) , antiderivative size = 2410, normalized size of antiderivative = 4.73

$$\int \frac{(e+fx)^2 \cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
-1/48*(6*b^2*d^2*f^2*x^2 + 6*b^2*d^2*e^2 + 6*b^2*d*e*f - 3*(2*b^2*d^2*f^2*
x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2
)*x)*cosh(d*x + c)^4 - 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f
+ b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*sinh(d*x + c)^4 + 3*b^2*f^2 +
 24*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*
e*f - a*b*d*f^2)*x)*cosh(d*x + c)^3 + 12*(2*a*b*d^2*f^2*x^2 + 2*a*b*d^2*e^
2 - 4*a*b*d*e*f + 4*a*b*f^2 + 4*(a*b*d^2*e*f - a*b*d*f^2)*x - (2*b^2*d^2*f
^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*
f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*((2*a^2 + b^2)*d^3*f^2*x^3 + 3*
(2*a^2 + b^2)*d^3*e*f*x^2 + 3*(2*a^2 + b^2)*d^3*e^2*x)*cosh(d*x + c)^2 - 2
*(4*(2*a^2 + b^2)*d^3*f^2*x^3 + 12*(2*a^2 + b^2)*d^3*e*f*x^2 + 12*(2*a^2 +
 b^2)*d^3*e^2*x + 9*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2
*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*cosh(d*x + c)^2 - 36*(a*b*d^2*f^2*
x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*
x)*cosh(d*x + c))*sinh(d*x + c)^2 + 96*((a*b*d*f^2*x + a*b*d*e*f)*cosh(d*x
 + c)^2 + 2*(a*b*d*f^2*x + a*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d
*f^2*x + a*b*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d
*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) - 96*((a*b*d*f^2*x + a*b*d*e*f)*cosh(d*x + c)^2 +
2*(a*b*d*f^2*x + a*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d*f^2*x ...
```

3.339.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.339.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^2\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/8*e^2*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) + 8*sqrt(a^2 + b^
2)*a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/(b^3*d) - 4*(2*a^2 + b^2)*(d*x + c)/(b^3*d) + (4*a*e^(-d*x -
 c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/48*(8*(2*a^2*d^3*f^2*e^(2*c) + b^2*
d^3*f^2*e^(2*c))*x^3 + 24*(2*a^2*d^3*e*f*e^(2*c) + b^2*d^3*e*f*e^(2*c))*x^
2 + 3*(2*b^2*d^2*f^2*x^2*e^(4*c) + 2*(2*d^2*e*f - d*f^2)*b^2*x*e^(4*c) - (
2*d*e*f - f^2)*b^2*e^(4*c))*e^(2*d*x) - 24*(a*b*d^2*f^2*x^2*e^(3*c) + 2*(d
^2*e*f - d*f^2)*a*b*x*e^(3*c) - 2*(d*e*f - f^2)*a*b*e^(3*c))*e^(d*x) - 24*
(a*b*d^2*f^2*x^2*e^c + 2*(d^2*e*f + d*f^2)*a*b*x*e^c + 2*(d*e*f + f^2)*a*b
*e^c)*e^(-d*x) - 3*(2*b^2*d^2*f^2*x^2 + 2*(2*d^2*e*f + d*f^2)*b^2*x + (2*d
*e*f + f^2)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^3) - integrate(2*((a^3*f^2*e^
c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e*f*e^c)*x)*e^(d*x)/(b^4*e
^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

3.339.8 Giac [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^2\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a),
 x)
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\sinh(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.340 $\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.340.1 Optimal result

Integrand size = 32, antiderivative size = 327

$$\begin{aligned}\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{a^2ex}{b^3}+\frac{ex}{2b}+\frac{a^2fx^2}{2b^3}+\frac{fx^2}{4b} \\ &\quad-\frac{a(e+fx)\cosh(c+dx)}{b^2d}-\frac{f\cosh^2(c+dx)}{4bd^2} \\ &\quad-\frac{a\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d} \\ &\quad+\frac{a\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d} \\ &\quad-\frac{a\sqrt{a^2+b^2}f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2} \\ &\quad+\frac{a\sqrt{a^2+b^2}f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2} \\ &\quad+\frac{af\sinh(c+dx)}{b^2d^2} \\ &\quad+\frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd}\end{aligned}$$

output
```
a^2*e*x/b^3+1/2*e*x/b+1/2*a^2*f*x^2/b^3+1/4*f*x^2/b-a*(f*x+e)*cosh(d*x+c)/
b^2/d-1/4*f*cosh(d*x+c)^2/b/d^2+a*f*sinh(d*x+c)/b^2/d^2+1/2*(f*x+e)*cosh(d
*x+c)*sinh(d*x+c)/b/d-a*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^
2+b^2)^(1/2)/b^3/d+a*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b
^2)^(1/2)/b^3/d-a*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)
^(1/2)/b^3/d^2+a*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^
(1/2)/b^3/d^2
```

3.340.2 Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.78

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{2b^2e\left(\frac{c}{d}+x-\frac{2a\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}d}\right)+b^2f\left(x^2-\frac{2a\left(dx\left(\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)+\mathrm{PolyLog}\left(2,\frac{\ }{-a}\right.\right.}{\sqrt{a^2+b^2}d^2}\right.}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
(2*b^2*e*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]
])/(Sqrt[-a^2 - b^2]*d)) + b^2*f*(x^2 - (2*a*(d*x*(Log[1 + (b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2])] - Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])
+ PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - PolyLog[2, -((b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^2)) + (2*e*((4*a^2
+ b^2)*(c + d*x) - (2*a*(4*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/S
qrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c +
 d*x)]))/d + (f*((4*a^2 + b^2)*(-c + d*x)*(c + d*x) - 8*a*b*d*x*Cosh[c + d
*x] - b^2*Cosh[2*(c + d*x)] - (2*a*(4*a^2 + 3*b^2)*(2*c*ArcTanh[(a + b*E^(
c + d*x))/Sqrt[a^2 + b^2]] + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2])] - (c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + P
olyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - PolyLog[2, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] + 8*a*b*Sinh[c + d*x] + 2
*b^2*d*x*Sinh[2*(c + d*x)]))/d^2)/(8*b^3)
```

3.340.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6113, 3042, 3791, 17, 6099, 17, 3042, 26, 3777, 3042, 3117, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)\cosh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{\frac{1}{2}\int (e+fx)dx-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6099}$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)dx}{b^2}+\frac{\int (e+fx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)\sinh(c+dx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int -i(e+fx)\sin(ic+idx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int(e+fx)\sin(ic+idx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}$$

$$\Big\downarrow \; 3117$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3803$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{2(a^2+b^2)\int-\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

3.340. $\displaystyle\int\frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 2694$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b^2}-a(\right.}{b}$$

$$\Big\downarrow \; 2715$$

3.340. $\displaystyle\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-$$

$$\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}}{a-\sqrt{a^2+\dots}}\right.}{bd^2}\right.}{2\sqrt{a^2+b^2}}}{b^2}\right.}{b}$$

$$\Big\downarrow \; 2838$$

$$\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-$$

$$\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b^2}-\dots\right.}{b}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((e + f*x)^2/(4*f) - (f*Cosh[c + d*x]^2)/(4*d^2) + ((e + f*x)*Cosh[c + d*x
]*Sinh[c + d*x])/(2*d))/b - (a*(-1/2*(a*(e + f*x)^2)/(b^2*f) - (2*(a^2 + b
^2)*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/
Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b
*d^2))/(2*Sqrt[a^2 + b^2])))/b^2 - (I*((I*(e + f*x)*Cosh[c + d*x])/d - (I
*f*Sinh[c + d*x])/d^2))/b))/b
```

3.340. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.340.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.340. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.340. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.340.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1011 vs. $2(297) = 594$.

Time = 6.05 (sec) , antiderivative size = 1012, normalized size of antiderivative = 3.09

method	result	size
risch	Expression too large to display	1012

input
```
int((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
1/2*a^2*f*x^2/b^3+1/4*f*x^2/b+a^2*e*x/b^3+1/2*e*x/b+1/16*(2*d*f*x+2*d*e-f)
/b/d^2*exp(2*d*x+2*c)-1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp(d*x+c)-1/2*a*(d*f*x+
d*e+f)/b^2/d^2*exp(-d*x-c)-1/16*(2*d*f*x+2*d*e+f)/b/d^2*exp(-2*d*x-2*c)+2/
d*a^3/b^3*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/
2))+2/d*a/b*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))-1/d*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(
-a+(a^2+b^2)^(1/2)))*x+1/d*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2
+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((
-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2*a^3/b^3*f/(
a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-
1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-
a+(a^2+b^2)^(1/2)))+1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d*a/b*f/(a^2+b^2)^(1/2)*ln((-b*exp
(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d*a/b*f/(a^2+b^2)^(1/
2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a/b*f/
(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
*c+1/d^2*a/b*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))*c-1/d^2*a/b*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)
^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2*a/b*f/(a^2+b^2)^(1/2)*dilog((b*exp(d
*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/d^2*a^3/b^3*f*c/(a^2+b^...
```

3.340.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1284 vs. $2(295) = 590$.

Time = 0.28 (sec) , antiderivative size = 1284, normalized size of antiderivative = 3.93

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-1/16*(2*b^2*d*f*x - (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^4 - (
2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*sinh(d*x + c)^4 + 2*b^2*d*e + 8*(a*b*d*f*
x + a*b*d*e - a*b*f)*cosh(d*x + c)^3 + 4*(2*a*b*d*f*x + 2*a*b*d*e - 2*a*b*
f - (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c))*sinh(d*x + c)^3 + b^2
*f - 4*((2*a^2 + b^2)*d^2*f*x^2 + 2*(2*a^2 + b^2)*d^2*e*x)*cosh(d*x + c)^2
 - 2*(2*(2*a^2 + b^2)*d^2*f*x^2 + 4*(2*a^2 + b^2)*d^2*e*x + 3*(2*b^2*d*f*x
 + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^2 - 12*(a*b*d*f*x + a*b*d*e - a*b*f)*c
osh(d*x + c))*sinh(d*x + c)^2 + 16*(a*b*f*cosh(d*x + c)^2 + 2*a*b*f*cosh(d
*x + c)*sinh(d*x + c) + a*b*f*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog
((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 16*(a*b*f*cosh(d*x + c)^2 + 2*a*b*f*co
sh(d*x + c)*sinh(d*x + c) + a*b*f*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*d
ilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 16*((a*b*d*e - a*b*c*f)*cosh(d*x +
 c)^2 + 2*(a*b*d*e - a*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d*e - a*b
*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*s
inh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*((a*b*d*e - a*b*c*f)*
cosh(d*x + c)^2 + 2*(a*b*d*e - a*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b
*d*e - a*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*((a*b*d*...
```

3.340.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.340.7 Maxima [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^2\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/16*(32*(a^3*e^c + a*b^2*e^c)*integrate(x*e^(d*x)/(b^4*e^(2*d*x + 2*c) +
 2*a*b^3*e^(d*x + c) - b^4), x) - (4*(2*a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))
*x^2 + (2*b^2*d*x*e^(4*c) - b^2*e^(4*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) -
a*b*e^(3*c))*e^(d*x) - 8*(a*b*d*x*e^c + a*b*e^c)*e^(-d*x) - (2*b^2*d*x + b
^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^2))*f - 1/8*e*((4*a*e^(-d*x - c) - b)*e^(2
*d*x + 2*c)/(b^2*d) + 8*sqrt(a^2 + b^2)*a*log((b*e^(-d*x - c) - a - sqrt(a
^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^3*d) - 4*(2*a^2 + b^
2)*(d*x + c)/(b^3*d) + (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d))
```

3.340.8 Giac [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^2\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a), x
)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\sinh(c+d\,x)\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.340. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.341 $\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.341.1 Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{(2a^2+b^2)\,x}{2b^3} + \frac{2a\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3d} - \frac{\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}$$

output
```
1/2*(2*a^2+b^2)*x/b^3-1/2*cosh(d*x+c)*(2*a-b*sinh(d*x+c))/b^2/d+2*a*arctan
h((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^3/d
```

3.341.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.15

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{4a^2c+2b^2c+4a^2dx+2b^2dx+8a\sqrt{-a^2-b^2}\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)-4ab\cosh(c+dx)+b^2\sinh(2(c}{4b^3d}$$

input
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(4*a^2*c + 2*b^2*c + 4*a^2*d*x + 2*b^2*d*x + 8*a*Sqrt[-a^2 - b^2]*ArcTan[(
b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] - 4*a*b*Cosh[c + d*x] + b^2*Sin
h[2*(c + d*x)])/(4*b^3*d)
```

3.341.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {3042, 26, 3344, 26, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)\cos(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ic+idx)^2\sin(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3344}$$

$$-i\left(-\frac{\int \frac{i(ab-(2a^2+b^2)\sinh(c+dx))}{a+b\sinh(c+dx)}dx}{2b^2}-\frac{i\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{i\int \frac{ab-(2a^2+b^2)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{2b^2}-\frac{i\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{i\int \frac{ab+i(2a^2+b^2)\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{2b^2}-\frac{i\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}\right)$$

$$\Big\downarrow \text{3214}$$

$$-i\left(-\frac{i\left(\frac{2a(a^2+b^2)\int \frac{1}{a+b\sinh(c+dx)}dx}{b}-\frac{x(2a^2+b^2)}{b}\right)}{2b^2}-\frac{i\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{i\left(-\frac{x(2a^2+b^2)}{b}+\frac{2a(a^2+b^2)\int\frac{1}{a-ib\sin(ic+idx)}dx}{b}\right)}{2b^2}-\frac{i\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}\right)$$

$$\Big\downarrow 3139$$

$$-i\left(-\frac{i\left(-\frac{x(2a^2+b^2)}{b}-\frac{4ia(a^2+b^2)\int\frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\right)}{2b^2}-\frac{i\cosh(c+dx)(2a-b\sinh(}{2b^2d}\right.$$

$$\Big\downarrow 1083$$

$$-i\left(-\frac{i\left(-\frac{x(2a^2+b^2)}{b}+\frac{8ia(a^2+b^2)\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{bd}\right)}{2b^2}-\frac{i\cosh(c+dx)(2a-b\sinh(c+}{2b^2d}\right.$$

$$\Big\downarrow 217$$

$$-i\left(-\frac{i\left(\frac{4a\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{x(2a^2+b^2)}{b}\right)}{2b^2}-\frac{i\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}\right)$$

input
```
Int[(Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-I)*(((-1/2*I)*(-(((2*a^2 + b^2)*x)/b) + (4*a*Sqrt[a^2 + b^2]*ArcTanh[Tan
h[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(b*d)))/b^2 - ((I/2)*Cosh[c + d*x]*(2
*a - b*Sinh[c + d*x]))/(b^2*d))
```

3.341. $\int\frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.341.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

3.341. $\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.341.4 Maple [A] (verified)

Time = 3.82 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.76

method	result
risch	$\frac{x\,a^2}{b^3}+\frac{x}{2b}+\frac{\mathrm{e}^{2dx+2c}}{8bd}-\frac{a\,\mathrm{e}^{dx+c}}{2b^2d}-\frac{a\,\mathrm{e}^{-dx-c}}{2b^2d}-\frac{\mathrm{e}^{-2dx-2c}}{8bd}+\frac{\sqrt{a^2+b^2}\,a\ln\left(\mathrm{e}^{dx+c}+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{d\,b^3}-\frac{\sqrt{a^2+b^2}\,a\ln}{}$
derivativedivides	$\frac{-\frac{2a\sqrt{a^2+b^2}\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^3}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-b+2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(2a^2+b^2)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^3}}{d}$
default	$\frac{-\frac{2a\sqrt{a^2+b^2}\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^3}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-b+2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(2a^2+b^2)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^3}}{d}$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
x/b^3*a^2+1/2*x/b+1/8/b/d*exp(2*d*x+2*c)-1/2*a/b^2/d*exp(d*x+c)-1/2*a/b^2/
d*exp(-d*x-c)-1/8/b/d*exp(-2*d*x-2*c)+(a^2+b^2)^(1/2)*a/d/b^3*ln(exp(d*x+c
)+(a+(a^2+b^2)^(1/2))/b)-(a^2+b^2)^(1/2)*a/d/b^3*ln(exp(d*x+c)-(-a+(a^2+b^
2)^(1/2))/b)
```

3.341.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. 2(87) = 174.

Time = 0.27 (sec) , antiderivative size = 446, normalized size of antiderivative = 4.69

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{b^2\cosh(dx+c)^4+b^2\sinh(dx+c)^4+4\,(2\,a^2+b^2)dx\cosh(dx+c)^2-4\,ab\cosh(dx+c)^3+4\,(b^2\cosh(dx}{}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/8*(b^2*cosh(d*x + c)^4 + b^2*sinh(d*x + c)^4 + 4*(2*a^2 + b^2)*d*x*cosh(
d*x + c)^2 - 4*a*b*cosh(d*x + c)^3 + 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x
+ c)^3 - 4*a*b*cosh(d*x + c) + 2*(3*b^2*cosh(d*x + c)^2 + 2*(2*a^2 + b^2)*
d*x - 6*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^2 + 2*a*co
sh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2)*sqrt(a^2 + b^2)*log((b^2*co
sh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 +
2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x
+ c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*
cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - b^2 + 4*(b^2
*cosh(d*x + c)^3 + 2*(2*a^2 + b^2)*d*x*cosh(d*x + c) - 3*a*b*cosh(d*x + c)
^2 - a*b)*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 + 2*b^3*d*cosh(d*x + c)*si
nh(d*x + c) + b^3*d*sinh(d*x + c)^2)
```

3.341.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.341.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.68

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^2d} - \frac{\sqrt{a^2+b^2}a\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{b^3d} + \frac{(2\,a^2+b^2)(dx+c)}{2\,b^3d} - \frac{4\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}}{8\,b^2d}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - sqrt(a^2 + b^2)*a*lo
g((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 +
b^2)))/(b^3*d) + 1/2*(2*a^2 + b^2)*(d*x + c)/(b^3*d) - 1/8*(4*a*e^(-d*x -
c) + b*e^(-2*d*x - 2*c))/(b^2*d)
```

3.341.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.63

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{4\,(2\,a^2+b^2)(dx+c)}{b^3}+\frac{be^{(2\,dx+2\,c)}-4\,ae^{(dx+c)}}{b^2}-\frac{(4\,abe^{(dx+c)}+b^2)e^{(-2\,dx-2\,c)}}{b^3}-\frac{8\,(a^3+ab^2)\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^3}}{8\,d}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/8*(4*(2*a^2 + b^2)*(d*x + c)/b^3 + (b*e^(2*d*x + 2*c) - 4*a*e^(d*x + c))
/b^2 - (4*a*b*e^(d*x + c) + b^2)*e^(-2*d*x - 2*c)/b^3 - 8*(a^3 + a*b^2)*lo
g(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) + 2*a
 + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3))/d
```

3.341.9 Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.23

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{2\,c+2\,d\,x}}{8\,b\,d}-\frac{e^{-2\,c-2\,d\,x}}{8\,b\,d}+\frac{x\,(2\,a^2+b^2)}{2\,b^3}-\frac{a\,e^{-c-d\,x}}{2\,b^2\,d}-\frac{a\,e^{c+d\,x}}{2\,b^2\,d} - \frac{a\,\ln\left(\frac{2\,a\,e^{c+d\,x}\,(a^2+b^2)}{b^4}-\frac{2\,a\,\sqrt{a^2+b^2}\,(b-a\,e^{c+d\,x})}{b^4}\right)\,\sqrt{a^2+b^2}}{b^3\,d} + \frac{a\,\ln\left(\frac{2\,a\,\sqrt{a^2+b^2}\,(b-a\,e^{c+d\,x})}{b^4}+\frac{2\,a\,e^{c+d\,x}\,(a^2+b^2)}{b^4}\right)\,\sqrt{a^2+b^2}}{b^3\,d}$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

output
```
exp(2*c + 2*d*x)/(8*b*d) - exp(- 2*c - 2*d*x)/(8*b*d) + (x*(2*a^2 + b^2))/
(2*b^3) - (a*exp(- c - d*x))/(2*b^2*d) - (a*exp(c + d*x))/(2*b^2*d) - (a*l
og((2*a*exp(c + d*x)*(a^2 + b^2))/b^4 - (2*a*(a^2 + b^2)^(1/2)*(b - a*exp(
c + d*x)))/b^4)*(a^2 + b^2)^(1/2))/(b^3*d) + (a*log((2*a*(a^2 + b^2)^(1/2)
*(b - a*exp(c + d*x)))/b^4 + (2*a*exp(c + d*x)*(a^2 + b^2))/b^4)*(a^2 + b^
2)^(1/2))/(b^3*d)
```

3.341. $\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.342 $\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.342.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \mathrm{Int}\left(\frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.342.2 Mathematica [N/A]

Not integrable

Time = 12.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

3.342.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

$$\Big\downarrow \text{6125}$$

$$\int \frac{\sinh(c+dx)\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]^2*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.342.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.342.4 Maple [N/A] (verified)

Not integrable

Time = 0.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(dx+c)^2 \sinh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.342.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^2\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(cosh(d*x + c)^2*sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

3.342.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.342.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 243, normalized size of antiderivative = 7.15

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^2\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-2*(a^3*e^c + a*b^2*e^c)*integrate(-e^(d*x)/(b^4*f*x + b^4*e - (b^4*f*x*e^
(2*c) + b^4*e*e^(2*c))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*b^3*e*e^c)*e^(d*x)
), x) - 1/4*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) -
1/2*a*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c
 - d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f
)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b*f) + 1/2*(2*a^2 + b^2)*log(f*x +
e)/(b^3*f)
```

3.342.8 Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate(cosh(d*x + c)^2*sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)),
 x)
```

3.342.9 Mupad [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^2\sinh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.343 $\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$

3.343.1 Optimal result

Integrand size = 34, antiderivative size = 864

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\begin{aligned}
&= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a(a^2+b^2)(e+fx)^4}{4b^4f} - \frac{6a^2f^3\cosh(c+dx)}{b^3d^4} \\
&- \frac{40f^3\cosh(c+dx)}{9bd^4} - \frac{3a^2f(e+fx)^2\cosh(c+dx)}{b^3d^2} \\
&- \frac{2f(e+fx)^2\cosh(c+dx)}{bd^2} - \frac{2f^3\cosh^3(c+dx)}{27bd^4} - \frac{f(e+fx)^2\cosh^3(c+dx)}{3bd^2} \\
&- \frac{a(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d} - \frac{a(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d} \\
&- \frac{3a(a^2+b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2} \\
&- \frac{3a(a^2+b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2} \\
&+ \frac{6a(a^2+b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3} \\
&+ \frac{6a(a^2+b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^3} \\
&- \frac{6a(a^2+b^2)f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^4} - \frac{6a(a^2+b^2)f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^4} \\
&+ \frac{6a^2f^2(e+fx)\sinh(c+dx)}{b^3d^3} + \frac{40f^2(e+fx)\sinh(c+dx)}{9bd^3} \\
&+ \frac{a^2(e+fx)^3\sinh(c+dx)}{b^3d} + \frac{2(e+fx)^3\sinh(c+dx)}{3bd} \\
&+ \frac{3af^3\cosh(c+dx)\sinh(c+dx)}{8b^2d^4} + \frac{3af(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{4b^2d^2} \\
&+ \frac{2f^2(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{9bd^3} + \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{3bd} \\
&- \frac{3af^2(e+fx)\sinh^2(c+dx)}{4b^2d^3} - \frac{a(e+fx)^3\sinh^2(c+dx)}{2b^2d}
\end{aligned}$$

output
```
-3*a^2*f*(f*x+e)^2*cosh(d*x+c)/b^3/d^2-2*f*(f*x+e)^2*cosh(d*x+c)/b/d^2+40/
9*f^2*(f*x+e)*sinh(d*x+c)/b/d^3-3/8*a*f^3*x/b^2/d^3+1/4*a*(a^2+b^2)*(f*x+e
)^4/b^4/f-6*a^2*f^3*cosh(d*x+c)/b^3/d^4-1/3*f*(f*x+e)^2*cosh(d*x+c)^3/b/d^
2+1/3*(f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/b/d-1/2*a*(f*x+e)^3*sinh(d*x+c)^
2/b^2/d+2/3*(f*x+e)^3*sinh(d*x+c)/b/d+a^2*(f*x+e)^3*sinh(d*x+c)/b^3/d-a*(a
^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d-a*(a^2+b^2)
*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d-1/4*a*(f*x+e)^3/b^
2/d-2/27*f^3*cosh(d*x+c)^3/b/d^4+3/4*a*f*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)
/b^2/d^2-3*a*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/b^4/d^2-3*a*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/b^4/d^2+6*a*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/b^4/d^3+6*a*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/
(a+(a^2+b^2)^(1/2)))/b^4/d^3-40/9*f^3*cosh(d*x+c)/b/d^4+6*a^2*f^2*(f*x+e)*
sinh(d*x+c)/b^3/d^3+3/8*a*f^3*cosh(d*x+c)*sinh(d*x+c)/b^2/d^4+2/9*f^2*(f*x
+e)*cosh(d*x+c)^2*sinh(d*x+c)/b/d^3-3/4*a*f^2*(f*x+e)*sinh(d*x+c)^2/b^2/d^
3-6*a*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^4-6
*a*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^4
```

3.343.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5656 vs. $2(864) = 1728$.

Time = 32.98 (sec) , antiderivative size = 5656, normalized size of antiderivative = 6.55

$$\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
Result too large to show
```

3.343.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \sinh(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)^3\cosh^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}$$

$$\Big\downarrow \text{ 3792}$$

$$\frac{\frac{2f^2\int (e+fx)\cosh^3(c+dx)dx}{3d^2}+\frac{2}{3}\int (e+fx)^3\cosh(c+dx)dx-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\sinh(c+dx)dx}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{(e+fx)^3\sinh(c}{}}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int (e+fx)^2\sinh(c+dx)dx}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{(e+fx)^3\sinh(c+dx}{3d}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\sin(ic+idx)dx}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{(e+fx)^3\sinh(c+}{}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\int (e+fx)^2\sin(ic+idx)dx}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{(e+fx)^3\sinh(c+}{3a}}{b}$$

$$\Big\downarrow \; 3777$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+}{3d^2}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)-\frac{f(e+fx)^2\cosh}{3d^2}}{b}$$

$$\Big\downarrow \; 3777$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)-}{b}$$

$$\Big\downarrow \; 26$$

3.343. $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)-f\cdots}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}\right)-\cdots}{b}$$

$$\Big\downarrow \textbf{26}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{d}\right)-\cdots}{b}$$

$$\Big\downarrow \textbf{3118}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\cdots\right)}{d}\cdots\right)}{d}\cdots\right)}{b}$$

$$\Big\downarrow \textbf{3791}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\left(\frac{2}{3}\int (e+fx)\cosh(c+dx)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\cdots\right)}{\cdots}\cdots\right)}{b}$$

3.343. $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\ldots\right.}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh\ldots}{d}\right.}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c\ldots}{d}\right.}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh\ldots}{d}\right.}{b}$$

$$\Big\downarrow \text{26}$$

3.343. $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c}{d}\right.}{b}$$

$$\downarrow \text{3118}$$

$$-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx}{d}\right.}{b}$$

$$\downarrow \text{6099}$$

$$-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^3\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}+$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx}{d}\right.}{b}$$

$$\downarrow \text{3042}$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx}{d}\right.}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\downarrow \text{3777}$$

3.343. $\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx}{d}\right.}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3if\int-i(e+fx)^2\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int(e+fx)^2\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}+$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx}{d}\right.}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx}{d}\right.}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int-i(e+fx)^2\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx}{d}\right.}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\int(e+fx)^2\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

3.343. $\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \color{blue}{3777}$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}\right.}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)}{b}\right.}{b}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{\frac{2f^2\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}\right.}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c-}{}\right.}{b}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.343. $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.343.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

3.343. $\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh(c+dx)}{a+b\sinh(c+dx)} \, dx$

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.343.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)^3 \sinh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.343.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7980 vs. $2(810) = 1620$.

Time = 0.36 (sec) , antiderivative size = 7980, normalized size of antiderivative = 9.24

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.343.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.343.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)^3\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/24*e^3*((3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))
*e^(3*d*x + 3*c)/(b^3*d) + 24*(a^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(
-2*d*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^
3*d) + 24*(a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b
^4*d)) - 1/864*(216*(a^3*d^4*f^3*e^(3*c) + a*b^2*d^4*f^3*e^(3*c))*x^4 + 86
4*(a^3*d^4*e*f^2*e^(3*c) + a*b^2*d^4*e*f^2*e^(3*c))*x^3 + 1296*(a^3*d^4*e^
2*f*e^(3*c) + a*b^2*d^4*e^2*f*e^(3*c))*x^2 - 4*(9*b^3*d^3*f^3*x^3*e^(6*c)
+ 9*(3*d^3*e*f^2 - d^2*f^3)*b^3*x^2*e^(6*c) + 3*(9*d^3*e^2*f - 6*d^2*e*f^2
 + 2*d*f^3)*b^3*x*e^(6*c) - (9*d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*b^3*e^(6*c))
*e^(3*d*x) + 27*(4*a*b^2*d^3*f^3*x^3*e^(5*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*a
*b^2*x^2*e^(5*c) + 6*(2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*a*b^2*x*e^(5*c) -
 3*(2*d^2*e^2*f - 2*d*e*f^2 + f^3)*a*b^2*e^(5*c))*e^(2*d*x) + 108*(12*(d^2
*e^2*f - 2*d*e*f^2 + 2*f^3)*a^2*b*e^(4*c) + 9*(d^2*e^2*f - 2*d*e*f^2 + 2*f
^3)*b^3*e^(4*c) - (4*a^2*b*d^3*f^3*e^(4*c) + 3*b^3*d^3*f^3*e^(4*c))*x^3 -
3*(4*(d^3*e*f^2 - d^2*f^3)*a^2*b*e^(4*c) + 3*(d^3*e*f^2 - d^2*f^3)*b^3*e^(
4*c))*x^2 - 3*(4*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a^2*b*e^(4*c) + 3*(d^
3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b^3*e^(4*c))*x)*e^(d*x) + 108*(12*(d^2*e^
2*f + 2*d*e*f^2 + 2*f^3)*a^2*b*e^(2*c) + 9*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)
*b^3*e^(2*c) + (4*a^2*b*d^3*f^3*e^(2*c) + 3*b^3*d^3*f^3*e^(2*c))*x^3 + 3*(
4*(d^3*e*f^2 + d^2*f^3)*a^2*b*e^(2*c) + 3*(d^3*e*f^2 + d^2*f^3)*b^3*e^(...
```

3.343.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh{(dx+c)}^3\sinh{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a),
 x)
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.344 $\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.344.1 Optimal result

Integrand size = 34, antiderivative size = 636

$$\begin{aligned}
&\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{aefx}{2b^2d}-\frac{af^2x^2}{4b^2d}+\frac{a(a^2+b^2)(e+fx)^3}{3b^4f}-\frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2} \\
&\quad -\frac{4f(e+fx)\cosh(c+dx)}{3bd^2}-\frac{2f(e+fx)\cosh^3(c+dx)}{9bd^2} \\
&\quad -\frac{a(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{a(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d} \\
&\quad -\frac{2a(a^2+b^2)f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2} \\
&\quad -\frac{2a(a^2+b^2)f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2} \\
&\quad +\frac{2a(a^2+b^2)f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3}+\frac{2a(a^2+b^2)f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^3} \\
&\quad +\frac{2a^2f^2\sinh(c+dx)}{b^3d^3}+\frac{14f^2\sinh(c+dx)}{9bd^3}+\frac{a^2(e+fx)^2\sinh(c+dx)}{b^3d} \\
&\quad +\frac{2(e+fx)^2\sinh(c+dx)}{3bd}+\frac{af(e+fx)\cosh(c+dx)\sinh(c+dx)}{2b^2d^2} \\
&\quad +\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{3bd}-\frac{af^2\sinh^2(c+dx)}{4b^2d^3} \\
&\quad -\frac{a(e+fx)^2\sinh^2(c+dx)}{2b^2d}+\frac{2f^2\sinh^3(c+dx)}{27bd^3}
\end{aligned}$$

output
```
-1/2*a*e*f*x/b^2/d-1/4*a*f^2*x^2/b^2/d+1/3*a*(a^2+b^2)*(f*x+e)^3/b^4/f-2*a
^2*f*(f*x+e)*cosh(d*x+c)/b^3/d^2-4/3*f*(f*x+e)*cosh(d*x+c)/b/d^2-2/9*f*(f*
x+e)*cosh(d*x+c)^3/b/d^2-a*(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/b^4/d-a*(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))/b^4/d-2*a*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/b^4/d^2-2*a*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/b^4/d^2+2*a*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/b^4/d^3+2*a*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)
))/b^4/d^3+2*a^2*f^2*sinh(d*x+c)/b^3/d^3+14/9*f^2*sinh(d*x+c)/b/d^3+a^2*(f
*x+e)^2*sinh(d*x+c)/b^3/d+2/3*(f*x+e)^2*sinh(d*x+c)/b/d+1/2*a*f*(f*x+e)*co
sh(d*x+c)*sinh(d*x+c)/b^2/d^2+1/3*(f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/b/d-
1/4*a*f^2*sinh(d*x+c)^2/b^2/d^3-1/2*a*(f*x+e)^2*sinh(d*x+c)^2/b^2/d+2/27*f
^2*sinh(d*x+c)^3/b/d^3
```

3.344.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1961 vs. $2(636) = 1272$.

Time = 11.52 (sec) , antiderivative size = 1961, normalized size of antiderivative = 3.08

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(f^2*((4*a*x^3)/(-1 + E^(2*c)) - 2*a*x^3*Coth[c] - (6*a*b^2*(d^2*x^2*Log[1
 + ((a - Sqrt[a^2 + b^2])*E^(-c - d*x))/b] - 2*d*x*PolyLog[2, ((-a + Sqrt[
a^2 + b^2])*E^(-c - d*x))/b] - 2*PolyLog[3, ((-a + Sqrt[a^2 + b^2])*E^(-c
- d*x))/b]))/(Sqrt[a^2 + b^2]*(-a + Sqrt[a^2 + b^2])*d^3) - (6*a*b^2*(d^2*
x^2*Log[1 + ((a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b] - 2*d*x*PolyLog[2, -((
(a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b)] - 2*PolyLog[3, -(((a + Sqrt[a^2 +
b^2])*E^(-c - d*x))/b)]))/(Sqrt[a^2 + b^2]*(a + Sqrt[a^2 + b^2])*d^3) + (6
*a^2*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d*x*PolyL
og[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*PolyLog[3, (b*E^(c + d*x
))/(-a + Sqrt[a^2 + b^2])]))/(Sqrt[a^2 + b^2]*d^3) - (6*a^2*(d^2*x^2*Log[1
 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^3) + (6*b*Cosh[d*x]*(-2*d*x*Cosh[c] + (2
+ d^2*x^2)*Sinh[c]))/d^3 + (6*b*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c])*Si
nh[d*x])/d^3))/(12*b^2) - (e^2*((a*Log[a + b*Sinh[c + d*x]])/b^2 - Sinh[c
+ d*x]/b))/(2*d) + (e*f*(-2*b*Cosh[c + d*x] - a*(2*c*(c + d*x) - (c + d*x)
^2 + 2*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*(c + d
*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*Log[b - 2*a*E^(c
+ d*x) - b*E^(2*(c + d*x))] + 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2
+ b^2])] + 2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + 2*...
```

3.344.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \ 6113$$

$$\frac{\int (e+fx)^2\cosh^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}$$

$$\Big\downarrow \ 3792$$

$$\frac{\frac{2f^2\int\cosh^3(c+dx)dx}{9d^2}+\frac{2}{3}\int(e+fx)^2\cosh(c+dx)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{9d^2}+\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if^2\int\left(\sinh^2(c+dx)+1\right)d(-i\sinh(c+dx))}{9d^3}+\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int-i(e+fx)\sinh(c+dx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

3.344. $\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow 26$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh}{}}{b}$$

$$\Big\downarrow 3777$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}}{b}$$

$$\Big\downarrow 3042$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c}{9d^2}}{b}$$

$$\Big\downarrow 3117$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\Big\downarrow 6099$$

$$-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^2\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}+\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\Big\downarrow 3042$$

3.344. $\displaystyle\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int-i(e+fx)\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}+$$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

3.344. $\displaystyle\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+}{b}\right.}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh}{b}\right.}{b}$$

$$\Big\downarrow \; 3117$$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right.}{b^2}\right.}{b}$$

$$\Big\downarrow \; 5969$$

3.344. $\displaystyle\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int(e+fx)\sinh^2(c+dx)dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(}{d^2}\right.}{d}\right.}{b^2}\right.}{b}$$

$$\downarrow \textbf{3042}$$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int-\left((e+fx)\sin(ic+idx)^2\right)dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if}{}\right.}{d}\right.}{b^2}\right.}{b}$$

$$\downarrow \textbf{25}$$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}+\frac{f\int(e+fx)\sin(ic+idx)^2dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh}{d}\right.}{d}\right.}{b^2}\right.}{b}$$

$$\downarrow \textbf{3791}$$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{f\left(\frac{1}{2}\int(e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c-}{d}\right.}{}\right.}{b}$$

$$\downarrow \textbf{17}$$

3.344. $\displaystyle\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)}{2d}\right.}{d}\right.}{b}$$

$$\Big\downarrow \text{6095}$$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}\right.}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$\frac{a\left(\frac{(a^2+b^2)\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\right.}{b^2}\right.}{}$$

$$\Big\downarrow \text{3011}$$

3.344. $\displaystyle\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+}{b}$$

$$a\left(\frac{(a^2+b^2)\left(-\frac{2f\left(\frac{f\int \mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{}{a}\right)}{d}\right)}{bd}\right.}{b^2}\right.$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.344.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.344. $\int \frac{(e+fx)^2 \cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.344.4 Maple [F]

$$\int \frac{(fx+e)^2 \cosh(dx+c)^3 \sinh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.344.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4887 vs. $2(592) = 1184$.

Time = 0.36 (sec) , antiderivative size = 4887, normalized size of antiderivative = 7.68

$$\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh(c+dx)}{a+b\sinh(c+dx)} \, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
-1/432*(18*b^3*d^2*f^2*x^2 + 18*b^3*d^2*e^2 - 2*(9*b^3*d^2*f^2*x^2 + 9*b^3
*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh
(d*x + c)^6 - 2*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f
^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*sinh(d*x + c)^6 + 12*b^3*d*e*f + 27*
(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*
a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c)^5 + 3*(18*a*b^2*d^2*f^2*x^2
+ 18*a*b^2*d^2*e^2 - 18*a*b^2*d*e*f + 9*a*b^2*f^2 + 18*(2*a*b^2*d^2*e*f -
a*b^2*d*f^2)*x - 4*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^
3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^5 +
4*b^3*f^2 - 54*((4*a^2*b + 3*b^3)*d^2*f^2*x^2 + (4*a^2*b + 3*b^3)*d^2*e^2
- 2*(4*a^2*b + 3*b^3)*d*e*f + 2*(4*a^2*b + 3*b^3)*f^2 + 2*((4*a^2*b + 3*b^
3)*d^2*e*f - (4*a^2*b + 3*b^3)*d*f^2)*x)*cosh(d*x + c)^4 - 3*(18*(4*a^2*b
+ 3*b^3)*d^2*f^2*x^2 + 18*(4*a^2*b + 3*b^3)*d^2*e^2 - 36*(4*a^2*b + 3*b^3)
*d*e*f + 36*(4*a^2*b + 3*b^3)*f^2 + 10*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2
- 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c)
^2 + 36*((4*a^2*b + 3*b^3)*d^2*e*f - (4*a^2*b + 3*b^3)*d*f^2)*x - 45*(2*a*
b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2
*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^4 - 144*((a^3 + a*
b^2)*d^3*f^2*x^3 + 3*(a^3 + a*b^2)*d^3*e*f*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*x
 + 6*(a^3 + a*b^2)*c*d^2*e^2 - 6*(a^3 + a*b^2)*c^2*d*e*f + 2*(a^3 + a*b...
```

3.344.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh(c+dx)}{a+b\sinh(c+dx)} \, dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.344.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh{(dx+c)}^3\sinh{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/24*e^2*((3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))
*e^(3*d*x + 3*c)/(b^3*d) + 24*(a^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(
-2*d*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^
3*d) + 24*(a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b
^4*d)) - 1/432*(144*(a^3*d^3*f^2*e^(3*c) + a*b^2*d^3*f^2*e^(3*c))*x^3 + 43
2*(a^3*d^3*e*f*e^(3*c) + a*b^2*d^3*e*f*e^(3*c))*x^2 - 2*(9*b^3*d^2*f^2*x^2
*e^(6*c) + 6*(3*d^2*e*f - d*f^2)*b^3*x*e^(6*c) - 2*(3*d*e*f - f^2)*b^3*e^(
6*c))*e^(3*d*x) + 27*(2*a*b^2*d^2*f^2*x^2*e^(5*c) + 2*(2*d^2*e*f - d*f^2)*
a*b^2*x*e^(5*c) - (2*d*e*f - f^2)*a*b^2*e^(5*c))*e^(2*d*x) + 54*(8*(d*e*f
- f^2)*a^2*b*e^(4*c) + 6*(d*e*f - f^2)*b^3*e^(4*c) - (4*a^2*b*d^2*f^2*e^(4
*c) + 3*b^3*d^2*f^2*e^(4*c))*x^2 - 2*(4*(d^2*e*f - d*f^2)*a^2*b*e^(4*c) +
3*(d^2*e*f - d*f^2)*b^3*e^(4*c))*x)*e^(d*x) + 54*(8*(d*e*f + f^2)*a^2*b*e^
(2*c) + 6*(d*e*f + f^2)*b^3*e^(2*c) + (4*a^2*b*d^2*f^2*e^(2*c) + 3*b^3*d^2
*f^2*e^(2*c))*x^2 + 2*(4*(d^2*e*f + d*f^2)*a^2*b*e^(2*c) + 3*(d^2*e*f + d*
f^2)*b^3*e^(2*c))*x)*e^(-d*x) + 27*(2*a*b^2*d^2*f^2*x^2*e^c + 2*(2*d^2*e*f
 + d*f^2)*a*b^2*x*e^c + (2*d*e*f + f^2)*a*b^2*e^c)*e^(-2*d*x) + 2*(9*b^3*d
^2*f^2*x^2 + 6*(3*d^2*e*f + d*f^2)*b^3*x + 2*(3*d*e*f + f^2)*b^3)*e^(-3*d*
x))*e^(-3*c)/(b^4*d^3) + integrate(-2*((a^3*b*f^2 + a*b^3*f^2)*x^2 + 2*(a^
3*b*e*f + a*b^3*e*f)*x - ((a^4*f^2*e^c + a^2*b^2*f^2*e^c)*x^2 + 2*(a^4*e*f
*e^c + a^2*b^2*e*f*e^c)*x)*e^(d*x))/(b^5*e^(2*d*x + 2*c) + 2*a*b^4*e^(d...
```

3.344.8 Giac [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh{(dx+c)}^3\sinh{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a),
 x)
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)\,{(e+f\,x)}^2}{a+b\sinh{(c+d\,x)}}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.345 $\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.345.1 Optimal result

Integrand size = 32, antiderivative size = 400

$$\begin{aligned}
&\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{afx}{4b^2d}+\frac{a(a^2+b^2)(e+fx)^2}{2b^4f}-\frac{a^2f\cosh(c+dx)}{b^3d^2}-\frac{2f\cosh(c+dx)}{3bd^2}-\frac{f\cosh^3(c+dx)}{9bd^2} \\
&\quad -\frac{a(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{a(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d} \\
&\quad -\frac{a(a^2+b^2)f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{a(a^2+b^2)f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2} \\
&\quad +\frac{a^2(e+fx)\sinh(c+dx)}{b^3d}+\frac{2(e+fx)\sinh(c+dx)}{3bd}+\frac{af\cosh(c+dx)\sinh(c+dx)}{4b^2d^2} \\
&\quad +\frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{3bd}-\frac{a(e+fx)\sinh^2(c+dx)}{2b^2d}
\end{aligned}$$

output
```
-1/4*a*f*x/b^2/d+1/2*a*(a^2+b^2)*(f*x+e)^2/b^4/f-a^2*f*cosh(d*x+c)/b^3/d^2
-2/3*f*cosh(d*x+c)/b/d^2-1/9*f*cosh(d*x+c)^3/b/d^2-a*(a^2+b^2)*(f*x+e)*ln(
1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d-a*(a^2+b^2)*(f*x+e)*ln(1+b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d-a*(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/b^4/d^2-a*(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/b^4/d^2+a^2*(f*x+e)*sinh(d*x+c)/b^3/d+2/3*(f*x+e)*sinh(d*x+c)/b
/d+1/4*a*f*cosh(d*x+c)*sinh(d*x+c)/b^2/d^2+1/3*(f*x+e)*cosh(d*x+c)^2*sinh(
d*x+c)/b/d-1/2*a*(f*x+e)*sinh(d*x+c)^2/b^2/d
```

3.345.2 Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.51

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{-36b^2de(-a\log(a+b\sinh(c+dx))+b\sinh(c+dx))+18b^2f\Big(2b\cosh(c+dx)+a\Big(2c(c+dx)-(c+$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
-1/72*(-36*b^2*d*e*(-(a*Log[a + b*Sinh[c + d*x]]) + b*Sinh[c + d*x]) + 18*
b^2*f*(2*b*Cosh[c + d*x] + a*(2*c*(c + d*x) - (c + d*x)^2 + 2*(c + d*x)*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*(c + d*x)*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c
+ d*x))] + 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*PolyLo
g[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 2*b*d*x*Sinh[c + d*x]) +
 12*d*e*(3*a*(2*a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - 3*b*(2*a^2 + b^2)*Si
nh[c + d*x] + 3*a*b^2*Sinh[c + d*x]^2 - 2*b^3*Sinh[c + d*x]^3) + f*(18*b*(
4*a^2 + b^2)*Cosh[c + d*x] + 18*a*b^2*d*x*Cosh[2*(c + d*x)] + 2*b^3*Cosh[3
*(c + d*x)] + 18*a*(2*a^2 + b^2)*(2*c*(c + d*x) - (c + d*x)^2 + 2*(c + d*x
)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*(c + d*x)*Log[1 + (b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*Log[b - 2*a*E^(c + d*x) - b*E^(2
*(c + d*x))] + 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*Po
lyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 18*b*(4*a^2 + b^2)*d
*x*Sinh[c + d*x] - 9*a*b^2*Sinh[2*(c + d*x)] - 6*b^3*d*x*Sinh[3*(c + d*x)]
))/(b^4*d^2)
```

3.345.3 Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.90, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.781$, Rules used = {6113, 3042, 3791, 3042, 3777, 26, 3042, 26, 3118, 6099, 3042, 3777, 26, 3042, 26, 3118, 5969, 3042, 25, 3115, 24, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)\cosh^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{\frac{2}{3}\int (e+fx)\cosh(c+dx)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}$$

$$\Big\downarrow \text{26}$$

3.345. $\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6099}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

3.345. $\displaystyle\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}$$

$$\Big\downarrow \ 3118$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{b}$$

$$\Big\downarrow \ 5969$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int\sinh^2(c+dx)dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int -\sin(ic+idx)^2dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{b}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\int\sin(ic+idx)^2dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{b}$$

3.345. $\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{f\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}\right)}{b}$$

$$\Big\downarrow \text{6095}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+f\left(\frac{x}{2}\cdots\right.}{b}\cdots\right.}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{b^2}-a\left(\cdots\right.\right.}{b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{(a^2+b^2)\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\cdots\right.}{bd}\cdots\right.}{b^2}\cdots\right.}{b}$$

3.345. $\displaystyle\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-$$

$$\frac{a\left(\frac{(a^2+b^2)\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{b^2}-\frac{a(\ldots}{}\right.}{b}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-1/9*(f*Cosh[c + d*x]^3)/d^2 + ((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/
(3*d) + (2*(-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d))/3)/b
- (a*(((a^2 + b^2)*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))])/(b*d^2)))/b^2 - (a*(-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c +
d*x])/d))/b^2 + (((e + f*x)*Sinh[c + d*x]^2)/(2*d) + (f*(x/2 - (Cosh[c + d
*x]*Sinh[c + d*x])/(2*d)))/(2*d))/b))/b
```

3.345.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.345. $\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.345.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1101 vs. $2(372) = 744$.

Time = 15.08 (sec) , antiderivative size = 1102, normalized size of antiderivative = 2.76

method	result	size
risch	Expression too large to display	1102

input
```
int((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
1/d^2*a^3/b^4*f*c^2+1/d^2*a/b^2*f*c^2-1/d^2*a^3/b^4*f*dilog((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*a^3/b^4*f*dilog((b*exp(d*x
+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d*a/b^2*e*ln(b*exp(2*d*x+2*c
)+2*a*exp(d*x+c)-b)+2/d*a^3/b^4*e*ln(exp(d*x+c))-1/d*a^3/b^4*e*ln(b*exp(2*
d*x+2*c)+2*a*exp(d*x+c)-b)+1/2*a*f*x^2/b^2-1/d^2*a/b^2*f*ln((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2*a/b^2*f*ln((b*exp(d*x+c)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d*a/b^2*f*ln((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+2/d*a/b^2*e*ln(exp(d*x+c))-1/d^2*a
/b^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2
*a/b^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d*a
^3/b^4*f*c*x-a^3*e*x/b^4-a*e*x/b^2-1/72*(3*d*f*x+3*d*e+f)/b/d^2*exp(-3*d*x
-3*c)-1/16*a*(2*d*f*x+2*d*e-f)/b^2/d^2*exp(2*d*x+2*c)-1/8*(4*a^2+3*b^2)*(d
*f*x+d*e+f)/b^3/d^2*exp(-d*x-c)-1/16*a*(2*d*f*x+2*d*e+f)/b^2/d^2*exp(-2*d*
x-2*c)+1/d^2*a/b^2*c*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d*a/b^2*f*c
*x+1/2*a^3*f*x^2/b^4-1/d*a/b^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+
(a^2+b^2)^(1/2)))*x-2/d^2*a/b^2*c*f*ln(exp(d*x+c))-2/d^2*a^3/b^4*c*f*ln(ex
p(d*x+c))+1/d^2*a^3/b^4*c*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d*a^3/
b^4*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d*a^3
/b^4*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a^
3/b^4*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/...
```

3.345.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2465 vs. $2(370) = 740$.

Time = 0.30 (sec) , antiderivative size = 2465, normalized size of antiderivative = 6.16

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
1/144*(2*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^6 + 2*(3*b^3*d*f*
x + 3*b^3*d*e - b^3*f)*sinh(d*x + c)^6 - 6*b^3*d*f*x - 9*(2*a*b^2*d*f*x +
2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^5 - 3*(6*a*b^2*d*f*x + 6*a*b^2*d*e -
3*a*b^2*f - 4*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c))*sinh(d*x +
c)^5 - 6*b^3*d*e + 18*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d*e - (
4*a^2*b + 3*b^3)*f)*cosh(d*x + c)^4 + 3*(6*(4*a^2*b + 3*b^3)*d*f*x + 6*(4*
a^2*b + 3*b^3)*d*e + 10*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^2
- 6*(4*a^2*b + 3*b^3)*f - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(
d*x + c))*sinh(d*x + c)^4 - 2*b^3*f + 72*((a^3 + a*b^2)*d^2*f*x^2 + 2*(a^3
 + a*b^2)*d^2*e*x + 4*(a^3 + a*b^2)*c*d*e - 2*(a^3 + a*b^2)*c^2*f)*cosh(d*
x + c)^3 + 2*(36*(a^3 + a*b^2)*d^2*f*x^2 + 72*(a^3 + a*b^2)*d^2*e*x + 144*
(a^3 + a*b^2)*c*d*e - 72*(a^3 + a*b^2)*c^2*f + 20*(3*b^3*d*f*x + 3*b^3*d*e
 - b^3*f)*cosh(d*x + c)^3 - 45*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cos
h(d*x + c)^2 + 36*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d*e - (4*a^
2*b + 3*b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^3 - 18*((4*a^2*b + 3*b^3)*d*f
*x + (4*a^2*b + 3*b^3)*d*e + (4*a^2*b + 3*b^3)*f)*cosh(d*x + c)^2 + 6*(5*(
3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^4 - 3*(4*a^2*b + 3*b^3)*d*f
*x - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^3 - 3*(4*a^2
*b + 3*b^3)*d*e + 18*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d*e - (4
*a^2*b + 3*b^3)*f)*cosh(d*x + c)^2 - 3*(4*a^2*b + 3*b^3)*f + 36*((a^3 +...
```

3.345.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.345.7 Maxima [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\cosh{(dx+c)}^3\sinh{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/24*e*((3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))*e
^(3*d*x + 3*c)/(b^3*d) + 24*(a^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2
*d*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^3*
d) + 24*(a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^4
*d)) - 1/144*f*((72*(a^3*d^2*e^(3*c) + a*b^2*d^2*e^(3*c))*x^2 - 2*(3*b^3*d
*x*e^(6*c) - b^3*e^(6*c))*e^(3*d*x) + 9*(2*a*b^2*d*x*e^(5*c) - a*b^2*e^(5*
c))*e^(2*d*x) + 18*(4*a^2*b*e^(4*c) + 3*b^3*e^(4*c) - (4*a^2*b*d*e^(4*c) +
 3*b^3*d*e^(4*c))*x)*e^(d*x) + 18*(4*a^2*b*e^(2*c) + 3*b^3*e^(2*c) + (4*a^
2*b*d*e^(2*c) + 3*b^3*d*e^(2*c))*x)*e^(-d*x) + 9*(2*a*b^2*d*x*e^c + a*b^2*
e^c)*e^(-2*d*x) + 2*(3*b^3*d*x + b^3)*e^(-3*d*x))*e^(-3*c)/(b^4*d^2) - 9*i
ntegrate(32*((a^4*e^c + a^2*b^2*e^c)*x*e^(d*x) - (a^3*b + a*b^3)*x)/(b^5*e
^(2*d*x + 2*c) + 2*a*b^4*e^(d*x + c) - b^5), x))
```

3.345.8 Giac [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\cosh{(dx+c)}^3\sinh{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a), x
)
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)\ (e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.346 $\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.346.1 Optimal result

Integrand size = 27, antiderivative size = 85

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a(a^2+b^2)\log(a+b\sinh(c+dx))}{b^4d} + \frac{(a^2+b^2)\sinh(c+dx)}{b^3d} - \frac{a\sinh^2(c+dx)}{2b^2d} + \frac{\sinh^3(c+dx)}{3bd}$$

output
```
-a*(a^2+b^2)*ln(a+b*sinh(d*x+c))/b^4/d+(a^2+b^2)*sinh(d*x+c)/b^3/d-1/2*a*s
inh(d*x+c)^2/b^2/d+1/3*sinh(d*x+c)^3/b/d
```

3.346.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-6a(a^2+b^2)\log(a+b\sinh(c+dx))+6b(a^2+b^2)\sinh(c+dx)-3ab^2\sinh^2(c+dx)+2b^3\sinh^3(c+dx)}{6b^4d}$$

input
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-6*a*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]] + 6*b*(a^2 + b^2)*Sinh[c + d*x]
 - 3*a*b^2*Sinh[c + d*x]^2 + 2*b^3*Sinh[c + d*x]^3)/(6*b^4*d)
```

3.346.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3042, 26, 3316, 26, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)\cos(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ic+idx)^3\sin(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{i\int -\frac{i\sinh(c+dx)(\sinh^2(c+dx)b^2+b^2)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\sinh(c+dx)(\sinh^2(c+dx)b^2+b^2)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b\sinh(c+dx)(\sinh^2(c+dx)b^2+b^2)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^4d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\left(\frac{b^2}{a^2}+1\right)a^2-b\sinh(c+dx)a-\frac{(a^2+b^2)a}{a+b\sinh(c+dx)}+b^2\sinh^2(c+dx)\right)d(b\sinh(c+dx))}{b^4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b(a^2+b^2)\sinh(c+dx)-a(a^2+b^2)\log(a+b\sinh(c+dx))-\frac{1}{2}ab^2\sinh^2(c+dx)+\frac{1}{3}b^3\sinh^3(c+dx)}{b^4d}$$

input
```
Int[(Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(a*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]]) + b*(a^2 + b^2)*Sinh[c + d*x] -
 (a*b^2*Sinh[c + d*x]^2)/2 + (b^3*Sinh[c + d*x]^3)/3)/(b^4*d)
```

3.346.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

3.346. $\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.346.4 Maple [A] (verified)

Time = 8.98 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{\frac{\sinh(dx+c)^3 b^2}{3}-\frac{a\sinh(dx+c)^2 b}{2}+a^2\sinh(dx+c)+b^2\sinh(dx+c)}{b^3}-\frac{a\left(a^2+b^2\right)\ln(a+b\sinh(dx+c))}{b^4}}{d}$
default	$\frac{\frac{\frac{\sinh(dx+c)^3 b^2}{3}-\frac{a\sinh(dx+c)^2 b}{2}+a^2\sinh(dx+c)+b^2\sinh(dx+c)}{b^3}-\frac{a\left(a^2+b^2\right)\ln(a+b\sinh(dx+c))}{b^4}}{d}$
risch	$\frac{a^3x}{b^4}+\frac{ax}{b^2}+\frac{e^{3dx+3c}}{24bd}-\frac{a\,e^{2dx+2c}}{8b^2d}+\frac{e^{dx+c}a^2}{2b^3d}+\frac{3\,e^{dx+c}}{8bd}-\frac{e^{-dx-c}a^2}{2b^3d}-\frac{3\,e^{-dx-c}}{8bd}-\frac{a\,e^{-2dx-2c}}{8b^2d}-\frac{e^{-3dx-}}{24ba}$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^3*(1/3*sinh(d*x+c)^3*b^2-1/2*a*sinh(d*x+c)^2*b+a^2*sinh(d*x+c)+b^
2*sinh(d*x+c))-a*(a^2+b^2)/b^4*ln(a+b*sinh(d*x+c)))
```

3.346.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. 2(81) = 162.

Time = 0.29 (sec) , antiderivative size = 652, normalized size of antiderivative = 7.67

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{b^3\cosh(dx+c)^6+b^3\sinh(dx+c)^6-3\,ab^2\cosh(dx+c)^5+24\,(a^3+ab^2)dx\cosh(dx+c)^3+3\,(2\,b^3\cosh}{}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/24*(b^3*cosh(d*x + c)^6 + b^3*sinh(d*x + c)^6 - 3*a*b^2*cosh(d*x + c)^5
+ 24*(a^3 + a*b^2)*d*x*cosh(d*x + c)^3 + 3*(2*b^3*cosh(d*x + c) - a*b^2)*s
inh(d*x + c)^5 + 3*(4*a^2*b + 3*b^3)*cosh(d*x + c)^4 + 3*(5*b^3*cosh(d*x +
 c)^2 - 5*a*b^2*cosh(d*x + c) + 4*a^2*b + 3*b^3)*sinh(d*x + c)^4 - 3*a*b^2
*cosh(d*x + c) + 2*(10*b^3*cosh(d*x + c)^3 - 15*a*b^2*cosh(d*x + c)^2 + 12
*(a^3 + a*b^2)*d*x + 6*(4*a^2*b + 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 -
b^3 - 3*(4*a^2*b + 3*b^3)*cosh(d*x + c)^2 + 3*(5*b^3*cosh(d*x + c)^4 - 10*
a*b^2*cosh(d*x + c)^3 + 24*(a^3 + a*b^2)*d*x*cosh(d*x + c) - 4*a^2*b - 3*b
^3 + 6*(4*a^2*b + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 24*((a^3 + a*b
^2)*cosh(d*x + c)^3 + 3*(a^3 + a*b^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a
^3 + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3 + a*b^2)*sinh(d*x + c)^3)
*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 3*(2*b^3*c
osh(d*x + c)^5 - 5*a*b^2*cosh(d*x + c)^4 + 24*(a^3 + a*b^2)*d*x*cosh(d*x +
 c)^2 + 4*(4*a^2*b + 3*b^3)*cosh(d*x + c)^3 - a*b^2 - 2*(4*a^2*b + 3*b^3)*
cosh(d*x + c))*sinh(d*x + c))/(b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x +
c)^2*sinh(d*x + c) + 3*b^4*d*cosh(d*x + c)*sinh(d*x + c)^2 + b^4*d*sinh(d*
x + c)^3)
```

3.346.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.346.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(81) = 162$.

Time = 0.20 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.15

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\left(3\,abe^{(-dx-c)} - b^2 - 3\,(4\,a^2+3\,b^2)e^{(-2\,dx-2\,c)}\right)e^{(3\,dx+3\,c)}}{24\,b^3d} - \frac{(a^3+ab^2)(dx+c)}{b^4d} - \frac{3\,abe^{(-2\,dx-2\,c)} + b^2e^{(-3\,dx-3\,c)} + 3\,(4\,a^2+3\,b^2)e^{(-dx-c)}}{24\,b^3d} - \frac{(a^3+ab^2)\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{b^4d}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-1/24*(3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))*e^(3
*d*x + 3*c)/(b^3*d) - (a^3 + a*b^2)*(d*x + c)/(b^4*d) - 1/24*(3*a*b*e^(-2*
d*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^3*d
) - (a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^4*d)
```

3.346.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.71

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3 - 3\,ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2 + 12\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right) + 12\,b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{b^3} - \frac{24\,(a^3+ab^2)\log\left(\left|b\left(e^{(dx+c)}-e^{(-\cdots}\right.\right.}{b^4}}{24\,d}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/24*((b^2*(e^(d*x + c) - e^(-d*x - c))^3 - 3*a*b*(e^(d*x + c) - e^(-d*x -
 c))^2 + 12*a^2*(e^(d*x + c) - e^(-d*x - c)) + 12*b^2*(e^(d*x + c) - e^(-d
*x - c)))/b^3 - 24*(a^3 + a*b^2)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) +
2*a))/b^4)/d
```

3.346.9 Mupad [B] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.12

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x\,(a^3+a\,b^2)}{b^4}-\frac{\mathrm{e}^{-3\,c-3\,d\,x}}{24\,b\,d}+\frac{\mathrm{e}^{3\,c+3\,d\,x}}{24\,b\,d}-\frac{a\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,b^2\,d}-\frac{a\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,b^2\,d}-\frac{\mathrm{e}^{-c-d\,x}\,(4\,a^2+3\,b^2)}{8\,b^3\,d}-\frac{\ln\left(2\,a\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-b+b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)\,(a^3+a\,b^2)}{b^4\,d}+\frac{\mathrm{e}^{c+d\,x}\,(4\,a^2+3\,b^2)}{8\,b^3\,d}$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

output
```
(x*(a*b^2 + a^3))/b^4 - exp(- 3*c - 3*d*x)/(24*b*d) + exp(3*c + 3*d*x)/(24
*b*d) - (a*exp(- 2*c - 2*d*x))/(8*b^2*d) - (a*exp(2*c + 2*d*x))/(8*b^2*d)
- (exp(- c - d*x)*(4*a^2 + 3*b^2))/(8*b^3*d) - (log(2*a*exp(d*x)*exp(c) -
b + b*exp(2*c)*exp(2*d*x))*(a*b^2 + a^3))/(b^4*d) + (exp(c + d*x)*(4*a^2 +
 3*b^2))/(8*b^3*d)
```

3.347 $\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.347.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.347.2 Mathematica [N/A]

Not integrable

Time = 38.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

3.347.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\sinh(c+dx)\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]^3*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.347.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.347.4 Maple [N/A] (verified)

Not integrable

Time = 0.85 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(dx+c)^3 \sinh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.347.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^3\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(cosh(d*x + c)^3*sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

3.347.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.347.7 Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 348, normalized size of antiderivative = 10.24

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^3\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/8*e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b*f) - 1/4*a*e
^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^2*f) + 1/4*a*e^(2*
c - 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^2*f) - 1/8*e^(3*c - 3*
d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b*f) - 1/8*(4*a^2 + 3*b^2)*e^(
-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^3*f) - 1/8*(4*a^2*e^c + 3*
b^2*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^3*f) - (a^3 + a*b
^2)*log(f*x + e)/(b^4*f) + 1/16*integrate(32*(a^3*b + a*b^3 - (a^4*e^c + a
^2*b^2*e^c)*e^(d*x))/(b^5*f*x + b^5*e - (b^5*f*x*e^(2*c) + b^5*e*e^(2*c))*
e^(2*d*x) - 2*(a*b^4*f*x*e^c + a*b^4*e*e^c)*e^(d*x)), x)
```

3.347.8 Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate(cosh(d*x + c)^3*sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)),
 x)
```

3.347.9 Mupad [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.348 $\int \frac{(e+fx)^3 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.348.1 Optimal result

Integrand size = 26, antiderivative size = 1021

$$
\begin{aligned}
\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{2(e+fx)^3\arctan\left(e^{c+dx}\right)}{bd}-\frac{2a^2(e+fx)^3\arctan\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} \\
&-\frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} \\
&-\frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} \\
&+\frac{a(e+fx)^3\log\left(1+e^{2(c+dx)}\right)}{\left(a^2+b^2\right)d} \\
&-\frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{bd^2} \\
&+\frac{3ia^2f(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^2} \\
&+\frac{3if(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{bd^2} \\
&-\frac{3ia^2f(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^2} \\
&-\frac{3af(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d^2} \\
&-\frac{3af(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d^2} \\
&+\frac{3af(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2\left(a^2+b^2\right)d^2} \\
&+\frac{6if^2(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{bd^3} \\
&-\frac{6ia^2f^2(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^3} \\
&-\frac{6if^2(e+fx)\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{bd^3} \\
&+\frac{6ia^2f^2(e+fx)\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^3} \\
&+\frac{6af^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d^3} \\
&+\frac{6af^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d^3} \\
&-\frac{3af^2(e+fx)\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2\left(a^2+b^2\right)d^3} \\
&-\frac{6if^3\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)}{bd^4}+\frac{6ia^2f^3\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^4}
\end{aligned}
$$

output
```
3/4*a*f^3*polylog(4,-exp(2*d*x+2*c))/(a^2+b^2)/d^4-6*a*f^3*polylog(4,-b*ex
p(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^4-6*a*f^3*polylog(4,-b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^4-6*I*f^3*polylog(4,-I*exp(d*x+c))/b/d^
4-6*I*a^2*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/b/(a^2+b^2)/d^3-3*I*a^2*f*(
f*x+e)^2*polylog(2,I*exp(d*x+c))/b/(a^2+b^2)/d^2+6*I*f^3*polylog(4,I*exp(d
*x+c))/b/d^4+a*(f*x+e)^3*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d-a*(f*x+e)^3*ln(1
+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d-a*(f*x+e)^3*ln(1+b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d+2*(f*x+e)^3*arctan(exp(d*x+c))/b/d+6*I
*a^2*f^3*polylog(4,-I*exp(d*x+c))/b/(a^2+b^2)/d^4+3*I*f*(f*x+e)^2*polylog(
2,I*exp(d*x+c))/b/d^2+6*I*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/b/d^3+3*I*a
^2*f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/b/(a^2+b^2)/d^2+6*I*a^2*f^2*(f*x+e
)*polylog(3,I*exp(d*x+c))/b/(a^2+b^2)/d^3-6*I*a^2*f^3*polylog(4,I*exp(d*x+
c))/b/(a^2+b^2)/d^4-2*a^2*(f*x+e)^3*arctan(exp(d*x+c))/b/(a^2+b^2)/d+3/2*a
*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^2-3/2*a*f^2*(f*x+e)*po
lylog(3,-exp(2*d*x+2*c))/(a^2+b^2)/d^3-3*a*f*(f*x+e)^2*polylog(2,-b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2-3*a*f*(f*x+e)^2*polylog(2,-b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2+6*a*f^2*(f*x+e)*polylog(3,-b*exp(
d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^3+6*a*f^2*(f*x+e)*polylog(3,-b*exp
(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^3-3*I*f*(f*x+e)^2*polylog(2,-I*ex
p(d*x+c))/b/d^2-6*I*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/b/d^3
```

3.348.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3078 vs. 2(1021) = 2042.

Time = 11.33 (sec) , antiderivative size = 3078, normalized size of antiderivative = 3.01

$$\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-8*a*d^4*e^3*E^(2*c)*x - 12*a*d^4*e^2*E^(2*c)*f*x^2 - 8*a*d^4*e*E^(2*c)*f
^2*x^3 - 2*a*d^4*E^(2*c)*f^3*x^4 + 8*b*d^3*e^3*ArcTan[E^(c + d*x)] + 8*b*d
^3*e^3*E^(2*c)*ArcTan[E^(c + d*x)] + (12*I)*b*d^3*e^2*f*x*Log[1 - I*E^(c +
 d*x)] + (12*I)*b*d^3*e^2*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (12*I)*b*d^
3*e*f^2*x^2*Log[1 - I*E^(c + d*x)] + (12*I)*b*d^3*e*E^(2*c)*f^2*x^2*Log[1
- I*E^(c + d*x)] + (4*I)*b*d^3*f^3*x^3*Log[1 - I*E^(c + d*x)] + (4*I)*b*d^
3*E^(2*c)*f^3*x^3*Log[1 - I*E^(c + d*x)] - (12*I)*b*d^3*e^2*f*x*Log[1 + I*
E^(c + d*x)] - (12*I)*b*d^3*e^2*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (12*I
)*b*d^3*e*f^2*x^2*Log[1 + I*E^(c + d*x)] - (12*I)*b*d^3*e*E^(2*c)*f^2*x^2*
Log[1 + I*E^(c + d*x)] - (4*I)*b*d^3*f^3*x^3*Log[1 + I*E^(c + d*x)] - (4*I
)*b*d^3*E^(2*c)*f^3*x^3*Log[1 + I*E^(c + d*x)] + 4*a*d^3*e^3*Log[1 + E^(2*
(c + d*x))] + 4*a*d^3*e^3*E^(2*c)*Log[1 + E^(2*(c + d*x))] + 12*a*d^3*e^2*
f*x*Log[1 + E^(2*(c + d*x))] + 12*a*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(2*(c +
d*x))] + 12*a*d^3*e*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 12*a*d^3*e*E^(2*c)*
f^2*x^2*Log[1 + E^(2*(c + d*x))] + 4*a*d^3*f^3*x^3*Log[1 + E^(2*(c + d*x))
] + 4*a*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(2*(c + d*x))] - (12*I)*b*d^2*(1 + E
^(2*c))*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)] + (12*I)*b*d^2*(1 + E^(
2*c))*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)] + 6*a*d^2*e^2*f*PolyLog[2, -
E^(2*(c + d*x))] + 6*a*d^2*e^2*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))] + 12
*a*d^2*e*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + 12*a*d^2*e*E^(2*c)*f^2*x*...
```

3.348.3 Rubi [A] (verified)

Time = 3.89 (sec) , antiderivative size = 886, normalized size of antiderivative = 0.87, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6101, 3042, 4668, 3011, 6107, 6095, 2620, 3011, 7163, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6101}$$

$$\frac{\int (e+fx)^3\text{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{4668}$$

$$\frac{-\frac{a\int\frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{3if\int(e+fx)^2\log(1-ie^{c+dx})dx}{d}+\frac{3if\int(e+fx)^2\log(1+ie^{c+dx})dx}{d}+\frac{2(e+fx)^3\arctan(e^{c+dx})}{d}}{b}}{}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{a\int\frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+\ldots}{b}$$

$$\Big\downarrow \text{6107}$$

$$-\frac{a\left(\frac{b^2\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+\ldots}{b}$$

$$\Big\downarrow \text{6095}$$

$$-\frac{a\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{a^2+b^2}+\frac{\int(e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+\ldots}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{a\left(\frac{b^2\left(-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\ldots\right.}{a^2+b^2}\ldots\right.}{b}+\frac{\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+\ldots}{b}$$

$$\Big\downarrow \text{3011}$$

3.348. $\displaystyle\int\frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\ \cdots}{}\right)}{bd}\cdots\right)}{a^2+b^2}\cdots\right)}{\frac{\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+\cdots}{b}}$$

$$\Big\downarrow \ 7163$$

$$-\frac{a\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyL}\cdots}{}\cdots\right)}{}\cdots\right)}{}\cdots\right)}{}\cdots\right)}{\frac{\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{d}-\frac{f\int\operatorname{Poly}\cdots}{}\right)}{d}\cdots\right)}{}\cdots}{b}}$$

$$\Big\downarrow \ 2720$$

3.348. $\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\cdots\right.}{}\right.}{}\right.}{}\right.}{\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,ie^{c+\cdots}\right.}{d}\right.}{}\right.}{}}{b}$$

$\downarrow$ 7143

3.348. $\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\int (e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyL}}{}}{bd}\right)}{}\right)}{\frac{2(e+fx)^3\arctan\left(e^{c+dx}\right)}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{Po}}{}\right.}{}\right.}{}}{b}$$

$$\Big\downarrow \text{7293}$$

3.348. $\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\int\left(a(e+fx)^3\operatorname{sech}(c+dx)-b(e+fx)^3\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+\cdots}{\cdots}\cdots\right)}{bd}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{\frac{2(e+fx)^3\arctan\left(e^{c+dx}\right)}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{Po}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}\cdots}{b}$$

$\downarrow$ 2009

3.348. $\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^3}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{Po}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\cdots}{b}$$

$$a\left(\frac{\left(-\frac{(e+fx)^4}{4bf}+\frac{\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+fx)^3}{bd}+\frac{\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)(e+fx)^3}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}\cdots\right.}{bd}\cdots\right.}{\cdots}\cdots\right.$$

input
```
Int[((e + f*x)^3*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.348. $\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
((2*(e + f*x)^3*ArcTan[E^(c + d*x)])/d + ((3*I)*f*(-(((e + f*x)^2*PolyLog[
2, (-I)*E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/
d - (f*PolyLog[4, (-I)*E^(c + d*x)])/d^2))/d))/d - ((3*I)*f*(-(((e + f*x)^
2*PolyLog[2, I*E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, I*E^(c + d*x
)])/d - (f*PolyLog[4, I*E^(c + d*x)])/d^2))/d))/d)/b - (a*((b^2*(-1/4*(e +
 f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b
*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2)
)/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)])/d^2))/d))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^4)/(4*f) + (2*a*(e + f*x
)^3*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])/d -
((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((3*I)*a*f*(e +
 f*x)^2*PolyLog[2, I*E^(c + d*x)])/d^2 - (3*b*f*(e + f*x)^2*PolyLog[2, -E^
(2*(c + d*x))])/(2*d^2) + ((6*I)*a*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*
x)])/d^3 - ((6*I)*a*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/d^3 + (3*b*f^
2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*d^3) - ((6*I)*a*f^3*PolyLo...
```

3.348.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.348. $\int \frac{(e+fx)^3 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

3.348. $\int \frac{(e+fx)^3 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.348.4 Maple [F]

$$\int \frac{(fx+e)^3 \tanh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.348.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1715, normalized size of antiderivative = 1.68

$$\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(6*a*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*a*f^3*polylog(4, (a*cosh
(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2))/b) + 3*(a*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*dilo
g((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3*(a*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x +
a*d^2*e^2*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(a*d^2*f^3*x^2 + I
*b*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + 2*I*b*d^2*e*f^2*x + a*d^2*e^2*f + I*b*d
^2*e^2*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - 3*(a*d^2*f^3*x^2 - I*
b*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x - 2*I*b*d^2*e*f^2*x + a*d^2*e^2*f - I*b*d^
2*e^2*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (a*d^3*e^3 - 3*a*c*d^
2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*
x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (a*d^3*e^3 - 3*a*c*d^2*e^2*f +
 3*a*c^2*d*e*f^2 - a*c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) -
2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*
a*d^3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*log(-(a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b) + (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*
f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*log(-(a*cosh(d*x +...
```

3.348.6 Sympy [F]

$$\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^3\tanh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.348.7 Maxima [F]

$$\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\tanh{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^3*(2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(-2*a*e^(-d*x - c) +
 b*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a
^2 + b^2)*d)) + integrate(2*f^3*x^3*(e^(d*x + c) - e^(-d*x - c))/((b*(e^(d
*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))) + 6*e*f^2*x^2
*(e^(d*x + c) - e^(-d*x - c))/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(
d*x + c) + e^(-d*x - c))) + 6*e^2*f*x*(e^(d*x + c) - e^(-d*x - c))/((b*(e^
(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))), x)
```

3.348.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.348.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\tanh(c+d\,x)\,{(e+f\,x)}^3}{a+b\,\sinh{(c+d\,x)}}\,dx$$

input
```
int((tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.349 $\int \frac{(e+fx)^2 \tanh(c+dx)}{a+b\sinh(c+dx)} \, dx$

3.349.1 Optimal result

Integrand size = 26, antiderivative size = 716

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^2\arctan\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d}$$
$$- \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}$$
$$- \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}$$
$$+ \frac{a(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{\left(a^2+b^2\right)d}$$
$$- \frac{2if(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{bd^2}$$
$$+ \frac{2ia^2f(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^2}$$
$$+ \frac{2if(e+fx)\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{bd^2}$$
$$- \frac{2ia^2f(e+fx)\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^2}$$
$$- \frac{2af(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d^2}$$
$$- \frac{2af(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d^2}$$
$$+ \frac{af(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{\left(a^2+b^2\right)d^2}$$
$$+ \frac{2if^2\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{bd^3} - \frac{2ia^2f^2\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^3}$$
$$- \frac{2if^2\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{bd^3} + \frac{2ia^2f^2\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^3}$$
$$+ \frac{2af^2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d^3}$$
$$+ \frac{2af^2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d^3}$$
$$- \frac{af^2\,\mathrm{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2\left(a^2+b^2\right)d^3}$$

output
```
2*(f*x+e)^2*arctan(exp(d*x+c))/b/d-2*a^2*(f*x+e)^2*arctan(exp(d*x+c))/b/(a
^2+b^2)/d+a*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d-a*(f*x+e)^2*ln(1+b*
exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d-a*(f*x+e)^2*ln(1+b*exp(d*x+c)/
(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d-2*I*a^2*f^2*polylog(3,-I*exp(d*x+c))/b/(a
^2+b^2)/d^3+2*I*f^2*polylog(3,-I*exp(d*x+c))/b/d^3-2*I*f^2*polylog(3,I*exp
(d*x+c))/b/d^3+2*I*a^2*f^2*polylog(3,I*exp(d*x+c))/b/(a^2+b^2)/d^3+a*f*(f*
x+e)*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^2-2*a*f*(f*x+e)*polylog(2,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2-2*a*f*(f*x+e)*polylog(2,-b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2+2*I*a^2*f*(f*x+e)*polylog(2,-I
*exp(d*x+c))/b/(a^2+b^2)/d^2-2*I*a^2*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b/(
a^2+b^2)/d^2+2*I*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b/d^2-2*I*f*(f*x+e)*pol
ylog(2,-I*exp(d*x+c))/b/d^2-1/2*a*f^2*polylog(3,-exp(2*d*x+2*c))/(a^2+b^2)
/d^3+2*a*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^3+2*
a*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^3
```

3.349.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1640 vs. $2(716) = 1432$.

Time = 10.38 (sec) , antiderivative size = 1640, normalized size of antiderivative = 2.29

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*f*x^2
 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 6*a*
d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d*e*(1
 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - Pol
yLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 + E^(2
*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x
))]) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2
*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyL
og[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(
c + d*x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d
*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x)
)]))/(6*(a^2 + b^2)*d^3*(1 + E^(2*c))) + (a*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)
*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c +
 d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2
)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2
)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqr
t[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2
*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) +
 (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c
)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 +...
```

3.349.3 Rubi [A] (verified)

Time = 2.98 (sec) , antiderivative size = 633, normalized size of antiderivative = 0.88, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6101, 3042, 4668, 3011, 2720, 6107, 6095, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6101}$$

$$\frac{\int (e+fx)^2\text{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{a\int\frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{2if\int(e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int(e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{a\int\frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\left(\frac{f\int\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+\frac{2(e+fx)^2\arctan(}{d}}{b}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{a\int\frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right.}{d}\right.}{d}}{b}$$

$$\Big\downarrow \text{6107}$$

$$-\frac{a\left(\frac{b^2\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right.}{d}\right.}{d}}{b}$$

$$\Big\downarrow \text{6095}$$

$$-\frac{a\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int(e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right.}{d}\right.}{d}}{b}$$

$$\Big\downarrow \text{2620}$$

3.349. $\int\frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{b^2\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+f}{3b}\right.}{a^2+b^2}\right.}{b}$$

$$\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}}{b}$$

$\downarrow$ 3011

$$-\frac{a\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c-}}{a+\sqrt{a}}\right.}{d}\right.}{bd}\right.}{a^2+b^2}\right.}{b}$$

$$\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}}{b}$$

$\downarrow$ 2720

$$-\frac{a\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\right.}{bd}\right.}{a^2+b^2}\right.}{}$$

$$\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}}{b}$$

$\downarrow$ 7143

3.349. $\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\cdots\right.}{d\cdots}\right.}{\cdots}\right.}{\cdots}\right)}{b}\cdots$$

$$\frac{\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}}{b}\cdots$$

$$\Big\downarrow \text{7293}$$

$$-\frac{a\left(\frac{\int \left(a(e+fx)^2\text{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{P}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right)}{b}\cdots$$

$$\frac{\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}}{b}\cdots$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}}{b}\cdots$$

$$a\left(\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}+\cdots\right.}{a^2+b^2}\cdots\right.$$

3.349. $\displaystyle\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((2*(e + f*x)^2*ArcTan[E^(c + d*x)])/d + ((2*I)*f*(-(((e + f*x)*PolyLog[2,
 (-I)*E^(c + d*x)])/d) + (f*PolyLog[3, (-I)*E^(c + d*x)])/d^2))/d - ((2*I)
*f*(-(((e + f*x)*PolyLog[2, I*E^(c + d*x)])/d) + (f*PolyLog[3, I*E^(c + d*
x)])/d^2))/d)/b - (a*((b^2*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -
((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3
)/(3*f) + (2*a*(e + f*x)^2*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 +
 E^(2*(c + d*x))])/d - ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/
d^2 + ((2*I)*a*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*(e + f*x)
*PolyLog[2, -E^(2*(c + d*x))])/d^2 + ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d
*x)])/d^3 - ((2*I)*a*f^2*PolyLog[3, I*E^(c + d*x)])/d^3 + (b*f^2*PolyLog[3
, -E^(2*(c + d*x))])/(2*d^3))/(a^2 + b^2)))/b
```

3.349.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.349. $\int \frac{(e+fx)^2 \tanh(c+dx)}{a+b \sinh(c+dx)}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

3.349. $\int \frac{(e+fx)^2 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.349.4 Maple [F]

$$\int \frac{(fx+e)^2 \tanh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.349.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1083, normalized size of antiderivative = 1.51

$$\int \frac{(e+fx)^2 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*a*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*a*f^2*polylog(3, (a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2))/b) - 2*(a*d*f^2*x + a*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh
(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b + 1) - 2*(a*d*f^2*x + a*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
 2*(a*d*f^2*x + I*b*d*f^2*x + a*d*e*f + I*b*d*e*f)*dilog(I*cosh(d*x + c) +
 I*sinh(d*x + c)) + 2*(a*d*f^2*x - I*b*d*f^2*x + a*d*e*f - I*b*d*e*f)*dilo
g(-I*cosh(d*x + c) - I*sinh(d*x + c)) - (a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f
^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2)
+ 2*a) - (a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b
*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (a*d^2*f^2*x^2 + 2*a*d
^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c
) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a
*d^2*f^2*x^2 + 2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*log(-(a*cosh(d*x +
 c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b) + (a*d^2*e^2 + I*b*d^2*e^2 - 2*a*c*d*e*f - 2*I*b*c*d*e*f +
 a*c^2*f^2 + I*b*c^2*f^2)*log(cosh(d*x + c) + sinh(d*x + c) + I) + (a*d^2*
e^2 - I*b*d^2*e^2 - 2*a*c*d*e*f + 2*I*b*c*d*e*f + a*c^2*f^2 - I*b*c^2*f...
```

3.349.6 Sympy [F]

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\tanh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.349.7 Maxima [F]

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\tanh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^2*(2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(-2*a*e^(-d*x - c) +
 b*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a
^2 + b^2)*d)) + integrate(2*f^2*x^2*(e^(d*x + c) - e^(-d*x - c))/((b*(e^(d
*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))) + 4*e*f*x*(e^
(d*x + c) - e^(-d*x - c))/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x
+ c) + e^(-d*x - c))), x)
```

3.349.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.350 $\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.350.1 Optimal result

Integrand size = 24, antiderivative size = 421

$$\begin{aligned}\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)\arctan\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} \\ &\quad - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} \\ &\quad - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} \\ &\quad + \frac{a(e+fx)\log\left(1+e^{2(c+dx)}\right)}{\left(a^2+b^2\right)d} - \frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{bd^2} \\ &\quad + \frac{ia^2f\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^2} + \frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{bd^2} \\ &\quad - \frac{ia^2f\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{b\left(a^2+b^2\right)d^2} - \frac{af\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d^2} \\ &\quad - \frac{af\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d^2} + \frac{af\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2\left(a^2+b^2\right)d^2}\end{aligned}$$

output
```
2*(f*x+e)*arctan(exp(d*x+c))/b/d-2*a^2*(f*x+e)*arctan(exp(d*x+c))/b/(a^2+b
^2)/d+a*(f*x+e)*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d-a*(f*x+e)*ln(1+b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d-a*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/(a^2+b^2)/d-I*f*polylog(2,-I*exp(d*x+c))/b/d^2+I*a^2*f*polylog(
2,-I*exp(d*x+c))/b/(a^2+b^2)/d^2+I*f*polylog(2,I*exp(d*x+c))/b/d^2-I*a^2*f
*polylog(2,I*exp(d*x+c))/b/(a^2+b^2)/d^2+1/2*a*f*polylog(2,-exp(2*d*x+2*c)
)/(a^2+b^2)/d^2-a*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)
/d^2-a*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2
```

3.350.2 Mathematica [A] (verified)

Time = 2.98 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.24

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{4bde\arctan\left(e^{c+dx}\right)-4bcf\arctan\left(e^{c+dx}\right)+\frac{4a^2(a^2+b^2)^{5/2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\left(-(a^2+b^2)^2\right)^{3/2}}+\frac{4a^2\sqrt{-(a^2+b^2)^2}de\,\mathrm{arctanh}\left(\frac{a+be^{c+d}}{\sqrt{a^2+b^2}}\right.}{(-a^2-b^2)^{3/2}}}{}$$

input
```
Integrate[((e + f*x)*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(4*b*d*e*ArcTan[E^(c + d*x)] - 4*b*c*f*ArcTan[E^(c + d*x)] + (4*a^2*(a^2 +
 b^2)^(5/2)*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(-(a^2 + b^2
)^2)^(3/2) + (4*a^2*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/S
qrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + (2*I)*b*f*(c + d*x)*Log[1 - I*E^(c +
 d*x)] - (2*I)*b*f*(c + d*x)*Log[1 + I*E^(c + d*x)] - 2*a*f*(c + d*x)*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*a*f*(c + d*x)*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*a*d*e*Log[1 + E^(2*(c + d*x))] - 2*a
*c*f*Log[1 + E^(2*(c + d*x))] + 2*a*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] +
 2*a*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] - 2*a*d*e*Log[2*a*E^
(c + d*x) + b*(-1 + E^(2*(c + d*x)))] - (2*I)*b*f*PolyLog[2, (-I)*E^(c + d
*x)] + (2*I)*b*f*PolyLog[2, I*E^(c + d*x)] - 2*a*f*PolyLog[2, (b*E^(c + d*
x))/(-a + Sqrt[a^2 + b^2])] - 2*a*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))] + a*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^2)
```

3.350.3 Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6101, 3042, 4668, 2715, 2838, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6101}$$

$$\frac{\int (e+fx)\text{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}$$

$$\Big\downarrow \text{6107}$$

$$-\frac{a\left(\frac{b^2\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}$$

$$\Big\downarrow \text{6095}$$

$$-\frac{a\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{a\left(\frac{b^2\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+fx)\ldots}{\ldots}\right)}{b}\ldots$$

$$\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{a\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\ldots\right)}{a^2+b^2}\ldots\right)}{b}\ldots$$

$$\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{a\left(\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+\ldots}{\ldots}\ldots\right)}{a^2+b^2}\ldots\right)}{b}\ldots$$

$$\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}$$

$$\Big\downarrow \text{7293}$$

3.350. $\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\int (a(e+fx)\operatorname{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+\right.}{bd}\right.}{a^2+b^2}\right.}{b}$$

$$\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}-\frac{a\left(\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{2a(e+fx)}{}\right.}{b}$$

input
```
Int[((e + f*x)*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((2*(e + f*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/
d^2 + (I*f*PolyLog[2, I*E^(c + d*x)])/d^2)/b - (a*((b^2*(-1/2*(e + f*x)^2/
(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) +
 ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*Pol
yLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2,
 -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2 + b^2) + ((b*(e
 + f*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)*Lo
g[1 + E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*
a*f*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2
*d^2))/(a^2 + b^2)))/b
```

3.350. $\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.350.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

3.350. $\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.350.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1286 vs. 2(393) = 786.

Time = 2.48 (sec) , antiderivative size = 1287, normalized size of antiderivative = 3.06

method	result	size
risch	Expression too large to display	1287

input
```
int((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/(a^2+b^2)^(1/2)/d*b^2*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/
(a^2+b^2)^(1/2))+2/(a^2+b^2)^(1/2)/d^2*b^2*c*f/(2*a^2+2*b^2)*arctanh(1/2*(
2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2*c*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/
2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2+2*I/d*f/(2*a^2+2*
b^2)*ln(1-I*exp(d*x+c))*b*x-2*I/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*b*c
+2*I/d^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*b*c-2*I/d*f/(2*a^2+2*b^2)*ln(1
+I*exp(d*x+c))*b*x-2/d^2*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1
/2)-a)/(-a+(a^2+b^2)^(1/2)))*a-2/d^2*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(
a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a+2/d^2*f/(2*a^2+2*b^2)*dilog(1+I*e
xp(d*x+c))*a+2/d^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))*a-2/d*e/(2*a^2+2*
b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d*e/(2*a^2+2*b^2)*a*ln(1+ex
p(2*d*x+2*c))+2/d*e/(2*a^2+2*b^2)*(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x
+c)+2*a)/(a^2+b^2)^(1/2))+4/d*e/(2*a^2+2*b^2)*b*arctan(exp(d*x+c))+2/d^2*f
/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*c-2*I/d^2*f/(2*a^2+2*b^2)*dilog(1+I*ex
p(d*x+c))*b+2*I/d^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))*b-2/d^2*f/(2*a^2
+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a*c-2/d
^2*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))
)*a*c+2/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*x+2/d*f/(2*a^2+2*b^2)*ln(1-
I*exp(d*x+c))*a*x+2/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*c-2/d^2*c*f/(
2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c))-4/d^2*c*f/(2*a^2+2*b^2)*b*arctan(ex...
```

3.350.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.40

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{af\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)+af\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh}{b}\right.}{}$$

input
```
integrate((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(a*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + a*f*dilog((a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b + 1) - (a*f + I*b*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) -
 (a*f - I*b*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (a*d*e - a*c*f)
*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) + (a*d*e - a*c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt
((a^2 + b^2)/b^2) + 2*a) + (a*d*f*x + a*c*f)*log(-(a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b) + (a*d*f*x + a*c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a*d*e + I*b*
d*e - a*c*f - I*b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) + I) - (a*d*e - I
*b*d*e - a*c*f + I*b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) - I) - (a*d*f*
x - I*b*d*f*x + a*c*f - I*b*c*f)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1
) - (a*d*f*x + I*b*d*f*x + a*c*f + I*b*c*f)*log(-I*cosh(d*x + c) - I*sinh(
d*x + c) + 1))/((a^2 + b^2)*d^2)
```

3.350.6 Sympy [F]

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)\tanh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.350.7 Maxima [F]

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\tanh{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e*(2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(-2*a*e^(-d*x - c) + b
*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a^2
 + b^2)*d)) + f*integrate(2*x*(e^(d*x + c) - e^(-d*x - c))/((b*(e^(d*x + c
) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))), x)
```

3.350.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.350.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.351 $\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.351.1 Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\arctan(\sinh(c+dx))}{(a^2+b^2)\,d} + \frac{a\log(\cosh(c+dx))}{(a^2+b^2)\,d} - \frac{a\log(a+b\sinh(c+dx))}{(a^2+b^2)\,d}$$

output
```
b*arctan(sinh(d*x+c))/(a^2+b^2)/d+a*ln(cosh(d*x+c))/(a^2+b^2)/d-a*ln(a+b*s
inh(d*x+c))/(a^2+b^2)/d
```

3.351.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{(a-ib)\log(i-\sinh(c+dx)) + (a+ib)\log(i+\sinh(c+dx)) - 2a\log(a+b\sinh(c+dx))}{2\,(a^2+b^2)\,d}$$

input
```
Integrate[Tanh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
((a - I*b)*Log[I - Sinh[c + d*x]] + (a + I*b)*Log[I + Sinh[c + d*x]] - 2*a
*Log[a + b*Sinh[c + d*x]])/(2*(a^2 + b^2)*d)
```

3.351.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {3042, 26, 3200, 25, 587, 16, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3200}$$

$$-\frac{\int -\frac{b\sinh(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b\sinh(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{587}$$

$$-\frac{\frac{a\int \frac{1}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{a^2+b^2}-\frac{\int \frac{b^2+a\sinh(c+dx)b}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))}{a^2+b^2}}{d}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\frac{a\log(a+b\sinh(c+dx))}{a^2+b^2}-\frac{\int \frac{b^2+a\sinh(c+dx)b}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))}{a^2+b^2}}{d}$$

$$\Big\downarrow \text{452}$$

$$-\frac{\frac{a\log(a+b\sinh(c+dx))}{a^2+b^2}-\frac{a\int \frac{b\sinh(c+dx)}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))+b^2\int \frac{1}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))}{a^2+b^2}}{d}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{a\log(a+b\sinh(c+dx))}{a^2+b^2}-\frac{a\int\frac{b\sinh(c+dx)}{\sinh^2(c+dx)b^2+b^2}d(b\sinh(c+dx))+b\arctan(\sinh(c+dx))}{a^2+b^2}}{d}$$

$$\Big\downarrow \textcolor{blue}{240}$$

$$-\frac{\frac{a\log(a+b\sinh(c+dx))}{a^2+b^2}-\frac{\frac{1}{2}a\log\left(b^2\sinh^2(c+dx)+b^2\right)+b\arctan(\sinh(c+dx))}{a^2+b^2}}{d}$$

input
```
Int[Tanh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
-(((a*Log[a + b*Sinh[c + d*x]])/(a^2 + b^2) - (b*ArcTan[Sinh[c + d*x]] + (
a*Log[b^2 + b^2*Sinh[c + d*x]^2])/2)/(a^2 + b^2))/d)
```

3.351.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.351.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{\frac{2a\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)+4b\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^{2}+2b^{2}}-\frac{2a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{2a^{2}+2b^{2}}}{d}$
default	$\frac{\frac{2a\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)+4b\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^{2}+2b^{2}}-\frac{2a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{2a^{2}+2b^{2}}}{d}$
risch	$-\frac{2a\,d^{2}x}{a^{2}d^{2}+b^{2}d^{2}}-\frac{2adc}{a^{2}d^{2}+b^{2}d^{2}}+\frac{2ax}{a^{2}+b^{2}}+\frac{2ac}{d(a^{2}+b^{2})}+\frac{i\ln(\mathrm{e}^{dx+c}+i)b}{(a^{2}+b^{2})d}+\frac{\ln(\mathrm{e}^{dx+c}+i)a}{(a^{2}+b^{2})d}-\frac{i\ln(\mathrm{e}^{dx+c}-i)b}{(a^{2}+b^{2})d}+$ ln

input
```
int(tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(4/(2*a^2+2*b^2)*(1/2*a*ln(1+tanh(1/2*d*x+1/2*c)^2)+b*arctan(tanh(1/2*
d*x+1/2*c)))-2*a/(2*a^2+2*b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x
+1/2*c)-a))
```

3.351.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2\,b\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)-a\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)+a\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^2+b^2)d}$$

input
```
integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*b*arctan(cosh(d*x + c) + sinh(d*x + c)) - a*log(2*(b*sinh(d*x + c) + a)
/(cosh(d*x + c) - sinh(d*x + c))) + a*log(2*cosh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))))/((a^2 + b^2)*d)
```

3.351.6 Sympy [F]

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.351.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d}-\frac{a\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^2+b^2)d}+\frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d}$$

input
```
integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(-2*a*e^(-d*x - c) + b*e^
(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)/((a^2 +
b^2)*d)
```

3.351.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.75

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2\,ab\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^2b+b^3}-\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)b}{a^2+b^2}-\frac{a\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2}}{2\,d}$$

input
```
integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*a*b*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^2*b + b^3) -
 (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*b/(a^2 + b^2) - a
*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2))/d
```

3.351.9 Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.88

$$\begin{aligned}\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\ln\left(e^{c+d\,x}+1\mathrm{i}\right)}{a\,d-b\,d\,1\mathrm{i}} \\ &\quad - \frac{a\,\ln\left(8\,a^3\,e^{d\,x}\,e^c-b^3-4\,a^2\,b+b^3\,e^{2\,c}\,e^{2\,d\,x}+4\,a^2\,b\,e^{2\,c}\,e^{2\,d\,x}+2\,a\,b^2\,e^{d\,x}\,e^c\right)}{d\,a^2+d\,b^2} \\ &\quad + \frac{\ln\left(1+e^{c+d\,x}\,1\mathrm{i}\right)\,1\mathrm{i}}{-b\,d+a\,d\,1\mathrm{i}}\end{aligned}$$

input
```
int(tanh(c + d*x)/(a + b*sinh(c + d*x)),x)
```

output
```
log(exp(c + d*x) + 1i)/(a*d - b*d*1i) + (log(exp(c + d*x)*1i + 1)*1i)/(a*d
*1i - b*d) - (a*log(8*a^3*exp(d*x)*exp(c) - b^3 - 4*a^2*b + b^3*exp(2*c)*e
xp(2*d*x) + 4*a^2*b*exp(2*c)*exp(2*d*x) + 2*a*b^2*exp(d*x)*exp(c)))/(a^2*d
 + b^2*d)
```

3.352 $\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.352.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.352.2 Mathematica [N/A]

Not integrable

Time = 11.39 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Tanh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Tanh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.352.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$\Big\downarrow \; 6111$$
$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Tanh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.352.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.352.4 Maple [N/A] (verified)

Not integrable

Time = 1.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\tanh\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.352.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.352.6 Sympy [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.352.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(tanh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.352.8 Giac [F(-1)]

Timed out.

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.352.9 Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(tanh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(tanh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.353 $\int \frac{(e+fx)^3 \operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.353.1 Optimal result

Integrand size = 32, antiderivative size = 917

$$\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{(e+fx)^3}{bd}-\frac{a^2(e+fx)^3}{b\left(a^2+b^2\right)d}+\frac{6af(e+fx)^2\arctan\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2}$$
$$-\frac{ab(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}+\frac{ab(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}$$
$$-\frac{3f(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{bd^2}+\frac{3a^2f(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{b\left(a^2+b^2\right)d^2}$$
$$-\frac{6iaf^2(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{\left(a^2+b^2\right)d^3}+\frac{6iaf^2(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{\left(a^2+b^2\right)d^3}$$
$$-\frac{3abf(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d^2}+\frac{3abf(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d^2}$$
$$-\frac{3f^2(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{bd^3}+\frac{3a^2f^2(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{b\left(a^2+b^2\right)d^3}$$
$$+\frac{6iaf^3\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{\left(a^2+b^2\right)d^4}-\frac{6iaf^3\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{\left(a^2+b^2\right)d^4}$$
$$+\frac{6abf^2(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d^3}-\frac{6abf^2(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d^3}$$
$$+\frac{3f^3\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2bd^4}-\frac{3a^2f^3\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2b\left(a^2+b^2\right)d^4}$$
$$-\frac{6abf^3\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d^4}+\frac{6abf^3\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d^4}$$
$$-\frac{a(e+fx)^3\text{sech}(c+dx)}{\left(a^2+b^2\right)d}+\frac{(e+fx)^3\tanh(c+dx)}{bd}-\frac{a^2(e+fx)^3\tanh(c+dx)}{b\left(a^2+b^2\right)d}$$

output
```
(f*x+e)^3/b/d-a^2*(f*x+e)^3/b/(a^2+b^2)/d+6*a*f*(f*x+e)^2*arctan(exp(d*x+c
))/(a^2+b^2)/d^2-3*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b/d^2+3*a^2*f*(f*x+e)^
2*ln(1+exp(2*d*x+2*c))/b/(a^2+b^2)/d^2-a*b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+a*b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+6*I*a*f^3*polylog(3,-I*exp(d*x+c))/(a^2+b
^2)/d^4+6*I*a*f^2*(f*x+e)*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^3-3*f^2*(f*x
+e)*polylog(2,-exp(2*d*x+2*c))/b/d^3+3*a^2*f^2*(f*x+e)*polylog(2,-exp(2*d*
x+2*c))/b/(a^2+b^2)/d^3-3*a*b*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2+3*a*b*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)
/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-6*I*a*f^3*polylog(3,I*exp(d*x+c)
)/(a^2+b^2)/d^4-6*I*a*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^3+3
/2*f^3*polylog(3,-exp(2*d*x+2*c))/b/d^4-3/2*a^2*f^3*polylog(3,-exp(2*d*x+2
*c))/b/(a^2+b^2)/d^4+6*a*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/(a^2+b^2)^(3/2)/d^3-6*a*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a
+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^3-6*a*b*f^3*polylog(4,-b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^4+6*a*b*f^3*polylog(4,-b*exp(d*x+c)/
(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^4-a*(f*x+e)^3*sech(d*x+c)/(a^2+b^2)
/d+(f*x+e)^3*tanh(d*x+c)/b/d-a^2*(f*x+e)^3*tanh(d*x+c)/b/(a^2+b^2)/d
```

3.353.2 Mathematica [A] (verified)

Time = 7.72 (sec) , antiderivative size = 1144, normalized size of antiderivative = 1.25

$$\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{f\left(12bd^3e^2e^{2c}x-12bd^3e^2(1+e^{2c})\,x-12bd^3efx^2-4bd^3f^2x^3+12ad^2e^2(1+e^{2c})\arctan\left(e^{c+dx}\right)+6bd^2e^2(\right.}{}$$
$$-\frac{ab\left(-2d^3e^3\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+3d^3e^2fx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+3d^3ef^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^3f^3x^3\,\text{l}\right.}{}$$
$$+\frac{\text{sech}(c)\text{sech}(c+dx)\left(-ae^3\cosh(c)-3ae^2fx\cosh(c)-3aef^2x^2\cosh(c)-af^3x^3\cosh(c)+be^3\sinh(dx)\right.}{(a^2+b^2)\,d}$$

input
```
Integrate[((e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
(f*(12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x - 12*b*d^3*e*f*x
^2 - 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] + 6*
b*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*a*d*e*
(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - P
olyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*b*d*e*(1 + E^
(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d
*x))]) + (6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x
^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*Pol
yLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E
^(c + d*x)]) + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c +
 d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*
x))])))/(2*(a^2 + b^2)*d^4*(1 + E^(2*c))) - (a*b*(-2*d^3*e^3*ArcTanh[(a +
b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] -
 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^3*e*f^
2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 +
 (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, (
b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*e*f^2*PolyLog[3, (b*E^(c ...
```

3.353.3 Rubi [A] (verified)

Time = 5.06 (sec) , antiderivative size = 792, normalized size of antiderivative = 0.86, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6117, 3042, 4672, 26, 3042, 26, 4201, 2620, 3011, 2720, 6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 7143, 7163, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\tanh(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \textit{6117}$$

$$\frac{\int (e+fx)^3\text{sech}^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \textit{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}$$

$$\Big\downarrow \textbf{4672}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3if\int -i(e+fx)^2\tanh(c+dx)dx}{d}}{b}$$

$$\Big\downarrow \textbf{26}$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3f\int (e+fx)^2\tanh(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3f\int -i(e+fx)^2\tan(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \textbf{26}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\int (e+fx)^2\tan(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \textbf{4201}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\int \frac{e^{2(c+dx)}(e+fx)^2}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}$$

$$\Big\downarrow \textbf{2620}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int (e+fx)\log\left(1+e^{2(c+dx)}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}$$

$$\Big\downarrow \textbf{3011}$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int \text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}$$

$$\Big\downarrow \textbf{2720}$$

3.353. $\displaystyle\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)\right)-}{d}}{b}-\frac{a\int\frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\Big\downarrow \text{6107}$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)\right)-}{d}}{b}-\frac{a\left(\frac{b^2\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)\right)-}{d}}{b}$$
$$\frac{a\left(\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{b}$$

$$\Big\downarrow \text{3803}$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)\right)-}{d}}{b}-\frac{a\left(\frac{2b^2\int-\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+$$

$$\Big\downarrow \text{25}$$

3.353. $\int\frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{a\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int \frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{b}+\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-}{d}}{b}$$

$$\Big\downarrow \text{2694}$$

$$\frac{-\frac{a\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}+\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-}{d}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{a\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}+\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-}{d}}{b}$$

$$\Big\downarrow \text{2620}$$

3.353. $\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx$

$$-\frac{a\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log(}{}\right.}{}\right.}{a^2+b^2}\right.}{b}$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\right.}{d}}{b}$$

$$\Big\downarrow \textbf{3011}$$

$$-\frac{a\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\text{P}}{}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.}{b}$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\right.}{d}}{b}$$

$$\Big\downarrow \textbf{7143}$$

3.353. $\displaystyle\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\ \text{P}\cdots}{\cdots}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\right)}{\cdots}$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}$$

$$\Big\downarrow \text{7163}$$

3.353. $\displaystyle\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \operatorname{PolyLog}\cdots}{\cdots}\right)}{d}\cdots\right)}{\cdots}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}$$

$$\Big\downarrow \text{2720}$$

3.353. $\int \frac{(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\cdots}{d}\right)}{d}\cdots\right)}{\cdots}\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\right)}{\cdots}\,\cdots}{\cdots}$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}$$

$$\Big\downarrow \; 7143$$

3.353. $\int \frac{(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\text{PolyLog}\left(4\cdots\right.}{\cdots}\right.}{d}\cdots}{\cdots}\right.}{2\sqrt{a^2+b^2}}\cdots}{\cdots}\right.}{\cdots}$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}$$

$$\Big\downarrow \text{7293}$$

3.353. $\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\int \left(a(e+fx)^3\operatorname{sech}^2(c+dx)-b(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{b\ldots}{a+\ldots}\right)}{d}\ldots\right.\right.}{\ldots}\right.\right.}{\ldots}\right)}{\ldots}$$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}$$

$\big\downarrow$ 2009

3.353. $\displaystyle\int \frac{(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\,\mathrm{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}-$$

$$a\left(-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\,\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b}}\right.}{d}\cdots}{bd}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{\cdots}\cdots\right.$$

input
```
Int[((e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.353. $\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(((3*I)*f*(((-1/3*I)*(e + f*x)^3)/f + (2*I)*(((e + f*x)^2*Log[1 + E^(2*(c
+ d*x))])/(2*d) - (f*(-1/2*((e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/d + (f
*PolyLog[3, -E^(2*(c + d*x))])/(4*d^2)))/d)))/d + ((e + f*x)^3*Tanh[c + d*
x])/d)/b - (a*((-2*b^2*(-1/2*(b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2]))])/d^2))/d))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^3*Log
[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*
PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x
)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(2*Sqrt[a^2 +
 b^2])))/(a^2 + b^2) + ((a*(e + f*x)^3)/d - (6*b*f*(e + f*x)^2*ArcTan[E^(c
 + d*x)])/d^2 - (3*a*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/d^2 + ((6*I)*
b*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^3 - ((6*I)*b*f^2*(e + f*x)
*PolyLog[2, I*E^(c + d*x)])/d^3 - (3*a*f^2*(e + f*x)*PolyLog[2, -E^(2*(c +
 d*x))])/d^3 - ((6*I)*b*f^3*PolyLog[3, (-I)*E^(c + d*x)])/d^4 + ((6*I)*b*f
^3*PolyLog[3, I*E^(c + d*x)])/d^4 + (3*a*f^3*PolyLog[3, -E^(2*(c + d*x))])
/(2*d^4) + (b*(e + f*x)^3*Sech[c + d*x])/d + (a*(e + f*x)^3*Tanh[c + d*x])
/d)/(a^2 + b^2)))/b
```

3.353.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.353. $\int \frac{(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.353. $\int \frac{(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.353. $\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.353.4 Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{sech}(dx+c)\tanh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.353.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6503 vs. 2(846) = 1692.

Time = 0.40 (sec) , antiderivative size = 6503, normalized size of antiderivative = 7.09

$$\int \frac{(e+fx)^3 \operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
Too large to include
```

3.353.6 Sympy [F]

$$\int \frac{(e+fx)^3 \operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3 \tanh(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*tanh(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

3.353.7 Maxima [F]

$$\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\text{sech}\,(dx+c)\tanh\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
3*b*e^2*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2
+ b^2)*d^2)) + 6*a*f^3*integrate(x^2*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) +
b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 6*b*f^3*integrate(x^2/(a^2*d*
e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 12*a*e*f^2*
integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a
^2*d + b^2*d), x) + 12*b*e*f^2*integrate(x/(a^2*d*e^(2*d*x + 2*c) + b^2*d*
e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - e^3*(a*b*log((b*e^(-d*x - c) - a -
sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2
)*d) + 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*
d)) + 6*a*e^2*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) - 2*(b*f^3*x^3 + 3*b
*e*f^2*x^2 + 3*b*e^2*f*x + (a*f^3*x^3*e^c + 3*a*e*f^2*x^2*e^c + 3*a*e^2*f*
x*e^c)*e^(d*x))/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)
) - integrate(-2*(a*b*f^3*x^3*e^c + 3*a*b*e*f^2*x^2*e^c + 3*a*b*e^2*f*x*e^
c)*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(2*c))*e^(2*d*x) - 2*(a^3
*e^c + a*b^2*e^c)*e^(d*x)), x)
```

3.353.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.353.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)^3}{\cosh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((tanh(c + d*x)*(e + f*x)^3)/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((tanh(c + d*x)*(e + f*x)^3)/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.354 $\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.354.1 Optimal result

Integrand size = 32, antiderivative size = 648

$$
\begin{aligned}
&\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{(e+fx)^2}{bd}-\frac{a^2(e+fx)^2}{b(a^2+b^2)d}+\frac{4af(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)d^2} \\
&\quad-\frac{ab(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d}+\frac{ab(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} \\
&\quad-\frac{2f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{bd^2}+\frac{2a^2f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{b(a^2+b^2)d^2} \\
&\quad-\frac{2iaf^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)d^3}+\frac{2iaf^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)d^3} \\
&\quad-\frac{2abf(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d^2}+\frac{2abf(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d^2} \\
&\quad-\frac{f^2\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{bd^3}+\frac{a^2f^2\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{b(a^2+b^2)d^3} \\
&\quad+\frac{2abf^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d^3}-\frac{2abf^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d^3} \\
&\quad-\frac{a(e+fx)^2\text{sech}(c+dx)}{(a^2+b^2)d}+\frac{(e+fx)^2\tanh(c+dx)}{bd}-\frac{a^2(e+fx)^2\tanh(c+dx)}{b(a^2+b^2)d}
\end{aligned}
$$

output
```
(f*x+e)^2/b/d-a^2*(f*x+e)^2/b/(a^2+b^2)/d+4*a*f*(f*x+e)*arctan(exp(d*x+c))
/(a^2+b^2)/d^2-2*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/b/d^2+2*a^2*f*(f*x+e)*ln(1
+exp(2*d*x+2*c))/b/(a^2+b^2)/d^2-a*b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/(a^2+b^2)^(3/2)/d+a*b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/(a^2+b^2)^(3/2)/d+2*I*a*f^2*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^3
-2*I*a*f^2*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^3-f^2*polylog(2,-exp(2*d*x
+2*c))/b/d^3+a^2*f^2*polylog(2,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^3-2*a*b*f*(f
*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2+2*a
*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/
d^2+2*a*b*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)
/d^3-2*a*b*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2
)/d^3-a*(f*x+e)^2*sech(d*x+c)/(a^2+b^2)/d+(f*x+e)^2*tanh(d*x+c)/b/d-a^2*(f
*x+e)^2*tanh(d*x+c)/b/(a^2+b^2)/d
```

3.354.2 Mathematica [A] (verified)

Time = 4.49 (sec) , antiderivative size = 633, normalized size of antiderivative = 0.98

$$\int \frac{(e+fx)^2 \text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{\frac{f\left(4bd^2ee^{2c}x-4bd^2e\left(1+e^{2c}\right)x+2bd^2e^{2c}fx^2-2bd^2\left(1+e^{2c}\right)fx^2+4ade\left(1+e^{2c}\right)\arctan\left(e^{c+dx}\right)+2bde\left(1+e^{2c}\right)\left(2dx-\log\left(1+e^{2(c+dx)}\right)\right)+2ia\left(1+e^{2c}\right)\ldots}{(a^2+\ldots}}{}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
((f*(4*b*d^2*e*E^(2*c)*x - 4*b*d^2*e*(1 + E^(2*c))*x + 2*b*d^2*E^(2*c)*f*x
^2 - 2*b*d^2*(1 + E^(2*c))*f*x^2 + 4*a*d*e*(1 + E^(2*c))*ArcTan[E^(c + d*x
)] + 2*b*d*e*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (2*I)*a*(1
 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - Pol
yLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + b*(1 + E^(2*c))*f
*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))])))
/((a^2 + b^2)*(1 + E^(2*c))) + (a*b*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))
/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d^2*
e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 +
 (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*E^
(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + S
qrt[a^2 + b^2])] - 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
))]))/(a^2 + b^2)^(3/2) + (d^2*(e + f*x)^2*Sech[c + d*x]*(-a + b*Sech[c]*S
inh[d*x]))/(a^2 + b^2))/d^3
```

3.354.3 Rubi [A] (verified)

Time = 3.63 (sec) , antiderivative size = 573, normalized size of antiderivative = 0.88, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {6117, 3042, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838, 6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6117}$$

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2if\int -i(e+fx)\tanh(c+dx)dx}{d}}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int(e+fx)\tanh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int-i(e+fx)\tan(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\int(e+fx)\tan(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \; 4201$$

$$-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\int\frac{e^{2(c+dx)}(e+fx)}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \; 2620$$

$$-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int\log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \; 2715$$

$$-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int e^{-2(c+dx)}\log\left(1+e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \; 2838$$

$$-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \; 6107$$

3.354. $\displaystyle\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{b^2\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{b}$$

$$\Big\downarrow \textbf{3803}$$

$$-\frac{a\left(\frac{2b^2\int-\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \textbf{2694}$$

$$-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

3.354. $\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \; 2620$$

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(-\right.}{b}\right.}{}\right.}{a^2+b^2}\right.}{b}$$

$$\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \; 3011$$

3.354. $\displaystyle\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\int (e+fx)^2\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2\cdots\right)}{d}\cdots\right)}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\cdots\right)}{b}-\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \; 2720$$

$$-\frac{a\left(\frac{\int (e+fx)^2\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+\cdots}{\cdots}\cdots\right)}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\cdots\right)}{b}-\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

3.354. $\displaystyle\int \frac{(e+fx)^2\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{7143}$$

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\cdots\right)}{d}\cdots\right)}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\cdots\right)}{b}\cdots$$

$$\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \text{7293}$$

3.354. $\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\int \left(a(e+fx)^2\text{sech}^2(c+dx)-b(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}\cdots\right.}{\cdots}\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}-$$

$$\frac{a\left(-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\cdots\right.}{\cdots}\right.}{a^2+b^2}\cdots\right.}{\cdots}$$

3.354. $\displaystyle\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(((2*I)*f*(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((e + f*x)*Log[1 + E^(2*(c +
d*x))])/(2*d) + (f*PolyLog[2, -E^(2*(c + d*x))])/(4*d^2))))/d + ((e + f*x)
^2*Tanh[c + d*x])/d)/b - (a*((-2*b^2*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x
)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e +
f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog
[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 +
 b^2])))/(a^2 + b^2) + ((a*(e + f*x)^2)/d - (4*b*f*(e + f*x)*ArcTan[E^(c +
 d*x)])/d^2 - (2*a*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/d^2 + ((2*I)*b*f^
2*PolyLog[2, (-I)*E^(c + d*x)])/d^3 - ((2*I)*b*f^2*PolyLog[2, I*E^(c + d*x
)])/d^3 - (a*f^2*PolyLog[2, -E^(2*(c + d*x))])/d^3 + (b*(e + f*x)^2*Sech[c
 + d*x])/d + (a*(e + f*x)^2*Tanh[c + d*x])/d)/(a^2 + b^2)))/b
```

3.354.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.354. $\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.354. $\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.354. $\int \frac{(e+fx)^2 \text{sech}(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$

3.354.4 Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)\tanh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.354.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3662 vs. 2(601) = 1202.

Time = 0.33 (sec) , antiderivative size = 3662, normalized size of antiderivative = 5.65

$$\int \frac{(e+fx)^2 \operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-(2*(a^2*b + b^3)*d^2*e^2 - 4*(a^2*b + b^3)*c*d*e*f + 2*(a^2*b + b^3)*c^2*
f^2 - 2*((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*d^2*e*f*x + 2*(a^2*b
+ b^3)*c*d*e*f - (a^2*b + b^3)*c^2*f^2)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)
*d^2*f^2*x^2 + 2*(a^2*b + b^3)*d^2*e*f*x + 2*(a^2*b + b^3)*c*d*e*f - (a^2*
b + b^3)*c^2*f^2)*sinh(d*x + c)^2 + 2*(a*b^2*d*f^2*x + a*b^2*d*e*f + (a*b^
2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*f^2*x + a*b^2*d*e*f)
*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*f^2*x + a*b^2*d*e*f)*sinh(d*x + c)
^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(a*b^
2*d*f^2*x + a*b^2*d*e*f + (a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^2 +
2*(a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*f^2
*x + a*b^2*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x
 + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) - (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2 +
 (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)^2 + 2*(a*
b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)
 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*sinh(d*x + c)^2)*sqrt
((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^
2 + b^2)/b^2) + 2*a) + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2 +
(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)^2 + 2*(...
```

3.354.6 Sympy [F]

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\tanh(c+dx)\,\text{sech}\,(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*tanh(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

3.354.7 Maxima [F]

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}\,(dx+c)\tanh\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
2*b*e*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 +
b^2)*d^2)) + 4*a*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*
d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 4*b*f^2*integrate(x/(a^2*d*e^(2*d
*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - e^2*(a*b*log((b*e
^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))
/((a^2 + b^2)^(3/2)*d) + 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^2)*
e^(-2*d*x - 2*c))*d)) + 4*a*e*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) - 2*
(b*f^2*x^2 + 2*b*e*f*x + (a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x))/(a^2*d +
 b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) - integrate(-2*(a*b*f^
2*x^2*e^c + 2*a*b*e*f*x*e^c)*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e
^(2*c))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x)
```

3.354.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.354.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)^2}{\cosh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((tanh(c + d*x)*(e + f*x)^2)/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((tanh(c + d*x)*(e + f*x)^2)/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.355 $\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.355.1 Optimal result

Integrand size = 30, antiderivative size = 335

$$\begin{aligned}
&\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{af\arctan(\sinh(c+dx))}{(a^2+b^2)\,d^2} - \frac{ab(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} \\
&\quad + \frac{ab(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{f\log(\cosh(c+dx))}{bd^2} + \frac{a^2f\log(\cosh(c+dx))}{b\,(a^2+b^2)\,d^2} \\
&\quad - \frac{abf\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} + \frac{abf\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} \\
&\quad - \frac{a(e+fx)\text{sech}(c+dx)}{(a^2+b^2)\,d} + \frac{(e+fx)\tanh(c+dx)}{bd} - \frac{a^2(e+fx)\tanh(c+dx)}{b\,(a^2+b^2)\,d}
\end{aligned}$$

output
```
a*f*arctan(sinh(d*x+c))/(a^2+b^2)/d^2-f*ln(cosh(d*x+c))/b/d^2+a^2*f*ln(cos
h(d*x+c))/b/(a^2+b^2)/d^2-a*b*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)
))/(a^2+b^2)^(3/2)/d+a*b*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a
^2+b^2)^(3/2)/d-a*b*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^
2)^(3/2)/d^2+a*b*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^
(3/2)/d^2-a*(f*x+e)*sech(d*x+c)/(a^2+b^2)/d+(f*x+e)*tanh(d*x+c)/b/d-a^2*(f
*x+e)*tanh(d*x+c)/b/(a^2+b^2)/d
```

3.355.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.65 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.99

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\frac{2f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a-ib}+\frac{2f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a+ib}+\frac{f\log(\cosh(c+dx))}{ia-b}-\frac{f\log(\cosh(c+dx))}{ia+b}-\frac{2ab\left(-2de\,\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)\right.}{}}{}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((2*f*ArcTan[Tanh[(c + d*x)/2]])/(a - I*b) + (2*f*ArcTan[Tanh[(c + d*x)/2]
])/(a + I*b) + (f*Log[Cosh[c + d*x]])/(I*a - b) - (f*Log[Cosh[c + d*x]])/(
I*a + b) - (2*a*b*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2
*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b
*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2 + b^2
)^(3/2) + (2*d*(e + f*x)*Sech[c + d*x]*(-a + b*Sinh[c + d*x]))/(a^2 + b^2)
)/(2*d^2)
```

3.355.3 Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.93, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6117, 3042, 4672, 26, 3042, 26, 3956, 6107, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$
$$\Big\downarrow \text{6117}$$
$$\frac{\int (e+fx)\text{sech}^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int -i\tanh(c+dx)dx}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int \tanh(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int -i\tan(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int \tan(ic+idx)dx}{d}}{b}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6107}$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{b^2\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{b}$$

$$\Big\downarrow \text{3803}$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{2b^2\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}$$

3.355. $\displaystyle \int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 25$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{b}$$

$$\Big\downarrow \; 2694$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int-\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{2\sqrt{a}}\right.}{a^2+b^2}\right.}{b}$$

$$\Big\downarrow \; 2715$$

3.355. $\displaystyle\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-$$

$$\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\cdots\right.}{b\cdots}\right.}{\cdots}\right.}{a^2+b^2}\right.}{b}$$

$$\Big\downarrow \; 2838$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-$$

$$\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}\cdots\right.}{2\sqrt{\cdots}}\right.}{a^2+b^2}\right.}{b}$$

$$\Big\downarrow \; 7293$$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-$$

$$\frac{a\left(\frac{\int \left(a(e+fx)\text{sech}^2(c+dx)-b(e+fx)\text{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\cdots}{\cdots}\right.}{\cdots}\right.}{a^2+b^2}\right.}{b}$$

$$\Big\downarrow \; 2009$$

3.355. $\displaystyle\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-$$

$$\frac{a\left(\frac{-\frac{af\log(\cosh(c+dx))}{d^2}+\frac{a(e+fx)\tanh(c+dx)}{d}-\frac{bf\arctan(\sinh(c+dx))}{d^2}+\frac{b(e+fx)\operatorname{sech}(c+dx)}{d}}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}}\ldots\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\ldots\right)}{\ldots}\right)}{b}$$

input
```
Int[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-((f*Log[Cosh[c + d*x]])/d^2) + ((e + f*x)*Tanh[c + d*x])/d)/b - (a*((-2*
b^2*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/
Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b
*d^2)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + (-((b*f*ArcTan[Sinh[c + d*x]])
/d^2) - (a*f*Log[Cosh[c + d*x]])/d^2 + (b*(e + f*x)*Sech[c + d*x])/d + (a*
(e + f*x)*Tanh[c + d*x])/d)/(a^2 + b^2)))/b
```

3.355.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.355. $\int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.355. $\int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.355.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1857 vs. 2(315) = 630.

Time = 5.27 (sec) , antiderivative size = 1858, normalized size of antiderivative = 5.55

method	result	size
risch	Expression too large to display	1858

input
```
int((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERB
OSE)
```

output
```
2/d^2/(a^2+b^2)^(5/2)*f*b*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)
)*a^3+2/d^2/(a^2+b^2)^(5/2)*f*b^3*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))*a+4/d^2/(a^2+b^2)*f*b^2/(2*a^2+2*b^2)*a*arctan(exp(d*x+c))-2/d^2
/(a^2+b^2)^(3/2)*a*b^3*c*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/
(a^2+b^2)^(1/2))-2/d^2/(a^2+b^2)^(3/2)*a^3*b*c*f/(2*a^2+2*b^2)*arctanh(1/2
*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d^2/(a^2+b^2)^(1/2)*a*b*c*f/(2*a^
2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d/(a^2+b^2)^(
3/2)*a^3*b*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b
^2)^(1/2)))*x+2/d/(a^2+b^2)^(3/2)*a^3*b*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(
a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-2/d/(a^2+b^2)^(3/2)*a*b^3*f/(2*a^
2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2*(f
*x+e)*(a*exp(d*x+c)+b)/d/(a^2+b^2)/(1+exp(2*d*x+2*c))-2/d^2/(a^2+b^2)^(3/2
)*a^3*b*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b
^2)^(1/2)))-2/d^2/(a^2+b^2)^(3/2)*a*b^3*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+
c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/d^2/(a^2+b^2)^(3/2)*a^3*b*f/
(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2/(a^2
+b^2)^(3/2)*a*b^3*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(
a+(a^2+b^2)^(1/2)))-2/d^2/(a^2+b^2)^(1/2)*a*b*f/(2*a^2+2*b^2)*arctanh(1/2*
(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d/(a^2+b^2)^(1/2)*a*b*e/(2*a^2+2*b
^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2/(a^2+b^2)^(...
```

3.355.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1338 vs. $2(313) = 626$.

Time = 0.31 (sec) , antiderivative size = 1338, normalized size of antiderivative = 3.99

$$\int \frac{(e + fx)\text{sech}(c + dx)\tanh(c + dx)}{a + b\sinh(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
(2*(a^2*b + b^3)*d*f*x*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*d*f*x*sinh(d*x +
c)^2 - 2*(a^2*b + b^3)*d*e - (a*b^2*f*cosh(d*x + c)^2 + 2*a*b^2*f*cosh(d*x
 + c)*sinh(d*x + c) + a*b^2*f*sinh(d*x + c)^2 + a*b^2*f)*sqrt((a^2 + b^2)/
b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a*b^2*f*cosh(d*x + c)^2 + 2
*a*b^2*f*cosh(d*x + c)*sinh(d*x + c) + a*b^2*f*sinh(d*x + c)^2 + a*b^2*f)*
sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a*b^2*d*e
- a*b^2*c*f + (a*b^2*d*e - a*b^2*c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*e - a*b
^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*e - a*b^2*c*f)*sinh(d*x + c
)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b
*sqrt((a^2 + b^2)/b^2) + 2*a) - (a*b^2*d*e - a*b^2*c*f + (a*b^2*d*e - a*b^
2*c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*e - a*b^2*c*f)*cosh(d*x + c)*sinh(d*x
+ c) + (a*b^2*d*e - a*b^2*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(
2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) -
 (a*b^2*d*f*x + a*b^2*c*f + (a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)^2 + 2*
(a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*f*x + a*b
^2*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*s
inh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
 b)/b) + (a*b^2*d*f*x + a*b^2*c*f + (a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x ...
```

3.355.6 Sympy [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\tanh{(c+dx)}\operatorname{sech}{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*tanh(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.355.7 Maxima [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{sech}\,(dx+c)\tanh\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
-(2*a*b*integrate(-x*e^(d*x + c)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(2*
c))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x) + 2*(a*x*e^(d*x + c)
+ b*x)/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) - 2*b*x
/((a^2 + b^2)*d) - 2*a*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) + b*log(e^(2*
d*x + 2*c) + 1)/((a^2 + b^2)*d^2))*f - e*(a*b*log((b*e^(-d*x - c) - a - sq
rt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*
d) + 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d)
)
```

3.355.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
Timed out
```

3.355.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)}{\cosh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((tanh(c + d*x)*(e + f*x))/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((tanh(c + d*x)*(e + f*x))/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.356 $\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.356.1 Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2ab\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{\operatorname{sech}(c+dx)(a-b\sinh(c+dx))}{\left(a^2+b^2\right)d}$$

output
```
2*a*b*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d
-sech(d*x+c)*(a-b*sinh(d*x+c))/(a^2+b^2)/d
```

3.356.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{-2ab\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right) - a\sqrt{-a^2-b^2}\operatorname{sech}(c+dx) + b\sqrt{-a^2-b^2}\tanh(c+dx)}{\left(-a^2-b^2\right)^{3/2}d}$$

input
```
Integrate[(Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-2*a*b*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] - a*Sqrt[-a^2
 - b^2]*Sech[c + d*x] + b*Sqrt[-a^2 - b^2]*Tanh[c + d*x])/((-a^2 - b^2)^(3
/2)*d))
```

3.356.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 26, 3345, 26, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)}{\cos(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)}{\cos(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3345}$$

$$-i\left(-\frac{\int \frac{iab}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{i\text{sech}(c+dx)(a-b\sinh(c+dx))}{d(a^2+b^2)}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{i\int \frac{ab}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{i\text{sech}(c+dx)(a-b\sinh(c+dx))}{d(a^2+b^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$-i\left(-\frac{iab\int \frac{1}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{i\text{sech}(c+dx)(a-b\sinh(c+dx))}{d(a^2+b^2)}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{iab\int \frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{i\text{sech}(c+dx)(a-b\sinh(c+dx))}{d(a^2+b^2)}\right)$$

$$\Big\downarrow \text{3139}$$

$$-i\left(-\frac{2ab\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(a^2+b^2\right)}-\frac{i\text{sech}(c+dx)(a-b\sinh(c+dx))}{d\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{1083}$$

$$-i\left(\frac{4ab\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{d\left(a^2+b^2\right)}-\frac{i\text{sech}(c+dx)(a-b\sinh(c+dx))}{d\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{217}$$

$$-i\left(-\frac{2iab\,\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{i\text{sech}(c+dx)(a-b\sinh(c+dx))}{d\left(a^2+b^2\right)}\right)$$

input
```
Int[(Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-I)*(((-2*I)*a*b*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/((a^2 +
b^2)^(3/2)*d) - (I*Sech[c + d*x]*(a - b*Sinh[c + d*x]))/((a^2 + b^2)*d))
```

3.356.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

3.356.4 Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{\frac{2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2a}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{4ab\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}}{d}$
default	$\frac{\frac{2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2a}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{4ab\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}}{d}$
risch	$-\frac{2\left(a\,\mathrm{e}^{dx+c}+b\right)}{d\left(a^2+b^2\right)\left(1+\mathrm{e}^{2dx+2c}\right)}+\frac{ba\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}-\frac{ba\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}$

input
```
int(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/(a^2+b^2)*(b*tanh(1/2*d*x+1/2*c)-a)/(1+tanh(1/2*d*x+1/2*c)^2)-4*a*b
/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(
a^2+b^2)^(1/2)))
```

3.356.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(75) = 150$.

Time = 0.26 (sec) , antiderivative size = 350, normalized size of antiderivative = 4.49

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{2\,a^2b+2\,b^3-\left(ab\cosh\left(dx+c\right)^2+2\,ab\cosh\left(dx+c\right)\sinh\left(dx+c\right)+ab\sinh\left(dx+c\right)^2+ab\right)\sqrt{a^2+b^2}\,\mathrm{lo}}{(a^4+2\,a^2b^2+b^4)d\cosh\left(dx+c\right)^2+2\,(a^4}$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*a^2*b + 2*b^3 - (a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x +
c) + a*b*sinh(d*x + c)^2 + a*b)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 +
 b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x
 + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d
*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) +
 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a^3 + a*b^2)*cosh(d*x +
c) + 2*(a^3 + a*b^2)*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x +
c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*
a^2*b^2 + b^4)*d*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)
```

3.356.6 Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh\left(c+dx\right)\operatorname{sech}\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.356.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.50

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{ab\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}d} - \frac{2\left(ae^{(-dx-c)}-b\right)}{(a^2+b^2+(a^2+b^2)e^{(-2\,dx-2\,c)})d}$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-a*b*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/((a^2 + b^2)^(3/2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 +
 (a^2 + b^2)*e^(-2*d*x - 2*c))*d)
```

3.356.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.36

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{ab\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2\left(ae^{(dx+c)}+b\right)}{(a^2+b^2)\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(a*b*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x +
c) + 2*a + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a*e^(d*x + c) + b)/(
(a^2 + b^2)*(e^(2*d*x + 2*c) + 1)))/d
```

3.356.9 Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.18

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a\,b\,\ln\left(\frac{2\,a\,\mathrm{e}^{c+d\,x}}{a^2+b^2}+\frac{2\,a\,\left(b-a\,\mathrm{e}^{c+d\,x}\right)}{{\left(a^2+b^2\right)}^{3/2}}\right)}{d\,{\left(a^2+b^2\right)}^{3/2}} - \frac{a\,b\,\ln\left(\frac{2\,a\,\mathrm{e}^{c+d\,x}}{a^2+b^2}-\frac{2\,a\,\left(b-a\,\mathrm{e}^{c+d\,x}\right)}{{\left(a^2+b^2\right)}^{3/2}}\right)}{d\,{\left(a^2+b^2\right)}^{3/2}} - \frac{\frac{2\,b}{d\,\left(a^2+b^2\right)}+\frac{2\,a\,\mathrm{e}^{c+d\,x}}{d\,\left(a^2+b^2\right)}}{\mathrm{e}^{2\,c+2\,d\,x}+1}$$

input
```
int(tanh(c + d*x)/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
(a*b*log((2*a*exp(c + d*x))/(a^2 + b^2) + (2*a*(b - a*exp(c + d*x)))/(a^2
+ b^2)^(3/2)))/(d*(a^2 + b^2)^(3/2)) - (a*b*log((2*a*exp(c + d*x))/(a^2 +
b^2) - (2*a*(b - a*exp(c + d*x)))/(a^2 + b^2)^(3/2)))/(d*(a^2 + b^2)^(3/2)
) - ((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(2*c
+ 2*d*x) + 1)
```

3.357 $\int \frac{\text{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.357.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\text{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\text{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.357.2 Mathematica [N/A]

Not integrable

Time = 71.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\text{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\text{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Sech[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

output
```
Integrate[(Sech[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

3.357.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\tanh(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Sech[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.357.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.357.4 Maple [N/A] (verified)

Not integrable

Time = 0.69 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}\left(dx+c\right)\tanh\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.357.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)\tanh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
integral(sech(d*x + c)*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
 + c)), x)
```

3.357.6 Sympy [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\tanh\left(c+dx\right)\operatorname{sech}\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)*sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.357.7 Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 386, normalized size of antiderivative = 12.06

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)\tanh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
-2*a*b*integrate(-e^(d*x + c)/(a^2*b*e + b^3*e + (a^2*b*f + b^3*f)*x - (a^
2*b*e*e^(2*c) + b^3*e*e^(2*c) + (a^2*b*f*e^(2*c) + b^3*f*e^(2*c))*x)*e^(2*
d*x) - 2*(a^3*e*e^c + a*b^2*e*e^c + (a^3*f*e^c + a*b^2*f*e^c)*x)*e^(d*x)),
 x) - 2*(a*e^(d*x + c) + b)/(a^2*d*e + b^2*d*e + (a^2*d*f + b^2*d*f)*x + (
a^2*d*e*e^(2*c) + b^2*d*e*e^(2*c) + (a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x)
*e^(2*d*x)) - 2*integrate((a*f*e^(d*x + c) + b*f)/(a^2*d*e^2 + b^2*d*e^2 +
 (a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*e*f + b^2*d*e*f)*x + (a^2*d*e^2*e^
(2*c) + b^2*d*e^2*e^(2*c) + (a^2*d*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 +
2*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x)*e^(2*d*x)), x)
```

3.357.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
Timed out
```

3.357.9 Mupad [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+d\,x)}{\cosh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(tanh(c + d*x)/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(tanh(c + d*x)/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.358 $\int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.358.1 Optimal result

Integrand size = 34, antiderivative size = 1176

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{(e+fx)^2\arctan\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)^2\arctan\left(e^{c+dx}\right)}{(a^2+b^2)^2 d}$$
$$- \frac{a^2(e+fx)^2\arctan\left(e^{c+dx}\right)}{b(a^2+b^2)d} - \frac{f^2\arctan(\sinh(c+dx))}{bd^3} + \frac{a^2f^2\arctan(\sinh(c+dx))}{b(a^2+b^2)d^3}$$
$$- \frac{ab^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} - \frac{ab^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d}$$
$$+ \frac{ab^2(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)^2 d} - \frac{af^2\log(\cosh(c+dx))}{(a^2+b^2)d^3}$$
$$- \frac{if(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{bd^2} + \frac{2ia^2bf(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)^2 d^2}$$
$$+ \frac{ia^2f(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{b(a^2+b^2)d^2} + \frac{if(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{bd^2}$$
$$- \frac{2ia^2bf(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)^2 d^2} - \frac{ia^2f(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{b(a^2+b^2)d^2}$$
$$- \frac{2ab^2f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d^2} - \frac{2ab^2f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d^2}$$
$$+ \frac{ab^2f(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{(a^2+b^2)^2 d^2} + \frac{if^2\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{bd^3}$$
$$- \frac{2ia^2bf^2\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{(a^2+b^2)^2 d^3} - \frac{ia^2f^2\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{b(a^2+b^2)d^3}$$
$$- \frac{if^2\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{bd^3} + \frac{2ia^2bf^2\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{(a^2+b^2)^2 d^3} + \frac{ia^2f^2\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{b(a^2+b^2)d^3}$$
$$+ \frac{2ab^2f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d^3} + \frac{2ab^2f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d^3}$$
$$- \frac{ab^2f^2\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2(a^2+b^2)^2 d^3} + \frac{f(e+fx)\text{sech}(c+dx)}{bd^2}$$
$$- \frac{a^2f(e+fx)\text{sech}(c+dx)}{b(a^2+b^2)d^2} - \frac{a(e+fx)^2\text{sech}^2(c+dx)}{2(a^2+b^2)d} + \frac{af(e+fx)\tanh(c+dx)}{(a^2+b^2)d^2}$$
$$+ \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{2bd} - \frac{a^2(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{2b(a^2+b^2)d}$$

output
```
-a^2*f*(f*x+e)*sech(d*x+c)/b/(a^2+b^2)/d^2+2*I*a^2*b*f*(f*x+e)*polylog(2,-
I*exp(d*x+c))/(a^2+b^2)^2/d^2-I*a^2*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b/(a
^2+b^2)/d^2-2*I*a^2*b*f*(f*x+e)*polylog(2,I*exp(d*x+c))/(a^2+b^2)^2/d^2+a*
b^2*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2-1/2*a*(f*x+e)^2*s
ech(d*x+c)^2/(a^2+b^2)/d+1/2*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/b/d-I*f^2*p
olylog(3,I*exp(d*x+c))/b/d^3+a^2*f^2*arctan(sinh(d*x+c))/b/(a^2+b^2)/d^3+I
*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b/d^2+a*f*(f*x+e)*tanh(d*x+c)/(a^2+b^2)
/d^2+I*f^2*polylog(3,-I*exp(d*x+c))/b/d^3+f*(f*x+e)*sech(d*x+c)/b/d^2+a*b^
2*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/(a^2+b^2)^2/d-a*b^2*(f*x+e)^2*ln(1+b*exp(
d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-a*b^2*(f*x+e)^2*ln(1+b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-a*f^2*ln(cosh(d*x+c))/(a^2+b^2)/d^3-2
*a^2*b*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)^2/d-1/2*a*b^2*f^2*polylog(3,
-exp(2*d*x+2*c))/(a^2+b^2)^2/d^3+2*a*b^2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/(a^2+b^2)^2/d^3+2*a*b^2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))/(a^2+b^2)^2/d^3-I*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b/d^2-
1/2*a^2*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/b/(a^2+b^2)/d-2*a*b^2*f*(f*x+e)*
polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-2*a*b^2*f*(f*
x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-2*I*a^2*
b*f^2*polylog(3,-I*exp(d*x+c))/(a^2+b^2)^2/d^3-I*a^2*f^2*polylog(3,-I*exp(
d*x+c))/b/(a^2+b^2)/d^3+(f*x+e)^2*arctan(exp(d*x+c))/b/d-f^2*arctan(sin...
```

3.358.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3390 vs. 2(1176) = 2352.

Time = 12.13 (sec) , antiderivative size = 3390, normalized size of antiderivative = 2.88

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(-12*a*b^2*d^3*e^2*E^(2*c)*x + 12*a^3*d*E^(2*c)*f^2*x + 12*a*b^2*d*E^(2*c)
*f^2*x - 12*a*b^2*d^3*e*E^(2*c)*f*x^2 - 4*a*b^2*d^3*E^(2*c)*f^2*x^3 - 6*a^
2*b*d^2*e^2*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*ArcTan[E^(c + d*x)] - 6*a^
2*b*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*E^(2*c)*ArcTan[E^(
c + d*x)] - 12*a^2*b*f^2*ArcTan[E^(c + d*x)] - 12*b^3*f^2*ArcTan[E^(c + d*
x)] - 12*a^2*b*E^(2*c)*f^2*ArcTan[E^(c + d*x)] - 12*b^3*E^(2*c)*f^2*ArcTan
[E^(c + d*x)] - (6*I)*a^2*b*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*b^3*d
^2*e*f*x*Log[1 - I*E^(c + d*x)] - (6*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 - I*
E^(c + d*x)] + (6*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] - (3*I)*
a^2*b*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*d^2*f^2*x^2*Log[1 - I
*E^(c + d*x)] - (3*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (
3*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (6*I)*a^2*b*d^2*e*f*
x*Log[1 + I*E^(c + d*x)] - (6*I)*b^3*d^2*e*f*x*Log[1 + I*E^(c + d*x)] + (6
*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (6*I)*b^3*d^2*e*E^(2*
c)*f*x*Log[1 + I*E^(c + d*x)] + (3*I)*a^2*b*d^2*f^2*x^2*Log[1 + I*E^(c + d
*x)] - (3*I)*b^3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] + (3*I)*a^2*b*d^2*E^(2
*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 +
 I*E^(c + d*x)] + 6*a*b^2*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 6*a*b^2*d^2*e
^2*E^(2*c)*Log[1 + E^(2*(c + d*x))] - 6*a^3*f^2*Log[1 + E^(2*(c + d*x))] -
 6*a*b^2*f^2*Log[1 + E^(2*(c + d*x))] - 6*a^3*E^(2*c)*f^2*Log[1 + E^(2*...
```

3.358.3 Rubi [A] (verified)

Time = 5.09 (sec) , antiderivative size = 958, normalized size of antiderivative = 0.81, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6117, 3042, 4674, 3042, 4257, 4668, 3011, 2720, 6107, 6107, 6095, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6117}$$

$$\frac{\int (e+fx)^2\text{sech}^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}$$

$$\Big\downarrow \ 4674$$

$$\frac{-\frac{f^2\int \text{sech}(c+dx)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\text{sech}(c+dx)dx+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{f^2\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$$\Big\downarrow \ 4257$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$$\Big\downarrow \ 4668$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{2if\int (e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}\right)-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{f(e+fx)\text{se}\cdots}{d^2}\cdots}{b}$$

$$\Big\downarrow \ 3011$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}+\frac{2(e+fx)^2\text{arc}\cdots}{d}\right.\cdots}{b}$$

$$\Big\downarrow \ 2720$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c}\cdots\right)}{d}\cdots\right.}{d}\right.\cdots}{b}$$

3.358. $\displaystyle\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 6107$$

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.}{b}$$

$$\Big\downarrow \; 6107$$

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}\right.}{b}$$

$$\Big\downarrow \; 6095$$

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)}{a^2+}\right.}{a^2+b^2}\right.}{b}$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}\right.}{b}$$

$$\Big\downarrow \; 2620$$

3.358. $\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\right.}{}\right.}{a^2+b^2}\right.}{}\right)}{b}$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}\right.}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{a\left(\frac{b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{}{a-}\right.}{d}\right.}{bd}\right.}{a^2+b^2}\right.}{a^2+b^2}\right.}{}$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c}\right.}{d}\right.}{d}\right.}{b}$$

$$\Big\downarrow \text{2720}$$

3.358. $\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+\cdots}}{d^2}\cdots\right.}{bd}\cdots\right.}{a^2+b^2}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c\cdots}\right)}{d}\cdots\right.}{d}\cdots\right.}{b}$$

$$\Big\downarrow \; 7143$$

3.358. $\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{b^2\left(\frac{\int (e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{\ }{a+}\right.}{d^2}\right.}{\ }\right.}{\ }\right)}{a^2+b^2}\right)}{-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{1}{2}\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,ie^{c+}\right.}{d^2}\right.}{\ }\right.}{b}}$$

$\downarrow$ 7293

3.358. $\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{b^2\left(\frac{\int\left(a(e+fx)^2\operatorname{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyL}}{}\right.}{}\right.}{}\right)}{a^2+b^2}\right.}{}$$

$$\frac{-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{1}{2}\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,ie^{c+}\right.}{d^2}\right.}{}\right.}{b}$$

$\downarrow$ 2009

3.358. $\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{\arctan(\sinh(c+dx))f^2}{d^3}+\frac{(e+fx)\operatorname{sech}(c+dx)f}{d^2}+\frac{1}{2}\left(\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^2}{d}+\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}\right.}{b}$$

$$\frac{a\left(\frac{\left(\frac{\left(-\frac{(e+fx)^3}{3bf}+\frac{\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+fx)^2}{bd}+\frac{\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)(e+fx)^2}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\right.}{a^2+b^2}\right.}{}\right.}{}$$

input
```
Int[((e + f*x)^2*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.358. $\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(-((f^2*ArcTan[Sinh[c + d*x]])/d^3) + ((2*(e + f*x)^2*ArcTan[E^(c + d*x)])
/d + ((2*I)*f*(-(((e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d) + (f*PolyLog[
3, (-I)*E^(c + d*x)])/d^2))/d - ((2*I)*f*(-(((e + f*x)*PolyLog[2, I*E^(c +
 d*x)])/d) + (f*PolyLog[3, I*E^(c + d*x)])/d^2))/d)/2 + (f*(e + f*x)*Sech[
c + d*x])/d^2 + ((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/b - (a*((
b^2*((b^2*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2]))])/d^2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3)/(3*f) + (2*a*(e
 + f*x)^2*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))]
)/d - ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((2*I)*a*f*
(e + f*x)*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*(e + f*x)*PolyLog[2, -E^(2
*(c + d*x))])/d^2 + ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/d^3 - ((2*I
)*a*f^2*PolyLog[3, I*E^(c + d*x)])/d^3 + (b*f^2*PolyLog[3, -E^(2*(c + d*x)
)])/(2*d^3))/(a^2 + b^2)))/(a^2 + b^2) + ((a*(e + f*x)^2*ArcTan[E^(c + d*x
)])/d - (a*f^2*ArcTan[Sinh[c + d*x]])/d^3 + (b*f^2*Log[Cosh[c + d*x]])/d^3
 - (I*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*a*f*(e + f*x...
```

3.358.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.358. $\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

3.358. $\int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.358.4 Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)^2 \tanh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.358.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 11164 vs. 2(1078) = 2156.

Time = 0.45 (sec) , antiderivative size = 11164, normalized size of antiderivative = 9.49

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.358.6 Sympy [F]

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\tanh(c+dx)\,\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*sech(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*tanh(c + d*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x))
, x)
```

3.358.7 Maxima [F]

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{sech}(dx+c)^2\tanh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-a^2*b*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*
b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^
2 + b^4*d^2), x) + b^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x
 + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^
2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a*b^2*d^2*f^2*integrate(x^2/(a^4*d^2*
e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c)
+ a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a^2*b*d^2*e*f*integrate(x*e^(
d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^
2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*b^3*d^2*e*f
*integrate(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x
 + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x)
 - 4*a*b^2*d^2*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^
(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^
2), x) + a^3*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x
 + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) + a*b^2*f^2*(2*(d*x + c)/((a^4
 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^
4)*d^3)) - (a*b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 +
2*a^2*b^2 + b^4)*d) - a*b^2*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 +
b^4)*d) - (a^2*b - b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) -
 (b*e^(-d*x - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b...
```

3.358.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.358.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)^2}{{\cosh\left(c+d\,x\right)}^2\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((tanh(c + d*x)*(e + f*x)^2)/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((tanh(c + d*x)*(e + f*x)^2)/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))), x
)
```

3.359 $\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.359.1 Optimal result

Integrand size = 32, antiderivative size = 711

$$\int \frac{(e+fx)\mathrm{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{(e+fx)\arctan\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)\arctan\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(e+fx)\arctan\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{ab^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d} - \frac{ab^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d} + \frac{ab^2(e+fx)\log\left(1+e^{2(c+dx)}\right)}{\left(a^2+b^2\right)^2 d} - \frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{2bd^2} + \frac{ia^2bf\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{\left(a^2+b^2\right)^2 d^2} + \frac{ia^2f\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{2b\left(a^2+b^2\right)d^2} + \frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{2bd^2} - \frac{ia^2bf\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{\left(a^2+b^2\right)^2 d^2} - \frac{ia^2f\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{2b\left(a^2+b^2\right)d^2} - \frac{ab^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d^2} - \frac{ab^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d^2} + \frac{ab^2f\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2\left(a^2+b^2\right)^2 d^2} + \frac{f\mathrm{sech}(c+dx)}{2bd^2} - \frac{a^2f\mathrm{sech}(c+dx)}{2b\left(a^2+b^2\right)d^2} - \frac{a(e+fx)\mathrm{sech}^2(c+dx)}{2\left(a^2+b^2\right)d} + \frac{af\tanh(c+dx)}{2\left(a^2+b^2\right)d^2} + \frac{(e+fx)\mathrm{sech}(c+dx)\tanh(c+dx)}{2bd} - \frac{a^2(e+fx)\mathrm{sech}(c+dx)\tanh(c+dx)}{2b\left(a^2+b^2\right)d}$$

output
```
(f*x+e)*arctan(exp(d*x+c))/b/d-2*a^2*b*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2
)^2/d-a^2*(f*x+e)*arctan(exp(d*x+c))/b/(a^2+b^2)/d+a*b^2*(f*x+e)*ln(1+exp(
2*d*x+2*c))/(a^2+b^2)^2/d-a*b^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/(a^2+b^2)^2/d-a*b^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a
^2+b^2)^2/d+1/2*I*a^2*f*polylog(2,-I*exp(d*x+c))/b/(a^2+b^2)/d^2-I*a^2*b*f
*polylog(2,I*exp(d*x+c))/(a^2+b^2)^2/d^2-1/2*I*f*polylog(2,-I*exp(d*x+c))/
b/d^2+I*a^2*b*f*polylog(2,-I*exp(d*x+c))/(a^2+b^2)^2/d^2-1/2*I*a^2*f*polyl
og(2,I*exp(d*x+c))/b/(a^2+b^2)/d^2+1/2*I*f*polylog(2,I*exp(d*x+c))/b/d^2+1
/2*a*b^2*f*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2-a*b^2*f*polylog(2,-b
*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-a*b^2*f*polylog(2,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2+1/2*f*sech(d*x+c)/b/d^2-1/2*a^
2*f*sech(d*x+c)/b/(a^2+b^2)/d^2-1/2*a*(f*x+e)*sech(d*x+c)^2/(a^2+b^2)/d+1/
2*a*f*tanh(d*x+c)/(a^2+b^2)/d^2+1/2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b/d-1/
2*a^2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b/(a^2+b^2)/d
```

3.359.2 Mathematica [A] (verified)

Time = 9.12 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.15

$$\int \frac{(e+fx)\mathrm{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{ab^2\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\,\mathrm{arctanh}\left(\frac{a+}{\sqrt{}}\right.}{(-a^2-b^2)^{3/2}}\right.}{}$$

$$+\frac{b\left(-2abde(c+dx)+2abcf(c+dx)-abf(c+dx)^2-2a^2de\arctan\left(e^{c+dx}\right)+2b^2de\arctan\left(e^{c+dx}\right)+2a\right.}{}$$

$$+\frac{\mathrm{sech}(c+dx)(bf+af\sinh(c+dx))}{2(a^2+b^2)d^2}$$

$$+\frac{\mathrm{sech}^2(c+dx)(-ade+acf-af(c+dx)+bde\sinh(c+dx)-bcf\sinh(c+dx)+bf(c+dx)\sinh(c+}{2(a^2+b^2)d^2}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
-1/2*(a*b^2*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqr
t[a^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2
+ b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt
[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Lo
g[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d
*x))/(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))]))/((a^2 + b^2)^2*d^2) + (b*(-2*a*b*d*e*(c + d*x) + 2*a*b*c*f
*(c + d*x) - a*b*f*(c + d*x)^2 - 2*a^2*d*e*ArcTan[E^(c + d*x)] + 2*b^2*d*e
*ArcTan[E^(c + d*x)] + 2*a^2*c*f*ArcTan[E^(c + d*x)] - 2*b^2*c*f*ArcTan[E^
(c + d*x)] - I*a^2*f*(c + d*x)*Log[1 - I*E^(c + d*x)] + I*b^2*f*(c + d*x)*
Log[1 - I*E^(c + d*x)] + I*a^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] - I*b^2*
f*(c + d*x)*Log[1 + I*E^(c + d*x)] + 2*a*b*d*e*Log[1 + E^(2*(c + d*x))] -
2*a*b*c*f*Log[1 + E^(2*(c + d*x))] + 2*a*b*f*(c + d*x)*Log[1 + E^(2*(c + d
*x))] + I*(a^2 - b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] - I*(a^2 - b^2)*f*Pol
yLog[2, I*E^(c + d*x)] + a*b*f*PolyLog[2, -E^(2*(c + d*x))]))/(2*(a^2 + b^
2)^2*d^2) + (Sech[c + d*x]*(b*f + a*f*Sinh[c + d*x]))/(2*(a^2 + b^2)*d^2)
+ (Sech[c + d*x]^2*(-(a*d*e) + a*c*f - a*f*(c + d*x) + b*d*e*Sinh[c + d*x]
 - b*c*f*Sinh[c + d*x] + b*f*(c + d*x)*Sinh[c + d*x]))/(2*(a^2 + b^2)*d...
```

3.359.3 Rubi [A] (verified)

Time = 2.89 (sec) , antiderivative size = 600, normalized size of antiderivative = 0.84, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {6117, 3042, 4673, 3042, 4668, 2715, 2838, 6107, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6117}$$

$$\frac{\int (e+fx)\text{sech}^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}$$

$$\Big\downarrow \; 4673$$

$$\frac{\frac{1}{2}\int (e+fx)\text{sech}(c+dx)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$$\Big\downarrow \; 4668$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$$\Big\downarrow \; 2715$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(}{}}{b}$$

$$\Big\downarrow \; 2838$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$$\Big\downarrow \; 6107$$

$$-\frac{a\left(\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$$\Big\downarrow \; 6107$$

3.359. $\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$$\Big\downarrow \; 6095$$

$$-\frac{a\left(\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a\cdots}{a^2+b^2}\cdots\right.}{a^2+b^2}\cdots\right.}{b}\cdots$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$$\Big\downarrow \; 2620$$

$$-\frac{a\left(\frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\cdots\right.}{a^2+b^2}\cdots\right.}{a^2+b^2}\cdots\right.}{b}\cdots$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}$$

$$\Big\downarrow \; 2715$$

3.359. $\displaystyle\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{b^2\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\cdots\right)}{bd}\cdots\right)}{a^2+b^2}\cdots\right)}{a^2+b^2}\cdots\right)}{b}\cdots$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\mathrm{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}}{b}$$

$$\Big\downarrow \; 2838$$

$$-\frac{a\left(\frac{b^2\left(\frac{\int(e+fx)\mathrm{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\cdots}{\cdots}\cdots\right)}{a^2+b^2}\cdots\right)}{a^2+b^2}\cdots\right)}{b}\cdots$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\mathrm{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}}{b}$$

$$\Big\downarrow \; 7293$$

$$-\frac{a\left(\frac{b^2\left(\frac{\int(a(e+fx)\mathrm{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\cdots\right)}{a^2+b^2}\cdots\right)}{a^2+b^2}\cdots\right)}{b}\cdots$$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\mathrm{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}}{b}$$

$$\Big\downarrow \; 2009$$

3.359. $\int \frac{(e+fx)\mathrm{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}-$$

$$a\left(\frac{b^2\left(\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{2a(e+fx}{}\right.}{a^2+b^2}\right.$$

input
```
Int[((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(((2*(e + f*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)])
/d^2 + (I*f*PolyLog[2, I*E^(c + d*x)])/d^2)/2 + (f*Sech[c + d*x])/(2*d^2)
+ ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/b - (a*((b^2*((b^2*(-1/2*
(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b
*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (
f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2 + b
^2) + ((b*(e + f*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d - (b*
(e + f*x)*Log[1 + E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*E^(c + d*x)
])/d^2 + (I*a*f*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2*(c
+ d*x))])/(2*d^2))/(a^2 + b^2)))/(a^2 + b^2) + ((a*(e + f*x)*ArcTan[E^(c +
 d*x)])/d - ((I/2)*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((I/2)*a*f*Poly
Log[2, I*E^(c + d*x)])/d^2 + (a*f*Sech[c + d*x])/(2*d^2) + (b*(e + f*x)*Se
ch[c + d*x]^2)/(2*d) - (b*f*Tanh[c + d*x])/(2*d^2) + (a*(e + f*x)*Sech[c +
 d*x]*Tanh[c + d*x])/(2*d))/(a^2 + b^2)))/b
```

3.359. $\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.359.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

3.359. $\int \frac{(e+fx)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.359.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2073 vs. $2(656) = 1312$.

Time = 13.61 (sec) , antiderivative size = 2074, normalized size of antiderivative = 2.92

method	result	size
risch	Expression too large to display	2074

input
```
int((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output

```
1/(a^2+b^2)^(3/2)/d^2*b^2*c*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))*a^2+2*b^2/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*dilog(1+I*exp(
d*x+c))*a+2*b^2/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))*a+I*b^
3/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))-2*b/d/(a^2+b^2)*a^2*
e/(2*a^2+2*b^2)*arctan(exp(d*x+c))-2*b^2/d/(a^2+b^2)*e/(2*a^2+2*b^2)*a*ln(
b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2*b^2/d/(a^2+b^2)*e/(2*a^2+2*b^2)*a*ln(
1+exp(2*d*x+2*c))-1/(a^2+b^2)^(3/2)/d*b^4*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b
*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(1/2)/d*b^2*e/(2*a^2+2*b^2)*
arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(1/2)/d^2*b^
2*c*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a
^2+b^2)^(3/2)/d^2*b^4*c*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(
a^2+b^2)^(1/2))-1/(a^2+b^2)^(3/2)/d*b^2*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*e
xp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2+2*b^3/d/(a^2+b^2)*e/(2*a^2+2*b^2)*arct
an(exp(d*x+c))+2*b^2/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*c-
2*b^2/d/(a^2+b^2)*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a
+(a^2+b^2)^(1/2)))*a*x-2*b^2/d/(a^2+b^2)*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+
(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*x+I*b^3/d/(a^2+b^2)*f/(2*a^2+2*b
^2)*ln(1-I*exp(d*x+c))*x+2*b^2/d^2/(a^2+b^2)*c*f/(2*a^2+2*b^2)*a*ln(b*exp(
2*d*x+2*c)+2*a*exp(d*x+c)-b)-2*b^2/d^2/(a^2+b^2)*c*f/(2*a^2+2*b^2)*a*ln(1+
exp(2*d*x+2*c))+I*b/d^2/(a^2+b^2)*a^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x...
```

3.359.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4993 vs. $2(635) = 1270$.

Time = 0.38 (sec) , antiderivative size = 4993, normalized size of antiderivative = 7.02

$$\int \frac{(e+fx)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input

```
integrate((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
1/2*(2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*cosh(d*
x + c)^3 + 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*s
inh(d*x + c)^3 - 2*(2*(a^3 + a*b^2)*d*f*x + 2*(a^3 + a*b^2)*d*e + (a^3 + a
*b^2)*f)*cosh(d*x + c)^2 - 2*(2*(a^3 + a*b^2)*d*f*x + 2*(a^3 + a*b^2)*d*e
+ (a^3 + a*b^2)*f - 3*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b +
b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(a^3 + a*b^2)*f - 2*((a^2*b + b
^3)*d*f*x + (a^2*b + b^3)*d*e - (a^2*b + b^3)*f)*cosh(d*x + c) - 2*(a*b^2*
f*cosh(d*x + c)^4 + 4*a*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^2*f*sinh
(d*x + c)^4 + 2*a*b^2*f*cosh(d*x + c)^2 + a*b^2*f + 2*(3*a*b^2*f*cosh(d*x
+ c)^2 + a*b^2*f)*sinh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c)^3 + a*b^2*f*c
osh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(a
*b^2*f*cosh(d*x + c)^4 + 4*a*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^2*f
*sinh(d*x + c)^4 + 2*a*b^2*f*cosh(d*x + c)^2 + a*b^2*f + 2*(3*a*b^2*f*cosh
(d*x + c)^2 + a*b^2*f)*sinh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c)^3 + a*b^
2*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
 ((2*a*b^2*f - I*(a^2*b - b^3)*f)*cosh(d*x + c)^4 + 4*(2*a*b^2*f - I*(a^2*
b - b^3)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b^2*f - I*(a^2*b - b^3)*f
)*sinh(d*x + c)^4 + 2*a*b^2*f + 2*(2*a*b^2*f - I*(a^2*b - b^3)*f)*cosh(...
```

3.359.6 Sympy [F]

$$\int \frac{(e+fx)\mathrm{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\tanh{(c+dx)}\,\mathrm{sech}^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*tanh(c + d*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x
)
```

3.359.7 Maxima [F]

$$\int \frac{(e+fx)\mathrm{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\,\mathrm{sech}\,(dx+c)^2\tanh\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(a*b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2
+ b^4)*d) - a*b^2*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) -
(a^2*b - b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (b*e^(-d*
x - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 +
 b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e + f*(((b*d*x*
e^(3*c) + b*e^(3*c))*e^(3*d*x) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d*x) -
 (b*d*x*e^c - b*e^c)*e^(d*x) - a)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) +
b^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*
x)) + 4*integrate(-1/2*(a^2*b^2*x*e^(d*x + c) - a*b^3*x)/(a^4*b + 2*a^2*b^
3 + b^5 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*
(a^5*e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - 4*integrate(1/4*(2*a*
b^2*x + (a^2*b*e^c - b^3*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(
2*c) + 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x))
```

3.359.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\mathrm{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.359.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)\ (e+f\,x)}{\cosh\left(c+d\,x\right)^2\ \left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((tanh(c + d*x)*(e + f*x))/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((tanh(c + d*x)*(e + f*x))/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.360 $\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.360.1 Optimal result

Integrand size = 27, antiderivative size = 122

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b(a^2-b^2)\arctan(\sinh(c+dx))}{2(a^2+b^2)^2 d} + \frac{ab^2\log(\cosh(c+dx))}{(a^2+b^2)^2 d} - \frac{ab^2\log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d} - \frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d}$$

output
```
-1/2*b*(a^2-b^2)*arctan(sinh(d*x+c))/(a^2+b^2)^2/d+a*b^2*ln(cosh(d*x+c))/(
a^2+b^2)^2/d-a*b^2*ln(a+b*sinh(d*x+c))/(a^2+b^2)^2/d-1/2*sech(d*x+c)^2*(a-
b*sinh(d*x+c))/(a^2+b^2)/d
```

3.360.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{i\left(-\frac{b\log(i-\sinh(c+dx))}{(a+ib)^2} + \frac{b\log(i+\sinh(c+dx))}{(a-ib)^2} - \frac{4iab^2\log(a+b\sinh(c+dx))}{(a^2+b^2)^2} - \frac{1}{(a+ib)(-i+\sinh(c+dx))} + \frac{1}{(a-ib)(i+\sinh(c+dx))}\right)}{4d}$$

input
```
Integrate[(Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((-1/4*I)*(-((b*Log[I - Sinh[c + d*x]])/(a + I*b)^2) + (b*Log[I + Sinh[c +
 d*x]])/(a - I*b)^2 - ((4*I)*a*b^2*Log[a + b*Sinh[c + d*x]])/(a^2 + b^2)^2
 - 1/((a + I*b)*(-I + Sinh[c + d*x])) + 1/((a - I*b)*(I + Sinh[c + d*x])))
)/d
```

3.360.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3316, 26, 27, 593, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)}{\cos(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)}{\cos(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$-\frac{ib^3\int \frac{i\sinh(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{b^3\int \frac{\sinh(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^2\int \frac{b\sinh(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{593}$$

$$\frac{b^2\left(\frac{\int -\frac{a-b\sinh(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2(a^2+b^2)}-\frac{a-b\sinh(c+dx)}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^2\left(-\frac{\int \frac{a-b\sinh(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2(a^2+b^2)}-\frac{a-b\sinh(c+dx)}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{657}$$

$$\frac{b^2\left(-\frac{\int \left(\frac{2a}{\left(a^2+b^2\right)(a+b\sinh(c+dx))}+\frac{a^2-2b\sinh(c+dx)a-b^2}{\left(a^2+b^2\right)\left(\sinh^2(c+dx)b^2+b^2\right)}\right)d(b\sinh(c+dx))}{2(a^2+b^2)}-\frac{a-b\sinh(c+dx)}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\left(-\frac{\frac{\left(a^2-b^2\right)\arctan(\sinh(c+dx))}{b\left(a^2+b^2\right)}-\frac{a\log\left(b^2\sinh^2(c+dx)+b^2\right)}{a^2+b^2}+\frac{2a\log(a+b\sinh(c+dx))}{a^2+b^2}}{2(a^2+b^2)}-\frac{a-b\sinh(c+dx)}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

input
```
Int[(Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^2*(-1/2*(((a^2 - b^2)*ArcTan[Sinh[c + d*x]])/(b*(a^2 + b^2)) + (2*a*Log
[a + b*Sinh[c + d*x]])/(a^2 + b^2) - (a*Log[b^2 + b^2*Sinh[c + d*x]^2])/(a
^2 + b^2))/(a^2 + b^2) - (a - b*Sinh[c + d*x])/(2*(a^2 + b^2)*(b^2 + b^2*S
inh[c + d*x]^2))))/d
```

3.360.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

3.360. $\int \frac{\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

3.360. $\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.360.4 Maple [A] (verified)

Time = 7.81 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.74

method	result
derivativedivides	$-\frac{\frac{2\left(\frac{\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^3-a\,b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{b\left(-ab\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+\left(\right.\right.}{2}}{a^4+2a^2b^2+b^4}}{d}$
default	$-\frac{\frac{2\left(\frac{\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^3-a\,b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{b\left(-ab\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+\left(\right.\right.}{2}}{a^4+2a^2b^2+b^4}}{d}$
risch	$-\frac{2a\,b^2d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2a\,b^2dc}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2a\,b^2x}{a^4+2a^2b^2+b^4}+\frac{2a\,b^2c}{d(a^4+2a^2b^2+b^4)}-\frac{\mathrm{e}^{dx+c}\left(-b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}\right.}{d(a^2+b^2)\left(1+\mathrm{e}^{2dx+2}\right.}$

input
```
int(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(a^4+2*a^2*b^2+b^4)*(((1/2*a^2*b+1/2*b^3)*tanh(1/2*d*x+1/2*c)^3+(-
a^3-a*b^2)*tanh(1/2*d*x+1/2*c)^2+(-1/2*a^2*b-1/2*b^3)*tanh(1/2*d*x+1/2*c))
/(1+tanh(1/2*d*x+1/2*c)^2)^2+1/2*b*(-a*b*ln(1+tanh(1/2*d*x+1/2*c)^2)+(a^2-
b^2)*arctan(tanh(1/2*d*x+1/2*c))))-2*a*b^2/(2*a^4+4*a^2*b^2+2*b^4)*ln(tanh
(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a))
```

3.360.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 926 vs. $2(119) = 238$.

Time = 0.27 (sec) , antiderivative size = 926, normalized size of antiderivative = 7.59

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
((a^2*b + b^3)*cosh(d*x + c)^3 + (a^2*b + b^3)*sinh(d*x + c)^3 - 2*(a^3 +
a*b^2)*cosh(d*x + c)^2 - (2*a^3 + 2*a*b^2 - 3*(a^2*b + b^3)*cosh(d*x + c))
*sinh(d*x + c)^2 - ((a^2*b - b^3)*cosh(d*x + c)^4 + 4*(a^2*b - b^3)*cosh(d
*x + c)*sinh(d*x + c)^3 + (a^2*b - b^3)*sinh(d*x + c)^4 + a^2*b - b^3 + 2*
(a^2*b - b^3)*cosh(d*x + c)^2 + 2*(a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 4*((a^2*b - b^3)*cosh(d*x + c)^3 + (a^2*b - b^3)
*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (a^
2*b + b^3)*cosh(d*x + c) - (a*b^2*cosh(d*x + c)^4 + 4*a*b^2*cosh(d*x + c)*
sinh(d*x + c)^3 + a*b^2*sinh(d*x + c)^4 + 2*a*b^2*cosh(d*x + c)^2 + a*b^2
+ 2*(3*a*b^2*cosh(d*x + c)^2 + a*b^2)*sinh(d*x + c)^2 + 4*(a*b^2*cosh(d*x
+ c)^3 + a*b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(
cosh(d*x + c) - sinh(d*x + c))) + (a*b^2*cosh(d*x + c)^4 + 4*a*b^2*cosh(d*
x + c)*sinh(d*x + c)^3 + a*b^2*sinh(d*x + c)^4 + 2*a*b^2*cosh(d*x + c)^2 +
 a*b^2 + 2*(3*a*b^2*cosh(d*x + c)^2 + a*b^2)*sinh(d*x + c)^2 + 4*(a*b^2*co
sh(d*x + c)^3 + a*b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(c
osh(d*x + c) - sinh(d*x + c))) - (a^2*b + b^3 - 3*(a^2*b + b^3)*cosh(d*x +
 c)^2 + 4*(a^3 + a*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 +
b^4)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*
x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2
+ b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)...
```

3.360.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\tanh\left(c+dx\right)\operatorname{sech}^2\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(sech(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.360.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.79

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{ab^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^4+2\,a^2b^2+b^4)d}+\frac{ab^2\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d}$$
$$+\frac{(a^2b-b^3)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d}+\frac{be^{(-dx-c)}-2\,ae^{(-2\,dx-2\,c)}-be^{(-3\,dx-3\,c)}}{(a^2+b^2+2\,(a^2+b^2)e^{(-2\,dx-2\,c)}+(a^2+b^2)e^{(-4\,dx-4\,c)})d}$$

input
```
integrate(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-a*b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 +
 b^4)*d) + a*b^2*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) + (
a^2*b - b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (b*e^(-d*x
 - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 +
b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d)
```

3.360.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(119) = 238$.

Time = 0.32 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.34

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{\frac{4\,ab^3\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{2\,ab^2\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(a^2b-b^3)}{a^4+2\,a^2b^2+b^4}+}{4\,d}$$

input
```
integrate(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(4*a*b^3*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b + 2*a^
2*b^3 + b^5) - 2*a*b^2*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a^
2*b^2 + b^4) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a^
2*b - b^3)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a*b^2*(e^(d*x + c) - e^(-d*x - c))
^2 - 2*a^2*b*(e^(d*x + c) - e^(-d*x - c)) - 2*b^3*(e^(d*x + c) - e^(-d*x -
 c)) + 4*a^3 + 8*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^(d*x + c) - e^(-d*x -
 c))^2 + 4)))/d
```

3.360.9 Mupad [B] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.76

$$
\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx
$$
$$
= \frac{\frac{2\,a}{d\,(a^2+b^2)}-\frac{2\,b\,\mathrm{e}^{c+d\,x}}{d\,(a^2+b^2)}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}-\frac{\frac{2\,(a^3+a\,b^2)}{d\,(a^2+b^2)^2}-\frac{\mathrm{e}^{c+d\,x}\,(a^2\,b+b^3)}{d\,(a^2+b^2)^2}}{\mathrm{e}^{2\,c+2\,d\,x}+1}+\frac{b\,\ln\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}{2\,(-1\mathrm{i}\,d\,a^2+2\,d\,a\,b+1\mathrm{i}\,d\,b^2)}
$$
$$
-\frac{a\,b^2\,\ln\left(b^6\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-14\,a^2\,b^4-a^4\,b^2-b^6+28\,a^3\,b^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+14\,a^2\,b^4\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+a^4\,b^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+2\,a\,b^5\,\mathrm{e}^{d\,x}\right.}{d\,a^4+2\,d\,a^2\,b^2+d\,b^4}
$$
$$
+\frac{b\,\ln\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)\,1\mathrm{i}}{2\,(-d\,a^2+2\mathrm{i}\,d\,a\,b+d\,b^2)}
$$

input
```
int(tanh(c + d*x)/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
((2*a)/(d*(a^2 + b^2)) - (2*b*exp(c + d*x))/(d*(a^2 + b^2)))/(2*exp(2*c +
2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(a*b^2 + a^3))/(d*(a^2 + b^2)^2) - (e
xp(c + d*x)*(a^2*b + b^3))/(d*(a^2 + b^2)^2))/(exp(2*c + 2*d*x) + 1) + (b*
log(exp(c + d*x) + 1i)*1i)/(2*(b^2*d - a^2*d + a*b*d*2i)) + (b*log(exp(c +
 d*x)*1i + 1))/(2*(b^2*d*1i - a^2*d*1i + 2*a*b*d)) - (a*b^2*log(b^6*exp(2*
c)*exp(2*d*x) - 14*a^2*b^4 - a^4*b^2 - b^6 + 28*a^3*b^3*exp(d*x)*exp(c) +
14*a^2*b^4*exp(2*c)*exp(2*d*x) + a^4*b^2*exp(2*c)*exp(2*d*x) + 2*a*b^5*exp
(d*x)*exp(c) + 2*a^5*b*exp(d*x)*exp(c)))/(a^4*d + b^4*d + 2*a^2*b^2*d)
```

3.361 $\int \frac{\text{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.361.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\text{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\text{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.361.2 Mathematica [N/A]

Not integrable

Time = 75.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\text{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\text{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Sech[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Sech[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

3.361.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\tanh(c+dx)\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Sech[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.361.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.361.4 Maple [N/A] (verified)

Not integrable

Time = 0.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}\left(dx+c\right)^{2}\tanh\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.361.5 Fricas [N/A]

Not integrable

Time = 4.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^2\tanh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(sech(d*x + c)^2*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

3.361.6 Sympy [N/A]

Not integrable

Time = 3.70 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\tanh\left(c+dx\right)\operatorname{sech}^2\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(sech(d*x+c)**2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)*sech(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)),
 x)
```

3.361.7 Maxima [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 1104, normalized size of antiderivative = 32.47

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^2\tanh\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
(a*f + (b*d*f*x*e^(3*c) + (d*e - f)*b*e^(3*c))*e^(3*d*x) - (2*a*d*f*x*e^(2
*c) + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) - (b*d*f*x*e^c + (d*e + f)*b*e^c)*e
^(d*x))/(a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(
a^2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^(4*c) + b^2*d^2*e^2*e^(4*c)
+ (a^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*e*f*e^(4*c)
 + b^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + 2*(a^2*d^2*e^2*e^(2*c) + b^2*d^2*e^
2*e^(2*c) + (a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*e
*f*e^(2*c) + b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) - 4*integrate(1/4*(2*a*b^2
*d^2*f^2*x^2 + 4*a*b^2*d^2*e*f*x - 2*a^3*f^2 + 2*(d^2*e^2 - f^2)*a*b^2 + (
(d^2*e^2 + 2*f^2)*a^2*b*e^c - (d^2*e^2 - 2*f^2)*b^3*e^c + (a^2*b*d^2*f^2*e
^c - b^3*d^2*f^2*e^c)*x^2 + 2*(a^2*b*d^2*e*f*e^c - b^3*d^2*e*f*e^c)*x)*e^(
d*x))/(a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^
2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2
+ b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^
2*f)*x + (a^4*d^2*e^3*e^(2*c) + 2*a^2*b^2*d^2*e^3*e^(2*c) + b^4*d^2*e^3*e^
(2*c) + (a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^(
2*c))*x^3 + 3*(a^4*d^2*e*f^2*e^(2*c) + 2*a^2*b^2*d^2*e*f^2*e^(2*c) + b^4*d
^2*e*f^2*e^(2*c))*x^2 + 3*(a^4*d^2*e^2*f*e^(2*c) + 2*a^2*b^2*d^2*e^2*f*e^(
2*c) + b^4*d^2*e^2*f*e^(2*c))*x)*e^(2*d*x)), x) + 4*integrate(-1/2*(a^2*b^
2*e^(d*x + c) - a*b^3)/(a^4*b*e + 2*a^2*b^3*e + b^5*e + (a^4*b*f + 2*a^...
```

3.361.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.361.9 Mupad [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+d\,x)}{\cosh\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(tanh(c + d*x)/(cosh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(tanh(c + d*x)/(cosh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.362 $\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.362.1 Optimal result

Integrand size = 34, antiderivative size = 606

$$\begin{aligned}
&\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{3f^3x}{8bd^3}+\frac{(e+fx)^3}{4bd}-\frac{a^2(e+fx)^4}{4b^3f}+\frac{6af^3\cosh(c+dx)}{b^2d^4}+\frac{3af(e+fx)^2\cosh(c+dx)}{b^2d^2} \\
&\quad+\frac{a^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{a^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d} \\
&\quad+\frac{3a^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{3a^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2} \\
&\quad-\frac{6a^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{6a^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3} \\
&\quad+\frac{6a^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^4}+\frac{6a^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^4} \\
&\quad-\frac{6af^2(e+fx)\sinh(c+dx)}{b^2d^3}-\frac{a(e+fx)^3\sinh(c+dx)}{b^2d} \\
&\quad-\frac{3f^3\cosh(c+dx)\sinh(c+dx)}{8bd^4}-\frac{3f(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{4bd^2} \\
&\quad+\frac{3f^2(e+fx)\sinh^2(c+dx)}{4bd^3}+\frac{(e+fx)^3\sinh^2(c+dx)}{2bd}
\end{aligned}$$

output
```
3/8*f^3*x/b/d^3+1/4*(f*x+e)^3/b/d-1/4*a^2*(f*x+e)^4/b^3/f+6*a*f^3*cosh(d*x
+c)/b^2/d^4+3*a*f*(f*x+e)^2*cosh(d*x+c)/b^2/d^2+a^2*(f*x+e)^3*ln(1+b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/b^3/d+3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/b^3/d^2+3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))/b^3/d^2-6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
/b^3/d^3-6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^
3/d^3+6*a^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^4+6*a^2
*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^4-6*a*f^2*(f*x+e)*
sinh(d*x+c)/b^2/d^3-a*(f*x+e)^3*sinh(d*x+c)/b^2/d-3/8*f^3*cosh(d*x+c)*sinh
(d*x+c)/b/d^4-3/4*f*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b/d^2+3/4*f^2*(f*x+e
)*sinh(d*x+c)^2/b/d^3+1/2*(f*x+e)^3*sinh(d*x+c)^2/b/d
```

3.362.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2684 vs. 2(606) = 1212.

Time = 15.55 (sec) , antiderivative size = 2684, normalized size of antiderivative = 4.43

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
-1/4*(e^3*Log[a + b*Sinh[c + d*x]])/(b*d) - (3*e^2*f*(-1/2*x^2/b + (x*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (x*Log[1 + (b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt
[a^2 + b^2])]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)]/(b*d^2)))/4 - (3*e*f^2*(-1/3*x^3/b + (x^2*Log[1 + (b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2])])/(b*d) + (x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(b*d) + (2*x*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(
b*d^2) + (2*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2
) - (2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])])/(b*d^3) - (2*Po
lyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^3)))/4 - (f^3*(-1
/4*x^4/b + (x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (x
^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (3*x^2*PolyLog[
2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (3*x^2*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2) - (6*x*PolyLog[3, -((b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^3) - (6*x*PolyLog[3, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^3) + (6*PolyLog[4, (b*E^(c + d*x))/(
-a + Sqrt[a^2 + b^2])])/(b*d^4) + (6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b*d^4)))/4 + (e*f^2*(2*(4*a^2 + b^2)*x^3*Coth[c] - (2*(4
*a^2 + b^2)*(2*x^3 - (3*b^2*(-1 + E^(2*c))*(d^2*x^2*Log[1 + ((a - Sqrt[a^2
 + b^2])*E^(-c - d*x))/b] - 2*d*x*PolyLog[2, ((-a + Sqrt[a^2 + b^2])*E^...
```

3.362.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.27 (sec) , antiderivative size = 591, normalized size of antiderivative = 0.98, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {6113, 5969, 3042, 25, 3792, 17, 25, 3042, 25, 3115, 24, 6113, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 6095, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \; 6113$$

$$\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 5969$$

$$\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}-\frac{3f\int(e+fx)^2\sinh^2(c+dx)dx}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}-\frac{3f\int-(e+fx)^2\sin(ic+idx)^2dx}{2d}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\int(e+fx)^2\sin(ic+idx)^2dx}{2d}}{b}$$

$$\Big\downarrow \text{3792}$$

$$\frac{\frac{3f\left(\frac{f^2\int-\sinh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^2dx+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{\frac{3f\left(\frac{f^2\int-\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3f\left(-\frac{f^2\int\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\left(-\frac{f^2\int-\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}$$

$$\Big\downarrow \text{25}$$

3.362. $\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\left(\frac{f^2\int\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\Big\downarrow \; 3115$$

$$\frac{\frac{3f\left(\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 6113$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\int(e+fx)^3\cosh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3777$$

3.362. $\displaystyle\int\frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3if\int-i(e+fx)^2\sinh(c+dx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int(e+fx)^2\sinh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int-i(e+fx)^2\sin(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\int(e+fx)^2\sin(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 3777$$

3.362. $\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \textcolor{blue}{3777}$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \textcolor{blue}{26}$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}}{b}\right)}{b}$$

3.362. $\displaystyle\int\frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{6095}$$

3.362. $\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-$$

$$\frac{a\left(-\frac{a\left(\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{b}+\frac{\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\cdots\right)}{d}\right)}{d}}{b}\cdots\right.}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-$$

$$\frac{a\left(-\frac{a\left(-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+\cdots}{4\cdots}\right.}{b}\cdots\right.}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-$$

$$\frac{a\left(-\frac{a\left(-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{P}\cdots}{\cdots}\right.}{bd}\cdots\right.}{b}\cdots\right.}{\cdots}$$

$$\Big\downarrow \text{7163}$$

3.362. $\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-$$

$$a\left(a-\frac{a\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyL}}{}\right.}{}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \; 2720$$

3.362. $\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-$$

$$a\left(a-\frac{\left(a-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\right.}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 7143

3.362. $\displaystyle\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-$$

$$\frac{a\left(-\frac{a\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}(3}{d}\right.\right.}{}\right.\right.}{}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(((e + f*x)^3*Sinh[c + d*x]^2)/(2*d) + (3*f*((e + f*x)^3/(6*f) - ((e + f*x
)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (f*(e + f*x)*Sinh[c + d*x]^2)/(2*
d^2) + (f^2*(x/2 - (Cosh[c + d*x]*Sinh[c + d*x])/(2*d)))/(2*d^2)))/(2*d))/
b - (a*(-((a*(-1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/b) + (((e + f*x)^3*Sinh[c + d
*x])/d + ((3*I)*f*((I*(e + f*x)^2*Cosh[c + d*x])/d - ((2*I)*f*(-((f*Cosh[c
 + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d))/d))/d)/b))/b
```

3.362. $\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.362.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.362. $\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.362. $\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.362.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c) \sinh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.362.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3891 vs. 2(566) = 1132.

Time = 0.33 (sec) , antiderivative size = 3891, normalized size of antiderivative = 6.42

$$\int \frac{(e+fx)^3 \cosh(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
1/32*(4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 + 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2
+ 3*b^2*f^3 + (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2
*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^
3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c)^4 + (4*b^2*d^3*f^3
*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*
b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 +
b^2*d*f^3)*x)*sinh(d*x + c)^4 - 16*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*b*
d^2*e^2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^
2 + 3*(a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c)^3 -
 4*(4*a*b*d^3*f^3*x^3 + 4*a*b*d^3*e^3 - 12*a*b*d^2*e^2*f + 24*a*b*d*e*f^2
- 24*a*b*f^3 + 12*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 12*(a*b*d^3*e^2*f -
2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x - (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*
b^2*d^2*e^2*f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f
^3)*x^2 + 6*(2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x +
c))*sinh(d*x + c)^3 + 6*(2*b^2*d^3*e*f^2 + b^2*d^2*f^3)*x^2 - 8*(a^2*d^4*f
^3*x^4 + 4*a^2*d^4*e*f^2*x^3 + 6*a^2*d^4*e^2*f*x^2 + 4*a^2*d^4*e^3*x + 8*a
^2*c*d^3*e^3 - 12*a^2*c^2*d^2*e^2*f + 8*a^2*c^3*d*e*f^2 - 2*a^2*c^4*f^3)*c
osh(d*x + c)^2 - 2*(4*a^2*d^4*f^3*x^4 + 16*a^2*d^4*e*f^2*x^3 + 24*a^2*d^4*
e^2*f*x^2 + 16*a^2*d^4*e^3*x + 32*a^2*c*d^3*e^3 - 48*a^2*c^2*d^2*e^2*f + 3
2*a^2*c^3*d*e*f^2 - 8*a^2*c^4*f^3 - 3*(4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^...
```

3.362.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.362.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
1/8*e^3*(8*(d*x + c)*a^2/(b^3*d) - (4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/
(b^2*d) + 8*a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) +
(4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/32*(8*a^2*d^4*f^3*x^4
*e^(2*c) + 32*a^2*d^4*e*f^2*x^3*e^(2*c) + 48*a^2*d^4*e^2*f*x^2*e^(2*c) + (
4*b^2*d^3*f^3*x^3*e^(4*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*b^2*x^2*e^(4*c) + 6*
(2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*b^2*x*e^(4*c) - 3*(2*d^2*e^2*f - 2*d*e
*f^2 + f^3)*b^2*e^(4*c))*e^(2*d*x) - 16*(a*b*d^3*f^3*x^3*e^(3*c) + 3*(d^3*
e*f^2 - d^2*f^3)*a*b*x^2*e^(3*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a
*b*x*e^(3*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b*e^(3*c))*e^(d*x) + 16
*(a*b*d^3*f^3*x^3*e^c + 3*(d^3*e*f^2 + d^2*f^3)*a*b*x^2*e^c + 3*(d^3*e^2*f
 + 2*d^2*e*f^2 + 2*d*f^3)*a*b*x*e^c + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a*
b*e^c)*e^(-d*x) + (4*b^2*d^3*f^3*x^3 + 6*(2*d^3*e*f^2 + d^2*f^3)*b^2*x^2 +
 6*(2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*b^2*x + 3*(2*d^2*e^2*f + 2*d*e*f^2
+ f^3)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^4) - integrate(-2*(a^2*b*f^3*x^3 +
 3*a^2*b*e*f^2*x^2 + 3*a^2*b*e^2*f*x - (a^3*f^3*x^3*e^c + 3*a^3*e*f^2*x^2*
e^c + 3*a^3*e^2*f*x*e^c)*e^(d*x))/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x +
c) - b^4), x)
```

3.362.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
 x)
```

3.362.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.363 $\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.363.1 Optimal result

Integrand size = 34, antiderivative size = 449

$$\begin{aligned}
&\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{efx}{2bd}+\frac{f^2x^2}{4bd}-\frac{a^2(e+fx)^3}{3b^3f}+\frac{2af(e+fx)\cosh(c+dx)}{b^2d^2}+\frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d} \\
&+\frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{2a^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2} \\
&+\frac{2a^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}-\frac{2a^2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3} \\
&-\frac{2a^2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{2af^2\sinh(c+dx)}{b^2d^3}-\frac{a(e+fx)^2\sinh(c+dx)}{b^2d} \\
&-\frac{f(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd^2}+\frac{f^2\sinh^2(c+dx)}{4bd^3}+\frac{(e+fx)^2\sinh^2(c+dx)}{2bd}
\end{aligned}$$

output
```
1/2*e*f*x/b/d+1/4*f^2*x^2/b/d-1/3*a^2*(f*x+e)^3/b^3/f+2*a*f*(f*x+e)*cosh(d
*x+c)/b^2/d^2+a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+a
^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d+2*a^2*f*(f*x+e)*
polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2+2*a^2*f*(f*x+e)*polyl
og(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^2-2*a^2*f^2*polylog(3,-b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^3-2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/b^3/d^3-2*a*f^2*sinh(d*x+c)/b^2/d^3-a*(f*x+e)^2*sinh(d*x
+c)/b^2/d-1/2*f*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b/d^2+1/4*f^2*sinh(d*x+c)^
2/b/d^3+1/2*(f*x+e)^2*sinh(d*x+c)^2/b/d
```

3.363.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1453 vs. $2(449) = 898$.

Time = 9.37 (sec) , antiderivative size = 1453, normalized size of antiderivative = 3.24

$$\int \frac{(e+fx)^2 \cosh(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
-1/4*(e^2*Log[a + b*Sinh[c + d*x]])/(b*d) - (e*f*(-1/2*x^2/b + (x*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (x*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])])/(b*d) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2
 + b^2])]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(
b*d^2)))/2 - (f^2*(-1/3*x^3/b + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2])])/(b*d) + (x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b
*d) + (2*x*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) +
 (2*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2) - (2*P
olyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])])/(b*d^3) - (2*PolyLog[3,
 -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^3)))/4 + (f^2*(2*(4*a^2 +
 b^2)*x^3*Coth[c] - (2*(4*a^2 + b^2)*(2*x^3 - (3*b^2*(-1 + E^(2*c))*(d^2*x
^2*Log[1 + ((a - Sqrt[a^2 + b^2])*E^(-c - d*x))/b] - 2*d*x*PolyLog[2, ((-a
 + Sqrt[a^2 + b^2])*E^(-c - d*x))/b] - 2*PolyLog[3, ((-a + Sqrt[a^2 + b^2]
)*E^(-c - d*x))/b]))/(Sqrt[a^2 + b^2]*(-a + Sqrt[a^2 + b^2])*d^3) - (3*b^2
*(-1 + E^(2*c))*(d^2*x^2*Log[1 + ((a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b] -
 2*d*x*PolyLog[2, -(((a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b)] - 2*PolyLog[3
, -(((a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b)]))/(Sqrt[a^2 + b^2]*(a + Sqrt[
a^2 + b^2])*d^3) + (3*a*(-1 + E^(2*c))*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b
^2])] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]))/(Sqrt[a^...
```

3.363.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.45 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.95, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {6113, 5969, 3042, 25, 3791, 17, 6113, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^2(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5969}$$

$$\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int (e+fx)\sinh^2(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int -((e+fx)\sin(ic+idx)^2)dx}{d}}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}+\frac{f\int (e+fx)\sin(ic+idx)^2dx}{d}}{b}$$

$$\Big\downarrow \text{ 3791}$$

$$\frac{\frac{f\left(\frac{1}{2}\int (e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\int (e+fx)^2\cosh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

3.363. $\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int-i(e+fx)\sinh(c+dx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 3777$$

3.363. $\displaystyle\int\frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 3117$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 6095$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx}{3bf}\right.}{b}\right.}{b}$$

3.363. $\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\downarrow 3011$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+}}{a+\sqrt{a^2}}\right.}{d}\right.}{bd}\right.}{b}\right.$$

$$\downarrow 2720$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\right.}{bd}\right.}{b}\right.$$

$$\downarrow 7143$$

$$\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-$$

$$\frac{a\left(-\frac{a\left(-\frac{2f\left(\frac{f\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right.}{b}\right.}{b}$$

3.363. $\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(((e + f*x)^2*Sinh[c + d*x]^2)/(2*d) + (f*((e + f*x)^2/(4*f) - ((e + f*x)*
Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (f*Sinh[c + d*x]^2)/(4*d^2)))/d)/b -
(a*(-((a*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/d^2))/(b*d)))/b) + (((e + f*x)^2*Sinh[c + d*x])/d + ((2*I)*f*
((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/d)/b))/b
```

3.363.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

3.363. $\int \frac{(e+fx)^2 \cosh(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)}\, dx$

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.363.4 Maple [F]

$$\int \frac{(fx+e)^2 \cosh(dx+c) \sinh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.363.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2414 vs. $2(417) = 834$.

Time = 0.29 (sec) , antiderivative size = 2414, normalized size of antiderivative = 5.38

$$\int \frac{(e+fx)^2 \cosh(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)} dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
1/48*(6*b^2*d^2*f^2*x^2 + 6*b^2*d^2*e^2 + 6*b^2*d*e*f + 3*(2*b^2*d^2*f^2*x
^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)
*x)*cosh(d*x + c)^4 + 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f +
 b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*sinh(d*x + c)^4 + 3*b^2*f^2 -
24*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e
*f - a*b*d*f^2)*x)*cosh(d*x + c)^3 - 12*(2*a*b*d^2*f^2*x^2 + 2*a*b*d^2*e^2
 - 4*a*b*d*e*f + 4*a*b*f^2 + 4*(a*b*d^2*e*f - a*b*d*f^2)*x - (2*b^2*d^2*f^
2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f
^2)*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 16*(a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*
f*x^2 + 3*a^2*d^3*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^
2)*cosh(d*x + c)^2 - 2*(8*a^2*d^3*f^2*x^3 + 24*a^2*d^3*e*f*x^2 + 24*a^2*d^
3*e^2*x + 48*a^2*c*d^2*e^2 - 48*a^2*c^2*d*e*f + 16*a^2*c^3*f^2 - 9*(2*b^2*
d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b
^2*d*f^2)*x)*cosh(d*x + c)^2 + 36*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d
*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c))*sinh(d*x
+ c)^2 + 6*(2*b^2*d^2*e*f + b^2*d*f^2)*x + 24*(a*b*d^2*f^2*x^2 + a*b*d^2*e
^2 + 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f + a*b*d*f^2)*x)*cosh(d*x + c
) + 96*((a^2*d*f^2*x + a^2*d*e*f)*cosh(d*x + c)^2 + 2*(a^2*d*f^2*x + a^2*d
*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d*f^2*x + a^2*d*e*f)*sinh(d*x + c
)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si...
```

3.363.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.363.7 Maxima [F]

$$\int \frac{(e+fx)^2 \cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
1/8*e^2*(8*(d*x + c)*a^2/(b^3*d) - (4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/
(b^2*d) + 8*a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) +
(4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/48*(16*a^2*d^3*f^2*x^
3*e^(2*c) + 48*a^2*d^3*e*f*x^2*e^(2*c) + 3*(2*b^2*d^2*f^2*x^2*e^(4*c) + 2*
(2*d^2*e*f - d*f^2)*b^2*x*e^(4*c) - (2*d*e*f - f^2)*b^2*e^(4*c))*e^(2*d*x)
 - 24*(a*b*d^2*f^2*x^2*e^(3*c) + 2*(d^2*e*f - d*f^2)*a*b*x*e^(3*c) - 2*(d*
e*f - f^2)*a*b*e^(3*c))*e^(d*x) + 24*(a*b*d^2*f^2*x^2*e^c + 2*(d^2*e*f + d
*f^2)*a*b*x*e^c + 2*(d*e*f + f^2)*a*b*e^c)*e^(-d*x) + 3*(2*b^2*d^2*f^2*x^2
 + 2*(2*d^2*e*f + d*f^2)*b^2*x + (2*d*e*f + f^2)*b^2)*e^(-2*d*x))*e^(-2*c)
/(b^3*d^3) - integrate(-2*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x - (a^3*f^2*x^2*e^
c + 2*a^3*e*f*x*e^c)*e^(d*x))/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) -
 b^4), x)
```

3.363.8 Giac [F]

$$\int \frac{(e+fx)^2 \cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
 x)
```

3.363.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.364 $\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.364.1 Optimal result

Integrand size = 32, antiderivative size = 278

$$\begin{aligned}\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{fx}{4bd}-\frac{a^2(e+fx)^2}{2b^3f}+\frac{af\cosh(c+dx)}{b^2d^2}+\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}\\&\quad+\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{a^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}\\&\quad+\frac{a^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}-\frac{a(e+fx)\sinh(c+dx)}{b^2d}\\&\quad-\frac{f\cosh(c+dx)\sinh(c+dx)}{4bd^2}+\frac{(e+fx)\sinh^2(c+dx)}{2bd}\end{aligned}$$

output
```
1/4*f*x/b/d-1/2*a^2*(f*x+e)^2/b^3/f+a*f*cosh(d*x+c)/b^2/d^2+a^2*(f*x+e)*ln
(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+a^2*(f*x+e)*ln(1+b*exp(d*x+c)/(
a+(a^2+b^2)^(1/2)))/b^3/d+a^2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)
))/b^3/d^2+a^2*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^2-a*(f
*x+e)*sinh(d*x+c)/b^2/d-1/4*f*cosh(d*x+c)*sinh(d*x+c)/b/d^2+1/2*(f*x+e)*si
nh(d*x+c)^2/b/d
```

3.364.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.74

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= -\frac{e\log(a+b\sinh(c+dx))}{4bd} - \frac{1}{4}f\left(-\frac{x^2}{2b} + \frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} + \frac{\operatorname{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)}{bd^2} + \frac{\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}\right)$$
$$+ \frac{e\left((4a^2+b^2)\log(a+b\sinh(c+dx)) - 4ab\sinh(c+dx) + 2b^2\sinh^2(c+dx)\right)}{4b^3d}$$
$$+ \frac{f\left(8ab\cosh(c+dx) + 2b^2dx\cosh(2(c+dx)) + (4a^2+b^2)\left(2c(c+dx) - (c+dx)^2 + 2(c+dx)\log\left(1+\right.\right.\right.}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),
x]
```

output
```
-1/4*(e*Log[a + b*Sinh[c + d*x]])/(b*d) - (f*(-1/2*x^2/b + (x*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (x*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])])/(b*d) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b
^2])]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b*d^
2)))/4 + (e*((4*a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - 4*a*b*Sinh[c + d*x]
+ 2*b^2*Sinh[c + d*x]^2))/(4*b^3*d) + (f*(8*a*b*Cosh[c + d*x] + 2*b^2*d*x*
Cosh[2*(c + d*x)] + (4*a^2 + b^2)*(2*c*(c + d*x) - (c + d*x)^2 + 2*(c + d*
x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*(c + d*x)*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*Log[b - 2*a*E^(c + d*x) - b*E^(
2*(c + d*x))] + 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*P
olyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 8*a*b*d*x*Sinh[c +
d*x] - b^2*Sinh[2*(c + d*x)]))/(8*b^3*d^2)
```

3.364.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.99, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.531$, Rules used = {6113, 5969, 3042, 25, 3115, 24, 6113, 3042, 3777, 26, 3042, 26, 3118, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5969}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int \sinh^2(c+dx)dx}{2d}}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int -\sin(ic+idx)^2dx}{2d}}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\int \sin(ic+idx)^2dx}{2d}}{b}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{f\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\int (e+fx)\cosh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{6095}$$

3.364. $\int\frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+}}{\sqrt{a^2+b}}\right.}{bd}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{bd}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c}}{\sqrt{a^2+}}\right.}{bd}\right.}{b}\right.}{b}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

3.364. $\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
-((a*(-((a*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
b*d^2)))/b) + (-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*Sinh[c + d*x])/d)/b))
/b) + (((e + f*x)*Sinh[c + d*x]^2)/(2*d) + (f*(x/2 - (Cosh[c + d*x]*Sinh[c
 + d*x])/(2*d)))/(2*d))/b
```

3.364.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.364. $\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.364. $\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.364.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(258) = 516.

Time = 8.82 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.03

method	result
risch	$-\frac{a^2 f\,x^2}{2b^3}+\frac{a^2 e x}{b^3}+\frac{(2dfx+2de-f)\mathrm{e}^{2dx+2c}}{16b\,d^2}-\frac{a(dfx+de-f)\mathrm{e}^{dx+c}}{2b^2d^2}+\frac{a(dfx+de+f)\mathrm{e}^{-dx-c}}{2b^2d^2}+\frac{(2dfx+2de+f)\mathrm{e}^{-2dx-2c}}{16b\,d^2}-$

input
```
int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
-1/2*a^2*f*x^2/b^3+a^2*e*x/b^3+1/16*(2*d*f*x+2*d*e-f)/b/d^2*exp(2*d*x+2*c)
-1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp(d*x+c)+1/2*a*(d*f*x+d*e+f)/b^2/d^2*exp(-d
*x-c)+1/16*(2*d*f*x+2*d*e+f)/b/d^2*exp(-2*d*x-2*c)-2/d/b^3*a^2*e*ln(exp(d*
x+c))+1/d/b^3*a^2*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d/b^3*a^2*f*ln
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d/b^3*a^2*f*l
n((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2/b^3*a^2*f*
dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2/b^3*a^
2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2/b^3*
a^2*f*c^2+1/d^2/b^3*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^
2)^(1/2)))*c+1/d^2/b^3*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b
^2)^(1/2)))*c-2/d/b^3*a^2*f*c*x+2/d^2/b^3*c*a^2*f*ln(exp(d*x+c))-1/d^2/b^3
*c*a^2*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)
```

3.364.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. 2(256) = 512.

Time = 0.28 (sec) , antiderivative size = 1248, normalized size of antiderivative = 4.49

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
1/16*(2*b^2*d*f*x + (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^4 + (2
*b^2*d*f*x + 2*b^2*d*e - b^2*f)*sinh(d*x + c)^4 + 2*b^2*d*e - 8*(a*b*d*f*x
 + a*b*d*e - a*b*f)*cosh(d*x + c)^3 - 4*(2*a*b*d*f*x + 2*a*b*d*e - 2*a*b*f
 - (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*
f - 8*(a^2*d^2*f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f)*cosh(d*x
 + c)^2 - 2*(4*a^2*d^2*f*x^2 + 8*a^2*d^2*e*x + 16*a^2*c*d*e - 8*a^2*c^2*f
- 3*(2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^2 + 12*(a*b*d*f*x + a*
b*d*e - a*b*f)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*(a*b*d*f*x + a*b*d*e + a
*b*f)*cosh(d*x + c) + 16*(a^2*f*cosh(d*x + c)^2 + 2*a^2*f*cosh(d*x + c)*si
nh(d*x + c) + a^2*f*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1
) + 16*(a^2*f*cosh(d*x + c)^2 + 2*a^2*f*cosh(d*x + c)*sinh(d*x + c) + a^2*
f*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 16*((a^2*d*e -
 a^2*c*f)*cosh(d*x + c)^2 + 2*(a^2*d*e - a^2*c*f)*cosh(d*x + c)*sinh(d*x +
 c) + (a^2*d*e - a^2*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sin
h(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*((a^2*d*e - a^2*c*f)*co
sh(d*x + c)^2 + 2*(a^2*d*e - a^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d
*e - a^2*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) -
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*((a^2*d*f*x + a^2*c*f)*cosh(d*x ...
```

3.364.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.364.7 Maxima [F]

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
1/8*e*(8*(d*x + c)*a^2/(b^3*d) - (4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b
^2*d) + 8*a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) + (4
*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/16*f*((8*a^2*d^2*x^2*e^
(2*c) + (2*b^2*d*x*e^(4*c) - b^2*e^(4*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) -
 a*b*e^(3*c))*e^(d*x) + 8*(a*b*d*x*e^c + a*b*e^c)*e^(-d*x) + (2*b^2*d*x +
b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^2) - 2*integrate(16*(a^3*x*e^(d*x + c) -
a^2*b*x)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x))
```

3.364.8 Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x
)
```

3.364.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.365 $\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.365.1 Optimal result

Integrand size = 27, antiderivative size = 55

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a^2\log(a+b\sinh(c+dx))}{b^3d} - \frac{a\sinh(c+dx)}{b^2d} + \frac{\sinh^2(c+dx)}{2bd}$$

output
```
a^2*ln(a+b*sinh(d*x+c))/b^3/d-a*sinh(d*x+c)/b^2/d+1/2*sinh(d*x+c)^2/b/d
```

3.365.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2a^2\log(a+b\sinh(c+dx)) - 2ab\sinh(c+dx) + b^2\sinh^2(c+dx)}{2b^3d}$$

input
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(2*a^2*Log[a + b*Sinh[c + d*x]] - 2*a*b*Sinh[c + d*x] + b^2*Sinh[c + d*x]^
2)/(2*b^3*d)
```

3.365.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3042, 25, 3312, 25, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\sin(ic+idx)^2\cos(ic+idx)}{a-ib\sin(ic+idx)}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{\cos(ic+idx)\sin(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$
$$\Big\downarrow \text{3312}$$
$$-\frac{\int -\frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{b^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$
$$\Big\downarrow \text{49}$$
$$\frac{\int \left(\frac{a^2}{a+b\sinh(c+dx)}-a+b\sinh(c+dx)\right)d(b\sinh(c+dx))}{b^3d}$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^2\log(a+b\sinh(c+dx))-ab\sinh(c+dx)+\frac{1}{2}b^2\sinh^2(c+dx)}{b^3d}$$

input
```
Int[(Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(a^2*Log[a + b*Sinh[c + d*x]] - a*b*Sinh[c + d*x] + (b^2*Sinh[c + d*x]^2)/
2)/(b^3*d)
```

3.365.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

3.365.4 Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{a^2\ln(a+b\sinh(dx+c))}{b^3d}-\frac{a\sinh(dx+c)}{b^2d}+\frac{\sinh(dx+c)^2}{2bd}$	54
default	$\frac{a^2\ln(a+b\sinh(dx+c))}{b^3d}-\frac{a\sinh(dx+c)}{b^2d}+\frac{\sinh(dx+c)^2}{2bd}$	54
risch	$-\frac{x\,a^2}{b^3}+\frac{e^{2dx+2c}}{8bd}-\frac{a\,e^{dx+c}}{2b^2d}+\frac{a\,e^{-dx-c}}{2b^2d}+\frac{e^{-2dx-2c}}{8bd}-\frac{2a^2c}{b^3d}+\frac{a^2\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{b^3d}$	124

input
```
int(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(a+b*sinh(d*x+c))/b^3/d-a*sinh(d*x+c)/b^2/d+1/2*sinh(d*x+c)^2/b/d
```

3.365.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(53) = 106$.

Time = 0.24 (sec) , antiderivative size = 309, normalized size of antiderivative = 5.62

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{8\,a^2dx\cosh\left(dx+c\right)^2 - b^2\cosh\left(dx+c\right)^4 - b^2\sinh\left(dx+c\right)^4 + 4\,ab\cosh\left(dx+c\right)^3 - 4\left(b^2\cosh\left(dx+c\right)\right.}{}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-1/8*(8*a^2*d*x*cosh(d*x + c)^2 - b^2*cosh(d*x + c)^4 - b^2*sinh(d*x + c)^
4 + 4*a*b*cosh(d*x + c)^3 - 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x + c)^3 -
4*a*b*cosh(d*x + c) + 2*(4*a^2*d*x - 3*b^2*cosh(d*x + c)^2 + 6*a*b*cosh(d*
x + c))*sinh(d*x + c)^2 - b^2 - 8*(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x +
c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2)*log(2*(b*sinh(d*x + c) + a)/(cosh(
d*x + c) - sinh(d*x + c))) + 4*(4*a^2*d*x*cosh(d*x + c) - b^2*cosh(d*x + c
)^3 + 3*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 +
 2*b^3*d*cosh(d*x + c)*sinh(d*x + c) + b^3*d*sinh(d*x + c)^2)
```

3.365.6 Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.58

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \begin{cases} \frac{x\sinh^2(c)\cosh(c)}{a} & \text{for}\ b=0 \wedge d=0 \\ \frac{\sinh^3(c+dx)}{3ad} & \text{for}\ b=0 \\ \frac{x\sinh^2(c)\cosh(c)}{a+b\sinh(c)} & \text{for}\ d=0 \\ \frac{a^2\log\left(\frac{a}{b}+\sinh(c+dx)\right)}{b^3d} - \frac{a\sinh(c+dx)}{b^2d} + \frac{\cosh^2(c+dx)}{2bd} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Piecewise((x*sinh(c)**2*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)**3
/(3*a*d), Eq(b, 0)), (x*sinh(c)**2*cosh(c)/(a + b*sinh(c)), Eq(d, 0)), (a*
*2*log(a/b + sinh(c + d*x))/(b**3*d) - a*sinh(c + d*x)/(b**2*d) + cosh(c +
 d*x)**2/(2*b*d), True))
```

3.365.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(53) = 106$.

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.16

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{(dx+c)a^2}{b^3d} - \frac{\left(4\,ae^{(-dx-c)} - b\right)e^{(2\,dx+2\,c)}}{8\,b^2d} + \frac{a^2\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{b^3d} + \frac{4\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)}}{8\,b^2d}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
(d*x + c)*a^2/(b^3*d) - 1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d)
 + a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) + 1/8*(4*a*
e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)
```

3.365.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.60

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{8\,a^2\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^3} + \frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-4\,a\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{b^2}}{8\,d}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/8*(8*a^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/b^3 + (b*(e^(d*x
 + c) - e^(-d*x - c))^2 - 4*a*(e^(d*x + c) - e^(-d*x - c)))/b^2)/d
```

3.365.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{a^2\,\ln\left(a+b\sinh(c+d\,x)\right)+\frac{b^2\sinh(c+d\,x)^2}{2}-a\,b\sinh(c+d\,x)}{b^3\,d}$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
(a^2*log(a + b*sinh(c + d*x)) + (b^2*sinh(c + d*x)^2)/2 - a*b*sinh(c + d*x
))/(b^3*d)
```

3.366 $\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.366.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \mathrm{Int}\left(\frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.366.2 Mathematica [N/A]

Not integrable

Time = 27.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

3.366.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$
$$\Big\downarrow \; 6125$$
$$\int \frac{\sinh^2(c+dx)\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.366.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.366.4 Maple [N/A] (verified)

Not integrable

Time = 0.77 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.366.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(cosh(d*x + c)*sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

3.366.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.366.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 235, normalized size of antiderivative = 6.91

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
1/4*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) + 1/2*a*e^
(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c - d*e/f
)*exp_integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f)*exp_in
tegral_e(1, -2*(f*x + e)*d/f)/(b*f) + a^2*log(f*x + e)/(b^3*f) - 1/8*integ
rate(-16*(a^3*e^(d*x + c) - a^2*b)/(b^4*f*x + b^4*e - (b^4*f*x*e^(2*c) + b
^4*e*e^(2*c))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*b^3*e*e^c)*e^(d*x)), x)
```

3.366.8 Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate(cosh(d*x + c)*sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)),
 x)
```

3.366.9 Mupad [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.367 $\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.367.1 Optimal result

Integrand size = 36, antiderivative size = 897

$$\begin{aligned}
&\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{3aef^2x}{4b^2d^2}-\frac{3af^3x^2}{8b^2d^2}-\frac{a^3(e+fx)^4}{4b^4f}-\frac{a(e+fx)^4}{8b^2f}+\frac{6a^2f^2(e+fx)\cosh(c+dx)}{b^3d^3} \\
&+\frac{4f^2(e+fx)\cosh(c+dx)}{3bd^3}+\frac{a^2(e+fx)^3\cosh(c+dx)}{b^3d}+\frac{3af^3\cosh^2(c+dx)}{8b^2d^4} \\
&+\frac{3af(e+fx)^2\cosh^2(c+dx)}{4b^2d^2}+\frac{2f^2(e+fx)\cosh^3(c+dx)}{9bd^3}+\frac{(e+fx)^3\cosh^3(c+dx)}{3bd} \\
&+\frac{a^2\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{a^2\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d} \\
&+\frac{3a^2\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2} \\
&-\frac{3a^2\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2} \\
&-\frac{6a^2\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3} \\
&+\frac{6a^2\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^3} \\
&+\frac{6a^2\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^4} \\
&-\frac{6a^2\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^4}-\frac{6a^2f^3\sinh(c+dx)}{b^3d^4} \\
&-\frac{14f^3\sinh(c+dx)}{9bd^4}-\frac{3a^2f(e+fx)^2\sinh(c+dx)}{b^3d^2}-\frac{2f(e+fx)^2\sinh(c+dx)}{3bd^2} \\
&-\frac{3af^2(e+fx)\cosh(c+dx)\sinh(c+dx)}{4b^2d^3}-\frac{a(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{2b^2d} \\
&-\frac{f(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{3bd^2}-\frac{2f^3\sinh^3(c+dx)}{27bd^4}
\end{aligned}$$

output
```
a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d
-a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/
d+a^2*(f*x+e)^3*cosh(d*x+c)/b^3/d-2/27*f^3*sinh(d*x+c)^3/b/d^4+4/3*f^2*(f*
x+e)*cosh(d*x+c)/b/d^3-2/3*f*(f*x+e)^2*sinh(d*x+c)/b/d^2-3/8*a*f^3*x^2/b^2
/d^2+3/8*a*f^3*cosh(d*x+c)^2/b^2/d^4+2/9*f^2*(f*x+e)*cosh(d*x+c)^3/b/d^3-6
*a^2*f^3*sinh(d*x+c)/b^3/d^4-3/4*a*f^2*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b^2
/d^3+3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b
^2)^(1/2)/b^4/d^2-3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))*(a^2+b^2)^(1/2)/b^4/d^2-6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d^3+6*a^2*f^2*(f*x+e)*polylog(3,-b
*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d^3-1/4*a^3*(f*x+e)^4
/b^4/f+1/3*(f*x+e)^3*cosh(d*x+c)^3/b/d-3/4*a*e*f^2*x/b^2/d^2+6*a^2*f^2*(f*
x+e)*cosh(d*x+c)/b^3/d^3+3/4*a*f*(f*x+e)^2*cosh(d*x+c)^2/b^2/d^2-3*a^2*f*(
f*x+e)^2*sinh(d*x+c)/b^3/d^2-1/2*a*(f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/b^2/d
-1/3*f*(f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/b/d^2+6*a^2*f^3*polylog(4,-b*ex
p(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d^4-6*a^2*f^3*polylog(4,
-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d^4-1/8*a*(f*x+e)^4
/b^2/f-14/9*f^3*sinh(d*x+c)/b/d^4
```

3.367.2 Mathematica [A] (verified)

Time = 4.64 (sec) , antiderivative size = 1667, normalized size of antiderivative = 1.86

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*
x]),x]
```

output
```
-1/432*(432*a^3*d^4*e^3*x + 216*a*b^2*d^4*e^3*x + 648*a^3*d^4*e^2*f*x^2 +
324*a*b^2*d^4*e^2*f*x^2 + 432*a^3*d^4*e*f^2*x^3 + 216*a*b^2*d^4*e*f^2*x^3
+ 108*a^3*d^4*f^3*x^4 + 54*a*b^2*d^4*f^3*x^4 + 864*a^2*Sqrt[a^2 + b^2]*d^3
*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 432*a^2*b*d^3*e^3*Cosh
[c + d*x] - 108*b^3*d^3*e^3*Cosh[c + d*x] - 2592*a^2*b*d*e*f^2*Cosh[c + d*
x] - 648*b^3*d*e*f^2*Cosh[c + d*x] - 1296*a^2*b*d^3*e^2*f*x*Cosh[c + d*x]
- 324*b^3*d^3*e^2*f*x*Cosh[c + d*x] - 2592*a^2*b*d*f^3*x*Cosh[c + d*x] - 6
48*b^3*d*f^3*x*Cosh[c + d*x] - 1296*a^2*b*d^3*e*f^2*x^2*Cosh[c + d*x] - 32
4*b^3*d^3*e*f^2*x^2*Cosh[c + d*x] - 432*a^2*b*d^3*f^3*x^3*Cosh[c + d*x] -
108*b^3*d^3*f^3*x^3*Cosh[c + d*x] - 162*a*b^2*d^2*e^2*f*Cosh[2*(c + d*x)]
- 81*a*b^2*f^3*Cosh[2*(c + d*x)] - 324*a*b^2*d^2*e*f^2*x*Cosh[2*(c + d*x)]
 - 162*a*b^2*d^2*f^3*x^2*Cosh[2*(c + d*x)] - 36*b^3*d^3*e^3*Cosh[3*(c + d*
x)] - 24*b^3*d*e*f^2*Cosh[3*(c + d*x)] - 108*b^3*d^3*e^2*f*x*Cosh[3*(c + d
*x)] - 24*b^3*d*f^3*x*Cosh[3*(c + d*x)] - 108*b^3*d^3*e*f^2*x^2*Cosh[3*(c
+ d*x)] - 36*b^3*d^3*f^3*x^3*Cosh[3*(c + d*x)] - 1296*a^2*Sqrt[a^2 + b^2]*
d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 1296*a^2*Sqrt
[a^2 + b^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] -
 432*a^2*Sqrt[a^2 + b^2]*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2])] + 1296*a^2*Sqrt[a^2 + b^2]*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])] + 1296*a^2*Sqrt[a^2 + b^2]*d^3*e*f^2*x^2*Log[1 + ...
```

3.367.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh^2(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)^3\cosh^2(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5970}$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\int (e+fx)^2\cosh^3(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{d}}{b}$$

$$\Big\downarrow \; 3792$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2f^2\int\cosh^3(c+dx)dx}{9d^2}+\frac{2}{3}\int(e+fx)^2\cosh(c+dx)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2f^2\int\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{9d^2}+\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{d}}{b}$$

$$\Big\downarrow \; 3113$$

$$-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\int\left(\sinh^2(c+dx)+1\right)d(-i\sinh(c+dx))}{9d^3}+\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cos}{3d}\right.}{d}}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2}{3d}\right.}{d}}{b}$$

$$\Big\downarrow \; 3777$$

$$-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int-i(e+fx)\sinh(c+dx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+(e\right.}{d}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+(e+fa\right.}{d}}{b}$$

3.367. $\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+(e+}{d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+(e+f}{d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx}{}}{d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-2f(}{d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\Big\downarrow \; 3117$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(}{d^2}\right.}{d}\right.\right.}{d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\Big\downarrow \; 6113$$

3.367. $\displaystyle\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\Bigg(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\Bigg(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\Big(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(}{d^2}}{d}}{d}}{b} -\frac{a\left(\frac{\int(e+fx)^3\cosh^2(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$\Big\downarrow \; 3042$$

$$\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\Bigg(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\Bigg(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\Big(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(}{d^2}}{d}}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3792$$

$$-\frac{a\left(\frac{\frac{3f^2\int(e+fx)\cosh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^3dx-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\Bigg(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\Bigg(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\Big(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(}{d^2}}{d}}{d}}{b}$$

$$\Big\downarrow \; 17$$

$$-\frac{a\left(\frac{\frac{3f^2\int(e+fx)\cosh^2(c+dx)dx}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\Bigg(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\Bigg(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\Big(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(}{d^2}}{d}}{d}}{b}$$

$$\Big\downarrow \; 3042$$

3.367. $\displaystyle\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(\cdots}{d^2}\cdots\right.}{d}\cdots\right.\cdots\right.}{b}\cdots$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{3f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}\right)}{b}$$

$$\Big\downarrow \text{3791}$$

$$-\frac{a\left(\frac{\frac{3f^2\left(\frac{1}{2}\int(e+fx)dx-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int\frac{(e\cdots}{\cdots}}{\cdots}\cdots\right.}{b}\cdots$$

$$\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(\cdots}{d^2}\cdots\right.}{d}\cdots\right.\cdots\right.}{b}\cdots$$

$$\Big\downarrow \text{17}$$

$$-\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int\frac{(e+f\cdots}{a\cdots}}{\cdots}\cdots\right.}{b}\cdots$$

$$\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(\cdots}{d^2}\cdots\right.}{d}\cdots\right.\cdots\right.}{b}\cdots$$

$$\Big\downarrow \text{6099}$$

$$-\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\left(a^2+\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots$$

$$\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(\cdots}{d^2}\cdots\right.}{d}\cdots\right.\cdots\right.}{b}\cdots$$

$$\Big\downarrow \text{17}$$

3.367. $\displaystyle\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\left(a^2+\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.}{b}$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(\cdots}{d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{b}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(\cdots}{d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{b}$$

$$\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.}{b}$$

$$\Big\downarrow 26$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(\cdots}{d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{b}$$

$$\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.}{b}$$

$$\Big\downarrow 3777$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(\cdots}{d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{b}$$

$$\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.}{b}$$

3.367. $\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\downarrow 3042$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(\cdots}{d^2}\cdots\right)}{d}\cdots\right)\cdots\right)}{d}\cdots}{b}$$

$$\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\cdots\right.}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{b}$$

$$\downarrow 3777$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(\cdots}{d^2}\cdots\right)}{d}\cdots\right)\cdots\right)}{d}\cdots}{b}$$

$$\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\cdots\right.}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{b}$$

$$\downarrow 26$$

3.367. $\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(\cdots}{d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{b}$$

$$\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(\cdots}{d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{b}$$

$$\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.367. $\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.367.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.367. $\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)} \, dx$

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.367.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)^2 \sinh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.367.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7042 vs. 2(825) = 1650.

Time = 0.36 (sec) , antiderivative size = 7042, normalized size of antiderivative = 7.85

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

3.367.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.367.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)^2\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
1/24*e^3*(24*sqrt(a^2 + b^2)*a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2)
)/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^4*d) - (3*a*b*e^(-d*x - c) -
b^2 - 3*(4*a^2 + b^2)*e^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) - 12*(2*a^
3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*x - 3
*c) + 3*(4*a^2 + b^2)*e^(-d*x - c))/(b^3*d)) - 1/864*(108*(2*a^3*d^4*f^3*e
^(3*c) + a*b^2*d^4*f^3*e^(3*c))*x^4 + 432*(2*a^3*d^4*e*f^2*e^(3*c) + a*b^2
*d^4*e*f^2*e^(3*c))*x^3 + 648*(2*a^3*d^4*e^2*f*e^(3*c) + a*b^2*d^4*e^2*f*e
^(3*c))*x^2 - 4*(9*b^3*d^3*f^3*x^3*e^(6*c) + 9*(3*d^3*e*f^2 - d^2*f^3)*b^3
*x^2*e^(6*c) + 3*(9*d^3*e^2*f - 6*d^2*e*f^2 + 2*d*f^3)*b^3*x*e^(6*c) - (9*
d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*b^3*e^(6*c))*e^(3*d*x) + 27*(4*a*b^2*d^3*f^
3*x^3*e^(5*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*a*b^2*x^2*e^(5*c) + 6*(2*d^3*e^2
*f - 2*d^2*e*f^2 + d*f^3)*a*b^2*x*e^(5*c) - 3*(2*d^2*e^2*f - 2*d*e*f^2 + f
^3)*a*b^2*e^(5*c))*e^(2*d*x) + 108*(12*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a^2
*b*e^(4*c) + 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b^3*e^(4*c) - (4*a^2*b*d^3*
f^3*e^(4*c) + b^3*d^3*f^3*e^(4*c))*x^3 - 3*(4*(d^3*e*f^2 - d^2*f^3)*a^2*b*
e^(4*c) + (d^3*e*f^2 - d^2*f^3)*b^3*e^(4*c))*x^2 - 3*(4*(d^3*e^2*f - 2*d^2
*e*f^2 + 2*d*f^3)*a^2*b*e^(4*c) + (d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b^3*
e^(4*c))*x)*e^(d*x) - 108*(12*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a^2*b*e^(2*c
) + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b^3*e^(2*c) + (4*a^2*b*d^3*f^3*e^(2*
c) + b^3*d^3*f^3*e^(2*c))*x^3 + 3*(4*(d^3*e*f^2 + d^2*f^3)*a^2*b*e^(2*c...
```

3.367.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^2}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a
), x)
```

3.367.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\,\sinh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x
)
```

3.368 $\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.368.1 Optimal result

Integrand size = 36, antiderivative size = 649

$$\begin{aligned}
&\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx\\
&= -\frac{af^2x}{4b^2d^2}-\frac{a^3(e+fx)^3}{3b^4f}-\frac{a(e+fx)^3}{6b^2f}+\frac{2a^2f^2\cosh(c+dx)}{b^3d^3}\\
&\quad+\frac{4f^2\cosh(c+dx)}{9bd^3}+\frac{a^2(e+fx)^2\cosh(c+dx)}{b^3d}\\
&\quad+\frac{af(e+fx)\cosh^2(c+dx)}{2b^2d^2}+\frac{2f^2\cosh^3(c+dx)}{27bd^3}+\frac{(e+fx)^2\cosh^3(c+dx)}{3bd}\\
&\quad+\frac{a^2\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{a^2\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d}\\
&\quad+\frac{2a^2\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}\\
&\quad-\frac{2a^2\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}\\
&\quad-\frac{2a^2\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3}+\frac{2a^2\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^3}\\
&\quad-\frac{2a^2f(e+fx)\sinh(c+dx)}{b^3d^2}-\frac{4f(e+fx)\sinh(c+dx)}{9bd^2}-\frac{af^2\cosh(c+dx)\sinh(c+dx)}{4b^2d^3}\\
&\quad-\frac{a(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{2b^2d}-\frac{2f(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{9bd^2}
\end{aligned}$$

output
```
-1/4*a*f^2*x/b^2/d^2-1/3*a^3*(f*x+e)^3/b^4/f-1/6*a*(f*x+e)^3/b^2/f+2*a^2*f
^2*cosh(d*x+c)/b^3/d^3+4/9*f^2*cosh(d*x+c)/b/d^3+a^2*(f*x+e)^2*cosh(d*x+c)
/b^3/d+1/2*a*f*(f*x+e)*cosh(d*x+c)^2/b^2/d^2+2/27*f^2*cosh(d*x+c)^3/b/d^3+
1/3*(f*x+e)^2*cosh(d*x+c)^3/b/d-2*a^2*f*(f*x+e)*sinh(d*x+c)/b^3/d^2-4/9*f*
(f*x+e)*sinh(d*x+c)/b/d^2-1/4*a*f^2*cosh(d*x+c)*sinh(d*x+c)/b^2/d^3-1/2*a*
(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b^2/d-2/9*f*(f*x+e)*cosh(d*x+c)^2*sinh(d
*x+c)/b/d^2+a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)
^(1/2)/b^4/d-a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2
)^(1/2)/b^4/d+2*a^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
*(a^2+b^2)^(1/2)/b^4/d^2-2*a^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d^2-2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d^3+2*a^2*f^2*polylog(3,-b*exp(d*x+c)
/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d^3
```

3.368.2 Mathematica [A] (verified)

Time = 2.97 (sec) , antiderivative size = 966, normalized size of antiderivative = 1.49

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{216a^3d^3e^2x+108ab^2d^3e^2x+216a^3d^3efx^2+108ab^2d^3efx^2+72a^3d^3f^2x^3+36ab^2d^3f^2x^3+432a^2\sqrt{a^2-}}{}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*
x]),x]
```

output
```
-1/216*(216*a^3*d^3*e^2*x + 108*a*b^2*d^3*e^2*x + 216*a^3*d^3*e*f*x^2 + 10
8*a*b^2*d^3*e*f*x^2 + 72*a^3*d^3*f^2*x^3 + 36*a*b^2*d^3*f^2*x^3 + 432*a^2*
Sqrt[a^2 + b^2]*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 216
*a^2*b*d^2*e^2*Cosh[c + d*x] - 54*b^3*d^2*e^2*Cosh[c + d*x] - 432*a^2*b*f^
2*Cosh[c + d*x] - 108*b^3*f^2*Cosh[c + d*x] - 432*a^2*b*d^2*e*f*x*Cosh[c +
 d*x] - 108*b^3*d^2*e*f*x*Cosh[c + d*x] - 216*a^2*b*d^2*f^2*x^2*Cosh[c + d
*x] - 54*b^3*d^2*f^2*x^2*Cosh[c + d*x] - 54*a*b^2*d*e*f*Cosh[2*(c + d*x)]
- 54*a*b^2*d*f^2*x*Cosh[2*(c + d*x)] - 18*b^3*d^2*e^2*Cosh[3*(c + d*x)] -
4*b^3*f^2*Cosh[3*(c + d*x)] - 36*b^3*d^2*e*f*x*Cosh[3*(c + d*x)] - 18*b^3*
d^2*f^2*x^2*Cosh[3*(c + d*x)] - 432*a^2*Sqrt[a^2 + b^2]*d^2*e*f*x*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 216*a^2*Sqrt[a^2 + b^2]*d^2*f^2*x
^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 432*a^2*Sqrt[a^2 + b^2
]*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 216*a^2*Sqrt[
a^2 + b^2]*d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 43
2*a^2*Sqrt[a^2 + b^2]*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[
a^2 + b^2])] + 432*a^2*Sqrt[a^2 + b^2]*d*f*(e + f*x)*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))] + 432*a^2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, (
b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 432*a^2*Sqrt[a^2 + b^2]*f^2*PolyL
og[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 432*a^2*b*d*e*f*Sinh[c +
 d*x] + 108*b^3*d*e*f*Sinh[c + d*x] + 432*a^2*b*d*f^2*x*Sinh[c + d*x] +...
```

3.368.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^2(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)^2\cosh^2(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5970}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\int (e+fx)\cosh^3(c+dx)dx}{3d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d}}{b}$$

$$\Big\downarrow \; 3791$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\int (e+fx)\cosh(c+dx)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \; 3777$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \; 3118$$

3.368. $\displaystyle \int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\cosh^2(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3792}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f^2\int\cosh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^2dx-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f^2\int\cosh^2(c+dx)dx}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

3.368. $\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{f^2\int\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}\right)}{b}$$

$$\Big\downarrow \; 3115$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f^2\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 6099$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)\cdots}{b^2}\cdots\right.}{b}\cdots\right.}{b}$$

$$\Big\downarrow \; 17$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)^2\cdots}{}\cdots\right.}{b}\cdots\right.}{b}$$

3.368. $\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}+\int -i(e+\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-i\int(e+fx\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-i\left(\frac{i(e+f}{}\right.\right.}{}\right.}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-i\left(\frac{i(e+f}{}\right.\right.}{}\right.}{b}$$

$$\Big\downarrow \text{3777}$$

3.368. $\displaystyle\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+f\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{b}$$

$\downarrow$ 26

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+f\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{b}$$

$\downarrow$ 3042

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+f\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{b}$$

$\downarrow$ 26

3.368. $\displaystyle\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+f\cdots}{\cdots}\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{b}$$

$$\Big\downarrow \color{blue}{3118}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^{\cdots}}{3b^2f}\cdots\right.}{\cdots}\cdots\right.}{b}$$

$$\Big\downarrow \color{blue}{3803}$$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(\frac{2\left(a^2+b^2\right)\int-\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}\cdots\right.}{\cdots}\cdots\right.}{b}$$

$$\Big\downarrow \color{blue}{25}$$

3.368. $\displaystyle\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\int\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}\right.}{}\right.}{b}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.368.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

3.368. $\int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.368.4 Maple [F]

$$\int \frac{(fx+e)^2 \cosh (dx+c)^2 \sinh (dx+c)^2}{a+b \sinh (dx+c)} dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.368.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4311 vs. $2(595) = 1190$.

Time = 0.33 (sec) , antiderivative size = 4311, normalized size of antiderivative = 6.64

$$\int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)} dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
1/432*(18*b^3*d^2*f^2*x^2 + 18*b^3*d^2*e^2 + 2*(9*b^3*d^2*f^2*x^2 + 9*b^3*
d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(
d*x + c)^6 + 2*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^
2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*sinh(d*x + c)^6 + 12*b^3*d*e*f - 27*(
2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a
*b^2*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c)^5 - 3*(18*a*b^2*d^2*f^2*x^2 +
 18*a*b^2*d^2*e^2 - 18*a*b^2*d*e*f + 9*a*b^2*f^2 + 18*(2*a*b^2*d^2*e*f - a
*b^2*d*f^2)*x - 4*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3
*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4
*b^3*f^2 + 54*((4*a^2*b + b^3)*d^2*f^2*x^2 + (4*a^2*b + b^3)*d^2*e^2 - 2*(
4*a^2*b + b^3)*d*e*f + 2*(4*a^2*b + b^3)*f^2 + 2*((4*a^2*b + b^3)*d^2*e*f
- (4*a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c)^4 + 3*(18*(4*a^2*b + b^3)*d^2*f^
2*x^2 + 18*(4*a^2*b + b^3)*d^2*e^2 - 36*(4*a^2*b + b^3)*d*e*f + 36*(4*a^2*
b + b^3)*f^2 + 10*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3
*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c)^2 + 36*((4*a^2*b + b
^3)*d^2*e*f - (4*a^2*b + b^3)*d*f^2)*x - 45*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2
*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x
)*cosh(d*x + c))*sinh(d*x + c)^4 - 72*((2*a^3 + a*b^2)*d^3*f^2*x^3 + 3*(2*
a^3 + a*b^2)*d^3*e*f*x^2 + 3*(2*a^3 + a*b^2)*d^3*e^2*x)*cosh(d*x + c)^3 -
2*(36*(2*a^3 + a*b^2)*d^3*f^2*x^3 + 108*(2*a^3 + a*b^2)*d^3*e*f*x^2 + 1...
```

3.368.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.368.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh{(dx+c)}^2\sinh{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
1/24*e^2*(24*sqrt(a^2 + b^2)*a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2)
)/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^4*d) - (3*a*b*e^(-d*x - c) -
b^2 - 3*(4*a^2 + b^2)*e^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) - 12*(2*a^
3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*x - 3
*c) + 3*(4*a^2 + b^2)*e^(-d*x - c))/(b^3*d)) - 1/432*(72*(2*a^3*d^3*f^2*e^
(3*c) + a*b^2*d^3*f^2*e^(3*c))*x^3 + 216*(2*a^3*d^3*e*f*e^(3*c) + a*b^2*d^
3*e*f*e^(3*c))*x^2 - 2*(9*b^3*d^2*f^2*x^2*e^(6*c) + 6*(3*d^2*e*f - d*f^2)*
b^3*x*e^(6*c) - 2*(3*d*e*f - f^2)*b^3*e^(6*c))*e^(3*d*x) + 27*(2*a*b^2*d^2
*f^2*x^2*e^(5*c) + 2*(2*d^2*e*f - d*f^2)*a*b^2*x*e^(5*c) - (2*d*e*f - f^2)
*a*b^2*e^(5*c))*e^(2*d*x) + 54*(8*(d*e*f - f^2)*a^2*b*e^(4*c) + 2*(d*e*f -
 f^2)*b^3*e^(4*c) - (4*a^2*b*d^2*f^2*e^(4*c) + b^3*d^2*f^2*e^(4*c))*x^2 -
2*(4*(d^2*e*f - d*f^2)*a^2*b*e^(4*c) + (d^2*e*f - d*f^2)*b^3*e^(4*c))*x)*e
^(d*x) - 54*(8*(d*e*f + f^2)*a^2*b*e^(2*c) + 2*(d*e*f + f^2)*b^3*e^(2*c) +
 (4*a^2*b*d^2*f^2*e^(2*c) + b^3*d^2*f^2*e^(2*c))*x^2 + 2*(4*(d^2*e*f + d*f
^2)*a^2*b*e^(2*c) + (d^2*e*f + d*f^2)*b^3*e^(2*c))*x)*e^(-d*x) - 27*(2*a*b
^2*d^2*f^2*x^2*e^c + 2*(2*d^2*e*f + d*f^2)*a*b^2*x*e^c + (2*d*e*f + f^2)*a
*b^2*e^c)*e^(-2*d*x) - 2*(9*b^3*d^2*f^2*x^2 + 6*(3*d^2*e*f + d*f^2)*b^3*x
+ 2*(3*d*e*f + f^2)*b^3)*e^(-3*d*x))*e^(-3*c)/(b^4*d^3) + integrate(2*((a^
4*f^2*e^c + a^2*b^2*f^2*e^c)*x^2 + 2*(a^4*e*f*e^c + a^2*b^2*e*f*e^c)*x)*e^
(d*x)/(b^5*e^(2*d*x + 2*c) + 2*a*b^4*e^(d*x + c) - b^5), x)
```

3.368.8 Giac [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh{(dx+c)}^2\sinh{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a
), x)
```

3.368.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\sinh(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh\,(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x
)
```

3.369 $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.369.1 Optimal result

Integrand size = 34, antiderivative size = 403

$$\begin{aligned}
&\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{a^3ex}{b^4}-\frac{aex}{2b^2}-\frac{a^3fx^2}{2b^4}-\frac{afx^2}{4b^2}+\frac{a^2(e+fx)\cosh(c+dx)}{b^3d}+\frac{af\cosh^2(c+dx)}{4b^2d^2} \\
&\quad+\frac{(e+fx)\cosh^3(c+dx)}{3bd}+\frac{a^2\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d} \\
&\quad-\frac{a^2\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d}+\frac{a^2\sqrt{a^2+b^2}f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2} \\
&\quad-\frac{a^2\sqrt{a^2+b^2}f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{a^2f\sinh(c+dx)}{b^3d^2} \\
&\quad-\frac{f\sinh(c+dx)}{3bd^2}-\frac{a(e+fx)\cosh(c+dx)\sinh(c+dx)}{2b^2d}-\frac{f\sinh^3(c+dx)}{9bd^2}
\end{aligned}$$

output
```
-a^3*e*x/b^4-1/2*a*e*x/b^2-1/2*a^3*f*x^2/b^4-1/4*a*f*x^2/b^2+a^2*(f*x+e)*c
osh(d*x+c)/b^3/d+1/4*a*f*cosh(d*x+c)^2/b^2/d^2+1/3*(f*x+e)*cosh(d*x+c)^3/b
/d-a^2*f*sinh(d*x+c)/b^3/d^2-1/3*f*sinh(d*x+c)/b/d^2-1/2*a*(f*x+e)*cosh(d*
x+c)*sinh(d*x+c)/b^2/d-1/9*f*sinh(d*x+c)^3/b/d^2+a^2*(f*x+e)*ln(1+b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d-a^2*(f*x+e)*ln(1+b*exp(d*x
+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d+a^2*f*polylog(2,-b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d^2-a^2*f*polylog(2,-b*exp(d*x
+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d^2
```

3.369.2 Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.55

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{72a^3cde+36ab^2cde-36a^3c^2f-18ab^2c^2f+72a^3d^2ex+36ab^2d^2ex+36a^3d^2fx^2+18ab^2d^2fx^2+144a}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
-1/72*(72*a^3*c*d*e + 36*a*b^2*c*d*e - 36*a^3*c^2*f - 18*a*b^2*c^2*f + 72*
a^3*d^2*e*x + 36*a*b^2*d^2*e*x + 36*a^3*d^2*f*x^2 + 18*a*b^2*d^2*f*x^2 + 1
44*a^2*Sqrt[a^2 + b^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] -
144*a^2*Sqrt[a^2 + b^2]*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] -
 72*a^2*b*d*e*Cosh[c + d*x] - 18*b^3*d*e*Cosh[c + d*x] - 72*a^2*b*d*f*x*Co
sh[c + d*x] - 18*b^3*d*f*x*Cosh[c + d*x] - 9*a*b^2*f*Cosh[2*(c + d*x)] - 6
*b^3*d*e*Cosh[3*(c + d*x)] - 6*b^3*d*f*x*Cosh[3*(c + d*x)] - 72*a^2*Sqrt[a
^2 + b^2]*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 72*a^2*Sqrt
[a^2 + b^2]*d*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 72*a^2*
Sqrt[a^2 + b^2]*c*f*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 72*a^
2*Sqrt[a^2 + b^2]*d*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 7
2*a^2*Sqrt[a^2 + b^2]*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 + 72*a^2*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))] + 72*a^2*b*f*Sinh[c + d*x] + 18*b^3*f*Sinh[c + d*x] + 18*a*b^2*d*e*
Sinh[2*(c + d*x)] + 18*a*b^2*d*f*x*Sinh[2*(c + d*x)] + 2*b^3*f*Sinh[3*(c +
 d*x)])/(b^4*d^2)
```

3.369.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.26 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.97, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.676$, Rules used = {6113, 5970, 3042, 3113, 2009, 6113, 3042, 3791, 17, 6099, 17, 3042, 26, 3777, 3042, 3117, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^2(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)\cosh^2(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5970}$$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{f\int \cosh^3(c+dx)dx}{3d}}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{f\int \sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d}}{b}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\int \left(\sinh^2(c+dx)+1\right)d(-i\sinh(c+dx))}{3d^2}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}$$

$$\Big\downarrow \text{6113}$$

$$-\frac{a\left(\frac{\int (e+fx)\cosh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3791}$$

3.369. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$-\frac{a\left(\frac{\frac{1}{2}\int (e+fx)dx-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}$$

$$\Big\downarrow \text{ 6099}$$

$$-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)dx}{b^2}+\frac{\int (e+fx)\sinh(c+dx)dx}{b}\right)}{b}\right)}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)\sinh(c+dx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}\right)}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int -i(e+fx)\sin(ic+idx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 26}$$

3.369. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int(e+fx)\sin(ic+idx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 3777$$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+\cdots}{2b^2\cdots}\right.}{b}\right.}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\cdots\right.}{b}\right.}{b}$$

$$\Big\downarrow \ 3117$$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\cdots\right.}{b}\right.}{b}$$

$$\Big\downarrow \ 3803$$

3.369. $\displaystyle\int\frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{2\left(a^2+b^2\right)\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\,\mathrm{si}}{}\right.}{b}\right.}{b}\right.}{b}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\,\mathrm{s}}{}\right.}{b}\right.}{b}\right.}{b}$$

$$\Big\downarrow \ 2694$$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\right.}{b}\right.}{b}$$

$$\Big\downarrow \ 27$$

3.369. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^2}{2b^2f}\cdots\right.}{b}\cdots\right.}{b}$$

$$\Big\downarrow \; 2620$$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e\cdots}{\cdots}\right.}{\cdots}\right.}{b^2}\cdots\right.}{\cdots}\cdots\right.}{b}$$

$$\Big\downarrow \; 2715$$

3.369. $\displaystyle\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-$$

$$a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+}\cdots}{bd^2}\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\cdots}{\cdots}\right.$$

$$\cdots \qquad b$$

$\big\downarrow$ 2838

$$\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-$$

$$a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(f\cdots\right.}{\cdots}\cdots\right)}{b^2}\cdots\right)}{\cdots}\right.$$

$$\cdots \qquad b$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

3.369. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(((e + f*x)*Cosh[c + d*x]^3)/(3*d) - ((I/3)*f*((-I)*Sinh[c + d*x] - (I/3)*
Sinh[c + d*x]^3))/d^2)/b - (a*(((e + f*x)^2/(4*f) - (f*Cosh[c + d*x]^2)/(4
*d^2) + ((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d))/b - (a*(-1/2*(a*(e
+ f*x)^2)/(b^2*f) - (2*(a^2 + b^2)*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/b^2 - (I*((I*
(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/b))/b))/b
```

3.369.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.369. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.369. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.369.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1127 vs. $2(367) = 734$.

Time = 21.24 (sec) , antiderivative size = 1128, normalized size of antiderivative = 2.80

method	result	size
risch	Expression too large to display	1128

input
```
int((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURN
VERBOSE)
```

output
```
-1/d^2/b^2*a^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a
+(a^2+b^2)^(1/2)))+1/8*(4*a^2+b^2)*(d*f*x+d*e+f)/b^3/d^2*exp(-d*x-c)+1/d/b
^2*a^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2
)^(1/2)))*x-1/d/b^2*a^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)
+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2/b^2*a^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+
c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2/b^2*a^2*f/(a^2+b^2)^(1
/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d^2/b^2*a
^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1
/d^2*a^4/b^4*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a
^2+b^2)^(1/2)))*c-1/d^2*a^4/b^4*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^
2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d^2*a^4/b^4*f*c/(a^2+b^2)^(1/2)*arcta
nh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/4*a*f*x^2/b^2+1/8*(4*a^2*d*
f*x+b^2*d*f*x+4*a^2*d*e+b^2*d*e-4*a^2*f-b^2*f)/b^3/d^2*exp(d*x+c)+1/d*a^4/
b^4*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(
1/2)))*x-1/d*a^4/b^4*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)
/(a+(a^2+b^2)^(1/2)))*x-a^3*e*x/b^4-2/d*a^4/b^4*e/(a^2+b^2)^(1/2)*arctanh(
1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d/b^2*a^2*e/(a^2+b^2)^(1/2)*ar
ctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d^2/b^2*a^2*f/(a^2+b^2)^
(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/2*a*
e*x/b^2+1/72*(3*d*f*x+3*d*e+f)/b/d^2*exp(-3*d*x-3*c)+1/d^2*a^4/b^4*f/(a...
```

3.369.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2195 vs. $2(365) = 730$.

Time = 0.31 (sec) , antiderivative size = 2195, normalized size of antiderivative = 5.45

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
1/144*(2*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^6 + 2*(3*b^3*d*f*
x + 3*b^3*d*e - b^3*f)*sinh(d*x + c)^6 + 6*b^3*d*f*x - 9*(2*a*b^2*d*f*x +
2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^5 - 3*(6*a*b^2*d*f*x + 6*a*b^2*d*e -
3*a*b^2*f - 4*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c))*sinh(d*x +
c)^5 + 6*b^3*d*e + 18*((4*a^2*b + b^3)*d*f*x + (4*a^2*b + b^3)*d*e - (4*a^
2*b + b^3)*f)*cosh(d*x + c)^4 + 3*(6*(4*a^2*b + b^3)*d*f*x + 6*(4*a^2*b +
b^3)*d*e + 10*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^2 - 6*(4*a^2
*b + b^3)*f - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c))*si
nh(d*x + c)^4 + 2*b^3*f - 36*((2*a^3 + a*b^2)*d^2*f*x^2 + 2*(2*a^3 + a*b^2
)*d^2*e*x)*cosh(d*x + c)^3 - 2*(18*(2*a^3 + a*b^2)*d^2*f*x^2 + 36*(2*a^3 +
 a*b^2)*d^2*e*x - 20*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^3 + 4
5*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^2 - 36*((4*a^2*b +
 b^3)*d*f*x + (4*a^2*b + b^3)*d*e - (4*a^2*b + b^3)*f)*cosh(d*x + c))*sinh
(d*x + c)^3 + 18*((4*a^2*b + b^3)*d*f*x + (4*a^2*b + b^3)*d*e + (4*a^2*b +
 b^3)*f)*cosh(d*x + c)^2 + 6*(5*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x
 + c)^4 + 3*(4*a^2*b + b^3)*d*f*x - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^
2*f)*cosh(d*x + c)^3 + 3*(4*a^2*b + b^3)*d*e + 18*((4*a^2*b + b^3)*d*f*x +
 (4*a^2*b + b^3)*d*e - (4*a^2*b + b^3)*f)*cosh(d*x + c)^2 + 3*(4*a^2*b + b
^3)*f - 18*((2*a^3 + a*b^2)*d^2*f*x^2 + 2*(2*a^3 + a*b^2)*d^2*e*x)*cosh(d*
x + c))*sinh(d*x + c)^2 + 144*(a^2*b*f*cosh(d*x + c)^3 + 3*a^2*b*f*cosh...
```

3.369.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.369.7 Maxima [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^2\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
1/144*(288*(a^4*e^c + a^2*b^2*e^c)*integrate(x*e^(d*x)/(b^5*e^(2*d*x + 2*c
) + 2*a*b^4*e^(d*x + c) - b^5), x) - (36*(2*a^3*d^2*e^(3*c) + a*b^2*d^2*e^
(3*c))*x^2 - 2*(3*b^3*d*x*e^(6*c) - b^3*e^(6*c))*e^(3*d*x) + 9*(2*a*b^2*d*
x*e^(5*c) - a*b^2*e^(5*c))*e^(2*d*x) + 18*(4*a^2*b*e^(4*c) + b^3*e^(4*c) -
 (4*a^2*b*d*e^(4*c) + b^3*d*e^(4*c))*x)*e^(d*x) - 18*(4*a^2*b*e^(2*c) + b^
3*e^(2*c) + (4*a^2*b*d*e^(2*c) + b^3*d*e^(2*c))*x)*e^(-d*x) - 9*(2*a*b^2*d
*x*e^c + a*b^2*e^c)*e^(-2*d*x) - 2*(3*b^3*d*x + b^3)*e^(-3*d*x))*e^(-3*c)/
(b^4*d^2))*f + 1/24*e*(24*sqrt(a^2 + b^2)*a^2*log((b*e^(-d*x - c) - a - sq
rt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^4*d) - (3*a*b*e^
(-d*x - c) - b^2 - 3*(4*a^2 + b^2)*e^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*
d) - 12*(2*a^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*
e^(-3*d*x - 3*c) + 3*(4*a^2 + b^2)*e^(-d*x - c))/(b^3*d))
```

3.369.8 Giac [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^2\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
 x)
```

3.369.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\sinh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.370 $\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.370.1 Optimal result

Integrand size = 29, antiderivative size = 141

$$
\begin{aligned}
\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = {} & -\frac{a(2a^2+b^2)\,x}{2b^4} \\
& - \frac{2a^2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^4d} \\
& + \frac{(3a^2+b^2)\cosh(c+dx)}{3b^3d} - \frac{a\cosh(c+dx)\sinh(c+dx)}{2b^2d} \\
& + \frac{\cosh(c+dx)\sinh^2(c+dx)}{3bd}
\end{aligned}
$$

output
```
-1/2*a*(2*a^2+b^2)*x/b^4+1/3*(3*a^2+b^2)*cosh(d*x+c)/b^3/d-1/2*a*cosh(d*x+
c)*sinh(d*x+c)/b^2/d+1/3*cosh(d*x+c)*sinh(d*x+c)^2/b/d-2*a^2*arctanh((b-a*
tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^4/d
```

3.370.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$
\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx
$$

$$
= \frac{3b(4a^2+b^2)\cosh(c+dx)+b^3\cosh(3(c+dx))-3a\left(2(2a^2+b^2)(c+dx)+8a\sqrt{-a^2-b^2}\arctan\left(\frac{b-a\tanh}{\sqrt{-}}\right.\right.}{12b^4d}
$$

input
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(3*b*(4*a^2 + b^2)*Cosh[c + d*x] + b^3*Cosh[3*(c + d*x)] - 3*a*(2*(2*a^2 +
 b^2)*(c + d*x) + 8*a*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sq
rt[-a^2 - b^2]] + b^2*Sinh[2*(c + d*x)]))/(12*b^4*d)
```

3.370.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.16, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.724$, Rules used = {3042, 25, 3368, 25, 3042, 25, 3529, 26, 3042, 26, 3528, 25, 3042, 3502, 27, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2\cos(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos(ic+idx)^2\sin(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3368}$$

$$-\int -\frac{\sinh^2(c+dx)\left(\sinh^2(c+dx)+1\right)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\sinh^2(c+dx)\left(\sinh^2(c+dx)+1\right)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2\left(1-\sin(ic+idx)^2\right)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2\left(1-\sin(ic+idx)^2\right)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3529}$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\int -\frac{i\sinh(c+dx)\left(3a\sinh^2(c+dx)-b\sinh(c+dx)+2a\right)}{a+b\sinh(c+dx)}dx}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{\int \frac{\sinh(c+dx)\left(3a\sinh^2(c+dx)-b\sinh(c+dx)+2a\right)}{a+b\sinh(c+dx)}dx}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{\int -\frac{i\sin(ic+idx)\left(-3a\sin(ic+idx)^2+ib\sin(ic+idx)+2a\right)}{a-ib\sin(ic+idx)}dx}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\int \frac{\sin(ic+idx)\left(-3a\sin(ic+idx)^2+ib\sin(ic+idx)+2a\right)}{a-ib\sin(ic+idx)}dx}{3b}$$

$$\Big\downarrow \text{3528}$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\left(\frac{i\int -\frac{3a^2-b\sinh(c+dx)a+2\left(3a^2+b^2\right)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{2b}+\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}\right)}{3b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\int \frac{3a^2-b\sinh(c+dx)a+2\left(3a^2+b^2\right)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{2b}\right)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\int \frac{3a^2+ib\sin(ic+idx)a-2\left(3a^2+b^2\right)\sin(ic+idx)^2}{a-ib\sin(ic+idx)}dx}{2b}\right)}{3b}$$

$$\Big\downarrow \text{3502}$$

3.370. $\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+}{\frac{i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{2\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{i\int -\frac{3i\left(a^2b-a\left(2a^2+b^2\right)\sinh(c+dx)\right)}{a+b\sinh(c+dx)}dx}{b}\right)}{2b}\right)}{3b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+}{\frac{i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{3\int \frac{a^2b-a\left(2a^2+b^2\right)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{2\left(3a^2+b^2\right)\cosh(c+dx)}{bd}\right)}{2b}\right)}{3b}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+}{\frac{i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{2\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{3\int \frac{ba^2+i\left(2a^2+b^2\right)\sin(ic+idx)a}{a-ib\sin(ic+idx)}dx}{b}\right)}{2b}\right)}{3b}}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+}{\frac{i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{3\left(\frac{2a^2\left(a^2+b^2\right)\int \frac{1}{a+b\sinh(c+dx)}dx}{b}-\frac{ax\left(2a^2+b^2\right)}{b}\right)}{b}+\frac{2\left(3a^2+b^2\right)\cosh(c+dx)}{bd}\right)}{2b}\right)}{3b}}$$

$$\Big\downarrow \text{3042}$$

3.370. $\displaystyle\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{2(3a^2+b^2)\cosh(c+dx)}{bd}+\frac{3\left(-\frac{ax(2a^2+b^2)}{b}+\frac{2a^2(a^2+b^2)\int\frac{1}{a-ib\sin(ic+idx)}dx}{b}\right)}{b}\right)}{2b}\right)}{3b}$$

$\downarrow$ 3139

$$\frac{\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{2(3a^2+b^2)\cosh(c+dx)}{bd}+\frac{3\left(-\frac{ax(2a^2+b^2)}{b}-\frac{4ia^2(a^2+b^2)\int\frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c\right.\right.}{bd}}{b}}{2b}}{3b}$$

$\downarrow$ 1083

$$\frac{\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{2(3a^2+b^2)\cosh(c+dx)}{bd}+\frac{3\left(-\frac{ax(2a^2+b^2)}{b}+\frac{8ia^2(a^2+b^2)\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right.}{bd}}{b}}{2b}}{3b}$$

$\downarrow$ 217

3.370. $\displaystyle\int\frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\left(\frac{3ia\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{3\left(\frac{4a^2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd}-\frac{ax(2a^2+b^2)}{b}\right)}{b}+\frac{2(3a^2+b^2)\cosh(c+dx)}{bd}\right)}{2b}\right)}{3b}$$

input
```
Int[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(Cosh[c + d*x]*Sinh[c + d*x]^2)/(3*b*d) + ((I/3)*(((-1/2*I)*((3*(-((a*(2*a
^2 + b^2)*x)/b) + (4*a^2*Sqrt[a^2 + b^2]*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt
[a^2 + b^2])])/(b*d)))/b + (2*(3*a^2 + b^2)*Cosh[c + d*x])/(b*d)))/b + (((
3*I)/2)*a*Cosh[c + d*x]*Sinh[c + d*x])/(b*d)))/b
```

3.370.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.370. $\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3529
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m + n + 2))   Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(
n + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*
(a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c,
0])))
```

3.370.4 Maple [A] (verified)

Time = 9.18 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.73

method	result
risch	$-\frac{a^{3}x}{b^{4}}-\frac{ax}{2b^{2}}+\frac{\mathrm{e}^{3dx+3c}}{24bd}-\frac{a\,\mathrm{e}^{2dx+2c}}{8b^{2}d}+\frac{\mathrm{e}^{dx+c}a^{2}}{2b^{3}d}+\frac{\mathrm{e}^{dx+c}}{8bd}+\frac{\mathrm{e}^{-dx-c}a^{2}}{2b^{3}d}+\frac{\mathrm{e}^{-dx-c}}{8bd}+\frac{a\,\mathrm{e}^{-2dx-2c}}{8b^{2}d}$
derivativedivides	$\frac{2a^{2}\sqrt{a^{2}+b^{2}}\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{b^{4}}+\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{b-a}{2b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{-2a^{2}+ab-b^{2}}{2b^{3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$
default	$\frac{2a^{2}\sqrt{a^{2}+b^{2}}\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{b^{4}}+\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{b-a}{2b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{-2a^{2}+ab-b^{2}}{2b^{3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-a^3*x/b^4-1/2*a*x/b^2+1/24/b/d*exp(3*d*x+3*c)-1/8*a/b^2/d*exp(2*d*x+2*c)+
1/2/b^3/d*exp(d*x+c)*a^2+1/8/b/d*exp(d*x+c)+1/2/b^3/d*exp(-d*x-c)*a^2+1/8/
b/d*exp(-d*x-c)+1/8*a/b^2/d*exp(-2*d*x-2*c)+1/24/b/d*exp(-3*d*x-3*c)+(a^2+
b^2)^(1/2)*a^2/d/b^4*ln(exp(d*x+c)-(-a+(a^2+b^2)^(1/2))/b)-(a^2+b^2)^(1/2)
*a^2/d/b^4*ln(exp(d*x+c)+(a+(a^2+b^2)^(1/2))/b)
```

3.370.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(130) = 260$.

Time = 0.26 (sec) , antiderivative size = 745, normalized size of antiderivative = 5.28

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{b^3\cosh{(dx+c)}^6 + b^3\sinh{(dx+c)}^6 - 3\,ab^2\cosh{(dx+c)}^5 - 12\,(2\,a^3+ab^2)dx\cosh{(dx+c)}^3 + 3\,(2\,b^3\cos}{}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fric
as")
```

output
```
1/24*(b^3*cosh(d*x + c)^6 + b^3*sinh(d*x + c)^6 - 3*a*b^2*cosh(d*x + c)^5
- 12*(2*a^3 + a*b^2)*d*x*cosh(d*x + c)^3 + 3*(2*b^3*cosh(d*x + c) - a*b^2)
*sinh(d*x + c)^5 + 3*(4*a^2*b + b^3)*cosh(d*x + c)^4 + 3*(5*b^3*cosh(d*x +
 c)^2 - 5*a*b^2*cosh(d*x + c) + 4*a^2*b + b^3)*sinh(d*x + c)^4 + 3*a*b^2*c
osh(d*x + c) + 2*(10*b^3*cosh(d*x + c)^3 - 15*a*b^2*cosh(d*x + c)^2 - 6*(2
*a^3 + a*b^2)*d*x + 6*(4*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + b^3
 + 3*(4*a^2*b + b^3)*cosh(d*x + c)^2 + 3*(5*b^3*cosh(d*x + c)^4 - 10*a*b^2
*cosh(d*x + c)^3 - 12*(2*a^3 + a*b^2)*d*x*cosh(d*x + c) + 4*a^2*b + b^3 +
6*(4*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 24*(a^2*cosh(d*x + c)
^3 + 3*a^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^2*cosh(d*x + c)*sinh(d*x +
c)^2 + a^2*sinh(d*x + c)^3)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2
*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c
) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x +
 c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(
b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 3*(2*b^3*cosh(d*x + c)^5 - 5*a*
b^2*cosh(d*x + c)^4 - 12*(2*a^3 + a*b^2)*d*x*cosh(d*x + c)^2 + 4*(4*a^2*b
+ b^3)*cosh(d*x + c)^3 + a*b^2 + 2*(4*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x
 + c))/(b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*
b^4*d*cosh(d*x + c)*sinh(d*x + c)^2 + b^4*d*sinh(d*x + c)^3)
```

3.370.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.370.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.48

$$\begin{aligned}\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\sqrt{a^2+b^2}a^2\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{b^4d} \\ &\quad - \frac{\left(3\,abe^{(-dx-c)}-b^2-3\,(4\,a^2+b^2)e^{(-2\,dx-2\,c)}\right)e^{(3\,dx+3\,c)}}{24\,b^3d} \\ &\quad - \frac{(2\,a^3+ab^2)(dx+c)}{2\,b^4d} \\ &\quad + \frac{3\,abe^{(-2\,dx-2\,c)}+b^2e^{(-3\,dx-3\,c)}+3\,(4\,a^2+b^2)e^{(-dx-c)}}{24\,b^3d}\end{aligned}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxi
ma")
```

output
```
sqrt(a^2 + b^2)*a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x
- c) - a + sqrt(a^2 + b^2)))/(b^4*d) - 1/24*(3*a*b*e^(-d*x - c) - b^2 - 3*
(4*a^2 + b^2)*e^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) - 1/2*(2*a^3 + a*b
^2)*(d*x + c)/(b^4*d) + 1/24*(3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*x - 3*c
) + 3*(4*a^2 + b^2)*e^(-d*x - c))/(b^3*d)
```

3.370.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.50

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{\frac{12\,(2\,a^3+ab^2)(dx+c)}{b^4}-\frac{b^2e^{(3\,dx+3\,c)}-3\,abe^{(2\,dx+2\,c)}+12\,a^2e^{(dx+c)}+3\,b^2e^{(dx+c)}}{b^3}-\frac{(3\,ab^2e^{(dx+c)}+b^3+3\,(4\,a^2b+b^3)e^{(2\,dx+2\,c)})e^{(-3\,dx-3\,c}}{b^4}}{24\,d}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac
")
```

output
```
-1/24*(12*(2*a^3 + a*b^2)*(d*x + c)/b^4 - (b^2*e^(3*d*x + 3*c) - 3*a*b*e^(
2*d*x + 2*c) + 12*a^2*e^(d*x + c) + 3*b^2*e^(d*x + c))/b^3 - (3*a*b^2*e^(d
*x + c) + b^3 + 3*(4*a^2*b + b^3)*e^(2*d*x + 2*c))*e^(-3*d*x - 3*c)/b^4 -
24*(a^4 + a^2*b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(
2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4))/d
```

3.370.9 Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.97

$$\begin{aligned}
&\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{\mathrm{e}^{-3\,c-3\,d\,x}}{24\,b\,d}-\frac{x\,(2\,a^3+a\,b^2)}{2\,b^4}+\frac{\mathrm{e}^{3\,c+3\,d\,x}}{24\,b\,d}+\frac{a\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,b^2\,d}-\frac{a\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,b^2\,d}+\frac{\mathrm{e}^{c+d\,x}\,(4\,a^2+b^2)}{8\,b^3\,d} \\
&\quad+\frac{\mathrm{e}^{-c-d\,x}\,(4\,a^2+b^2)}{8\,b^3\,d}-\frac{a^2\,\ln\left(-\frac{2\,a^2\,\sqrt{a^2+b^2}\,(b-a\,\mathrm{e}^{c+d\,x})}{b^5}-\frac{2\,a^2\,\mathrm{e}^{c+d\,x}\,(a^2+b^2)}{b^5}\right)\sqrt{a^2+b^2}}{b^4\,d} \\
&\quad+\frac{a^2\,\ln\left(\frac{2\,a^2\,\sqrt{a^2+b^2}\,(b-a\,\mathrm{e}^{c+d\,x})}{b^5}-\frac{2\,a^2\,\mathrm{e}^{c+d\,x}\,(a^2+b^2)}{b^5}\right)\sqrt{a^2+b^2}}{b^4\,d}
\end{aligned}$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
exp(- 3*c - 3*d*x)/(24*b*d) - (x*(a*b^2 + 2*a^3))/(2*b^4) + exp(3*c + 3*d*
x)/(24*b*d) + (a*exp(- 2*c - 2*d*x))/(8*b^2*d) - (a*exp(2*c + 2*d*x))/(8*b
^2*d) + (exp(c + d*x)*(4*a^2 + b^2))/(8*b^3*d) + (exp(- c - d*x)*(4*a^2 +
b^2))/(8*b^3*d) - (a^2*log(- (2*a^2*(a^2 + b^2)^(1/2)*(b - a*exp(c + d*x))
)/b^5 - (2*a^2*exp(c + d*x)*(a^2 + b^2))/b^5)*(a^2 + b^2)^(1/2))/(b^4*d) +
 (a^2*log((2*a^2*(a^2 + b^2)^(1/2)*(b - a*exp(c + d*x)))/b^5 - (2*a^2*exp(
c + d*x)*(a^2 + b^2))/b^5)*(a^2 + b^2)^(1/2))/(b^4*d)
```

3.370. $\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.371 $\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.371.1 Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\mathrm{Int}\left(\frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.371.2 Mathematica [N/A]

Not integrable

Time = 11.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])),x]
```

output
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

3.371.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\sinh^2(c+dx)\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.371.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.371.4 Maple [N/A] (verified)

Not integrable

Time = 0.77 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.371.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
integral(cosh(d*x + c)^2*sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh
(d*x + c)), x)
```

3.371.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2*sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.371.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 334, normalized size of antiderivative = 9.28

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
2*(a^4*e^c + a^2*b^2*e^c)*integrate(-e^(d*x)/(b^5*f*x + b^5*e - (b^5*f*x*e
^(2*c) + b^5*e*e^(2*c))*e^(2*d*x) - 2*(a*b^4*f*x*e^c + a*b^4*e*e^c)*e^(d*x
)), x) + 1/8*e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b*f) +
 1/4*a*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^2*f) + 1/4
*a*e^(2*c - 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^2*f) - 1/8*e^(
3*c - 3*d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b*f) + 1/8*(4*a^2 + b^
2)*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^3*f) - 1/8*(4*a^2*e^
c + b^2*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^3*f) - 1/2*(2
*a^3 + a*b^2)*log(f*x + e)/(b^4*f)
```

3.371.8 Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^2\sinh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate(cosh(d*x + c)^2*sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)
), x)
```

3.371.9 Mupad [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^2\sinh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))), x
)
```

3.372 $\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} \, dx$

3.372.1 Optimal result

Integrand size = 36, antiderivative size = 1123

$$\begin{aligned}
&\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{3a^2f^3x}{8b^3d^3}-\frac{45f^3x}{256bd^3}+\frac{a^2(e+fx)^3}{4b^3d}-\frac{3(e+fx)^3}{32bd}-\frac{a^2(a^2+b^2)(e+fx)^4}{4b^5f} \\
&+\frac{6a^3f^3\cosh(c+dx)}{b^4d^4}+\frac{40af^3\cosh(c+dx)}{9b^2d^4}+\frac{3a^3f(e+fx)^2\cosh(c+dx)}{b^4d^2} \\
&+\frac{2af(e+fx)^2\cosh(c+dx)}{b^2d^2}+\frac{9f^2(e+fx)\cosh^2(c+dx)}{32bd^3}+\frac{2af^3\cosh^3(c+dx)}{27b^2d^4} \\
&+\frac{af(e+fx)^2\cosh^3(c+dx)}{3b^2d^2}+\frac{3f^2(e+fx)\cosh^4(c+dx)}{32bd^3}+\frac{(e+fx)^3\cosh^4(c+dx)}{4bd} \\
&+\frac{a^2(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d}+\frac{a^2(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d} \\
&+\frac{3a^2(a^2+b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2} \\
&+\frac{3a^2(a^2+b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^2} \\
&-\frac{6a^2(a^2+b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^3} \\
&-\frac{6a^2(a^2+b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^3} \\
&+\frac{6a^2(a^2+b^2)f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^4}+\frac{6a^2(a^2+b^2)f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^4} \\
&-\frac{6a^3f^2(e+fx)\sinh(c+dx)}{b^4d^3}-\frac{40af^2(e+fx)\sinh(c+dx)}{9b^2d^3} \\
&-\frac{a^3(e+fx)^3\sinh(c+dx)}{b^4d}-\frac{2a(e+fx)^3\sinh(c+dx)}{3b^2d} \\
&-\frac{3a^2f^3\cosh(c+dx)\sinh(c+dx)}{8b^3d^4}-\frac{45f^3\cosh(c+dx)\sinh(c+dx)}{256bd^4} \\
&-\frac{3a^2f(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{4b^3d^2}-\frac{9f(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{32bd^2} \\
&-\frac{2af^2(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{9b^2d^3}-\frac{a(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{3b^2d} \\
&-\frac{3f^3\cosh^3(c+dx)\sinh(c+dx)}{128bd^4}-\frac{3f(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{16bd^2} \\
&+\frac{3a^2f^2(e+fx)\sinh^2(c+dx)}{4b^3d^3}+\frac{a^2(e+fx)^3\sinh^2(c+dx)}{2b^3d}
\end{aligned}$$

output
```
-2/3*a*(f*x+e)^3*sinh(d*x+c)/b^2/d+40/9*a*f^3*cosh(d*x+c)/b^2/d^4-45/256*f
^3*cosh(d*x+c)*sinh(d*x+c)/b/d^4-40/9*a*f^2*(f*x+e)*sinh(d*x+c)/b^2/d^3-9/
32*f*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b/d^2+3*a^2*(a^2+b^2)*f*(f*x+e)^2*p
olylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d^2+3*a^2*(a^2+b^2)*f*(f*x
+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d^2-6*a^2*(a^2+b^2)
*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d^3-6*a^2*(a
^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d^3+2
*a*f*(f*x+e)^2*cosh(d*x+c)/b^2/d^2+1/4*a^2*(f*x+e)^3/b^3/d+1/4*(f*x+e)^3*c
osh(d*x+c)^4/b/d+3/8*a^2*f^3*x/b^3/d^3-1/4*a^2*(a^2+b^2)*(f*x+e)^4/b^5/f+6
*a^3*f^3*cosh(d*x+c)/b^4/d^4+9/32*f^2*(f*x+e)*cosh(d*x+c)^2/b/d^3+2/27*a*f
^3*cosh(d*x+c)^3/b^2/d^4+3/32*f^2*(f*x+e)*cosh(d*x+c)^4/b/d^3-3/128*f^3*co
sh(d*x+c)^3*sinh(d*x+c)/b/d^4+1/2*a^2*(f*x+e)^3*sinh(d*x+c)^2/b^3/d-3/4*a^
2*f*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b^3/d^2-2/9*a*f^2*(f*x+e)*cosh(d*x+c
)^2*sinh(d*x+c)/b^2/d^3-3/32*(f*x+e)^3/b/d+3*a^3*f*(f*x+e)^2*cosh(d*x+c)/b
^4/d^2+1/3*a*f*(f*x+e)^2*cosh(d*x+c)^3/b^2/d^2-6*a^3*f^2*(f*x+e)*sinh(d*x+
c)/b^4/d^3-3/8*a^2*f^3*cosh(d*x+c)*sinh(d*x+c)/b^3/d^4-1/3*a*(f*x+e)^3*cos
h(d*x+c)^2*sinh(d*x+c)/b^2/d-3/16*f*(f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/b/
d^2+3/4*a^2*f^2*(f*x+e)*sinh(d*x+c)^2/b^3/d^3+6*a^2*(a^2+b^2)*f^3*polylog(
4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d^4+6*a^2*(a^2+b^2)*f^3*polylog(4
,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d^4+a^2*(a^2+b^2)*(f*x+e)^3*ln(...
```

3.372.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 8401 vs. $2(1123) = 2246$.

Time = 28.85 (sec) , antiderivative size = 8401, normalized size of antiderivative = 7.48

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*
x]),x]
```

output
```
Result too large to show
```

3.372.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh^2(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)^3\cosh^3(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5970}$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\int (e+fx)^2\cosh^4(c+dx)dx}{4d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^4dx}{4d}}{b}$$

$$\Big\downarrow \text{ 3792}$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\int \cosh^4(c+dx)dx}{8d^2}+\frac{3}{4}\int (e+fx)^2\cosh^2(c+dx)dx-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{4d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\int \sin\left(ic+idx+\frac{\pi}{2}\right)^4dx}{8d^2}+\frac{3}{4}\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{4d}}{b}$$

$$\Big\downarrow \text{ 3115}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\left(\frac{3}{4}\int \cosh^2(c+dx)dx+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{8d^2}+\frac{3}{4}\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{(e+fx)^2\sin}{}\right.}{4d}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{3f\left(\frac{f^2\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\int\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx\right)}{8d^2}+\frac{3}{4}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{(e+fx}{}\right.}{4d}}{b}$$

$\downarrow$ 3115

$$\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{3f\left(\frac{f^2\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{8d^2}+\frac{3}{4}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\cosh^4(c+}{8d^2}\right.}{4d}}{b}$$

$\downarrow$ 24

$$\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{3f\left(\frac{3}{4}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{8d^2}\right.}{4d}}{b}$$

$\downarrow$ 3792

$$\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{\frac{3f\left(\frac{3}{4}\left(\frac{f^2\int\cosh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^2dx-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}\right.}{4d}}{b}$$

$$\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\downarrow$ 17

$$\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{\frac{3f\left(\frac{3}{4}\left(\frac{f^2\int\cosh^2(c+dx)dx}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{f^2(}{}\right.}{4d}}{b}$$

$$\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$\downarrow$ 3042

3.372. $\displaystyle\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(\frac{f^2\int \sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\right.}{4d}}{b}$$

$$\Big\downarrow \; 3115$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(\frac{f^2\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\text{co}}{8d}\right.}{4d}}{b}$$

$$\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}\right.}{4d}}{b}$$

$$\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 6113$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}\right.}{4d}}{b}$$

$$\frac{a\left(\frac{\int (e+fx)^3\cosh^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}\right.}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}\right)}{b}$$

3.372. $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 3792$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}}{4d}}{b}$$

$$\frac{a\left(\frac{\frac{2f^2\int(e+fx)\cosh^3(c+dx)dx}{3d^2}+\frac{2}{3}\int(e+fx)^3\cosh(c+dx)dx-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\int(e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{(e+fx)^3\sinh(c+dx)\cos}{3d}}{b}\right.}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3if\int-i(e+fx)^2\sinh(c+dx)dx}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}}{b}\right.}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int(e+fx)^2\sinh(c+dx)dx}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+(}{b}\right.}{b}$$

$$\Big\downarrow \; 3042$$

3.372. $\displaystyle\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}\right.}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int-i(e+fx)^2\sin(ic+idx)dx}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}}{b}\right.}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}\right.}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\int(e+fx)^2\sin(ic+idx)dx}{d}\right)-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+}{b}\right.}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}\right.}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right)}{d}\right.}{b}\right.}{b}$$

$$\Big\downarrow \; 3042$$

3.372. $\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)}{d}\right.}{d}\right.}{b}\right.}{b}$$

$\downarrow$ 3777

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-if\right.}{d}\right.}{d}\right.}{b}\right.}{b}$$

$\downarrow$ 26

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-f\int\right.}{d}\right.}{d}\right.}{b}\right.}{b}$$

3.372. $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f}{}\right.}{d}\right.}{d}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if}{}\right.}{d}\right.}{d}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{3118}$$

3.372. $\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if}{}\right.}{}\right.}{b}\right.}{b}$$

$$\Big\downarrow \; 3791$$

$$\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh}{8d^2}}{4d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\left(\frac{2}{3}\int(e+fx)\cosh(c+dx)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d^2}-\frac{f(e+fx)^2\cosh^3(c+dx)}{3d^2}+\frac{2}{3}\left((e+f\right.}{}\right.}{b}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.372. $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.372.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.372. $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.372.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)^3 \sinh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.372.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12603 vs. $2(1051) = 2102$.

Time = 0.43 (sec) , antiderivative size = 12603, normalized size of antiderivative = 11.22

$$\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

3.372.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.372.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)^3\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
-1/192*e^3*((8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3)*e^(-2*d*x -
 2*c) + 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) - 1
92*(a^4 + a^2*b^2)*(d*x + c)/(b^5*d) - (8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e
^(-4*d*x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*b + b^3)*e
^(-2*d*x - 2*c))/(b^4*d) - 192*(a^4 + a^2*b^2)*log(-2*a*e^(-d*x - c) + b*e
^(-2*d*x - 2*c) - b)/(b^5*d)) + 1/55296*(13824*(a^4*d^4*f^3*e^(4*c) + a^2*
b^2*d^4*f^3*e^(4*c))*x^4 + 55296*(a^4*d^4*e*f^2*e^(4*c) + a^2*b^2*d^4*e*f^
2*e^(4*c))*x^3 + 82944*(a^4*d^4*e^2*f*e^(4*c) + a^2*b^2*d^4*e^2*f*e^(4*c))
*x^2 + 27*(32*b^4*d^3*f^3*x^3*e^(8*c) + 24*(4*d^3*e*f^2 - d^2*f^3)*b^4*x^2
*e^(8*c) + 12*(8*d^3*e^2*f - 4*d^2*e*f^2 + d*f^3)*b^4*x*e^(8*c) - 3*(8*d^2
*e^2*f - 4*d*e*f^2 + f^3)*b^4*e^(8*c))*e^(4*d*x) - 256*(9*a*b^3*d^3*f^3*x^
3*e^(7*c) + 9*(3*d^3*e*f^2 - d^2*f^3)*a*b^3*x^2*e^(7*c) + 3*(9*d^3*e^2*f -
 6*d^2*e*f^2 + 2*d*f^3)*a*b^3*x*e^(7*c) - (9*d^2*e^2*f - 6*d*e*f^2 + 2*f^3
)*a*b^3*e^(7*c))*e^(3*d*x) - 864*(6*(2*d^2*e^2*f - 2*d*e*f^2 + f^3)*a^2*b^
2*e^(6*c) + 3*(2*d^2*e^2*f - 2*d*e*f^2 + f^3)*b^4*e^(6*c) - 4*(2*a^2*b^2*d
^3*f^3*e^(6*c) + b^4*d^3*f^3*e^(6*c))*x^3 - 6*(2*(2*d^3*e*f^2 - d^2*f^3)*a
^2*b^2*e^(6*c) + (2*d^3*e*f^2 - d^2*f^3)*b^4*e^(6*c))*x^2 - 6*(2*(2*d^3*e^
2*f - 2*d^2*e*f^2 + d*f^3)*a^2*b^2*e^(6*c) + (2*d^3*e^2*f - 2*d^2*e*f^2 +
d*f^3)*b^4*e^(6*c))*x)*e^(2*d*x) + 6912*(12*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3
)*a^3*b*e^(5*c) + 9*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b^3*e^(5*c) - (4*...
```

3.372.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh(dx+c)^3\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a
), x)
```

3.372.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\,(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x
)
```

3.373 $\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)} \, dx$

3.373.1 Optimal result

Integrand size $=36$, antiderivative size $=819$

$$\begin{aligned}
&\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{a^2efx}{2b^3d}-\frac{3efx}{16bd}+\frac{a^2f^2x^2}{4b^3d}-\frac{3f^2x^2}{32bd}-\frac{a^2(a^2+b^2)(e+fx)^3}{3b^5f} \\
&+\frac{2a^3f(e+fx)\cosh(c+dx)}{b^4d^2}+\frac{4af(e+fx)\cosh(c+dx)}{3b^2d^2}+\frac{3f^2\cosh^2(c+dx)}{32bd^3} \\
&+\frac{2af(e+fx)\cosh^3(c+dx)}{9b^2d^2}+\frac{f^2\cosh^4(c+dx)}{32bd^3}+\frac{(e+fx)^2\cosh^4(c+dx)}{4bd} \\
&+\frac{a^2(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d}+\frac{a^2(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d} \\
&+\frac{2a^2(a^2+b^2)f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2} \\
&+\frac{2a^2(a^2+b^2)f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^2} \\
&-\frac{2a^2(a^2+b^2)f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^3}-\frac{2a^2(a^2+b^2)f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^3} \\
&-\frac{2a^3f^2\sinh(c+dx)}{b^4d^3}-\frac{14af^2\sinh(c+dx)}{9b^2d^3}-\frac{a^3(e+fx)^2\sinh(c+dx)}{b^4d} \\
&-\frac{2a(e+fx)^2\sinh(c+dx)}{3b^2d}-\frac{a^2f(e+fx)\cosh(c+dx)\sinh(c+dx)}{2b^3d^2} \\
&-\frac{3f(e+fx)\cosh(c+dx)\sinh(c+dx)}{16bd^2}-\frac{a(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{3b^2d} \\
&-\frac{f(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{8bd^2}+\frac{a^2f^2\sinh^2(c+dx)}{4b^3d^3} \\
&+\frac{a^2(e+fx)^2\sinh^2(c+dx)}{2b^3d}-\frac{2af^2\sinh^3(c+dx)}{27b^2d^3}
\end{aligned}$$

output
```
-2/3*a*(f*x+e)^2*sinh(d*x+c)/b^2/d-3/16*e*f*x/b/d-14/9*a*f^2*sinh(d*x+c)/b
^2/d^3+4/3*a*f*(f*x+e)*cosh(d*x+c)/b^2/d^2-3/16*f*(f*x+e)*cosh(d*x+c)*sinh
(d*x+c)/b/d^2-1/2*a^2*f*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b^3/d^2+2*a^2*(a^2
+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d^2+2*a^2
*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d^2+
3/32*f^2*cosh(d*x+c)^2/b/d^3+1/32*f^2*cosh(d*x+c)^4/b/d^3+1/4*(f*x+e)^2*co
sh(d*x+c)^4/b/d+1/4*a^2*f^2*x^2/b^3/d-1/3*a^2*(a^2+b^2)*(f*x+e)^3/b^5/f-2*
a^3*f^2*sinh(d*x+c)/b^4/d^3+1/4*a^2*f^2*sinh(d*x+c)^2/b^3/d^3+1/2*a^2*(f*x
+e)^2*sinh(d*x+c)^2/b^3/d-2/27*a*f^2*sinh(d*x+c)^3/b^2/d^3+1/2*a^2*e*f*x/b
^3/d+2*a^3*f*(f*x+e)*cosh(d*x+c)/b^4/d^2+2/9*a*f*(f*x+e)*cosh(d*x+c)^3/b^2
/d^2-1/3*a*(f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/b^2/d-1/8*f*(f*x+e)*cosh(d*
x+c)^3*sinh(d*x+c)/b/d^2-2*a^2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/b^5/d^3-2*a^2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))/b^5/d^3+a^2*(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/b^5/d+a^2*(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/b^5/d-a^3*(f*x+e)^2*sinh(d*x+c)/b^4/d-3/32*f^2*x^2/b/d
```

3.373.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5113 vs. 2(819) = 1638.

Time = 15.71 (sec) , antiderivative size = 5113, normalized size of antiderivative = 6.24

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*
x]),x]
```

output
```
Result too large to show
```

3.373.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^2(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6113$$

$$\frac{\int (e+fx)^2\cosh^3(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5970}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\int (e+fx)\cosh^4(c+dx)dx}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^4dx}{2d}}{b}$$

$$\Big\downarrow \text{ 3791}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\int (e+fx)\cosh^2(c+dx)dx-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}$$

$$\Big\downarrow \text{ 3791}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(\frac{1}{2}\int (e+fx)dx-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 6113}$$

3.373. $\displaystyle\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\frac{\int (e+fx)^2\cosh^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3792$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{2f^2\int\cosh^3(c+dx)dx}{9d^2}+\frac{2}{3}\int(e+fx)^2\cosh(c+dx)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2f^2\int\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{9d^2}+\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+}{3d}}{b}\right)}{b}$$

$$\Big\downarrow \; 3113$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if^2\int\left(\sinh^2(c+dx)+1\right)d(-i\sinh(c+dx))}{9d^3}+\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c}{}}{b}\right)}{b}$$

$$\Big\downarrow \; 2009$$

3.373. $\displaystyle\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+d}{3d}}{b}\right.}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int-i(e+fx)\sinh(c+dx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+}{9d^2}}{b}\right.}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}}{b}\right.}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+}{9d^2}}{b}\right.}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx}{9d^2}}{b}\right.}{b}$$

3.373. $\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\cdots}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx\cdots\right)}{9d^3}\cdots}{b}\right)}{b}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\cdots\right)}{d}\cdots\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{6099}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^2\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}+\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\cdots}{\cdots}\right)}{b}$$

3.373. $\displaystyle\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+\frac{(e+fx)^2\sinh(c\text{-}}{}}{b}\right.}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+\frac{(e+fx)^2\sinh(c\text{-}}{}}{b}\right.}{}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{b}+\frac{2if^2\left(-\frac{1}{3}i\sinh\right.}{}\right.}{}$$

$$\Big\downarrow \text{3042}$$

3.373. $\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$a\left(\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+\frac{(e+fx)^2\sinh(c-}{}}{b}\right.$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$a\left(\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+\frac{(e+fx)^2\sinh(c-}{}}{b}\right.$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$a\left(\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+\frac{(e+fx)^2\sinh(c-}{}}{b}\right.$$

$$\Big\downarrow \; 3042$$

3.373. $\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$a\left(\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+\frac{(e+fx)^2\sinh(c-}{}}{b}\right.$$

$$\Big\downarrow \text{3117}$$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$a\left(\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+\frac{(e+fx)^2\sinh(c-}{}}{b}\right.$$

$$\Big\downarrow \text{5969}$$

3.373. $\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-$$

$$a\left(\frac{\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)+\frac{(e+fx)^2\sinh(c\text{-}}{}}{b}\right.$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.373.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

3.373. $\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} \, dx$

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.373.4 Maple [F]

$$\int \frac{(fx+e)^2 \cosh(dx+c)^3 \sinh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.373.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7645 vs. $2(761) = 1522$.

Time = 0.38 (sec) , antiderivative size = 7645, normalized size of antiderivative = 9.33

$$\int \frac{(e+fx)^2 \cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

3.373.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.373.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^3\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
-1/192*e^2*((8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3)*e^(-2*d*x -
 2*c) + 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) - 1
92*(a^4 + a^2*b^2)*(d*x + c)/(b^5*d) - (8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e
^(-4*d*x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*b + b^3)*e
^(-2*d*x - 2*c))/(b^4*d) - 192*(a^4 + a^2*b^2)*log(-2*a*e^(-d*x - c) + b*e
^(-2*d*x - 2*c) - b)/(b^5*d)) + 1/13824*(4608*(a^4*d^3*f^2*e^(4*c) + a^2*b
^2*d^3*f^2*e^(4*c))*x^3 + 13824*(a^4*d^3*e*f*e^(4*c) + a^2*b^2*d^3*e*f*e^(
4*c))*x^2 + 27*(8*b^4*d^2*f^2*x^2*e^(8*c) + 4*(4*d^2*e*f - d*f^2)*b^4*x*e^
(8*c) - (4*d*e*f - f^2)*b^4*e^(8*c))*e^(4*d*x) - 64*(9*a*b^3*d^2*f^2*x^2*e
^(7*c) + 6*(3*d^2*e*f - d*f^2)*a*b^3*x*e^(7*c) - 2*(3*d*e*f - f^2)*a*b^3*e
^(7*c))*e^(3*d*x) - 432*(2*(2*d*e*f - f^2)*a^2*b^2*e^(6*c) + (2*d*e*f - f^
2)*b^4*e^(6*c) - 2*(2*a^2*b^2*d^2*f^2*e^(6*c) + b^4*d^2*f^2*e^(6*c))*x^2 -
 2*(2*(2*d^2*e*f - d*f^2)*a^2*b^2*e^(6*c) + (2*d^2*e*f - d*f^2)*b^4*e^(6*c
))*x)*e^(2*d*x) + 1728*(8*(d*e*f - f^2)*a^3*b*e^(5*c) + 6*(d*e*f - f^2)*a*
b^3*e^(5*c) - (4*a^3*b*d^2*f^2*e^(5*c) + 3*a*b^3*d^2*f^2*e^(5*c))*x^2 - 2*
(4*(d^2*e*f - d*f^2)*a^3*b*e^(5*c) + 3*(d^2*e*f - d*f^2)*a*b^3*e^(5*c))*x)
*e^(d*x) + 1728*(8*(d*e*f + f^2)*a^3*b*e^(3*c) + 6*(d*e*f + f^2)*a*b^3*e^(
3*c) + (4*a^3*b*d^2*f^2*e^(3*c) + 3*a*b^3*d^2*f^2*e^(3*c))*x^2 + 2*(4*(d^2
*e*f + d*f^2)*a^3*b*e^(3*c) + 3*(d^2*e*f + d*f^2)*a*b^3*e^(3*c))*x)*e^(-d*
x) + 432*(2*(2*d*e*f + f^2)*a^2*b^2*e^(2*c) + (2*d*e*f + f^2)*b^4*e^(2*...
```

3.373.8 Giac [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^3\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a
), x)
```

3.373.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x
)
```

3.374 $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.374.1 Optimal result

Integrand size = 34, antiderivative size = 499

$$\begin{aligned}
&\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{a^2fx}{4b^3d}-\frac{3fx}{32bd}-\frac{a^2(a^2+b^2)(e+fx)^2}{2b^5f}+\frac{a^3f\cosh(c+dx)}{b^4d^2}+\frac{2af\cosh(c+dx)}{3b^2d^2} \\
&\quad+\frac{af\cosh^3(c+dx)}{9b^2d^2}+\frac{(e+fx)\cosh^4(c+dx)}{4bd}+\frac{a^2(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d} \\
&\quad+\frac{a^2(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d}+\frac{a^2(a^2+b^2)f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2} \\
&\quad+\frac{a^2(a^2+b^2)f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^2}-\frac{a^3(e+fx)\sinh(c+dx)}{b^4d} \\
&\quad-\frac{2a(e+fx)\sinh(c+dx)}{3b^2d}-\frac{a^2f\cosh(c+dx)\sinh(c+dx)}{4b^3d^2} \\
&\quad-\frac{3f\cosh(c+dx)\sinh(c+dx)}{32bd^2}-\frac{a(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{3b^2d} \\
&\quad-\frac{f\cosh^3(c+dx)\sinh(c+dx)}{16bd^2}+\frac{a^2(e+fx)\sinh^2(c+dx)}{2b^3d}
\end{aligned}$$

output
```
1/4*a^2*f*x/b^3/d-3/32*f*x/b/d-1/2*a^2*(a^2+b^2)*(f*x+e)^2/b^5/f+a^3*f*cos
h(d*x+c)/b^4/d^2+2/3*a*f*cosh(d*x+c)/b^2/d^2+1/9*a*f*cosh(d*x+c)^3/b^2/d^2
+1/4*(f*x+e)*cosh(d*x+c)^4/b/d+a^2*(a^2+b^2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/b^5/d+a^2*(a^2+b^2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/b^5/d+a^2*(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2
)))/b^5/d^2+a^2*(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b
^5/d^2-a^3*(f*x+e)*sinh(d*x+c)/b^4/d-2/3*a*(f*x+e)*sinh(d*x+c)/b^2/d-1/4*a
^2*f*cosh(d*x+c)*sinh(d*x+c)/b^3/d^2-3/32*f*cosh(d*x+c)*sinh(d*x+c)/b/d^2-
1/3*a*(f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/b^2/d-1/16*f*cosh(d*x+c)^3*sinh(d*
x+c)/b/d^2+1/2*a^2*(f*x+e)*sinh(d*x+c)^2/b^3/d
```

3.374.2 Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 904, normalized size of antiderivative = 1.81

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-144b^4de\log(a+b\sinh(c+dx))+72b^4f\left(dx\left(dx-2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)-2\,\mathrm{Po}\right.}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(-144*b^4*d*e*Log[a + b*Sinh[c + d*x]] + 72*b^4*f*(d*x*(d*x - 2*Log[1 + (b
*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])]) - 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*
PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + 72*b^2*d*e*((4*a^2
 + b^2)*Log[a + b*Sinh[c + d*x]] - 4*a*b*Sinh[c + d*x] + 2*b^2*Sinh[c + d*
x]^2) + 24*d*e*(3*(16*a^4 + 12*a^2*b^2 + b^4)*Log[a + b*Sinh[c + d*x]] - 1
2*a*b*(4*a^2 + 3*b^2)*Sinh[c + d*x] + 6*b^2*(4*a^2 + 3*b^2)*Sinh[c + d*x]^
2 - 16*a*b^3*Sinh[c + d*x]^3 + 12*b^4*Sinh[c + d*x]^4) + 36*b^2*f*(8*a*b*C
osh[c + d*x] + 2*b^2*d*x*Cosh[2*(c + d*x)] + (4*a^2 + b^2)*(2*c*(c + d*x)
- (c + d*x)^2 + 2*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]
 + 2*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*Log[b
- 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*PolyLog[2, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] + 2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)]) - 8*a*b*d*x*Sinh[c + d*x] - b^2*Sinh[2*(c + d*x)]) + f*(576*a*b*(2*a^2
 + b^2)*Cosh[c + d*x] + 72*b^2*(4*a^2 + b^2)*d*x*Cosh[2*(c + d*x)] + 32*a*
b^3*Cosh[3*(c + d*x)] + 36*b^4*d*x*Cosh[4*(c + d*x)] + 36*(16*a^4 + 12*a^2
*b^2 + b^4)*(2*c*(c + d*x) - (c + d*x)^2 + 2*(c + d*x)*Log[1 + (b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2])] + 2*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])] - 2*c*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*Poly
Log[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*PolyLog[2, -((b*E^(c...
```

3.374.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^2(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6113$$

$$\frac{\int (e+fx)\cosh^3(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 5970$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\int \cosh^4(c+dx)dx}{4d}}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\int \sin\left(ic+idx+\frac{\pi}{2}\right)^4dx}{4d}}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\int\cosh^2(c+dx)dx+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{4d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\int\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx\right)}{4d}}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{4d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6113}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\int(e+fx)\cosh^3(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3791}$$

3.374. $\displaystyle\int\frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\int(e+fx)\cosh(c+dx)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

3.374. $\int\frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{6099}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)\cosh(c+da}{b^2}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)\sin\left(ic+id\right.}{b^2}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+da}{d}\right.}{}\right.}{}\right.}{b}$$

3.374. $\int\frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \ 26$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+da}{d}\right.}{}\right.}{}\right)}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+da}{d}\right.}{}\right.}{}\right)}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+da}{d}\right.}{}\right.}{}\right)}{b}$$

$$\Big\downarrow \ 3118$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)\cosh(c+dx)}{b}\right.}{b}\right)}{b}$$

$$\Big\downarrow \ 5969$$

3.374. $\displaystyle\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-$$

$$\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{(e+fx)\sinh^2(c+dx)}{2d}\cdots\right.}{\cdots}\right.}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-$$

$$\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{(e+fx)\sinh^2(c+dx)}{2d}\cdots\right.}{\cdots}\right.}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-$$

$$\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{(e+fx)\sinh^2(c+dx)}{2d}\cdots\right.}{\cdots}\right.}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-$$

$$\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{f\left(\frac{\int 1dx}{2}-\sinh(c+d\cdots\right.}{2d\cdots}\right.}{\cdots}\right.}{b}$$

$$\Big\downarrow \text{24}$$

3.374. $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-$$

$$a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx}{d}\right.}{b^2}\right.}{}\right.$$

$$b$$

$$\Big\downarrow \text{6095}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-$$

$$a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx}{a+be^{c+dx}+\sqrt{a^2}}\right.}{b^2}\right.}{}\right.$$

$$b$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-$$

$$a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+}}{a+\sqrt{a}}\right.}{bd}\right.}{}\right.}{}\right.$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.374. $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.374.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.374. $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.374. $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.374.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. $2(463) = 926$.

Time = 59.06 (sec) , antiderivative size = 1217, normalized size of antiderivative = 2.44

method	result	size
risch	Expression too large to display	1217

input
```
int((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURN
VERBOSE)
```

output
```
-1/d^2/b^3*a^2*f*c^2+1/d^2/b^3*a^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-
a)/(-a+(a^2+b^2)^(1/2)))+1/d^2/b^3*a^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/
2)+a)/(a+(a^2+b^2)^(1/2)))-2/d/b^3*a^2*e*ln(exp(d*x+c))+1/d/b^3*a^2*e*ln(b
*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d/b^3*a^2*f*c*x+1/256*(4*d*f*x+4*d*e-f
)/b/d^2*exp(4*d*x+4*c)-1/2*a^2*f*x^2/b^3-1/d^2/b^3*c*a^2*f*ln(b*exp(2*d*x+
2*c)+2*a*exp(d*x+c)-b)+1/256*(4*d*f*x+4*d*e+f)/b/d^2*exp(-4*d*x-4*c)+1/d^2
/b^3*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/
d^2/b^3*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2
/d^2/b^3*c*a^2*f*ln(exp(d*x+c))-1/2*a^4*f*x^2/b^5+1/d/b^3*a^2*f*ln((-b*exp
(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d/b^3*a^2*f*ln((b*exp
(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/32*(4*a^2*d*f*x+2*b^2*
d*f*x+4*a^2*d*e+2*b^2*d*e-2*a^2*f-b^2*f)/b^3/d^2*exp(2*d*x+2*c)+a^2*e*x/b^
3+a^4*e*x/b^5-1/8*a*(4*a^2*d*f*x+3*b^2*d*f*x+4*a^2*d*e+3*b^2*d*e-4*a^2*f-3
*b^2*f)/b^4/d^2*exp(d*x+c)-1/d^2*a^4/b^5*f*c^2-2/d*a^4/b^5*e*ln(exp(d*x+c)
)+1/d*a^4/b^5*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d^2*a^4/b^5*f*dilo
g((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2*a^4/b^5*f*
dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/8*a*(4*a^2+3
*b^2)*(d*f*x+d*e+f)/b^4/d^2*exp(-d*x-c)-2/d*a^4/b^5*f*c*x+1/d*a^4/b^5*f*ln
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d*a^4/b^5*f*l
n((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*a^4/b^5...
```

3.374.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3795 vs. $2(461) = 922$.

Time = 0.32 (sec) , antiderivative size = 3795, normalized size of antiderivative = 7.61

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
1/2304*(9*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c)^8 + 9*(4*b^4*d*f
*x + 4*b^4*d*e - b^4*f)*sinh(d*x + c)^8 - 32*(3*a*b^3*d*f*x + 3*a*b^3*d*e
- a*b^3*f)*cosh(d*x + c)^7 - 8*(12*a*b^3*d*f*x + 12*a*b^3*d*e - 4*a*b^3*f
- 9*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c))*sinh(d*x + c)^7 + 36*
b^4*d*f*x + 72*(2*(2*a^2*b^2 + b^4)*d*f*x + 2*(2*a^2*b^2 + b^4)*d*e - (2*a
^2*b^2 + b^4)*f)*cosh(d*x + c)^6 + 4*(36*(2*a^2*b^2 + b^4)*d*f*x + 36*(2*a
^2*b^2 + b^4)*d*e + 63*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c)^2 -
 18*(2*a^2*b^2 + b^4)*f - 56*(3*a*b^3*d*f*x + 3*a*b^3*d*e - a*b^3*f)*cosh(
d*x + c))*sinh(d*x + c)^6 + 36*b^4*d*e - 288*((4*a^3*b + 3*a*b^3)*d*f*x +
(4*a^3*b + 3*a*b^3)*d*e - (4*a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^5 - 24*(12*
(4*a^3*b + 3*a*b^3)*d*f*x - 21*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x
+ c)^3 + 12*(4*a^3*b + 3*a*b^3)*d*e + 28*(3*a*b^3*d*f*x + 3*a*b^3*d*e - a*
b^3*f)*cosh(d*x + c)^2 - 12*(4*a^3*b + 3*a*b^3)*f - 18*(2*(2*a^2*b^2 + b^4
)*d*f*x + 2*(2*a^2*b^2 + b^4)*d*e - (2*a^2*b^2 + b^4)*f)*cosh(d*x + c))*si
nh(d*x + c)^5 + 9*b^4*f - 1152*((a^4 + a^2*b^2)*d^2*f*x^2 + 2*(a^4 + a^2*b
^2)*d^2*e*x + 4*(a^4 + a^2*b^2)*c*d*e - 2*(a^4 + a^2*b^2)*c^2*f)*cosh(d*x
+ c)^4 - 2*(576*(a^4 + a^2*b^2)*d^2*f*x^2 + 1152*(a^4 + a^2*b^2)*d^2*e*x -
 315*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c)^4 + 2304*(a^4 + a^2*b
^2)*c*d*e - 1152*(a^4 + a^2*b^2)*c^2*f + 560*(3*a*b^3*d*f*x + 3*a*b^3*d*e
- a*b^3*f)*cosh(d*x + c)^3 - 540*(2*(2*a^2*b^2 + b^4)*d*f*x + 2*(2*a^2*...
```

3.374.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.374.7 Maxima [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh{(dx+c)}^3\sinh{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/192*e*((8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3)*e^(-2*d*x - 2
*c) + 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) - 192
*(a^4 + a^2*b^2)*(d*x + c)/(b^5*d) - (8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(
-4*d*x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*b + b^3)*e^(
-2*d*x - 2*c))/(b^4*d) - 192*(a^4 + a^2*b^2)*log(-2*a*e^(-d*x - c) + b*e^(
-2*d*x - 2*c) - b)/(b^5*d)) + 1/2304*f*((1152*(a^4*d^2*e^(4*c) + a^2*b^2*d
^2*e^(4*c))*x^2 + 9*(4*b^4*d*x*e^(8*c) - b^4*e^(8*c))*e^(4*d*x) - 32*(3*a*
b^3*d*x*e^(7*c) - a*b^3*e^(7*c))*e^(3*d*x) - 72*(2*a^2*b^2*e^(6*c) + b^4*e
^(6*c) - 2*(2*a^2*b^2*d*e^(6*c) + b^4*d*e^(6*c))*x)*e^(2*d*x) + 288*(4*a^3
*b*e^(5*c) + 3*a*b^3*e^(5*c) - (4*a^3*b*d*e^(5*c) + 3*a*b^3*d*e^(5*c))*x)*
e^(d*x) + 288*(4*a^3*b*e^(3*c) + 3*a*b^3*e^(3*c) + (4*a^3*b*d*e^(3*c) + 3*
a*b^3*d*e^(3*c))*x)*e^(-d*x) + 72*(2*a^2*b^2*e^(2*c) + b^4*e^(2*c) + 2*(2*
a^2*b^2*d*e^(2*c) + b^4*d*e^(2*c))*x)*e^(-2*d*x) + 32*(3*a*b^3*d*x*e^c + a
*b^3*e^c)*e^(-3*d*x) + 9*(4*b^4*d*x + b^4)*e^(-4*d*x))*e^(-4*c)/(b^5*d^2)
- 72*integrate(64*((a^5*e^c + a^3*b^2*e^c)*x*e^(d*x) - (a^4*b + a^2*b^3)*x
)/(b^6*e^(2*d*x + 2*c) + 2*a*b^5*e^(d*x + c) - b^6), x))
```

3.374.8 Giac [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh{(dx+c)}^3\sinh{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
 x)
```

3.374.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.375 $\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.375.1 Optimal result

Integrand size = 29, antiderivative size = 113

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a^2(a^2+b^2)\log(a+b\sinh(c+dx))}{b^5d} - \frac{a(a^2+b^2)\sinh(c+dx)}{b^4d} + \frac{(a^2+b^2)\sinh^2(c+dx)}{2b^3d} - \frac{a\sinh^3(c+dx)}{3b^2d} + \frac{\sinh^4(c+dx)}{4bd}$$

output
```
a^2*(a^2+b^2)*ln(a+b*sinh(d*x+c))/b^5/d-a*(a^2+b^2)*sinh(d*x+c)/b^4/d+1/2*
(a^2+b^2)*sinh(d*x+c)^2/b^3/d-1/3*a*sinh(d*x+c)^3/b^2/d+1/4*sinh(d*x+c)^4/
b/d
```

3.375.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{12a^2(a^2+b^2)\log(a+b\sinh(c+dx)) - 12ab(a^2+b^2)\sinh(c+dx) + 6b^2(a^2+b^2)\sinh^2(c+dx) - 4ab^3\sin}{12b^5d}$$

input
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(12*a^2*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - 12*a*b*(a^2 + b^2)*Sinh[c +
 d*x] + 6*b^2*(a^2 + b^2)*Sinh[c + d*x]^2 - 4*a*b^3*Sinh[c + d*x]^3 + 3*b^
4*Sinh[c + d*x]^4)/(12*b^5*d)
```

3.375.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2\cos(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos(ic+idx)^3\sin(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{\int \frac{\sinh^2(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b^2\sinh^2(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^5d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(b^3\sinh^3(c+dx)-ab^2\sinh^2(c+dx)+b\left(a^2+b^2\right)\sinh(c+dx)-a\left(a^2+b^2\right)+\frac{a^2\left(a^2+b^2\right)}{a+b\sinh(c+dx)}\right)d(b\sinh(c+dx))}{b^5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}b^2\left(a^2+b^2\right)\sinh^2(c+dx)-ab\left(a^2+b^2\right)\sinh(c+dx)+a^2\left(a^2+b^2\right)\log(a+b\sinh(c+dx))-\frac{1}{3}ab^3\sinh^3(c+dx)}{b^5d}$$

input
```
Int[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(a^2*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - a*b*(a^2 + b^2)*Sinh[c + d*x]
+ (b^2*(a^2 + b^2)*Sinh[c + d*x]^2)/2 - (a*b^3*Sinh[c + d*x]^3)/3 + (b^4*S
inh[c + d*x]^4)/4)/(b^5*d)
```

3.375.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

3.375. $\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.375.4 Maple [A] (verified)

Time = 25.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-\frac{-\frac{\sinh(dx+c)^4 b^3}{4}+\frac{a\sinh(dx+c)^3 b^2}{3}-\frac{\left(a^2+b^2\right)\sinh(dx+c)^2 b}{2}+a\left(a^2+b^2\right)\sinh(dx+c)}{b^4}+\frac{a^2\left(a^2+b^2\right)\ln(a+b\sinh(dx+c))}{b^5}}{d}$
default	$\frac{-\frac{-\frac{\sinh(dx+c)^4 b^3}{4}+\frac{a\sinh(dx+c)^3 b^2}{3}-\frac{\left(a^2+b^2\right)\sinh(dx+c)^2 b}{2}+a\left(a^2+b^2\right)\sinh(dx+c)}{b^4}+\frac{a^2\left(a^2+b^2\right)\ln(a+b\sinh(dx+c))}{b^5}}{d}$
risch	$-\frac{a^4x}{b^5}-\frac{x\,a^2}{b^3}+\frac{\mathrm{e}^{4dx+4c}}{64bd}-\frac{a\,\mathrm{e}^{3dx+3c}}{24b^2d}+\frac{\mathrm{e}^{2dx+2c}a^2}{8b^3d}+\frac{\mathrm{e}^{2dx+2c}}{16bd}-\frac{a^3\mathrm{e}^{dx+c}}{2b^4d}-\frac{3a\,\mathrm{e}^{dx+c}}{8b^2d}+\frac{a^3\mathrm{e}^{-dx-c}}{2b^4d}+\frac{3a\,\mathrm{e}}{8}$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^4*(-1/4*sinh(d*x+c)^4*b^3+1/3*a*sinh(d*x+c)^3*b^2-1/2*(a^2+b^2)*
sinh(d*x+c)^2*b+a*(a^2+b^2)*sinh(d*x+c))+a^2*(a^2+b^2)/b^5*ln(a+b*sinh(d*x
+c)))
```

3.375.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1069 vs. 2(107) = 214.

Time = 0.27 (sec) , antiderivative size = 1069, normalized size of antiderivative = 9.46

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fric
as")
```

output
```
1/192*(3*b^4*cosh(d*x + c)^8 + 3*b^4*sinh(d*x + c)^8 - 8*a*b^3*cosh(d*x +
c)^7 + 8*(3*b^4*cosh(d*x + c) - a*b^3)*sinh(d*x + c)^7 - 192*(a^4 + a^2*b^
2)*d*x*cosh(d*x + c)^4 + 12*(2*a^2*b^2 + b^4)*cosh(d*x + c)^6 + 4*(21*b^4*
cosh(d*x + c)^2 - 14*a*b^3*cosh(d*x + c) + 6*a^2*b^2 + 3*b^4)*sinh(d*x + c
)^6 - 24*(4*a^3*b + 3*a*b^3)*cosh(d*x + c)^5 + 24*(7*b^4*cosh(d*x + c)^3 -
 7*a*b^3*cosh(d*x + c)^2 - 4*a^3*b - 3*a*b^3 + 3*(2*a^2*b^2 + b^4)*cosh(d*
x + c))*sinh(d*x + c)^5 + 8*a*b^3*cosh(d*x + c) + 2*(105*b^4*cosh(d*x + c)
^4 - 140*a*b^3*cosh(d*x + c)^3 - 96*(a^4 + a^2*b^2)*d*x + 90*(2*a^2*b^2 +
b^4)*cosh(d*x + c)^2 - 60*(4*a^3*b + 3*a*b^3)*cosh(d*x + c))*sinh(d*x + c)
^4 + 3*b^4 + 24*(4*a^3*b + 3*a*b^3)*cosh(d*x + c)^3 + 8*(21*b^4*cosh(d*x +
 c)^5 - 35*a*b^3*cosh(d*x + c)^4 + 12*a^3*b + 9*a*b^3 - 96*(a^4 + a^2*b^2)
*d*x*cosh(d*x + c) + 30*(2*a^2*b^2 + b^4)*cosh(d*x + c)^3 - 30*(4*a^3*b +
3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(2*a^2*b^2 + b^4)*cosh(d*x
+ c)^2 + 12*(7*b^4*cosh(d*x + c)^6 - 14*a*b^3*cosh(d*x + c)^5 - 96*(a^4 +
a^2*b^2)*d*x*cosh(d*x + c)^2 + 15*(2*a^2*b^2 + b^4)*cosh(d*x + c)^4 + 2*a^
2*b^2 + b^4 - 20*(4*a^3*b + 3*a*b^3)*cosh(d*x + c)^3 + 6*(4*a^3*b + 3*a*b^
3)*cosh(d*x + c))*sinh(d*x + c)^2 + 192*((a^4 + a^2*b^2)*cosh(d*x + c)^4 +
 4*(a^4 + a^2*b^2)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^4 + a^2*b^2)*cosh(
d*x + c)^2*sinh(d*x + c)^2 + 4*(a^4 + a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)
^3 + (a^4 + a^2*b^2)*sinh(d*x + c)^4)*log(2*(b*sinh(d*x + c) + a)/(cosh...
```

3.375.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.375.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(107) = 214$.

Time = 0.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.07

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{\left(8\,ab^2e^{(-dx-c)} - 3\,b^3 - 12\,(2\,a^2b + b^3)e^{(-2\,dx-2\,c)} + 24\,(4\,a^3 + 3\,ab^2)e^{(-3\,dx-3\,c)}\right)e^{(4\,dx+4\,c)}}{192\,b^4d}$$
$$+ \frac{(a^4 + a^2b^2)(dx + c)}{b^5d}$$
$$+ \frac{8\,ab^2e^{(-3\,dx-3\,c)} + 3\,b^3e^{(-4\,dx-4\,c)} + 24\,(4\,a^3 + 3\,ab^2)e^{(-dx-c)} + 12\,(2\,a^2b + b^3)e^{(-2\,dx-2\,c)}}{192\,b^4d}$$
$$+ \frac{(a^4 + a^2b^2)\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{b^5d}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxi
ma")
```

output
```
-1/192*(8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3)*e^(-2*d*x - 2*c)
 + 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) + (a^4 +
 a^2*b^2)*(d*x + c)/(b^5*d) + 1/192*(8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-
4*d*x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*b + b^3)*e^(-
2*d*x - 2*c))/(b^4*d) + (a^4 + a^2*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*
x - 2*c) - b)/(b^5*d)
```

3.375.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.79

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\frac{3\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^4-8\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+24\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+24\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-96\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)-96}{b^4}}{192\,d}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac
")
```

output
```
1/192*((3*b^3*(e^(d*x + c) - e^(-d*x - c))^4 - 8*a*b^2*(e^(d*x + c) - e^(-
d*x - c))^3 + 24*a^2*b*(e^(d*x + c) - e^(-d*x - c))^2 + 24*b^3*(e^(d*x + c
) - e^(-d*x - c))^2 - 96*a^3*(e^(d*x + c) - e^(-d*x - c)) - 96*a*b^2*(e^(d
*x + c) - e^(-d*x - c)))/b^4 + 192*(a^4 + a^2*b^2)*log(abs(b*(e^(d*x + c)
- e^(-d*x - c)) + 2*a))/b^5)/d
```

3.375.9 Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.11

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{-4c-4dx}}{64\,b\,d} - \frac{x\,(a^4+a^2\,b^2)}{b^5} + \frac{e^{4c+4dx}}{64\,b\,d} + \frac{a\,e^{-3c-3dx}}{24\,b^2\,d} - \frac{a\,e^{3c+3dx}}{24\,b^2\,d} + \frac{\ln\left(2\,a\,e^{dx}\,e^{c} - b + b\,e^{2c}\,e^{2dx}\right)\,(a^4+a^2\,b^2)}{b^5\,d} - \frac{e^{c+dx}\,(4\,a^3+3\,a\,b^2)}{8\,b^4\,d} + \frac{e^{-c-dx}\,(4\,a^3+3\,a\,b^2)}{8\,b^4\,d} + \frac{e^{-2c-2dx}\,(2\,a^2+b^2)}{16\,b^3\,d} + \frac{e^{2c+2dx}\,(2\,a^2+b^2)}{16\,b^3\,d}$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
exp(- 4*c - 4*d*x)/(64*b*d) - (x*(a^4 + a^2*b^2))/b^5 + exp(4*c + 4*d*x)/(
64*b*d) + (a*exp(- 3*c - 3*d*x))/(24*b^2*d) - (a*exp(3*c + 3*d*x))/(24*b^2
*d) + (log(2*a*exp(d*x)*exp(c) - b + b*exp(2*c)*exp(2*d*x))*(a^4 + a^2*b^2
))/(b^5*d) - (exp(c + d*x)*(3*a*b^2 + 4*a^3))/(8*b^4*d) + (exp(- c - d*x)*
(3*a*b^2 + 4*a^3))/(8*b^4*d) + (exp(- 2*c - 2*d*x)*(2*a^2 + b^2))/(16*b^3*
d) + (exp(2*c + 2*d*x)*(2*a^2 + b^2))/(16*b^3*d)
```

3.376 $\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.376.1 Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Int}\left(\frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.376.2 Mathematica [N/A]

Not integrable

Time = 30.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])),x]
```

output
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

3.376.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\sinh^2(c+dx)\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.376.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.376.4 Maple [N/A] (verified)

Not integrable

Time = 0.87 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(dx+c\right)^{3}\sinh\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.376.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^3\sinh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
integral(cosh(d*x + c)^3*sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh
(d*x + c)), x)
```

3.376.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3*sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.376.7 Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 442, normalized size of antiderivative = 12.28

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^3\sinh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
1/16*e^(-4*c + 4*d*e/f)*exp_integral_e(1, 4*(f*x + e)*d/f)/(b*f) + 1/8*a*e
^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b^2*f) + 1/8*a*e^(3*
c - 3*d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b^2*f) - 1/16*e^(4*c - 4
*d*e/f)*exp_integral_e(1, -4*(f*x + e)*d/f)/(b*f) + 1/8*(2*a^2 + b^2)*e^(-
2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^3*f) - 1/8*(2*a^2*e^(
2*c) + b^2*e^(2*c))*e^(-2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^3*
f) + 1/8*(4*a^3 + 3*a*b^2)*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)
/(b^4*f) + 1/8*(4*a^3*e^c + 3*a*b^2*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*
x + e)*d/f)/(b^4*f) + (a^4 + a^2*b^2)*log(f*x + e)/(b^5*f) - 1/32*integrat
e(64*(a^4*b + a^2*b^3 - (a^5*e^c + a^3*b^2*e^c)*e^(d*x))/(b^6*f*x + b^6*e
- (b^6*f*x*e^(2*c) + b^6*e*e^(2*c))*e^(2*d*x) - 2*(a*b^5*f*x*e^c + a*b^5*e
*e^c)*e^(d*x)), x)
```

3.376.8 Giac [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate(cosh(d*x + c)^3*sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)
), x)
```

3.376.9 Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))), x
)
```

3.376. $\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.377 $\int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} \, dx$

3.377.1 Optimal result

Integrand size = 32, antiderivative size = 1218

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\begin{aligned}
&= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3\arctan\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^3\arctan\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} \\
&+ \frac{a^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d} + \frac{a^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d} \\
&+ \frac{(e+fx)^3\log\left(1+e^{2(c+dx)}\right)}{bd} - \frac{a^2(e+fx)^3\log\left(1+e^{2(c+dx)}\right)}{b\left(a^2+b^2\right)d} \\
&+ \frac{3iaf(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{b^2d^2} - \frac{3ia^3f(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{b^2\left(a^2+b^2\right)d^2} \\
&- \frac{3iaf(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{b^2d^2} + \frac{3ia^3f(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{b^2\left(a^2+b^2\right)d^2} \\
&+ \frac{3a^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^2} + \frac{3a^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^2} \\
&+ \frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2bd^2} - \frac{3a^2f(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2b\left(a^2+b^2\right)d^2} \\
&- \frac{6iaf^2(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{b^2d^3} + \frac{6ia^3f^2(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{b^2\left(a^2+b^2\right)d^3} \\
&+ \frac{6iaf^2(e+fx)\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{b^2d^3} - \frac{6ia^3f^2(e+fx)\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{b^2\left(a^2+b^2\right)d^3} \\
&- \frac{6a^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3} - \frac{6a^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3} \\
&- \frac{3f^2(e+fx)\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2bd^3} + \frac{3a^2f^2(e+fx)\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2b\left(a^2+b^2\right)d^3} \\
&+ \frac{6iaf^3\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)}{b^2d^4} - \frac{6ia^3f^3\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)}{b^2\left(a^2+b^2\right)d^4} \\
&- \frac{6iaf^3\operatorname{PolyLog}\left(4,ie^{c+dx}\right)}{b^2d^4} + \frac{6ia^3f^3\operatorname{PolyLog}\left(4,ie^{c+dx}\right)}{b^2\left(a^2+b^2\right)d^4} \\
&+ \frac{6a^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^4} + \frac{6a^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^4} \\
&+ \frac{3f^3\operatorname{PolyLog}\left(4,-e^{2(c+dx)}\right)}{4bd^4} - \frac{3a^2f^3\operatorname{PolyLog}\left(4,-e^{2(c+dx)}\right)}{4b\left(a^2+b^2\right)d^4}
\end{aligned}$$

output
```
a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d-6*I*a^3
*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/b^2/(a^2+b^2)/d^3-3*I*a^3*f*(f*x+e)^2
*polylog(2,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^2+(f*x+e)^3*ln(1+exp(2*d*x+2*c))
/b/d-3/2*a^2*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^2+3/2*a^
2*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^3+3*a^2*f*(f*x+e)^2
*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^2+3*a^2*f*(f*x
+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^2-6*a^2*f
^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^3+6*
I*a*f^3*polylog(4,-I*exp(d*x+c))/b^2/d^4-1/4*(f*x+e)^4/b/f-6*a^2*f^2*(f*x+
e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^3-3*I*a*f*(f
*x+e)^2*polylog(2,I*exp(d*x+c))/b^2/d^2-6*I*a*f^2*(f*x+e)*polylog(3,-I*exp
(d*x+c))/b^2/d^3-6*I*a^3*f^3*polylog(4,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^4+3/
4*f^3*polylog(4,-exp(2*d*x+2*c))/b/d^4-2*a*(f*x+e)^3*arctan(exp(d*x+c))/b^
2/d+3/2*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/b/d^2-3/2*f^2*(f*x+e)*polyl
og(3,-exp(2*d*x+2*c))/b/d^3+3*I*a*f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/b^2
/d^2+6*I*a*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/b^2/d^3+6*I*a^3*f^3*polylog
(4,I*exp(d*x+c))/b^2/(a^2+b^2)/d^4+3*I*a^3*f*(f*x+e)^2*polylog(2,I*exp(d*x
+c))/b^2/(a^2+b^2)/d^2+6*I*a^3*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/b^2/(a
^2+b^2)/d^3+2*a^3*(f*x+e)^3*arctan(exp(d*x+c))/b^2/(a^2+b^2)/d-3/4*a^2*f^3
*polylog(4,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^4+6*a^2*f^3*polylog(4,-b*exp(...
```

3.377.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3251 vs. $2(1218) = 2436$.

Time = 11.15 (sec) , antiderivative size = 3251, normalized size of antiderivative = 2.67

$$\int \frac{(e+fx)^3 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
-1/4*(8*b*d^4*e^3*E^(2*c)*x + 12*b*d^4*e^2*E^(2*c)*f*x^2 + 8*b*d^4*e*E^(2*
c)*f^2*x^3 + 2*b*d^4*E^(2*c)*f^3*x^4 + 8*a*d^3*e^3*ArcTan[E^(c + d*x)] + 8
*a*d^3*e^3*E^(2*c)*ArcTan[E^(c + d*x)] + (12*I)*a*d^3*e^2*f*x*Log[1 - I*E^
(c + d*x)] + (12*I)*a*d^3*e^2*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (12*I)*
a*d^3*e*f^2*x^2*Log[1 - I*E^(c + d*x)] + (12*I)*a*d^3*e*E^(2*c)*f^2*x^2*Lo
g[1 - I*E^(c + d*x)] + (4*I)*a*d^3*f^3*x^3*Log[1 - I*E^(c + d*x)] + (4*I)*
a*d^3*E^(2*c)*f^3*x^3*Log[1 - I*E^(c + d*x)] - (12*I)*a*d^3*e^2*f*x*Log[1
+ I*E^(c + d*x)] - (12*I)*a*d^3*e^2*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (
12*I)*a*d^3*e*f^2*x^2*Log[1 + I*E^(c + d*x)] - (12*I)*a*d^3*e*E^(2*c)*f^2*
x^2*Log[1 + I*E^(c + d*x)] - (4*I)*a*d^3*f^3*x^3*Log[1 + I*E^(c + d*x)] -
(4*I)*a*d^3*E^(2*c)*f^3*x^3*Log[1 + I*E^(c + d*x)] - 4*b*d^3*e^3*Log[1 + E
^(2*(c + d*x))] - 4*b*d^3*e^3*E^(2*c)*Log[1 + E^(2*(c + d*x))] - 12*b*d^3*
e^2*f*x*Log[1 + E^(2*(c + d*x))] - 12*b*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(2*(
c + d*x))] - 12*b*d^3*e*f^2*x^2*Log[1 + E^(2*(c + d*x))] - 12*b*d^3*e*E^(2
*c)*f^2*x^2*Log[1 + E^(2*(c + d*x))] - 4*b*d^3*f^3*x^3*Log[1 + E^(2*(c + d
*x))] - 4*b*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(2*(c + d*x))] - (12*I)*a*d^2*(1
 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)] + (12*I)*a*d^2*(1 +
 E^(2*c))*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)] - 6*b*d^2*e^2*f*PolyLog[
2, -E^(2*(c + d*x))] - 6*b*d^2*e^2*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))]
- 12*b*d^2*e*f^2*x*PolyLog[2, -E^(2*(c + d*x))] - 12*b*d^2*e*E^(2*c)*f^...
```

3.377.3 Rubi [A] (verified)

Time = 5.14 (sec) , antiderivative size = 1036, normalized size of antiderivative = 0.85, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.594, Rules used = {6115, 3042, 26, 4201, 2620, 3011, 6101, 3042, 4668, 3011, 6107, 6095, 2620, 3011, 7163, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6115}$$

$$\frac{\int (e+fx)^3\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)^3\tan(ic+idx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)^3\tan(ic+idx)dx}{b}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{a\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\int \frac{e^{2(c+dx)}(e+fx)^3}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{a\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\int (e+fx)^2\log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{a\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$$\Big\downarrow \text{6101}$$

$$-\frac{a\left(\frac{\int (e+fx)^3\text{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$$\Big\downarrow \text{4668}$$

3.377. $\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{3if\int(e+fx)^2\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{3if\int(e+fx)^2\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^3\arctan\left(e^{c+dx}\right)}{d}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$\downarrow$ 3011

$$-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)d}{d}\right.}{d}}{b}\right.}{b}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$\downarrow$ 6107

$$-\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{3if\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-i\right.}{d}\right.}{d}\right.}{b}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$\downarrow$ 6095

3.377. $\int\frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx$

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLo}\cdots}{d}\right.}{\cdots}\right)}{b\cdots}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\cdots\right.}{a^2+b^2}\cdots\right.}{b}\cdots\right.}{\cdots}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

3.377. $\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+\cdots}{\cdots}\right)}{bd}\cdots\right)}{a^2+b^2}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}$$

$$\Big\downarrow \; 7163$$

3.377. $\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$a\left(-\frac{a\left(b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots$$

$$-\frac{\cdots}{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2d}-\frac{f\int \operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)dx}{2d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\cdots\right.}$$

$$\frac{\cdots}{b}$$

$$\Big\downarrow \text{2720}$$

3.377. $\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2(c+dx)}\right)}{2d}-\frac{3f\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2d}-\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2\right.}{2d}\right.}{2d}\right.\right.}{b}$$

$$a\left(\frac{\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^3}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}}{b}-\frac{3if\left(\right.}{}\right.$$

$\downarrow$ 7143

3.377. $\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$b^2 - \frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLo}}{}\right.}{bd}$$

$$a\left(a\left(\frac{\int(e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\right.\right.$$

$$-\frac{a\left(-\right.}{}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(4,-e^{2(c+dx)}\right)}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e}{}\right.}{b}$$

$$\Big\downarrow \; 7293$$

3.377. $\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$a\left(-\frac{a\left(\frac{\int \left(a(e+fx)^3\operatorname{sech}(c+dx)-b(e+fx)^3\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx\cdots}{\cdots}\right)}{bd}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)$$

$$-\frac{\cdots}{\cdots}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(4,-e^{2(c+dx)}\right)}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e\cdots}{\cdots}\right)}{b}$$

$$\downarrow 2009$$

3.377. $\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^3}{d}+\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\left(\frac{(e+fx)\operatorname{Pol}}{}\right.}{}\right.}{}}{b}\right.}{\ldots}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2(c+dx)}\right)}{2d}-\frac{3f\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(4,-e^{2(c+dx)}\right)}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e}{}\right.}{b}$$

3.377. $\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^3*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((a*(((2*(e + f*x)^3*ArcTan[E^(c + d*x)])/d + ((3*I)*f*(-(((e + f*x)^2*Po
lyLog[2, (-I)*E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, (-I)*E^(c + d
*x)])/d - (f*PolyLog[4, (-I)*E^(c + d*x)])/d^2))/d))/d - ((3*I)*f*(-(((e +
 f*x)^2*PolyLog[2, I*E^(c + d*x)])/d) + (2*f*(((e + f*x)*PolyLog[3, I*E^(c
 + d*x)])/d - (f*PolyLog[4, I*E^(c + d*x)])/d^2))/d))/d)/b - (a*((b^2*(-1/
4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/d^2))/d))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^4)/(4*f) + (2*a*(e
 + f*x)^3*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^3*Log[1 + E^(2*(c + d*x))]
)/d - ((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((3*I)*a*
f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/d^2 - (3*b*f*(e + f*x)^2*PolyLog[
2, -E^(2*(c + d*x))])/(2*d^2) + ((6*I)*a*f^2*(e + f*x)*PolyLog[3, (-I)*E^(
c + d*x)])/d^3 - ((6*I)*a*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/d^3 + (
3*b*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*d^3) - ((6*I)*a*f^3*...
```

3.377.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

3.377. $\int \frac{(e+fx)^3 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6115
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - S
imp[a/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.377. $\int \frac{(e+fx)^3 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.377.4 Maple [F]

$$\int \frac{(fx+e)^3 \sinh(dx+c)\tanh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.377.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1962, normalized size of antiderivative = 1.61

$$\int \frac{(e+fx)^3 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-1/4*((a^2 + b^2)*d^4*f^3*x^4 + 4*(a^2 + b^2)*d^4*e*f^2*x^3 + 6*(a^2 + b^2
)*d^4*e^2*f*x^2 + 4*(a^2 + b^2)*d^4*e^3*x - 24*a^2*f^3*polylog(4, (a*cosh(
d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2))/b) - 24*a^2*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c
) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(a^
2*d^2*f^3*x^2 + 2*a^2*d^2*e*f^2*x + a^2*d^2*e^2*f)*dilog((a*cosh(d*x + c)
+ a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b + 1) - 12*(a^2*d^2*f^3*x^2 + 2*a^2*d^2*e*f^2*x + a^2*d^2*e^2*f)
*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*(I*a*b*d^2*f^3*x^2 - b^2*d^2*
f^3*x^2 + 2*I*a*b*d^2*e*f^2*x - 2*b^2*d^2*e*f^2*x + I*a*b*d^2*e^2*f - b^2*
d^2*e^2*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + 12*(-I*a*b*d^2*f^3*x
^2 - b^2*d^2*f^3*x^2 - 2*I*a*b*d^2*e*f^2*x - 2*b^2*d^2*e*f^2*x - I*a*b*d^2
*e^2*f - b^2*d^2*e^2*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - 4*(a^2
*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2 - a^2*c^3*f^3)*log(2*b*co
sh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(a^
2*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2 - a^2*c^3*f^3)*log(2*b*c
osh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(a
^2*d^3*f^3*x^3 + 3*a^2*d^3*e*f^2*x^2 + 3*a^2*d^3*e^2*f*x + 3*a^2*c*d^2*e^2
*f - 3*a^2*c^2*d*e*f^2 + a^2*c^3*f^3)*log(-(a*cosh(d*x + c) + a*sinh(d*...
```

3.377.6 Sympy [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3\sinh\left(c+dx\right)\tanh\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

3.377.7 Maxima [F]

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\sinh\left(dx+c\right)\tanh\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
e^3*(a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b + b^3)*d)
 + 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/
((a^2 + b^2)*d) + (d*x + c)/(b*d)) + 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*
x^2)/b - integrate(2*(a^2*b*f^3*x^3 + 3*a^2*b*e*f^2*x^2 + 3*a^2*b*e^2*f*x
- (a^3*f^3*x^3*e^c + 3*a^3*e*f^2*x^2*e^c + 3*a^3*e^2*f*x*e^c)*e^(d*x))/(a^
2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x) - 2*(a^3*b*e^c + a
*b^3*e^c)*e^(d*x)), x) - integrate(2*(b*f^3*x^3 + 3*b*e*f^2*x^2 + 3*b*e^2*
f*x + (a*f^3*x^3*e^c + 3*a*e*f^2*x^2*e^c + 3*a*e^2*f*x*e^c)*e^(d*x))/(a^2
+ b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

3.377.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.377.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.378 $\int \frac{(e+fx)^2 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.378.1 Optimal result

Integrand size = 32, antiderivative size = 861

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\begin{aligned}
&= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2\arctan\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\arctan\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} \\
&+ \frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d} + \frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d} \\
&+ \frac{(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{bd} - \frac{a^2(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{b\left(a^2+b^2\right)d} \\
&+ \frac{2iaf(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{b^2d^2} - \frac{2ia^3f(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{b^2\left(a^2+b^2\right)d^2} \\
&- \frac{2iaf(e+fx)\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{b^2d^2} + \frac{2ia^3f(e+fx)\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{b^2\left(a^2+b^2\right)d^2} \\
&+ \frac{2a^2f(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^2} + \frac{2a^2f(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^2} \\
&+ \frac{f(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{bd^2} - \frac{a^2f(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{b\left(a^2+b^2\right)d^2} \\
&- \frac{2iaf^2\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{b^2d^3} + \frac{2ia^3f^2\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{b^2\left(a^2+b^2\right)d^3} \\
&+ \frac{2iaf^2\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{b^2d^3} - \frac{2ia^3f^2\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{b^2\left(a^2+b^2\right)d^3} \\
&- \frac{2a^2f^2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3} - \frac{2a^2f^2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3} \\
&- \frac{f^2\,\mathrm{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2bd^3} + \frac{a^2f^2\,\mathrm{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2b\left(a^2+b^2\right)d^3}
\end{aligned}$$

output
```
-1/3*(f*x+e)^3/b/f-2*a*(f*x+e)^2*arctan(exp(d*x+c))/b^2/d+2*a^3*(f*x+e)^2*
arctan(exp(d*x+c))/b^2/(a^2+b^2)/d+(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b/d-a^2*
(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b/(a^2+b^2)/d+a^2*(f*x+e)^2*ln(1+b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d+a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(
a^2+b^2)^(1/2)))/b/(a^2+b^2)/d-2*I*a^3*f^2*polylog(3,I*exp(d*x+c))/b^2/(a^
2+b^2)/d^3-2*I*a*f^2*polylog(3,-I*exp(d*x+c))/b^2/d^3+2*I*a*f*(f*x+e)*poly
log(2,-I*exp(d*x+c))/b^2/d^2+2*I*a^3*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b^2
/(a^2+b^2)/d^2+f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/b/d^2-a^2*f*(f*x+e)*po
lylog(2,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^2+2*a^2*f*(f*x+e)*polylog(2,-b*exp(
d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^2+2*a^2*f*(f*x+e)*polylog(2,-b*e
xp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^2+2*I*a*f^2*polylog(3,I*exp(d
*x+c))/b^2/d^3+2*I*a^3*f^2*polylog(3,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^3-2*I*
a*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b^2/d^2-2*I*a^3*f*(f*x+e)*polylog(2,-I
*exp(d*x+c))/b^2/(a^2+b^2)/d^2-1/2*f^2*polylog(3,-exp(2*d*x+2*c))/b/d^3+1/
2*a^2*f^2*polylog(3,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^3-2*a^2*f^2*polylog(3,-
b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^3-2*a^2*f^2*polylog(3,-b*e
xp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^3
```

3.378.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1759 vs. 2(861) = 1722.

Time = 10.57 (sec) , antiderivative size = 1759, normalized size of antiderivative = 2.04

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
-1/6*(12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x - 12*b*d^3*e*f
*x^2 - 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] +
6*b*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*a*d*
e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) -
 PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*b*d*e*(1 +
E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c +
 d*x))]) + (6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2
*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*P
olyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I
*E^(c + d*x)]) + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c
 + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c +
d*x))]))/((a^2 + b^2)*d^3*(1 + E^(2*c))) - (a^2*(6*e^2*E^(2*c)*x + 6*e*E^(
2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^
(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 +
 b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 +
 b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))
/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*
E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*
d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^
(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log...
```

3.378.3 Rubi [A] (verified)

Time = 4.11 (sec) , antiderivative size = 748, normalized size of antiderivative = 0.87, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6115, 3042, 26, 4201, 2620, 3011, 2720, 6101, 3042, 4668, 3011, 2720, 6107, 6095, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)} \, dx$$

$$\Big\downarrow \text{6115}$$

$$\frac{\int (e+fx)^2 \tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2 \tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2 \tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)^2 \tan(ic+idx)dx}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)^2\tan(ic+idx)dx}{b}$$

$$\Big\downarrow \; 4201$$

$$-\frac{a\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\int \frac{e^{2(c+dx)}(e+fx)^2}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$-\frac{a\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int (e+fx)\log\left(1+e^{2(c+dx)}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \; 3011$$

$$-\frac{a\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int \operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{2d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \; 2720$$

$$-\frac{a\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \; 6101$$

$$-\frac{a\left(\frac{\int (e+fx)^2\operatorname{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \; 3042$$

3.378. $\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{2if\int(e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int(e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLo}}{d}\right.}{d}}{b}\right.}{b}$$
$$-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+c}\right.}{d^2}\right.}{}}{b}\right.}{b}$$
$$-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \text{6107}$$

3.378. $\int\frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx$

$$-\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}(2}{d}\right.}{d}\right.}{b}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\downarrow \text{6095}$$

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int(e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLo}}{}\right.}{}\right.}{}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\downarrow \text{2620}$$

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\right.}{a^2+b^2}\right.}{b}\right.}{}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\downarrow \text{3011}$$

3.378. $\int\frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\cdots\right.}{d}\right.}{bd}\right.}{a^2+b^2}\right.}{b}\right.}{}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\downarrow \text{2720}$$

3.378. $\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(a-\frac{a\left(b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c-}}{d^2}\right.}{bd}\right.}{a^2+b^2}\right.}{}\right.}{}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \text{7143}$$

3.378. $\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(-\frac{a\left(\frac{\int (e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\cdots\right.}{d^2}\right.}{\cdots}\right.}{\cdots}\right.}{b}\right.}{\cdots}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\downarrow 7293$$

3.378. $\displaystyle\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(-\frac{a\left(\frac{\int \left(a(e+fx)^2\operatorname{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{Poly}}{}\right.}{}\right.}{}\right.}{b}\right.}{}$$

$$-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}$$

$$\Big\downarrow \text{2009}$$

3.378. $\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}}{b}\right.}{\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\,\mathrm{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}}$$

input
```
Int[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.378. $\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
((-I)*(((-1/3*I)*(e + f*x)^3)/f + (2*I)*(((e + f*x)^2*Log[1 + E^(2*(c + d*
x))])/(2*d) - (f*(-1/2*((e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/d + (f*Pol
yLog[3, -E^(2*(c + d*x))])/(4*d^2)))/d)))/b - (a*(((2*(e + f*x)^2*ArcTan[E
^(c + d*x)])/d + ((2*I)*f*(-(((e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d) +
 (f*PolyLog[3, (-I)*E^(c + d*x)])/d^2))/d - ((2*I)*f*(-(((e + f*x)*PolyLog
[2, I*E^(c + d*x)])/d) + (f*PolyLog[3, I*E^(c + d*x)])/d^2))/d)/b - (a*((b
^2*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))])/d^2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3)/(3*f) + (2*a*(e + f*x)
^2*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/d - (
(2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((2*I)*a*f*(e + f*
x)*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*(e + f*x)*PolyLog[2, -E^(2*(c + d
*x))])/d^2 + ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/d^3 - ((2*I)*a*f^2
*PolyLog[3, I*E^(c + d*x)])/d^3 + (b*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*
d^3))/(a^2 + b^2)))/b))/b
```

3.378.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.378. $\int \frac{(e+fx)^2 \sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} \, dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.378. $\int \frac{(e+fx)^2 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)} \, dx$

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6115
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - S
imp[a/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.378.4 Maple [F]

$$\int \frac{(fx+e)^2 \sinh(dx+c)\tanh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.378.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1248, normalized size of antiderivative = 1.45

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-1/3*((a^2 + b^2)*d^3*f^2*x^3 + 3*(a^2 + b^2)*d^3*e*f*x^2 + 3*(a^2 + b^2)*
d^3*e^2*x + 6*a^2*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*a^2*f^2*poly
log(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(a^2*d*f^2*x + a^2*d*e*f)*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b + 1) - 6*(a^2*d*f^2*x + a^2*d*e*f)*dilog((a*cosh(d*x
 + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) + 6*(I*a*b*d*f^2*x - b^2*d*f^2*x + I*a*b*d*e*f - b^2
*d*e*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + 6*(-I*a*b*d*f^2*x - b^2
*d*f^2*x - I*a*b*d*e*f - b^2*d*e*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x +
c)) - 3*(a^2*d^2*e^2 - 2*a^2*c*d*e*f + a^2*c^2*f^2)*log(2*b*cosh(d*x + c)
+ 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a^2*d^2*e^2 -
2*a^2*c*d*e*f + a^2*c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2
*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + 2
*a^2*c*d*e*f - a^2*c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(a^2*d^2
*f^2*x^2 + 2*a^2*d^2*e*f*x + 2*a^2*c*d*e*f - a^2*c^2*f^2)*log(-(a*cosh(d*x
 + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b) + 3*(I*a*b*d^2*e^2 - b^2*d^2*e^2 - 2*I*a*b*c*d*e*f + ...
```

3.378.6 Sympy [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

3.378.7 Maxima [F]

$$\int \frac{(e+fx)^2 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \sinh(dx+c)\tanh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
e^2*(a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b + b^3)*d)
 + 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/
((a^2 + b^2)*d) + (d*x + c)/(b*d)) + 1/3*(f^2*x^3 + 3*e*f*x^2)/b - integra
te(2*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x - (a^3*f^2*x^2*e^c + 2*a^3*e*f*x*e^c)*
e^(d*x))/(a^2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x) - 2*(a
^3*b*e^c + a*b^3*e^c)*e^(d*x)), x) - integrate(2*(b*f^2*x^2 + 2*b*e*f*x +
(a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e
^(2*c))*e^(2*d*x)), x)
```

3.378.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.378.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.379 $\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.379.1 Optimal result

Integrand size = 30, antiderivative size = 516

$$\begin{aligned}
\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{(e+fx)^2}{2bf}-\frac{2a(e+fx)\arctan\left(e^{c+dx}\right)}{b^2d}+\frac{2a^3(e+fx)\arctan\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d}\\
&\quad+\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d}+\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d}\\
&\quad+\frac{(e+fx)\log\left(1+e^{2(c+dx)}\right)}{bd}-\frac{a^2(e+fx)\log\left(1+e^{2(c+dx)}\right)}{b\left(a^2+b^2\right)d}+\frac{iaf\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{b^2d^2}\\
&\quad-\frac{ia^3f\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{b^2\left(a^2+b^2\right)d^2}-\frac{iaf\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{b^2d^2}+\frac{ia^3f\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{b^2\left(a^2+b^2\right)d^2}\\
&\quad+\frac{a^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^2}+\frac{a^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^2}\\
&\quad+\frac{f\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2bd^2}-\frac{a^2f\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2b\left(a^2+b^2\right)d^2}
\end{aligned}$$

output
```
-1/2*(f*x+e)^2/b/f-2*a*(f*x+e)*arctan(exp(d*x+c))/b^2/d+2*a^3*(f*x+e)*arct
an(exp(d*x+c))/b^2/(a^2+b^2)/d+(f*x+e)*ln(1+exp(2*d*x+2*c))/b/d-a^2*(f*x+e
)*ln(1+exp(2*d*x+2*c))/b/(a^2+b^2)/d+a^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))/b/(a^2+b^2)/d+a^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))/b/(a^2+b^2)/d+I*a*f*polylog(2,-I*exp(d*x+c))/b^2/d^2-I*a^3*f*polylog(
2,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^2-I*a*f*polylog(2,I*exp(d*x+c))/b^2/d^2+I
*a^3*f*polylog(2,I*exp(d*x+c))/b^2/(a^2+b^2)/d^2+1/2*f*polylog(2,-exp(2*d*
x+2*c))/b/d^2-1/2*a^2*f*polylog(2,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^2+a^2*f*p
olylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^2+a^2*f*polylog(
2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^2
```

3.379.2 Mathematica [A] (verified)

Time = 3.92 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.12

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{-2bde(c+dx)+2bcf(c+dx)-bf(c+dx)^2-4ade\arctan\left(e^{c+dx}\right)+4acf\arctan\left(e^{c+dx}\right)-2iaf(c+dx}{}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-2*b*d*e*(c + d*x) + 2*b*c*f*(c + d*x) - b*f*(c + d*x)^2 - 4*a*d*e*ArcTan
[E^(c + d*x)] + 4*a*c*f*ArcTan[E^(c + d*x)] - (2*I)*a*f*(c + d*x)*Log[1 -
I*E^(c + d*x)] + (2*I)*a*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + 2*b*d*e*Log[
1 + E^(2*(c + d*x))] - 2*b*c*f*Log[1 + E^(2*(c + d*x))] + 2*b*f*(c + d*x)*
Log[1 + E^(2*(c + d*x))] + (2*I)*a*f*PolyLog[2, (-I)*E^(c + d*x)] - (2*I)*
a*f*PolyLog[2, I*E^(c + d*x)] + (a^2*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) -
 f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[
-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh
[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^
(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2
*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/b + b*f*PolyLog[2, -E^(2*(c + d*x
))])/(2*(a^2 + b^2)*d^2)
```

3.379.3 Rubi [A] (verified)

Time = 2.55 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.91, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.633$, Rules used = {6115, 3042, 26, 4201, 2620, 2715, 2838, 6101, 3042, 4668, 2715, 2838, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6115}$$

$$\frac{\int (e+fx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)\tan(ic+idx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)\tan(ic+idx)dx}{b}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\int \frac{e^{2(c+dx)}(e+fx)}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int \log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int e^{-2(c+dx)}\log\left(1+e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \; 6101$$

$$-\frac{a\left(\frac{\int (e+fx)\operatorname{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \; 4668$$

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \; 2715$$

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \; 2838$$

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \; 6107$$

3.379. $\displaystyle\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie\right.}{d^2}}{b}\right.}{b}$$

$$\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \; 6095$$

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right.}{b}$$

$$\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int(e+f}{}\right.}{b}\right.}{b}$$

$$\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \; 2715$$

3.379. $\displaystyle\int\frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+\cdots}\right.}{bd}\cdots\right.}{a^2+b^2}\cdots\right.}{b}\cdots\right.}{b}\cdots-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \ 2838$$

$$-\frac{a\left(-\frac{a\left(\frac{\int (e+fx)\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\cdots\right.}{a^2+b^2}\cdots\right.}{b}\cdots\right.}{b}\cdots-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \ 7293$$

$$-\frac{a\left(-\frac{a\left(\frac{\int (a(e+fx)\operatorname{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\cdots\right.}{a^2+b^2}\cdots\right.}{b}\cdots\right.}{b}\cdots-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

$$\Big\downarrow \ 2009$$

3.379. $\displaystyle\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}-\frac{a\left(\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e \cdots}{\cdots}\right)}{a^2 \cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}$$

input
```
Int[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((-I)*(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((e + f*x)*Log[1 + E^(2*(c + d*x)
)])/(2*d) + (f*PolyLog[2, -E^(2*(c + d*x))])/(4*d^2))))/b - (a*(((2*(e + f
*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*f
*PolyLog[2, I*E^(c + d*x)])/d^2)/b - (a*((b^2*(-1/2*(e + f*x)^2/(b*f) + ((
e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x
)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2 + b^2) + ((b*(e + f*x)^2)
/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)*Log[1 + E^(2
*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*a*f*PolyLo
g[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*d^2))/(a^
2 + b^2)))/b))/b
```

3.379.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

3.379. $\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6115
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - S
imp[a/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.379.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3881 vs. 2(484) = 968.

Time = 2.07 (sec) , antiderivative size = 3882, normalized size of antiderivative = 7.52

method	result	size
risch	Expression too large to display	3882

input
```
int((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERB
OSE)
```

output
```
2/d*e/(2*a^2+2*b^2)*b*ln(1+exp(2*d*x+2*c))-4/d*e/(2*a^2+2*b^2)*a*arctan(ex
p(d*x+c))-1/d*e*b/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/b/
d^2*f/(a^2+b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))
)*a^2+1/b/d^2*f/(a^2+b^2)^(3/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-
a+(a^2+b^2)^(1/2)))*a^3-1/b/d^2*f/(a^2+b^2)^(3/2)*dilog((b*exp(d*x+c)+(a^2
+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^3+1/b/d*e/(a^2+b^2)*ln(b*exp(2*d*x+2
*c)+2*a*exp(d*x+c)-b)*a^2-2/b/d*e/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x
+c)+2*a)/(a^2+b^2)^(1/2))*a^3-2/d^2/(a^2+b^2)^(1/2)*a*b*c*f/(2*a^2+2*b^2)*
arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/b/d^2*f/(a^2+b^2)*ln((
b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2*c+2/b/d^2*a^3*f/(
2*a^2+2*b^2)/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^
2+b^2)^(1/2)))-2/b/d^2*a^3*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*dilog((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/b/d*f/(a^2+b^2)*ln((-b*ex
p(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2*x+1/b/d^2*f/(a^2+b^2
)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2*c+1/b/d*f
/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2*x+
1/b/d*f/(a^2+b^2)^(3/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))*a^3*x+1/b/d^2*f/(a^2+b^2)^(3/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*a^3*c+b/d*f/(a^2+b^2)^(3/2)*ln((-b*exp(d*x+c)+(a
^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a*x+b/d^2*f/(a^2+b^2)^(3/2)*ln((...
```

3.379.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 682, normalized size of antiderivative = 1.32

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{(a^2+b^2)d^2fx^2+2\,(a^2+b^2)d^2ex-2\,a^2f\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right.}{}$$

input
```
integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
-1/2*((a^2 + b^2)*d^2*f*x^2 + 2*(a^2 + b^2)*d^2*e*x - 2*a^2*f*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b + 1) - 2*a^2*f*dilog((a*cosh(d*x + c) + a*sinh(d*x +
 c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1
) + 2*(I*a*b*f - b^2*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + 2*(-I*a
*b*f - b^2*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - 2*(a^2*d*e - a^2
*c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2
) + 2*a) - 2*(a^2*d*e - a^2*c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c)
 - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(a^2*d*f*x + a^2*c*f)*log(-(a*cosh
(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b) - 2*(a^2*d*f*x + a^2*c*f)*log(-(a*cosh(d*x + c) + a*
sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b) + 2*(I*a*b*d*e - b^2*d*e - I*a*b*c*f + b^2*c*f)*log(cosh(d*x + c)
+ sinh(d*x + c) + I) + 2*(-I*a*b*d*e - b^2*d*e + I*a*b*c*f + b^2*c*f)*log(
cosh(d*x + c) + sinh(d*x + c) - I) + 2*(-I*a*b*d*f*x - b^2*d*f*x - I*a*b*c
*f - b^2*c*f)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + 2*(I*a*b*d*f*x
- b^2*d*f*x + I*a*b*c*f - b^2*c*f)*log(-I*cosh(d*x + c) - I*sinh(d*x + c)
+ 1))/((a^2*b + b^3)*d^2)
```

3.379.6 Sympy [F]

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\sinh{(c+dx)}\tanh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.379.7 Maxima [F]

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\sinh{(dx+c)}\tanh{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
1/2*f*(x^2/b - integrate(-4*(a^3*x*e^(d*x + c) - a^2*b*x)/(a^2*b^2 + b^4 -
 (a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x) - 2*(a^3*b*e^c + a*b^3*e^c)*e^(
d*x)), x) - integrate(4*(a*x*e^(d*x + c) + b*x)/(a^2 + b^2 + (a^2*e^(2*c)
+ b^2*e^(2*c))*e^(2*d*x)), x)) + e*(a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*
x - 2*c) - b)/((a^2*b + b^3)*d) + 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d)
 + b*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b^2)*d) + (d*x + c)/(b*d))
```

3.379.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
Timed out
```

3.379.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)\,\left(e+f\,x\right)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.380 $\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.380.1 Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\arctan(\sinh(c+dx))}{(a^2+b^2)\,d}+\frac{b\log(\cosh(c+dx))}{(a^2+b^2)\,d}+\frac{a^2\log(a+b\sinh(c+dx))}{b\,(a^2+b^2)\,d}$$

output
```
-a*arctan(sinh(d*x+c))/(a^2+b^2)/d+b*ln(cosh(d*x+c))/(a^2+b^2)/d+a^2*ln(a+
b*sinh(d*x+c))/b/(a^2+b^2)/d
```

3.380.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b(ia+b)\log(i-\sinh(c+dx))+b(-ia+b)\log(i+\sinh(c+dx))+2a^2\log(a+b\sinh(c+dx))}{2b\,(a^2+b^2)\,d}$$

input
```
Integrate[(Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b*(I*a + b)*Log[I - Sinh[c + d*x]] + b*((-I)*a + b)*Log[I + Sinh[c + d*x]
] + 2*a^2*Log[a + b*Sinh[c + d*x]])/(2*b*(a^2 + b^2)*d)
```

3.380.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 25, 3316, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2}{\cos(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2}{\cos(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{b\int \frac{\sinh^2(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b^2\sinh^2(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{615}$$

$$\frac{\int \left(\frac{a^2}{(a^2+b^2)(a+b\sinh(c+dx))}-\frac{b^2(a-b\sinh(c+dx))}{(a^2+b^2)(\sinh^2(c+dx)b^2+b^2)}\right)d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{ab\arctan(\sinh(c+dx))}{a^2+b^2}+\frac{b^2\log(b^2\sinh^2(c+dx)+b^2)}{2(a^2+b^2)}+\frac{a^2\log(a+b\sinh(c+dx))}{a^2+b^2}}{bd}$$

input
```
Int[(Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-((a*b*ArcTan[Sinh[c + d*x]])/(a^2 + b^2)) + (a^2*Log[a + b*Sinh[c + d*x]
])/(a^2 + b^2) + (b^2*Log[b^2 + b^2*Sinh[c + d*x]^2])/(2*(a^2 + b^2)))/(b*
d)
```

3.380.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

3.380.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.78

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{4b\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-8a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2+4b^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}+\frac{a^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\right.}{b\left(a^2+b^2\right.}}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{4b\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-8a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2+4b^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}+\frac{a^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\right.}{b\left(a^2+b^2\right.}}{d}$
risch	$\frac{x}{b}-\frac{2b\,d^2x}{a^2d^2+b^2d^2}-\frac{2bdc}{a^2d^2+b^2d^2}-\frac{2a^2x}{b(a^2+b^2)}-\frac{2a^2c}{bd(a^2+b^2)}+\frac{i\ln(\mathrm{e}^{dx+c}-i)a}{(a^2+b^2)d}+\frac{\ln(\mathrm{e}^{dx+c}-i)b}{(a^2+b^2)d}-\frac{i\ln(\mathrm{e}^{dx+c}+i)}{(a^2+b^2)d}$

input
```
int(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b*ln(tanh(1/2*d*x+1/2*c)-1)+8/(4*a^2+4*b^2)*(1/2*b*ln(1+tanh(1/2*d
*x+1/2*c)^2)-a*arctan(tanh(1/2*d*x+1/2*c)))-1/b*ln(tanh(1/2*d*x+1/2*c)+1)+
a^2/b/(a^2+b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a))
```

3.380.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.50

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(a^2+b^2)dx + 2\,ab\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right) - a^2\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right) - b^2\log\left(\frac{2}{\cosh(d}\right.}{(a^2b+b^3)d}$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-((a^2 + b^2)*d*x + 2*a*b*arctan(cosh(d*x + c) + sinh(d*x + c)) - a^2*log(
2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) - b^2*log(2*cosh(
d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a^2*b + b^3)*d)
```

3.380.6 Sympy [F]

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh\left(c+dx\right)\tanh\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.380.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.49

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^2b+b^3)d} + \frac{2\,a\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d} + \frac{b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d} + \frac{dx+c}{bd}$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b + b^3)*d) + 2*
a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2
 + b^2)*d) + (d*x + c)/(b*d)
```

3.380.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.64

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,a^2\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^2b+b^3} - \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)a}{a^2+b^2} + \frac{b\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2}}{2\,d}$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*a^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^2*b + b^3) -
(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*a/(a^2 + b^2) + b*
log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2))/d
```

3.380.9 Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.35

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\ln\left(e^{c+d\,x}+1\mathrm{i}\right)}{b\,d+a\,d\,1\mathrm{i}} - \frac{x}{b} + \frac{a^2\,\ln\left(a^2\,b^3-b^5-a^4\,b+2\,a^5\,e^{d\,x}\,e^{c}+b^5\,e^{2\,c}\,e^{2\,d\,x}+a^4\,b\,e^{2\,c}\,e^{2\,d\,x}-2\,a^3\,b^2\,e^{d\,x}\,e^{c}-a^2\,b^3\,e^{2\,c}\,e^{2\,d\,x}+2\,a\,b^4\,e\right.}{d\,a^2\,b+d\,b^3} + \frac{\ln\left(1+e^{c+d\,x}\,1\mathrm{i}\right)\,1\mathrm{i}}{a\,d+b\,d\,1\mathrm{i}}$$

input
```
int((sinh(c + d*x)*tanh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

output
```
log(exp(c + d*x) + 1i)/(a*d*1i + b*d) - x/b + (log(exp(c + d*x)*1i + 1)*1i
)/(a*d + b*d*1i) + (a^2*log(a^2*b^3 - b^5 - a^4*b + 2*a^5*exp(d*x)*exp(c)
+ b^5*exp(2*c)*exp(2*d*x) + a^4*b*exp(2*c)*exp(2*d*x) - 2*a^3*b^2*exp(d*x)
*exp(c) - a^2*b^3*exp(2*c)*exp(2*d*x) + 2*a*b^4*exp(d*x)*exp(c)))/(b^3*d +
 a^2*b*d)
```

3.381 $\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.381.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.381.2 Mathematica [N/A]

Not integrable

Time = 15.54 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Sinh[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

output
```
Integrate[(Sinh[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

3.381.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Sinh[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.381.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.381.4 Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(dx+c)\tanh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.381.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh{(dx+c)}\tanh{(dx+c)}}{(fx+e)(b\sinh{(dx+c)}+a)}\,dx$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
integral(sinh(d*x + c)*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
 + c)), x)
```

3.381.6 Sympy [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh{(c+dx)}\tanh{(c+dx)}}{(a+b\sinh{(c+dx)})\,(e+fx)}\,dx$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)*tanh(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.381.7 Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 252, normalized size of antiderivative = 7.88

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh{(dx+c)}\tanh{(dx+c)}}{(fx+e)(b\sinh{(dx+c)}+a)}\,dx$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
log(f*x + e)/(b*f) - 1/2*integrate(-4*(a^3*e^(d*x + c) - a^2*b)/(a^2*b^2*e
 + b^4*e + (a^2*b^2*f + b^4*f)*x - (a^2*b^2*e*e^(2*c) + b^4*e*e^(2*c) + (a
^2*b^2*f*e^(2*c) + b^4*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^3*b*e*e^c + a*b^3*e*
e^c + (a^3*b*f*e^c + a*b^3*f*e^c)*x)*e^(d*x)), x) - 1/2*integrate(4*(a*e^(
d*x + c) + b)/(a^2*e + b^2*e + (a^2*f + b^2*f)*x + (a^2*e*e^(2*c) + b^2*e*
e^(2*c) + (a^2*f*e^(2*c) + b^2*f*e^(2*c))*x)*e^(2*d*x)), x)
```

3.381.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
Timed out
```

3.381.9 Mupad [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((sinh(c + d*x)*tanh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((sinh(c + d*x)*tanh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.382 $\int \frac{(e+fx)^3 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\, dx$

3.382.1 Optimal result

Integrand size = 28, antiderivative size = 1118

$$\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a(e+fx)^3}{b^2d}+\frac{a^3(e+fx)^3}{b^2(a^2+b^2)d}+\frac{6f(e+fx)^2\arctan\left(e^{c+dx}\right)}{bd^2}$$
$$-\frac{6a^2f(e+fx)^2\arctan\left(e^{c+dx}\right)}{b(a^2+b^2)d^2}$$
$$+\frac{a^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d}$$
$$-\frac{a^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d}$$
$$+\frac{3af(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{b^2d^2}$$
$$-\frac{3a^3f(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{b^2(a^2+b^2)d^2}$$
$$-\frac{6if^2(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{bd^3}$$
$$+\frac{6ia^2f^2(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{b(a^2+b^2)d^3}$$
$$+\frac{6if^2(e+fx)\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{bd^3}$$
$$-\frac{6ia^2f^2(e+fx)\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{b(a^2+b^2)d^3}$$
$$+\frac{3a^2f(e+fx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d^2}$$
$$-\frac{3a^2f(e+fx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d^2}$$
$$+\frac{3af^2(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{b^2d^3}$$
$$-\frac{3a^3f^2(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{b^2(a^2+b^2)d^3}$$
$$+\frac{6if^3\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{bd^4}-\frac{6ia^2f^3\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{b(a^2+b^2)d^4}$$
$$-\frac{6if^3\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{bd^4}+\frac{6ia^2f^3\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{b(a^2+b^2)d^4}$$
$$-\frac{6a^2f^2(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d^3}$$
$$+\frac{6a^2f^2(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d^3}$$
$$3af^3\,\mathrm{PolyLog}\left(3,-e^{2(c+dx)}\right)$$

output
```
-a*(f*x+e)^3*tanh(d*x+c)/b^2/d+a^3*(f*x+e)^3/b^2/(a^2+b^2)/d+6*I*f^3*polyl
og(3,-I*exp(d*x+c))/b/d^4+a^2*(f*x+e)^3*sech(d*x+c)/b/(a^2+b^2)/d+a^3*(f*x
+e)^3*tanh(d*x+c)/b^2/(a^2+b^2)/d+a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/(a^2+b^2)^(3/2)/d+6*I*a^2*f^3*polylog(3,I*exp(d*x+c))/b/(a^2+b^2
)/d^4+6*I*a^2*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/b/(a^2+b^2)/d^3+6*I*f^2
*(f*x+e)*polylog(2,I*exp(d*x+c))/b/d^3-6*I*a^2*f^2*(f*x+e)*polylog(2,I*exp
(d*x+c))/b/(a^2+b^2)/d^3-6*a^2*f*(f*x+e)^2*arctan(exp(d*x+c))/b/(a^2+b^2)/
d^2-3*a^3*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^3-3*a^3*f
*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^2+3*a*f^2*(f*x+e)*polylog(
2,-exp(2*d*x+2*c))/b^2/d^3+3/2*a^3*f^3*polylog(3,-exp(2*d*x+2*c))/b^2/(a^2
+b^2)/d^4+3*a*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b^2/d^2+3*a^2*f*(f*x+e)^2*p
olylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-3*a^2*f*(f
*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-6
*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3
/2)/d^3+6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^
2+b^2)^(3/2)/d^3-6*I*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/b/d^3-6*I*a^2*f^
3*polylog(3,-I*exp(d*x+c))/b/(a^2+b^2)/d^4-a*(f*x+e)^3/b^2/d+6*f*(f*x+e)^2
*arctan(exp(d*x+c))/b/d^2-3/2*a*f^3*polylog(3,-exp(2*d*x+2*c))/b^2/d^4+6*a
^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^4...
```

3.382.2 Mathematica [A] (verified)

Time = 8.18 (sec) , antiderivative size = 1147, normalized size of antiderivative = 1.03

$$\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{f\left(-12ad^3e^2e^{2c}x+12ad^3e^2(1+e^{2c})\,x+12ad^3efx^2+4ad^3f^2x^3+12bd^2e^2(1+e^{2c})\arctan\left(e^{c+dx}\right)-6ad^2e\right.}{}$$
$$+\frac{a^2\left(-2d^3e^3\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+3d^3e^2fx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+3d^3ef^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^3f^3x^3\,\mathrm{l}\right.}{}$$
$$+\frac{\operatorname{sech}(c)\operatorname{sech}(c+dx)\left(-be^3\cosh(c)-3be^2fx\cosh(c)-3bef^2x^2\cosh(c)-bf^3x^3\cosh(c)-ae^3\sinh(dx)\right.}{(a^2+b^2)\,d}$$

input
```
Integrate[((e + f*x)^3*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(f*(-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*f*
x^2 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 6
*a*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d*e
*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) -
PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 + E
^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c +
d*x))]) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*
x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*Po
lyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*
E^(c + d*x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c
+ d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d
*x))]))))/(2*(a^2 + b^2)*d^4*(1 + E^(2*c))) + (a^2*(-2*d^3*e^3*ArcTanh[(a +
 b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]
- 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^3*e*f
^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2,
(b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*e*f^2*PolyLog[3, (b*E^(c...
```

3.382.3 Rubi [A] (verified)

Time = 6.36 (sec) , antiderivative size = 949, normalized size of antiderivative = 0.85, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {6101, 5974, 3042, 4668, 3011, 2720, 6117, 3042, 4672, 26, 3042, 26, 4201, 2620, 3011, 2720, 6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 7143, 7163, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$$
$$\Big\downarrow \text{6101}$$
$$\frac{\int (e+fx)^3\text{sech}(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$
$$\Big\downarrow \text{5974}$$
$$\frac{\frac{3f\int (e+fx)^2\text{sech}(c+dx)dx}{d}-\frac{(e+fx)^3\text{sech}(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{b}$$

$$\Big\downarrow \; 4668$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(-\frac{2if\int (e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \; 3011$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c\cdots}\right.}{d}\right.}{d}\cdots\right.}{d}}{b}$$

$$\Big\downarrow \; 2720$$

$$-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c\cdots}}{d^2}\right.}{d}\cdots\right.}{d}}{b}$$

$$\Big\downarrow \; 6117$$

$$-\frac{a\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c\cdots}}{d^2}\right.}{d}\cdots\right.}{d}}{b}$$

$$\Big\downarrow \; 3042$$

3.382. $\displaystyle\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+\cdots}}{d^2}\cdots\right)}{d}\cdots\right)}{d}\cdots$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}\right)}{b}$$

$$\downarrow \text{4672}$$

$$-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+\cdots}}{d^2}\cdots\right)}{d}\cdots\right)}{d}\cdots$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3if\int-i(e+fx)^2\tanh(c+dx)dx}{d}}{b}\right)}{b}$$

$$\downarrow \text{26}$$

$$-\frac{a\left(\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3f\int(e+fx)^2\tanh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+\cdots}}{d^2}\cdots\right)}{d}\cdots\right)}{d}\cdots$$

$$\downarrow \text{3042}$$

$$-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+\cdots}}{d^2}\cdots\right)}{d}\cdots\right)}{d}\cdots$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3f\int-i(e+fx)^2\tan(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\downarrow \text{26}$$

3.382. $\int\frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+\cdots}}{d^2}\cdots\right.}{d}\cdots\right.}{d}\cdots}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\int(e+fx)^2\tan(ic+idx)dx}{d}}{b}\right)}{b}$$

$\Big\downarrow$ 4201

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+\cdots}}{d^2}\cdots\right.}{d}\cdots\right.}{d}\cdots}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\int\frac{e^{2(c+dx)}(e+fx)^2}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}\right)}{b}$$

$\Big\downarrow$ 2620

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+\cdots}}{d^2}\cdots\right.}{d}\cdots\right.}{d}\cdots}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2(c+dx)}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}\right)}{b}$$

$\Big\downarrow$ 3011

3.382. $\displaystyle\int\frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{2d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e\right)}{2d}\right)}{d}\right)\right)}{d}}{b}\right)}{b}$$

$$\downarrow 2720$$

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e}{}\right)}{d}\right)\right)}{d}}{b}\right)}{b}$$

$$\downarrow 6107$$

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+}}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2}}{}\right)}{}\right)\right)}{}\right)}{b}$$

3.382. $\int\frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$\Big\downarrow$ 3042

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+}}{d^2}\right.}{d}\right.}{d}}{b}$$

$$\frac{a\left(\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e}{}\right.}{d}}{b}\right.}{b}$$

$\Big\downarrow$ 3803

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+}}{d^2}\right.}{d}\right.}{d}}{b}$$

$$\frac{a\left(-\frac{a\left(\frac{2b^2\int -\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}\right.\right.}{}\right.}{b}$$

$\Big\downarrow$ 25

3.382. $\displaystyle\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+}}{d^2}\right.}{d}\right.}{d}}{b}$$

$$\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{b}+\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\right.\right.}{}\right.}{b}$$

$$\Big\downarrow \text{2694}$$

$$\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+}}{d^2}\right.}{d}\right.}{d}}{b}$$

$$\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}+\frac{(e+fx)^3\tanh(c+dx)}{d}\right.}{b}$$

$$\Big\downarrow \text{27}$$

3.382. $\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}\cdots\right)}{d}\cdots\right)}{d}}{b}$$

$$\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}+\frac{(e+fx)^3\tanh(c+dx)}{d}+\frac{3if\left(2i\cdots\right)}{\cdots}\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}\cdots\right)}{d}\cdots\right)}{d}}{b}$$

$$\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be\cdots}{a-\cdots}\right)}{bd}\cdots\right)}{\cdots}\right)}{a^2+b^2}\cdots\right)}{b}\cdots\right)}{\cdots}$$

$$\Big\downarrow \; 3011$$

3.382. $\displaystyle\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+\cdots}}{d^2}\cdots\right)}{d}\cdots\right)}{d}\cdots$$

$$b$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyL}\cdots}{\cdots}\right)}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\right)}{\cdots}\cdots\right)$$

$\downarrow$ 7143

3.382. $\displaystyle\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\frac{(e\cdots}{\cdots}\right)}{d}\cdots\right.}{d}\cdots}{b}$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyL}\cdots}{\cdots}\right.}{bd}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

$\downarrow$ 7163

3.382. $\int\frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\frac{(e\cdots}{\cdots}\right.}{d}\right.}{d}}{b}$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,\cdots\right.}{\cdots}\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{\cdots}\right.}{\cdots}\right.$$

3.382. $\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow 2720$$

3.382. $\int \frac{(e+fx)^3 \tanh^2(c+dx)}{a+b \sinh(c+dx)} \, dx$

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\frac{(e\cdots}{}\right.}{d}\cdots\right.}{d}\cdots}{b}$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\,\text{Pol}\cdots}{}\right.}{d}\cdots\right.}{}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{}\cdots\right.}{}\cdots\right.$$

3.382. $\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\downarrow \text{7143}$$

3.382. $\int \frac{(e+fx)^3 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\, dx$

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\frac{(e\cdots}{}\right.}{d}\right.}{d}}{b}$$

$$a\left(-\frac{a\left(\frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\,\mathrm{PolyLog}\left(4,-\cdots\right)}{d^2}\right)}{d}\cdots\right.}{bd}\cdots\right.}{2\sqrt{a^2+b^2}}\cdots\right.}{}\cdots\right.}{}\cdots\right.$$

3.382. $\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\downarrow 7293$$

3.382. $\int \frac{(e+fx)^3 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\frac{(e\cdots}{}\right)}{d}\cdots\right.}{d}}{b}$$

$$a\left(-\frac{a\left(\frac{\int\left(a(e+fx)^3\operatorname{sech}^2(c+dx)-b(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2\cdots}}\right)}{d}\cdots\right.}{\cdots}\right.}{\cdots}\right)}{2\cdots}\right.}{\cdots}\cdots}{\cdots}\right.$$

3.382. $\int\frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.381.

$$\downarrow \text{2009}$$

3.382. $\int \frac{(e+fx)^3 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f\left(\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^2}{d}+\frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}\right)}{d}}{b}$$

$$a\left(\frac{\frac{\tanh(c+dx)(e+fx)^3}{d}+\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{2d}-\frac{f\left(\frac{f\,\mathrm{PolyLog}\left(3,-e^{2(c+dx)}\right)}{4d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{b}-\frac{a\left(-\frac{6ib\ \cdots}{\cdots}\right.}{\cdots}\right.$$

3.382. $\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^3*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
((3*f*((2*(e + f*x)^2*ArcTan[E^(c + d*x)])/d + ((2*I)*f*(-(((e + f*x)*Poly
Log[2, (-I)*E^(c + d*x)])/d) + (f*PolyLog[3, (-I)*E^(c + d*x)])/d^2))/d -
((2*I)*f*(-(((e + f*x)*PolyLog[2, I*E^(c + d*x)])/d) + (f*PolyLog[3, I*E^(
c + d*x)])/d^2))/d))/d - ((e + f*x)^3*Sech[c + d*x])/d)/b - (a*((((3*I)*f*
(((-1/3*I)*(e + f*x)^3)/f + (2*I)*(((e + f*x)^2*Log[1 + E^(2*(c + d*x))])/
(2*d) - (f*(-1/2*((e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/d + (f*PolyLog[3
, -E^(2*(c + d*x))])/(4*d^2)))/d)))/d + ((e + f*x)^3*Tanh[c + d*x])/d)/b -
 (a*((-2*b^2*(-1/2*(b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/d^2))/d))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^3*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2,
 -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[
3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(
a^2 + b^2) + ((a*(e + f*x)^3)/d - (6*b*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/
d^2 - (3*a*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/d^2 + ((6*I)*b*f^2*(e +
 f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^3 - ((6*I)*b*f^2*(e + f*x)*PolyLog[2
, I*E^(c + d*x)])/d^3 - (3*a*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))]...
```

3.382.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.382. $\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.382. $\int \frac{(e+fx)^3 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.382.4 Maple [F]

$$\int \frac{(fx+e)^3 \tanh (dx+c)^2}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.382.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6494 vs. $2(1024) = 2048$.

Time = 0.41 (sec) , antiderivative size = 6494, normalized size of antiderivative = 5.81

$$\int \frac{(e+fx)^3 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

3.382.6 Sympy [F]

$$\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^3\tanh^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.382.7 Maxima [F]

$$\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\tanh{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-3*a*e^2*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2
 + b^2)*d^2)) + 6*b*f^3*integrate(x^2*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) +
 b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 6*a*f^3*integrate(x^2/(a^2*d
*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 12*b*e*f^2
*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) +
a^2*d + b^2*d), x) - 12*a*e*f^2*integrate(x/(a^2*d*e^(2*d*x + 2*c) + b^2*d
*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + e^3*(a^2*log((b*e^(-d*x - c) - a -
 sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/
2)*d) - 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))
*d)) + 6*b*e^2*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) + 2*(a*f^3*x^3 + 3*
a*e*f^2*x^2 + 3*a*e^2*f*x - (b*f^3*x^3*e^c + 3*b*e*f^2*x^2*e^c + 3*b*e^2*f
*x*e^c)*e^(d*x))/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x
)) + integrate(-2*(a^2*f^3*x^3*e^c + 3*a^2*e*f^2*x^2*e^c + 3*a^2*e^2*f*x*e
^c)*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(2*c))*e^(2*d*x) - 2*(a^
3*e^c + a*b^2*e^c)*e^(d*x)), x)
```

3.382.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.382.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((tanh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((tanh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.383 $\int \frac{(e+fx)^2 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.383.1 Optimal result

Integrand size = 28, antiderivative size = 772

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a(e+fx)^2}{b^2d}+\frac{a^3(e+fx)^2}{b^2(a^2+b^2)\,d}+\frac{4f(e+fx)\arctan\left(e^{c+dx}\right)}{bd^2} - \frac{4a^2f(e+fx)\arctan\left(e^{c+dx}\right)}{b(a^2+b^2)\,d^2} + \frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} + \frac{2af(e+fx)\log\left(1+e^{2(c+dx)}\right)}{b^2d^2} - \frac{2a^3f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{b^2(a^2+b^2)\,d^2} - \frac{2if^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{bd^3}+\frac{2ia^2f^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{b(a^2+b^2)\,d^3} + \frac{2if^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{bd^3}-\frac{2ia^2f^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{b(a^2+b^2)\,d^3} + \frac{2a^2f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} - \frac{2a^2f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2} + \frac{af^2\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{b^2d^3}-\frac{a^3f^2\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{b^2(a^2+b^2)\,d^3} - \frac{2a^2f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^3} + \frac{2a^2f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^3} - \frac{(e+fx)^2\text{sech}(c+dx)}{bd}+\frac{a^2(e+fx)^2\text{sech}(c+dx)}{b(a^2+b^2)\,d} - \frac{a(e+fx)^2\tanh(c+dx)}{b^2d}+\frac{a^3(e+fx)^2\tanh(c+dx)}{b^2(a^2+b^2)\,d}$$

output
```
-a*(f*x+e)^2/b^2/d+a^3*(f*x+e)^2/b^2/(a^2+b^2)/d+4*f*(f*x+e)*arctan(exp(d*
x+c))/b/d^2-4*a^2*f*(f*x+e)*arctan(exp(d*x+c))/b/(a^2+b^2)/d^2+2*a*f*(f*x+
e)*ln(1+exp(2*d*x+2*c))/b^2/d^2-2*a^3*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/b^2/(
a^2+b^2)/d^2+a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2
)^(3/2)/d-a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(
3/2)/d+2*I*a^2*f^2*polylog(2,-I*exp(d*x+c))/b/(a^2+b^2)/d^3-2*I*f^2*polylo
g(2,-I*exp(d*x+c))/b/d^3+2*I*f^2*polylog(2,I*exp(d*x+c))/b/d^3-2*I*a^2*f^2
*polylog(2,I*exp(d*x+c))/b/(a^2+b^2)/d^3+a*f^2*polylog(2,-exp(2*d*x+2*c))/
b^2/d^3-a^3*f^2*polylog(2,-exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^3+2*a^2*f*(f*x+
e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-2*a^2*
f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2
-2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^
3+2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d
^3-(f*x+e)^2*sech(d*x+c)/b/d+a^2*(f*x+e)^2*sech(d*x+c)/b/(a^2+b^2)/d-a*(f*
x+e)^2*tanh(d*x+c)/b^2/d+a^3*(f*x+e)^2*tanh(d*x+c)/b^2/(a^2+b^2)/d
```

3.383.2 Mathematica [A] (verified)

Time = 5.06 (sec) , antiderivative size = 633, normalized size of antiderivative = 0.82

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{f\left(4ad^2ee^{2c}x-4ad^2e\left(1+e^{2c}\right)x+2ad^2e^{2c}fx^2-2ad^2\left(1+e^{2c}\right)fx^2-4bde\left(1+e^{2c}\right)\arctan\left(e^{c+dx}\right)+2ade\left(1+e^{2c}\right)\left(2dx-\log\left(1+e^{2(c+dx)}\right)\right)+2ib(1+\cdots\right.}{\cdots}$$

input
```
Integrate[((e + f*x)^2*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-(((f*(4*a*d^2*e*E^(2*c)*x - 4*a*d^2*e*(1 + E^(2*c))*x + 2*a*d^2*E^(2*c)*f
*x^2 - 2*a*d^2*(1 + E^(2*c))*f*x^2 - 4*b*d*e*(1 + E^(2*c))*ArcTan[E^(c + d
*x)] + 2*a*d*e*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (2*I)*b*
(1 + E^(2*c))*f*(d*x*(-Log[1 - I*E^(c + d*x)] + Log[1 + I*E^(c + d*x)]) +
PolyLog[2, (-I)*E^(c + d*x)] - PolyLog[2, I*E^(c + d*x)]) + a*(1 + E^(2*c)
)*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]
)))/((a^2 + b^2)*(1 + E^(2*c))) + (a^2*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*
x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d
^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b
*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] - 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))]))/(a^2 + b^2)^(3/2) + (d^2*(e + f*x)^2*Sech[c + d*x]*(b + a*Sech[c]
*Sinh[d*x]))/(a^2 + b^2))/d^3)
```

3.383.3 Rubi [A] (verified)

Time = 4.79 (sec) , antiderivative size = 670, normalized size of antiderivative = 0.87, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6101, 5974, 3042, 4668, 2715, 2838, 6117, 3042, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838, 6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{6101}$$
$$\frac{\int (e+fx)^2\text{sech}(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$
$$\Big\downarrow \text{5974}$$
$$\frac{\frac{2f\int (e+fx)\text{sech}(c+dx)dx}{d}-\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{b}$$

$$\Big\downarrow \; 4668$$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(-\frac{if\int\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)}{d}}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\Big\downarrow \; 2715$$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)}{d}}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\Big\downarrow \; 2838$$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\Big\downarrow \; 6117$$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}\right)}{b}$$

$$\Big\downarrow \; 4672$$

3.383. $\int\frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2if\int-i(e+fx)\tanh(c+dx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \color{blue}{26}$$

$$-\frac{a\left(\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int(e+fx)\tanh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int-i(e+fx)\tan(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \color{blue}{26}$$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\int(e+fx)\tan(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \color{blue}{4201}$$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\int\frac{e^{2(c+dx)}(e+fx)}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \color{blue}{2620}$$

3.383. $\int\frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int\log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \textbf{2715}$$

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int e^{-2(c+dx)}\log\left(1+e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \textbf{2838}$$

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \textbf{6107}$$

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e\ldots\right.}{2\ldots}\right.\right.}{d}}{b}\right.}{b}$$

$$\Big\downarrow \textbf{3042}$$

3.383. $\int\frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\cdots\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{3803}$$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\left(\frac{2b^2\int-\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\cdots\right.\right.}{\cdots}}{b}\right.}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{b}+\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\cdots\right.\right.}{\cdots}}{b}\right.}{b}$$

$$\Big\downarrow \text{2694}$$

3.383. $\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}+\frac{(e+fx)^2\tanh(c+dx)}{d}\right.}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}+\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\right.}{}\right.}{b}$$

$$\Big\downarrow \; 2620$$

3.383. $\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{\cdots}}\right.}{bd}\cdots\right.}{\cdots}\right.}{a^2+b^2}\cdots\right.}{b}\cdots\right.$$

$$\Big\downarrow \; 3011$$

3.383. $\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.$$

$$a\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\right.$$

$$a\left(-\frac{\quad}{b}\right.$$

↓ 2720

3.383. $\int\frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{P}\cdots}{\cdots}\right)}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots}{\cdots}\right)}{\cdots}\cdots\right)$$

$\downarrow$ 7143

3.383. $\displaystyle\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c}}{a+\sqrt{}}\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.}{b}\right.$$

$$\Big\downarrow \; 7293$$

3.383. $\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$a\left(-\frac{a\left(\frac{\int\left(a(e+fx)^2\operatorname{sech}^2(c+dx)-b(e+fx)^2\operatorname{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+\ldots}{}\right)}{bd}\ldots\right)}{2\sqrt{a^2+b^2}}\ldots\right)}{}\ldots\right)}{b\ldots}\ldots\right)$$

↓ 2009

3.383. $\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}}{b}-\frac{a-\frac{2b^2\left(b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f}{\cdots}\right.\right.}{\cdots}}{\cdots}\right.$$

input
```
Int[((e + f*x)^2*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

3.383. $\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
((2*f*((2*(e + f*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d
*x)])/d^2 + (I*f*PolyLog[2, I*E^(c + d*x)])/d^2))/d - ((e + f*x)^2*Sech[c
+ d*x])/d)/b - (a*((((2*I)*f*(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((e + f*x)
*Log[1 + E^(2*(c + d*x))])/(2*d) + (f*PolyLog[2, -E^(2*(c + d*x))])/(4*d^2
))))/d + ((e + f*x)^2*Tanh[c + d*x])/d)/b - (a*((-2*b^2*(-1/2*(b*(((e + f*
x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e +
 f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLo
g[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/Sqrt[a^2 + b
^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*
d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b
*d)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + ((a*(e + f*x)^2)/d - (4*b*f*(e +
 f*x)*ArcTan[E^(c + d*x)])/d^2 - (2*a*f*(e + f*x)*Log[1 + E^(2*(c + d*x))]
)/d^2 + ((2*I)*b*f^2*PolyLog[2, (-I)*E^(c + d*x)])/d^3 - ((2*I)*b*f^2*Poly
Log[2, I*E^(c + d*x)])/d^3 - (a*f^2*PolyLog[2, -E^(2*(c + d*x))])/d^3 + (b
*(e + f*x)^2*Sech[c + d*x])/d + (a*(e + f*x)^2*Tanh[c + d*x])/d)/(a^2 + b^
2)))/b))/b
```

3.383.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.383. $\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.383. $\int \frac{(e+fx)^2 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

3.383. $\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.383.4 Maple [F]

$$\int \frac{(fx+e)^2 \tanh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.383.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3661 vs. 2(712) = 1424.

Time = 0.34 (sec) , antiderivative size = 3661, normalized size of antiderivative = 4.74

$$\int \frac{(e+fx)^2 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*(a^3 + a*b^2)*d^2*e^2 - 4*(a^3 + a*b^2)*c*d*e*f + 2*(a^3 + a*b^2)*c^2*f
^2 - 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a
*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)^2 - 2*((a^3 + a*b^2)*
d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 +
 a*b^2)*c^2*f^2)*sinh(d*x + c)^2 + 2*(a^2*b*d*f^2*x + a^2*b*d*e*f + (a^2*b
*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*f^2*x + a^2*b*d*e*f)*
cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*f^2*x + a^2*b*d*e*f)*sinh(d*x + c)^
2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(a^2*b
*d*f^2*x + a^2*b*d*e*f + (a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^2 + 2
*(a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*f^2*
x + a^2*b*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b + 1) - (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2 +
(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)^2 + 2*(a^2
*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)
+ (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*sinh(d*x + c)^2)*sqrt(
(a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2
 + b^2)/b^2) + 2*a) + (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2 + (
a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)^2 + 2*(a...
```

3.383.6 Sympy [F]

$$\int \frac{(e+fx)^2 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2 \tanh^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.383.7 Maxima [F]

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\tanh{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*a*e*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 +
 b^2)*d^2)) + 4*b*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2
*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 4*a*f^2*integrate(x/(a^2*d*e^(2*
d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + e^2*(a^2*log((b*
e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2))
)/((a^2 + b^2)^(3/2)*d) - 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)
*e^(-2*d*x - 2*c))*d)) + 4*b*e*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) + 2
*(a*f^2*x^2 + 2*a*e*f*x - (b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^(d*x))/(a^2*d
+ b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) + integrate(-2*(a^2*f
^2*x^2*e^c + 2*a^2*e*f*x*e^c)*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*
e^(2*c))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x)
```

3.383.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.383.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{{\mathrm{tanh}\left(c+d\,x\right)}^2\,{\left(e+f\,x\right)}^2}{a+b\,\mathrm{sinh}\left(c+d\,x\right)}\,dx$$

input
```
int((tanh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((tanh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.384 $\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.384.1 Optimal result

Integrand size = 26, antiderivative size = 385

$$\begin{aligned}\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =& \frac{f\arctan(\sinh(c+dx))}{bd^2}-\frac{a^2f\arctan(\sinh(c+dx))}{b\,(a^2+b^2)\,d^2}\\&+\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}\\&-\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}+\frac{af\log(\cosh(c+dx))}{b^2d^2}\\&-\frac{a^3f\log(\cosh(c+dx))}{b^2\,(a^2+b^2)\,d^2}+\frac{a^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2}\\&-\frac{a^2f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d^2}\\&-\frac{(e+fx)\mathrm{sech}(c+dx)}{bd}+\frac{a^2(e+fx)\mathrm{sech}(c+dx)}{b\,(a^2+b^2)\,d}\\&-\frac{a(e+fx)\tanh(c+dx)}{b^2d}+\frac{a^3(e+fx)\tanh(c+dx)}{b^2\,(a^2+b^2)\,d}\end{aligned}$$

output
```
f*arctan(sinh(d*x+c))/b/d^2-a^2*f*arctan(sinh(d*x+c))/b/(a^2+b^2)/d^2+a*f*
ln(cosh(d*x+c))/b^2/d^2-a^3*f*ln(cosh(d*x+c))/b^2/(a^2+b^2)/d^2+a^2*(f*x+e
)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-a^2*(f*x+e)*ln(
1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+a^2*f*polylog(2,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-a^2*f*polylog(2,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-(f*x+e)*sech(d*x+c)/b/d+a^
2*(f*x+e)*sech(d*x+c)/b/(a^2+b^2)/d-a*(f*x+e)*tanh(d*x+c)/b^2/d+a^3*(f*x+e
)*tanh(d*x+c)/b^2/(a^2+b^2)/d
```

3.384.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.06 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.86

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{-\frac{2if\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a-ib}+\frac{2if\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a+ib}+\frac{f\log(\cosh(c+dx))}{a-ib}+\frac{f\log(\cosh(c+dx))}{a+ib}+\frac{2a^2\left(-2de\,\text{arctanh}\left(\frac{a+be^{c+}}{\sqrt{a^2+}}\right.\right.}{}}{}$$

input
```
Integrate[((e + f*x)*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(((-2*I)*f*ArcTan[Tanh[(c + d*x)/2]])/(a - I*b) + ((2*I)*f*ArcTan[Tanh[(c
+ d*x)/2]])/(a + I*b) + (f*Log[Cosh[c + d*x]])/(a - I*b) + (f*Log[Cosh[c +
 d*x]])/(a + I*b) + (2*a^2*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
b^2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt
[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(
a^2 + b^2)^(3/2) - (2*d*(e + f*x)*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a^
2 + b^2))/(2*d^2)
```

3.384.3 Rubi [A] (verified)

Time = 2.55 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.92, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {6101, 5974, 3042, 4257, 6117, 3042, 4672, 26, 3042, 26, 3956, 6107, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6101}$$

$$\frac{\int (e+fx)\text{sech}(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5974}$$

$$\frac{\frac{f\int \text{sech}(c+dx)dx}{d}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)\text{sech}(c+dx)}{d}+\frac{f\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6117}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}\right)}{b}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int-i\tanh(c+dx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int\tanh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int-i\tan(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int\tan(ic+idx)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{6107}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{b^2\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

3.384. $\displaystyle\int\frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3803}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{2b^2\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 2694}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 27}$$

3.384. $\displaystyle\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\cdots\right)}{a^2+b^2}\cdots\right)}{b}\cdots\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+\cdots\right)}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\cdots\right)}{b}\cdots\right)}{b}$$

$$\Big\downarrow \text{2715}$$

3.384. $\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+\cdots}}{a+\sqrt{\cdots}}\right)\cdots}{bd^2}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\right)}{b}\cdots\right)}{b}$$

$$\Big\downarrow \; 2838$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+\cdots}\cdots\right)}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\right)}{b}\cdots\right)}{b}$$

$$\Big\downarrow \; 7293$$

3.384. $\displaystyle\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int\left(a(e+fx)\text{sech}^2(c+dx)-b(e+fx)\text{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\cdots\right)}{2\sqrt{a^2+\cdots}}\cdots\right)}{\cdots}\right)}{b}\cdots\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{-\frac{af\log(\cosh(c+dx))}{d^2}+\frac{a(e+fx)\tanh(c+dx)}{d}-\frac{bf\arctan(\sinh(c+dx))}{d^2}+\frac{b(e+fx)\text{sech}(c+dx)}{d}}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{Po}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}\cdots\right)$$

input
```
Int[((e + f*x)*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

3.384. $\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
((f*ArcTan[Sinh[c + d*x]])/d^2 - ((e + f*x)*Sech[c + d*x])/d)/b - (a*((-((
f*Log[Cosh[c + d*x]])/d^2) + ((e + f*x)*Tanh[c + d*x])/d)/b - (a*((-2*b^2*
(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d)
 + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt
[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2
)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + (-((b*f*ArcTan[Sinh[c + d*x]])/d^2
) - (a*f*Log[Cosh[c + d*x]])/d^2 + (b*(e + f*x)*Sech[c + d*x])/d + (a*(e +
 f*x)*Tanh[c + d*x])/d)/(a^2 + b^2)))/b))/b
```

3.384.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

3.384. $\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.384. $\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.384.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1927 vs. $2(365) = 730$.

Time = 2.50 (sec) , antiderivative size = 1928, normalized size of antiderivative = 5.01

method	result	size
risch	Expression too large to display	1928

input
```
int((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/(a^2+b^2)^(3/2)/d*a^4*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+
a)/(a+(a^2+b^2)^(1/2)))*x+2/(a^2+b^2)^(3/2)/d^2*b^2*c*f/(2*a^2+2*b^2)*arct
anh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2+2/(a^2+b^2)^(3/2)/d*b^2*
f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
*a^2*x-2/(a^2+b^2)^(3/2)/d*b^2*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^
(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2*x+2/(a^2+b^2)^(3/2)/d^2*b^2*f/(2*a^2+2*b
^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2*c-2/(a^
2+b^2)^(3/2)/d^2*b^2*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(
a+(a^2+b^2)^(1/2)))*a^2*c-2/(a^2+b^2)^(5/2)/d^2*a^2*b^2*f*arctanh(1/2*(2*b
*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+4/(a^2+b^2)^(1/2)/d^2*a^2*f/(2*a^2+2*b^2
)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/(a^2+b^2)^(3/2)/d^2*
b^4*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a
^2+b^2)/d^2*b^2*f/(2*a^2+2*b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/
(a^2+b^2)^(1/2)/d^2*b^2*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(
a^2+b^2)^(1/2))+2/(a^2+b^2)/d^2*b^2*f/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c))
+4/(a^2+b^2)/d^2*a^2*b*f/(2*a^2+2*b^2)*arctan(exp(d*x+c))-2/(a^2+b^2)^(3/2
)/d^2*b^2*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)
)*a^2+2/(a^2+b^2)^(3/2)/d^2*b^2*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+
b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2-2/(a^2+b^2)^(3/2)/d^2*b^2*f/(2*a^2
+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2...
```

3.384.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1337 vs. $2(363) = 726$.

Time = 0.31 (sec) , antiderivative size = 1337, normalized size of antiderivative = 3.47

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*(a^3 + a*b^2)*d*f*x*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*d*f*x*sinh(d*x +
 c)^2 - 2*(a^3 + a*b^2)*d*e - (a^2*b*f*cosh(d*x + c)^2 + 2*a^2*b*f*cosh(d*
x + c)*sinh(d*x + c) + a^2*b*f*sinh(d*x + c)^2 + a^2*b*f)*sqrt((a^2 + b^2)
/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a^2*b*f*cosh(d*x + c)^2 +
2*a^2*b*f*cosh(d*x + c)*sinh(d*x + c) + a^2*b*f*sinh(d*x + c)^2 + a^2*b*f)
*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a^2*b*d*e
 - a^2*b*c*f + (a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*e - a^
2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*e - a^2*b*c*f)*sinh(d*x +
c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*
b*sqrt((a^2 + b^2)/b^2) + 2*a) - (a^2*b*d*e - a^2*b*c*f + (a^2*b*d*e - a^2
*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)*sinh(d*x
 + c) + (a^2*b*d*e - a^2*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log
(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
- (a^2*b*d*f*x + a^2*b*c*f + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2 + 2
*(a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*f*x + a^
2*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*
sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b) + (a^2*b*d*f*x + a^2*b*c*f + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x...
```

3.384.6 Sympy [F]

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.384.7 Maxima [F]

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\tanh{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
(2*a^2*integrate(-x*e^(d*x + c)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(2*c
))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x) - 2*(b*x*e^(d*x + c) -
 a*x)/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) - 2*a*x/
((a^2 + b^2)*d) + 2*b*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) + a*log(e^(2*d
*x + 2*c) + 1)/((a^2 + b^2)*d^2))*f + e*(a^2*log((b*e^(-d*x - c) - a - sqr
t(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*d
) - 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d))
```

3.384.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.384.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh{(c+d\,x)}}\,dx$$

input
```
int((tanh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((tanh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.385 $\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.385.1 Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=-\frac{2a^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{b\operatorname{sech}(c+dx)}{(a^2+b^2)\,d}-\frac{a\tanh(c+dx)}{(a^2+b^2)\,d}$$

output
```
-2*a^2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/
d-b*sech(d*x+c)/(a^2+b^2)/d-a*tanh(d*x+c)/(a^2+b^2)/d
```

3.385.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.18

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{-b\sqrt{-a^2-b^2}\operatorname{sech}(c+dx)+a\left(2a\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)-\sqrt{-a^2-b^2}\tanh(c+dx)\right)}{(-a^2-b^2)^{3/2}\,d}$$

input
```
Integrate[Tanh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-(b*Sqrt[-a^2 - b^2]*Sech[c + d*x]) + a*(2*a*ArcTan[(b - a*Tanh[(c + d*
x)/2])/Sqrt[-a^2 - b^2]] - Sqrt[-a^2 - b^2]*Tanh[c + d*x]))/((-a^2 - b^2)^
(3/2)*d))
```

3.385.3 Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 25, 3206, 26, 3042, 26, 3086, 24, 3139, 1083, 217, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3206}$$

$$\frac{a^2\int \frac{1}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{a\int \text{sech}^2(c+dx)dx}{a^2+b^2}-\frac{ib\int i\text{sech}(c+dx)\tanh(c+dx)dx}{a^2+b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{a^2\int \frac{1}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{a\int \text{sech}^2(c+dx)dx}{a^2+b^2}+\frac{b\int \text{sech}(c+dx)\tanh(c+dx)dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{a\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{b\int -i\sec(ic+idx)\tan(ic+idx)dx}{a^2+b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{a^2\int \frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{a\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{ib\int \sec(ic+idx)\tan(ic+idx)dx}{a^2+b^2}$$

$$\Big\downarrow \text{3086}$$

$$\frac{a^2\int \frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{a\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{b\int 1d\text{sech}(c+dx)}{d\,(a^2+b^2)}$$

$$\Big\downarrow \text{24}$$

$$\frac{a^2\int \frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{a\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{b\text{sech}(c+dx)}{d\,(a^2+b^2)}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{a\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{2ia^2\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(a^2+b^2\right)}-\frac{b\text{sech}(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{a\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{4ia^2\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{d\left(a^2+b^2\right)}-\frac{b\text{sech}(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{217}$$

$$-\frac{a\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{2a^2\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{b\text{sech}(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{ia\int 1d(-i\tanh(c+dx))}{d\left(a^2+b^2\right)}+\frac{2a^2\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{b\text{sech}(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{24}$$

$$\frac{2a^2\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{a\tanh(c+dx)}{d\left(a^2+b^2\right)}-\frac{b\text{sech}(c+dx)}{d\left(a^2+b^2\right)}$$

input
```
Int[Tanh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
(2*a^2*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*
d) - (b*Sech[c + d*x])/((a^2 + b^2)*d) - (a*Tanh[c + d*x])/((a^2 + b^2)*d)
```

3.385.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3206
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[a/(a^2 - b^2)   Int[(g*Tan[e + f*x])^p/Sin[e + f*x]
^2, x], x] + (-Simp[b*(g/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 1)/Cos[e
+ f*x], x], x] - Simp[a^2*(g^2/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 2)/
(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2
, 0] && IntegersQ[2*p] && GtQ[p, 1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.385. $\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.385.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\frac{-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{8a^2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(4a^2+4b^2\right)\sqrt{a^2+b^2}}}{d}$
default	$\frac{\frac{-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{8a^2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(4a^2+4b^2\right)\sqrt{a^2+b^2}}}{d}$
risch	$\frac{-2b\,\mathrm{e}^{dx+c}+2a}{d(a^2+b^2)(1+\mathrm{e}^{2dx+2c})}+\frac{a^2\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}d}-\frac{a^2\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}d}$

input
```
int(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/(a^2+b^2)*(-a*tanh(1/2*d*x+1/2*c)-b)/(1+tanh(1/2*d*x+1/2*c)^2)+8*a^
2/(4*a^2+4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/
(a^2+b^2)^(1/2)))
```

3.385.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(87) = 174.

Time = 0.26 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.90

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{2\,a^3+2\,ab^2+\left(a^2\cosh\left(dx+c\right)^2+2\,a^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a^2\sinh\left(dx+c\right)^2+a^2\right)\sqrt{a^2+b^2}\log\left(\right.}{(a^4+2\,a^2b^2+b^4)d\cosh\left(dx+c\right)^2+2\,(a^4+}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*a^3 + 2*a*b^2 + (a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c
) + a^2*sinh(d*x + c)^2 + a^2)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 +
b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x
+ c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*
x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) +
2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(a^2*b + b^3)*cosh(d*x + c
) - 2*(a^2*b + b^3)*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c
)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a
^2*b^2 + b^4)*d*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)
```

3.385.6 Sympy [F]

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.385.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.28

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a^2\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}d} - \frac{2\left(be^{(-dx-c)}+a\right)}{(a^2+b^2+(a^2+b^2)e^{(-2\,dx-2\,c)})d}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/((a^2 + b^2)^(3/2)*d) - 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 +
(a^2 + b^2)*e^(-2*d*x - 2*c))*d)
```

3.385.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{a^2\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\,\left(be^{(dx+c)}-a\right)}{(a^2+b^2)\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
(a^2*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c
) + 2*a + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^(d*x + c) - a)/((
a^2 + b^2)*(e^(2*d*x + 2*c) + 1)))/d
```

3.385.9 Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 422, normalized size of antiderivative = 4.69

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,a}{d\,(a^2+b^2)} - \frac{2\,b\,\mathrm{e}^{c+d\,x}}{d\,(a^2+b^2)}}{\mathrm{e}^{2\,c+2\,d\,x}+1}$$
$$- \frac{2\,\mathrm{atan}\left(\left(\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\left(\frac{2\,a^2}{b^2\,d\,\sqrt{a^4}\,(a^2+b^2)^2} + \frac{2\left(a^3\,d\,\sqrt{a^4}+a\,b^2\,d\,\sqrt{a^4}\right)}{a\,b^2\,\sqrt{-d^2\,(a^2+b^2)^3}\,(a^2+b^2)\,\sqrt{-a^6\,d^2-3\,a^4\,b^2\,d^2-3\,a^2\,b^4\,d^2-b^6\,d^2}}\right) - \frac{\cdots}{a\,b^2\,\sqrt{-d^2\,(a^2+b^2)\cdots}}\right.\right.\right.}{\sqrt{-a^6\,d^2-3\,a^4\cdots}}$$

input
```
int(tanh(c + d*x)^2/(a + b*sinh(c + d*x)),x)
```

output
```
((2*a)/(d*(a^2 + b^2)) - (2*b*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(2*c + 2*
d*x) + 1) - (2*atan((exp(d*x)*exp(c)*((2*a^2)/(b^2*d*(a^4)^(1/2)*(a^2 + b^
2)^2) + (2*(a^3*d*(a^4)^(1/2) + a*b^2*d*(a^4)^(1/2)))/(a*b^2*(-d^2*(a^2 +
b^2)^3)^(1/2)*(a^2 + b^2)*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2
*d^2)^(1/2))) - (2*(b^3*d*(a^4)^(1/2) + a^2*b*d*(a^4)^(1/2)))/(a*b^2*(-d^2
*(a^2 + b^2)^3)^(1/2)*(a^2 + b^2)*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3
*a^4*b^2*d^2)^(1/2)))*((b^3*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b
^2*d^2)^(1/2))/2 + (a^2*b*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2
*d^2)^(1/2))/2))*(a^4)^(1/2))/(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4
*b^2*d^2)^(1/2)
```

3.386 $\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.386.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Int}\left(\frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.386.2 Mathematica [N/A]

Not integrable

Time = 52.79 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Tanh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Tanh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.386.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Tanh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.386.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.386.4 Maple [N/A] (verified)

Not integrable

Time = 0.62 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\tanh \left(d x +c \right)^{2}}{\left(f x +e \right) \left(a +b \sinh \left(d x +c \right)\right)}d x$$

input
```
int(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.386.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(tanh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.386.6 Sympy [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh^2(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(tanh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.386.7 Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 391, normalized size of antiderivative = 13.96

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2*a^2*integrate(-e^(d*x + c)/(a^2*b*e + b^3*e + (a^2*b*f + b^3*f)*x - (a^2
*b*e*e^(2*c) + b^3*e*e^(2*c) + (a^2*b*f*e^(2*c) + b^3*f*e^(2*c))*x)*e^(2*d
*x) - 2*(a^3*e*e^c + a*b^2*e*e^c + (a^3*f*e^c + a*b^2*f*e^c)*x)*e^(d*x)),
x) - 2*(b*e^(d*x + c) - a)/(a^2*d*e + b^2*d*e + (a^2*d*f + b^2*d*f)*x + (a
^2*d*e*e^(2*c) + b^2*d*e*e^(2*c) + (a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x)*
e^(2*d*x)) - integrate(2*(b*f*e^(d*x + c) - a*f)/(a^2*d*e^2 + b^2*d*e^2 +
(a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*e*f + b^2*d*e*f)*x + (a^2*d*e^2*e^(
2*c) + b^2*d*e^2*e^(2*c) + (a^2*d*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 + 2
*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x)*e^(2*d*x)), x)
```

3.386.8 Giac [F(-1)]

Timed out.

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.386.9 Mupad [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(tanh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(tanh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.387 $\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.387.1 Optimal result

Integrand size = 34, antiderivative size = 1256

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
-I*a^3*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^2-1/2*(f*x+e)^2*
sech(d*x+c)^2/b/d+2*I*a^3*f*(f*x+e)*polylog(2,I*exp(d*x+c))/(a^2+b^2)^2/d^
2+I*a^3*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b^2/(a^2+b^2)/d^2+2*I*a^3*f^2*po
lylog(3,-I*exp(d*x+c))/(a^2+b^2)^2/d^3+I*a*f*(f*x+e)*polylog(2,-I*exp(d*x+
c))/b^2/d^2-a^2*b*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2+I*a
^3*f^2*polylog(3,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^3+a^3*f*(f*x+e)*sech(d*x+c
)/b^2/(a^2+b^2)/d^2-a^2*f*(f*x+e)*tanh(d*x+c)/b/(a^2+b^2)/d^2+a^2*b*(f*x+e
)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d+a^2*b*(f*x+e)^2*l
n(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d+1/2*a^2*b*f^2*polylog(
3,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^3+1/2*a^2*(f*x+e)^2*sech(d*x+c)^2/b/(a^2+
b^2)/d-1/2*a*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/b^2/d-2*a^2*b*f^2*polylog(3
,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^3-2*a^2*b*f^2*polylog(3,
-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^3-I*a*f^2*polylog(3,-I*ex
p(d*x+c))/b^2/d^3-2*I*a^3*f^2*polylog(3,I*exp(d*x+c))/(a^2+b^2)^2/d^3+a^3*
(f*x+e)^2*arctan(exp(d*x+c))/b^2/(a^2+b^2)/d+1/2*a^3*(f*x+e)^2*sech(d*x+c)
*tanh(d*x+c)/b^2/(a^2+b^2)/d+2*a^2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2+2*a^2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-2*I*a^3*f*(f*x+e)*polylog(2,-I*exp(
d*x+c))/(a^2+b^2)^2/d^2-I*a*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b^2/d^2-I*a^
3*f^2*polylog(3,I*exp(d*x+c))/b^2/(a^2+b^2)/d^3-a*(f*x+e)^2*arctan(exp(...
```

3.387.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3390 vs. $2(1256) = 2512$.

Time = 12.07 (sec) , antiderivative size = 3390, normalized size of antiderivative = 2.70

$$\int \frac{(e+fx)^2 \operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(12*a^2*b*d^3*e^2*E^(2*c)*x + 12*a^2*b*d*E^(2*c)*f^2*x + 12*b^3*d*E^(2*c)*
f^2*x + 12*a^2*b*d^3*e*E^(2*c)*f*x^2 + 4*a^2*b*d^3*E^(2*c)*f^2*x^3 + 6*a^3
*d^2*e^2*ArcTan[E^(c + d*x)] - 6*a*b^2*d^2*e^2*ArcTan[E^(c + d*x)] + 6*a^3
*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] - 6*a*b^2*d^2*e^2*E^(2*c)*ArcTan[E^(c
 + d*x)] + 12*a^3*f^2*ArcTan[E^(c + d*x)] + 12*a*b^2*f^2*ArcTan[E^(c + d*x
)] + 12*a^3*E^(2*c)*f^2*ArcTan[E^(c + d*x)] + 12*a*b^2*E^(2*c)*f^2*ArcTan[
E^(c + d*x)] + (6*I)*a^3*d^2*e*f*x*Log[1 - I*E^(c + d*x)] - (6*I)*a*b^2*d^
2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*a^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(
c + d*x)] - (6*I)*a*b^2*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (3*I)*a
^3*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] - (3*I)*a*b^2*d^2*f^2*x^2*Log[1 - I*
E^(c + d*x)] + (3*I)*a^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (3*I
)*a*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (6*I)*a^3*d^2*e*f*x*L
og[1 + I*E^(c + d*x)] + (6*I)*a*b^2*d^2*e*f*x*Log[1 + I*E^(c + d*x)] - (6*
I)*a^3*d^2*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] + (6*I)*a*b^2*d^2*e*E^(2*c
)*f*x*Log[1 + I*E^(c + d*x)] - (3*I)*a^3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)
] + (3*I)*a*b^2*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*a^3*d^2*E^(2*c)
*f^2*x^2*Log[1 + I*E^(c + d*x)] + (3*I)*a*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 +
I*E^(c + d*x)] - 6*a^2*b*d^2*e^2*Log[1 + E^(2*(c + d*x))] - 6*a^2*b*d^2*e^
2*E^(2*c)*Log[1 + E^(2*(c + d*x))] - 6*a^2*b*f^2*Log[1 + E^(2*(c + d*x))]
- 6*b^3*f^2*Log[1 + E^(2*(c + d*x))] - 6*a^2*b*E^(2*c)*f^2*Log[1 + E^(2...
```

3.387.3 Rubi [A] (verified)

Time = 6.38 (sec) , antiderivative size = 1026, normalized size of antiderivative = 0.82, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.735, Rules used = {6117, 5974, 3042, 4672, 26, 3042, 26, 3956, 6117, 3042, 4674, 3042, 4257, 4668, 3011, 2720, 6107, 6107, 6095, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\tanh^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6117}$$

$$\frac{\int (e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5974}$$

$$\frac{\frac{f\int (e+fx)\text{sech}^2(c+dx)dx}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{d}}{b}$$

$$\Big\downarrow \text{ 4672}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int -i\tanh(c+dx)dx}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int \tanh(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int -i\tan(ic+idx)dx}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int \tan(ic+idx)dx}{d}\right)}{d}}{b}$$

$$\Big\downarrow \; 3956$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 6117$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^3(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}\right)}{b}$$

$$\Big\downarrow \; 4674$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{-\frac{f^2\int\text{sech}(c+dx)dx}{d^2}+\frac{1}{2}\int(e+fx)^2\text{sech}(c+dx)dx+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{f^2\int\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+}{2d}}{b}\right)}{b}$$

$$\Big\downarrow \; 4257$$

3.387. $\displaystyle\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f^2\arctan(\sinh(c+dx))}{d^3}+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}\right)}{b}$$

$$\Big\downarrow \text{4668}$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{2if\int(e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int(e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}\right)-\frac{f^2\arctan(\sinh(c}{d^3}}{b}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{Poly}}{}\right)}{d}\right)}{}\right)}{b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c}\right.}{d^2}\right)}{}\right)}{}\right)}{}$$

$$\Big\downarrow \text{6107}$$

3.387. $\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyL}}{}\right.}{d}\right.}{}\right.$$

$$\Big\downarrow \textbf{6107}$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-}}{}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \textbf{6095}$$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int(e+fx)^2\text{sech}(c+dx)(a-}{a^2+b^2}\right.}{a^2+b^2}\right.}{b}\right.$$

$$\Big\downarrow \textbf{2620}$$

3.387. $\displaystyle\int\frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{b}{a-\ldots}\right.}{bd}\right.}{a^2+b^2}\right.}{a^2\ldots}\right.}{b}\right.$$

$\downarrow$ 3011

3.387. $\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,\cdot\right)}{d}\right.}{bd}\right.}{a^2+b^2}}{a^2+b^2}\right.}{}\right.$$

$$\Big\downarrow \text{2720}$$

3.387. $\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\cdots\right.}{bd}\cdots\right)}{a^2+b^2}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\Big\downarrow \; 7143$$

3.387. $\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{b^2\left(\frac{\int(e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\right.}{d}\right.}{}\right.}{}\right.}{a^2+b^2}}{}\right.}{}\right.$$

$\downarrow$ 7293

3.387. $\displaystyle\int \frac{(e+fx)^2\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{b^2\left(\frac{\int\left(a(e+fx)^2\operatorname{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{Po}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right)}{a^2+b^2\cdots}\right)}{\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

3.387. $\int \frac{(e+fx)^2\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(\frac{-\frac{\arctan(\sinh(c+dx))f^2}{d^3}+\frac{(e+fx)\text{sech}(c+dx)f}{d^2}+\frac{1}{2}\left(\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^2}{d}+\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\,\text{P}}{}\right.}{}\right.}{b}\right.$$

input
```
Int[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

3.387. $\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(-1/2*((e + f*x)^2*Sech[c + d*x]^2)/d + (f*(-((f*Log[Cosh[c + d*x]])/d^2)
+ ((e + f*x)*Tanh[c + d*x])/d))/d)/b - (a*((-((f^2*ArcTan[Sinh[c + d*x]])/
d^3) + ((2*(e + f*x)^2*ArcTan[E^(c + d*x)])/d + ((2*I)*f*(-(((e + f*x)*Pol
yLog[2, (-I)*E^(c + d*x)])/d) + (f*PolyLog[3, (-I)*E^(c + d*x)])/d^2))/d -
 ((2*I)*f*(-(((e + f*x)*PolyLog[2, I*E^(c + d*x)])/d) + (f*PolyLog[3, I*E^
(c + d*x)])/d^2))/d)/2 + (f*(e + f*x)*Sech[c + d*x])/d^2 + ((e + f*x)^2*Se
ch[c + d*x]*Tanh[c + d*x])/(2*d))/b - (a*((b^2*((b^2*(-1/3*(e + f*x)^3/(b*
f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) +
((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*
(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (
f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d) - (2*f
*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) +
(f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(a^
2 + b^2) + ((b*(e + f*x)^3)/(3*f) + (2*a*(e + f*x)^2*ArcTan[E^(c + d*x)])/
d - (b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/d - ((2*I)*a*f*(e + f*x)*Poly
Log[2, (-I)*E^(c + d*x)])/d^2 + ((2*I)*a*f*(e + f*x)*PolyLog[2, I*E^(c + d
*x)])/d^2 - (b*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/d^2 + ((2*I)*a*f^
2*PolyLog[3, (-I)*E^(c + d*x)])/d^3 - ((2*I)*a*f^2*PolyLog[3, I*E^(c + d*x
)])/d^3 + (b*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*d^3))/(a^2 + b^2)))/(a^2
 + b^2) + ((a*(e + f*x)^2*ArcTan[E^(c + d*x)])/d - (a*f^2*ArcTan[Sinh[c...
```

3.387.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.387. $\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.387. $\int \frac{(e+fx)^2 \operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.387. $\int \frac{(e+fx)^2\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.387.4 Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c) \tanh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.387.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10934 vs. 2(1156) = 2312.

Time = 0.46 (sec) , antiderivative size = 10934, normalized size of antiderivative = 8.71

$$\int \frac{(e+fx)^2 \text{sech}(c+dx) \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.387.6 Sympy [F]

$$\int \frac{(e+fx)^2 \text{sech}(c+dx) \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2 \tanh^2(c+dx) \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*sech(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*tanh(c + d*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x))
, x)
```

3.387.7 Maxima [F]

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\,\text{sech}\,(dx+c)\tanh\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
a^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2
*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 +
 b^4*d^2), x) - a*b^2*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x
+ 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2
 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^2*b*d^2*f^2*integrate(x^2/(a^4*d^2*e
^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) +
 a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^3*d^2*e*f*integrate(x*e^(d*x
 + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e
^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a*b^2*d^2*e*f*
integrate(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x
+ 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x)
+ 4*a^2*b*d^2*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(
2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2
), x) + a^2*b*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*
x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) + b^3*f^2*(2*(d*x + c)/((a^4
+ 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4
)*d^3)) + (a^2*b*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2
*a^2*b^2 + b^4)*d) - a^2*b*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b
^4)*d) - (a^3 - a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) -
(a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^...
```

3.387.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.387.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)^2\,(e+f\,x)^2}{\cosh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((tanh(c + d*x)^2*(e + f*x)^2)/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((tanh(c + d*x)^2*(e + f*x)^2)/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x
)
```

3.388 $\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.388.1 Optimal result

Integrand size = 32, antiderivative size = 760

$$
\begin{aligned}
\int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = & -\frac{a(e+fx)\arctan\left(e^{c+dx}\right)}{b^2d} \\
& + \frac{2a^3(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} \\
& + \frac{a^3(e+fx)\arctan\left(e^{c+dx}\right)}{b^2(a^2+b^2) d} \\
& + \frac{a^2b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} \\
& + \frac{a^2b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} \\
& - \frac{a^2b(e+fx)\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)^2 d} \\
& + \frac{iaf\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{2b^2d^2} \\
& - \frac{ia^3f\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)^2 d^2} \\
& - \frac{ia^3f\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{2b^2(a^2+b^2) d^2} \\
& - \frac{iaf\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{2b^2d^2} \\
& + \frac{ia^3f\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)^2 d^2} \\
& + \frac{ia^3f\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{2b^2(a^2+b^2) d^2} \\
& + \frac{a^2bf\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d^2} \\
& + \frac{a^2bf\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d^2} \\
& - \frac{a^2bf\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2(a^2+b^2)^2 d^2} \\
& - \frac{af\operatorname{sech}(c+dx)}{2b^2d^2} + \frac{a^3f\operatorname{sech}(c+dx)}{2b^2(a^2+b^2)d^2} \\
& - \frac{(e+fx)\operatorname{sech}^2(c+dx)}{2bd} \\
& + \frac{a^2(e+fx)\operatorname{sech}^2(c+dx)}{2b(a^2+b^2)d} \\
& + \frac{f\tanh(c+dx)}{2bd^2} - \frac{a^2f\tanh(c+dx)}{2b(a^2+b^2)d^2}
\end{aligned}
$$

output
```
-a*(f*x+e)*arctan(exp(d*x+c))/b^2/d+2*a^3*(f*x+e)*arctan(exp(d*x+c))/(a^2+
b^2)^2/d+a^3*(f*x+e)*arctan(exp(d*x+c))/b^2/(a^2+b^2)/d-a^2*b*(f*x+e)*ln(1
+exp(2*d*x+2*c))/(a^2+b^2)^2/d+a^2*b*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/(a^2+b^2)^2/d+a^2*b*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)
))/(a^2+b^2)^2/d-I*a^3*f*polylog(2,-I*exp(d*x+c))/(a^2+b^2)^2/d^2+1/2*I*a^
3*f*polylog(2,I*exp(d*x+c))/b^2/(a^2+b^2)/d^2+I*a^3*f*polylog(2,I*exp(d*x+
c))/(a^2+b^2)^2/d^2-1/2*I*a*f*polylog(2,I*exp(d*x+c))/b^2/d^2+1/2*I*a*f*po
lylog(2,-I*exp(d*x+c))/b^2/d^2-1/2*I*a^3*f*polylog(2,-I*exp(d*x+c))/b^2/(a
^2+b^2)/d^2-1/2*a^2*b*f*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2+a^2*b*f
*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2+a^2*b*f*poly
log(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-1/2*a*f*sech(d*x+
c)/b^2/d^2+1/2*a^3*f*sech(d*x+c)/b^2/(a^2+b^2)/d^2-1/2*(f*x+e)*sech(d*x+c)
^2/b/d+1/2*a^2*(f*x+e)*sech(d*x+c)^2/b/(a^2+b^2)/d+1/2*f*tanh(d*x+c)/b/d^2
-1/2*a^2*f*tanh(d*x+c)/b/(a^2+b^2)/d^2-1/2*a*(f*x+e)*sech(d*x+c)*tanh(d*x+
c)/b^2/d+1/2*a^3*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b^2/(a^2+b^2)/d
```

3.388.2 Mathematica [A] (verified)

Time = 9.27 (sec) , antiderivative size = 823, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{a^2b\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\,\text{arctanh}\left(\frac{a+be^{c}}{\sqrt{a^2-}}\right.}{(-a^2-b^2)^{3/2}}\right.}{}$$
$$+\frac{a\left(2abde(c+dx)-2abcf(c+dx)+abf(c+dx)^2+2a^2de\arctan\left(e^{c+dx}\right)-2b^2de\arctan\left(e^{c+dx}\right)-2a^2c\right.}{}$$
$$+\frac{\text{sech}(c+dx)(-af+bf\sinh(c+dx))}{2(a^2+b^2)d^2}$$
$$+\frac{\text{sech}^2(c+dx)(-bde+bcf-bf(c+dx)-ade\sinh(c+dx)+acf\sinh(c+dx)-af(c+dx)\sinh(c+}{2(a^2+b^2)d^2}$$

input
```
Integrate[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),
x]
```

output
```
(a^2*b*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2
 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2
)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2
+ b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a
*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/
(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))]))/(2*(a^2 + b^2)^2*d^2) + (a*(2*a*b*d*e*(c + d*x) - 2*a*b*c*f*(c
+ d*x) + a*b*f*(c + d*x)^2 + 2*a^2*d*e*ArcTan[E^(c + d*x)] - 2*b^2*d*e*Arc
Tan[E^(c + d*x)] - 2*a^2*c*f*ArcTan[E^(c + d*x)] + 2*b^2*c*f*ArcTan[E^(c +
 d*x)] + I*a^2*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - I*b^2*f*(c + d*x)*Log[
1 - I*E^(c + d*x)] - I*a^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + I*b^2*f*(c
 + d*x)*Log[1 + I*E^(c + d*x)] - 2*a*b*d*e*Log[1 + E^(2*(c + d*x))] + 2*a*
b*c*f*Log[1 + E^(2*(c + d*x))] - 2*a*b*f*(c + d*x)*Log[1 + E^(2*(c + d*x))
] - I*(a^2 - b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] + I*(a^2 - b^2)*f*PolyLog
[2, I*E^(c + d*x)] - a*b*f*PolyLog[2, -E^(2*(c + d*x))]))/(2*(a^2 + b^2)^2
*d^2) + (Sech[c + d*x]*(-(a*f) + b*f*Sinh[c + d*x]))/(2*(a^2 + b^2)*d^2) +
 (Sech[c + d*x]^2*(-(b*d*e) + b*c*f - b*f*(c + d*x) - a*d*e*Sinh[c + d*x]
+ a*c*f*Sinh[c + d*x] - a*f*(c + d*x)*Sinh[c + d*x]))/(2*(a^2 + b^2)*d^...
```

3.388.3 Rubi [A] (verified)

Time = 3.56 (sec) , antiderivative size = 646, normalized size of antiderivative = 0.85, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6117, 5974, 3042, 4254, 24, 6117, 3042, 4673, 3042, 4668, 2715, 2838, 6107, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\tanh^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6117}$$

$$\frac{\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5974}$$

$$\frac{\frac{f\int \text{sech}^2(c+dx)dx}{2d}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}+\frac{f\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d}}{b}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}+\frac{if\int 1d(-i\tanh(c+dx))}{2d^2}}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6117}$$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}\right)}{b}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\frac{1}{2}\int (e+fx)\text{sech}(c+dx)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d}}{b}\right)}{b}$$

$$\Big\downarrow \text{4668}$$

3.388. $\displaystyle\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{if\int\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+da}{2d}}{b}\right.}{b}$$

$$\Big\downarrow \; 2715$$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{f\text{sech}}{2c}}{b}\right.}{b}$$

$$\Big\downarrow \; 2838$$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\text{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx}{2d}}{b}\right.}{b}$$

$$\Big\downarrow \; 6107$$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}(}{d^2}\right.}{b}\right.}{b}$$

$$\Big\downarrow \; 6107$$

3.388. $\displaystyle\int\frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(\cdots\right.}{d}\cdots\right.}{\cdots}\right)}{b}$$

$\downarrow$ 6095

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh\cdots}{a^2+b^2}\cdots\right.}{a^2+b^2}\cdots\right.}{b}\cdots\right.}{b\cdots}$$

$\downarrow$ 2620

3.388. $\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+\cdots}{\cdots}\right.}{a^2+b^2}\right.}{a^2+b^2}\right.}{b}\right.$$

$$\Big\downarrow \; 2715$$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{b^2\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}\cdots}\right.}{bd}\right.}{a^2+b^2}\right.}{a^2+b^2}\right.}{b}\right.$$

$$\Big\downarrow \; 2838$$

3.388. $\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{b^2\left(\frac{\int(e+fx)\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+\cdots}{\cdots}\right.}{a^2+b^2}}{a^2+b^2}\right.}{b}\right.$$

$$\downarrow \text{7293}$$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(-\frac{a\left(\frac{b^2\left(\frac{\int(a(e+fx)\operatorname{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\cdots\right.}{bd}\cdots\right.}{a^2+b^2}}{a^2+b^2}\right.}{b}\right.$$

$$\downarrow \text{2009}$$

3.388. $\int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\mathrm{sech}^2(c+dx)}{2d}}{b}-$$

$$a\left(\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{f\mathrm{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}}{b}-\frac{a\left(\frac{b^2\left(\frac{b^2\left(\frac{f\,\mathrm{Po}}{}\right.}{}\right.}{}\right.}{}\right.$$

input
```
Int[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-1/2*((e + f*x)*Sech[c + d*x]^2)/d + (f*Tanh[c + d*x])/(2*d^2))/b - (a*((
((2*(e + f*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/
d^2 + (I*f*PolyLog[2, I*E^(c + d*x)])/d^2)/2 + (f*Sech[c + d*x])/(2*d^2) +
 ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/b - (a*((b^2*((b^2*(-1/2*(
e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
)])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*
d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f
*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2 + b^
2) + ((b*(e + f*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d - (b*(
e + f*x)*Log[1 + E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*E^(c + d*x)]
)/d^2 + (I*a*f*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2*(c +
 d*x))])/(2*d^2))/(a^2 + b^2)))/(a^2 + b^2) + ((a*(e + f*x)*ArcTan[E^(c +
d*x)])/d - ((I/2)*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((I/2)*a*f*PolyL
og[2, I*E^(c + d*x)])/d^2 + (a*f*Sech[c + d*x])/(2*d^2) + (b*(e + f*x)*Sec
h[c + d*x]^2)/(2*d) - (b*f*Tanh[c + d*x])/(2*d^2) + (a*(e + f*x)*Sech[c +
d*x]*Tanh[c + d*x])/(2*d))/(a^2 + b^2)))/b))/b
```

3.388. $\int \frac{(e+fx)\mathrm{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.388.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.388. $\int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.388. $\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.388.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2067 vs. $2(701) = 1402$.

Time = 7.20 (sec) , antiderivative size = 2068, normalized size of antiderivative = 2.72

method	result	size
risch	Expression too large to display	2068

input
```
int((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
-1/d^2/(a^2+b^2)^(3/2)*a*b^3*c*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)
+2*a)/(a^2+b^2)^(1/2))-1/d^2/(a^2+b^2)^(3/2)*a^3*b*c*f/(2*a^2+2*b^2)*arcta
nh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d^2/(a^2+b^2)^(1/2)*a*b*c*f
/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d/(a^2+
b^2)^(1/2)*a*b*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^
(1/2))+1/d/(a^2+b^2)^(3/2)*a*b^3*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+
c)+2*a)/(a^2+b^2)^(1/2))+1/d/(a^2+b^2)^(3/2)*a^3*b*e/(2*a^2+2*b^2)*arctanh
(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2*a^2/d^2/(a^2+b^2)*c*f*b/(2*a^
2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2*a^2/d^2/(a^2+b^2)*c*f*b/(
2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))-I*a/d^2/(a^2+b^2)*f*b^2/(2*a^2+2*b^2)*di
log(1-I*exp(d*x+c))-I*a^3/d/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x
-I*a^3/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c-I*a/d/(a^2+b^2)*
f*b^2/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x-I*a/d^2/(a^2+b^2)*f*b^2/(2*a^2+2*
b^2)*ln(1-I*exp(d*x+c))*c-2*a^2/d^2/(a^2+b^2)*f*b/(2*a^2+2*b^2)*ln(1+I*exp
(d*x+c))*c-2*a^2/d^2/(a^2+b^2)*f*b/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c+I*a/
d^2/(a^2+b^2)*f*b^2/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+I*a^3/d/(a^2+b^2)*
f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x+I*a^3/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*l
n(1-I*exp(d*x+c))*c+2*a/d^2/(a^2+b^2)*c*f*b^2/(2*a^2+2*b^2)*arctan(exp(d*x
+c))+2*a^2/d/(a^2+b^2)*f*b/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*x-2*a^2/d/(a^2+b^2)*f*b/(2*a^2+2*b^2)*ln(1+I*...
```

3.388.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4903 vs. 2(680) = 1360.

Time = 0.37 (sec) , antiderivative size = 4903, normalized size of antiderivative = 6.45

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-1/2*(2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*cosh(d
*x + c)^3 + 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*
sinh(d*x + c)^3 + 2*(2*(a^2*b + b^3)*d*f*x + 2*(a^2*b + b^3)*d*e + (a^2*b
+ b^3)*f)*cosh(d*x + c)^2 + 2*(2*(a^2*b + b^3)*d*f*x + 2*(a^2*b + b^3)*d*e
 + (a^2*b + b^3)*f + 3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a
*b^2)*f)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a^2*b + b^3)*f - 2*((a^3 + a*
b^2)*d*f*x + (a^3 + a*b^2)*d*e - (a^3 + a*b^2)*f)*cosh(d*x + c) - 2*(a^2*b
*f*cosh(d*x + c)^4 + 4*a^2*b*f*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*b*f*sin
h(d*x + c)^4 + 2*a^2*b*f*cosh(d*x + c)^2 + a^2*b*f + 2*(3*a^2*b*f*cosh(d*x
 + c)^2 + a^2*b*f)*sinh(d*x + c)^2 + 4*(a^2*b*f*cosh(d*x + c)^3 + a^2*b*f*
cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(
a^2*b*f*cosh(d*x + c)^4 + 4*a^2*b*f*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*b*
f*sinh(d*x + c)^4 + 2*a^2*b*f*cosh(d*x + c)^2 + a^2*b*f + 2*(3*a^2*b*f*cos
h(d*x + c)^2 + a^2*b*f)*sinh(d*x + c)^2 + 4*(a^2*b*f*cosh(d*x + c)^3 + a^2
*b*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c
) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
+ ((2*a^2*b*f - I*(a^3 - a*b^2)*f)*cosh(d*x + c)^4 + 4*(2*a^2*b*f - I*(a^3
 - a*b^2)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b*f - I*(a^3 - a*b^2)*
f)*sinh(d*x + c)^4 + 2*a^2*b*f + 2*(2*a^2*b*f - I*(a^3 - a*b^2)*f)*cosh...
```

3.388.6 Sympy [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\tanh^2{(c+dx)}\,\text{sech}\,(c+dx)}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*tanh(c + d*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

3.388.7 Maxima [F]

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{sech}\,(dx+c)\tanh{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
(a^2*b*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 +
 b^4)*d) - a^2*b*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (
a^3 - a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (a*e^(-d*x
 - c) + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 +
b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e - f*(((a*d*x*e
^(3*c) + a*e^(3*c))*e^(3*d*x) + (2*b*d*x*e^(2*c) + b*e^(2*c))*e^(2*d*x) -
(a*d*x*e^c - a*e^c)*e^(d*x) + b)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b
^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x
)) + 2*integrate(-(a^3*b*x*e^(d*x + c) - a^2*b^2*x)/(a^4*b + 2*a^2*b^3 + b
^5 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*
e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - 2*integrate(1/2*(2*a^2*b*x
 + (a^3*e^c - a*b^2*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c)
+ 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x))
```

3.388.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.388.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)^2\,(e+f\,x)}{\cosh(c+d\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((tanh(c + d*x)^2*(e + f*x))/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((tanh(c + d*x)^2*(e + f*x))/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.389 $\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.389.1 Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a(a^2-b^2)\arctan(\sinh(c+dx))}{2\,(a^2+b^2)^2\,d} - \frac{a^2b\log(\cosh(c+dx))}{(a^2+b^2)^2\,d} + \frac{a^2b\log(a+b\sinh(c+dx))}{(a^2+b^2)^2\,d} - \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2\,(a^2+b^2)\,d}$$

output
```
1/2*a*(a^2-b^2)*arctan(sinh(d*x+c))/(a^2+b^2)^2/d-a^2*b*ln(cosh(d*x+c))/(a
^2+b^2)^2/d+a^2*b*ln(a+b*sinh(d*x+c))/(a^2+b^2)^2/d-1/2*sech(d*x+c)^2*(b+a
*sinh(d*x+c))/(a^2+b^2)/d
```

3.389.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a((a^2+b^2)\arctan(\sinh(c+dx)) + a((ia+b)\log(i-\sinh(c+dx)) + (-ia+b)\log(i+\sinh(c+dx)) -}{2\,(a^2+b^2)}$$

input
```
Integrate[(Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(a*((a^2 + b^2)*ArcTan[Sinh[c + d*x]] + a*((I*a + b)*Log[I - Sinh[c +
 d*x]] + ((-I)*a + b)*Log[I + Sinh[c + d*x]] - 2*b*Log[a + b*Sinh[c + d*x]
])) + b*(a^2 + b^2)*Sech[c + d*x]^2 + a*(a^2 + b^2)*Sech[c + d*x]*Tanh[c +
 d*x])/((a^2 + b^2)^2*d)
```

3.389.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {3042, 25, 3316, 25, 27, 601, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2}{\cos(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2}{\cos(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$-\frac{b^3\int -\frac{\sinh^2(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^3\int \frac{\sinh^2(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{b^2\sinh^2(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{601}$$

$$\frac{b\left(-\frac{\int -\frac{ab^2(a-b\sinh(c+dx))}{(a^2+b^2)(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{2b^2}-\frac{ab\sinh(c+dx)+b^2}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\int \frac{ab^2(a-b\sinh(c+dx))}{(a^2+b^2)(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{2b^2}-\frac{ab\sinh(c+dx)+b^2}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{a\int \frac{a-b\sinh(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{2(a^2+b^2)}-\frac{ab\sinh(c+dx)+b^2}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{657}$$

$$\frac{b\left(\frac{a\int \left(\frac{2a}{(a^2+b^2)(a+b\sinh(c+dx))}+\frac{a^2-2b\sinh(c+dx)a-b^2}{(a^2+b^2)(\sinh^2(c+dx)b^2+b^2)}\right)d(b\sinh(c+dx))}{2(a^2+b^2)}-\frac{ab\sinh(c+dx)+b^2}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{a\left(\frac{(a^2-b^2)\arctan(\sinh(c+dx))}{b(a^2+b^2)}-\frac{a\log(b^2\sinh^2(c+dx)+b^2)}{a^2+b^2}+\frac{2a\log(a+b\sinh(c+dx))}{a^2+b^2}\right)}{2(a^2+b^2)}-\frac{ab\sinh(c+dx)+b^2}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}\right)}{d}$$

input
```
Int[(Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b*((a*(((a^2 - b^2)*ArcTan[Sinh[c + d*x]])/(b*(a^2 + b^2)) + (2*a*Log[a +
 b*Sinh[c + d*x]])/(a^2 + b^2) - (a*Log[b^2 + b^2*Sinh[c + d*x]^2])/(a^2 +
 b^2)))/(2*(a^2 + b^2)) - (b^2 + a*b*Sinh[c + d*x])/(2*(a^2 + b^2)*(b^2 +
b^2*Sinh[c + d*x]^2))))/d
```

3.389. $\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.389.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

3.389. $\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.389.4 Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.73

method	result
derivativedivides	$\frac{\frac{\frac{2\left(\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(a^2b+b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{1}{2}a^3-\frac{1}{2}a\,b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+a\left(-ab\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+\left(a^2\right.\right.}{a^4+2a^2b^2+b^4}}{d}$
default	$\frac{\frac{\frac{2\left(\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(a^2b+b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{1}{2}a^3-\frac{1}{2}a\,b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+a\left(-ab\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+\left(a^2\right.\right.}{a^4+2a^2b^2+b^4}}{d}$
risch	$\frac{2a^2b\,d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2a^2bdc}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2a^2bx}{a^4+2a^2b^2+b^4}-\frac{2a^2bc}{d(a^4+2a^2b^2+b^4)}-\frac{\mathrm{e}^{dx+c}\left(a\,\mathrm{e}^{2dx+2c}+2b\,\mathrm{e}^{dx+c}\right.}{d(a^2+b^2)\left(1+\mathrm{e}^{2dx+2c}\right)^2}$

input
```
int(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/(a^4+2*a^2*b^2+b^4)*(((1/2*a^3+1/2*a*b^2)*tanh(1/2*d*x+1/2*c)^3+(a^
2*b+b^3)*tanh(1/2*d*x+1/2*c)^2+(-1/2*a^3-1/2*a*b^2)*tanh(1/2*d*x+1/2*c))/(
1+tanh(1/2*d*x+1/2*c)^2)^2+1/2*a*(-a*b*ln(1+tanh(1/2*d*x+1/2*c)^2)+(a^2-b^
2)*arctan(tanh(1/2*d*x+1/2*c))))+4*a^2*b/(4*a^4+8*a^2*b^2+4*b^4)*ln(tanh(1
/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a))
```

3.389.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 917 vs. $2(117) = 234$.

Time = 0.28 (sec) , antiderivative size = 917, normalized size of antiderivative = 7.58

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-((a^3 + a*b^2)*cosh(d*x + c)^3 + (a^3 + a*b^2)*sinh(d*x + c)^3 + 2*(a^2*b
 + b^3)*cosh(d*x + c)^2 + (2*a^2*b + 2*b^3 + 3*(a^3 + a*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^2 - ((a^3 - a*b^2)*cosh(d*x + c)^4 + 4*(a^3 - a*b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^3 - a*b^2)*sinh(d*x + c)^4 + a^3 - a*b^2 + 2
*(a^3 - a*b^2)*cosh(d*x + c)^2 + 2*(a^3 - a*b^2 + 3*(a^3 - a*b^2)*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + 4*((a^3 - a*b^2)*cosh(d*x + c)^3 + (a^3 - a*b^2
)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (a
^3 + a*b^2)*cosh(d*x + c) - (a^2*b*cosh(d*x + c)^4 + 4*a^2*b*cosh(d*x + c)
*sinh(d*x + c)^3 + a^2*b*sinh(d*x + c)^4 + 2*a^2*b*cosh(d*x + c)^2 + a^2*b
 + 2*(3*a^2*b*cosh(d*x + c)^2 + a^2*b)*sinh(d*x + c)^2 + 4*(a^2*b*cosh(d*x
 + c)^3 + a^2*b*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/
(cosh(d*x + c) - sinh(d*x + c))) + (a^2*b*cosh(d*x + c)^4 + 4*a^2*b*cosh(d
*x + c)*sinh(d*x + c)^3 + a^2*b*sinh(d*x + c)^4 + 2*a^2*b*cosh(d*x + c)^2
+ a^2*b + 2*(3*a^2*b*cosh(d*x + c)^2 + a^2*b)*sinh(d*x + c)^2 + 4*(a^2*b*c
osh(d*x + c)^3 + a^2*b*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(
cosh(d*x + c) - sinh(d*x + c))) - (a^3 + a*b^2 - 3*(a^3 + a*b^2)*cosh(d*x
+ c)^2 - 4*(a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 +
 b^4)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d
*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2
 + b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c...
```

3.389.6 Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh^2{(c+dx)}\operatorname{sech}{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.389.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{a^2b\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^4+2\,a^2b^2+b^4)d} - \frac{a^2b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d} - \frac{(a^3-ab^2)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d} - \frac{ae^{(-dx-c)}+2\,be^{(-2\,dx-2\,c)}-ae^{(-3\,dx-3\,c)}}{(a^2+b^2+2\,(a^2+b^2)e^{(-2\,dx-2\,c)}+(a^2+b^2)e^{(-4\,dx-4\,c)})d}$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
a^2*b*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 +
b^4)*d) - a^2*b*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a
^3 - a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (a*e^(-d*x
- c) + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b
^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d)
```

3.389.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(117) = 234$.

Time = 0.32 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.32

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\frac{4\,a^2b^2\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5} - \frac{2\,a^2b\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^4+2\,a^2b^2+b^4} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(a^3-ab^2)}{a^4+2\,a^2b^2+b^4} + \frac{2}{}}{4\,d}$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(4*a^2*b^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b + 2*a
^2*b^3 + b^5) - 2*a^2*b*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a
^2*b^2 + b^4) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a
^3 - a*b^2)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^2*b*(e^(d*x + c) - e^(-d*x - c)
)^2 - 2*a^3*(e^(d*x + c) - e^(-d*x - c)) - 2*a*b^2*(e^(d*x + c) - e^(-d*x
- c)) - 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*((e^(d*x + c) - e^(-d*x - c))^2 +
4)))/d
```

3.389.9 Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.80

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\frac{2b}{d(a^2+b^2)}+\frac{2a\,\mathrm{e}^{c+d\,x}}{d(a^2+b^2)}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}-\frac{\frac{2\,(a^2\,b+b^3)}{d\,(a^2+b^2)^2}+\frac{\mathrm{e}^{c+d\,x}\,(a^3+a\,b^2)}{d\,(a^2+b^2)^2}}{\mathrm{e}^{2\,c+2\,d\,x}+1}-\frac{a\,\ln\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}{2\,\left(-1\mathrm{i}\,d\,a^2+2\,d\,a\,b+1\mathrm{i}\,d\,b^2\right)}$$
$$+\frac{a^2\,b\,\ln\left(2\,a^7\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-a^2\,b^5-14\,a^4\,b^3-a^6\,b+a^6\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+2\,a^3\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+28\,a^5\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+a^2\,b^5\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right.}{d\,a^4+2\,d\,a^2\,b^2+d\,b^4}$$
$$-\frac{a\,\ln\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(-d\,a^2+2\mathrm{i}\,d\,a\,b+d\,b^2\right)}$$

input
```
int(tanh(c + d*x)^2/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c + d*x))/(d*(a^2 + b^2)))/(2*exp(2*c +
2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(a^2*b + b^3))/(d*(a^2 + b^2)^2) + (e
xp(c + d*x)*(a*b^2 + a^3))/(d*(a^2 + b^2)^2))/(exp(2*c + 2*d*x) + 1) - (a*
log(exp(c + d*x) + 1i)*1i)/(2*(b^2*d - a^2*d + a*b*d*2i)) - (a*log(exp(c +
 d*x)*1i + 1))/(2*(b^2*d*1i - a^2*d*1i + 2*a*b*d)) + (a^2*b*log(2*a^7*exp(
d*x)*exp(c) - a^2*b^5 - 14*a^4*b^3 - a^6*b + a^6*b*exp(2*c)*exp(2*d*x) + 2
*a^3*b^4*exp(d*x)*exp(c) + 28*a^5*b^2*exp(d*x)*exp(c) + a^2*b^5*exp(2*c)*e
xp(2*d*x) + 14*a^4*b^3*exp(2*c)*exp(2*d*x)))/(a^4*d + b^4*d + 2*a^2*b^2*d)
```

3.390 $\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.390.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.390.2 Mathematica [N/A]

Not integrable

Time = 68.83 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Sech[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Sech[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

3.390.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

$$\Big\downarrow \ 6125$$

$$\int \frac{\tanh^2(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Sech[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.390.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.390.4 Maple [N/A] (verified)

Not integrable

Time = 0.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}\left(dx+c\right)\tanh\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.390.5 Fricas [N/A]

Not integrable

Time = 4.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)\tanh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(sech(d*x + c)*tanh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

3.390.6 Sympy [N/A]

Not integrable

Time = 2.91 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh^2(c+dx)\operatorname{sech}(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)**2*sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)),
 x)
```

3.390.7 Maxima [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 1102, normalized size of antiderivative = 32.41

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(dx+c)\tanh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
(b*f - (a*d*f*x*e^(3*c) + (d*e - f)*a*e^(3*c))*e^(3*d*x) - (2*b*d*f*x*e^(2
*c) + (2*d*e - f)*b*e^(2*c))*e^(2*d*x) + (a*d*f*x*e^c + (d*e + f)*a*e^c)*e
^(d*x))/(a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(
a^2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^(4*c) + b^2*d^2*e^2*e^(4*c)
+ (a^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*e*f*e^(4*c)
 + b^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + 2*(a^2*d^2*e^2*e^(2*c) + b^2*d^2*e^
2*e^(2*c) + (a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*e
*f*e^(2*c) + b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) + 2*integrate(1/2*(2*a^2*b
*d^2*f^2*x^2 + 4*a^2*b*d^2*e*f*x + 2*b^3*f^2 + 2*(d^2*e^2 + f^2)*a^2*b + (
(d^2*e^2 + 2*f^2)*a^3*e^c - (d^2*e^2 - 2*f^2)*a*b^2*e^c + (a^3*d^2*f^2*e^c
 - a*b^2*d^2*f^2*e^c)*x^2 + 2*(a^3*d^2*e*f*e^c - a*b^2*d^2*e*f*e^c)*x)*e^(
d*x))/(a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^
2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2
+ b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^
2*f)*x + (a^4*d^2*e^3*e^(2*c) + 2*a^2*b^2*d^2*e^3*e^(2*c) + b^4*d^2*e^3*e^
(2*c) + (a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^(
2*c))*x^3 + 3*(a^4*d^2*e*f^2*e^(2*c) + 2*a^2*b^2*d^2*e*f^2*e^(2*c) + b^4*d
^2*e*f^2*e^(2*c))*x^2 + 3*(a^4*d^2*e^2*f*e^(2*c) + 2*a^2*b^2*d^2*e^2*f*e^(
2*c) + b^4*d^2*e^2*f*e^(2*c))*x)*e^(2*d*x)), x) - 2*integrate(-(a^3*b*e^(d
*x + c) - a^2*b^2)/(a^4*b*e + 2*a^2*b^3*e + b^5*e + (a^4*b*f + 2*a^2*b^...
```

3.390.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.390.9 Mupad [N/A]

Not integrable

Time = 3.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+d\,x)^2}{\cosh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(tanh(c + d*x)^2/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(tanh(c + d*x)^2/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.391 $\int \frac{(e+fx)^3 \cosh(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)} \, dx$

3.391.1 Optimal result

Integrand size = 34, antiderivative size = 792

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} - \frac{6a^2f^3\cosh(c+dx)}{b^3d^4} + \frac{14f^3\cosh(c+dx)}{9bd^4}$$
$$- \frac{3a^2f(e+fx)^2\cosh(c+dx)}{b^3d^2} + \frac{2f(e+fx)^2\cosh(c+dx)}{3bd^2} - \frac{2f^3\cosh^3(c+dx)}{27bd^4}$$
$$- \frac{a^3(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d} - \frac{a^3(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d}$$
$$- \frac{3a^3f(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2} - \frac{3a^3f(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}$$
$$+ \frac{6a^3f^2(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3} + \frac{6a^3f^2(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^3}$$
$$- \frac{6a^3f^3\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^4} - \frac{6a^3f^3\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^4}$$
$$+ \frac{6a^2f^2(e+fx)\sinh(c+dx)}{b^3d^3} - \frac{4f^2(e+fx)\sinh(c+dx)}{3bd^3}$$
$$+ \frac{a^2(e+fx)^3\sinh(c+dx)}{b^3d} + \frac{3af^3\cosh(c+dx)\sinh(c+dx)}{8b^2d^4}$$
$$+ \frac{3af(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{4b^2d^2} - \frac{3af^2(e+fx)\sinh^2(c+dx)}{4b^2d^3}$$
$$- \frac{a(e+fx)^3\sinh^2(c+dx)}{2b^2d} - \frac{f(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{3bd^2}$$
$$+ \frac{2f^2(e+fx)\sinh^3(c+dx)}{9bd^3} + \frac{(e+fx)^3\sinh^3(c+dx)}{3bd}$$

output
```
-1/3*f*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/b/d^2-3*a^3*f*(f*x+e)^2*polylog
(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^2-3*a^3*f*(f*x+e)^2*polylog(2,
-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^2+6*a^3*f^2*(f*x+e)*polylog(3,-b*
exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^3+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp
(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^3-3*a^2*f*(f*x+e)^2*cosh(d*x+c)/b^3/d^2
+2/3*f*(f*x+e)^2*cosh(d*x+c)/b/d^2-4/3*f^2*(f*x+e)*sinh(d*x+c)/b/d^3-3/8*a
*f^3*x/b^2/d^3-6*a^2*f^3*cosh(d*x+c)/b^3/d^4-1/2*a*(f*x+e)^3*sinh(d*x+c)^2
/b^2/d+1/3*(f*x+e)^3*sinh(d*x+c)^3/b/d+a^2*(f*x+e)^3*sinh(d*x+c)/b^3/d-a^3
*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d-a^3*(f*x+e)^3*ln(1
+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d+1/4*a^3*(f*x+e)^4/b^4/f+2/9*f^2*(
f*x+e)*sinh(d*x+c)^3/b/d^3-6*a^3*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))/b^4/d^4-6*a^3*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4
/d^4-1/4*a*(f*x+e)^3/b^2/d-2/27*f^3*cosh(d*x+c)^3/b/d^4+3/4*a*f*(f*x+e)^2*
cosh(d*x+c)*sinh(d*x+c)/b^2/d^2+14/9*f^3*cosh(d*x+c)/b/d^4+6*a^2*f^2*(f*x+
e)*sinh(d*x+c)/b^3/d^3+3/8*a*f^3*cosh(d*x+c)*sinh(d*x+c)/b^2/d^4-3/4*a*f^2
*(f*x+e)*sinh(d*x+c)^2/b^2/d^3
```

3.391.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5656 vs. 2(792) = 1584.

Time = 28.49 (sec) , antiderivative size = 5656, normalized size of antiderivative = 7.14

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]
),x]
```

output
```
Result too large to show
```

3.391.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh^3(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)^3\cosh(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5969}$$

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{f\int (e+fx)^2\sinh^3(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{f\int i(e+fx)^2\sin(ic+idx)^3dx}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\int (e+fx)^2\sin(ic+idx)^3dx}{d}}{b}$$

$$\Big\downarrow \text{3792}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2f^2\int -i\sinh^3(c+dx)dx}{9d^2}+\frac{2}{3}\int i(e+fx)^2\sinh(c+dx)dx+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)^2\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(-\frac{2if^2\int \sinh^3(c+dx)dx}{9d^2}+\frac{2}{3}i\int (e+fx)^2\sinh(c+dx)dx+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)^2\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(-\frac{2if^2\int i\sin(ic+idx)^3dx}{9d^2}+\frac{2}{3}i\int -i(e+fx)^2\sin(ic+idx)dx+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)^2\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2f^2\int \sin(ic+idx)^3dx}{9d^2}+\frac{2}{3}\int (e+fx)^2\sin(ic+idx)dx+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)^2\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{d}}{b}$$

3.391. $\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}dx$

$$\Big\downarrow \text{ 3113}$$

$$-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\Bigg(\frac{2if^2\int\left(1-\cosh^2(c+dx)\right)d\cosh(c+dx)}{9d^3}+\frac{2}{3}\int(e+fx)^2\sin(ic+idx)dx+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)^2\sinh^2(c+dx)\cosh(c+}{3d}}{d}}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\Bigg(\frac{2}{3}\int(e+fx)^2\sin(ic+idx)dx+\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)^2\sinh^2(c+dx)\cosh(c+dx}{3d}}{d}}{b}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\Bigg(\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right)+\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}-i(e+f}{d}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\Bigg(\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)+\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}-i}{d}}{b}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\Bigg(\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)}{d}\right)+\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+2if(e+f}{d}}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\Bigg(\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{d}\right)+\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\,s}{9d}}{d}}{b}$$

3.391. $\int\frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)+\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+f\cdots}{\cdots}\right.}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)+\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\cdots}{9\cdots}\right.}{d}}{b}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+\cdots}{d^2}\right.}{d}\right.\right.}{d}}{b}$$

$$\Big\downarrow \text{6113}$$

$$-\frac{a\left(\frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+\cdots}{d^2}\right.}{d}\right.\right.}{d}}{b}$$

$$\Big\downarrow \text{5969}$$

$$-\frac{a\left(\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}-\frac{3f\int (e+fx)^2\sinh^2(c+dx)dx}{2d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+\cdots}{d^2}\right.}{d}\right.\right.}{d}}{b}$$

$$\Big\downarrow \text{3042}$$

3.391. $\displaystyle\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+}{d^2}\right.}{d}\right.\right.}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}-\frac{3f\int-(e+fx)^2\sin(ic+idx)^2dx}{2d}}{b}\right)}{b}$$

$\downarrow$ 25

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+}{d^2}\right.}{d}\right.\right.}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\int(e+fx)^2\sin(ic+idx)^2dx}{2d}}{b}\right)}{b}$$

$\downarrow$ 3792

$$-\frac{a\left(\frac{\frac{3f\left(\frac{f^2\int-\sinh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^2dx+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+d}{a+b\sinh(c}}{b}\right.}{b}$$

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+}{d^2}\right.}{d}\right.\right.}{d}}{b}$$

$\downarrow$ 17

$$-\frac{a\left(\frac{\frac{3f\left(\frac{f^2\int-\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sin}{a+b\sinh(c+dx)}}{b}\right.}{b}$$

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+}{d^2}\right.}{d}\right.\right.}{d}}{b}$$

$\downarrow$ 25

3.391. $\int\frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\frac{3f\left(-\frac{f^2\int\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh}{a+b\sinh(c+dx)}}{b}\right.}{b}$$

$$\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+}{d^2}\right.}{d}\right.\right.}{d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+}{d^2}\right.}{d}\right.\right.}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\left(-\frac{f^2\int-\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right.}{2d}}{b}\right.}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+}{d^2}\right.}{d}\right.\right.}{d}}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\sinh^2(c+dx)}{2d}+\frac{3f\left(\frac{f^2\int\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e}{}\right.}{2d}}{b}\right.}{b}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{a\left(\frac{\frac{3f\left(\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3}{}}{}\right.}{b}$$

$$\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+}{d^2}\right.}{d}\right.\right.}{d}}{b}$$

$$\Big\downarrow \text{24}$$

3.391. $\int\frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cos\cdots}{a+b\cdots}\cdots}{\cdots}\cdots\right.}{b}\cdots$$

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+\cdots}{d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{b}\cdots$$

$$\Big\downarrow \; 6113$$

$$-\frac{a\left(\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\int(e+fx)^3\cos\cdots}{b\cdots}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots$$

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+\cdots}{d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{b}\cdots$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(e+fx)^3\sinh^3(c+dx)}{3d}-\frac{if\left(\frac{2if^2\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{9d^3}+\frac{2if(e+fx)\sinh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+\cdots}{d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{b}\cdots$$

$$\frac{a\left(\frac{\frac{3f\left(\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}+\frac{(e+fx)^3\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cdots}{a+b\sin\cdots}\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.391. $\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.391.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.391. $\int \frac{(e+fx)^3 \cosh(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)} dx$

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.391.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)\sinh(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.391.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7020 vs. $2(738) = 1476$.

Time = 0.36 (sec) , antiderivative size = 7020, normalized size of antiderivative = 8.86

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.391.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.391.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/24*e^3*(24*(d*x + c)*a^3/(b^4*d) + 24*a^3*log(-2*a*e^(-d*x - c) + b*e^(
-2*d*x - 2*c) - b)/(b^4*d) + (3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 - b^2)*e
^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e
^(-3*d*x - 3*c) + 3*(4*a^2 - b^2)*e^(-d*x - c))/(b^3*d)) - 1/864*(216*a^3*
d^4*f^3*x^4*e^(3*c) + 864*a^3*d^4*e*f^2*x^3*e^(3*c) + 1296*a^3*d^4*e^2*f*x
^2*e^(3*c) - 4*(9*b^3*d^3*f^3*x^3*e^(6*c) + 9*(3*d^3*e*f^2 - d^2*f^3)*b^3*
x^2*e^(6*c) + 3*(9*d^3*e^2*f - 6*d^2*e*f^2 + 2*d*f^3)*b^3*x*e^(6*c) - (9*d
^2*e^2*f - 6*d*e*f^2 + 2*f^3)*b^3*e^(6*c))*e^(3*d*x) + 27*(4*a*b^2*d^3*f^3
*x^3*e^(5*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*a*b^2*x^2*e^(5*c) + 6*(2*d^3*e^2*
f - 2*d^2*e*f^2 + d*f^3)*a*b^2*x*e^(5*c) - 3*(2*d^2*e^2*f - 2*d*e*f^2 + f^
3)*a*b^2*e^(5*c))*e^(2*d*x) + 108*(12*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a^2*
b*e^(4*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b^3*e^(4*c) - (4*a^2*b*d^3*f
^3*e^(4*c) - b^3*d^3*f^3*e^(4*c))*x^3 - 3*(4*(d^3*e*f^2 - d^2*f^3)*a^2*b*e
^(4*c) - (d^3*e*f^2 - d^2*f^3)*b^3*e^(4*c))*x^2 - 3*(4*(d^3*e^2*f - 2*d^2*
e*f^2 + 2*d*f^3)*a^2*b*e^(4*c) - (d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b^3*e
^(4*c))*x)*e^(d*x) + 108*(12*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a^2*b*e^(2*c)
 - 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b^3*e^(2*c) + (4*a^2*b*d^3*f^3*e^(2*c
) - b^3*d^3*f^3*e^(2*c))*x^3 + 3*(4*(d^3*e*f^2 + d^2*f^3)*a^2*b*e^(2*c) -
(d^3*e*f^2 + d^2*f^3)*b^3*e^(2*c))*x^2 + 3*(4*(d^3*e^2*f + 2*d^2*e*f^2 + 2
*d*f^3)*a^2*b*e^(2*c) - (d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*b^3*e^(2*c)...
```

3.391.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh(dx+c)\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a),
 x)
```

3.391.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^3\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.392 $\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.392.1 Optimal result

Integrand size = 34, antiderivative size = 578

$$\begin{aligned}
&\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{aefx}{2b^2d}-\frac{af^2x^2}{4b^2d}+\frac{a^3(e+fx)^3}{3b^4f}-\frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2}+\frac{4f(e+fx)\cosh(c+dx)}{9bd^2} \\
&\quad -\frac{a^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{a^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d} \\
&\quad -\frac{2a^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{2a^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2} \\
&\quad +\frac{2a^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3}+\frac{2a^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^3} \\
&\quad +\frac{2a^2f^2\sinh(c+dx)}{b^3d^3}-\frac{4f^2\sinh(c+dx)}{9bd^3}+\frac{a^2(e+fx)^2\sinh(c+dx)}{b^3d} \\
&\quad +\frac{af(e+fx)\cosh(c+dx)\sinh(c+dx)}{2b^2d^2}-\frac{af^2\sinh^2(c+dx)}{4b^2d^3}-\frac{a(e+fx)^2\sinh^2(c+dx)}{2b^2d} \\
&\quad -\frac{2f(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{9bd^2}+\frac{2f^2\sinh^3(c+dx)}{27bd^3}+\frac{(e+fx)^2\sinh^3(c+dx)}{3bd}
\end{aligned}$$

output
```
-1/2*a*e*f*x/b^2/d-1/4*a*f^2*x^2/b^2/d+1/3*a^3*(f*x+e)^3/b^4/f-2*a^2*f*(f*
x+e)*cosh(d*x+c)/b^3/d^2+4/9*f*(f*x+e)*cosh(d*x+c)/b/d^2-a^3*(f*x+e)^2*ln(
1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d-a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/
(a+(a^2+b^2)^(1/2)))/b^4/d-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))/b^4/d^2-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/b^4/d^2+2*a^3*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^
4/d^3+2*a^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^3+2*a^2
*f^2*sinh(d*x+c)/b^3/d^3-4/9*f^2*sinh(d*x+c)/b/d^3+a^2*(f*x+e)^2*sinh(d*x+
c)/b^3/d+1/2*a*f*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b^2/d^2-1/4*a*f^2*sinh(d*
x+c)^2/b^2/d^3-1/2*a*(f*x+e)^2*sinh(d*x+c)^2/b^2/d-2/9*f*(f*x+e)*cosh(d*x+
c)*sinh(d*x+c)^2/b/d^2+2/27*f^2*sinh(d*x+c)^3/b/d^3+1/3*(f*x+e)^2*sinh(d*x
+c)^3/b/d
```

3.392.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1961 vs. $2(578) = 1156$.

Time = 11.41 (sec) , antiderivative size = 1961, normalized size of antiderivative = 3.39

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]
),x]
```

output
```
-1/12*(f^2*((4*a*x^3)/(-1 + E^(2*c)) - 2*a*x^3*Coth[c] - (6*a*b^2*(d^2*x^2
*Log[1 + ((a - Sqrt[a^2 + b^2])*E^(-c - d*x))/b] - 2*d*x*PolyLog[2, ((-a +
 Sqrt[a^2 + b^2])*E^(-c - d*x))/b] - 2*PolyLog[3, ((-a + Sqrt[a^2 + b^2])*
E^(-c - d*x))/b]))/(Sqrt[a^2 + b^2]*(-a + Sqrt[a^2 + b^2])*d^3) - (6*a*b^2
*(d^2*x^2*Log[1 + ((a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b] - 2*d*x*PolyLog[
2, -(((a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b)] - 2*PolyLog[3, -(((a + Sqrt[
a^2 + b^2])*E^(-c - d*x))/b)]))/(Sqrt[a^2 + b^2]*(a + Sqrt[a^2 + b^2])*d^3
) + (6*a^2*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d*x
*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*PolyLog[3, (b*E^(c
 + d*x))/(-a + Sqrt[a^2 + b^2])]))/(Sqrt[a^2 + b^2]*d^3) - (6*a^2*(d^2*x^2
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^3) + (6*b*Cosh[d*x]*(-2*d*x*Cosh[c]
 + (2 + d^2*x^2)*Sinh[c]))/d^3 + (6*b*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[
c])*Sinh[d*x])/d^3))/b^2 + (e^2*((a*Log[a + b*Sinh[c + d*x]])/b^2 - Sinh[c
 + d*x]/b))/(2*d) - (e*f*(-2*b*Cosh[c + d*x] - a*(2*c*(c + d*x) - (c + d*x
)^2 + 2*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*(c +
d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*Log[b - 2*a*E^(c
 + d*x) - b*E^(2*(c + d*x))] + 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2
 + b^2])] + 2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + 2...
```

3.392.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^3(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6113$$

$$\frac{\int (e+fx)^2\cosh(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 5969$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2f\int (e+fx)\sinh^3(c+dx)dx}{3d}}{b}-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2f\int i(e+fx)\sin(ic+idx)^3dx}{3d}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\int (e+fx)\sin(ic+idx)^3dx}{3d}}{b}$$

$$\Big\downarrow \; 3791$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\int i(e+fx)\sinh(c+dx)dx+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}i\int (e+fx)\sinh(c+dx)dx+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}i\int -i(e+fx)\sin(ic+idx)dx+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\int (e+fx)\sin(ic+idx)dx+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \; 3777$$

$$-\frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \; 3042$$

3.392. $\displaystyle \int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\Big\downarrow \textbf{3117}$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+$$

$$\Big\downarrow \textbf{6113}$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$\Big\downarrow \textbf{5969}$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int(e+fx)\sinh^2(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int-\left((e+fx)\sin(ic+idx)^2\right)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}+\frac{f\int(e+fx)\sin(ic+idx)^2dx}{d}}{b}\right)}{b}$$

3.392. $\displaystyle\int\frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 3791$$

$$-\frac{a\left(\frac{\frac{f\left(\frac{1}{2}\int(e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \; 17$$

$$-\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \; 6113$$

$$-\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\cosh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}+\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3777$$

3.392. $\int\frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int\cdots}{\cdots}}{b}\cdots\right.}{b}\cdots\right.}{b}$$

$$\Big\downarrow \ 26$$

$$-\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e\cdots}{\cdots}}{\cdots}\cdots\right.}{b}\cdots\right.}{b}$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -\cdots}{\cdots}}{b}\cdots\right.}{b}\cdots\right.}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(\cdots}{\cdots}}{b}\cdots\right.}{b}\cdots\right.}{b}$$

$$\Big\downarrow \ 3777$$

3.392. $\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\cdots\right)}{\cdots}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\cdots\right)}{\cdots}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 3117$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\cdots\right)}{\cdots}\right)}{b}\right)}{b}$$

$$\Big\downarrow \ 6095$$

3.392. $\displaystyle\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\right.}{b}\right.}{}\right.}{b}$$

$$\Big\downarrow \; 2620$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-$$

$$\frac{a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log}{}\right.}{}\right.}{}\right.}{}$$

$$\Big\downarrow \; 3011$$

3.392. $\int\frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-$$

$$a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(a-\frac{a\left(-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyL}}{}\right.}{bd}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{2720}$$

$$\frac{\frac{(e+fx)^2\sinh^3(c+dx)}{3d}-\frac{2if\left(\frac{2}{3}\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)+\frac{if\sinh^3(c+dx)}{9d^2}-\frac{i(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{3d}\right)}{3d}}{b}-$$

$$a\left(\frac{\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(a-\frac{a\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\right.}{bd}\right.}{}\right.}{}\right.$$

3.392. $\displaystyle\int\frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.392.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.392. $\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.392. $\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.392.4 Maple [F]

$$\int \frac{(fx+e)^2 \cosh (dx+c) \sinh (dx+c)^3}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.392.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4263 vs. $2(536) = 1072$.

Time = 0.33 (sec) , antiderivative size = 4263, normalized size of antiderivative = 7.38

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
-1/432*(18*b^3*d^2*f^2*x^2 + 18*b^3*d^2*e^2 - 2*(9*b^3*d^2*f^2*x^2 + 9*b^3
*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh
(d*x + c)^6 - 2*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f
^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*sinh(d*x + c)^6 + 12*b^3*d*e*f + 27*
(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*
a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c)^5 + 3*(18*a*b^2*d^2*f^2*x^2
+ 18*a*b^2*d^2*e^2 - 18*a*b^2*d*e*f + 9*a*b^2*f^2 + 18*(2*a*b^2*d^2*e*f -
a*b^2*d*f^2)*x - 4*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^
3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^5 +
4*b^3*f^2 - 54*((4*a^2*b - b^3)*d^2*f^2*x^2 + (4*a^2*b - b^3)*d^2*e^2 - 2*
(4*a^2*b - b^3)*d*e*f + 2*(4*a^2*b - b^3)*f^2 + 2*((4*a^2*b - b^3)*d^2*e*f
 - (4*a^2*b - b^3)*d*f^2)*x)*cosh(d*x + c)^4 - 3*(18*(4*a^2*b - b^3)*d^2*f
^2*x^2 + 18*(4*a^2*b - b^3)*d^2*e^2 - 36*(4*a^2*b - b^3)*d*e*f + 36*(4*a^2
*b - b^3)*f^2 + 10*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^
3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c)^2 + 36*((4*a^2*b -
b^3)*d^2*e*f - (4*a^2*b - b^3)*d*f^2)*x - 45*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^
2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*
x)*cosh(d*x + c))*sinh(d*x + c)^4 - 144*(a^3*d^3*f^2*x^3 + 3*a^3*d^3*e*f*x
^2 + 3*a^3*d^3*e^2*x + 6*a^3*c*d^2*e^2 - 6*a^3*c^2*d*e*f + 2*a^3*c^3*f^2)*
cosh(d*x + c)^3 - 2*(72*a^3*d^3*f^2*x^3 + 216*a^3*d^3*e*f*x^2 + 216*a^3...
```

3.392.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.392.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/24*e^2*(24*(d*x + c)*a^3/(b^4*d) + 24*a^3*log(-2*a*e^(-d*x - c) + b*e^(
-2*d*x - 2*c) - b)/(b^4*d) + (3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 - b^2)*e
^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e
^(-3*d*x - 3*c) + 3*(4*a^2 - b^2)*e^(-d*x - c))/(b^3*d)) - 1/432*(144*a^3*
d^3*f^2*x^3*e^(3*c) + 432*a^3*d^3*e*f*x^2*e^(3*c) - 2*(9*b^3*d^2*f^2*x^2*e
^(6*c) + 6*(3*d^2*e*f - d*f^2)*b^3*x*e^(6*c) - 2*(3*d*e*f - f^2)*b^3*e^(6*
c))*e^(3*d*x) + 27*(2*a*b^2*d^2*f^2*x^2*e^(5*c) + 2*(2*d^2*e*f - d*f^2)*a*
b^2*x*e^(5*c) - (2*d*e*f - f^2)*a*b^2*e^(5*c))*e^(2*d*x) + 54*(8*(d*e*f -
f^2)*a^2*b*e^(4*c) - 2*(d*e*f - f^2)*b^3*e^(4*c) - (4*a^2*b*d^2*f^2*e^(4*c
) - b^3*d^2*f^2*e^(4*c))*x^2 - 2*(4*(d^2*e*f - d*f^2)*a^2*b*e^(4*c) - (d^2
*e*f - d*f^2)*b^3*e^(4*c))*x)*e^(d*x) + 54*(8*(d*e*f + f^2)*a^2*b*e^(2*c)
- 2*(d*e*f + f^2)*b^3*e^(2*c) + (4*a^2*b*d^2*f^2*e^(2*c) - b^3*d^2*f^2*e^(
2*c))*x^2 + 2*(4*(d^2*e*f + d*f^2)*a^2*b*e^(2*c) - (d^2*e*f + d*f^2)*b^3*e
^(2*c))*x)*e^(-d*x) + 27*(2*a*b^2*d^2*f^2*x^2*e^c + 2*(2*d^2*e*f + d*f^2)*
a*b^2*x*e^c + (2*d*e*f + f^2)*a*b^2*e^c)*e^(-2*d*x) + 2*(9*b^3*d^2*f^2*x^2
 + 6*(3*d^2*e*f + d*f^2)*b^3*x + 2*(3*d*e*f + f^2)*b^3)*e^(-3*d*x))*e^(-3*
c)/(b^4*d^3) + integrate(-2*(a^3*b*f^2*x^2 + 2*a^3*b*e*f*x - (a^4*f^2*x^2*
e^c + 2*a^4*e*f*x*e^c)*e^(d*x))/(b^5*e^(2*d*x + 2*c) + 2*a*b^4*e^(d*x + c)
 - b^5), x)
```

3.392.8 Giac [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a),
 x)
```

3.392.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^3\,{(e+f\,x)}^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.393 $\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.393.1 Optimal result

Integrand size = 32, antiderivative size = 348

$$\begin{aligned}\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{afx}{4b^2d}+\frac{a^3(e+fx)^2}{2b^4f}-\frac{a^2f\cosh(c+dx)}{b^3d^2}+\frac{f\cosh(c+dx)}{3bd^2}-\frac{f\cosh^3(c+dx)}{9bd^2}\\&\quad-\frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d}\\&\quad-\frac{a^3f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{a^3f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}\\&\quad+\frac{a^2(e+fx)\sinh(c+dx)}{b^3d}+\frac{af\cosh(c+dx)\sinh(c+dx)}{4b^2d^2}\\&\quad-\frac{a(e+fx)\sinh^2(c+dx)}{2b^2d}+\frac{(e+fx)\sinh^3(c+dx)}{3bd}\end{aligned}$$

output
```
-1/4*a*f*x/b^2/d+1/2*a^3*(f*x+e)^2/b^4/f-a^2*f*cosh(d*x+c)/b^3/d^2+1/3*f*c
osh(d*x+c)/b/d^2-1/9*f*cosh(d*x+c)^3/b/d^2-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))/b^4/d-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)
))/b^4/d-a^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^2-a^3*f*
polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^2+a^2*(f*x+e)*sinh(d*x+
c)/b^3/d+1/4*a*f*cosh(d*x+c)*sinh(d*x+c)/b^2/d^2-1/2*a*(f*x+e)*sinh(d*x+c)
^2/b^2/d+1/3*(f*x+e)*sinh(d*x+c)^3/b/d
```

3.393.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.30

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{36a^3c^2f - 36a^3d^2fx^2 + 72a^2bf\cosh(c+dx) - 18b^3f\cosh(c+dx) + 18ab^2dfx\cosh(2(c+dx)) + 2b^3f}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),
x]
```

output
```
-1/72*(36*a^3*c^2*f - 36*a^3*d^2*f*x^2 + 72*a^2*b*f*Cosh[c + d*x] - 18*b^3
*f*Cosh[c + d*x] + 18*a*b^2*d*f*x*Cosh[2*(c + d*x)] + 2*b^3*f*Cosh[3*(c +
d*x)] + 72*a^3*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 72*a^3
*d*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 72*a^3*c*f*Log[1 +
 (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 72*a^3*d*f*x*Log[1 + (b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2])] - 72*a^3*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*
(c + d*x))] + 72*a^3*d*e*Log[a + b*Sinh[c + d*x]] + 72*a^3*f*PolyLog[2, (b
*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 72*a^3*f*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))] - 72*a^2*b*d*e*Sinh[c + d*x] - 72*a^2*b*d*f*x*
Sinh[c + d*x] + 18*b^3*d*f*x*Sinh[c + d*x] + 36*a*b^2*d*e*Sinh[c + d*x]^2
- 24*b^3*d*e*Sinh[c + d*x]^3 - 9*a*b^2*f*Sinh[2*(c + d*x)] - 6*b^3*d*f*x*S
inh[3*(c + d*x)])/(b^4*d^2)
```

3.393.3 Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.96, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.719$, Rules used = {6113, 5969, 3042, 26, 3113, 2009, 6113, 5969, 3042, 25, 3115, 24, 6113, 3042, 3777, 26, 3042, 26, 3118, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^3(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6113$$

$$\frac{\int (e+fx)\cosh(c+dx)\sinh^2(c+dx)dx}{b} - \frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5969}$$

$$\frac{\frac{(e+fx)\sinh^3(c+dx)}{3d} - \frac{f\int \sinh^3(c+dx)dx}{3d}}{b} - \frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b} + \frac{\frac{(e+fx)\sinh^3(c+dx)}{3d} - \frac{f\int i\sin(ic+idx)^3dx}{3d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b} + \frac{\frac{(e+fx)\sinh^3(c+dx)}{3d} - \frac{if\int \sin(ic+idx)^3dx}{3d}}{b}$$

$$\Big\downarrow \text{3113}$$

$$\frac{\frac{f\int (1-\cosh^2(c+dx))d\cosh(c+dx)}{3d^2} + \frac{(e+fx)\sinh^3(c+dx)}{3d}}{b} - \frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2} + \frac{(e+fx)\sinh^3(c+dx)}{3d}}{b} - \frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6113}$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2} + \frac{(e+fx)\sinh^3(c+dx)}{3d}}{b} - \frac{a\left(\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b} - \frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{5969}$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2} + \frac{(e+fx)\sinh^3(c+dx)}{3d}}{b} - \frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d} - \frac{f\int \sinh^2(c+dx)dx}{2d}}{b} - \frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2} + \frac{(e+fx)\sinh^3(c+dx)}{3d}}{b} - \frac{a\left(-\frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b} + \frac{\frac{(e+fx)\sinh^2(c+dx)}{2d} - \frac{f\int -\sin(ic+idx)^2dx}{2d}}{b}\right)}{b}$$

3.393. $\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 25$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\int\sin(ic+idx)^2dx}{2d}}{b}\right)}{b}$$

$$\Big\downarrow \; 3115$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{f\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\int\frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 6113$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\int(e+fx)\cosh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3777$$

3.393. $\displaystyle\int\frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3118$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b}-\frac{a\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}$$

3.393. $\displaystyle\int\frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}dx$

$$\Big\downarrow \text{6095}$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b} - \frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b} - \frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b} - \frac{a\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}d\right.}{b}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b} - \frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b} - \frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b} - \frac{a\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}\right.}{bd}\right.}{}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b} - \frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b} - \frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b} - \frac{a\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-}}{}\right.}{}\right.}{}\right.}{b}$$

$$\Big\downarrow \text{2838}$$

3.393. $\displaystyle\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\left(\cosh(c+dx)-\frac{1}{3}\cosh^3(c+dx)\right)}{3d^2}+\frac{(e+fx)\sinh^3(c+dx)}{3d}}{b} - \frac{a\left(\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b} - \frac{a\left(\frac{\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}}{b} - \frac{a\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}\right.}{b}\right)}{b}\right)}{b}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
((f*(Cosh[c + d*x] - Cosh[c + d*x]^3/3))/(3*d^2) + ((e + f*x)*Sinh[c + d*x
]^3)/(3*d))/b - (a*(-((a*(-((a*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/(b*d^2)))/b) + (-((f*Cosh[c + d*x])/d^2) + ((e + f*x)*S
inh[c + d*x])/d)/b))/b) + (((e + f*x)*Sinh[c + d*x]^2)/(2*d) + (f*(x/2 - (
Cosh[c + d*x]*Sinh[c + d*x])/(2*d)))/(2*d))/b))/b
```

3.393.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.393. $\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.393.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(322) = 644$.

Time = 28.60 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.93

method	result
risch	$\frac{a^3 f\,x^2}{2b^4}-\frac{a^3 ex}{b^4}+\frac{(3dfx+3de-f)\mathrm{e}^{3dx+3c}}{72b\,d^2}-\frac{a(2dfx+2de-f)\mathrm{e}^{2dx+2c}}{16b^2d^2}+\frac{(4a^2dfx-b^2dfx+4de\,a^2-b^2de-4a^2f+f\,b^2)\mathrm{e}^{dx+c}}{8b^3d^2}-$

input
```
int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
1/2*a^3*f*x^2/b^4-a^3*e*x/b^4+1/72*(3*d*f*x+3*d*e-f)/b/d^2*exp(3*d*x+3*c)-
1/16*a*(2*d*f*x+2*d*e-f)/b^2/d^2*exp(2*d*x+2*c)+1/8*(4*a^2*d*f*x-b^2*d*f*x
+4*a^2*d*e-b^2*d*e-4*a^2*f+b^2*f)/b^3/d^2*exp(d*x+c)-1/8*(4*a^2-b^2)*(d*f*
x+d*e+f)/b^3/d^2*exp(-d*x-c)-1/16*a*(2*d*f*x+2*d*e+f)/b^2/d^2*exp(-2*d*x-2
*c)-1/72*(3*d*f*x+3*d*e+f)/b/d^2*exp(-3*d*x-3*c)+2/d*a^3/b^4*e*ln(exp(d*x+
c))-2/d^2*a^3/b^4*c*f*ln(exp(d*x+c))+1/d^2*a^3/b^4*c*f*ln(b*exp(2*d*x+2*c)
+2*a*exp(d*x+c)-b)+1/d^2*a^3/b^4*f*c^2-1/d^2*a^3/b^4*f*ln((-b*exp(d*x+c)+(
a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2*a^3/b^4*f*ln((b*exp(d*x+c)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d*a^3/b^4*f*ln((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d*a^3/b^4*f*ln((b*exp(d*x+c)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a^3/b^4*f*dilog((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*a^3/b^4*f*dilog((b*ex
p(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d*a^3/b^4*c*f*x-1/d*a^3
/b^4*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)
```

3.393.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2129 vs. $2(320) = 640$.

Time = 0.30 (sec) , antiderivative size = 2129, normalized size of antiderivative = 6.12

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
1/144*(2*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^6 + 2*(3*b^3*d*f*
x + 3*b^3*d*e - b^3*f)*sinh(d*x + c)^6 - 6*b^3*d*f*x - 9*(2*a*b^2*d*f*x +
2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^5 - 3*(6*a*b^2*d*f*x + 6*a*b^2*d*e -
3*a*b^2*f - 4*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c))*sinh(d*x +
c)^5 - 6*b^3*d*e + 18*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*e - (4*a^
2*b - b^3)*f)*cosh(d*x + c)^4 + 3*(6*(4*a^2*b - b^3)*d*f*x + 6*(4*a^2*b -
b^3)*d*e + 10*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^2 - 6*(4*a^2
*b - b^3)*f - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c))*si
nh(d*x + c)^4 - 2*b^3*f + 72*(a^3*d^2*f*x^2 + 2*a^3*d^2*e*x + 4*a^3*c*d*e
- 2*a^3*c^2*f)*cosh(d*x + c)^3 + 2*(36*a^3*d^2*f*x^2 + 72*a^3*d^2*e*x + 14
4*a^3*c*d*e - 72*a^3*c^2*f + 20*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x
 + c)^3 - 45*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^2 + 36*
((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*e - (4*a^2*b - b^3)*f)*cosh(d*x
 + c))*sinh(d*x + c)^3 - 18*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*e +
 (4*a^2*b - b^3)*f)*cosh(d*x + c)^2 + 6*(5*(3*b^3*d*f*x + 3*b^3*d*e - b^3*
f)*cosh(d*x + c)^4 - 3*(4*a^2*b - b^3)*d*f*x - 15*(2*a*b^2*d*f*x + 2*a*b^2
*d*e - a*b^2*f)*cosh(d*x + c)^3 - 3*(4*a^2*b - b^3)*d*e + 18*((4*a^2*b - b
^3)*d*f*x + (4*a^2*b - b^3)*d*e - (4*a^2*b - b^3)*f)*cosh(d*x + c)^2 - 3*(
4*a^2*b - b^3)*f + 36*(a^3*d^2*f*x^2 + 2*a^3*d^2*e*x + 4*a^3*c*d*e - 2*a^3
*c^2*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 9*(2*a*b^2*d*f*x + 2*a*b^2*d*e...
```

3.393.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.393.7 Maxima [F]

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/24*e*(24*(d*x + c)*a^3/(b^4*d) + 24*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2
*d*x - 2*c) - b)/(b^4*d) + (3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 - b^2)*e^(
-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(
-3*d*x - 3*c) + 3*(4*a^2 - b^2)*e^(-d*x - c))/(b^3*d)) - 1/144*f*((72*a^3*
d^2*x^2*e^(3*c) - 2*(3*b^3*d*x*e^(6*c) - b^3*e^(6*c))*e^(3*d*x) + 9*(2*a*b
^2*d*x*e^(5*c) - a*b^2*e^(5*c))*e^(2*d*x) + 18*(4*a^2*b*e^(4*c) - b^3*e^(4
*c) - (4*a^2*b*d*e^(4*c) - b^3*d*e^(4*c))*x)*e^(d*x) + 18*(4*a^2*b*e^(2*c)
 - b^3*e^(2*c) + (4*a^2*b*d*e^(2*c) - b^3*d*e^(2*c))*x)*e^(-d*x) + 9*(2*a*
b^2*d*x*e^c + a*b^2*e^c)*e^(-2*d*x) + 2*(3*b^3*d*x + b^3)*e^(-3*d*x))*e^(-
3*c)/(b^4*d^2) - 9*integrate(32*(a^4*x*e^(d*x + c) - a^3*b*x)/(b^5*e^(2*d*
x + 2*c) + 2*a*b^4*e^(d*x + c) - b^5), x))
```

3.393.8 Giac [F]

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x
)
```

3.393.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^3\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.394 $\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.394.1 Optimal result

Integrand size = 27, antiderivative size = 76

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a^3\log(a+b\sinh(c+dx))}{b^4d}+\frac{a^2\sinh(c+dx)}{b^3d} - \frac{a\sinh^2(c+dx)}{2b^2d}+\frac{\sinh^3(c+dx)}{3bd}$$

output
```
-a^3*ln(a+b*sinh(d*x+c))/b^4/d+a^2*sinh(d*x+c)/b^3/d-1/2*a*sinh(d*x+c)^2/b
^2/d+1/3*sinh(d*x+c)^3/b/d
```

3.394.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-6a^3\log(a+b\sinh(c+dx))+6a^2b\sinh(c+dx)-3ab^2\sinh^2(c+dx)+2b^3\sinh^3(c+dx)}{6b^4d}$$

input
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-6*a^3*Log[a + b*Sinh[c + d*x]] + 6*a^2*b*Sinh[c + d*x] - 3*a*b^2*Sinh[c
+ d*x]^2 + 2*b^3*Sinh[c + d*x]^3)/(6*b^4*d)
```

3.394.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 26, 3312, 26, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ic+idx)^3\cos(ic+idx)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos(ic+idx)\sin(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3312}$$

$$\frac{i\int -\frac{i\sinh^3(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^4d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(-\frac{a^3}{a+b\sinh(c+dx)}+a^2-b\sinh(c+dx)a+b^2\sinh^2(c+dx)\right)d(b\sinh(c+dx))}{b^4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(-\log(a+b\sinh(c+dx)))+a^2b\sinh(c+dx)-\frac{1}{2}ab^2\sinh^2(c+dx)+\frac{1}{3}b^3\sinh^3(c+dx)}{b^4d}$$

input
```
Int[(Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(a^3*Log[a + b*Sinh[c + d*x]]) + a^2*b*Sinh[c + d*x] - (a*b^2*Sinh[c + d
*x]^2)/2 + (b^3*Sinh[c + d*x]^3)/3)/(b^4*d)
```

3.394.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

3.394. $\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.394.4 Maple [A] (verified)

Time = 13.60 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{\frac{\sinh(dx+c)^3 b^2}{3}-\frac{a\sinh(dx+c)^2 b}{2}+a^2\sinh(dx+c)}{b^3}-\frac{a^3\ln(a+b\sinh(dx+c))}{b^4}}{d}$
default	$\frac{\frac{\frac{\sinh(dx+c)^3 b^2}{3}-\frac{a\sinh(dx+c)^2 b}{2}+a^2\sinh(dx+c)}{b^3}-\frac{a^3\ln(a+b\sinh(dx+c))}{b^4}}{d}$
risch	$\frac{a^3x}{b^4}+\frac{e^{3dx+3c}}{24bd}-\frac{a\,e^{2dx+2c}}{8b^2d}+\frac{e^{dx+c}a^2}{2b^3d}-\frac{e^{dx+c}}{8bd}-\frac{e^{-dx-c}a^2}{2b^3d}+\frac{e^{-dx-c}}{8bd}-\frac{a\,e^{-2dx-2c}}{8b^2d}-\frac{e^{-3dx-3c}}{24bd}+\frac{2a^3}{b^4}$

input
```
int(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^3*(1/3*sinh(d*x+c)^3*b^2-1/2*a*sinh(d*x+c)^2*b+a^2*sinh(d*x+c))-a
^3/b^4*ln(a+b*sinh(d*x+c)))
```

3.394.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(72) = 144.

Time = 0.27 (sec) , antiderivative size = 602, normalized size of antiderivative = 7.92

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{b^3\cosh(dx+c)^6+b^3\sinh(dx+c)^6+24\,a^3dx\cosh(dx+c)^3-3\,ab^2\cosh(dx+c)^5+3\,(2\,b^3\cosh(dx+c)}{}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/24*(b^3*cosh(d*x + c)^6 + b^3*sinh(d*x + c)^6 + 24*a^3*d*x*cosh(d*x + c)
^3 - 3*a*b^2*cosh(d*x + c)^5 + 3*(2*b^3*cosh(d*x + c) - a*b^2)*sinh(d*x +
c)^5 + 3*(4*a^2*b - b^3)*cosh(d*x + c)^4 + 3*(5*b^3*cosh(d*x + c)^2 - 5*a*
b^2*cosh(d*x + c) + 4*a^2*b - b^3)*sinh(d*x + c)^4 - 3*a*b^2*cosh(d*x + c)
 + 2*(10*b^3*cosh(d*x + c)^3 + 12*a^3*d*x - 15*a*b^2*cosh(d*x + c)^2 + 6*(
4*a^2*b - b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - b^3 - 3*(4*a^2*b - b^3)*co
sh(d*x + c)^2 + 3*(5*b^3*cosh(d*x + c)^4 + 24*a^3*d*x*cosh(d*x + c) - 10*a
*b^2*cosh(d*x + c)^3 - 4*a^2*b + b^3 + 6*(4*a^2*b - b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 - 24*(a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c)^2*sinh(d*x
 + c) + 3*a^3*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*sinh(d*x + c)^3)*log(2*(
b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 3*(2*b^3*cosh(d*x
+ c)^5 + 24*a^3*d*x*cosh(d*x + c)^2 - 5*a*b^2*cosh(d*x + c)^4 + 4*(4*a^2*b
 - b^3)*cosh(d*x + c)^3 - a*b^2 - 2*(4*a^2*b - b^3)*cosh(d*x + c))*sinh(d*
x + c))/(b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c)^2*sinh(d*x + c) + 3
*b^4*d*cosh(d*x + c)*sinh(d*x + c)^2 + b^4*d*sinh(d*x + c)^3)
```

3.394.6 Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.38

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \begin{cases} \frac{x\sinh^3(c)\cosh(c)}{a} & \text{for}\ b=0\wedge d=0 \\ \frac{\sinh^4(c+dx)}{4ad} & \text{for}\ b=0 \\ \frac{x\sinh^3(c)\cosh(c)}{a+b\sinh(c)} & \text{for}\ d=0 \\ -\frac{a^3\log\left(\frac{a}{b}+\sinh(c+dx)\right)}{b^4d}+\frac{a^2\sinh(c+dx)}{b^3d}-\frac{a\cosh^2(c+dx)}{2b^2d}+\frac{\sinh^3(c+dx)}{3bd} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Piecewise((x*sinh(c)**3*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)**4
/(4*a*d), Eq(b, 0)), (x*sinh(c)**3*cosh(c)/(a + b*sinh(c)), Eq(d, 0)), (-a
**3*log(a/b + sinh(c + d*x))/(b**4*d) + a**2*sinh(c + d*x)/(b**3*d) - a*co
sh(c + d*x)**2/(2*b**2*d) + sinh(c + d*x)**3/(3*b*d), True))
```

3.394.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(72) = 144$.

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.25

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(dx+c)a^3}{b^4d} - \frac{a^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{b^4d} - \frac{\left(3\,abe^{(-dx-c)}-b^2-3\,(4\,a^2-b^2)e^{(-2\,dx-2\,c)}\right)e^{(3\,dx+3\,c)}}{24\,b^3d} - \frac{3\,abe^{(-2\,dx-2\,c)}+b^2e^{(-3\,dx-3\,c)}+3\,(4\,a^2-b^2)e^{(-dx-c)}}{24\,b^3d}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-(d*x + c)*a^3/(b^4*d) - a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) -
b)/(b^4*d) - 1/24*(3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 - b^2)*e^(-2*d*x -
2*c))*e^(3*d*x + 3*c)/(b^3*d) - 1/24*(3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d
*x - 3*c) + 3*(4*a^2 - b^2)*e^(-d*x - c))/(b^3*d)
```

3.394.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.54

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{24\,a^3\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^4} - \frac{b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3-3\,ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+12\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{b^3}}{24\,d}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/24*(24*a^3*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/b^4 - (b^2*(e
^(d*x + c) - e^(-d*x - c))^3 - 3*a*b*(e^(d*x + c) - e^(-d*x - c))^2 + 12*a
^2*(e^(d*x + c) - e^(-d*x - c)))/b^3)/d
```

3.394.9 Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{a^3\ \ln\left(a+b\sinh(c+d\,x)\right)-\frac{b^3\sinh(c+d\,x)^3}{3}+\frac{a\,b^2\sinh(c+d\,x)^2}{2}-a^2\,b\sinh(c+d\,x)}{b^4\,d}$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
-(a^3*log(a + b*sinh(c + d*x)) - (b^3*sinh(c + d*x)^3)/3 + (a*b^2*sinh(c +
 d*x)^2)/2 - a^2*b*sinh(c + d*x))/(b^4*d)
```

3.395 $\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.395.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \mathrm{Int}\left(\frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.395.2 Mathematica [N/A]

Not integrable

Time = 35.62 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

3.395.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\sinh^3(c+dx)\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.395.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.395.4 Maple [N/A] (verified)

Not integrable

Time = 0.88 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(dx+c)\sinh(dx+c)^3}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.395.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)\sinh(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(cosh(d*x + c)*sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

3.395.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.395.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 326, normalized size of antiderivative = 9.59

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)\sinh(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/8*e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b*f) - 1/4*a*e
^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^2*f) + 1/4*a*e^(2*
c - 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^2*f) - 1/8*e^(3*c - 3*
d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b*f) - 1/8*(4*a^2 - b^2)*e^(-c
 + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^3*f) - 1/8*(4*a^2*e^c - b^2*
e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^3*f) - a^3*log(f*x +
e)/(b^4*f) + 1/16*integrate(-32*(a^4*e^(d*x + c) - a^3*b)/(b^5*f*x + b^5*e
 - (b^5*f*x*e^(2*c) + b^5*e*e^(2*c))*e^(2*d*x) - 2*(a*b^4*f*x*e^c + a*b^4*
e*e^c)*e^(d*x)), x)
```

3.395.8 Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)\sinh(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate(cosh(d*x + c)*sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a)),
 x)
```

3.395.9 Mupad [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^3}{(e+f\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((cosh(c + d*x)*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.396 $\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} \, dx$

3.396.1 Optimal result

Integrand size = 36, antiderivative size = 1038

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\begin{aligned}
&= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+fx)^4}{32bf} \\
&- \frac{6a^3f^2(e+fx)\cosh(c+dx)}{b^4d^3} - \frac{4af^2(e+fx)\cosh(c+dx)}{3b^2d^3} - \frac{a^3(e+fx)^3\cosh(c+dx)}{b^4d} \\
&- \frac{3a^2f^3\cosh^2(c+dx)}{8b^3d^4} - \frac{3a^2f(e+fx)^2\cosh^2(c+dx)}{4b^3d^2} - \frac{2af^2(e+fx)\cosh^3(c+dx)}{9b^2d^3} \\
&- \frac{a(e+fx)^3\cosh^3(c+dx)}{3b^2d} - \frac{3f^3\cosh(4c+4dx)}{1024bd^4} - \frac{3f(e+fx)^2\cosh(4c+4dx)}{128bd^2} \\
&- \frac{a^3\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d} + \frac{a^3\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d} \\
&- \frac{3a^3\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2} \\
&+ \frac{3a^3\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^2} \\
&+ \frac{6a^3\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^3} \\
&- \frac{6a^3\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^3} \\
&- \frac{6a^3\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^4} + \frac{6a^3\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^4} \\
&+ \frac{6a^3f^3\sinh(c+dx)}{b^4d^4} + \frac{14af^3\sinh(c+dx)}{9b^2d^4} + \frac{3a^3f(e+fx)^2\sinh(c+dx)}{b^4d^2} \\
&+ \frac{2af(e+fx)^2\sinh(c+dx)}{3b^2d^2} + \frac{3a^2f^2(e+fx)\cosh(c+dx)\sinh(c+dx)}{4b^3d^3} \\
&+ \frac{a^2(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{2b^3d} + \frac{af(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{3b^2d^2} \\
&+ \frac{2af^3\sinh^3(c+dx)}{27b^2d^4} + \frac{3f^2(e+fx)\sinh(4c+4dx)}{256bd^3} + \frac{(e+fx)^3\sinh(4c+4dx)}{32bd}
\end{aligned}$$

output
```
3/4*a^2*e*f^2*x/b^3/d^2-6*a^3*f^2*(f*x+e)*cosh(d*x+c)/b^4/d^3-1/32*(f*x+e)
^4/b/f-3/4*a^2*f*(f*x+e)^2*cosh(d*x+c)^2/b^3/d^2-2/9*a*f^2*(f*x+e)*cosh(d*
x+c)^3/b^2/d^3+3*a^3*f*(f*x+e)^2*sinh(d*x+c)/b^4/d^2+1/2*a^2*(f*x+e)^3*cos
h(d*x+c)*sinh(d*x+c)/b^3/d-6*a^3*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))*(a^2+b^2)^(1/2)/b^5/d^4+6*a^3*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+
b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d^4+14/9*a*f^3*sinh(d*x+c)/b^2/d^4+1/4*a^
4*(f*x+e)^4/b^5/f-3/1024*f^3*cosh(4*d*x+4*c)/b/d^4+1/32*(f*x+e)^3*sinh(4*d
*x+4*c)/b/d+3/8*a^2*f^3*x^2/b^3/d^2-3/8*a^2*f^3*cosh(d*x+c)^2/b^3/d^4-1/3*
a*(f*x+e)^3*cosh(d*x+c)^3/b^2/d-3/128*f*(f*x+e)^2*cosh(4*d*x+4*c)/b/d^2+6*
a^3*f^3*sinh(d*x+c)/b^4/d^4+2/27*a*f^3*sinh(d*x+c)^3/b^2/d^4+3/256*f^2*(f*
x+e)*sinh(4*d*x+4*c)/b/d^3+3/4*a^2*f^2*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b^3
/d^3+1/3*a*f*(f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/b^2/d^2-3*a^3*f*(f*x+e)^2
*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d^2+3*a^
3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)
/b^5/d^2+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a
^2+b^2)^(1/2)/b^5/d^3-6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d^3-a^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d+a^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d-a^3*(f*x+e)^3*cosh(d*x+c)/b^4/d+1/8*a
^2*(f*x+e)^4/b^3/f-4/3*a*f^2*(f*x+e)*cosh(d*x+c)/b^2/d^3+2/3*a*f*(f*x+e...
```

3.396.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5984 vs. $2(1038) = 2076$.

Time = 21.24 (sec) , antiderivative size = 5984, normalized size of antiderivative = 5.76

$$\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)} \, dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*
x]),x]
```

output
```
Result too large to show
```

3.396.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh^3(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\int \left(\frac{1}{8}(e+fx)^3\cosh(4c+4dx)-\frac{1}{8}(e+fx)^3\right)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6113}$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(\frac{\int (e+fx)^3\cosh^2(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{5970}$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\int (e+fx)^2\cosh^3(c+dx)dx}{d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3792}$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2f^2\int\cosh^3(c+dx)dx}{9d^2}+\frac{2}{3}\int(e+fx)^2\cosh(c+dx)dx-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{d}}{b}-\frac{a\int\frac{(e+fx)^3}{}}{}\right.}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2f^2\int\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{9d^2}+\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f(e+fx)\cosh^3(c+d}{9d^2}\right.}{d}}{b}\right.}{b}$$

$$\Big\downarrow \text{3113}$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\int\left(\sinh^2(c+dx)+1\right)d(-i\sinh(c+dx))}{9d^3}+\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f(e}{}\right.}{d}}{b}\right.}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+f}{}\right.}{d}}{b}\right.}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int-i(e+fx)\sinh(c+dx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx\right.}{9d^3}\right.}{d}}{b}\right.}{b}$$

3.396. $\displaystyle\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\cdots\right)}{9d^3}\cdots\right)}{d}}{b}\cdots\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int-i(e+fx)\sin(ic+idx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx\cdots\right)}{9d^3}\cdots\right)}{d}}{b}\cdots\right)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)+\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-\cdots\right)}{9d^3}\cdots\right)}{d}}{b}\cdots\right)}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)+\frac{2if\cdots}{\cdots}\right)\cdots}{\cdots}}{b}\cdots\right)}{b}$$

$$\Big\downarrow \; 3042$$

3.396. $\displaystyle\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)\right)}{b}\right)}{b}$$

$\downarrow$ 3117

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sin}{d}\right.\right.}{b}\right.}{b}$$

$\downarrow$ 6113

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^3\cosh^2(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c}{9d^2}\right.}{}\right.}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$\frac{a\left(\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+}{d^2}\right.}{d}\right.\right.}{b}\right.}{b}$$

$\downarrow$ 3792

3.396. $\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(-\frac{a\left(\frac{\frac{3f^2\int(e+fx)\cosh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^3dx-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{(e+fx\cdots}{\cdots}\cdots\right.$$

$\downarrow$ 17

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(-\frac{a\left(\frac{\frac{3f^2\int(e+fx)\cosh^2(c+dx)dx}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{(e+fx)^3\cos\cdots}{3d\cdots}\cdots\right.$$

$\downarrow$ 3042

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+\cdots}{d^2}\cdots\right.}{d}\cdots\right.\right.}{d}\cdots}{b}\cdots\right.$$

$\downarrow$ 3791

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(-\frac{a\left(\frac{\frac{3f^2\left(\frac{1}{2}\int(e+fx)dx-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int\frac{(e\cdots}{\cdots}}{\cdots}\cdots\right.}{b}\cdots\right.$$

$\downarrow$ 17

3.396. $\displaystyle\int\frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(-\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\int\frac{(e+}{a}}{}\right.}{b}\right.$$

$$\Big\downarrow \; 6099$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(-\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2-}{}\right.}{}\right.}{b}\right.$$

$$\Big\downarrow \; 17$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(-\frac{a\left(\frac{\frac{3f^2\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)}{2d^2}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^4}{8f}}{b}-\frac{a\left(\frac{(a^2-}{}\right.}{}\right.}{b}\right.$$

$$\Big\downarrow \; 3042$$

3.396. $\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+\cdots}{d^2}\cdots\right)}{d}\cdots\right)\cdots\right)}{d}\cdots}{b}\cdots\right.$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+\cdots}{d^2}\cdots\right)}{d}\cdots\right)\cdots\right)}{d}\cdots}{b}\cdots\right.$$

$$\Big\downarrow \; 3777$$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+\cdots}{d^2}\cdots\right)}{d}\cdots\right)\cdots\right)}{d}\cdots}{b}\cdots\right.$$

$$\Big\downarrow \; 3042$$

3.396. $\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{3f^3\cosh(4c+4dx)}{1024d^4}+\frac{3f^2(e+fx)\sinh(4c+4dx)}{256d^3}-\frac{3f(e+fx)^2\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)^3\sinh(4c+4dx)}{32d}-\frac{(e+fx)^4}{32f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^3(c+dx)}{3d}-\frac{f\left(\frac{2if^2\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{9d^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9d^2}+\frac{2}{3}\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+}{d^2}\right.}{d}\right.\right.}{d}}{b}\right.$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.396.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

3.396. $\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)} \, dx$

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.396.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)^2 \sinh(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.396.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10658 vs. 2(956) = 1912.

Time = 0.45 (sec) , antiderivative size = 10658, normalized size of antiderivative = 10.27

$$\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

3.396.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.396.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh(dx+c)^2\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
-1/192*e^3*(192*sqrt(a^2 + b^2)*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b
^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^5*d) + (8*a*b^2*e^(-d*x -
c) - 24*a^2*b*e^(-2*d*x - 2*c) - 3*b^3 + 24*(4*a^3 + a*b^2)*e^(-3*d*x - 3*
c))*e^(4*d*x + 4*c)/(b^4*d) - 24*(8*a^4 + 4*a^2*b^2 - b^4)*(d*x + c)/(b^5*
d) + (24*a^2*b*e^(-2*d*x - 2*c) + 8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-4*d
*x - 4*c) + 24*(4*a^3 + a*b^2)*e^(-d*x - c))/(b^4*d)) + 1/55296*(1728*(8*a
^4*d^4*f^3*e^(4*c) + 4*a^2*b^2*d^4*f^3*e^(4*c) - b^4*d^4*f^3*e^(4*c))*x^4
+ 6912*(8*a^4*d^4*e*f^2*e^(4*c) + 4*a^2*b^2*d^4*e*f^2*e^(4*c) - b^4*d^4*e*
f^2*e^(4*c))*x^3 + 10368*(8*a^4*d^4*e^2*f*e^(4*c) + 4*a^2*b^2*d^4*e^2*f*e^
(4*c) - b^4*d^4*e^2*f*e^(4*c))*x^2 + 27*(32*b^4*d^3*f^3*x^3*e^(8*c) + 24*(
4*d^3*e*f^2 - d^2*f^3)*b^4*x^2*e^(8*c) + 12*(8*d^3*e^2*f - 4*d^2*e*f^2 + d
*f^3)*b^4*x*e^(8*c) - 3*(8*d^2*e^2*f - 4*d*e*f^2 + f^3)*b^4*e^(8*c))*e^(4*
d*x) - 256*(9*a*b^3*d^3*f^3*x^3*e^(7*c) + 9*(3*d^3*e*f^2 - d^2*f^3)*a*b^3*
x^2*e^(7*c) + 3*(9*d^3*e^2*f - 6*d^2*e*f^2 + 2*d*f^3)*a*b^3*x*e^(7*c) - (9
*d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*a*b^3*e^(7*c))*e^(3*d*x) + 1728*(4*a^2*b^2
*d^3*f^3*x^3*e^(6*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*a^2*b^2*x^2*e^(6*c) + 6*(
2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*a^2*b^2*x*e^(6*c) - 3*(2*d^2*e^2*f - 2*
d*e*f^2 + f^3)*a^2*b^2*e^(6*c))*e^(2*d*x) + 6912*(12*(d^2*e^2*f - 2*d*e*f^
2 + 2*f^3)*a^3*b*e^(5*c) + 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b^3*e^(5*c)
 - (4*a^3*b*d^3*f^3*e^(5*c) + a*b^3*d^3*f^3*e^(5*c))*x^3 - 3*(4*(d^3*e*...
```

3.396.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a
), x)
```

3.396.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\mathrm{cosh}(c+d\,x)^2\,\mathrm{sinh}(c+d\,x)^3\,{(e+f\,x)}^3}{a+b\,\mathrm{sinh}\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x
)
```

3.397 $\int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.397.1 Optimal result

Integrand size = 36, antiderivative size = 755

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\begin{aligned}
&= \frac{a^2f^2x}{4b^3d^2}+\frac{a^4(e+fx)^3}{3b^5f}+\frac{a^2(e+fx)^3}{6b^3f}-\frac{(e+fx)^3}{24bf}-\frac{2a^3f^2\cosh(c+dx)}{b^4d^3}\\
&-\frac{4af^2\cosh(c+dx)}{9b^2d^3}-\frac{a^3(e+fx)^2\cosh(c+dx)}{b^4d}-\frac{a^2f(e+fx)\cosh^2(c+dx)}{2b^3d^2}\\
&-\frac{2af^2\cosh^3(c+dx)}{27b^2d^3}-\frac{a(e+fx)^2\cosh^3(c+dx)}{3b^2d}-\frac{f(e+fx)\cosh(4c+4dx)}{64bd^2}\\
&-\frac{a^3\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d}+\frac{a^3\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d}\\
&-\frac{2a^3\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2}\\
&+\frac{2a^3\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^2}\\
&+\frac{2a^3\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^3}\\
&-\frac{2a^3\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^3}+\frac{2a^3f(e+fx)\sinh(c+dx)}{b^4d^2}\\
&+\frac{4af(e+fx)\sinh(c+dx)}{9b^2d^2}+\frac{a^2f^2\cosh(c+dx)\sinh(c+dx)}{4b^3d^3}\\
&+\frac{a^2(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{2b^3d}+\frac{2af(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{9b^2d^2}\\
&+\frac{f^2\sinh(4c+4dx)}{256bd^3}+\frac{(e+fx)^2\sinh(4c+4dx)}{32bd}
\end{aligned}$$

output
```
1/4*a^2*f^2*x/b^3/d^2+1/3*a^4*(f*x+e)^3/b^5/f+1/6*a^2*(f*x+e)^3/b^3/f-1/24
*(f*x+e)^3/b/f-2*a^3*f^2*cosh(d*x+c)/b^4/d^3-4/9*a*f^2*cosh(d*x+c)/b^2/d^3
-a^3*(f*x+e)^2*cosh(d*x+c)/b^4/d-1/2*a^2*f*(f*x+e)*cosh(d*x+c)^2/b^3/d^2-2
/27*a*f^2*cosh(d*x+c)^3/b^2/d^3-1/3*a*(f*x+e)^2*cosh(d*x+c)^3/b^2/d-1/64*f
*(f*x+e)*cosh(4*d*x+4*c)/b/d^2+2*a^3*f*(f*x+e)*sinh(d*x+c)/b^4/d^2+4/9*a*f
*(f*x+e)*sinh(d*x+c)/b^2/d^2+1/4*a^2*f^2*cosh(d*x+c)*sinh(d*x+c)/b^3/d^3+1
/2*a^2*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b^3/d+2/9*a*f*(f*x+e)*cosh(d*x+c)
^2*sinh(d*x+c)/b^2/d^2+1/256*f^2*sinh(4*d*x+4*c)/b/d^3+1/32*(f*x+e)^2*sinh
(4*d*x+4*c)/b/d-a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+
b^2)^(1/2)/b^5/d+a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2
+b^2)^(1/2)/b^5/d-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))*(a^2+b^2)^(1/2)/b^5/d^2+2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a
^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d^2+2*a^3*f^2*polylog(3,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d^3-2*a^3*f^2*polylog(3,-b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d^3
```

3.397.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3579 vs. $2(755) = 1510$.

Time = 13.14 (sec) , antiderivative size = 3579, normalized size of antiderivative = 4.74

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*
x]),x]
```

output
```
-1/8*(e^2*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2
]])/(Sqrt[-a^2 - b^2]*d)))/b - (e*f*(x^2 - (2*a*(d*x*(Log[1 + (b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2])] - Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
]) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^2)))/(8*b) - (f^
2*(x^3 - (3*a*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^
2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b
*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b
^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2
 + b^2]*d^3)))/(24*b) - (f^2*(2*(4*a^2 + b^2)*x^3 - (6*a*(4*a^2 + 3*b^2)*(
d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*x^2*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*E^(c + d*x))/(
-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*PolyL
og[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^3) -
(24*a*b*Cosh[d*x]*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c]))/d^3 + (3*b^2*Co
sh[2*d*x]*(-2*d*x*Cosh[2*c] + (1 + 2*d^2*x^2)*Sinh[2*c]))/d^3 - (24*a*b*(-
2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*((1 + 2*d^2
*x^2)*Cosh[2*c] - 2*d*x*Sinh[2*c])*Sinh[2*d*x])/d^3))/(96*b^3) - (e^2*(...
```

3.397.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 \sinh^3(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\int \left(\frac{1}{8}(e+fx)^2\cosh(4c+4dx)-\frac{1}{8}(e+fx)^2\right)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6113}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\int(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{5970}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\int(e+fx)\cosh^3(c+dx)dx}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\int(e+fx)\cosh(c+dx)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right.}{3d}}{b}\right.}{b}$$

3.397. $\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}dx$

$$\Big\downarrow \text{3777}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c\cdots}{}\right.}{3d}}{b}\cdots\right.}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\,s\cdots}{a+b\sinh(c+d\cdots}}{b}\cdots\right.}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c\cdots}{}\right.}{3d}}{b}\cdots\right.}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+d\cdots}{3d\cdots}\right.}{3d}}{b}\cdots\right.}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh(\cdots}{a+b\sinh(c+dx)}}{b}\cdots\right.}{b}$$

$$\Big\downarrow \text{6113}$$

3.397. $\displaystyle\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\cosh^2(c+dx)dx}{b}-\right.}{}\right.}{b}$$

$$\downarrow \text{3042}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}}{b}\right.}{}\right.}{b}$$

$$\downarrow \text{3792}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f^2\int\cosh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(\right.}{}\right.}{b}$$

$$\downarrow \text{17}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{\frac{f^2\int\cosh^2(c+dx)dx}{2d^2}-\frac{f(e}{}\right.}{}\right.}{b}$$

$$\downarrow \text{3042}$$

3.397. $\displaystyle\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}}{b}\cdots\right.}{\cdots}\right.}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{f^2\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cos\cdots}{2d}\right.}{2d^2}\cdots\right.}{\cdots}\right.}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\cdots\right.}{\cdots}\right.}{b}$$

$$\Big\downarrow \text{6099}$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\cdots\right.}{\cdots}\right.}{\cdots}$$

3.397. $\int\frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\downarrow 17$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\right.}{}\right.}{b}$$

$$\downarrow 3042$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\right.}{}\right.}{}$$

$$\downarrow 26$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\right.}{}\right.}{}$$

$$\downarrow 3777$$

3.397. $\displaystyle\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\cdots\right.}{\cdots}\right.$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\cdots\right.}{\cdots}\right.$$

$$\Big\downarrow \; 3777$$

3.397. $\displaystyle\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}$$

$$\Big\downarrow \; 3042$$

3.397. $\displaystyle\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\cdots}{\cdots}\right)}{\cdots}\right)$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{f^2\sinh(4c+4dx)}{256d^3}-\frac{f(e+fx)\cosh(4c+4dx)}{64d^2}+\frac{(e+fx)^2\sinh(4c+4dx)}{32d}-\frac{(e+fx)^3}{24f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^3(c+dx)}{3d}-\frac{2f\left(\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}\right)}{3d}}{b}-\frac{a\left(\frac{-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\cdots}{\cdots}\right)}{\cdots}\right)$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

3.397. $\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
$Aborted
```

3.397.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.397. $\int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} \, dx$

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.397. $\int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} \, dx$

3.397.4 Maple [F]

$$\int \frac{(fx+e)^2 \cosh(dx+c)^2 \sinh(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.397.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6459 vs. $2(693) = 1386$.

Time = 0.37 (sec) , antiderivative size = 6459, normalized size of antiderivative = 8.55

$$\int \frac{(e+fx)^2 \cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

3.397.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.397.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^2\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
-1/192*e^2*(192*sqrt(a^2 + b^2)*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b
^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^5*d) + (8*a*b^2*e^(-d*x -
c) - 24*a^2*b*e^(-2*d*x - 2*c) - 3*b^3 + 24*(4*a^3 + a*b^2)*e^(-3*d*x - 3*
c))*e^(4*d*x + 4*c)/(b^4*d) - 24*(8*a^4 + 4*a^2*b^2 - b^4)*(d*x + c)/(b^5*
d) + (24*a^2*b*e^(-2*d*x - 2*c) + 8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-4*d
*x - 4*c) + 24*(4*a^3 + a*b^2)*e^(-d*x - c))/(b^4*d)) + 1/13824*(576*(8*a^
4*d^3*f^2*e^(4*c) + 4*a^2*b^2*d^3*f^2*e^(4*c) - b^4*d^3*f^2*e^(4*c))*x^3 +
 1728*(8*a^4*d^3*e*f*e^(4*c) + 4*a^2*b^2*d^3*e*f*e^(4*c) - b^4*d^3*e*f*e^(
4*c))*x^2 + 27*(8*b^4*d^2*f^2*x^2*e^(8*c) + 4*(4*d^2*e*f - d*f^2)*b^4*x*e^
(8*c) - (4*d*e*f - f^2)*b^4*e^(8*c))*e^(4*d*x) - 64*(9*a*b^3*d^2*f^2*x^2*e
^(7*c) + 6*(3*d^2*e*f - d*f^2)*a*b^3*x*e^(7*c) - 2*(3*d*e*f - f^2)*a*b^3*e
^(7*c))*e^(3*d*x) + 864*(2*a^2*b^2*d^2*f^2*x^2*e^(6*c) + 2*(2*d^2*e*f - d*
f^2)*a^2*b^2*x*e^(6*c) - (2*d*e*f - f^2)*a^2*b^2*e^(6*c))*e^(2*d*x) + 1728
*(8*(d*e*f - f^2)*a^3*b*e^(5*c) + 2*(d*e*f - f^2)*a*b^3*e^(5*c) - (4*a^3*b
*d^2*f^2*e^(5*c) + a*b^3*d^2*f^2*e^(5*c))*x^2 - 2*(4*(d^2*e*f - d*f^2)*a^3
*b*e^(5*c) + (d^2*e*f - d*f^2)*a*b^3*e^(5*c))*x)*e^(d*x) - 1728*(8*(d*e*f
+ f^2)*a^3*b*e^(3*c) + 2*(d*e*f + f^2)*a*b^3*e^(3*c) + (4*a^3*b*d^2*f^2*e^
(3*c) + a*b^3*d^2*f^2*e^(3*c))*x^2 + 2*(4*(d^2*e*f + d*f^2)*a^3*b*e^(3*c)
+ (d^2*e*f + d*f^2)*a*b^3*e^(3*c))*x)*e^(-d*x) - 864*(2*a^2*b^2*d^2*f^2*x^
2*e^(2*c) + 2*(2*d^2*e*f + d*f^2)*a^2*b^2*x*e^(2*c) + (2*d*e*f + f^2)*a...
```

3.397.8 Giac [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^2\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a
), x)
```

3.397.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\sinh(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x
)
```

3.398 $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.398.1 Optimal result

Integrand size = 34, antiderivative size = 474

$$\begin{aligned}\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{a^4ex}{b^5}+\frac{a^2ex}{2b^3}+\frac{a^4fx^2}{2b^5}+\frac{a^2fx^2}{4b^3}\\
&-\frac{(e+fx)^2}{16bf}-\frac{a^3(e+fx)\cosh(c+dx)}{b^4d}\\
&-\frac{a^2f\cosh^2(c+dx)}{4b^3d^2}\\
&-\frac{a(e+fx)\cosh^3(c+dx)}{3b^2d}-\frac{f\cosh(4c+4dx)}{128bd^2}\\
&-\frac{a^3\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d}\\
&+\frac{a^3\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d}\\
&-\frac{a^3\sqrt{a^2+b^2}f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2}\\
&+\frac{a^3\sqrt{a^2+b^2}f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^2}\\
&+\frac{a^3f\sinh(c+dx)}{b^4d^2}+\frac{af\sinh(c+dx)}{3b^2d^2}\\
&+\frac{a^2(e+fx)\cosh(c+dx)\sinh(c+dx)}{2b^3d}\\
&+\frac{af\sinh^3(c+dx)}{9b^2d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32bd}\end{aligned}$$

output
```
a^4*e*x/b^5+1/2*a^2*e*x/b^3+1/2*a^4*f*x^2/b^5+1/4*a^2*f*x^2/b^3-1/16*(f*x+
e)^2/b/f-a^3*(f*x+e)*cosh(d*x+c)/b^4/d-1/4*a^2*f*cosh(d*x+c)^2/b^3/d^2-1/3
*a*(f*x+e)*cosh(d*x+c)^3/b^2/d-1/128*f*cosh(4*d*x+4*c)/b/d^2+a^3*f*sinh(d*
x+c)/b^4/d^2+1/3*a*f*sinh(d*x+c)/b^2/d^2+1/2*a^2*(f*x+e)*cosh(d*x+c)*sinh(
d*x+c)/b^3/d+1/9*a*f*sinh(d*x+c)^3/b^2/d^2+1/32*(f*x+e)*sinh(4*d*x+4*c)/b/
d-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d
+a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d-
a^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d^2
+a^3*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d^
2
```

3.398.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1627 vs. $2(474) = 948$.

Time = 4.22 (sec) , antiderivative size = 1627, normalized size of antiderivative = 3.43

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(1152*a^4*Sqrt[-(a^2 + b^2)^2]*c*d*e + 576*a^2*b^2*Sqrt[-(a^2 + b^2)^2]*c*
d*e - 144*b^4*Sqrt[-(a^2 + b^2)^2]*c*d*e - 576*a^4*Sqrt[-(a^2 + b^2)^2]*c^
2*f - 288*a^2*b^2*Sqrt[-(a^2 + b^2)^2]*c^2*f + 1152*a^4*Sqrt[-(a^2 + b^2)^
2]*d^2*e*x + 576*a^2*b^2*Sqrt[-(a^2 + b^2)^2]*d^2*e*x - 144*b^4*Sqrt[-(a^2
 + b^2)^2]*d^2*e*x + 576*a^4*Sqrt[-(a^2 + b^2)^2]*d^2*f*x^2 + 288*a^2*b^2*
Sqrt[-(a^2 + b^2)^2]*d^2*f*x^2 - 72*b^4*Sqrt[-(a^2 + b^2)^2]*d^2*f*x^2 - 2
304*a^5*Sqrt[a^2 + b^2]*d*e*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b
^2]] - 2304*a^3*b^2*Sqrt[a^2 + b^2]*d*e*ArcTan[(b - a*Tanh[(c + d*x)/2])/S
qrt[-a^2 - b^2]] - 2304*a^5*Sqrt[-a^2 - b^2]*c*f*ArcTanh[(a + b*E^(c + d*x
))/Sqrt[a^2 + b^2]] - 2304*a^3*b^2*Sqrt[-a^2 - b^2]*c*f*ArcTanh[(a + b*E^(
c + d*x))/Sqrt[a^2 + b^2]] - 288*a*b^4*Sqrt[-a^2 - b^2]*c*f*ArcTanh[(a + b
*E^(c + d*x))/Sqrt[a^2 + b^2]] - 1152*a^3*b*Sqrt[-(a^2 + b^2)^2]*d*e*Cosh[
c + d*x] - 288*a*b^3*Sqrt[-(a^2 + b^2)^2]*d*e*Cosh[c + d*x] - 1152*a^3*b*S
qrt[-(a^2 + b^2)^2]*d*f*x*Cosh[c + d*x] - 288*a*b^3*Sqrt[-(a^2 + b^2)^2]*d
*f*x*Cosh[c + d*x] - 144*a^2*b^2*Sqrt[-(a^2 + b^2)^2]*f*Cosh[2*(c + d*x)]
- 96*a*b^3*Sqrt[-(a^2 + b^2)^2]*d*e*Cosh[3*(c + d*x)] - 96*a*b^3*Sqrt[-(a^
2 + b^2)^2]*d*f*x*Cosh[3*(c + d*x)] - 9*b^4*Sqrt[-(a^2 + b^2)^2]*f*Cosh[4*
(c + d*x)] - 1152*a^5*Sqrt[-a^2 - b^2]*c*f*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])] - 1152*a^3*b^2*Sqrt[-a^2 - b^2]*c*f*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])] - 144*a*b^4*Sqrt[-a^2 - b^2]*c*f*Log[1 + (b*E^...
```

3.398.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.87 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.96, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {6113, 5971, 2009, 6113, 5970, 3042, 3113, 2009, 6113, 3042, 3791, 17, 6099, 17, 3042, 26, 3777, 3042, 3117, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^3(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6113}$$

$$\frac{\int (e+fx)\cosh^2(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5971}$$

$$\frac{\int \left(\frac{1}{8}(-e-fx)+\frac{1}{8}(e+fx)\cosh(4c+4dx)\right)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(\frac{\int (e+fx)\cosh^2(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 5970}$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{f\int \cosh^3(c+dx)dx}{3d}}{b}-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{f\int \sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{3d}}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3113}$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\int \left(\sinh^2(c+dx)+1\right)d(-i\sinh(c+dx))}{3d^2}}{b}\right)}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}\right)}{b}$$

3.398. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 6113$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\cosh^2(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3791$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(-\frac{a\left(\frac{\frac{1}{2}\int(e+fx)dx-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}\right)}{b}$$

$$\Big\downarrow \; 17$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-\frac{a\left(-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}\right)}{b}$$

$$\Big\downarrow \; 6099$$

3.398. $\displaystyle\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)dx}{b^2}+\frac{\int(e+fx)\sinh(c+dx)dx}{b}\right)}{b}\right)}{b}+\frac{(e+\cdots}{\cdots}\right.}{b}$$

$$\Big\downarrow \; 17$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)\sinh(c+dx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{b}\right)}{b}+\frac{(e+fx\cdots}{\cdots}\right.}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{(a^2+b^2)\int\frac{e+f\cdots}{a-ib\sin(i\cdots}}{b^2}\cdots\right.}{\cdots}\right.}{b}\right.}{b}$$

$$\Big\downarrow \; 26$$

3.398. $\displaystyle\int\frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{e+f\cdots}{a-ib\sin(i\cdots}}{b^2}\cdots\right.}{\cdots}\right.}{b}\right.}{b}$$

$\downarrow$ 3777

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{e+f\cdots}{a-ib\sin(i\cdots}}{b^2}\cdots\right.}{\cdots}\right.}{b}\right.}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{e+f\cdots}{a-ib\sin(i\cdots}}{b^2}\cdots\right.}{\cdots}\right.}{b}\right.}{b}$$

$\downarrow$ 3117

3.398. $\displaystyle\int\frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{\left(a^2+b^2\right)\int\frac{e+f}{a-ib\sin(i}}{b^2}\right.}{}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{3803}$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(\frac{2\left(a^2+b^2\right)\int-\frac{e}{-2e^{c+}}}{b^2}\right.}{}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\int\frac{e}{-2e^{c+}}}{b}\right.}{}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{2694}$$

3.398. $\displaystyle\int\frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\int-\frac{}{2(}}{}\right.}{}\right.}{}\right.}{}\right.$$

$$b$$

$\downarrow$ 27

3.398. $\int\frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\int\frac{e}{a+b}}{}\right.}{}\right.}{}\right.}{}\right.}{b}$$

$$\Big\downarrow \; 2620$$

3.398. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+f}{}\right.}{}\right.}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 2715

3.398. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\left(\frac{(e+f}{}\right.}{}\right.}{}\right.}{}\right.}{}\right.}{}$$

$\downarrow$ 2838

3.398. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f\cosh(4c+4dx)}{128d^2}+\frac{(e+fx)\sinh(4c+4dx)}{32d}-\frac{(e+fx)^2}{16f}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^3(c+dx)}{3d}-\frac{if\left(-\frac{1}{3}i\sinh^3(c+dx)-i\sinh(c+dx)\right)}{3d^2}}{b}-\frac{a\left(\frac{-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}}{b}-\frac{a\left(-\frac{2\left(a^2+b^2\right)\left(b\left(\frac{f\,\mathrm{Po}}{}\right.\right.}{}\right.}{}\right.}{}\right.$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

3.398. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
-((a*((((e + f*x)*Cosh[c + d*x]^3)/(3*d) - ((I/3)*f*((-I)*Sinh[c + d*x] -
(I/3)*Sinh[c + d*x]^3))/d^2)/b - (a*(((e + f*x)^2/(4*f) - (f*Cosh[c + d*x]
^2)/(4*d^2) + ((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d))/b - (a*(-1/2*
(a*(e + f*x)^2)/(b^2*f) - (2*(a^2 + b^2)*(-1/2*(b*(((e + f*x)*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/b^2 - (
I*((I*(e + f*x)*Cosh[c + d*x])/d - (I*f*Sinh[c + d*x])/d^2))/b))/b))/b))/b
) + (-1/16*(e + f*x)^2/f - (f*Cosh[4*c + 4*d*x])/(128*d^2) + ((e + f*x)*Si
nh[4*c + 4*d*x])/(32*d))/b
```

3.398.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.398. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.398. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.398.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1212 vs. $2(432) = 864$.

Time = 76.74 (sec) , antiderivative size = 1213, normalized size of antiderivative = 2.56

method	result	size
risch	Expression too large to display	1213

3.398. $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
int((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURN
VERBOSE)
```

output
```
-1/8*a*(4*a^2*d*f*x+b^2*d*f*x+4*a^2*d*e+b^2*d*e-4*a^2*f-b^2*f)/b^4/d^2*exp
(d*x+c)+1/256*(4*d*f*x+4*d*e-f)/b/d^2*exp(4*d*x+4*c)-1/d*a^5/b^5*f/(a^2+b^
2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d*
a^5/b^5*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)
^(1/2)))*x-1/d^2*a^5/b^5*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2*a^5/b^5*f/(a^2+b^2)^(1/2)*ln((b*exp(d*
x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d*a^3/b^3*f/(a^2+b^2)^(1/
2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d*a^3/b^
3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)
))*x-1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))*c+1/4*a^2*f*x^2/b^3+1/16*a^2*(2*d*f*x+2*d*e-f)/b^3/d
^2*exp(2*d*x+2*c)-1/16*a^2*(2*d*f*x+2*d*e+f)/b^3/d^2*exp(-2*d*x-2*c)+2/d*a
^3/b^3*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
-1/d^2*a^5/b^5*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(
-a+(a^2+b^2)^(1/2)))+1/d^2*a^5/b^5*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(
a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*dil
og((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2*a^3/b^3*f
/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)
))-1/16*f*x^2/b-1/256*(4*d*f*x+4*d*e+f)/b/d^2*exp(-4*d*x-4*c)+1/2*a^4*f*x^
2/b^5-1/8*e*x/b+1/2*a^2*e*x/b^3+1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((b*e...
```

3.398.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3228 vs. $2(430) = 860$.

Time = 0.33 (sec) , antiderivative size = 3228, normalized size of antiderivative = 6.81

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
1/2304*(9*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c)^8 + 9*(4*b^4*d*f
*x + 4*b^4*d*e - b^4*f)*sinh(d*x + c)^8 - 32*(3*a*b^3*d*f*x + 3*a*b^3*d*e
- a*b^3*f)*cosh(d*x + c)^7 - 8*(12*a*b^3*d*f*x + 12*a*b^3*d*e - 4*a*b^3*f
- 9*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c))*sinh(d*x + c)^7 - 36*
b^4*d*f*x + 144*(2*a^2*b^2*d*f*x + 2*a^2*b^2*d*e - a^2*b^2*f)*cosh(d*x + c
)^6 + 4*(72*a^2*b^2*d*f*x + 72*a^2*b^2*d*e - 36*a^2*b^2*f + 63*(4*b^4*d*f*
x + 4*b^4*d*e - b^4*f)*cosh(d*x + c)^2 - 56*(3*a*b^3*d*f*x + 3*a*b^3*d*e -
 a*b^3*f)*cosh(d*x + c))*sinh(d*x + c)^6 - 36*b^4*d*e - 288*((4*a^3*b + a*
b^3)*d*f*x + (4*a^3*b + a*b^3)*d*e - (4*a^3*b + a*b^3)*f)*cosh(d*x + c)^5
- 24*(12*(4*a^3*b + a*b^3)*d*f*x - 21*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*co
sh(d*x + c)^3 + 12*(4*a^3*b + a*b^3)*d*e + 28*(3*a*b^3*d*f*x + 3*a*b^3*d*e
 - a*b^3*f)*cosh(d*x + c)^2 - 12*(4*a^3*b + a*b^3)*f - 36*(2*a^2*b^2*d*f*x
 + 2*a^2*b^2*d*e - a^2*b^2*f)*cosh(d*x + c))*sinh(d*x + c)^5 - 9*b^4*f + 1
44*((8*a^4 + 4*a^2*b^2 - b^4)*d^2*f*x^2 + 2*(8*a^4 + 4*a^2*b^2 - b^4)*d^2*
e*x)*cosh(d*x + c)^4 + 2*(72*(8*a^4 + 4*a^2*b^2 - b^4)*d^2*f*x^2 + 144*(8*
a^4 + 4*a^2*b^2 - b^4)*d^2*e*x + 315*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cos
h(d*x + c)^4 - 560*(3*a*b^3*d*f*x + 3*a*b^3*d*e - a*b^3*f)*cosh(d*x + c)^3
 + 1080*(2*a^2*b^2*d*f*x + 2*a^2*b^2*d*e - a^2*b^2*f)*cosh(d*x + c)^2 - 72
0*((4*a^3*b + a*b^3)*d*f*x + (4*a^3*b + a*b^3)*d*e - (4*a^3*b + a*b^3)*f)*
cosh(d*x + c))*sinh(d*x + c)^4 - 288*((4*a^3*b + a*b^3)*d*f*x + (4*a^3*...
```

3.398.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.398.7 Maxima [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh{(dx+c)}^2\sinh{(dx+c)}^3}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/2304*(4608*(a^5*e^c + a^3*b^2*e^c)*integrate(x*e^(d*x)/(b^6*e^(2*d*x +
2*c) + 2*a*b^5*e^(d*x + c) - b^6), x) - (144*(8*a^4*d^2*e^(4*c) + 4*a^2*b^
2*d^2*e^(4*c) - b^4*d^2*e^(4*c))*x^2 + 9*(4*b^4*d*x*e^(8*c) - b^4*e^(8*c))
*e^(4*d*x) - 32*(3*a*b^3*d*x*e^(7*c) - a*b^3*e^(7*c))*e^(3*d*x) + 144*(2*a
^2*b^2*d*x*e^(6*c) - a^2*b^2*e^(6*c))*e^(2*d*x) + 288*(4*a^3*b*e^(5*c) + a
*b^3*e^(5*c) - (4*a^3*b*d*e^(5*c) + a*b^3*d*e^(5*c))*x)*e^(d*x) - 288*(4*a
^3*b*e^(3*c) + a*b^3*e^(3*c) + (4*a^3*b*d*e^(3*c) + a*b^3*d*e^(3*c))*x)*e^
(-d*x) - 144*(2*a^2*b^2*d*x*e^(2*c) + a^2*b^2*e^(2*c))*e^(-2*d*x) - 32*(3*
a*b^3*d*x*e^c + a*b^3*e^c)*e^(-3*d*x) - 9*(4*b^4*d*x + b^4)*e^(-4*d*x))*e^
(-4*c)/(b^5*d^2))*f - 1/192*e*(192*sqrt(a^2 + b^2)*a^3*log((b*e^(-d*x - c)
 - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^5*d) +
(8*a*b^2*e^(-d*x - c) - 24*a^2*b*e^(-2*d*x - 2*c) - 3*b^3 + 24*(4*a^3 + a*
b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) - 24*(8*a^4 + 4*a^2*b^2 - b
^4)*(d*x + c)/(b^5*d) + (24*a^2*b*e^(-2*d*x - 2*c) + 8*a*b^2*e^(-3*d*x - 3
*c) + 3*b^3*e^(-4*d*x - 4*c) + 24*(4*a^3 + a*b^2)*e^(-d*x - c))/(b^4*d))
```

3.398.8 Giac [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh{(dx+c)}^2\sinh{(dx+c)}^3}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a),
 x)
```

3.398.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\sinh(c+d\,x)^3\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.399 $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.399.1 Optimal result

Integrand size = 29, antiderivative size = 184

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{(8a^4+4a^2b^2-b^4)\,x}{8b^5} + \frac{2a^3\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^5d} - \frac{a(3a^2+b^2)\cosh(c+dx)}{3b^4d} + \frac{(4a^2+b^2)\cosh(c+dx)\sinh(c+dx)}{8b^3d} - \frac{a\cosh(c+dx)\sinh^2(c+dx)}{3b^2d} + \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd}$$

output
```
1/8*(8*a^4+4*a^2*b^2-b^4)*x/b^5-1/3*a*(3*a^2+b^2)*cosh(d*x+c)/b^4/d+1/8*(4
*a^2+b^2)*cosh(d*x+c)*sinh(d*x+c)/b^3/d-1/3*a*cosh(d*x+c)*sinh(d*x+c)^2/b^
2/d+1/4*cosh(d*x+c)*sinh(d*x+c)^3/b/d+2*a^3*arctanh((b-a*tanh(1/2*d*x+1/2*
c))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^5/d
```

3.399.2 Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.83

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-24ab(4a^2+b^2)\cosh(c+dx)-8ab^3\cosh(3(c+dx))+3\Big(4(8a^4+4a^2b^2-b^4)(c+dx)+64a^3\sqrt{-a^2-b^2}}{96b^5d}$$

input
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-24*a*b*(4*a^2 + b^2)*Cosh[c + d*x] - 8*a*b^3*Cosh[3*(c + d*x)] + 3*(4*(8
*a^4 + 4*a^2*b^2 - b^4)*(c + d*x) + 64*a^3*Sqrt[-a^2 - b^2]*ArcTan[(b - a*
Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] + 8*a^2*b^2*Sinh[2*(c + d*x)] + b^4*S
inh[4*(c + d*x)]))/(96*b^5*d)
```

3.399.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.21, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.828, Rules used = {3042, 26, 3368, 26, 3042, 26, 3529, 3042, 25, 3528, 26, 3042, 26, 3528, 25, 3042, 3502, 27, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{i\sin(ic+idx)^3\cos(ic+idx)^2}{a-ib\sin(ic+idx)}dx$$
$$\Big\downarrow \ 26$$
$$i\int \frac{\cos(ic+idx)^2\sin(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$
$$\Big\downarrow \ 3368$$

$$i\int -\frac{i\sinh^3(c+dx)\left(\sinh^2(c+dx)+1\right)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\sinh^3(c+dx)\left(\sinh^2(c+dx)+1\right)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\sin(ic+idx)^3\left(1-\sin(ic+idx)^2\right)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\sin(ic+idx)^3\left(1-\sin(ic+idx)^2\right)}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{ 3529}$$

$$i\left(\frac{i\int \frac{\sinh^2(c+dx)\left(4a\sinh^2(c+dx)-b\sinh(c+dx)+3a\right)}{a+b\sinh(c+dx)}dx}{4b}-\frac{i\sinh^3(c+dx)\cosh(c+dx)}{4bd}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$i\left(\frac{i\int -\frac{\sin(ic+idx)^2\left(-4a\sin(ic+idx)^2+ib\sin(ic+idx)+3a\right)}{a-ib\sin(ic+idx)}dx}{4b}-\frac{i\sinh^3(c+dx)\cosh(c+dx)}{4bd}\right)$$

$$\Big\downarrow \text{ 25}$$

$$i\left(-\frac{i\int \frac{\sin(ic+idx)^2\left(-4a\sin(ic+idx)^2+ib\sin(ic+idx)+3a\right)}{a-ib\sin(ic+idx)}dx}{4b}-\frac{i\sinh^3(c+dx)\cosh(c+dx)}{4bd}\right)$$

$$\Big\downarrow \text{ 3528}$$

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{i\int -\frac{i\sinh(c+dx)\left(8a^2-b\sinh(c+dx)a+3\left(4a^2+b^2\right)\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}dx}{3b}\right)}{4b}-\frac{i\sinh^3(c+dx)\cosh(c+a}{4bd}\right.$$

$$\Big\downarrow \text{ 26}$$

3.399. $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$i\left(-\frac{i\left(\frac{\int \frac{\sinh(c+dx)\left(8a^2-b\sinh(c+dx)a+3\left(4a^2+b^2\right)\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}dx}{3b}-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}\right)}{4b}-\frac{i\sinh^3(c+dx)\cosh(c+dx)}{4bd}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}+\frac{\int -\frac{i\sin(ic+idx)\left(8a^2+ib\sin(ic+idx)a-3\left(4a^2+b^2\right)\sin(ic+idx)^2\right)}{a-ib\sin(ic+idx)}dx}{3b}\right)}{4b}-\frac{i\sinh^3(c+dx)\cosh(c+dx)}{4bd}\right)$$

$$\Big\downarrow \textbf{26}$$

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\int \frac{\sin(ic+idx)\left(8a^2+ib\sin(ic+idx)a-3\left(4a^2+b^2\right)\sin(ic+idx)^2\right)}{a-ib\sin(ic+idx)}dx}{3b}\right)}{4b}-\frac{i\sinh^3(c+dx)\cosh(c+dx)}{4bd}\right)$$

$$\Big\downarrow \textbf{3528}$$

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{i\int -\frac{8a\left(3a^2+b^2\right)\sinh^2(c+dx)-b\left(4a^2-3b^2\right)\sinh(c+dx)+3a\left(4a^2+b^2\right)}{a+b\sinh(c+dx)}dx}{2b}+\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}\right)}{3b}\right)}{4b}\right.$$

$$\Big\downarrow \textbf{25}$$

3.399. $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\int\frac{8a\left(3a^2+b^2\right)\sinh^2(c+dx)-b\left(4a^2-3b^2\right)\sinh(c+dx)+3a\left(4a^2+b^2\right)}{a+b\sinh(c+dx)}dx}{2b}\right.}{3b}\right.}{4b}\right.$$

$$\downarrow \text{3042}$$

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\int\frac{-8a\left(3a^2+b^2\right)\sin(ic+idx)^2+ib\left(4a^2-3b^2\right)\sin(ic+idx)+3a\left(4a^2+b^2\right)}{a-ib\sin(ic+idx)}}{2b}\right.}{3b}\right.}{4b}\right.$$

$$\downarrow \text{3502}$$

3.399. $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{8a\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{i\int-\frac{3i\left(ab\left(4a^2+b^2\right)-\left(8a^4+4b^2a^2-b^4\right)\mathrm{s}\cdots\right.}{a+b\sinh(c+dx)}\cdots}{b}\right.}{2b}\right.}{3b}\right.}{4b}\right.$$

$$\Big\downarrow \ 27$$

3.399. $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{3\int\frac{ab\left(4a^2+b^2\right)-\left(8a^4+4b^2a^2-b^4\right)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{8a\left(3a^2+b^2\right)\cos}{bd}\right.}{2b}\right.}{3b}\right.}{4b}\right.$$

$$\Big\downarrow \textcolor{blue}{3042}$$

3.399. $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{8a\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{3\int\frac{ab\left(4a^2+b^2\right)+i\left(8a^4+4b^2a^2-b^4\right)\sin(i}{a-ib\sin(ic+idx)}}{b}\right.}{2b}\right.}{3b}\right.}{4b}\right.$$

$$\Big\downarrow \text{3214}$$

3.399. $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{3\left(\frac{8a^3\left(a^2+b^2\right)\int\frac{1}{a+b\sinh(c+dx)}dx}{b}-\frac{x\left(8a^4+4a^2b^2-b^4\right)}{b}\right)}{b}+\frac{8a\left(3a\cdots\right.}{\cdots}\right.}{2b}\right.}{3b}\right.}{4b}\right.$$

$\Big\downarrow$ 3042

3.399. $\displaystyle\int\frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{8a\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{3\left(-\frac{x\left(8a^4+4a^2b^2-b^4\right)}{b}+\frac{8a^3\left(a^2+b^2\right)\int}{}\right.}{b}\right.}{2b}\right.}{3b}\right.}{4b}\right.$$

$\downarrow$ 3139

3.399. $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{8a\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{3\left(-\frac{x\left(8a^4+4a^2b^2-b^4\right)}{b}-\frac{16ia^3\left(a^2+b^2\right)}{}\right.}{}\right.}{2}\right.}{3b}\right.}{4b}\right.$$

↓ 1083

3.399. $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{8a\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{3\left(-\frac{x\left(8a^4+4a^2b^2-b^4\right)}{b}+\frac{32ia^3\left(a^2+b^2\right)\cdots}{\cdots}\right.}{\cdots}\right.}{2b}\right.}{3b}\right.}{4b}\right.$$

$\downarrow$ 217

3.399. $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$i\left(-\frac{i\left(-\frac{4a\sinh^2(c+dx)\cosh(c+dx)}{3bd}-\frac{i\left(\frac{3i\left(4a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{i\left(\frac{8a\left(3a^2+b^2\right)\cosh(c+dx)}{bd}+\frac{3\left(\frac{16a^3\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+\cdots\right.}{2\sqrt{a^2+b\cdots}}\right.}{bd}\cdots\right.}{b}\cdots\right.}{2b}\cdots\right.}{3b}\cdots\right.}{4b}\cdots\right.$$

3.399. $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
I*(((-1/4*I)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(b*d) - ((I/4)*((-4*a*Cosh[c +
 d*x]*Sinh[c + d*x]^2)/(3*b*d) - ((I/3)*(((-1/2*I)*((3*(-(((8*a^4 + 4*a^2*
b^2 - b^4)*x)/b) + (16*a^3*Sqrt[a^2 + b^2]*ArcTanh[Tanh[(c + d*x)/2]/(2*Sq
rt[a^2 + b^2])])/(b*d)))/b + (8*a*(3*a^2 + b^2)*Cosh[c + d*x])/(b*d)))/b +
 (((3*I)/2)*(4*a^2 + b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(b*d)))/b))/b)
```

3.399.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

3.399. $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

rule 3529
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m + n + 2))   Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(
n + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*
(a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c,
0])))
```

3.399. $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.399.4 Maple [A] (verified)

Time = 36.97 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.59

method	result
risch	$\frac{x\,a^4}{b^5}+\frac{x\,a^2}{2b^3}-\frac{x}{8b}+\frac{e^{4dx+4c}}{64bd}-\frac{a\,e^{3dx+3c}}{24b^2d}+\frac{a^2e^{2dx+2c}}{8b^3d}-\frac{a^3e^{dx+c}}{2b^4d}-\frac{a\,e^{dx+c}}{8b^2d}-\frac{a^3e^{-dx-c}}{2b^4d}-\frac{a\,e^{-dx-c}}{8b^2d}-$
derivativedivides	$-\frac{1}{4b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{-3b+2a}{6b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{4a^2-4ab+3b^2}{8b^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{\left(8a^4+4a^2b^2-b^4\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8b^5}-\frac{8a^3-4a}{8b^4(\tanh}$
default	$-\frac{1}{4b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{-3b+2a}{6b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{4a^2-4ab+3b^2}{8b^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{\left(8a^4+4a^2b^2-b^4\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8b^5}-\frac{8a^3-4a}{8b^4(\tanh}$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
x/b^5*a^4+1/2*x/b^3*a^2-1/8*x/b+1/64/b/d*exp(4*d*x+4*c)-1/24*a/b^2/d*exp(3
*d*x+3*c)+1/8/b^3*a^2/d*exp(2*d*x+2*c)-1/2*a^3/b^4/d*exp(d*x+c)-1/8*a/b^2/
d*exp(d*x+c)-1/2*a^3/b^4/d*exp(-d*x-c)-1/8*a/b^2/d*exp(-d*x-c)-1/8/b^3*a^2
/d*exp(-2*d*x-2*c)-1/24*a/b^2/d*exp(-3*d*x-3*c)-1/64/b/d*exp(-4*d*x-4*c)+(
a^2+b^2)^(1/2)*a^3/d/b^5*ln(exp(d*x+c)+(a+(a^2+b^2)^(1/2))/b)-(a^2+b^2)^(1
/2)*a^3/d/b^5*ln(exp(d*x+c)-(-a+(a^2+b^2)^(1/2))/b)
```

3.399.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. 2(171) = 342.

Time = 0.26 (sec) , antiderivative size = 1134, normalized size of antiderivative = 6.16

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fric
as")
```

output
```
1/192*(3*b^4*cosh(d*x + c)^8 + 3*b^4*sinh(d*x + c)^8 - 8*a*b^3*cosh(d*x +
c)^7 + 24*a^2*b^2*cosh(d*x + c)^6 + 8*(3*b^4*cosh(d*x + c) - a*b^3)*sinh(d
*x + c)^7 + 24*(8*a^4 + 4*a^2*b^2 - b^4)*d*x*cosh(d*x + c)^4 + 4*(21*b^4*c
osh(d*x + c)^2 - 14*a*b^3*cosh(d*x + c) + 6*a^2*b^2)*sinh(d*x + c)^6 - 24*
a^2*b^2*cosh(d*x + c)^2 - 24*(4*a^3*b + a*b^3)*cosh(d*x + c)^5 + 24*(7*b^4
*cosh(d*x + c)^3 - 7*a*b^3*cosh(d*x + c)^2 + 6*a^2*b^2*cosh(d*x + c) - 4*a
^3*b - a*b^3)*sinh(d*x + c)^5 - 8*a*b^3*cosh(d*x + c) + 2*(105*b^4*cosh(d*
x + c)^4 - 140*a*b^3*cosh(d*x + c)^3 + 180*a^2*b^2*cosh(d*x + c)^2 + 12*(8
*a^4 + 4*a^2*b^2 - b^4)*d*x - 60*(4*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^4 - 3*b^4 - 24*(4*a^3*b + a*b^3)*cosh(d*x + c)^3 + 8*(21*b^4*cosh(d*
x + c)^5 - 35*a*b^3*cosh(d*x + c)^4 + 60*a^2*b^2*cosh(d*x + c)^3 - 12*a^3*
b - 3*a*b^3 + 12*(8*a^4 + 4*a^2*b^2 - b^4)*d*x*cosh(d*x + c) - 30*(4*a^3*b
 + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(7*b^4*cosh(d*x + c)^6 - 1
4*a*b^3*cosh(d*x + c)^5 + 30*a^2*b^2*cosh(d*x + c)^4 + 12*(8*a^4 + 4*a^2*b
^2 - b^4)*d*x*cosh(d*x + c)^2 - 2*a^2*b^2 - 20*(4*a^3*b + a*b^3)*cosh(d*x
+ c)^3 - 6*(4*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 192*(a^3*cos
h(d*x + c)^4 + 4*a^3*cosh(d*x + c)^3*sinh(d*x + c) + 6*a^3*cosh(d*x + c)^2
*sinh(d*x + c)^2 + 4*a^3*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)
^4)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b
*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c...
```

3.399.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.399.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.40

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{\sqrt{a^2+b^2}a^3\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{b^5d} - \frac{\left(8\,ab^2e^{(-dx-c)}-24\,a^2be^{(-2\,dx-2\,c)}-3\,b^3+24\,(4\,a^3+ab^2)e^{(-3\,dx-3\,c)}\right)e^{(4\,dx+4\,c)}}{192\,b^4d} + \frac{(8\,a^4+4\,a^2b^2-b^4)(dx+c)}{8\,b^5d} - \frac{24\,a^2be^{(-2\,dx-2\,c)}+8\,ab^2e^{(-3\,dx-3\,c)}+3\,b^3e^{(-4\,dx-4\,c)}+24\,(4\,a^3+ab^2)e^{(-dx-c)}}{192\,b^4d}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxi
ma")
```

output
```
-sqrt(a^2 + b^2)*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x
 - c) - a + sqrt(a^2 + b^2)))/(b^5*d) - 1/192*(8*a*b^2*e^(-d*x - c) - 24*a
^2*b*e^(-2*d*x - 2*c) - 3*b^3 + 24*(4*a^3 + a*b^2)*e^(-3*d*x - 3*c))*e^(4*
d*x + 4*c)/(b^4*d) + 1/8*(8*a^4 + 4*a^2*b^2 - b^4)*(d*x + c)/(b^5*d) - 1/1
92*(24*a^2*b*e^(-2*d*x - 2*c) + 8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-4*d*x
 - 4*c) + 24*(4*a^3 + a*b^2)*e^(-d*x - c))/(b^4*d)
```

3.399.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.40

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\frac{24\,(8\,a^4+4\,a^2b^2-b^4)(dx+c)}{b^5} + \frac{3\,b^3e^{(4\,dx+4\,c)}-8\,ab^2e^{(3\,dx+3\,c)}+24\,a^2be^{(2\,dx+2\,c)}-96\,a^3e^{(dx+c)}-24\,ab^2e^{(dx+c)}}{b^4} - \frac{(24\,a^2b^2e^{(2\,dx+2\,c)}+8\,ab^3e^{(}}{}}{192\,d}$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac
")
```

output
```
1/192*(24*(8*a^4 + 4*a^2*b^2 - b^4)*(d*x + c)/b^5 + (3*b^3*e^(4*d*x + 4*c)
 - 8*a*b^2*e^(3*d*x + 3*c) + 24*a^2*b*e^(2*d*x + 2*c) - 96*a^3*e^(d*x + c)
 - 24*a*b^2*e^(d*x + c))/b^4 - (24*a^2*b^2*e^(2*d*x + 2*c) + 8*a*b^3*e^(d*
x + c) + 3*b^4 + 24*(4*a^3*b + a*b^3)*e^(3*d*x + 3*c))*e^(-4*d*x - 4*c)/b^
5 - 192*(a^5 + a^3*b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))
/abs(2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^5))/d
```

3.399.9 Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.79

$$\begin{aligned}
&\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{x\,(8\,a^4+4\,a^2\,b^2-b^4)}{8\,b^5} - \frac{\mathrm{e}^{-4\,c-4\,d\,x}}{64\,b\,d} + \frac{\mathrm{e}^{4\,c+4\,d\,x}}{64\,b\,d} - \frac{a\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,b^2\,d} - \frac{a\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,b^2\,d} \\
&\quad - \frac{\mathrm{e}^{c+d\,x}\,(4\,a^3+a\,b^2)}{8\,b^4\,d} - \frac{a^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,b^3\,d} + \frac{a^2\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,b^3\,d} - \frac{\mathrm{e}^{-c-d\,x}\,(4\,a^3+a\,b^2)}{8\,b^4\,d} \\
&\quad - \frac{a^3\,\ln\left(\frac{2\,a^3\,\mathrm{e}^{c+d\,x}\,(a^2+b^2)}{b^6} - \frac{2\,a^3\,\sqrt{a^2+b^2}\,(b-a\,\mathrm{e}^{c+d\,x})}{b^6}\right)\,\sqrt{a^2+b^2}}{b^5\,d} \\
&\quad + \frac{a^3\,\ln\left(\frac{2\,a^3\,\sqrt{a^2+b^2}\,(b-a\,\mathrm{e}^{c+d\,x})}{b^6} + \frac{2\,a^3\,\mathrm{e}^{c+d\,x}\,(a^2+b^2)}{b^6}\right)\,\sqrt{a^2+b^2}}{b^5\,d}
\end{aligned}$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
(x*(8*a^4 - b^4 + 4*a^2*b^2))/(8*b^5) - exp(- 4*c - 4*d*x)/(64*b*d) + exp(
4*c + 4*d*x)/(64*b*d) - (a*exp(- 3*c - 3*d*x))/(24*b^2*d) - (a*exp(3*c + 3
*d*x))/(24*b^2*d) - (exp(c + d*x)*(a*b^2 + 4*a^3))/(8*b^4*d) - (a^2*exp(-
2*c - 2*d*x))/(8*b^3*d) + (a^2*exp(2*c + 2*d*x))/(8*b^3*d) - (exp(- c - d*
x)*(a*b^2 + 4*a^3))/(8*b^4*d) - (a^3*log((2*a^3*exp(c + d*x)*(a^2 + b^2))/
b^6 - (2*a^3*(a^2 + b^2)^(1/2)*(b - a*exp(c + d*x)))/b^6)*(a^2 + b^2)^(1/2
))/(b^5*d) + (a^3*log((2*a^3*(a^2 + b^2)^(1/2)*(b - a*exp(c + d*x)))/b^6 +
 (2*a^3*exp(c + d*x)*(a^2 + b^2))/b^6)*(a^2 + b^2)^(1/2))/(b^5*d)
```

3.400 $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.400.1 Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.400.2 Mathematica [N/A]

Not integrable

Time = 15.78 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])),x]
```

output
```
Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

3.400.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

$$\Big\downarrow \text{6125}$$

$$\int \frac{\sinh^3(c+dx)\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.400.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.400.4 Maple [N/A] (verified)

Not integrable

Time = 0.89 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.400.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^2\sinh(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
integral(cosh(d*x + c)^2*sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh
(d*x + c)), x)
```

3.400.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2*sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.400.7 Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 417, normalized size of antiderivative = 11.58

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^2\sinh(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-2*(a^5*e^c + a^3*b^2*e^c)*integrate(-e^(d*x)/(b^6*f*x + b^6*e - (b^6*f*x*
e^(2*c) + b^6*e*e^(2*c))*e^(2*d*x) - 2*(a*b^5*f*x*e^c + a*b^5*e*e^c)*e^(d*
x)), x) - 1/16*e^(-4*c + 4*d*e/f)*exp_integral_e(1, 4*(f*x + e)*d/f)/(b*f)
 - 1/8*a*e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b^2*f) - 1
/4*a^2*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^3*f) - 1/4
*a^2*e^(2*c - 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^3*f) + 1/8*a
*e^(3*c - 3*d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b^2*f) - 1/16*e^(4
*c - 4*d*e/f)*exp_integral_e(1, -4*(f*x + e)*d/f)/(b*f) - 1/8*(4*a^3 + a*b
^2)*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^4*f) + 1/8*(4*a^3*e
^c + a*b^2*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^4*f) + 1/8
*(8*a^4 + 4*a^2*b^2 - b^4)*log(f*x + e)/(b^5*f)
```

3.400.8 Giac [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^2\sinh(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate(cosh(d*x + c)^2*sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a)
), x)
```

3.400.9 Mupad [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^2\sinh(c+d\,x)^3}{(e+f\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)^2*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))), x
)
```

3.401 $\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} \, dx$

3.401.1 Optimal result

Integrand size = 36, antiderivative size = 1443

$$
\begin{aligned}
&\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{3a^3f^3x}{8b^4d^3}+\frac{45af^3x}{256b^2d^3}-\frac{a^3(e+fx)^3}{4b^4d}+\frac{3a(e+fx)^3}{32b^2d}+\frac{a^3(a^2+b^2)(e+fx)^4}{4b^6f} \\
&\quad -\frac{6a^4f^3\cosh(c+dx)}{b^5d^4}-\frac{40a^2f^3\cosh(c+dx)}{9b^3d^4}+\frac{3f^3\cosh(c+dx)}{4bd^4} \\
&\quad -\frac{3a^4f(e+fx)^2\cosh(c+dx)}{b^5d^2}-\frac{2a^2f(e+fx)^2\cosh(c+dx)}{b^3d^2}+\frac{3f(e+fx)^2\cosh(c+dx)}{8bd^2} \\
&\quad -\frac{9af^2(e+fx)\cosh^2(c+dx)}{32b^2d^3}-\frac{2a^2f^3\cosh^3(c+dx)}{27b^3d^4}-\frac{a^2f(e+fx)^2\cosh^3(c+dx)}{3b^3d^2} \\
&\quad -\frac{3af^2(e+fx)\cosh^4(c+dx)}{32b^2d^3}-\frac{a(e+fx)^3\cosh^4(c+dx)}{4b^2d}-\frac{f^3\cosh(3c+3dx)}{216bd^4} \\
&\quad -\frac{f(e+fx)^2\cosh(3c+3dx)}{48bd^2}-\frac{3f^3\cosh(5c+5dx)}{5000bd^4}-\frac{3f(e+fx)^2\cosh(5c+5dx)}{400bd^2} \\
&\quad -\frac{a^3(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^6d}-\frac{a^3(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^6d} \\
&\quad -\frac{3a^3(a^2+b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^6d^2} \\
&\quad -\frac{3a^3(a^2+b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^6d^2} \\
&\quad +\frac{6a^3(a^2+b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^6d^3} \\
&\quad +\frac{6a^3(a^2+b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^6d^3} \\
&\quad -\frac{6a^3(a^2+b^2)f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^6d^4}-\frac{6a^3(a^2+b^2)f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^6d^4} \\
&\quad +\frac{6a^4f^2(e+fx)\sinh(c+dx)}{b^5d^3}+\frac{40a^2f^2(e+fx)\sinh(c+dx)}{9b^3d^3}-\frac{3f^2(e+fx)\sinh(c+dx)}{4bd^3} \\
&\quad +\frac{a^4(e+fx)^3\sinh(c+dx)}{b^5d}+\frac{2a^2(e+fx)^3\sinh(c+dx)}{3b^3d}-\frac{(e+fx)^3\sinh(c+dx)}{8bd} \\
&\quad +\frac{3a^3f^3\cosh(c+dx)\sinh(c+dx)}{8b^4d^4}+\frac{45af^3\cosh(c+dx)\sinh(c+dx)}{256b^2d^4} \\
&\quad +\frac{3a^3f(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{4b^4d^2}+\frac{9af(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{32b^2d^2} \\
&\quad +\frac{2a^2f^2(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{9b^3d^3}+\frac{a^2(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{3b^3d} \\
&\quad +\frac{3af^3\cosh^3(c+dx)\sinh(c+dx)}{128b^2d^4}+\frac{3af(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{16b^2d^2} \\
&\quad -\frac{3a^3f^2(e+fx)\sinh^2(c+dx)}{4b^4d^3}-\frac{a^3(e+fx)^3\sinh^2(c+dx)}{2b^4d}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72bd^3} \\
&\quad +\frac{(e+fx)^3\sinh(3c+3dx)}{\cdots}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{\cdots}+\frac{(e+fx)^3\sinh(5c+5dx)}{\cdots}
\end{aligned}
$$

output
```
-2*a^2*f*(f*x+e)^2*cosh(d*x+c)/b^3/d^2+3/8*f*(f*x+e)^2*cosh(d*x+c)/b/d^2-3
/4*f^2*(f*x+e)*sinh(d*x+c)/b/d^3+45/256*a*f^3*x/b^2/d^3-40/9*a^2*f^3*cosh(
d*x+c)/b^3/d^4-1/8*(f*x+e)^3*sinh(d*x+c)/b/d+2/3*a^2*(f*x+e)^3*sinh(d*x+c)
/b^3/d-3*a^4*f*(f*x+e)^2*cosh(d*x+c)/b^5/d^2-9/32*a*f^2*(f*x+e)*cosh(d*x+c
)^2/b^2/d^3-1/3*a^2*f*(f*x+e)^2*cosh(d*x+c)^3/b^3/d^2-3/32*a*f^2*(f*x+e)*c
osh(d*x+c)^4/b^2/d^3+6*a^4*f^2*(f*x+e)*sinh(d*x+c)/b^5/d^3+3/8*a^3*f^3*cos
h(d*x+c)*sinh(d*x+c)/b^4/d^4+1/3*a^2*(f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/b
^3/d+3/128*a*f^3*cosh(d*x+c)^3*sinh(d*x+c)/b^2/d^4-3/4*a^3*f^2*(f*x+e)*sin
h(d*x+c)^2/b^4/d^3-6*a^3*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/b^6/d^4-6*a^3*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/b^6/d^4+1/48*(f*x+e)^3*sinh(3*d*x+3*c)/b/d+1/80*(f*x+e)^3*sinh(5*
d*x+5*c)/b/d-1/4*a^3*(f*x+e)^3/b^4/d-1/216*f^3*cosh(3*d*x+3*c)/b/d^4-3/500
0*f^3*cosh(5*d*x+5*c)/b/d^4+3/4*a^3*f*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b^
4/d^2+2/9*a^2*f^2*(f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/b^3/d^3+3/16*a*f*(f*x+
e)^2*cosh(d*x+c)^3*sinh(d*x+c)/b^2/d^2-3*a^3*(a^2+b^2)*f*(f*x+e)^2*polylog
(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/d^2-3*a^3*(a^2+b^2)*f*(f*x+e)^2*
polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^6/d^2+6*a^3*(a^2+b^2)*f^2*(
f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/d^3+6*a^3*(a^2+b^2
)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^6/d^3+3/32*a*
(f*x+e)^3/b^2/d-a^3*(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^...
```

3.401.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5147 vs. 2(1443) = 2886.

Time = 11.07 (sec) , antiderivative size = 5147, normalized size of antiderivative = 3.57

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*
x]),x]
```

output
```
Result too large to show
```

3.401.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\sinh^3(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\int (e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{\int \left(-\frac{1}{8}\cosh(c+dx)(e+fx)^3+\frac{1}{16}\cosh(3c+3dx)(e+fx)^3+\frac{1}{16}\cosh(5c+5dx)(e+fx)^3\right)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}}{}$$

$$\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 6113}$$

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}}{}$$

$$\frac{a\left(\frac{\int (e+fx)^3\cosh^3(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 5970}$$

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}}{}$$

$$\frac{a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\int (e+fx)^2\cosh^4(c+dx)dx}{4d}}{b}-\frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^4dx}{4d}}{b}\right)}{b}$$

$$\Big\downarrow\ 3792$$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$\frac{a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\int\cosh^4(c+dx)dx}{8d^2}+\frac{3}{4}\int(e+fx)^2\cosh^2(c+dx)dx-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{4d}}{b}-\frac{a\int\frac{(e+fx)^3}{}\cdots}{}\right.}{b}$$

$$\Big\downarrow\ 3042$$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\int\sin\left(ic+idx+\frac{\pi}{2}\right)^4dx}{8d^2}+\frac{3}{4}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\cosh^4(c+\cdots}{8d^2}\right.}{4d}}{b}\right.}{b}$$

$$\Big\downarrow\ 3115$$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\left(\frac{3}{4}\int\cosh^2(c+dx)dx+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{8d^2}+\frac{3}{4}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\cdots\right.\right.}{4d}}{b}\right.}{b}$$

$$\Big\downarrow\ 3042$$

3.401. $\displaystyle\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\int\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx\right)}{8d^2}+\frac{3}{4}\int(e+fx)^2\sin\left(ic+i\right.\right.}{4d}}{b}\right.}{b}$$

$\downarrow$ 3115

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{f^2\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{8d^2}+\frac{3}{4}\int(e+f\right.}{4}}{b}\right.}{b}$$

$\downarrow$ 24

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh^3}{4d}\right.}{}\right.}{4d}}{b}\right.}{b}$$

$\downarrow$ 3792

3.401. $\displaystyle\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(\frac{f^2\int\cosh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^2dx-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\right.}{4d}}{b}\right.$$

$$b$$

$\big\downarrow$ 17

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(\frac{f^2\int\cosh^2(c+dx)dx}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c+dx)}{8d^2}+\right.}{4d}}{b}\right.$$

$$b$$

$\big\downarrow$ 3042

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$a\left(-\frac{a\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(\frac{f^2\int\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right.\right.}{}\right.$$

$$b$$

$\big\downarrow$ 3115

3.401. $\displaystyle\int\frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(\frac{f^2\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4}{8d^2}\right.}{4d}}{b}\right.$$

$$b$$

$$\Big\downarrow \; 24$$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c}{8d^2}\right.}{4d}}{b}\right.$$

$$b$$

$$\Big\downarrow \; 6113$$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c}{8d^2}\right.}{4d}}{b}\right.$$

$$\Big\downarrow \; 3042$$

3.401. $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}}{a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c}{8d^2}\right.}{4d}}{b}\right.}$$

$\downarrow$ 3792

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}}{a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c}{8d^2}\right.}{4d}}{b}\right.}$$

$\downarrow$ 3042

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}}{a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c}{8d^2}\right.}{4d}}{b}\right.}$$

$\downarrow$ 3777

3.401. $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}\cdots}{a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c\cdots}{8d^2}\cdots\right)}{4d}}{b}\cdots\right.}$$

$$\downarrow 26$$

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}\cdots}{a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c\cdots}{8d^2}\cdots\right)}{4d}}{b}\cdots\right.}$$

$$\downarrow 3042$$

$$\frac{\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}\cdots}{a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c\cdots}{8d^2}\cdots\right)}{4d}}{b}\cdots\right.}$$

$$\downarrow 26$$

3.401. $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}\cdots$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c\cdots}{8d^2}\cdots\right)}{4d}}{b}\cdots\right.$$

$$\Big\downarrow \; 3777$$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}\cdots$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c\cdots}{8d^2}\cdots\right)}{4d}}{b}\cdots\right.$$

$$\Big\downarrow \; 3042$$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}\cdots$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c\cdots}{8d^2}\cdots\right)}{4d}}{b}\cdots\right.$$

3.401. $\displaystyle\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3777}$$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c}{8d^2}}{4d}}{b}\right.$$

$$\Big\downarrow \text{26}$$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c}{8d^2}}{4d}}{b}\right.$$

3.401. $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{3f^3\cosh(c+dx)}{4d^4}-\frac{f^3\cosh(3c+3dx)}{216d^4}-\frac{3f^3\cosh(5c+5dx)}{5000d^4}-\frac{3f^2(e+fx)\sinh(c+dx)}{4d^3}+\frac{f^2(e+fx)\sinh(3c+3dx)}{72d^3}+\frac{3f^2(e+fx)\sinh(5c+5dx)}{1000d^3}$$

$$a\left(\frac{\frac{(e+fx)^3\cosh^4(c+dx)}{4d}-\frac{3f\left(\frac{3}{4}\left(-\frac{f(e+fx)\cosh^2(c+dx)}{2d^2}+\frac{f^2\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)}{2d^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)-\frac{f(e+fx)\cosh^4(c}{8d^2}\right.}{4d}}{b}\right.$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.401.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.401. $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.401. $\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.401.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)^3 \sinh(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.401.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18801 vs. $2(1347) = 2694$.

Time = 0.53 (sec) , antiderivative size = 18801, normalized size of antiderivative = 13.03

$$\int \frac{(e+fx)^3 \cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

3.401.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.401.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh(dx+c)^3\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
-1/960*e^3*((15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*
x - 2*c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 -
 b^4)*e^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) + 960*(a^5 + a^3*b^2)*(d*x
 + c)/(b^6*d) + (15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(
8*a^4 + 6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2
*c) + 10*(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) + 960*(a^5 + a^3*b^2)
*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d)) - 1/34560000*(86
40000*(a^5*d^4*f^3*e^(5*c) + a^3*b^2*d^4*f^3*e^(5*c))*x^4 + 34560000*(a^5*
d^4*e*f^2*e^(5*c) + a^3*b^2*d^4*e*f^2*e^(5*c))*x^3 + 51840000*(a^5*d^4*e^2
*f*e^(5*c) + a^3*b^2*d^4*e^2*f*e^(5*c))*x^2 - 1728*(125*b^5*d^3*f^3*x^3*e^
(10*c) + 75*(5*d^3*e*f^2 - d^2*f^3)*b^5*x^2*e^(10*c) + 15*(25*d^3*e^2*f -
10*d^2*e*f^2 + 2*d*f^3)*b^5*x*e^(10*c) - 3*(25*d^2*e^2*f - 10*d*e*f^2 + 2*
f^3)*b^5*e^(10*c))*e^(5*d*x) + 16875*(32*a*b^4*d^3*f^3*x^3*e^(9*c) + 24*(4
*d^3*e*f^2 - d^2*f^3)*a*b^4*x^2*e^(9*c) + 12*(8*d^3*e^2*f - 4*d^2*e*f^2 +
d*f^3)*a*b^4*x*e^(9*c) - 3*(8*d^2*e^2*f - 4*d*e*f^2 + f^3)*a*b^4*e^(9*c))*
e^(4*d*x) + 40000*(4*(9*d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*a^2*b^3*e^(8*c) + (
9*d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*b^5*e^(8*c) - 9*(4*a^2*b^3*d^3*f^3*e^(8*c
) + b^5*d^3*f^3*e^(8*c))*x^3 - 9*(4*(3*d^3*e*f^2 - d^2*f^3)*a^2*b^3*e^(8*c
) + (3*d^3*e*f^2 - d^2*f^3)*b^5*e^(8*c))*x^2 - 3*(4*(9*d^3*e^2*f - 6*d^2*e
*f^2 + 2*d*f^3)*a^2*b^3*e^(8*c) + (9*d^3*e^2*f - 6*d^2*e*f^2 + 2*d*f^3)...
```

3.401.8 Giac [F]

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh(dx+c)^3\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
integrate((f*x + e)^3*cosh(d*x + c)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a
), x)
```

3.401.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)^3\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x
)
```

3.402 $\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} \, dx$

3.402.1 Optimal result

Integrand size = 36, antiderivative size = 1049

$$\begin{aligned}
&\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{a^3efx}{2b^4d}+\frac{3aefx}{16b^2d}-\frac{a^3f^2x^2}{4b^4d}+\frac{3af^2x^2}{32b^2d}+\frac{a^3(a^2+b^2)(e+fx)^3}{3b^6f} \\
&-\frac{2a^4f(e+fx)\cosh(c+dx)}{b^5d^2}-\frac{4a^2f(e+fx)\cosh(c+dx)}{3b^3d^2}+\frac{f(e+fx)\cosh(c+dx)}{4bd^2} \\
&-\frac{3af^2\cosh^2(c+dx)}{32b^2d^3}-\frac{2a^2f(e+fx)\cosh^3(c+dx)}{9b^3d^2}-\frac{af^2\cosh^4(c+dx)}{32b^2d^3} \\
&-\frac{a(e+fx)^2\cosh^4(c+dx)}{4b^2d}-\frac{f(e+fx)\cosh(3c+3dx)}{72bd^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200bd^2} \\
&-\frac{a^3(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^6d}-\frac{a^3(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^6d} \\
&-\frac{2a^3(a^2+b^2)f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^6d^2} \\
&-\frac{2a^3(a^2+b^2)f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^6d^2} \\
&+\frac{2a^3(a^2+b^2)f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^6d^3}+\frac{2a^3(a^2+b^2)f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^6d^3} \\
&+\frac{2a^4f^2\sinh(c+dx)}{b^5d^3}+\frac{14a^2f^2\sinh(c+dx)}{9b^3d^3}-\frac{f^2\sinh(c+dx)}{4bd^3} \\
&+\frac{a^4(e+fx)^2\sinh(c+dx)}{b^5d}+\frac{2a^2(e+fx)^2\sinh(c+dx)}{3b^3d} \\
&-\frac{(e+fx)^2\sinh(c+dx)}{8bd}+\frac{a^3f(e+fx)\cosh(c+dx)\sinh(c+dx)}{2b^4d^2} \\
&+\frac{3af(e+fx)\cosh(c+dx)\sinh(c+dx)}{16b^2d^2}+\frac{a^2(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{3b^3d} \\
&+\frac{af(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{8b^2d^2}-\frac{a^3f^2\sinh^2(c+dx)}{4b^4d^3} \\
&-\frac{a^3(e+fx)^2\sinh^2(c+dx)}{2b^4d}+\frac{2a^2f^2\sinh^3(c+dx)}{27b^3d^3}+\frac{f^2\sinh(3c+3dx)}{216bd^3} \\
&+\frac{(e+fx)^2\sinh(3c+3dx)}{48bd}+\frac{f^2\sinh(5c+5dx)}{1000bd^3}+\frac{(e+fx)^2\sinh(5c+5dx)}{80bd}
\end{aligned}$$

output
```
1/4*f*(f*x+e)*cosh(d*x+c)/b/d^2+3/32*a*f^2*x^2/b^2/d+14/9*a^2*f^2*sinh(d*x
+c)/b^3/d^3+2/3*a^2*(f*x+e)^2*sinh(d*x+c)/b^3/d-1/8*(f*x+e)^2*sinh(d*x+c)/
b/d-1/2*a^3*e*f*x/b^4/d-2*a^4*f*(f*x+e)*cosh(d*x+c)/b^5/d^2+1/216*f^2*sinh
(3*d*x+3*c)/b/d^3+1/48*(f*x+e)^2*sinh(3*d*x+3*c)/b/d+1/1000*f^2*sinh(5*d*x
+5*c)/b/d^3+1/80*(f*x+e)^2*sinh(5*d*x+5*c)/b/d+1/2*a^3*f*(f*x+e)*cosh(d*x+
c)*sinh(d*x+c)/b^4/d^2+1/8*a*f*(f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/b^2/d^2-2
*a^3*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/
d^2-2*a^3*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))
/b^6/d^2-a^3*(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^
6/d-a^3*(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^6/d+a
^4*(f*x+e)^2*sinh(d*x+c)/b^5/d+3/16*a*f*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b^
2/d^2-1/4*f^2*sinh(d*x+c)/b/d^3-2/9*a^2*f*(f*x+e)*cosh(d*x+c)^3/b^3/d^2+1/
3*a^2*(f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/b^3/d+2*a^3*(a^2+b^2)*f^2*polylo
g(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/d^3+2*a^3*(a^2+b^2)*f^2*polylog
(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^6/d^3+3/16*a*e*f*x/b^2/d-4/3*a^2*f
*(f*x+e)*cosh(d*x+c)/b^3/d^2-1/4*a^3*f^2*x^2/b^4/d+1/3*a^3*(a^2+b^2)*(f*x+
e)^3/b^6/f-3/32*a*f^2*cosh(d*x+c)^2/b^2/d^3-1/32*a*f^2*cosh(d*x+c)^4/b^2/d
^3-1/4*a*(f*x+e)^2*cosh(d*x+c)^4/b^2/d-1/72*f*(f*x+e)*cosh(3*d*x+3*c)/b/d^
2-1/200*f*(f*x+e)*cosh(5*d*x+5*c)/b/d^2+2*a^4*f^2*sinh(d*x+c)/b^5/d^3-1/4*
a^3*f^2*sinh(d*x+c)^2/b^4/d^3-1/2*a^3*(f*x+e)^2*sinh(d*x+c)^2/b^4/d+2/2...
```

3.402.2 Mathematica [A] (verified)

Time = 9.73 (sec) , antiderivative size = 1811, normalized size of antiderivative = 1.73

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*
x]),x]
```

output
```
((-8*a^3*(a^2 + b^2)*e^2*x*Coth[c])/b^6 - (8*a^3*(a^2 + b^2)*e*f*x^2*Coth[
c])/b^6 - (8*a^3*(a^2 + b^2)*f^2*x^3*Coth[c])/(3*b^6) + (8*a^3*(a^2 + b^2)
*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2
+ b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2
)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))
/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*
ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*
a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*
(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d
*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E
^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sq
rt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x)
)/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d
*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b
*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1
 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*
f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d
- (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c -...
```

3.402.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^3(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6113$$

$$\frac{\int (e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 5971$$

$$\frac{\int \left(-\frac{1}{8}\cosh(c+dx)(e+fx)^2+\frac{1}{16}\cosh(3c+3dx)(e+fx)^2+\frac{1}{16}\cosh(5c+5dx)(e+fx)^2\right)dx}{b}-\frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e}{b}$$

$$\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 6113$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e}{b}$$

$$\frac{a\left(\frac{\int(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 5970$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e}{b}$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\int(e+fx)\cosh^4(c+dx)dx}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^4dx}{2d}}{b}\right)}{b}$$

$$\Big\downarrow \; 3791$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e}{b}$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\int(e+fx)\cosh^2(c+dx)dx-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

3.402. $\displaystyle\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e\cdots}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\cdots}{2d}}{b}\cdots\right.}{b}$$

$$\Big\downarrow \; 3791$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e\cdots}{b}$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(\frac{1}{2}\int(e+fx)dx-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\int\cdots}{\cdots}\right.}{b}$$

$$\Big\downarrow \; 17$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e\cdots}{b}$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\int\frac{(e\cdots}{\cdots}}{\cdots}\right.}{b}$$

$$\Big\downarrow \; 6113$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e\cdots}{b}$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\int(\cdots\right.}{\cdots}\right.}{b}$$

$$\Big\downarrow \; 3042$$

3.402. $\displaystyle\int\frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\frac{(e\cdots}{\cdots}}{b}$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\cdots\right.}{\cdots}\right.}{b}$$

$\downarrow$ 3792

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\frac{(e\cdots}{\cdots}}{b}$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\frac{2\cdots}{\cdots}\right.}{\cdots}\right.}{b}$$

$\downarrow$ 3042

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\frac{(e\cdots}{\cdots}}{b}$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\cdots\right.}{\cdots}\right.}{b}$$

$\downarrow$ 3113

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\frac{(e\cdots}{\cdots}}{b}$$

$$\frac{a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\cdots\right.}{\cdots}\right.}{\cdots}$$

$\downarrow$ 2009

3.402. $\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e \ldots \over b$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\ldots\right.}{\ldots}\right.$$

$$\Big\downarrow \text{3777}$$

$$-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e \ldots \over b$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\ldots\right.}{\ldots}\right.$$

$$\Big\downarrow \text{26}$$

$$-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e \ldots \over b$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\ldots\right.}{\ldots}\right.$$

$$\Big\downarrow \text{3042}$$

$$-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e \ldots \over b$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\ldots\right.}{\ldots}\right.$$

$$\Big\downarrow \text{26}$$

3.402. $\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\frac{(e}{}}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\right.}{}\right.$$

$$\Big\downarrow \; 3777$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\frac{(e}{}}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\right.}{}\right.$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\frac{(e}{}}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\right.}{}\right.$$

$$\Big\downarrow \; 3117$$

3.402. $\displaystyle\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e\cdots}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\cdots\right.}{\cdots}\right.$$

$\big\downarrow$ 6099

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e\cdots}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\cdots\right.}{\cdots}\right.$$

$\big\downarrow$ 3042

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e\cdots}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\frac{2i\cdots}{\cdots}-\cdots\right.}{\cdots}\right.$$

$\big\downarrow$ 3777

3.402. $\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\frac{2i}{}\ -\right.}{}\right.$$

$\downarrow$ 26

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-(e}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(-\right.}{}\right.$$

$\downarrow$ 3042

3.402. $\displaystyle\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\frac{(e}{}}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\frac{2i}{}\ -\right.}{}\right.$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\frac{(e}{}}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\frac{2i}{}\ -\right.}{}\right.$$

$$\Big\downarrow \text{3777}$$

3.402. $\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{f^2\sinh(c+dx)}{4d^3}+\frac{f^2\sinh(3c+3dx)}{216d^3}+\frac{f^2\sinh(5c+5dx)}{1000d^3}+\frac{f(e+fx)\cosh(c+dx)}{4d^2}-\frac{f(e+fx)\cosh(3c+3dx)}{72d^2}-\frac{f(e+fx)\cosh(5c+5dx)}{200d^2}-\frac{(e\cdots}{b}$$

$$a\left(\frac{\frac{(e+fx)^2\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(-\frac{f\cosh^2(c+dx)}{4d^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^2}{4f}\right)-\frac{f\cosh^4(c+dx)}{16d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{2d}}{b}-\frac{a\left(\frac{2i\cdots}{\cdots}\right)}{\cdots}\right.$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.402.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.402. $\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.402.4 Maple [F]

$$\int \frac{(fx+e)^2 \cosh(dx+c)^3 \sinh(dx+c)^3}{a+b\sinh(dx+c)}\,dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.402.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11318 vs. 2(973) = 1946.

Time = 0.41 (sec) , antiderivative size = 11318, normalized size of antiderivative = 10.79

$$\int \frac{(e+fx)^2 \cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

3.402.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.402.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh(dx+c)^3\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
-1/960*e^2*((15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*
x - 2*c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 -
 b^4)*e^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) + 960*(a^5 + a^3*b^2)*(d*x
 + c)/(b^6*d) + (15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(
8*a^4 + 6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2
*c) + 10*(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) + 960*(a^5 + a^3*b^2)
*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d)) - 1/1728000*(576
000*(a^5*d^3*f^2*e^(5*c) + a^3*b^2*d^3*f^2*e^(5*c))*x^3 + 1728000*(a^5*d^3
*e*f*e^(5*c) + a^3*b^2*d^3*e*f*e^(5*c))*x^2 - 432*(25*b^5*d^2*f^2*x^2*e^(1
0*c) + 10*(5*d^2*e*f - d*f^2)*b^5*x*e^(10*c) - 2*(5*d*e*f - f^2)*b^5*e^(10
*c))*e^(5*d*x) + 3375*(8*a*b^4*d^2*f^2*x^2*e^(9*c) + 4*(4*d^2*e*f - d*f^2)
*a*b^4*x*e^(9*c) - (4*d*e*f - f^2)*a*b^4*e^(9*c))*e^(4*d*x) + 2000*(8*(3*d
*e*f - f^2)*a^2*b^3*e^(8*c) + 2*(3*d*e*f - f^2)*b^5*e^(8*c) - 9*(4*a^2*b^3
*d^2*f^2*e^(8*c) + b^5*d^2*f^2*e^(8*c))*x^2 - 6*(4*(3*d^2*e*f - d*f^2)*a^2
*b^3*e^(8*c) + (3*d^2*e*f - d*f^2)*b^5*e^(8*c))*x)*e^(3*d*x) - 54000*(2*(2
*d*e*f - f^2)*a^3*b^2*e^(7*c) + (2*d*e*f - f^2)*a*b^4*e^(7*c) - 2*(2*a^3*b
^2*d^2*f^2*e^(7*c) + a*b^4*d^2*f^2*e^(7*c))*x^2 - 2*(2*(2*d^2*e*f - d*f^2)
*a^3*b^2*e^(7*c) + (2*d^2*e*f - d*f^2)*a*b^4*e^(7*c))*x)*e^(2*d*x) + 10800
0*(16*(d*e*f - f^2)*a^4*b*e^(6*c) + 12*(d*e*f - f^2)*a^2*b^3*e^(6*c) - 2*(
d*e*f - f^2)*b^5*e^(6*c) - (8*a^4*b*d^2*f^2*e^(6*c) + 6*a^2*b^3*d^2*f^2...
```

3.402.8 Giac [F]

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^3\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
integrate((f*x + e)^2*cosh(d*x + c)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a
), x)
```

3.402.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x
)
```

3.403 $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.403.1 Optimal result

Integrand size = 34, antiderivative size = 641

$$\begin{aligned}
&\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{a^3fx}{4b^4d}+\frac{3afx}{32b^2d}+\frac{a^3(a^2+b^2)\,(e+fx)^2}{2b^6f}-\frac{a^4f\cosh(c+dx)}{b^5d^2} \\
&\quad-\frac{2a^2f\cosh(c+dx)}{3b^3d^2}+\frac{f\cosh(c+dx)}{8bd^2}-\frac{a^2f\cosh^3(c+dx)}{9b^3d^2} \\
&\quad-\frac{a(e+fx)\cosh^4(c+dx)}{4b^2d}-\frac{f\cosh(3c+3dx)}{144bd^2}-\frac{f\cosh(5c+5dx)}{400bd^2} \\
&\quad-\frac{a^3(a^2+b^2)\,(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^6d}-\frac{a^3(a^2+b^2)\,(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^6d} \\
&\quad-\frac{a^3(a^2+b^2)\,f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^6d^2}-\frac{a^3(a^2+b^2)\,f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^6d^2} \\
&\quad+\frac{a^4(e+fx)\sinh(c+dx)}{b^5d}+\frac{2a^2(e+fx)\sinh(c+dx)}{3b^3d}-\frac{(e+fx)\sinh(c+dx)}{8bd} \\
&\quad+\frac{a^3f\cosh(c+dx)\sinh(c+dx)}{4b^4d^2}+\frac{3af\cosh(c+dx)\sinh(c+dx)}{32b^2d^2} \\
&\quad+\frac{a^2(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{3b^3d}+\frac{af\cosh^3(c+dx)\sinh(c+dx)}{16b^2d^2} \\
&\quad-\frac{a^3(e+fx)\sinh^2(c+dx)}{2b^4d}+\frac{(e+fx)\sinh(3c+3dx)}{48bd}+\frac{(e+fx)\sinh(5c+5dx)}{80bd}
\end{aligned}$$

output
```
-1/4*a^3*f*x/b^4/d+3/32*a*f*x/b^2/d+1/2*a^3*(a^2+b^2)*(f*x+e)^2/b^6/f-a^4*
f*cosh(d*x+c)/b^5/d^2-2/3*a^2*f*cosh(d*x+c)/b^3/d^2+1/8*f*cosh(d*x+c)/b/d^
2-1/9*a^2*f*cosh(d*x+c)^3/b^3/d^2-1/4*a*(f*x+e)*cosh(d*x+c)^4/b^2/d-1/144*
f*cosh(3*d*x+3*c)/b/d^2-1/400*f*cosh(5*d*x+5*c)/b/d^2-a^3*(a^2+b^2)*(f*x+e
)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/d-a^3*(a^2+b^2)*(f*x+e)*ln(1+
b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^6/d-a^3*(a^2+b^2)*f*polylog(2,-b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/d^2-a^3*(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)
/(a+(a^2+b^2)^(1/2)))/b^6/d^2+a^4*(f*x+e)*sinh(d*x+c)/b^5/d+2/3*a^2*(f*x+e
)*sinh(d*x+c)/b^3/d-1/8*(f*x+e)*sinh(d*x+c)/b/d+1/4*a^3*f*cosh(d*x+c)*sinh
(d*x+c)/b^4/d^2+3/32*a*f*cosh(d*x+c)*sinh(d*x+c)/b^2/d^2+1/3*a^2*(f*x+e)*c
osh(d*x+c)^2*sinh(d*x+c)/b^3/d+1/16*a*f*cosh(d*x+c)^3*sinh(d*x+c)/b^2/d^2-
1/2*a^3*(f*x+e)*sinh(d*x+c)^2/b^4/d+1/48*(f*x+e)*sinh(3*d*x+3*c)/b/d+1/80*
(f*x+e)*sinh(5*d*x+5*c)/b/d
```

3.403.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2704 vs. $2(641) = 1282$.

Time = 8.52 (sec) , antiderivative size = 2704, normalized size of antiderivative = 4.22

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]
),x]
```

output
```
((-4*a^3*(a^2 + b^2)*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 +
 (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sq
rt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d
*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] +
 2*d*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b
*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))]]))/(b^6*d^2) + ((Cosh[5*(c + d*x)]/(7200*b^5*d) - Si
nh[5*(c + d*x)]/(7200*b^5*d))*(-360*b^4*d*e - 72*b^4*f + 360*b^4*c*f - 360
*b^4*f*(c + d*x) - 900*a*b^3*d*e*Cosh[c + d*x] - 225*a*b^3*f*Cosh[c + d*x]
 + 900*a*b^3*c*f*Cosh[c + d*x] - 900*a*b^3*f*(c + d*x)*Cosh[c + d*x] - 240
0*a^2*b^2*d*e*Cosh[2*(c + d*x)] - 600*b^4*d*e*Cosh[2*(c + d*x)] - 800*a^2*
b^2*f*Cosh[2*(c + d*x)] - 200*b^4*f*Cosh[2*(c + d*x)] + 2400*a^2*b^2*c*f*C
osh[2*(c + d*x)] + 600*b^4*c*f*Cosh[2*(c + d*x)] - 2400*a^2*b^2*f*(c + d*x
)*Cosh[2*(c + d*x)] - 600*b^4*f*(c + d*x)*Cosh[2*(c + d*x)] - 7200*a^3*b*d
*e*Cosh[3*(c + d*x)] - 3600*a*b^3*d*e*Cosh[3*(c + d*x)] - 3600*a^3*b*f*Cos
h[3*(c + d*x)] - 1800*a*b^3*f*Cosh[3*(c + d*x)] + 7200*a^3*b*c*f*Cosh[3*(c
 + d*x)] + 3600*a*b^3*c*f*Cosh[3*(c + d*x)] - 7200*a^3*b*f*(c + d*x)*Cosh[
3*(c + d*x)] - 3600*a*b^3*f*(c + d*x)*Cosh[3*(c + d*x)] - 28800*a^4*d*e...
```

3.403.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^3(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6113$$

$$\frac{\int (e+fx)\cosh^3(c+dx)\sinh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 5971$$

$$\frac{\int \left(-\frac{1}{8}(e+fx)\cosh(c+dx)+\frac{1}{16}(e+fx)\cosh(3c+3dx)+\frac{1}{16}(e+fx)\cosh(5c+5dx)\right)dx}{b}-\frac{a\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{6113}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\int(e+fx)\cosh^3(c+dx)\sinh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{5970}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\int\cosh^4(c+dx)dx}{4d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\int\sin\left(ic+idx+\frac{\pi}{2}\right)^4dx}{4d}}{b}\right)}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\int\cosh^2(c+dx)dx+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{4d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

3.403. $\displaystyle\int\frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\int\sin\left(ic+idx+\frac{\pi}{2}\right)^2dx\right)}{4d}}{b}\right)}{b}$$

$$\Big\downarrow \; 3115$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}\right)}{4d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 6113$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\int(e+fx)\cosh^3(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

3.403. $\int\frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 3791

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\int(e+fx)\cosh(c+dx)dx-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3d}}{b}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx \ldots}{b}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 3777

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int \ldots}{\ldots}\right) \ldots}{b}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 26

3.403. $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)-\frac{f\cosh^3(c+dx)}{9d^2}\cdots}{b}\cdots\right.}{\cdots}\right.}{b}$$

$\downarrow$ 3042

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-f\int-\cdots\right.}{\cdots}\right.}{\cdots}\right.}{b}$$

$\downarrow$ 26

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}+if\int\cdots\right.}{\cdots}\right.}{\cdots}\right.}{b}$$

$\downarrow$ 3118

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+(e\cdots}{b}\cdots\right.}{b}\right.}{b}$$

$\downarrow$ 6099

3.403. $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+(e}{b}\right.}{}\right.$$

$$b$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+(e}{b}\right.}{}\right.$$

$$b$$

$$\Big\downarrow \; 3777$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+(e}{b}\right.}{}\right.$$

$$\Big\downarrow \; 26$$

3.403. $\displaystyle\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e\cdots}{\cdots}}{b}\cdots\right.}{\cdots}\cdots\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e\cdots}{\cdots}}{b}\cdots\right.}{\cdots}\cdots\right)}{\cdots}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e\cdots}{\cdots}}{b}\cdots\right.}{\cdots}\cdots\right)}{b}$$

$$\Big\downarrow \; 3118$$

3.403. $\displaystyle \int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e\cdots}{\cdots}}{b}\cdots\right)}{\cdots}\right)}{b}$$

$$\Big\downarrow \textbf{5969}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e\cdots}{\cdots}}{b}\cdots\right)}{\cdots}\right)}{\cdots}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$\frac{a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e\cdots}{\cdots}}{b}\cdots\right)}{\cdots}\right)}{\cdots}$$

$$\Big\downarrow \textbf{25}$$

3.403. $\displaystyle\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e\cdots}{}}{b}\right.}{}\right.$$

$$\Big\downarrow \color{blue}{3115}$$

$$\frac{\frac{f\cosh(c+dx)}{8d^2}-\frac{f\cosh(3c+3dx)}{144d^2}-\frac{f\cosh(5c+5dx)}{400d^2}-\frac{(e+fx)\sinh(c+dx)}{8d}+\frac{(e+fx)\sinh(3c+3dx)}{48d}+\frac{(e+fx)\sinh(5c+5dx)}{80d}}{b}-$$

$$a\left(\frac{\frac{(e+fx)\cosh^4(c+dx)}{4d}-\frac{f\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)\right)}{4d}}{b}-\frac{a\left(\frac{\frac{2}{3}\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)-\frac{f\cosh^3(c+dx)}{9d^2}+\frac{(e\cdots}{}}{b}\right.}{}\right.$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.403. $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.403.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.403. $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5970
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6113
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - S
imp[a/b   Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.403.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. $2(593) = 1186$.

Time = 187.96 (sec) , antiderivative size = 1363, normalized size of antiderivative = 2.13

method	result	size
risch	Expression too large to display	1363

input
```
int((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURN
VERBOSE)
```

output
```
-1/d^2*a^3/b^4*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))
)*c-1/d^2*a^3/b^4*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)
))*c-1/d*a^3/b^4*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))*x-1/d*a^3/b^4*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)
))*x-1/256*a*(4*d*f*x+4*d*e-f)/b^2/d^2*exp(4*d*x+4*c)-1/32*a*(4*a^2*d*f*x+
2*b^2*d*f*x+4*a^2*d*e+2*b^2*d*e-2*a^2*f-b^2*f)/b^4/d^2*exp(2*d*x+2*c)+1/d^
2*a^5/b^6*c*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d*a^5/b^6*f*ln((-b*e
xp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d*a^5/b^6*f*ln((b*e
xp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a^5/b^6*f*ln((-b
*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2*a^5/b^6*f*ln(
(b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2*a^5/b^6*f*c^
2+1/16*(8*a^4*d*f*x+6*a^2*b^2*d*f*x-b^4*d*f*x+8*a^4*d*e+6*a^2*b^2*d*e-b^4*
d*e-8*a^4*f-6*a^2*b^2*f+b^4*f)/b^5/d^2*exp(d*x+c)-1/16*(8*a^4+6*a^2*b^2-b^
4)*(d*f*x+d*e+f)/b^5/d^2*exp(-d*x-c)-1/288*(4*a^2+b^2)*(3*d*f*x+3*d*e+f)/b
^3/d^2*exp(-3*d*x-3*c)-1/256*a*(4*d*f*x+4*d*e+f)/b^2/d^2*exp(-4*d*x-4*c)-1
/d^2*a^3/b^4*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))
-1/d*a^3/b^4*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-a^3*e*x/b^4+1/d^2*a^3
/b^4*f*c^2+2/d*a^3/b^4*e*ln(exp(d*x+c))-1/d^2*a^3/b^4*f*dilog((-b*exp(d*x+
c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*a^5/b^6*f*dilog((-b*exp(
d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*a^5/b^6*f*dilog((...
```

3.403.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5548 vs. $2(591) = 1182$.

Time = 0.33 (sec) , antiderivative size = 5548, normalized size of antiderivative = 8.66

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.403.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.403.7 Maxima [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^3\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/960*e*((15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*x
- 2*c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 - b
^4)*e^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) + 960*(a^5 + a^3*b^2)*(d*x +
 c)/(b^6*d) + (15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(8*
a^4 + 6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2*c
) + 10*(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) + 960*(a^5 + a^3*b^2)*l
og(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d)) - 1/57600*f*((2880
0*(a^5*d^2*e^(5*c) + a^3*b^2*d^2*e^(5*c))*x^2 - 72*(5*b^5*d*x*e^(10*c) - b
^5*e^(10*c))*e^(5*d*x) + 225*(4*a*b^4*d*x*e^(9*c) - a*b^4*e^(9*c))*e^(4*d*
x) + 200*(4*a^2*b^3*e^(8*c) + b^5*e^(8*c) - 3*(4*a^2*b^3*d*e^(8*c) + b^5*d
*e^(8*c))*x)*e^(3*d*x) - 1800*(2*a^3*b^2*e^(7*c) + a*b^4*e^(7*c) - 2*(2*a^
3*b^2*d*e^(7*c) + a*b^4*d*e^(7*c))*x)*e^(2*d*x) + 3600*(8*a^4*b*e^(6*c) +
6*a^2*b^3*e^(6*c) - b^5*e^(6*c) - (8*a^4*b*d*e^(6*c) + 6*a^2*b^3*d*e^(6*c)
 - b^5*d*e^(6*c))*x)*e^(d*x) + 3600*(8*a^4*b*e^(4*c) + 6*a^2*b^3*e^(4*c) -
 b^5*e^(4*c) + (8*a^4*b*d*e^(4*c) + 6*a^2*b^3*d*e^(4*c) - b^5*d*e^(4*c))*x
)*e^(-d*x) + 1800*(2*a^3*b^2*e^(3*c) + a*b^4*e^(3*c) + 2*(2*a^3*b^2*d*e^(3
*c) + a*b^4*d*e^(3*c))*x)*e^(-2*d*x) + 200*(4*a^2*b^3*e^(2*c) + b^5*e^(2*c
) + 3*(4*a^2*b^3*d*e^(2*c) + b^5*d*e^(2*c))*x)*e^(-3*d*x) + 225*(4*a*b^4*d
*x*e^c + a*b^4*e^c)*e^(-4*d*x) + 72*(5*b^5*d*x + b^5)*e^(-5*d*x))*e^(-5*c)
/(b^6*d^2) - 900*integrate(128*((a^6*e^c + a^4*b^2*e^c)*x*e^(d*x) - (a^...
```

3.403.8 Giac [F]

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate((f*x + e)*cosh(d*x + c)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a),
 x)
```

3.403.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3\sinh(c+d\,x)^3\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.404 $\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.404.1 Optimal result

Integrand size = 29, antiderivative size = 141

$$\begin{aligned}\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = &-\frac{a^3(a^2+b^2)\log(a+b\sinh(c+dx))}{b^6d} \\ &+\frac{a^2(a^2+b^2)\sinh(c+dx)}{b^5d} \\ &-\frac{a(a^2+b^2)\sinh^2(c+dx)}{2b^4d}+\frac{(a^2+b^2)\sinh^3(c+dx)}{3b^3d} \\ &-\frac{a\sinh^4(c+dx)}{4b^2d}+\frac{\sinh^5(c+dx)}{5bd}\end{aligned}$$

output
```
-a^3*(a^2+b^2)*ln(a+b*sinh(d*x+c))/b^6/d+a^2*(a^2+b^2)*sinh(d*x+c)/b^5/d-1
/2*a*(a^2+b^2)*sinh(d*x+c)^2/b^4/d+1/3*(a^2+b^2)*sinh(d*x+c)^3/b^3/d-1/4*a
*sinh(d*x+c)^4/b^2/d+1/5*sinh(d*x+c)^5/b/d
```

3.404.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{\frac{60a^3(a^2+b^2)\log(a+b\sinh(c+dx))}{b^6}-\frac{60a^2(a^2+b^2)\sinh(c+dx)}{b^5}+\frac{30a(a^2+b^2)\sinh^2(c+dx)}{b^4}-\frac{20(a^2+b^2)\sinh^3(c+dx)}{b^3}+\frac{15a\sinh^4(c+}{b^2}}{60d}$$

input
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/60*((60*a^3*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/b^6 - (60*a^2*(a^2 +
b^2)*Sinh[c + d*x])/b^5 + (30*a*(a^2 + b^2)*Sinh[c + d*x]^2)/b^4 - (20*(a^
2 + b^2)*Sinh[c + d*x]^3)/b^3 + (15*a*Sinh[c + d*x]^4)/b^2 - (12*Sinh[c +
d*x]^5)/b)/d
```

3.404.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {3042, 26, 3316, 26, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ic+idx)^3\cos(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos(ic+idx)^3\sin(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3316}$$

$$-\frac{i\int \frac{i\sinh^3(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\sinh^3(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b^3\sinh^3(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^6d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(b^4 \sinh^4(c+dx) - ab^3 \sinh^3(c+dx) + b^2\left(a^2+b^2\right)\sinh^2(c+dx) - ab\left(a^2+b^2\right)\sinh(c+dx) + a^2\left(a^2+b^2\right) - \frac{}{a}\right.}{b^6 d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}ab^2\left(a^2+b^2\right)\sinh^2(c+dx) + a^2b\left(a^2+b^2\right)\sinh(c+dx) + \frac{1}{3}b^3\left(a^2+b^2\right)\sinh^3(c+dx) - a^3\left(a^2+b^2\right)\log(a+b\,\mathrm{si}}{b^6 d}$$

input
```
Int[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(a^3*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]]) + a^2*b*(a^2 + b^2)*Sinh[c +
d*x] - (a*b^2*(a^2 + b^2)*Sinh[c + d*x]^2)/2 + (b^3*(a^2 + b^2)*Sinh[c + d
*x]^3)/3 - (a*b^4*Sinh[c + d*x]^4)/4 + (b^5*Sinh[c + d*x]^5)/5)/(b^6*d)
```

3.404.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

3.404.4 Maple [A] (verified)

Time = 99.05 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{\frac{\sinh(dx+c)^5 b^4}{5}-\frac{a\sinh(dx+c)^4 b^3}{4}+\frac{a^2b^2\sinh(dx+c)^3}{3}+\frac{b^4\sinh(dx+c)^3}{3}-\frac{a^3b\sinh(dx+c)^2}{2}-\frac{a\,b^3\sinh(dx+c)^2}{2}+a^4\sinh(dx+c)+a^2b^2}{b^5}}{d}$
default	$\frac{\frac{\frac{\sinh(dx+c)^5 b^4}{5}-\frac{a\sinh(dx+c)^4 b^3}{4}+\frac{a^2b^2\sinh(dx+c)^3}{3}+\frac{b^4\sinh(dx+c)^3}{3}-\frac{a^3b\sinh(dx+c)^2}{2}-\frac{a\,b^3\sinh(dx+c)^2}{2}+a^4\sinh(dx+c)+a^2b^2}{b^5}}{d}$
risch	$-\frac{e^{-3dx-3c}a^2}{24b^3d}+\frac{2a^5c}{b^6d}-\frac{a^5\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{b^6d}-\frac{a^3e^{2dx+2c}}{8b^4d}-\frac{e^{-3dx-3c}}{96bd}+\frac{e^{dx+c}a^4}{2b^5d}-\frac{e^{-dx-c}a^4}{2b^5d}-$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^5*(1/5*sinh(d*x+c)^5*b^4-1/4*a*sinh(d*x+c)^4*b^3+1/3*a^2*b^2*sinh
(d*x+c)^3+1/3*b^4*sinh(d*x+c)^3-1/2*a^3*b*sinh(d*x+c)^2-1/2*a*b^3*sinh(d*x
+c)^2+a^4*sinh(d*x+c)+a^2*b^2*sinh(d*x+c))-a^3*(a^2+b^2)/b^6*ln(a+b*sinh(d
*x+c)))
```

3.404.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1660 vs. $2(133) = 266$.

Time = 0.28 (sec) , antiderivative size = 1660, normalized size of antiderivative = 11.77

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fric
as")
```

output
```
1/960*(6*b^5*cosh(d*x + c)^10 + 6*b^5*sinh(d*x + c)^10 - 15*a*b^4*cosh(d*x
 + c)^9 + 15*(4*b^5*cosh(d*x + c) - a*b^4)*sinh(d*x + c)^9 + 10*(4*a^2*b^3
 + b^5)*cosh(d*x + c)^8 + 5*(54*b^5*cosh(d*x + c)^2 - 27*a*b^4*cosh(d*x +
c) + 8*a^2*b^3 + 2*b^5)*sinh(d*x + c)^8 + 960*(a^5 + a^3*b^2)*d*x*cosh(d*x
 + c)^5 - 60*(2*a^3*b^2 + a*b^4)*cosh(d*x + c)^7 + 20*(36*b^5*cosh(d*x + c
)^3 - 27*a*b^4*cosh(d*x + c)^2 - 6*a^3*b^2 - 3*a*b^4 + 4*(4*a^2*b^3 + b^5)
*cosh(d*x + c))*sinh(d*x + c)^7 + 60*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x
+ c)^6 + 20*(63*b^5*cosh(d*x + c)^4 - 63*a*b^4*cosh(d*x + c)^3 + 24*a^4*b
+ 18*a^2*b^3 - 3*b^5 + 14*(4*a^2*b^3 + b^5)*cosh(d*x + c)^2 - 21*(2*a^3*b^
2 + a*b^4)*cosh(d*x + c))*sinh(d*x + c)^6 - 15*a*b^4*cosh(d*x + c) + 2*(75
6*b^5*cosh(d*x + c)^5 - 945*a*b^4*cosh(d*x + c)^4 + 280*(4*a^2*b^3 + b^5)*
cosh(d*x + c)^3 + 480*(a^5 + a^3*b^2)*d*x - 630*(2*a^3*b^2 + a*b^4)*cosh(d
*x + c)^2 + 180*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c))*sinh(d*x + c)^5
 - 6*b^5 - 60*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c)^4 + 10*(126*b^5*co
sh(d*x + c)^6 - 189*a*b^4*cosh(d*x + c)^5 - 48*a^4*b - 36*a^2*b^3 + 6*b^5
+ 70*(4*a^2*b^3 + b^5)*cosh(d*x + c)^4 + 480*(a^5 + a^3*b^2)*d*x*cosh(d*x
+ c) - 210*(2*a^3*b^2 + a*b^4)*cosh(d*x + c)^3 + 90*(8*a^4*b + 6*a^2*b^3 -
 b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 60*(2*a^3*b^2 + a*b^4)*cosh(d*x +
 c)^3 + 20*(36*b^5*cosh(d*x + c)^7 - 63*a*b^4*cosh(d*x + c)^6 + 28*(4*a^2*
b^3 + b^5)*cosh(d*x + c)^5 - 6*a^3*b^2 - 3*a*b^4 + 480*(a^5 + a^3*b^2)*...
```

3.404.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.404.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(133) = 266$.

Time = 0.22 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.13

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(15\,ab^3e^{(-dx-c)} - 6\,b^4 - 10\,(4\,a^2b^2+b^4)e^{(-2\,dx-2\,c)} + 60\,(2\,a^3b+ab^3)e^{(-3\,dx-3\,c)} - 60\,(8\,a^4+6\,a^2b^2-b^4)}{960\,b^5d} - \frac{(a^5+a^3b^2)(dx+c)}{b^6d} - \frac{15\,ab^3e^{(-4\,dx-4\,c)} + 6\,b^4e^{(-5\,dx-5\,c)} + 60\,(8\,a^4+6\,a^2b^2-b^4)e^{(-dx-c)} + 60\,(2\,a^3b+ab^3)e^{(-2\,dx-2\,c)} + 10\,(4\,a}{960\,b^5d} - \frac{(a^5+a^3b^2)\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{b^6d}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxi
ma")
```

output
```
-1/960*(15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*x - 2
*c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 - b^4)
*e^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) - (a^5 + a^3*b^2)*(d*x + c)/(b^
6*d) - 1/960*(15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(8*a
^4 + 6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2*c)
 + 10*(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) - (a^5 + a^3*b^2)*log(-2
*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d)
```

3.404.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.83

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{6\,b^4\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5 - 15\,ab^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^4 + 40\,a^2b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3 + 40\,b^4\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3 - 120\,a^3b\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{b^5}}{96}$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac
")
```

output
```
1/960*((6*b^4*(e^(d*x + c) - e^(-d*x - c))^5 - 15*a*b^3*(e^(d*x + c) - e^(
-d*x - c))^4 + 40*a^2*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 40*b^4*(e^(d*x
+ c) - e^(-d*x - c))^3 - 120*a^3*b*(e^(d*x + c) - e^(-d*x - c))^2 - 120*a*
b^3*(e^(d*x + c) - e^(-d*x - c))^2 + 480*a^4*(e^(d*x + c) - e^(-d*x - c))
+ 480*a^2*b^2*(e^(d*x + c) - e^(-d*x - c)))/b^5 - 960*(a^5 + a^3*b^2)*log(
abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/b^6)/d
```

3.404.9 Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.18

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{5c+5dx}}{160\,b\,d} - \frac{e^{-5c-5dx}}{160\,b\,d} - \frac{a\,e^{-4c-4dx}}{64\,b^2\,d} - \frac{a\,e^{4c+4dx}}{64\,b^2\,d} - \frac{\ln\left(2\,a\,e^{dx}\,e^{c} - b + b\,e^{2c}\,e^{2dx}\right)\left(a^5 + a^3\,b^2\right)}{b^6\,d} - \frac{e^{-c-dx}\left(8\,a^4 + 6\,a^2\,b^2 - b^4\right)}{16\,b^5\,d} + \frac{a^3\,x\left(a^2 + b^2\right)}{b^6} - \frac{e^{-2c-2dx}\left(2\,a^3 + a\,b^2\right)}{16\,b^4\,d} - \frac{e^{2c+2dx}\left(2\,a^3 + a\,b^2\right)}{16\,b^4\,d} + \frac{e^{c+dx}\left(8\,a^4 + 6\,a^2\,b^2 - b^4\right)}{16\,b^5\,d} - \frac{e^{-3c-3dx}\left(4\,a^2 + b^2\right)}{96\,b^3\,d} + \frac{e^{3c+3dx}\left(4\,a^2 + b^2\right)}{96\,b^3\,d}$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
exp(5*c + 5*d*x)/(160*b*d) - exp(- 5*c - 5*d*x)/(160*b*d) - (a*exp(- 4*c -
 4*d*x))/(64*b^2*d) - (a*exp(4*c + 4*d*x))/(64*b^2*d) - (log(2*a*exp(d*x)*
exp(c) - b + b*exp(2*c)*exp(2*d*x))*(a^5 + a^3*b^2))/(b^6*d) - (exp(- c -
d*x)*(8*a^4 - b^4 + 6*a^2*b^2))/(16*b^5*d) + (a^3*x*(a^2 + b^2))/b^6 - (ex
p(- 2*c - 2*d*x)*(a*b^2 + 2*a^3))/(16*b^4*d) - (exp(2*c + 2*d*x)*(a*b^2 +
2*a^3))/(16*b^4*d) + (exp(c + d*x)*(8*a^4 - b^4 + 6*a^2*b^2))/(16*b^5*d) -
 (exp(- 3*c - 3*d*x)*(4*a^2 + b^2))/(96*b^3*d) + (exp(3*c + 3*d*x)*(4*a^2
+ b^2))/(96*b^3*d)
```

3.405 $\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.405.1 Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.405.2 Mathematica [N/A]

Not integrable

Time = 28.68 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])),x]
```

output
```
Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

3.405.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\sinh^3(c+dx)\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.405.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.405.4 Maple [N/A] (verified)

Not integrable

Time = 0.80 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(dx+c\right)^{3}\sinh\left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.405.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^3\sinh(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
integral(cosh(d*x + c)^3*sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh
(d*x + c)), x)
```

3.405.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3*sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.405.7 Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 551, normalized size of antiderivative = 15.31

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)^3\sinh(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/32*e^(-5*c + 5*d*e/f)*exp_integral_e(1, 5*(f*x + e)*d/f)/(b*f) - 1/16*a
*e^(-4*c + 4*d*e/f)*exp_integral_e(1, 4*(f*x + e)*d/f)/(b^2*f) + 1/16*a*e^
(4*c - 4*d*e/f)*exp_integral_e(1, -4*(f*x + e)*d/f)/(b^2*f) - 1/32*e^(5*c
- 5*d*e/f)*exp_integral_e(1, -5*(f*x + e)*d/f)/(b*f) - 1/32*(4*a^2 + b^2)*
e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b^3*f) - 1/32*(4*a^
2*e^(3*c) + b^2*e^(3*c))*e^(-3*d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/
(b^3*f) - 1/8*(2*a^3 + a*b^2)*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x
+ e)*d/f)/(b^4*f) + 1/8*(2*a^3*e^(2*c) + a*b^2*e^(2*c))*e^(-2*d*e/f)*exp_i
ntegral_e(1, -2*(f*x + e)*d/f)/(b^4*f) - 1/16*(8*a^4 + 6*a^2*b^2 - b^4)*e^
(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^5*f) - 1/16*(8*a^4*e^c +
6*a^2*b^2*e^c - b^4*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^5
*f) - (a^5 + a^3*b^2)*log(f*x + e)/(b^6*f) + 1/64*integrate(128*(a^5*b + a
^3*b^3 - (a^6*e^c + a^4*b^2*e^c)*e^(d*x))/(b^7*f*x + b^7*e - (b^7*f*x*e^(2
*c) + b^7*e*e^(2*c))*e^(2*d*x) - 2*(a*b^6*f*x*e^c + a*b^6*e*e^c)*e^(d*x)),
 x)
```

3.405.8 Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
integrate(cosh(d*x + c)^3*sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a)
), x)
```

3.405.9 Mupad [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\mathrm{cosh}(c+d\,x)^3\,\mathrm{sinh}(c+d\,x)^3}{(e+f\,x)\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}\,dx$$

input
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)^3*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))), x
)
```

3.405. $\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.406 $\int \frac{(e+fx)^3 \sinh^2(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)} dx$

3.406.1 Optimal result

Integrand size = 34, antiderivative size = 1519

$$\int \frac{(e+fx)^3 \sinh^2(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)} dx = \text{Too large to display}$$

output
```
-3*f*(f*x+e)^2*cosh(d*x+c)/b/d^2+6*f^2*(f*x+e)*sinh(d*x+c)/b/d^3-3*I*a^4*f
*(f*x+e)^2*polylog(2,I*exp(d*x+c))/b^3/(a^2+b^2)/d^2-6*I*a^4*f^2*(f*x+e)*p
olylog(3,-I*exp(d*x+c))/b^3/(a^2+b^2)/d^3+6*I*f^3*polylog(4,-I*exp(d*x+c))
/b/d^4+(f*x+e)^3*sinh(d*x+c)/b/d+a^3*(f*x+e)^3*ln(1+exp(2*d*x+2*c))/b^2/(a
^2+b^2)/d-a^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^
2)/d-a^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d-
a*(f*x+e)^3*ln(1+exp(2*d*x+2*c))/b^2/d+3/2*a^3*f*(f*x+e)^2*polylog(2,-exp(
2*d*x+2*c))/b^2/(a^2+b^2)/d^2-3/2*a^3*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c
))/b^2/(a^2+b^2)/d^3-3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/b^2/(a^2+b^2)/d^2-3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(
a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^2+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^3+6*a^3*f^2*(f*x+e)*polylog(3,-b*e
xp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^3-3*I*a^2*f*(f*x+e)^2*polyl
og(2,-I*exp(d*x+c))/b^3/d^2-6*I*a^2*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/b^
3/d^3-2*(f*x+e)^3*arctan(exp(d*x+c))/b/d+3*I*f*(f*x+e)^2*polylog(2,-I*exp(
d*x+c))/b/d^2+6*I*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/b/d^3-6*I*a^4*f^3*po
lylog(4,I*exp(d*x+c))/b^3/(a^2+b^2)/d^4+1/4*a*(f*x+e)^4/b^2/f+3*I*a^2*f*(f
*x+e)^2*polylog(2,I*exp(d*x+c))/b^3/d^2+6*I*a^2*f^2*(f*x+e)*polylog(3,-I*e
xp(d*x+c))/b^3/d^3+6*I*a^4*f^3*polylog(4,-I*exp(d*x+c))/b^3/(a^2+b^2)/d^4+
3*I*a^4*f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/b^3/(a^2+b^2)/d^2+6*I*a^4*...
```

3.406.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4239 vs. 2(1519) = 3038.

Time = 11.36 (sec) , antiderivative size = 4239, normalized size of antiderivative = 2.79

$$\int \frac{(e+fx)^3 \sinh^2(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
-1/4*(-8*a*d^4*e^3*E^(2*c)*x - 12*a*d^4*e^2*E^(2*c)*f*x^2 - 8*a*d^4*e*E^(2
*c)*f^2*x^3 - 2*a*d^4*E^(2*c)*f^3*x^4 + 8*b*d^3*e^3*ArcTan[E^(c + d*x)] +
8*b*d^3*e^3*E^(2*c)*ArcTan[E^(c + d*x)] + (12*I)*b*d^3*e^2*f*x*Log[1 - I*E
^(c + d*x)] + (12*I)*b*d^3*e^2*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (12*I)
*b*d^3*e*f^2*x^2*Log[1 - I*E^(c + d*x)] + (12*I)*b*d^3*e*E^(2*c)*f^2*x^2*L
og[1 - I*E^(c + d*x)] + (4*I)*b*d^3*f^3*x^3*Log[1 - I*E^(c + d*x)] + (4*I)
*b*d^3*E^(2*c)*f^3*x^3*Log[1 - I*E^(c + d*x)] - (12*I)*b*d^3*e^2*f*x*Log[1
 + I*E^(c + d*x)] - (12*I)*b*d^3*e^2*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] -
(12*I)*b*d^3*e*f^2*x^2*Log[1 + I*E^(c + d*x)] - (12*I)*b*d^3*e*E^(2*c)*f^2
*x^2*Log[1 + I*E^(c + d*x)] - (4*I)*b*d^3*f^3*x^3*Log[1 + I*E^(c + d*x)] -
 (4*I)*b*d^3*E^(2*c)*f^3*x^3*Log[1 + I*E^(c + d*x)] + 4*a*d^3*e^3*Log[1 +
E^(2*(c + d*x))] + 4*a*d^3*e^3*E^(2*c)*Log[1 + E^(2*(c + d*x))] + 12*a*d^3
*e^2*f*x*Log[1 + E^(2*(c + d*x))] + 12*a*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(2*
(c + d*x))] + 12*a*d^3*e*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 12*a*d^3*e*E^(
2*c)*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 4*a*d^3*f^3*x^3*Log[1 + E^(2*(c +
d*x))] + 4*a*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(2*(c + d*x))] - (12*I)*b*d^2*(
1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)] + (12*I)*b*d^2*(1
+ E^(2*c))*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)] + 6*a*d^2*e^2*f*PolyLog
[2, -E^(2*(c + d*x))] + 6*a*d^2*e^2*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))]
 + 12*a*d^2*e*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + 12*a*d^2*e*E^(2*c)*f...
```

3.406.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \sinh^2(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \ 6115$$

$$\frac{\int (e+fx)^3 \sinh(c+dx) \tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 5972$$

$$\frac{\int (e+fx)^3 \cosh(c+dx)dx-\int (e+fx)^3 \text{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^3 \sin\left(ic+idx+\frac{\pi}{2}\right)dx-\int (e+fx)^3 \csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \ 3777$$

$$-\frac{a\int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^3 \csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{3if\int -i(e+fx)^2\sinh(c+dx)dx}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \ 26$$

$$-\frac{a\int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^3 \csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{3f\int (e+fx)^2\sinh(c+dx)dx}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{3f\int -i(e+fx)^2\sin(ic+idx)dx}{d}-\int (e+fx)^3 \csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{3if\int (e+fx)^2\sin(ic+idx)dx}{d}-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}}{}$$

$$\Big\downarrow \; 3777$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 3777$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 26$$

3.406. $\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int(e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 3118$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int(e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 4668$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{3if\int(e+fx)^2\log(1-ie^{c+dx})dx}{d}-\frac{3if\int(e+fx)^2\log(1+ie^{c+dx})dx}{d}-\frac{2(e+fx)^3\arctan(e^{c+dx})}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c\cdots}{d}\right.}{}\right.}{d}}{b}$$

$$\Big\downarrow \; 3011$$

$$-\frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,-ie^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-ie^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,ie^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,ie^{c+dx})}{d}\right)}{d}-\cdots}{b}$$

$$\Big\downarrow \; 6115$$

$$-\frac{a\left(\frac{\int(e+fx)^3\tanh(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,-ie^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-ie^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,ie^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,ie^{c+dx})}{d}\right)}{d}-\cdots}{b}$$

$$\Big\downarrow \; 3042$$

3.406. $\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int-i(e+fx)^3\tan(ic+idx)dx}{b}\right)}{b}$$

$\Big\downarrow$ 26

$$-\frac{\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int(e+fx)^3\tan(ic+idx)dx}{b}\right)}{b}$$

$\Big\downarrow$ 4201

$$-\frac{\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\int\frac{e^{2(c+dx)}(e+fx)^3}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^4}{4f}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 2620

$$-\frac{\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\int(e+fx)^2\log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 3011

3.406. $\displaystyle\int\frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$a\left(-\frac{a\int\frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\ldots\right)}{b}\right)$$
$$\frac{-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-\ldots}{b}$$
$$\overline{\qquad b\qquad}$$

$$\Big\downarrow \text{6101}$$

$$a\left(-\frac{a\left(\frac{\int(e+fx)^3\text{sech}(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\ldots\right)}{2d}\right)\ldots\right)}{b}\ldots\right)$$
$$\frac{-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-\ldots}{b}$$
$$\overline{\qquad b\qquad}$$

$$\Big\downarrow \text{3042}$$

$$a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^3\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\ldots\right)}{2d}\right)\ldots\right)}{b}\ldots\right)$$
$$\frac{-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-\ldots}{b}$$
$$\overline{\qquad b\qquad}$$

$$\Big\downarrow \text{4668}$$

3.406. $\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-$$

$$b$$

$$a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{3if\int(e+fx)^2\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{3if\int(e+fx)^2\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^3\arctan\left(e^{c+dx}\right)}{d}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+f}{}\right.\right.}{}\right.$$

$$b$$

$$\Big\downarrow \; 3011$$

$$-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-$$

$$b$$

$$a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)d}{d}\right.}{d}}{b}\right.}{b}\right.$$

$$\Big\downarrow \; 6107$$

$$-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-$$

$$b$$

$$a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-i\right.}{d}\right.}{d}\right.}{b}\right.$$

$$\Big\downarrow \; 6095$$

3.406. $\displaystyle\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-$$

$$b$$

$$a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{a^2+b^2}+\frac{\int(e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLo}}{d}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{2620}$$

$$-\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-$$

$$b$$

$$a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+\right.}{bd}\right.}{a^2+b^2}\right.}{b}\right.}{}\right.$$

$$\Big\downarrow \text{3011}$$

3.406. $\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^3}{d}+\frac{\sinh(c+dx)(e+fx)^3}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}-\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{Pol}\cdots}{\cdots}\right.}{\cdots}$$

$$b$$

$$a\left(-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2(c+dx)}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{b}-\frac{b\left(a\,\frac{2\arctan\left(e^{c+\cdots}\right)}{\cdots}\cdots\right.}{\cdots}\right.$$

input
```
Int[((e + f*x)^3*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.406. $\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.406.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.406. $\int \frac{(e+fx)^3 \sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6115
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - S
imp[a/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.406. $\int \frac{(e+fx)^3 \sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.406.4 Maple [F]

$$\int \frac{(fx+e)^3 \sinh(dx+c)^2 \tanh(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.406.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4522 vs. $2(1378) = 2756$.

Time = 0.37 (sec) , antiderivative size = 4522, normalized size of antiderivative = 2.98

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
-1/4*(2*(a^2*b + b^3)*d^3*f^3*x^3 + 2*(a^2*b + b^3)*d^3*e^3 + 6*(a^2*b + b
^3)*d^2*e^2*f + 12*(a^2*b + b^3)*d*e*f^2 + 12*(a^2*b + b^3)*f^3 + 6*((a^2*
b + b^3)*d^3*e*f^2 + (a^2*b + b^3)*d^2*f^3)*x^2 - 2*((a^2*b + b^3)*d^3*f^3
*x^3 + (a^2*b + b^3)*d^3*e^3 - 3*(a^2*b + b^3)*d^2*e^2*f + 6*(a^2*b + b^3)
*d*e*f^2 - 6*(a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^2*b + b^3
)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^2*b + b^3)*d^2*e*f^2 +
2*(a^2*b + b^3)*d*f^3)*x)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^3*f^3*x^3 +
 (a^2*b + b^3)*d^3*e^3 - 3*(a^2*b + b^3)*d^2*e^2*f + 6*(a^2*b + b^3)*d*e*f
^2 - 6*(a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^2*b + b^3)*d^2*
f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^2*b + b^3)*d^2*e*f^2 + 2*(a^2
*b + b^3)*d*f^3)*x)*sinh(d*x + c)^2 + 6*((a^2*b + b^3)*d^3*e^2*f + 2*(a^2*
b + b^3)*d^2*e*f^2 + 2*(a^2*b + b^3)*d*f^3)*x - ((a^3 + a*b^2)*d^4*f^3*x^4
 + 4*(a^3 + a*b^2)*d^4*e*f^2*x^3 + 6*(a^3 + a*b^2)*d^4*e^2*f*x^2 + 4*(a^3
+ a*b^2)*d^4*e^3*x + 8*(a^3 + a*b^2)*c*d^3*e^3 - 12*(a^3 + a*b^2)*c^2*d^2*
e^2*f + 8*(a^3 + a*b^2)*c^3*d*e*f^2 - 2*(a^3 + a*b^2)*c^4*f^3)*cosh(d*x +
c) + 12*((a^3*d^2*f^3*x^2 + 2*a^3*d^2*e*f^2*x + a^3*d^2*e^2*f)*cosh(d*x +
c) + (a^3*d^2*f^3*x^2 + 2*a^3*d^2*e*f^2*x + a^3*d^2*e^2*f)*sinh(d*x + c))*
dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x +
 c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*((a^3*d^2*f^3*x^2 + 2*a^3*d^2*
e*f^2*x + a^3*d^2*e^2*f)*cosh(d*x + c) + (a^3*d^2*f^3*x^2 + 2*a^3*d^2*e...
```

3.406.6 Sympy [F]

$$\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^3\sinh^2{(c+dx)}\tanh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x))
, x)
```

3.406.7 Maxima [F]

$$\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\sinh{(dx+c)}^2\tanh{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/2*(2*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^
4)*d) - 4*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + 2*a*log(e^(-2*d*x - 2*c
) + 1)/((a^2 + b^2)*d) + 2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d
*x - c)/(b*d))*e^3 - 1/4*(a*d^4*f^3*x^4*e^c + 4*a*d^4*e*f^2*x^3*e^c + 6*a*
d^4*e^2*f*x^2*e^c - 2*(b*d^3*f^3*x^3*e^(2*c) + 3*(d^3*e*f^2 - d^2*f^3)*b*x
^2*e^(2*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b*x*e^(2*c) - 3*(d^2*e^
2*f - 2*d*e*f^2 + 2*f^3)*b*e^(2*c))*e^(d*x) + 2*(b*d^3*f^3*x^3 + 3*(d^3*e*
f^2 + d^2*f^3)*b*x^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*b*x + 3*(d^2*
e^2*f + 2*d*e*f^2 + 2*f^3)*b)*e^(-d*x))*e^(-c)/(b^2*d^4) + integrate(2*(a^
3*b*f^3*x^3 + 3*a^3*b*e*f^2*x^2 + 3*a^3*b*e^2*f*x - (a^4*f^3*x^3*e^c + 3*a
^4*e*f^2*x^2*e^c + 3*a^4*e^2*f*x*e^c)*e^(d*x))/(a^2*b^3 + b^5 - (a^2*b^3*e
^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x)
 - integrate(-2*(a*f^3*x^3 + 3*a*e*f^2*x^2 + 3*a*e^2*f*x - (b*f^3*x^3*e^c
+ 3*b*e*f^2*x^2*e^c + 3*b*e^2*f*x*e^c)*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c)
+ b^2*e^(2*c))*e^(2*d*x)), x)
```

3.406.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.406.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)^2\tanh(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.407 $\int \frac{(e+fx)^2 \sinh^2(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.407.1 Optimal result

Integrand size = 34, antiderivative size = 1067

$$
\begin{aligned}
&\int \frac{(e+fx)^2 \sinh^2(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{a(e+fx)^3}{3b^2f} + \frac{2a^2(e+fx)^2 \arctan\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^2 \arctan\left(e^{c+dx}\right)}{bd} \\
&\quad - \frac{2a^4(e+fx)^2 \arctan\left(e^{c+dx}\right)}{b^3\left(a^2+b^2\right)d} - \frac{2f(e+fx)\cosh(c+dx)}{bd^2} \\
&\quad - \frac{a^3(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)d} - \frac{a^3(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)d} \\
&\quad - \frac{a(e+fx)^2 \log\left(1+e^{2(c+dx)}\right)}{b^2d} + \frac{a^3(e+fx)^2 \log\left(1+e^{2(c+dx)}\right)}{b^2\left(a^2+b^2\right)d} \\
&\quad - \frac{2ia^2f(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{b^3d^2} + \frac{2if(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{bd^2} \\
&\quad + \frac{2ia^4f(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{b^3\left(a^2+b^2\right)d^2} + \frac{2ia^2f(e+fx)\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{b^3d^2} \\
&\quad - \frac{2if(e+fx)\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{bd^2} - \frac{2ia^4f(e+fx)\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{b^3\left(a^2+b^2\right)d^2} \\
&\quad - \frac{2a^3f(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)d^2} - \frac{2a^3f(e+fx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)d^2} \\
&\quad - \frac{af(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{b^2d^2} + \frac{a^3f(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{b^2\left(a^2+b^2\right)d^2} \\
&\quad + \frac{2ia^2f^2\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{b^3d^3} - \frac{2if^2\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{bd^3} \\
&\quad - \frac{2ia^4f^2\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{b^3\left(a^2+b^2\right)d^3} - \frac{2ia^2f^2\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{b^3d^3} + \frac{2if^2\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{bd^3} \\
&\quad + \frac{2ia^4f^2\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{b^3\left(a^2+b^2\right)d^3} + \frac{2a^3f^2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)d^3} \\
&\quad + \frac{2a^3f^2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)d^3} + \frac{af^2\,\mathrm{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2b^2d^3} \\
&\quad - \frac{a^3f^2\,\mathrm{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2b^2\left(a^2+b^2\right)d^3} + \frac{2f^2\sinh(c+dx)}{bd^3} + \frac{(e+fx)^2\sinh(c+dx)}{bd}
\end{aligned}
$$

output
```
-2*f*(f*x+e)*cosh(d*x+c)/b/d^2-2*I*a^4*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b
^3/(a^2+b^2)/d^2+(f*x+e)^2*sinh(d*x+c)/b/d-a*f*(f*x+e)*polylog(2,-exp(2*d*
x+2*c))/b^2/d^2+a^3*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b^2/(a^2+b^2)/d-a^3*(f*
x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d-a^3*(f*x+e)^
2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d-a*(f*x+e)^2*ln(1+
exp(2*d*x+2*c))/b^2/d+2*I*f^2*polylog(3,I*exp(d*x+c))/b/d^3-2*I*a^2*f^2*po
lylog(3,I*exp(d*x+c))/b^3/d^3-2*(f*x+e)^2*arctan(exp(d*x+c))/b/d+2*I*f*(f*
x+e)*polylog(2,-I*exp(d*x+c))/b/d^2-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c
)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^2-2*a^3*f*(f*x+e)*polylog(2,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^2-2*I*a^2*f*(f*x+e)*polylog(2,
-I*exp(d*x+c))/b^3/d^2-2*I*a^4*f^2*polylog(3,-I*exp(d*x+c))/b^3/(a^2+b^2)/
d^3+1/3*a*(f*x+e)^3/b^2/f+2*I*a^2*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b^3/d^
2+2*I*a^4*f^2*polylog(3,I*exp(d*x+c))/b^3/(a^2+b^2)/d^3+2*I*a^4*f*(f*x+e)*
polylog(2,-I*exp(d*x+c))/b^3/(a^2+b^2)/d^2+a^3*f*(f*x+e)*polylog(2,-exp(2*
d*x+2*c))/b^2/(a^2+b^2)/d^2+2*I*a^2*f^2*polylog(3,-I*exp(d*x+c))/b^3/d^3+2
*f^2*sinh(d*x+c)/b/d^3-2*a^4*(f*x+e)^2*arctan(exp(d*x+c))/b^3/(a^2+b^2)/d-
1/2*a^3*f^2*polylog(3,-exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^3+2*a^3*f^2*polylog
(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^3+2*a^3*f^2*polylog(
3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^3-2*I*f*(f*x+e)*polyl
og(2,I*exp(d*x+c))/b/d^2+2*a^2*(f*x+e)^2*arctan(exp(d*x+c))/b^3/d+1/2*a...
```

3.407.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3418 vs. 2(1067) = 2134.

Time = 10.88 (sec) , antiderivative size = 3418, normalized size of antiderivative = 3.20

$$\int \frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
-1/6*(-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*
f*x^2 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] -
 6*a*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d
*e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)])
- PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 +
 E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c
+ d*x))]) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^
2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*
PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3,
I*E^(c + d*x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(
c + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c +
 d*x))]))/((a^2 + b^2)*d^3*(1 + E^(2*c))) + (a^3*(6*e^2*E^(2*c)*x + 6*e*E^
(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E
^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2
+ b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2
+ b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x)
)/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2
*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)
*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E
^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Lo...
```

3.407.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6115$$

$$\frac{\int (e+fx)^2\sinh(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 5972$$

$$\frac{\int (e+fx)^2\cosh(c+dx)dx-\int (e+fx)^2\text{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}-\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}-\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}}{b}$$

3.407. $\int \frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 3117$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 4668$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\int (e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}-\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}-\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+}{}}{b}$$

$$\Big\downarrow \; 3011$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-\frac{2(e+fx)^2\text{arcta}}{d}}{b}$$

$$\Big\downarrow \; 2720$$

$$-\frac{a\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+c}\right.}{d}\right.}{d}}{b}$$

$$\Big\downarrow \; 6115$$

$$-\frac{a\left(\frac{\int (e+fx)^2\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+c}\right.}{d}\right.}{d}}{b}$$

$$\Big\downarrow \; 3042$$

3.407. $$\int \frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+c}\right.}{d}\right.}{d}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)^2\tan(ic+idx)dx}{b}\right)}{b}$$

$\Big\downarrow$ 26

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+c}\right.}{d}\right.}{d}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int(e+fx)^2\tan(ic+idx)dx}{b}\right)}{b}$$

$\Big\downarrow$ 4201

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+c}\right.}{d}\right.}{d}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\int\frac{e^{2(c+dx)}(e+fx)^2}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^3}{3f}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 2620

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+c}\right.}{d}\right.}{d}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2(c+dx)}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 3011

3.407. $\quad \displaystyle\int\frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}$$
$$b$$
$$a\left(-\frac{a\int\frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int\text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{2d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right.}{b}\right.$$
$$b$$

$$\Big\downarrow \text{2720}$$

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}$$
$$b$$
$$a\left(-\frac{a\int\frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right.}{2d}\right.}{d}\right.\right.}{b}\right.$$
$$b$$

$$\Big\downarrow \text{6101}$$

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}$$
$$b$$
$$a\left(-\frac{a\left(\frac{\int(e+fx)^2\text{sech}(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+\right.}{4d^2}\right.}{d}\right.\right.}{b}\right.$$
$$b$$

$$\Big\downarrow \text{3042}$$

3.407. $\int\frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}$$
$$\overline{\qquad\qquad\qquad b}$$
$$\frac{a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)d\right.}{4d^2}\right.}{}\right.\right.}{b}\right.}{b}$$

$$\downarrow \text{4668}$$

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}$$
$$\overline{\qquad\qquad\qquad b}$$
$$\frac{a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{2if\int(e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int(e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{(e+fx)^2}{}\right.\right.}{}\right.}{b}$$

$$\downarrow \text{3011}$$

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}$$
$$\overline{\qquad\qquad\qquad b}$$
$$a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int\text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}}{d}\right.}{d}}{b}\right.}{b}\right.$$

$$\downarrow \text{2720}$$

3.407. $\int\frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+}\right.}{d}\right.}{d}$$
$$b$$
$$a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+}\right.}{d^2}\right.}{b}\right.}{b}\right.$$

$\downarrow$ 6107

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+}\right.}{d}\right.}{d}$$
$$b$$
$$a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(\right.}{d}\right.}{d}\right.}{b}\right.$$

$\downarrow$ 6095

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+}\right.}{d}\right.}{d}$$
$$b$$
$$a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int(e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLo}}{}\right.}{}\right.}{}\right.$$

3.407. $\int\frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$\downarrow$ 2620

$$-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,ie^{c+c}\right.}{d}\right.}{d}$$

$$b$$

$$a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right.}{a^2+b^2}\right.}{b}\right.}{}\right.$$

$\downarrow$ 3011

3.407. $\int \frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^2}{d}+\frac{\sinh(c+dx)(e+fx)^2}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\cdots\right.}{d}$$

$$b$$

$$a\left(-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}-\frac{a\left(\frac{2a\cdots}{\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \text{2720}$$

3.407. $\int \frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^2}{d}+\frac{\sinh(c+dx)(e+fx)^2}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(\cdots}{\cdots}\right.}{d}}{b}$$

$$a\left(-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{b}-\frac{a\left(\frac{2a\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[((e + f*x)^2*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.407. $\int \frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.407.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.407. $\int \frac{(e+fx)^2 \sinh^2(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

3.407. $\int \frac{(e+fx)^2 \sinh^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} \, dx$

rule 6115
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - S
imp[a/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.407.4 Maple [F]

$$\int \frac{(fx+e)^2 \sinh (dx+c)^2 \tanh (dx+c)}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.407.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2775 vs. 2(970) = 1940.

Time = 0.34 (sec) , antiderivative size = 2775, normalized size of antiderivative = 2.60

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
-1/6*(3*(a^2*b + b^3)*d^2*f^2*x^2 + 3*(a^2*b + b^3)*d^2*e^2 + 6*(a^2*b + b
^3)*d*e*f + 6*(a^2*b + b^3)*f^2 - 3*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b +
b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*(a^2*b + b^3)*f^2 + 2*((a^2*b + b
^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c)^2 - 3*((a^2*b + b^3)*d
^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*(a^2*b + b^
3)*f^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*sinh(d*x + c)^
2 + 6*((a^2*b + b^3)*d^2*e*f + (a^2*b + b^3)*d*f^2)*x - 2*((a^3 + a*b^2)*d
^3*f^2*x^3 + 3*(a^3 + a*b^2)*d^3*e*f*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*x + 6*(
a^3 + a*b^2)*c*d^2*e^2 - 6*(a^3 + a*b^2)*c^2*d*e*f + 2*(a^3 + a*b^2)*c^3*f
^2)*cosh(d*x + c) + 12*((a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x + c) + (a^3*d*f
^2*x + a^3*d*e*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
12*((a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x + c) + (a^3*d*f^2*x + a^3*d*e*f)*si
nh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*((a*b^2*d*f^2*x +
 I*b^3*d*f^2*x + a*b^2*d*e*f + I*b^3*d*e*f)*cosh(d*x + c) + (a*b^2*d*f^2*x
 + I*b^3*d*f^2*x + a*b^2*d*e*f + I*b^3*d*e*f)*sinh(d*x + c))*dilog(I*cosh(
d*x + c) + I*sinh(d*x + c)) + 12*((a*b^2*d*f^2*x - I*b^3*d*f^2*x + a*b^2*d
*e*f - I*b^3*d*e*f)*cosh(d*x + c) + (a*b^2*d*f^2*x - I*b^3*d*f^2*x + a*b^2
*d*e*f - I*b^3*d*e*f)*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*...
```

3.407.6 Sympy [F]

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2 \sinh^2{(c+dx)} \tanh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x))
, x)
```

3.407.7 Maxima [F]

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \sinh(dx+c)^2 \tanh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/2*(2*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^
4)*d) - 4*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + 2*a*log(e^(-2*d*x - 2*c
) + 1)/((a^2 + b^2)*d) + 2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d
*x - c)/(b*d))*e^2 - 1/6*(2*a*d^3*f^2*x^3*e^c + 6*a*d^3*e*f*x^2*e^c - 3*(b
*d^2*f^2*x^2*e^(2*c) + 2*(d^2*e*f - d*f^2)*b*x*e^(2*c) - 2*(d*e*f - f^2)*b
*e^(2*c))*e^(d*x) + 3*(b*d^2*f^2*x^2 + 2*(d^2*e*f + d*f^2)*b*x + 2*(d*e*f
+ f^2)*b)*e^(-d*x))*e^(-c)/(b^2*d^3) + integrate(2*(a^3*b*f^2*x^2 + 2*a^3*
b*e*f*x - (a^4*f^2*x^2*e^c + 2*a^4*e*f*x*e^c)*e^(d*x))/(a^2*b^3 + b^5 - (a
^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^3*b^2*e^c + a*b^4*e^c)*e^(d
*x)), x) - integrate(-2*(a*f^2*x^2 + 2*a*e*f*x - (b*f^2*x^2*e^c + 2*b*e*f*
x*e^c)*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

3.407.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.407.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)^2\tanh(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.408 $\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.408.1 Optimal result

Integrand size $= 32$, antiderivative size $= 631$

$$
\begin{aligned}
\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = {} & \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\arctan\left(e^{c+dx}\right)}{b^3d} \\
& - \frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{bd} \\
& - \frac{2a^4(e+fx)\arctan\left(e^{c+dx}\right)}{b^3\left(a^2+b^2\right)d} - \frac{f\cosh(c+dx)}{bd^2} \\
& - \frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)d} \\
& - \frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)d} \\
& - \frac{a(e+fx)\log\left(1+e^{2(c+dx)}\right)}{b^2d} \\
& + \frac{a^3(e+fx)\log\left(1+e^{2(c+dx)}\right)}{b^2\left(a^2+b^2\right)d} \\
& - \frac{ia^2f\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{b^3d^2} \\
& + \frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{bd^2} \\
& + \frac{ia^4f\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{b^3\left(a^2+b^2\right)d^2} \\
& + \frac{ia^2f\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{b^3d^2} \\
& - \frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{bd^2} \\
& - \frac{ia^4f\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{b^3\left(a^2+b^2\right)d^2} \\
& - \frac{a^3f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)d^2} \\
& - \frac{a^3f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)d^2} \\
& - \frac{af\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2b^2d^2} \\
& + \frac{a^3f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2b^2\left(a^2+b^2\right)d^2} \\
& + \frac{(e+fx)\sinh(c+dx)}{bd}
\end{aligned}
$$

output
```
1/2*a*(f*x+e)^2/b^2/f+2*a^2*(f*x+e)*arctan(exp(d*x+c))/b^3/d-2*(f*x+e)*arc
tan(exp(d*x+c))/b/d-2*a^4*(f*x+e)*arctan(exp(d*x+c))/b^3/(a^2+b^2)/d-f*cos
h(d*x+c)/b/d^2-a*(f*x+e)*ln(1+exp(2*d*x+2*c))/b^2/d+a^3*(f*x+e)*ln(1+exp(2
*d*x+2*c))/b^2/(a^2+b^2)/d-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2
)))/b^2/(a^2+b^2)/d-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2
/(a^2+b^2)/d+I*a^2*f*polylog(2,I*exp(d*x+c))/b^3/d^2+I*f*polylog(2,-I*exp(
d*x+c))/b/d^2-I*a^4*f*polylog(2,I*exp(d*x+c))/b^3/(a^2+b^2)/d^2+I*a^4*f*po
lylog(2,-I*exp(d*x+c))/b^3/(a^2+b^2)/d^2-I*f*polylog(2,I*exp(d*x+c))/b/d^2
-I*a^2*f*polylog(2,-I*exp(d*x+c))/b^3/d^2-1/2*a*f*polylog(2,-exp(2*d*x+2*c
))/b^2/d^2+1/2*a^3*f*polylog(2,-exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^2-a^3*f*po
lylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^2-a^3*f*polylog
(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^2+(f*x+e)*sinh(d*x+c
)/b/d
```

3.408.2 Mathematica [A] (verified)

Time = 8.64 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.01

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{f\cosh(c+dx)}{bd^2}$$
$$-\frac{a^3\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\,\mathrm{arctanh}\left(\frac{a+b}{\sqrt{a}}\right.}{(-a^2-b^2)^{3/2}}\right.}{}$$
$$-\frac{-ade(c+dx)+acf(c+dx)-\frac{1}{2}af(c+dx)^2+2bde\arctan\left(e^{c+dx}\right)-2bcf\arctan\left(e^{c+dx}\right)+ibf(c+dx}{}$$
$$+\frac{(de-cf+f(c+dx))\sinh(c+dx)}{bd^2}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
-((f*Cosh[c + d*x])/(b*d^2)) - (a^3*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) -
f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-
a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[
(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(
2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*
f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(2*b^2*(a^2 + b^2)*d^2) - (-(a*d*e
*(c + d*x)) + a*c*f*(c + d*x) - (a*f*(c + d*x)^2)/2 + 2*b*d*e*ArcTan[E^(c
+ d*x)] - 2*b*c*f*ArcTan[E^(c + d*x)] + I*b*f*(c + d*x)*Log[1 - I*E^(c + d
*x)] - I*b*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + a*d*e*Log[1 + E^(2*(c + d*
x))] - a*c*f*Log[1 + E^(2*(c + d*x))] + a*f*(c + d*x)*Log[1 + E^(2*(c + d*
x))] - I*b*f*PolyLog[2, (-I)*E^(c + d*x)] + I*b*f*PolyLog[2, I*E^(c + d*x)
] + (a*f*PolyLog[2, -E^(2*(c + d*x))])/2)/((a^2 + b^2)*d^2) + ((d*e - c*f
+ f*(c + d*x))*Sinh[c + d*x])/(b*d^2)
```

3.408.3 Rubi [A] (verified)

Time = 3.60 (sec) , antiderivative size = 570, normalized size of antiderivative = 0.90, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {6115, 5972, 3042, 3777, 26, 3042, 26, 3118, 4668, 2715, 2838, 6115, 3042, 26, 4201, 2620, 2715, 2838, 6101, 3042, 4668, 2715, 2838, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6115}$$

$$\frac{\int (e+fx)\sinh(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5972}$$

$$\frac{\int (e+fx)\cosh(c+dx)dx-\int (e+fx)\text{sech}(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \; 3777$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{if\int -i\sinh(c+dx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f\int \sinh(c+dx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f\int -i\sin(ic+idx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{if\int \sin(ic+idx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 3118$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 4668$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 2715$$

3.408. $\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}}{}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{ 6115}$$

$$-\frac{a\left(\frac{\int (e+fx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -i(e+fx)\tan(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\int (e+fx)\tan(ic+idx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 4201}$$

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\int \frac{e^{2(c+dx)}(e+fx)}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^2}{2f}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 2620}$$

3.408. $\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int\log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 2715$$

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int e^{-2(c+dx)}\log\left(1+e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 2838$$

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 6101$$

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\text{sech}(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 4668$$

3.408. $\displaystyle\int\frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\operatorname{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\operatorname{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{if\int\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\right.\right.}{}\right.}{b}$$

$$\Big\downarrow \; 2715$$

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\operatorname{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\operatorname{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}}{b}\right)}{b}-\frac{i\left(2i\left(\right.\right.}{}\right.}{b}$$

$$\Big\downarrow \; 2838$$

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\operatorname{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\operatorname{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\left(-\frac{a\int\frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}\right)}{b}-\frac{i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\right.\right.}{}\right.}{b}$$

$$\Big\downarrow \; 6107$$

$$\frac{-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\operatorname{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\operatorname{PolyLog}(2,ie^{c+dx})}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}\right.}{b}\right.}{b}$$

$$\Big\downarrow \; 6095$$

3.408. $\displaystyle\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-$$

$$\frac{a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{b}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\cdots}{b}\cdots\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-$$

$$\frac{a\left(-\frac{a\left(-\frac{a\left(\frac{b^2\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int(e\cdots}{\cdots}\right)}{b}\cdots}{b}\cdots\right)}{\cdots}$$

$$\Big\downarrow \; 2715$$

3.408. $\displaystyle\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-$$

$$a\left(a-\frac{a\left(a-\frac{a\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+d}}{\sqrt{a^2+b^2}}\right.}{bd}\right.}{a^2+b^2}\right.}{b}\right.}{b}\right.$$

$$\Big\downarrow \text{2838}$$

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-$$

$$a\left(a-\frac{a\left(a-\frac{a\left(\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e\text{-}}{}\right.}{a^2+b^2}\right.}{b}\right.}{b}\right.$$

$$\Big\downarrow \text{7293}$$

3.408. $\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(a-\frac{a\left(-\frac{a\left(\frac{\int(a(e+fx)\text{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+\cdots\right)}{bd}\cdots\right)}{a^2+b^2}\right)}{b}\cdots\right)}{b}\cdots\right)}{\cdots}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{b}-\frac{a\left(a-\frac{a\left(\frac{\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}-\frac{a\left(\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\cdots\right)}{a\cdots}\cdots\right)}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}$$

input
```
Int[((e + f*x)*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.408. $\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
-((a*(((-I)*(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((e + f*x)*Log[1 + E^(2*(c
+ d*x))])/(2*d) + (f*PolyLog[2, -E^(2*(c + d*x))])/(4*d^2))))/b - (a*(((2*
(e + f*x)*ArcTan[E^(c + d*x)])/d - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2
+ (I*f*PolyLog[2, I*E^(c + d*x)])/d^2)/b - (a*((b^2*(-1/2*(e + f*x)^2/(b*f
) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e
 + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog
[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2 + b^2) + ((b*(e + f
*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)*Log[1
+ E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*a*f*
PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*d^2
))/(a^2 + b^2)))/b))/b))/b) + ((-2*(e + f*x)*ArcTan[E^(c + d*x)])/d - (f*C
osh[c + d*x])/d^2 + (I*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 - (I*f*PolyLog[
2, I*E^(c + d*x)])/d^2 + ((e + f*x)*Sinh[c + d*x])/d)/b
```

3.408.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.408. $\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5972
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

3.408. $\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6115
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - S
imp[a/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.408.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4065 vs. $2(592) = 1184$.

Time = 3.15 (sec) , antiderivative size = 4066, normalized size of antiderivative = 6.44

method	result	size
risch	Expression too large to display	4066

input
```
int((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
2/(a^2+b^2)^(1/2)/d*b^2*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(
a^2+b^2)^(1/2))+2/d^2/b^2*c*a^4*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/
2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d/b^2*a^4*f/(2*a^2+2*b^2)/(a^2+b
^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2/d
/b^2*a^4*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+
a)/(a+(a^2+b^2)^(1/2)))*x+2/d^2/b^2*a^4*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/d^2/b^2*a^4*f
/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))*c+2/d^2/b^2*c*a^2*f/(2*a^2+2*b^2)*(a^2+b^2)^(1/2)*arctanh(1/
2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/(a^2+b^2)^(1/2)/d^2*b^2*c*f/(2*a
^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d^2*f/(2*a^2
+2*b^2)*dilog(1+I*exp(d*x+c))*a-2/d^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c)
)*a+2/d/b^2*e*a*ln(exp(d*x+c))-4/d*b*e/(2*a^2+2*b^2)*arctan(exp(d*x+c))+1/
d*e/(2*a^2+2*b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d*e/(2*a^2+2*b
^2)*(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d*
e/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c))+1/d^2*a*f/(2*a^2+2*b^2)*dilog((-b*e
xp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2*a*f/(2*a^2+2*b^2)
*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/2/d^2*a*f/(
a^2+b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/2/d
^2*a*f/(a^2+b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^...
```

3.408.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1410 vs. $2(570) = 1140$.

Time = 0.31 (sec) , antiderivative size = 1410, normalized size of antiderivative = 2.23

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-1/2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e - ((a^2*b + b^3)*d*f*x + (a^
2*b + b^3)*d*e - (a^2*b + b^3)*f)*cosh(d*x + c)^2 - ((a^2*b + b^3)*d*f*x +
 (a^2*b + b^3)*d*e - (a^2*b + b^3)*f)*sinh(d*x + c)^2 + (a^2*b + b^3)*f -
((a^3 + a*b^2)*d^2*f*x^2 + 2*(a^3 + a*b^2)*d^2*e*x + 4*(a^3 + a*b^2)*c*d*e
 - 2*(a^3 + a*b^2)*c^2*f)*cosh(d*x + c) + 2*(a^3*f*cosh(d*x + c) + a^3*f*s
inh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a^3*f*cosh(d*x +
 c) + a^3*f*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*((a*
b^2*f + I*b^3*f)*cosh(d*x + c) + (a*b^2*f + I*b^3*f)*sinh(d*x + c))*dilog(
I*cosh(d*x + c) + I*sinh(d*x + c)) + 2*((a*b^2*f - I*b^3*f)*cosh(d*x + c)
+ (a*b^2*f - I*b^3*f)*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x +
 c)) + 2*((a^3*d*e - a^3*c*f)*cosh(d*x + c) + (a^3*d*e - a^3*c*f)*sinh(d*x
 + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^
2) + 2*a) + 2*((a^3*d*e - a^3*c*f)*cosh(d*x + c) + (a^3*d*e - a^3*c*f)*sin
h(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^
2)/b^2) + 2*a) + 2*((a^3*d*f*x + a^3*c*f)*cosh(d*x + c) + (a^3*d*f*x + a^3
*c*f)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*((a^3*d*f*x + a
^3*c*f)*cosh(d*x + c) + (a^3*d*f*x + a^3*c*f)*sinh(d*x + c))*log(-(a*co...
```

3.408.6 Sympy [F]

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\sinh^2{(c+dx)}\tanh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

3.408.7 Maxima [F]

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\sinh\left(dx+c\right)^2\tanh\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/2*(2*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^
4)*d) - 4*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + 2*a*log(e^(-2*d*x - 2*c
) + 1)/((a^2 + b^2)*d) + 2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d
*x - c)/(b*d))*e - 1/4*f*(2*(a*d^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*e
^(d*x) + (b*d*x + b)*e^(-d*x))*e^(-c)/(b^2*d^2) - integrate(-8*(a^4*x*e^(d
*x + c) - a^3*b*x)/(a^2*b^3 + b^5 - (a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d
*x) - 2*(a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) + integrate(8*(b*x*e^(d*x +
 c) - a*x)/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x))
```

3.408.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.408.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)^2\tanh(c+d\,x)\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.409 $\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.409.1 Optimal result

Integrand size = 27, antiderivative size = 89

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b\arctan(\sinh(c+dx))}{(a^2+b^2)\,d} - \frac{a\log(\cosh(c+dx))}{(a^2+b^2)\,d} - \frac{a^3\log(a+b\sinh(c+dx))}{b^2\,(a^2+b^2)\,d} + \frac{\sinh(c+dx)}{bd}$$

output
```
-b*arctan(sinh(d*x+c))/(a^2+b^2)/d-a*ln(cosh(d*x+c))/(a^2+b^2)/d-a^3*ln(a+
b*sinh(d*x+c))/b^2/(a^2+b^2)/d+sinh(d*x+c)/b/d
```

3.409.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{\log(i-\sinh(c+dx))}{a+ib} + \frac{\log(i+\sinh(c+dx))}{a-ib} + \frac{2a^3\log(a+b\sinh(c+dx))}{b^2(a^2+b^2)} - \frac{2\sinh(c+dx)}{b}}{2d}$$

input
```
Integrate[(Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(Log[I - Sinh[c + d*x]]/(a + I*b) + Log[I + Sinh[c + d*x]]/(a - I*b)
+ (2*a^3*Log[a + b*Sinh[c + d*x]])/(b^2*(a^2 + b^2)) - (2*Sinh[c + d*x])/b
)/d
```

3.409.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3316, 26, 27, 604, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ic+idx)^3}{\cos(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ic+idx)^3}{\cos(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$-\frac{ib\int \frac{i\sinh^3(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{b\int \frac{\sinh^3(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b^3\sinh^3(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{b^2d}$$

$$\Big\downarrow \text{604}$$

$$\frac{\int -\frac{\sinh(c+dx)b^3+a\sinh^2(c+dx)b^2+ab^2}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))+a+b\sinh(c+dx)}{b^2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\int \frac{\sinh(c+dx)b^3+a\sinh^2(c+dx)b^2+ab^2}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))+a+b\sinh(c+dx)}{b^2d}$$

$$\Big\downarrow \text{2160}$$

$$\frac{-\int \left(\frac{a^3}{(a^2+b^2)(a+b\sinh(c+dx))}+\frac{b^4+a\sinh(c+dx)b^3}{(a^2+b^2)\left(\sinh^2(c+dx)b^2+b^2\right)}\right)d(b\sinh(c+dx))+a+b\sinh(c+dx)}{b^2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^3\arctan(\sinh(c+dx))}{a^2+b^2}-\frac{ab^2\log\left(b^2\sinh^2(c+dx)+b^2\right)}{2(a^2+b^2)}-\frac{a^3\log(a+b\sinh(c+dx))}{a^2+b^2}+a+b\sinh(c+dx)}{b^2d}$$

input
```
Int[(Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(a - (b^3*ArcTan[Sinh[c + d*x]])/(a^2 + b^2) - (a^3*Log[a + b*Sinh[c + d*x
]])/(a^2 + b^2) - (a*b^2*Log[b^2 + b^2*Sinh[c + d*x]^2])/(2*(a^2 + b^2)) +
 b*Sinh[c + d*x])/(b^2*d)
```

3.409.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 604
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(b*d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*
x^2)^p*ExpandToSum[b*d^m*(m + n + 2*p + 1)*x^m - b*(m + n + 2*p + 1)*(c + d
*x)^m - (c + d*x)^(m - 2)*(a*d^2*(m + n - 1) - b*c^2*(m + n + 2*p + 1) - 2*
b*c*d*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m,
 1] && NeQ[m + n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

3.409.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.90

method	result
derivativedivides	$\frac{-\frac{a^3 \ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{b^2(a^2+b^2)}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{-8a\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-16b}{8a^2+8b^2}}{d}$
default	$\frac{-\frac{a^3 \ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{b^2(a^2+b^2)}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{-8a\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-16b}{8a^2+8b^2}}{d}$
risch	$-\frac{ax}{b^2}+\frac{e^{dx+c}}{2bd}-\frac{e^{-dx-c}}{2bd}+\frac{2a\,d^2x}{a^2d^2+b^2d^2}+\frac{2adc}{a^2d^2+b^2d^2}+\frac{2a^3x}{b^2(a^2+b^2)}+\frac{2a^3c}{b^2d(a^2+b^2)}+\frac{i\ln(e^{dx+c}-i)b}{(a^2+b^2)d}-\frac{\ln(e}{(a}$

input
```
int(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3/b^2/(a^2+b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)
-a)-1/b/(tanh(1/2*d*x+1/2*c)+1)+a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)+16/(8*a^2+
8*b^2)*(-1/2*a*ln(1+tanh(1/2*d*x+1/2*c)^2)-b*arctan(tanh(1/2*d*x+1/2*c)))-
1/b/(tanh(1/2*d*x+1/2*c)-1)+a/b^2*ln(tanh(1/2*d*x+1/2*c)-1))
```

3.409.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(89) = 178$.

Time = 0.27 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.24

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{2\left(a^3+ab^2\right)dx\cosh\left(dx+c\right)-a^2b-b^3+\left(a^2b+b^3\right)\cosh\left(dx+c\right)^2+\left(a^2b+b^3\right)\sinh\left(dx+c\right)^2-4\left(b^3\cosh\right.}{}$$

input
```
integrate(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/2*(2*(a^3 + a*b^2)*d*x*cosh(d*x + c) - a^2*b - b^3 + (a^2*b + b^3)*cosh(
d*x + c)^2 + (a^2*b + b^3)*sinh(d*x + c)^2 - 4*(b^3*cosh(d*x + c) + b^3*si
nh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(a^3*cosh(d*x + c)
+ a^3*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x
 + c))) - 2*(a*b^2*cosh(d*x + c) + a*b^2*sinh(d*x + c))*log(2*cosh(d*x + c
)/(cosh(d*x + c) - sinh(d*x + c))) + 2*((a^3 + a*b^2)*d*x + (a^2*b + b^3)*
cosh(d*x + c))*sinh(d*x + c))/((a^2*b^2 + b^4)*d*cosh(d*x + c) + (a^2*b^2
+ b^4)*d*sinh(d*x + c))
```

3.409.6 Sympy [F]

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh^2\left(c+dx\right)\tanh\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.409.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.65

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^2b^2+b^4)d} + \frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d} - \frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d} - \frac{(dx+c)a}{b^2d} + \frac{e^{(dx+c)}}{2\,bd} - \frac{e^{(-dx-c)}}{2\,bd}$$

input
```
integrate(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^4)*d) +
 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((
a^2 + b^2)*d) - (d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) - 1/2*e^(-d*x
- c)/(b*d)
```

3.409.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.63

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2\,a^3\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^2b^2+b^4} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)b}{a^2+b^2} + \frac{a\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2} - \frac{e^{(dx+c)}-e^{(-dx-c)}}{b}}{2\,d}$$

input
```
integrate(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*a^3*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^2*b^2 + b^4)
 + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*b/(a^2 + b^2) +
 a*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2) - (e^(d*x + c) - e^
(-d*x - c))/b)/d
```

3.409.9 Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.80

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\mathrm{e}^{c+d\,x}}{2\,b\,d} - \frac{\ln\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)}{a\,d-b\,d\,1\mathrm{i}}$$
$$-\frac{a^3\,\ln\left(2\,a^4\,b^3-b^7-a^2\,b^5-a^6\,b+2\,a^7\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+b^7\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+a^6\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+2\,a^3\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-4\,a^5\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right.}{d\,a^2\,b^2+d\,b^4}$$
$$-\frac{\mathrm{e}^{-c-d\,x}}{2\,b\,d}+\frac{a\,x}{b^2}-\frac{\ln\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)\,1\mathrm{i}}{-b\,d+a\,d\,1\mathrm{i}}$$

input
```
int((sinh(c + d*x)^2*tanh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

output
```
exp(c + d*x)/(2*b*d) - (log(exp(c + d*x)*1i + 1)*1i)/(a*d*1i - b*d) - log(
exp(c + d*x) + 1i)/(a*d - b*d*1i) - (a^3*log(2*a^4*b^3 - b^7 - a^2*b^5 - a
^6*b + 2*a^7*exp(d*x)*exp(c) + b^7*exp(2*c)*exp(2*d*x) + a^6*b*exp(2*c)*ex
p(2*d*x) + 2*a^3*b^4*exp(d*x)*exp(c) - 4*a^5*b^2*exp(d*x)*exp(c) + a^2*b^5
*exp(2*c)*exp(2*d*x) - 2*a^4*b^3*exp(2*c)*exp(2*d*x) + 2*a*b^6*exp(d*x)*ex
p(c)))/(b^4*d + a^2*b^2*d) - exp(- c - d*x)/(2*b*d) + (a*x)/b^2
```

3.410 $\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.410.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.410.2 Mathematica [N/A]

Not integrable

Time = 39.99 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Sinh[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Sinh[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

3.410.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{ 6125}$$
$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Sinh[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.410.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.410.4 Maple [N/A] (verified)

Not integrable

Time = 1.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(dx+c\right)^{2}\tanh\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.410.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\sinh(dx+c)^2\tanh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(sinh(d*x + c)^2*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

3.410.6 Sympy [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(sinh(d*x+c)**2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)**2*tanh(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)),
 x)
```

3.410.7 Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 322, normalized size of antiderivative = 9.47

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\sinh(dx+c)^2\tanh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/2*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d*
e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f) + 1/
4*integrate(-8*(a^4*e^(d*x + c) - a^3*b)/(a^2*b^3*e + b^5*e + (a^2*b^3*f +
 b^5*f)*x - (a^2*b^3*e*e^(2*c) + b^5*e*e^(2*c) + (a^2*b^3*f*e^(2*c) + b^5*
f*e^(2*c))*x)*e^(2*d*x) - 2*(a^3*b^2*e*e^c + a*b^4*e*e^c + (a^3*b^2*f*e^c
+ a*b^4*f*e^c)*x)*e^(d*x)), x) - 1/4*integrate(8*(b*e^(d*x + c) - a)/(a^2*
e + b^2*e + (a^2*f + b^2*f)*x + (a^2*e*e^(2*c) + b^2*e*e^(2*c) + (a^2*f*e^
(2*c) + b^2*f*e^(2*c))*x)*e^(2*d*x)), x)
```

3.410.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.410.9 Mupad [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)^2\tanh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((sinh(c + d*x)^2*tanh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((sinh(c + d*x)^2*tanh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.411 $\int \frac{(e+fx)^3 \sinh(c+dx) \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.411.1 Optimal result

Integrand size = 34, antiderivative size = 1294

$$\int \frac{(e+fx)^3 \sinh(c+dx) \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output

```
6*a^3*f*(f*x+e)^2*arctan(exp(d*x+c))/b^2/(a^2+b^2)/d^2+3*a^4*f^2*(f*x+e)*p
olylog(2,-exp(2*d*x+2*c))/b^3/(a^2+b^2)/d^3-6*I*a^3*f^2*(f*x+e)*polylog(2,
-I*exp(d*x+c))/b^2/(a^2+b^2)/d^3+3*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b/d^2+
3*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/b/d^3-3/2*f^3*polylog(3,-exp(2*d*
x+2*c))/b/d^4+3*a^4*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b^3/(a^2+b^2)/d^2-3*a
^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3
/2)/d^2+3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(
a^2+b^2)^(3/2)/d^2+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))/b/(a^2+b^2)^(3/2)/d^3-6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a
+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-6*I*a*f^2*(f*x+e)*polylog(2,I*exp
(d*x+c))/b^2/d^3-6*I*a^3*f^3*polylog(3,I*exp(d*x+c))/b^2/(a^2+b^2)/d^4+1/4
*(f*x+e)^4/b/f+a^2*(f*x+e)^3/b^3/d-(f*x+e)^3/b/d-a^4*(f*x+e)^3/b^3/(a^2+b^
2)/d+a*(f*x+e)^3*sech(d*x+c)/b^2/d+a^2*(f*x+e)^3*tanh(d*x+c)/b^3/d-a^3*(f*
x+e)^3*sech(d*x+c)/b^2/(a^2+b^2)/d-a^4*(f*x+e)^3*tanh(d*x+c)/b^3/(a^2+b^2)
/d-a^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/
d+a^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d
-6*a*f*(f*x+e)^2*arctan(exp(d*x+c))/b^2/d^2-3*a^2*f^2*(f*x+e)*polylog(2,-e
xp(2*d*x+2*c))/b^3/d^3-3/2*a^4*f^3*polylog(3,-exp(2*d*x+2*c))/b^3/(a^2+b^2
)/d^4-3*a^2*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b^3/d^2-6*a^3*f^3*polylog(4,-
b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^4+6*a^3*f^3*polyl...
```

3.411.2 Mathematica [A] (verified)

Time = 5.13 (sec) , antiderivative size = 1111, normalized size of antiderivative = 0.86

$$\int \frac{(e+fx)^3 \sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x(4e^3+6e^2fx+4ef^2x^2+f^3x^3)}{4b}$$

$$-\frac{f\left(12bd^3e^2e^{2c}x-12bd^3e^2(1+e^{2c})\,x-12bd^3efx^2-4bd^3f^2x^3+12ad^2e^2(1+e^{2c})\arctan\left(e^{c+dx}\right)+6bd^2e\right.}{}$$

$$+\frac{a^3\left(2d^3e^3\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-3d^3e^2fx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-3d^3ef^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-d^3f^3x^3\log\right.}{}$$

$$+\frac{(e+fx)^3\operatorname{sech}(c+dx)(a-b\operatorname{sech}(c)\sinh(dx))}{(a^2+b^2)\,d}$$

input
```
Integrate[((e + f*x)^3*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(4*b) - (f*(12*b*d^3*e^2*E
^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x - 12*b*d^3*e*f*x^2 - 4*b*d^3*f^2*x
^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] + 6*b*d^2*e^2*(1 + E^(
2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*a*d*e*(1 + E^(2*c))*f*(d
*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(
c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*b*d*e*(1 + E^(2*c))*f*(2*d*x*(d
*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]) + (6*I)*a*(
1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(c
+ d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*E^(c + d
*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)]) + b*(
1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6*d*x*P
olyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))])))/(2*(a^2 +
b^2)*d^4*(1 + E^(2*c))) + (a^3*(2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt
[a^2 + b^2]] - 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
] - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^3*f
^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e^2*f*x*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/...
```

3.411.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6115$$

$$\frac{\int (e+fx)^3\tanh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -(e+fx)^3\tan(ic+idx)^2dx}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\int (e+fx)^3\tan(ic+idx)^2dx}{b}$$

$$\Big\downarrow \; 4203$$

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3if\int i(e+fx)^2\tanh(c+dx)dx}{d}-\int (e+fx)^3dx+\frac{(e+fx)^3\tanh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 17$$

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3if\int i(e+fx)^2\tanh(c+dx)dx}{d}+\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{3f\int (e+fx)^2\tanh(c+dx)dx}{d}+\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{3f\int -i(e+fx)^2\tan(ic+idx)dx}{d}+\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3if\int (e+fx)^2\tan(ic+idx)dx}{d}+\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{b}$$

$$\Big\downarrow \; 4201$$

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3if\left(2i\int \frac{e^{2(c+dx)}(e+fx)^2}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{b}$$

$$\Big\downarrow \ 2620$$

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int (e+fx)\log\left(1+e^{2(c+dx)}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{b}$$

$$\Big\downarrow \ 3011$$

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int \text{PolyLog}\left(2,-e^{2(c+dx)}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\tanh(c+dx)}{d}-(e\cdots}{b}$$

$$\Big\downarrow \ 2720$$

$$-\frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\text{t}\cdots}{\cdots}}{b}$$

$$\Big\downarrow \ 6101$$

$$-\frac{a\left(\frac{\int (e+fx)^3\text{sech}(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\text{t}\cdots}{\cdots}}{b}$$

$$\Big\downarrow \ 5974$$

3.411. $\displaystyle\int \frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\frac{3f\int (e+fx)^2\text{sech}(c+dx)dx}{d}-\frac{(e+fx)^3\text{sech}(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3 \text{t}}{}}{b}$$

$\downarrow$ 3042

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{b}\right)}{b}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3 \text{t}}{}}{b}$$

$\downarrow$ 4668

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(-\frac{2if\int (e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+f}{}\right.}{d}}{b}\right.}{b}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3 \text{t}}{}}{b}$$

$\downarrow$ 3011

3.411. $\displaystyle\int \frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if(}{}\right.}{}}{b}\right.}{b}$$

$$\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\operatorname{t}}{}$$
$$\overline{\qquad b\qquad}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{a\left(-\frac{a\int\frac{(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+}\right.}{d}\right.}{d}\right.}{}}{}\right.}{b}$$

$$\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\operatorname{t}}{}$$
$$\overline{\qquad b\qquad}$$

$$\Big\downarrow \text{6117}$$

$$-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\right.}{d}\right.}{}}{}\right.}{b}$$

$$\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\operatorname{t}}{}$$
$$\overline{\qquad b\qquad}$$

3.411. $\int\frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}\right.}{d}}{d}}{b}\right)}{b}$$

$$\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\,\text{t}}{}$$
$$b$$

$$\Big\downarrow \text{4672}$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\text{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}\right.}{d}}{d}}{b}\right)}{b}$$

$$\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\text{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\,\text{t}}{}$$
$$b$$

$$\Big\downarrow \text{26}$$

3.411. $\int \frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(-\frac{a\left(\frac{\frac{(e+fx)^3\tanh(c+dx)}{d}-\frac{3f\int(e+fx)^2\tanh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\cdots}{d\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}\cdots\right.}{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\operatorname{t}\cdots}{\cdots}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{\cdots}}{d^2}\cdots\right.}{d}\cdots\right.}{d}}{b}\cdots\right.}{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\operatorname{t}\cdots}{\cdots}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{\cdots}}{d^2}\cdots\right.}{d}\cdots\right.}{d}}{b}\cdots\right.}{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\operatorname{t}\cdots}{\cdots}}{b}$$

3.411. $\displaystyle\int\frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 4201$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{\cdots}}{d^2}\cdots\right.}{d}\cdots\right.}{d}}{b}\cdots\right.}{\cdots}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\,\mathrm{t}\cdots}{\cdots}}{b}$$

$$\Big\downarrow \; 2620$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{\cdots}}{d^2}\cdots\right.}{d}\cdots\right.}{d}}{b}\cdots\right.}{\cdots}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\,\mathrm{t}\cdots}{\cdots}}{b}$$

$$\Big\downarrow \; 3011$$

3.411. $\displaystyle\int \frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{\cdots}}{d^2}\cdots\right.}{d}\cdots\right.}{d}\cdots}{b}\cdots\right.}{\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\,t\cdots}{\cdots}}{b}\cdots}$$

$$\Big\downarrow \; 2720$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{\cdots}}{d^2}\cdots\right.}{d}\cdots\right.}{d}\cdots}{b}\cdots\right.}{\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\,t\cdots}{\cdots}}{b}\cdots}$$

$$\Big\downarrow \; 6107$$

3.411. $\displaystyle\int \frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}\cdots\right)}{d}\cdots\right)}{d}\cdots}{b}\cdots\right)}{\cdots}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\tanh\cdots}{\cdots}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}+\frac{3f\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}\cdots\right)}{d}\cdots\right)}{d}\cdots}{b}\cdots\right)}{\cdots}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\left(\frac{f\int e^{-2(c+dx)}\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\tanh\cdots}{\cdots}}{b}$$

3.411. $\int \frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^3*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.411.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.411. $\int \frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6115
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - S
imp[a/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.411.4 Maple [F]

$$\int \frac{(fx+e)^3 \sinh(dx+c) \tanh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.411.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7331 vs. 2(1195) = 2390.

Time = 0.40 (sec) , antiderivative size = 7331, normalized size of antiderivative = 5.67

$$\int \frac{(e+fx)^3 \sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.411.6 Sympy [F]

$$\int \frac{(e+fx)^3 \sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3 \sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**3*sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x))
, x)
```

3.411.7 Maxima [F]

$$\int \frac{(e+fx)^3 \sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \sinh(dx+c)\tanh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-3*b*e^2*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2
 + b^2)*d^2)) - 6*a*f^3*integrate(x^2*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) +
 b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 6*b*f^3*integrate(x^2/(a^2*d
*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 12*a*e*f^2
*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) +
a^2*d + b^2*d), x) - 12*b*e*f^2*integrate(x/(a^2*d*e^(2*d*x + 2*c) + b^2*d
*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - (a^3*log((b*e^(-d*x - c) - a - sqr
t(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2*b + b^3)*sqrt(
a^2 + b^2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x
 - 2*c))*d) - (d*x + c)/(b*d))*e^3 - 6*a*e^2*f*arctan(e^(d*x + c))/((a^2 +
 b^2)*d^2) + 1/4*(24*b^2*e^2*f*x + (a^2*d*f^3 + b^2*d*f^3)*x^4 + 4*(a^2*d*
e*f^2 + (d*e*f^2 + 2*f^3)*b^2)*x^3 + 6*(a^2*d*e^2*f + (d*e^2*f + 4*e*f^2)*
b^2)*x^2 + ((a^2*d*f^3*e^(2*c) + b^2*d*f^3*e^(2*c))*x^4 + 4*(a^2*d*e*f^2*e
^(2*c) + b^2*d*e*f^2*e^(2*c))*x^3 + 6*(a^2*d*e^2*f*e^(2*c) + b^2*d*e^2*f*e
^(2*c))*x^2)*e^(2*d*x) + 8*(a*b*f^3*x^3*e^c + 3*a*b*e*f^2*x^2*e^c + 3*a*b*
e^2*f*x*e^c)*e^(d*x))/(a^2*b*d + b^3*d + (a^2*b*d*e^(2*c) + b^3*d*e^(2*c))
*e^(2*d*x)) - integrate(-2*(a^3*f^3*x^3*e^c + 3*a^3*e*f^2*x^2*e^c + 3*a^3*
e^2*f*x*e^c)*e^(d*x)/(a^2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2
*d*x) - 2*(a^3*b*e^c + a*b^3*e^c)*e^(d*x)), x)
```

3.411.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.411.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\,(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.412 $\int \frac{(e+fx)^2 \sinh(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} \, dx$

3.412.1 Optimal result

Integrand size = 34, antiderivative size = 904

$$
\begin{aligned}
&\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{a^2(e+fx)^2}{b^3d}-\frac{(e+fx)^2}{bd}-\frac{a^4(e+fx)^2}{b^3(a^2+b^2)\,d}+\frac{(e+fx)^3}{3bf}-\frac{4af(e+fx)\arctan\left(e^{c+dx}\right)}{b^2d^2} \\
&+\frac{4a^3f(e+fx)\arctan\left(e^{c+dx}\right)}{b^2(a^2+b^2)\,d^2}-\frac{a^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d} \\
&+\frac{a^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d}-\frac{2a^2f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{b^3d^2} \\
&+\frac{2f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{bd^2}+\frac{2a^4f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{b^3(a^2+b^2)\,d^2} \\
&+\frac{2iaf^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{b^2d^3}-\frac{2ia^3f^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{b^2(a^2+b^2)\,d^3} \\
&-\frac{2iaf^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{b^2d^3}+\frac{2ia^3f^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{b^2(a^2+b^2)\,d^3} \\
&-\frac{2a^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2}+\frac{2a^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2} \\
&-\frac{a^2f^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{b^3d^3}+\frac{f^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{bd^3} \\
&+\frac{a^4f^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{b^3(a^2+b^2)\,d^3}+\frac{2a^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^3} \\
&-\frac{2a^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^3}+\frac{a(e+fx)^2\operatorname{sech}(c+dx)}{b^2d}-\frac{a^3(e+fx)^2\operatorname{sech}(c+dx)}{b^2(a^2+b^2)\,d} \\
&+\frac{a^2(e+fx)^2\tanh(c+dx)}{b^3d}-\frac{(e+fx)^2\tanh(c+dx)}{bd}-\frac{a^4(e+fx)^2\tanh(c+dx)}{b^3(a^2+b^2)\,d}
\end{aligned}
$$

output
```
2*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/b/d^2+a^2*(f*x+e)^2/b^3/d-4*a*f*(f*x+e)*a
rctan(exp(d*x+c))/b^2/d^2-2*a^2*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/b^3/d^2+2*a
^3*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-
2*a^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d
^3-2*I*a*f^2*polylog(2,I*exp(d*x+c))/b^2/d^3+1/3*(f*x+e)^3/b/f-a^2*f^2*pol
ylog(2,-exp(2*d*x+2*c))/b^3/d^3-a^4*(f*x+e)^2/b^3/(a^2+b^2)/d+a*(f*x+e)^2*
sech(d*x+c)/b^2/d+a^2*(f*x+e)^2*tanh(d*x+c)/b^3/d+a^4*f^2*polylog(2,-exp(2
*d*x+2*c))/b^3/(a^2+b^2)/d^3-a^3*(f*x+e)^2*sech(d*x+c)/b^2/(a^2+b^2)/d-a^4
*(f*x+e)^2*tanh(d*x+c)/b^3/(a^2+b^2)/d-a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d+a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(
a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d+2*I*a^3*f^2*polylog(2,I*exp(d*x+c))/b
^2/(a^2+b^2)/d^3+2*I*a*f^2*polylog(2,-I*exp(d*x+c))/b^2/d^3+f^2*polylog(2,
-exp(2*d*x+2*c))/b/d^3-(f*x+e)^2*tanh(d*x+c)/b/d+4*a^3*f*(f*x+e)*arctan(ex
p(d*x+c))/b^2/(a^2+b^2)/d^2+2*a^4*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/b^3/(a^2+
b^2)/d^2-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a
^2+b^2)^(3/2)/d^2+2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))/b/(a^2+b^2)^(3/2)/d^2-2*I*a^3*f^2*polylog(2,-I*exp(d*x+c))/b^2/(a^2+b
^2)/d^3-(f*x+e)^2/b/d
```

3.412.2 Mathematica [A] (verified)

Time = 2.92 (sec) , antiderivative size = 665, normalized size of antiderivative = 0.74

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x(3e^2+3efx+f^2x^2)}{3b}$$

$$- \frac{f\left(4bd^2ee^{2c}x - 4bd^2e(1+e^{2c})\,x + 2bd^2e^{2c}fx^2 - 2bd^2(1+e^{2c})\,fx^2 + 4ade(1+e^{2c})\arctan\left(e^{c+dx}\right) + 2bde\right.}{}$$

$$+ \frac{a^3\left(2d^2e^2\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) - 2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) - d^2f^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) + 2d^2efx\log\left(1\right.\right.}{}$$

$$+ \frac{(e+fx)^2\text{sech}(c+dx)(a-b\text{sech}(c)\sinh(dx))}{(a^2+b^2)\,d}$$

input
```
Integrate[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) - (f*(4*b*d^2*e*E^(2*c)*x - 4*b*d^2*
e*(1 + E^(2*c))*x + 2*b*d^2*E^(2*c)*f*x^2 - 2*b*d^2*(1 + E^(2*c))*f*x^2 +
4*a*d*e*(1 + E^(2*c))*ArcTan[E^(c + d*x)] + 2*b*d*e*(1 + E^(2*c))*(2*d*x -
 Log[1 + E^(2*(c + d*x))]) + (2*I)*a*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c
+ d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog
[2, I*E^(c + d*x)]) + b*(1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*
x))]) - PolyLog[2, -E^(2*(c + d*x))])))/((a^2 + b^2)*d^3*(1 + E^(2*c))) +
(a^3*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x
*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2
])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
 + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*f^2*PolyLo
g[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*(a^2 + b^2)^(3/2)*d^3)
 + ((e + f*x)^2*Sech[c + d*x]*(a - b*Sech[c]*Sinh[d*x]))/((a^2 + b^2)*d)
```

3.412.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6115}$$

$$\frac{\int (e+fx)^2\tanh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -(e+fx)^2\tan(ic+idx)^2dx}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\int (e+fx)^2\tan(ic+idx)^2dx}{b}$$

$$\Big\downarrow \text{ 4203}$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{2if\int i(e+fx)\tanh(c+dx)dx}{d}-\int (e+fx)^2dx+\frac{(e+fx)^2\tanh(c+dx)}{d}}{b}$$

$$\Big\downarrow \; 17$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{2if\int i(e+fx)\tanh(c+dx)dx}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{2f\int (e+fx)\tanh(c+dx)dx}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{2f\int -i(e+fx)\tan(ic+idx)dx}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{2if\int (e+fx)\tan(ic+idx)dx}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \; 4201$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{2if\left(2i\int \frac{e^{2(c+dx)}(e+fx)}{1+e^{2(c+dx)}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \; 2620$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int \log\left(1+e^{2(c+dx)}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \; 2715$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{2if\left(2i\left(\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}-\frac{f\int e^{-2(c+dx)}\log\left(1+e^{2(c+dx)}\right)de^{2(c+dx)}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \; 2838$$

$$-\frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

3.412. $\displaystyle\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 6101$$

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \; 5974$$

$$-\frac{a\left(\frac{\frac{2f\int (e+fx)\text{sech}(c+dx)dx}{d}-\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{b}\right)}{b}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \; 4668$$

$$-\frac{a\left(-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right.}{d}\cdots\right.}{d}}{b}\cdots\right)}{b}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \; 2715$$

3.412. $\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}\right.}{d}}{b}\right.}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right.}{d}}{b}\right.}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \text{6117}$$

$$-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}\right.}{d}}{b}\right.}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(\right.}{}\right.}{b}\right.}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \text{4672}$$

3.412. $\int\frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\cdots\right.}{\cdots}\cdots\right.}{b}\cdots$$

$$\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow 26$$

$$-\frac{a\left(-\frac{a\left(\frac{\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{2f\int(e+fx)\tanh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\cdots\right.}{\cdots}}{b}\cdots\right.}{b}\cdots$$

$$\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow 3042$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\cdots\right.}{\cdots}\cdots\right.}{b}\cdots$$

$$\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow 26$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\cdots\right.}{\cdots}\cdots\right.}{b}\cdots$$

$$\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

3.412. $\int\frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\Big\downarrow \ 4201$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\cdots}{\cdots}\right)}{\cdots}\right)}{b}\cdots$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \ 2620$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\cdots}{\cdots}\right)}{\cdots}\right)}{b}\cdots$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \ 2715$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\text{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\cdots}{\cdots}\right)}{\cdots}\right)}{b}\cdots$$

$$\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \ 2838$$

3.412. $\displaystyle\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{\frac{a\int\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{(e\cdots}{\cdots}}{\cdots}\right)}{\cdots}\right)}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \; 6107$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(-\frac{a\left(\frac{b^2\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\left(\frac{-\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}+\frac{2f\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\tanh(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{Po}\cdots}{\cdots}\right)\cdots\right)}{\cdots}\right)}{\cdots}\right)}{b}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{4d^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\tanh(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{b}$$

3.412. $\int\frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.412.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

3.412. $\int \frac{(e+fx)^2 \sinh(c+dx) \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6115
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - S
imp[a/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.412.4 Maple [F]

$$\int \frac{(fx+e)^2 \sinh(dx+c)\tanh(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.412.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4196 vs. 2(841) = 1682.

Time = 0.37 (sec) , antiderivative size = 4196, normalized size of antiderivative = 4.64

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
1/3*((a^4 + 2*a^2*b^2 + b^4)*d^3*f^2*x^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*e
*f*x^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*e^2*x + 6*(a^2*b^2 + b^4)*d^2*e^2 -
 12*(a^2*b^2 + b^4)*c*d*e*f + 6*(a^2*b^2 + b^4)*c^2*f^2 + ((a^4 + 2*a^2*b^
2 + b^4)*d^3*f^2*x^3 - 12*(a^2*b^2 + b^4)*c*d*e*f + 6*(a^2*b^2 + b^4)*c^2*
f^2 + 3*((a^4 + 2*a^2*b^2 + b^4)*d^3*e*f - 2*(a^2*b^2 + b^4)*d^2*f^2)*x^2
+ 3*((a^4 + 2*a^2*b^2 + b^4)*d^3*e^2 - 4*(a^2*b^2 + b^4)*d^2*e*f)*x)*cosh(
d*x + c)^2 + ((a^4 + 2*a^2*b^2 + b^4)*d^3*f^2*x^3 - 12*(a^2*b^2 + b^4)*c*d
*e*f + 6*(a^2*b^2 + b^4)*c^2*f^2 + 3*((a^4 + 2*a^2*b^2 + b^4)*d^3*e*f - 2*
(a^2*b^2 + b^4)*d^2*f^2)*x^2 + 3*((a^4 + 2*a^2*b^2 + b^4)*d^3*e^2 - 4*(a^2
*b^2 + b^4)*d^2*e*f)*x)*sinh(d*x + c)^2 - 6*(a^3*b*d*f^2*x + a^3*b*d*e*f +
 (a^3*b*d*f^2*x + a^3*b*d*e*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*f^2*x + a^3*b*
d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*f^2*x + a^3*b*d*e*f)*sinh(d*
x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) +
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 6
*(a^3*b*d*f^2*x + a^3*b*d*e*f + (a^3*b*d*f^2*x + a^3*b*d*e*f)*cosh(d*x + c
)^2 + 2*(a^3*b*d*f^2*x + a^3*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b
*d*f^2*x + a^3*b*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*co
sh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
a^2 + b^2)/b^2) - b)/b + 1) + 3*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c
^2*f^2 + (a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*cosh(d*x + c...
```

3.412.6 Sympy [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x))
, x)
```

3.412.7 Maxima [F]

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\sinh(dx+c)\tanh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-2*b*e*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 +
 b^2)*d^2)) - 4*a*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2
*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 4*b*f^2*integrate(x/(a^2*d*e^(2*
d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - (a^3*log((b*e^(-
d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((
a^2*b + b^3)*sqrt(a^2 + b^2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^
2 + b^2)*e^(-2*d*x - 2*c))*d) - (d*x + c)/(b*d))*e^2 - 4*a*e*f*arctan(e^(d
*x + c))/((a^2 + b^2)*d^2) + 1/3*(12*b^2*e*f*x + (a^2*d*f^2 + b^2*d*f^2)*x
^3 + 3*(a^2*d*e*f + (d*e*f + 2*f^2)*b^2)*x^2 + ((a^2*d*f^2*e^(2*c) + b^2*d
*f^2*e^(2*c))*x^3 + 3*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x^2)*e^(2*d*
x) + 6*(a*b*f^2*x^2*e^c + 2*a*b*e*f*x*e^c)*e^(d*x))/(a^2*b*d + b^3*d + (a^
2*b*d*e^(2*c) + b^3*d*e^(2*c))*e^(2*d*x)) - integrate(-2*(a^3*f^2*x^2*e^c
+ 2*a^3*e*f*x*e^c)*e^(d*x)/(a^2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^(2*c)
)*e^(2*d*x) - 2*(a^3*b*e^c + a*b^3*e^c)*e^(d*x)), x)
```

3.412.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.412.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.413 $\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.413.1 Optimal result

Integrand size = 32, antiderivative size = 454

$$\begin{aligned}
&\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx\\
&=\frac{ex}{b}+\frac{fx^2}{2b}-\frac{af\arctan(\sinh(c+dx))}{b^2d^2}+\frac{a^3f\arctan(\sinh(c+dx))}{b^2(a^2+b^2)\,d^2}\\
&\quad-\frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}\,d}+\frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}\,d}\\
&\quad-\frac{a^2f\log(\cosh(c+dx))}{b^3d^2}+\frac{f\log(\cosh(c+dx))}{bd^2}+\frac{a^4f\log(\cosh(c+dx))}{b^3(a^2+b^2)\,d^2}\\
&\quad-\frac{a^3f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}\,d^2}+\frac{a^3f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}\,d^2}\\
&\quad+\frac{a(e+fx)\mathrm{sech}(c+dx)}{b^2d}-\frac{a^3(e+fx)\mathrm{sech}(c+dx)}{b^2(a^2+b^2)\,d}+\frac{a^2(e+fx)\tanh(c+dx)}{b^3d}\\
&\quad-\frac{(e+fx)\tanh(c+dx)}{bd}-\frac{a^4(e+fx)\tanh(c+dx)}{b^3(a^2+b^2)\,d}
\end{aligned}$$

output
```
e*x/b+1/2*f*x^2/b-a*f*arctan(sinh(d*x+c))/b^2/d^2+a^3*f*arctan(sinh(d*x+c)
)/b^2/(a^2+b^2)/d^2-a^2*f*ln(cosh(d*x+c))/b^3/d^2+f*ln(cosh(d*x+c))/b/d^2+
a^4*f*ln(cosh(d*x+c))/b^3/(a^2+b^2)/d^2-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d+a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d-a^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+a^3*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+a*(f*x+e)*sech(d*x+c)/b^2/d-a^3*(f*x+e)*se
ch(d*x+c)/b^2/(a^2+b^2)/d+a^2*(f*x+e)*tanh(d*x+c)/b^3/d-(f*x+e)*tanh(d*x+c
)/b/d-a^4*(f*x+e)*tanh(d*x+c)/b^3/(a^2+b^2)/d
```

3.413.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.67 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.79

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{\frac{(c+dx)(cf-d(2e+fx))}{b}+\frac{2f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a-ib}+\frac{2f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a+ib}+\frac{f\log(\cosh(c+dx))}{ia-b}-\frac{f\log(\cosh(c+dx))}{ia+b}+\frac{2a^3}{}}{}$$

input
```
Integrate[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),
x]
```

output
```
-1/2*(((c + d*x)*(c*f - d*(2*e + f*x)))/b + (2*f*ArcTan[Tanh[(c + d*x)/2]]
)/(a - I*b) + (2*f*ArcTan[Tanh[(c + d*x)/2]])/(a + I*b) + (f*Log[Cosh[c +
d*x]])/(I*a - b) - (f*Log[Cosh[c + d*x]])/(I*a + b) + (2*a^3*(-2*d*e*ArcTa
nh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*x)
)/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + f*Pol
yLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*(a^2 + b^2)^(3/2)) + (2*d*(e + f*x)*Se
ch[c + d*x]*(-a + b*Sinh[c + d*x]))/(a^2 + b^2))/d^2
```

3.413.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6115}$$

$$\frac{\int (e+fx)\tanh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int -\left((e+fx)\tan(ic+idx)^2\right)dx}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\int (e+fx)\tan(ic+idx)^2dx}{b}$$

$$\Big\downarrow \text{ 4203}$$

$$-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{if\int i\tanh(c+dx)dx}{d}-\int (e+fx)dx+\frac{(e+fx)\tanh(c+dx)}{d}}{b}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{if\int i\tanh(c+dx)dx}{d}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{f\int \tanh(c+dx)dx}{d}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{f\int -i\tan(ic+idx)dx}{d}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{\frac{if\int \tan(ic+idx)dx}{d}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \text{ 3956}$$

$$-\frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \text{6101}$$

$$-\frac{a\left(\frac{\int (e+fx)\text{sech}(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \text{5974}$$

$$-\frac{a\left(\frac{\frac{f\int \text{sech}(c+dx)dx}{d}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}-\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)\text{sech}(c+dx)}{d}+\frac{f\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \text{6117}$$

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \text{3042}$$

3.413. $\displaystyle\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 4672$$

$$-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int-i\tanh(c+dx)dx}{d}}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int\tanh(c+dx)dx}{d}}{b}-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int-i\tan(ic+idx)dx}{d}}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

3.413. $\int\frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int\tan(ic+idx)dx}{d}}{b}\right)}{b}\right)}{b}$$

$\downarrow$ 3956

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$\downarrow$ 6107

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{b^2\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\,s\cdots}{a^2+b^2}\cdots\right.}{b}\cdots\right.}{b}\cdots\right.}{b}\cdots-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$\downarrow$ 3042

$$-\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{e\cdots}{a-ib\sin\cdots}}{a^2\cdots}\right.}{b}\cdots\right.}{b}\cdots\right.}{b}\cdots$$

$\downarrow$ 3803

3.413. $\int\frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{2b^2\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)\operatorname{sech}^2\cdots}{\cdots}\right)}{b}\cdots\right)}{b}\cdots\right)}{b}\cdots$$

$$\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int (e+fx)\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int \frac{\cdots}{-2e\cdots}}{\cdots}\right)}{b}\cdots\right)}{b}\cdots\right)}{b}\cdots$$

$$\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \; 2694$$

3.413. $\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\text{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots\right.}{b}\cdots$$

$$\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow 27$$

3.413. $\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{b\cdots}\cdots\right.}{b\cdots}\cdots\right.}{b\cdots}$$

$$\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \; 2620$$

3.413. $\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots$$

$$\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \text{2715}$$

3.413. $\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \; 2838$$

3.413. $\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{sech}(c+dx)}{d}}{b}-\frac{a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}-\frac{a\left(\frac{\int(e+fx)\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(b\left(\cdots\right)\right.}{\cdots}\right.}{\cdots}\right)}{\cdots}\right)}{b}$$

$$\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}$$

$$\Big\downarrow \text{7293}$$

3.413. $\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$a\frac{\int\left(a(e+fx)\operatorname{sech}^2(c+dx)-b(e+fx)\operatorname{sech}(c+dx)\tanh(\right.}{a^2+b^2}$$

$$a\left(\frac{\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}}{b}\right)-$$

$$a\left(\frac{\frac{f\arctan(\sinh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{sech}(c+dx)}{d}}{b}\right)-$$

$$-\frac{}{\frac{-\frac{f\log(\cosh(c+dx))}{d^2}+\frac{(e+fx)\tanh(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{b}}$$

input
```
Int[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.413. $\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.413.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.413. $\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.413. $\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6115
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - S
imp[a/b   Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.413.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1896 vs. $2(432) = 864$.

Time = 2.36 (sec) , antiderivative size = 1897, normalized size of antiderivative = 4.18

method	result	size
risch	Expression too large to display	1897

input
```
int((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
2/d^2/(a^2+b^2)*b*a^2*f/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))-2/d^2/(a^2+b^2)
^(5/2)*b*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^3-2/d^2/(a^
2+b^2)^(5/2)*b^3*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a-4/d
^2/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*a*arctan(exp(d*x+c))+2*(f*x+e)*(a*exp(d*x
+c)+b)/d/(a^2+b^2)/(1+exp(2*d*x+2*c))-2/d^2/(a^2+b^2)*b*a^2*f/(2*a^2+2*b^2
)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d/(a^2+b^2)^(3/2)*b*a^3*e/(2*a^2
+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/2*f*x^2/b+2/d^
2/(a^2+b^2)^(3/2)/b*a^5*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2
)+a)/(a+(a^2+b^2)^(1/2)))-2/d^2/(a^2+b^2)^(3/2)/b*a^5*f/(2*a^2+2*b^2)*dilo
g((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+2/d^2/(a^2+b^2)^
(3/2)*b^3*a*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/
2))+2/d^2/(a^2+b^2)^(3/2)*b*a^3*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c
)+2*a)/(a^2+b^2)^(1/2))+2/d^2/(a^2+b^2)^(3/2)*b*a^3*f/(2*a^2+2*b^2)*dilog(
(b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d^2/(a^2+b^2)^(1/2
)*b*a*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/
d/(a^2+b^2)^(3/2)/b*a^5*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(
a^2+b^2)^(1/2))-2/d^2/(a^2+b^2)^(3/2)*b*a^3*f/(2*a^2+2*b^2)*dilog((-b*exp(
d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/d/(a^2+b^2)^(3/2)/b*a^5*
f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
*x+2/d/(a^2+b^2)^(3/2)/b*a^5*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)...
```

3.413.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1571 vs. $2(430) = 860$.

Time = 0.31 (sec) , antiderivative size = 1571, normalized size of antiderivative = 3.46

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
1/2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*x
 + 4*(a^2*b^2 + b^4)*d*e + ((a^4 + 2*a^2*b^2 + b^4)*d^2*f*x^2 + 2*((a^4 +
2*a^2*b^2 + b^4)*d^2*e - 2*(a^2*b^2 + b^4)*d*f)*x)*cosh(d*x + c)^2 + ((a^4
 + 2*a^2*b^2 + b^4)*d^2*f*x^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*e - 2*(a^2*
b^2 + b^4)*d*f)*x)*sinh(d*x + c)^2 - 2*(a^3*b*f*cosh(d*x + c)^2 + 2*a^3*b*
f*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh(d*x + c)^2 + a^3*b*f)*sqrt((a
^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a^3*b*f*cosh(d*
x + c)^2 + 2*a^3*b*f*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh(d*x + c)^2
 + a^3*b*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
 2*(a^3*b*d*e - a^3*b*c*f + (a^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)^2 + 2*(a
^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*e - a^3*b*c*f
)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(
d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(a^3*b*d*e - a^3*b*c*f + (
a^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*e - a^3*b*c*f)*cosh(d*
x + c)*sinh(d*x + c) + (a^3*b*d*e - a^3*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2
+ b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^
2)/b^2) + 2*a) - 2*(a^3*b*d*f*x + a^3*b*c*f + (a^3*b*d*f*x + a^3*b*c*f)*co
sh(d*x + c)^2 + 2*(a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c)...
```

3.413.6 Sympy [F]

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\sinh{(c+dx)}\tanh^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x
)
```

3.413.7 Maxima [F]

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\sinh(dx+c)\tanh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqr
t(a^2 + b^2)))/((a^2*b + b^3)*sqrt(a^2 + b^2)*d) - 2*(a*e^(-d*x - c) - b)/
((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d) - (d*x + c)/(b*d))*e - 1/2*
(4*a^3*integrate(-x*e^(d*x + c)/(a^2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^
(2*c))*e^(2*d*x) - 2*(a^3*b*e^c + a*b^3*e^c)*e^(d*x)), x) - ((a^2*d*e^(2*c
) + b^2*d*e^(2*c))*x^2*e^(2*d*x) + 4*a*b*x*e^(d*x + c) + 4*b^2*x + (a^2*d
+ b^2*d)*x^2)/(a^2*b*d + b^3*d + (a^2*b*d*e^(2*c) + b^3*d*e^(2*c))*e^(2*d*
x)) + 4*b*x/((a^2 + b^2)*d) + 4*a*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) -
2*b*log(e^(2*d*x + 2*c) + 1)/((a^2 + b^2)*d^2))*f
```

3.413.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.413.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh\,(c+d\,x)}\,dx$$

input
```
int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.414 $\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.414.1 Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{a^2x}{b(a^2+b^2)}+\frac{bx}{a^2+b^2}+\frac{2a^3\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d}+\frac{a\operatorname{sech}(c+dx)}{(a^2+b^2)d}-\frac{b\tanh(c+dx)}{(a^2+b^2)d}$$

output
```
a^2*x/b/(a^2+b^2)+b*x/(a^2+b^2)+2*a^3*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a
^2+b^2)^(1/2))/b/(a^2+b^2)^(3/2)/d+a*sech(d*x+c)/(a^2+b^2)/d-b*tanh(d*x+c)
/(a^2+b^2)/d
```

3.414.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{c+dx}{b}+\frac{2a^3\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{b(-a^2-b^2)^{3/2}}+\frac{\operatorname{sech}(c+dx)(a-b\sinh(c+dx))}{a^2+b^2}}{d}$$

input
```
Integrate[(Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
((c + d*x)/b + (2*a^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/
(b*(-a^2 - b^2)^(3/2)) + (Sech[c + d*x]*(a - b*Sinh[c + d*x]))/(a^2 + b^2)
)/d
```

3.414.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {3042, 26, 3381, 25, 26, 3042, 25, 26, 3086, 24, 3214, 3042, 3139, 1083, 217, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ic+idx)^3}{\cos(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ic+idx)^3}{\cos(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3381}$$

$$i\left(-\frac{a^2\int \frac{i\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{ib\int -\tanh^2(c+dx)dx}{a^2+b^2}+\frac{a\int i\text{sech}(c+dx)\tanh(c+dx)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\frac{a^2\int \frac{i\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{ib\int \tanh^2(c+dx)dx}{a^2+b^2}+\frac{a\int i\text{sech}(c+dx)\tanh(c+dx)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{ia^2\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{ib\int \tanh^2(c+dx)dx}{a^2+b^2}+\frac{ia\int \text{sech}(c+dx)\tanh(c+dx)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{ia^2\int -\frac{i\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{a^2+b^2}-\frac{ib\int -\tan(ic+idx)^2dx}{a^2+b^2}+\frac{ia\int -i\sec(ic+idx)\tan(ic+idx)dx}{a^2+b^2}\right)$$

$$\Big\downarrow 25$$

$$i\left(-\frac{ia^2\int -\frac{i\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}+\frac{ia\int -i\sec(ic+idx)\tan(ic+idx)dx}{a^2+b^2}\right)$$

$$\Big\downarrow 26$$

$$i\left(-\frac{a^2\int \frac{\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}+\frac{a\int \sec(ic+idx)\tan(ic+idx)dx}{a^2+b^2}\right)$$

$$\Big\downarrow 3086$$

$$i\left(-\frac{a^2\int \frac{\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}-\frac{ia\int 1d\text{sech}(c+dx)}{d\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow 24$$

$$i\left(-\frac{a^2\int \frac{\sin(ic+idx)}{a-ib\sin(ic+idx)}dx}{a^2+b^2}+\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}-\frac{ia\text{sech}(c+dx)}{d\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow 3214$$

$$i\left(\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}-\frac{a^2\left(\frac{ix}{b}-\frac{ia\int \frac{1}{a+b\sinh(c+dx)}dx}{b}\right)}{a^2+b^2}-\frac{ia\text{sech}(c+dx)}{d\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow 3042$$

$$i\left(-\frac{a^2\left(\frac{ix}{b}-\frac{ia\int \frac{1}{a-ib\sin(ic+idx)}dx}{b}\right)}{a^2+b^2}+\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}-\frac{ia\text{sech}(c+dx)}{d\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow 3139$$

$$i\left(\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}-\frac{a^2\left(\frac{ix}{b}-\frac{2a\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\right)}{a^2+b^2}-\frac{ia\text{sech}(c+dx)}{d\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow 1083$$

3.414. $\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$i\left(\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}-\frac{a^2\left(\frac{4a\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{bd}+\frac{ix}{b}\right)}{a^2+b^2}-\frac{ia\text{sech}(c+dx)}{d\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{217}$$

$$i\left(\frac{ib\int \tan(ic+idx)^2dx}{a^2+b^2}-\frac{a^2\left(\frac{ix}{b}-\frac{2ia\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}}\right)}{a^2+b^2}-\frac{ia\text{sech}(c+dx)}{d\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{3954}$$

$$i\left(\frac{ib\left(\frac{\tanh(c+dx)}{d}-\int 1dx\right)}{a^2+b^2}-\frac{a^2\left(\frac{ix}{b}-\frac{2ia\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}}\right)}{a^2+b^2}-\frac{ia\text{sech}(c+dx)}{d\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{24}$$

$$i\left(-\frac{a^2\left(\frac{ix}{b}-\frac{2ia\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}}\right)}{a^2+b^2}+\frac{ib\left(\frac{\tanh(c+dx)}{d}-x\right)}{a^2+b^2}-\frac{ia\text{sech}(c+dx)}{d\left(a^2+b^2\right)}\right)$$

input
```
Int[(Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
I*(-((a^2*((I*x)/b - ((2*I)*a*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2]
)])/(b*Sqrt[a^2 + b^2]*d)))/(a^2 + b^2)) - (I*a*Sech[c + d*x])/((a^2 + b^2
)*d) + (I*b*(-x + Tanh[c + d*x]/d))/(a^2 + b^2))
```

3.414. $\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.414.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.414. $\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3381
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a*(d^2/(a^2
- b^2))   Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Simp[
b*(d/(a^2 - b^2))   Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x]
 - Simp[a^2*(d^2/(g^2*(a^2 - b^2)))   Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[
e + f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1
]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.414.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{2\left(b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^{2}+b^{2}\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}-\frac{2a^{3}\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{2\left(b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^{2}+b^{2}\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}-\frac{2a^{3}\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}}{d}$
risch	$\frac{x}{b}+\frac{2a\,\mathrm{e}^{dx+c}+2b}{d\left(a^{2}+b^{2}\right)\left(1+\mathrm{e}^{2dx+2c}\right)}+\frac{a^{3}\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}+a^{4}+2a^{2}b^{2}+b^{4}}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}db}-\frac{a^{3}\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}-a^{4}-2a^{2}b^{2}-}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right.}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}db}$

input
```
int(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b*ln(tanh(1/2*d*x+1/2*c)-1)-2/(a^2+b^2)*(b*tanh(1/2*d*x+1/2*c)-a)/
(1+tanh(1/2*d*x+1/2*c)^2)-2/b*a^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/
2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+1/b*ln(tanh(1/2*d*x+1/2*c)+1))
```

3.414.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(118) = 236$.

Time = 0.26 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.79

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{(a^4+2a^2b^2+b^4)dx\cosh(dx+c)^2+(a^4+2a^2b^2+b^4)dx\sinh(dx+c)^2+2a^2b^2+2b^4+(a^4+2a^2b^2+b^4)}{}$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
((a^4 + 2*a^2*b^2 + b^4)*d*x*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d*x
*sinh(d*x + c)^2 + 2*a^2*b^2 + 2*b^4 + (a^4 + 2*a^2*b^2 + b^4)*d*x + (a^3*
cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x + c)^2
+ a^3)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*
a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c
) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x
 + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*
sinh(d*x + c) - b)) + 2*(a^3*b + a*b^3)*cosh(d*x + c) + 2*(a^3*b + a*b^3 +
 (a^4 + 2*a^2*b^2 + b^4)*d*x*cosh(d*x + c))*sinh(d*x + c))/((a^4*b + 2*a^2
*b^3 + b^5)*d*cosh(d*x + c)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*d*cosh(d*x + c
)*sinh(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sinh(d*x + c)^2 + (a^4*b + 2
*a^2*b^3 + b^5)*d)
```

3.414.6 Sympy [F]

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.414.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.17

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a^3\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^2b+b^3)\sqrt{a^2+b^2}d} + \frac{2\left(ae^{(-dx-c)}-b\right)}{(a^2+b^2+(a^2+b^2)e^{(-2dx-2c)})d} + \frac{dx+c}{bd}$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/((a^2*b + b^3)*sqrt(a^2 + b^2)*d) + 2*(a*e^(-d*x - c) - b)/(
(a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d) + (d*x + c)/(b*d)
```

3.414.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{a^3\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2b+b^3)\sqrt{a^2+b^2}} - \frac{dx+c}{b} - \frac{2\left(ae^{(dx+c)}+b\right)}{(a^2+b^2)\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(a^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x +
c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b + b^3)*sqrt(a^2 + b^2)) - (d*x + c)
/b - 2*(a*e^(d*x + c) + b)/((a^2 + b^2)*(e^(2*d*x + 2*c) + 1)))/d
```

3.414.9 Mupad [B] (verification not implemented)

Time = 2.87 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.87

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2b}{d\,(a^2+b^2)}+\frac{2\,a\,e^{c+d\,x}}{d\,(a^2+b^2)}}{e^{2\,c+2\,d\,x}+1}+\frac{x}{b}$$
$$+\frac{2\,\mathrm{atan}\left(\left(e^{d\,x}\,e^{c}\left(\frac{2\,a^3}{b^3\,d\,(a^2\,b+b^3)\,\sqrt{a^6}\,(a^2+b^2)}+\frac{2\left(a\,b^3\,d\,\sqrt{a^6}+a^3\,b\,d\,\sqrt{a^6}\right)}{a^2\,b^2\,(a^2\,b+b^3)\,\sqrt{-b^2\,d^2\,(a^2+b^2)^3}\,\sqrt{-a^6\,b^2\,d^2-3\,a^4\,b^4\,d^2-3\,a^2\,b^6\,d^2-b^8\,d^2}}\right)-\frac{\quad}{a^2 \cdots}\right.\right.}{\sqrt{-a^6 \cdots}}$$

input

```
int((sinh(c + d*x)*tanh(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
```

output

```
((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(2*c + 2*
d*x) + 1) + x/b + (2*atan((exp(d*x)*exp(c)*((2*a^3)/(b^3*d*(a^2*b + b^3)*(
a^6)^(1/2)*(a^2 + b^2)) + (2*(a*b^3*d*(a^6)^(1/2) + a^3*b*d*(a^6)^(1/2)))/
(a^2*b^2*(a^2*b + b^3)*(-b^2*d^2*(a^2 + b^2)^3)^(1/2)*(- b^8*d^2 - 3*a^2*b
^6*d^2 - 3*a^4*b^4*d^2 - a^6*b^2*d^2)^(1/2))) - (2*(b^4*d*(a^6)^(1/2) + a^
2*b^2*d*(a^6)^(1/2)))/(a^2*b^2*(a^2*b + b^3)*(-b^2*d^2*(a^2 + b^2)^3)^(1/2
)*(- b^8*d^2 - 3*a^2*b^6*d^2 - 3*a^4*b^4*d^2 - a^6*b^2*d^2)^(1/2)))*((b^4*
(- b^8*d^2 - 3*a^2*b^6*d^2 - 3*a^4*b^4*d^2 - a^6*b^2*d^2)^(1/2))/2 + (a^2*
b^2*(- b^8*d^2 - 3*a^2*b^6*d^2 - 3*a^4*b^4*d^2 - a^6*b^2*d^2)^(1/2))/2))*(
a^6)^(1/2))/(- b^8*d^2 - 3*a^2*b^6*d^2 - 3*a^4*b^4*d^2 - a^6*b^2*d^2)^(1/2
)
```

3.415 $\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.415.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.415.2 Mathematica [N/A]

Not integrable

Time = 37.65 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Sinh[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Sinh[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

3.415.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Sinh[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.415.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.415.4 Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(dx+c)\tanh(dx+c)^2}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.415.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)\tanh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(sinh(d*x + c)*tanh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

3.415.6 Sympy [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)*tanh(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)),
 x)
```

3.415.7 Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 411, normalized size of antiderivative = 12.09

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(dx+c)\tanh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-2*a^3*integrate(-e^(d*x + c)/(a^2*b^2*e + b^4*e + (a^2*b^2*f + b^4*f)*x -
 (a^2*b^2*e*e^(2*c) + b^4*e*e^(2*c) + (a^2*b^2*f*e^(2*c) + b^4*f*e^(2*c))*
x)*e^(2*d*x) - 2*(a^3*b*e*e^c + a*b^3*e*e^c + (a^3*b*f*e^c + a*b^3*f*e^c)*
x)*e^(d*x)), x) + 2*(a*e^(d*x + c) + b)/(a^2*d*e + b^2*d*e + (a^2*d*f + b^
2*d*f)*x + (a^2*d*e*e^(2*c) + b^2*d*e*e^(2*c) + (a^2*d*f*e^(2*c) + b^2*d*f
*e^(2*c))*x)*e^(2*d*x)) + log(f*x + e)/(b*f) + 1/2*integrate(4*(a*f*e^(d*x
 + c) + b*f)/(a^2*d*e^2 + b^2*d*e^2 + (a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2
*d*e*f + b^2*d*e*f)*x + (a^2*d*e^2*e^(2*c) + b^2*d*e^2*e^(2*c) + (a^2*d*f^
2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 + 2*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2
*c))*x)*e^(2*d*x)), x)
```

3.415.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.415.9 Mupad [N/A]

Not integrable

Time = 3.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((sinh(c + d*x)*tanh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((sinh(c + d*x)*tanh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.416 $\int \frac{(e+fx)^2 \tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.416.1 Optimal result

Integrand size = 28, antiderivative size = 1479

$$\int \frac{(e+fx)^2 \tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
-2*I*a^4*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b/(a^2+b^2)^2/d^2-I*a^4*f*(f*x+
e)*polylog(2,I*exp(d*x+c))/b^3/(a^2+b^2)/d^2-1/2*(f*x+e)^2*sech(d*x+c)*tan
h(d*x+c)/b/d-I*f^2*polylog(3,I*exp(d*x+c))/b/d^3+I*f*(f*x+e)*polylog(2,I*e
xp(d*x+c))/b/d^2+I*f^2*polylog(3,-I*exp(d*x+c))/b/d^3-f*(f*x+e)*sech(d*x+c
)/b/d^2-I*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b/d^2-2*a^4*(f*x+e)^2*arctan(
exp(d*x+c))/b/(a^2+b^2)^2/d-1/2*a^3*(f*x+e)^2*sech(d*x+c)^2/b^2/(a^2+b^2)/
d+1/2*a^2*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/b^3/d-2*a^3*f*(f*x+e)*polylog(
2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-2*a^3*f*(f*x+e)*polyl
og(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-I*a^2*f^2*polylog(
3,I*exp(d*x+c))/b^3/d^3+I*a^4*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b^3/(a^2+
b^2)/d^2+2*I*a^4*f^2*polylog(3,I*exp(d*x+c))/b/(a^2+b^2)^2/d^3+2*I*a^4*f*(
f*x+e)*polylog(2,-I*exp(d*x+c))/b/(a^2+b^2)^2/d^2+I*a^2*f*(f*x+e)*polylog(
2,I*exp(d*x+c))/b^3/d^2+I*a^4*f^2*polylog(3,I*exp(d*x+c))/b^3/(a^2+b^2)/d^
3-a^4*f*(f*x+e)*sech(d*x+c)/b^3/(a^2+b^2)/d^2+(f*x+e)^2*arctan(exp(d*x+c))
/b/d+f^2*arctan(sinh(d*x+c))/b/d^3-a^2*f^2*arctan(sinh(d*x+c))/b^3/d^3+a^4
*f^2*arctan(sinh(d*x+c))/b^3/(a^2+b^2)/d^3+a^3*f*(f*x+e)*polylog(2,-exp(2*
d*x+2*c))/(a^2+b^2)^2/d^2+I*a^2*f^2*polylog(3,-I*exp(d*x+c))/b^3/d^3+a^2*f
*(f*x+e)*sech(d*x+c)/b^3/d^2-a*f*(f*x+e)*tanh(d*x+c)/b^2/d^2-a^3*f^2*ln(co
sh(d*x+c))/b^2/(a^2+b^2)/d^3+a^3*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/(a^2+b^2)^
2/d+a*f^2*ln(cosh(d*x+c))/b^2/d^3-a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a...
```

3.416.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3368 vs. 2(1479) = 2958.

Time = 11.93 (sec) , antiderivative size = 3368, normalized size of antiderivative = 2.28

$$\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-12*a^3*d^3*e^2*E^(2*c)*x - 12*a^3*d*E^(2*c)*f^2*x - 12*a*b^2*d*E^(2*c)*f
^2*x - 12*a^3*d^3*e*E^(2*c)*f*x^2 - 4*a^3*d^3*E^(2*c)*f^2*x^3 + 18*a^2*b*d
^2*e^2*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*ArcTan[E^(c + d*x)] + 18*a^2*b*
d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*E^(2*c)*ArcTan[E^(c +
d*x)] + 12*a^2*b*f^2*ArcTan[E^(c + d*x)] + 12*b^3*f^2*ArcTan[E^(c + d*x)]
+ 12*a^2*b*E^(2*c)*f^2*ArcTan[E^(c + d*x)] + 12*b^3*E^(2*c)*f^2*ArcTan[E^(
c + d*x)] + (18*I)*a^2*b*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*b^3*d^2*
e*f*x*Log[1 - I*E^(c + d*x)] + (18*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 - I*E^
(c + d*x)] + (6*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (9*I)*a^
2*b*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*d^2*f^2*x^2*Log[1 - I*E
^(c + d*x)] + (9*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*
I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (18*I)*a^2*b*d^2*e*f*x
*Log[1 + I*E^(c + d*x)] - (6*I)*b^3*d^2*e*f*x*Log[1 + I*E^(c + d*x)] - (18
*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (6*I)*b^3*d^2*e*E^(2*
c)*f*x*Log[1 + I*E^(c + d*x)] - (9*I)*a^2*b*d^2*f^2*x^2*Log[1 + I*E^(c + d
*x)] - (3*I)*b^3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (9*I)*a^2*b*d^2*E^(2
*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 +
 I*E^(c + d*x)] + 6*a^3*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 6*a^3*d^2*e^2*E
^(2*c)*Log[1 + E^(2*(c + d*x))] + 6*a^3*f^2*Log[1 + E^(2*(c + d*x))] + 6*a
*b^2*f^2*Log[1 + E^(2*(c + d*x))] + 6*a^3*E^(2*c)*f^2*Log[1 + E^(2*(c +...
```

3.416.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6101}$$

$$\frac{\int (e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{5978}$$

$$\frac{\int (e+fx)^2\text{sech}(c+dx)dx-\int (e+fx)^2\text{sech}^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{2if\int (e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}-\int (e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-\int (e+fx)^2\text{cs}}{b}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}}{b}$$

$$\Big\downarrow \text{4674}$$

$$-\frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}}{}$$

3.416. $\displaystyle\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 3042$$

$$-\frac{a\int \frac{(e+fx)^2\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}$$

$$\Big\downarrow \; 4257$$

$$-\frac{a\int \frac{(e+fx)^2\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right.}{d}$$

$$\Big\downarrow \; 4668$$

$$-\frac{a\int \frac{(e+fx)^2\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{1}{2}\left(\frac{2if\int (e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}-\frac{2if\int (e+fx)\log\left(1+ie^{c+dx}\right)dx}{d}-\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+}}{d^2}\right.}{d}$$

$$\Big\downarrow \; 3011$$

$$-\frac{a\int \frac{(e+fx)^2\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{1}{2}\left(-\frac{2if\left(\frac{f\int \operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \operatorname{PolyLog}\left(2,ie^{c+dx}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d}\right)}{d}-\frac{2(e+fx)^2 a}{}\right.$$

$$\Big\downarrow \; 2720$$

$$-\frac{a\int \frac{(e+fx)^2\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\right.}{d}\right.}{d}\right.$$

$$\Big\downarrow \; 6117$$

3.416. $\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,\right.}{d}\right.}{d}\right.$$

$$\Big\downarrow \text{5974}$$

$$-\frac{a\left(\frac{\frac{f\int (e+fx)\text{sech}^2(c+dx)dx}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,\right.}{d}\right.}{d}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,\right.}{d}\right.}{d}\right.$$

$$\frac{a\left(-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{4672}$$

$$\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,\right.}{d}\right.}{d}\right.$$

$$\frac{a\left(-\frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{if\int -i\tanh(c+dx)dx}{d}\right)}{d}}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

3.416. $\displaystyle\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int\tanh(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\right.}{d}\right.}{d}\right.$$

$$\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\right.}{d}\right.}{d}\right.$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\int -i\tan(ic+idx)dx}{d}\right)}{d}}{b}\right)}{b}$$

$$\downarrow \text{26}$$

$$\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\right.}{d}\right.}{d}\right.$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}+\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}+\frac{if\int\tan(ic+idx)dx}{d}\right)}{d}}{b}\right)}{b}$$

$$\downarrow \text{3956}$$

$$-\frac{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\right.}{d}\right.}{d}\right.$$

$$\downarrow \text{6117}$$

3.416. $\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\int(e+fx)^2\text{sech}^3(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}+$$

$$\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,\right.}{d}\right.}{d}\right.$$

$$\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,\right.}{d}\right.}{d}\right.$$

$$\frac{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}\right)}{b}\right)}{b}$$

$$\downarrow \text{4674}$$

$$-\frac{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{-\frac{f^2\int\text{sech}(c+dx)dx}{d^2}+\frac{1}{2}\int(e+fx)^2\text{sech}(c+dx)dx+\frac{f(e+fx)\text{sech}(c+dx)}{d^2}}{b}\right.}{b}\right.}{b}$$

$$\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,\right.}{d}\right.}{d}\right.$$

$$\downarrow \text{3042}$$

3.416. $\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\right.}{d}\right.}{d}\right.}{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{f^2\int\csc\left(ic+idx+\frac{\pi}{2}\right)dx}{d^2}+\frac{1}{2}\int(e+fx)^2\csc\right.}{}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\right.}{d}\right.}{d}\right.}{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\int(e+fx)^2\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{f^2\arctan(}{}\right.}{}\right.}{b}\right.}{b}$$

$$\Big\downarrow \text{4668}$$

$$\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\right.}{d}\right.}{d}\right.}{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{2if\int(e+fx)\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{2if}{}\right.}{}\right.}{}\right.}{b}$$

$$\Big\downarrow \text{3011}$$

3.416. $\int\frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\cdots\right.}{d}\right.}{d}\cdots\right.}{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}}{b}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)dx}{d}-\frac{(e+\cdots}{}\right.}{d}\right.}{\cdots}\right.}{\cdots}\right.}$$

$$\Big\downarrow \; 2720$$

$$\frac{\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\cdots\right.}{d}\right.}{d}\cdots\right.}{a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}}{b}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}-\frac{a\left(-\frac{a\int\frac{(e+fx)^2\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)\cdots}{d^2}\right.}{}\right.}{\cdots}\right.}{\cdots}\right.}$$

$$\Big\downarrow \; 6107$$

3.416. $\displaystyle\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{1}{2}\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,\right.}{d}\right.}{d}\right.$$

$$a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)^2\operatorname{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{(e+fx)^2\operatorname{sech}(c+dx}{a+b\sinh(c+dx)}}{a^2+b^2}\right.}{b}\right.}{}\right.$$

$$\Big\downarrow \; 6107$$

$$\frac{\arctan(\sinh(c+dx))f^2}{d^3}-\frac{(e+fx)\operatorname{sech}(c+dx)f}{d^2}+\frac{2i\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d}\right)f}{d}-\frac{2i\left(\frac{f\int e^{-c-dx}}{}\right.}{}$$

$$a\left(\frac{\frac{f\left(\frac{(e+fx)\tanh(c+dx)}{d}-\frac{f\log(\cosh(c+dx))}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{-\frac{\arctan(\sinh(c+dx))f^2}{d^3}+\frac{(e+fx)\operatorname{sech}(c+dx)f}{d^2}+\frac{(e+fx)^2\operatorname{sech}(c+dx)\tanh(}{2d}}{}\right.}{}\right.$$

3.416. $\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^2*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.416.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.416. $\int \frac{(e+fx)^2 \tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5978
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.416. $\int \frac{(e+fx)^2 \tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.416.4 Maple [F]

$$\int \frac{(fx+e)^2 \tanh(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.416.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10574 vs. 2(1354) = 2708.

Time = 0.46 (sec) , antiderivative size = 10574, normalized size of antiderivative = 7.15

$$\int \frac{(e+fx)^2 \tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

3.416.6 Sympy [F]

$$\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\tanh^3{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*tanh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*tanh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

3.416.7 Maxima [F]

$$\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\tanh{(dx+c)}^3}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
3*a^2*b*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2
*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d
^2 + b^4*d^2), x) + b^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*
x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d
^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a^3*d^2*f^2*integrate(x^2/(a^4*d^2*e
^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) +
 a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 6*a^2*b*d^2*e*f*integrate(x*e^(d
*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2
*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*b^3*d^2*e*f*
integrate(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x
+ 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x)
- 4*a^3*d^2*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*
d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2),
 x) - a^3*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x +
2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) - a*b^2*f^2*(2*(d*x + c)/((a^4 +
2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*
d^3)) - (a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2
*b^2 + b^4)*d) - a^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d)
 + (3*a^2*b + b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (b*e
^(-d*x - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + ...
```

3.416.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.416.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((tanh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((tanh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.417 $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.417.1 Optimal result

Integrand size = 26, antiderivative size = 894

$$
\begin{aligned}
\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx =\ & \frac{a^2(e+fx)\arctan\left(e^{c+dx}\right)}{b^3d}+\frac{(e+fx)\arctan\left(e^{c+dx}\right)}{bd} \\
& -\frac{2a^4(e+fx)\arctan\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)^2d}-\frac{a^4(e+fx)\arctan\left(e^{c+dx}\right)}{b^3\left(a^2+b^2\right)d} \\
& -\frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2d} \\
& -\frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2d} \\
& +\frac{a^3(e+fx)\log\left(1+e^{2(c+dx)}\right)}{\left(a^2+b^2\right)^2d}-\frac{ia^2f\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{2b^3d^2} \\
& -\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{2bd^2}+\frac{ia^4f\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{b\left(a^2+b^2\right)^2d^2} \\
& +\frac{ia^4f\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{2b^3\left(a^2+b^2\right)d^2}+\frac{ia^2f\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{2b^3d^2} \\
& +\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{2bd^2}-\frac{ia^4f\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{b\left(a^2+b^2\right)^2d^2} \\
& -\frac{ia^4f\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{2b^3\left(a^2+b^2\right)d^2}-\frac{a^3f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2d^2} \\
& -\frac{a^3f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2d^2}+\frac{a^3f\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2\left(a^2+b^2\right)^2d^2} \\
& +\frac{a^2f\,\mathrm{sech}(c+dx)}{2b^3d^2}-\frac{f\,\mathrm{sech}(c+dx)}{2bd^2}-\frac{a^4f\,\mathrm{sech}(c+dx)}{2b^3\left(a^2+b^2\right)d^2} \\
& +\frac{a(e+fx)\mathrm{sech}^2(c+dx)}{2b^2d}-\frac{a^3(e+fx)\mathrm{sech}^2(c+dx)}{2b^2\left(a^2+b^2\right)d} \\
& -\frac{af\tanh(c+dx)}{2b^2d^2}+\frac{a^3f\tanh(c+dx)}{2b^2\left(a^2+b^2\right)d^2} \\
& +\frac{a^2(e+fx)\mathrm{sech}(c+dx)\tanh(c+dx)}{2b^3d} \\
& -\frac{(e+fx)\mathrm{sech}(c+dx)\tanh(c+dx)}{2bd} \\
& -\frac{a^4(e+fx)\mathrm{sech}(c+dx)\tanh(c+dx)}{2b^3\left(a^2+b^2\right)d}
\end{aligned}
$$

output
```
-1/2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b/d-1/2*I*f*polylog(2,-I*exp(d*x+c))/
b/d^2+1/2*a^3*f*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2+1/2*a^2*f*sech(
d*x+c)/b^3/d^2+1/2*a*(f*x+e)*sech(d*x+c)^2/b^2/d-1/2*a*f*tanh(d*x+c)/b^2/d
^2-2*a^4*(f*x+e)*arctan(exp(d*x+c))/b/(a^2+b^2)^2/d-1/2*a^4*f*sech(d*x+c)/
b^3/(a^2+b^2)/d^2-1/2*a^3*(f*x+e)*sech(d*x+c)^2/b^2/(a^2+b^2)/d+1/2*a^3*f*
tanh(d*x+c)/b^2/(a^2+b^2)/d^2+1/2*a^2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b^3/
d-1/2*I*a^2*f*polylog(2,-I*exp(d*x+c))/b^3/d^2+1/2*I*f*polylog(2,I*exp(d*x
+c))/b/d^2-a^4*(f*x+e)*arctan(exp(d*x+c))/b^3/(a^2+b^2)/d+(f*x+e)*arctan(e
xp(d*x+c))/b/d+a^3*(f*x+e)*ln(1+exp(2*d*x+2*c))/(a^2+b^2)^2/d-a^3*(f*x+e)*
ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-a^3*(f*x+e)*ln(1+b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-a^3*f*polylog(2,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-a^3*f*polylog(2,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/(a^2+b^2)^2/d^2-1/2*f*sech(d*x+c)/b/d^2+1/2*I*a^4*f*polylog(
2,-I*exp(d*x+c))/b^3/(a^2+b^2)/d^2+I*a^4*f*polylog(2,-I*exp(d*x+c))/b/(a^2
+b^2)^2/d^2+1/2*I*a^2*f*polylog(2,I*exp(d*x+c))/b^3/d^2-1/2*a^4*(f*x+e)*se
ch(d*x+c)*tanh(d*x+c)/b^3/(a^2+b^2)/d-I*a^4*f*polylog(2,I*exp(d*x+c))/b/(a
^2+b^2)^2/d^2-1/2*I*a^4*f*polylog(2,I*exp(d*x+c))/b^3/(a^2+b^2)/d^2+a^2*(f
*x+e)*arctan(exp(d*x+c))/b^3/d
```

3.417.2 Mathematica [A] (verified)

Time = 9.10 (sec) , antiderivative size = 834, normalized size of antiderivative = 0.93

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{a^3\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\operatorname{arctanh}\left(\frac{a+b}{\sqrt{a}}\right.}{(-a^2-b^2)^{3/2}}\right.}{}$$

$$+\frac{-2a^3de(c+dx)+2a^3cf(c+dx)-a^3f(c+dx)^2+6a^2bde\arctan\left(e^{c+dx}\right)+2b^3de\arctan\left(e^{c+dx}\right)-6a^2}{}$$

$$+\frac{\operatorname{sech}(c+dx)(-bf-af\sinh(c+dx))}{2\left(a^2+b^2\right)d^2}$$

$$+\frac{\operatorname{sech}^2(c+dx)(ade-acf+af(c+dx)-bde\sinh(c+dx)+bcf\sinh(c+dx)-bf(c+dx)\sinh(c+dx}{2\left(a^2+b^2\right)d^2}$$

input
```
Integrate[((e + f*x)*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(a^3*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[
a^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 +
b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a
^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[
2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x
))/(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))]))/((a^2 + b^2)^2*d^2) + (-2*a^3*d*e*(c + d*x) + 2*a^3*c*f*(c +
 d*x) - a^3*f*(c + d*x)^2 + 6*a^2*b*d*e*ArcTan[E^(c + d*x)] + 2*b^3*d*e*Ar
cTan[E^(c + d*x)] - 6*a^2*b*c*f*ArcTan[E^(c + d*x)] - 2*b^3*c*f*ArcTan[E^(
c + d*x)] + (3*I)*a^2*b*f*(c + d*x)*Log[1 - I*E^(c + d*x)] + I*b^3*f*(c +
d*x)*Log[1 - I*E^(c + d*x)] - (3*I)*a^2*b*f*(c + d*x)*Log[1 + I*E^(c + d*x
)] - I*b^3*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + 2*a^3*d*e*Log[1 + E^(2*(c
+ d*x))] - 2*a^3*c*f*Log[1 + E^(2*(c + d*x))] + 2*a^3*f*(c + d*x)*Log[1 +
E^(2*(c + d*x))] - I*b*(3*a^2 + b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] + I*b*
(3*a^2 + b^2)*f*PolyLog[2, I*E^(c + d*x)] + a^3*f*PolyLog[2, -E^(2*(c + d*
x))])/(2*(a^2 + b^2)^2*d^2) + (Sech[c + d*x]*(-(b*f) - a*f*Sinh[c + d*x]))
/(2*(a^2 + b^2)*d^2) + (Sech[c + d*x]^2*(a*d*e - a*c*f + a*f*(c + d*x) - b
*d*e*Sinh[c + d*x] + b*c*f*Sinh[c + d*x] - b*f*(c + d*x)*Sinh[c + d*x])...
```

3.417.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6101}$$

$$\frac{\int (e+fx)\text{sech}(c+dx)\tanh^2(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 5978}$$

$$\frac{\int (e+fx)\text{sech}(c+dx)dx-\int (e+fx)\text{sech}^3(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}$$

$$\Big\downarrow \ \mathbf{4668}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx-\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}}{b}$$

$$\Big\downarrow \ \mathbf{2715}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{2(e+fx)\arctan\left(e^{c+}\right.}{d}}{b}$$

$$\Big\downarrow \ \mathbf{2838}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}}{b}$$

$$\Big\downarrow \ \mathbf{4673}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{1}{2}\int (e+fx)\text{sech}(c+dx)dx+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\text{sech}(c+dx)}{2d^2}-\frac{(e+}{}}{b}$$

$$\Big\downarrow \ \mathbf{3042}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{1}{2}\int (e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}-\frac{f\text{sech}(c+dx)}{2d^2}}{b}$$

$$\Big\downarrow \ \mathbf{4668}$$

$$-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{if\int \log\left(1-ie^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+ie^{c+dx}\right)dx}{d}-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\,\text{P}}{}}{b}$$

$$\Big\downarrow \ \mathbf{2715}$$

3.417. $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{1}{2}\left(\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c+dx}}{d^2}-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{ Poly}}{}}{b}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{-\frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{ PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\Big\downarrow \text{ 6117}$$

$$\frac{-\frac{a\left(\frac{\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)dx}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{ PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\Big\downarrow \text{ 5974}$$

$$\frac{-\frac{a\left(\frac{\frac{f\int \text{sech}^2(c+dx)dx}{2d}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{ PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\text{ PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\text{ PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\text{ PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$
$$\frac{a\left(-\frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}+\frac{f\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d}}{b}\right)}{b}$$

$$\Big\downarrow \text{ 4254}$$

3.417. $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}+\frac{if\int 1d(-i\tanh(c+dx))}{2d^2}}{b}\right)}{b}$$

$$\Big\downarrow \; 24$$

$$-\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}+$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\Big\downarrow \; 6117$$

$$-\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\int(e+fx)\text{sech}^3(c+dx)dx}{b}-\frac{a\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}\right)}{b}\right)}{b}+$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)^3dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 4673$$

3.417. $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{\frac{1}{2}\int(e+fx)\operatorname{sech}(c+dx)dx+\frac{f\operatorname{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}}{b}-\frac{a\int\frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}d}{b}\right.}{b}\right.}{b}$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\int(e+fx)\csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{f\operatorname{sech}(c+dx)}{2d^2}+\frac{(e+fx)\tanh(c+dx)\operatorname{sech}(c-}{2d}}{b}\right.}{b}\right.}{b}$$

$$\Big\downarrow \textbf{4668}$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{if\int\log\left(1-ie^{c+dx}\right)dx}{d}+\frac{if\int\log\left(1+ie^{c+dx}\right)dx}{d}+\frac{2(e+fx)\arctan\left(e^{c+d}\right.}{d}\right.}{b}\right.}{b}\right.}{b}$$

$$\Big\downarrow \textbf{2715}$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(-\frac{if\int e^{-c-dx}\log\left(1-ie^{c+dx}\right)de^{c+dx}}{d^2}+\frac{if\int e^{-c-dx}\log\left(1+ie^{c+dx}\right)de^{c}}{d^2}\right.}{b}\right.}{b}\right.}{b}$$

3.417. $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\cdots}{b}$$

$$\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\int\frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{b}+\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\cdots\right.}{b}\cdots\right.}{b}\cdots\right.}{b}$$

$$\Big\downarrow \text{6107}$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\cdots}{b}$$

$$\frac{a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\operatorname{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}\right)}{b}+\frac{\frac{1}{2}\left(\frac{2(e+fx)\arctan\left(e^{c+dx}\cdots\right.\right.}{d}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots\right.}{b}$$

$$\Big\downarrow \text{6107}$$

3.417. $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a-}{a^2+b^2}\right.}{a^2+b^2}\right.}{b}\right.}{}\right.$$

$$\Big\downarrow \; 6095$$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}\right.}{a^2+b^2}\right.}{a^2}\right.}{b}\right.}{}\right.$$

3.417. $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\downarrow \text{2620}$$

$$\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+$$

$$b$$

$$a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a-\frac{a\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(b^2\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b}}\right.}{bd}\right.\right.}{}\right.}{}}{}\right.$$

$$\downarrow \text{2715}$$

3.417. $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2d}}{b}-\frac{a\left(-\frac{a\left(\frac{b^2\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx}{}\right.}{a^2+b^2}\right.}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 2838

3.417. $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}+\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)+\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{if\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+}{b}$$

$$a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a-\frac{a\left(\frac{b^2\left(\frac{\int(e+fx)\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c}}{a+\sqrt{}}\right)}{bd^2}\right.}{}\right.}{a^2+b^2}\right.}{}}{}\right.$$

↓ 7293

3.417. $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{1}{2}\left(-\frac{2(e+fx)\arctan(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)+\frac{2(e+fx)\arctan(e^{c+dx})}{d}-\frac{if\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+}{b}$$

$$a\left(\frac{\frac{f\tanh(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2d}}{b}-\frac{a-\frac{a\left(\frac{b^2\left(\frac{\int(a(e+fx)\operatorname{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2\right.}{}\right.}{}\right.}{a^2+b^2}}{}}{}\right.$$

input
```
Int[((e + f*x)*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.417. $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.417.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

3.417. $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5974
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5978
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6101
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/b   Int[(e + f*x)^m*Sech[
c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Simp[a/b   Int[(e + f*x)^m*Sech[c
+ d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6117
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x],
x] - Simp[a/b   Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1
)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.417. $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.417.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2283 vs. $2(820) = 1640$.

Time = 2.71 (sec) , antiderivative size = 2284, normalized size of antiderivative = 2.55

method	result	size
risch	Expression too large to display	2284

input
```
int((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3/(a^2+b^2)^(3/2)/d^2*b^2*c*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))*a^2+I*b^3/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*dilog(1-I*exp(
d*x+c))+6*b/d/(a^2+b^2)*a^2*e/(2*a^2+2*b^2)*arctan(exp(d*x+c))-1/(a^2+b^2)
^(3/2)/d*b^4*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1
/2))+1/(a^2+b^2)^(1/2)/d*b^2*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2
*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(1/2)/d^2*b^2*c*f/(2*a^2+2*b^2)*arctanh(1
/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(3/2)/d^2*b^4*c*f/(2*
a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-3/(a^2+b^2)^(
3/2)/d*b^2*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2
))*a^2+2*b^3/d/(a^2+b^2)*e/(2*a^2+2*b^2)*arctan(exp(d*x+c))+3*I/d^2/(a^2+b
^2)*a^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))*b-3*I*b/d^2/(a^2+b^2)*a^2*f/
(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+2/(a^2+b^2)^(3/2)/d^2*c*a^4*f/(2*a^2+2
*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+I*b^3/d/(a^2+b^2)*
f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x-I*b^3/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*l
n(1+I*exp(d*x+c))*c-I*b^3/d/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x
-6*b/d^2/(a^2+b^2)*c*a^2*f/(2*a^2+2*b^2)*arctan(exp(d*x+c))+I*b^3/d^2/(a^2
+b^2)*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c-2/d^2*c*f/(2*a^2+2*b^2)/(a^2+b^
2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2-2/(a^2+b^2)
^(3/2)/d*e*a^4/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1
/2))+(-b*d*f*x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2*c)-b*d*e*exp(3*d*x+...
```

3.417.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4729 vs. $2(793) = 1586$.

Time = 0.37 (sec) , antiderivative size = 4729, normalized size of antiderivative = 5.29

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*cosh(d
*x + c)^3 + 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*
sinh(d*x + c)^3 - 2*(2*(a^3 + a*b^2)*d*f*x + 2*(a^3 + a*b^2)*d*e + (a^3 +
a*b^2)*f)*cosh(d*x + c)^2 - 2*(2*(a^3 + a*b^2)*d*f*x + 2*(a^3 + a*b^2)*d*e
 + (a^3 + a*b^2)*f - 3*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b +
 b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(a^3 + a*b^2)*f - 2*((a^2*b +
b^3)*d*f*x + (a^2*b + b^3)*d*e - (a^2*b + b^3)*f)*cosh(d*x + c) + 2*(a^3*f
*cosh(d*x + c)^4 + 4*a^3*f*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*f*sinh(d*x
+ c)^4 + 2*a^3*f*cosh(d*x + c)^2 + a^3*f + 2*(3*a^3*f*cosh(d*x + c)^2 + a^
3*f)*sinh(d*x + c)^2 + 4*(a^3*f*cosh(d*x + c)^3 + a^3*f*cosh(d*x + c))*sin
h(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a^3*f*cosh(d*x + c
)^4 + 4*a^3*f*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*f*sinh(d*x + c)^4 + 2*a^
3*f*cosh(d*x + c)^2 + a^3*f + 2*(3*a^3*f*cosh(d*x + c)^2 + a^3*f)*sinh(d*x
 + c)^2 + 4*(a^3*f*cosh(d*x + c)^3 + a^3*f*cosh(d*x + c))*sinh(d*x + c))*d
ilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - ((2*a^3*f + I*(3*a^2*b + b^3)*f)*c
osh(d*x + c)^4 + 4*(2*a^3*f + I*(3*a^2*b + b^3)*f)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (2*a^3*f + I*(3*a^2*b + b^3)*f)*sinh(d*x + c)^4 + 2*a^3*f + 2*(2*
a^3*f + I*(3*a^2*b + b^3)*f)*cosh(d*x + c)^2 + 2*(2*a^3*f + 3*(2*a^3*f ...
```

3.417.6 Sympy [F]

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\tanh^3{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*tanh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*tanh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

3.417.7 Maxima [F]

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\tanh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 +
b^4)*d) - a^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) + (3*a
^2*b + b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (b*e^(-d*x
- c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b
^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e - f*(((b*d*x*e^
(3*c) + b*e^(3*c))*e^(3*d*x) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d*x) - (
b*d*x*e^c - b*e^c)*e^(d*x) - a)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^
2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x)
) - integrate(-2*(a^4*x*e^(d*x + c) - a^3*b*x)/(a^4*b + 2*a^2*b^3 + b^5 -
(a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c +
 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - integrate(-(2*a^3*x - (3*a^2*b*
e^c + b^3*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) + 2*a^2*b^
2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x))
```

3.417.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.417.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh(c+d\,x)^3\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((tanh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((tanh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.418 $\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.418.1 Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b(3a^2+b^2)\arctan(\sinh(c+dx))}{2\left(a^2+b^2\right)^2 d} + \frac{a^3\log(\cosh(c+dx))}{\left(a^2+b^2\right)^2 d} - \frac{a^3\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)^2 d} + \frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2\left(a^2+b^2\right) d}$$

output
```
1/2*b*(3*a^2+b^2)*arctan(sinh(d*x+c))/(a^2+b^2)^2/d+a^3*ln(cosh(d*x+c))/(a
^2+b^2)^2/d-a^3*ln(a+b*sinh(d*x+c))/(a^2+b^2)^2/d+1/2*sech(d*x+c)^2*(a-b*s
inh(d*x+c))/(a^2+b^2)/d
```

3.418.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.27

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b(a^2+b^2)\arctan(\sinh(c+dx)) - (a^3 - i(2a^2b+b^3))\log(i-\sinh(c+dx)) - (a^3 + i(2a^2b+b^3))\log(i}{2(}$$

input
```
Integrate[Tanh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(b*(a^2 + b^2)*ArcTan[Sinh[c + d*x]] - (a^3 - I*(2*a^2*b + b^3))*Log[
I - Sinh[c + d*x]] - (a^3 + I*(2*a^2*b + b^3))*Log[I + Sinh[c + d*x]] + 2*
a^3*Log[a + b*Sinh[c + d*x]] - a*(a^2 + b^2)*Sech[c + d*x]^2 + b*(a^2 + b^
2)*Sech[c + d*x]*Tanh[c + d*x])/((a^2 + b^2)^2*d)
```

3.418.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 26, 3200, 601, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\tan(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\tan(ic+idx)^3}{a-ib\sin(ic+idx)}dx$$
$$\Big\downarrow \text{3200}$$
$$\frac{\int \frac{b^3\sinh^3(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$
$$\Big\downarrow \text{601}$$
$$\frac{\frac{b^2(a-b\sinh(c+dx))}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}-\frac{\int -\frac{b^2\left(ab^2+\left(2a^2+b^2\right)\sinh(c+dx)b\right)}{\left(a^2+b^2\right)(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2b^2}}{d}$$
$$\Big\downarrow \text{25}$$
$$\frac{\frac{\int \frac{b^2\left(ab^2+\left(2a^2+b^2\right)\sinh(c+dx)b\right)}{\left(a^2+b^2\right)(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2b^2}+\frac{b^2(a-b\sinh(c+dx))}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}}{d}$$
$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{ab^2+\left(2a^2+b^2\right)\sinh(c+dx)b}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{2(a^2+b^2)}+\frac{b^2(a-b\sinh(c+dx))}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}}{d}$$

$$\Big\downarrow \text{657}$$

$$\frac{\frac{\int \left(\frac{b^4+3a^2b^2+2a^3\sinh(c+dx)b}{\left(a^2+b^2\right)\left(\sinh^2(c+dx)b^2+b^2\right)}-\frac{2a^3}{\left(a^2+b^2\right)(a+b\sinh(c+dx))}\right)d(b\sinh(c+dx))}{2(a^2+b^2)}+\frac{b^2(a-b\sinh(c+dx))}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2(a-b\sinh(c+dx))}{2(a^2+b^2)(b^2\sinh^2(c+dx)+b^2)}+\frac{\frac{b\left(3a^2+b^2\right)\arctan(\sinh(c+dx))}{a^2+b^2}+\frac{a^3\log\left(b^2\sinh^2(c+dx)+b^2\right)}{a^2+b^2}-\frac{2a^3\log(a+b\sinh(c+dx))}{a^2+b^2}}{2(a^2+b^2)}}{d}$$

input
```
Int[Tanh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
(((b*(3*a^2 + b^2)*ArcTan[Sinh[c + d*x]])/(a^2 + b^2) - (2*a^3*Log[a + b*S
inh[c + d*x]])/(a^2 + b^2) + (a^3*Log[b^2 + b^2*Sinh[c + d*x]^2])/(a^2 + b
^2))/(2*(a^2 + b^2)) + (b^2*(a - b*Sinh[c + d*x]))/(2*(a^2 + b^2)*(b^2 + b
^2*Sinh[c + d*x]^2)))/d
```

3.418.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.418. $\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.418.4 Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.75

method	result
derivativedivides	$\frac{-\frac{8a^3 \ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{8a^4+16a^2b^2+8b^4}+\frac{\frac{2\left(\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^3-a\,b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}}{a^4+2a^2}}{d}$
default	$\frac{-\frac{8a^3 \ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{8a^4+16a^2b^2+8b^4}+\frac{\frac{2\left(\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^3-a\,b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}}{a^4+2a^2}}{d}$
risch	$-\frac{2d^2a^3x}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2d\,a^3c}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2a^3x}{a^4+2a^2b^2+b^4}+\frac{2a^3c}{d(a^4+2a^2b^2+b^4)}+\frac{\mathrm{e}^{dx+c}\left(-b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}\right.}{d(a^2+b^2)\left(1+\mathrm{e}^{2dx+2}\right.}$

3.418. $\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
int(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-8*a^3/(8*a^4+16*a^2*b^2+8*b^4)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1
/2*d*x+1/2*c)-a)+2/(a^4+2*a^2*b^2+b^4)*(((1/2*a^2*b+1/2*b^3)*tanh(1/2*d*x+
1/2*c)^3+(-a^3-a*b^2)*tanh(1/2*d*x+1/2*c)^2+(-1/2*a^2*b-1/2*b^3)*tanh(1/2*
d*x+1/2*c))/(1+tanh(1/2*d*x+1/2*c)^2)^2+1/2*a^3*ln(1+tanh(1/2*d*x+1/2*c)^2
)+1/2*(3*a^2*b+b^3)*arctan(tanh(1/2*d*x+1/2*c))))
```

3.418.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(117) = 234$.

Time = 0.28 (sec) , antiderivative size = 896, normalized size of antiderivative = 7.47

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-((a^2*b + b^3)*cosh(d*x + c)^3 + (a^2*b + b^3)*sinh(d*x + c)^3 - 2*(a^3 +
 a*b^2)*cosh(d*x + c)^2 - (2*a^3 + 2*a*b^2 - 3*(a^2*b + b^3)*cosh(d*x + c)
)*sinh(d*x + c)^2 - ((3*a^2*b + b^3)*cosh(d*x + c)^4 + 4*(3*a^2*b + b^3)*c
osh(d*x + c)*sinh(d*x + c)^3 + (3*a^2*b + b^3)*sinh(d*x + c)^4 + 3*a^2*b +
 b^3 + 2*(3*a^2*b + b^3)*cosh(d*x + c)^2 + 2*(3*a^2*b + b^3 + 3*(3*a^2*b +
 b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((3*a^2*b + b^3)*cosh(d*x + c)^
3 + (3*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + s
inh(d*x + c)) - (a^2*b + b^3)*cosh(d*x + c) + (a^3*cosh(d*x + c)^4 + 4*a^3
*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)^4 + 2*a^3*cosh(d*x + c)
^2 + a^3 + 2*(3*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 4*(a^3*cosh(d
*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/
(cosh(d*x + c) - sinh(d*x + c))) - (a^3*cosh(d*x + c)^4 + 4*a^3*cosh(d*x +
 c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)^4 + 2*a^3*cosh(d*x + c)^2 + a^3 +
2*(3*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 4*(a^3*cosh(d*x + c)^3 +
 a^3*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - si
nh(d*x + c))) - (a^2*b + b^3 - 3*(a^2*b + b^3)*cosh(d*x + c)^2 + 4*(a^3 +
a*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x +
 c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 +
 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x
 + c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + (a^4 + 2*a^2...
```

3.418.6 Sympy [F]

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(tanh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

3.418.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.81

$$\begin{aligned}\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = &-\frac{a^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^4+2\,a^2b^2+b^4)d}\\&+\frac{a^3\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d}-\frac{(3\,a^2b+b^3)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d}\\&-\frac{be^{(-dx-c)}-2\,ae^{(-2\,dx-2\,c)}-be^{(-3\,dx-3\,c)}}{(a^2+b^2+2\,(a^2+b^2)e^{(-2\,dx-2\,c)}+(a^2+b^2)e^{(-4\,dx-4\,c)})d}\end{aligned}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b
^4)*d) + a^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (3*a^
2*b + b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (b*e^(-d*x -
 c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^
2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d)
```

3.418.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(117) = 234$.

Time = 0.34 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.32

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{\frac{4\,a^3b\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{2\,a^3\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^4+2\,a^2b^2+b^4}-\frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(3\,a^2b+b^3)}{a^4+2\,a^2b^2+b^4}+}{4\,d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(4*a^3*b*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b + 2*a^
2*b^3 + b^5) - 2*a^3*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a^2*
b^2 + b^4) - (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(3*a^
2*b + b^3)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^3*(e^(d*x + c) - e^(-d*x - c))^2
 + 2*a^2*b*(e^(d*x + c) - e^(-d*x - c)) + 2*b^3*(e^(d*x + c) - e^(-d*x - c
)) - 4*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^(d*x + c) - e^(-d*x - c))^2 + 4
)))/d
```

3.418.9 Mupad [B] (verification not implemented)

Time = 3.22 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.18

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,(a^3+a\,b^2)}{d\,(a^2+b^2)^2} - \frac{e^{c+d\,x}\,(a^2\,b+b^3)}{d\,(a^2+b^2)^2}}{e^{2\,c+2\,d\,x}+1} - \frac{\frac{2\,a}{d\,(a^2+b^2)} - \frac{2\,b\,e^{c+d\,x}}{d\,(a^2+b^2)}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1} + \frac{\ln\left(1+e^{c+d\,x}\,1\mathrm{i}\right)\,\left(2\,a+b\,1\mathrm{i}\right)}{2\,\left(d\,a^2+2\mathrm{i}\,d\,a\,b-d\,b^2\right)} + \frac{\ln\left(e^{c+d\,x}+1\mathrm{i}\right)\,\left(b+a\,2\mathrm{i}\right)}{2\,\left(1\mathrm{i}\,d\,a^2+2\,d\,a\,b-1\mathrm{i}\,d\,b^2\right)} - \frac{a^3\,\ln\left(32\,a^7\,e^{d\,x}\,e^c-b^7-6\,a^2\,b^5-9\,a^4\,b^3-16\,a^6\,b+b^7\,e^{2\,c}\,e^{2\,d\,x}+16\,a^6\,b\,e^{2\,c}\,e^{2\,d\,x}+12\,a^3\,b^4\,e^{d\,x}\,e^c+18 \right.}{d\,a^4+2\,d\,a^2\,b^2+d\,b^4}$$

input
```
int(tanh(c + d*x)^3/(a + b*sinh(c + d*x)),x)
```

output
```
((2*(a*b^2 + a^3))/(d*(a^2 + b^2)^2) - (exp(c + d*x)*(a^2*b + b^3))/(d*(a^
2 + b^2)^2))/(exp(2*c + 2*d*x) + 1) - ((2*a)/(d*(a^2 + b^2)) - (2*b*exp(c
+ d*x))/(d*(a^2 + b^2)))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + (lo
g(exp(c + d*x)*1i + 1)*(2*a + b*1i))/(2*(a^2*d - b^2*d + a*b*d*2i)) + (log
(exp(c + d*x) + 1i)*(a*2i + b))/(2*(a^2*d*1i - b^2*d*1i + 2*a*b*d)) - (a^3
*log(32*a^7*exp(d*x)*exp(c) - b^7 - 6*a^2*b^5 - 9*a^4*b^3 - 16*a^6*b + b^7
*exp(2*c)*exp(2*d*x) + 16*a^6*b*exp(2*c)*exp(2*d*x) + 12*a^3*b^4*exp(d*x)*
exp(c) + 18*a^5*b^2*exp(d*x)*exp(c) + 6*a^2*b^5*exp(2*c)*exp(2*d*x) + 9*a^
4*b^3*exp(2*c)*exp(2*d*x) + 2*a*b^6*exp(d*x)*exp(c)))/(a^4*d + b^4*d + 2*a
^2*b^2*d)
```

3.419 $\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.419.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \mathrm{Int}\left(\frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.419.2 Mathematica [N/A]

Not integrable

Time = 60.76 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Tanh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Tanh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.419.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Tanh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.419.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.419.4 Maple [N/A] (verified)

Not integrable

Time = 0.66 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\tanh\left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.419.5 Fricas [N/A]

Not integrable

Time = 4.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(tanh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.419.6 Sympy [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh^3\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(tanh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(tanh(c + d*x)**3/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.419.7 Maxima [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 1095, normalized size of antiderivative = 39.11

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(a*f + (b*d*f*x*e^(3*c) + (d*e - f)*b*e^(3*c))*e^(3*d*x) - (2*a*d*f*x*e^(
2*c) + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) - (b*d*f*x*e^c + (d*e + f)*b*e^c)*
e^(d*x))/(a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*
(a^2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^(4*c) + b^2*d^2*e^2*e^(4*c)
 + (a^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*e*f*e^(4*c
) + b^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + 2*(a^2*d^2*e^2*e^(2*c) + b^2*d^2*e
^2*e^(2*c) + (a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*
e*f*e^(2*c) + b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) + integrate(-(2*a^3*d^2*f
^2*x^2 + 4*a^3*d^2*e*f*x + 2*a*b^2*f^2 + 2*(d^2*e^2 + f^2)*a^3 - ((3*d^2*e
^2 + 2*f^2)*a^2*b*e^c + (d^2*e^2 + 2*f^2)*b^3*e^c + (3*a^2*b*d^2*f^2*e^c +
 b^3*d^2*f^2*e^c)*x^2 + 2*(3*a^2*b*d^2*e*f*e^c + b^3*d^2*e*f*e^c)*x)*e^(d*
x))/(a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^2*
b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2 +
b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^2*
f)*x + (a^4*d^2*e^3*e^(2*c) + 2*a^2*b^2*d^2*e^3*e^(2*c) + b^4*d^2*e^3*e^(2
*c) + (a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^(2*
c))*x^3 + 3*(a^4*d^2*e*f^2*e^(2*c) + 2*a^2*b^2*d^2*e*f^2*e^(2*c) + b^4*d^2
*e*f^2*e^(2*c))*x^2 + 3*(a^4*d^2*e^2*f*e^(2*c) + 2*a^2*b^2*d^2*e^2*f*e^(2*
c) + b^4*d^2*e^2*f*e^(2*c))*x)*e^(2*d*x)), x) + integrate(-2*(a^4*e^(d*x +
 c) - a^3*b)/(a^4*b*e + 2*a^2*b^3*e + b^5*e + (a^4*b*f + 2*a^2*b^3*f + ...
```

3.419.8 Giac [F(-1)]

Timed out.

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.419.9 Mupad [N/A]

Not integrable

Time = 3.82 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+d\,x)^3}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(tanh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(tanh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.420 $\int \frac{(e+fx)^3 \coth(c+dx)}{a+b \sinh(c+dx)} \, dx$

3.420.1 Optimal result

Integrand size = 26, antiderivative size = 451

$$\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx)^3\log\left(1-e^{2(c+dx)}\right)}{ad} - \frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2} - \frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2} + \frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^2} + \frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3} + \frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^3} - \frac{3f^2(e+fx)\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^3} - \frac{6f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^4} - \frac{6f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^4} + \frac{3f^3\operatorname{PolyLog}\left(4,e^{2(c+dx)}\right)}{4ad^4}$$

output
```
(f*x+e)^3*ln(1-exp(2*d*x+2*c))/a/d-(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/a/d-(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d+3/2*f*(
f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a/d^2-3*f*(f*x+e)^2*polylog(2,-b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/a/d^2-3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(
a^2+b^2)^(1/2)))/a/d^2-3/2*f^2*(f*x+e)*polylog(3,exp(2*d*x+2*c))/a/d^3+6*f
^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^3+6*f^2*(f*x+e
)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^3+3/4*f^3*polylog(4,exp
(2*d*x+2*c))/a/d^4-6*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^
4-6*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^4
```

3.420.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1914 vs. $2(451) = 902$.

Time = 9.79 (sec) , antiderivative size = 1914, normalized size of antiderivative = 4.24

$$\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)} \, dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(E^(2*c)*((e + f*x)^4/(E^(2*c)*f) - (2*(1 - E^(-2*c))*(e + f*x)^3*Log
[1 - E^(-c - d*x)])/d - (2*(1 - E^(-2*c))*(e + f*x)^3*Log[1 + E^(-c - d*x)
])/d + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2, -E^(-c - d*x)] + 2*
f*(d*(e + f*x)*PolyLog[3, -E^(-c - d*x)] + f*PolyLog[4, -E^(-c - d*x)])))/
(d^4*E^(2*c)) + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2, E^(-c - d*
x)] + 2*f*(d*(e + f*x)*PolyLog[3, E^(-c - d*x)] + f*PolyLog[4, E^(-c - d*x
)])))/(d^4*E^(2*c))))/(a*(-1 + E^(2*c))) + (4*e^3*E^(2*c)*x + 6*e^2*E^(2*c
)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)*f^3*x^4 + (4*a*Sqrt[a^2 + b^2]*e^3
*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) +
(4*a*Sqrt[-a^2 - b^2]*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(S
qrt[-(a^2 + b^2)^2]*d) - (2*e^3*E^(2*c)*Log[b - 2*a*E^(c + d*x) - b*E^(2*(
c + d*x))])/d + (2*e^3*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d
+ (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])
])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b
^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(
a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/
(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[1 + (b*E^(2*c + d
*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*c)*f^3*x^3*Log[1 +
(b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e^2*f*x*Log
[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2...
```

3.420.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.30 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6103, 3042, 26, 4201, 2620, 3011, 6095, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx$$
$$\Big\downarrow \text{6103}$$
$$\frac{\int (e+fx)^3\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$
$$\Big\downarrow \text{26}$$
$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int (e+fx)^3\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}$$
$$\Big\downarrow \text{4201}$$
$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)^3}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^4}{4f}\right)}{a}$$
$$\Big\downarrow \text{2620}$$
$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\int (e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{a}$$
$$\Big\downarrow \text{3011}$$
$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{a}$$
$$\Big\downarrow \text{6095}$$
$$-\frac{b\left(\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{a}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{a}$$
$$\Big\downarrow \text{2620}$$

3.420. $\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx$

$$-\frac{b\left(-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}\cdots}\right)}{bd}\cdots\right.}{a}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{a}$$

$$\Big\downarrow \textbf{3011}$$

$$-\frac{b\left(-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\cdots}{\cdots}\right)}{bd}\cdots\right.}{a}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{a}$$

$$\Big\downarrow \textbf{7163}$$

$$-\frac{b\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyL}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int\operatorname{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)dx}{2d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx\cdots}\right)}{2d}\cdots\right)}{2d}\cdots\right.\right.}{a}$$

$$\Big\downarrow \textbf{2720}$$

3.420. $\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(2f\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)}{d}-\frac{(e+f\cdots}{\cdots}\right)}{2d}\right.\right.}{a}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{b\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(4,-e^{2c+2dx-i\pi}\right)}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\cdots\right)}{2d}\right.\right.}{a}$$

input
```
Int[((e + f*x)^3*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.420. $\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
-((b*(-1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/a) - (I*(((-1/4*I)*(e + f*x)^4)/f + (
2*I)*(((e + f*x)^3*Log[1 + E^(2*c - I*Pi + 2*d*x)])/(2*d) - (3*f*(-1/2*((e
 + f*x)^2*PolyLog[2, -E^(2*c - I*Pi + 2*d*x)])/d + (f*(((e + f*x)*PolyLog[
3, -E^(2*c - I*Pi + 2*d*x)])/(2*d) - (f*PolyLog[4, -E^(2*c - I*Pi + 2*d*x)
])/(4*d^2)))/d))/(2*d))))/a
```

3.420.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.420. $\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.420. $\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.420.4 Maple [F]

$$\int \frac{(fx+e)^3 \coth(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.420.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1228 vs. $2(418) = 836$.

Time = 0.29 (sec) , antiderivative size = 1228, normalized size of antiderivative = 2.72

$$\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(6*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*f^3*polylog(4, (a*cosh(d*x
 + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2))/b) - 6*f^3*polylog(4, cosh(d*x + c) + sinh(d*x + c)) - 6*f^3*po
lylog(4, -cosh(d*x + c) - sinh(d*x + c)) + 3*(d^2*f^3*x^2 + 2*d^2*e*f^2*x
+ d^2*e^2*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3*(d^2*f^3*x^2 + 2*d
^2*e*f^2*x + d^2*e^2*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(d^2*f^
3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) -
3*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*dilog(-cosh(d*x + c) - sinh(d*
x + c)) + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(2*b*cosh
(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (d^3*e^
3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*s
inh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (d^3*f^3*x^3 + 3*d^3*e*f
^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*log(-(a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b) + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x
 + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*log(-(a*cosh(d*x + c) + a*sinh
(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -...
```

3.420.6 Sympy [F]

$$\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^3\coth{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.420.7 Maxima [F]

$$\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\coth{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^3*(log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*d) - log(e^(-d*x
- c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d)) + 3*(d*x*log(e^(d*x + c) +
1) + dilog(-e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d*x*log(-e^(d*x + c) + 1) + d
ilog(e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x
*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*e*f^2/(a*d^3) + 3*(d^2*
x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x
 + c)))*e*f^2/(a*d^3) + (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e
^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))
*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c
)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*f^3/(a*d^4
) - 1/2*(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2)/(a*d^4) + integr
ate(-2*(b*f^3*x^3 + 3*b*e*f^2*x^2 + 3*b*e^2*f*x - (a*f^3*x^3*e^c + 3*a*e*f
^2*x^2*e^c + 3*a*e^2*f*x*e^c)*e^(d*x))/(a*b*e^(2*d*x + 2*c) + 2*a^2*e^(d*x
 + c) - a*b), x)
```

3.420.8 Giac [F]

$$\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*coth(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.420.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)\,(e+f\,x)^3}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.421 $\int \frac{(e+fx)^2 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.421.1 Optimal result

Integrand size = 26, antiderivative size = 325

$$
\begin{aligned}
\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = &-\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} \\
&-\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} \\
&+\frac{(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad} \\
&-\frac{2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2} \\
&-\frac{2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2} \\
&+\frac{f(e+fx)\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^2} \\
&+\frac{2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3} \\
&+\frac{2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^3}-\frac{f^2\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^3}
\end{aligned}
$$

output
```
(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a/d-(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/a/d-(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d+f*(f*x+
e)*polylog(2,exp(2*d*x+2*c))/a/d^2-2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/a/d^2-2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/a/d^2-1/2*f^2*polylog(3,exp(2*d*x+2*c))/a/d^3+2*f^2*polylog(3,-b*ex
p(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^3+2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+
b^2)^(1/2)))/a/d^3
```

3.421.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1296 vs. $2(325) = 650$.

Time = 5.70 (sec) , antiderivative size = 1296, normalized size of antiderivative = 3.99

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{6e^2e^{2c}x + 6ee^{2c}fx^2 + 2e^{2c}f^2x^3 - \frac{2(e+fx)^3}{f} + \frac{6a\sqrt{a^2+b^2}e^2\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}d} + \frac{6a\sqrt{-(a^2+b^2)^2}e^2e^{2c}\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{(a^2+b^2)^{3/2}d} -}{}$$

input
```
Integrate[((e + f*x)^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 - (2*(e + f*x)^3)
/f + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]]
)/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[
(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-
(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 -
b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c
+ d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*(-1 + E^(2*c))*(e +
f*x)^2*Log[1 - E^(-c - d*x)])/d + (3*(-1 + E^(2*c))*(e + f*x)^2*Log[1 + E^
(-c - d*x)])/d + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d
 - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*
e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d -
(6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*
c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*
E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt
[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + S
qrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))
/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c +
d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 +
 (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2
*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^...
```

3.421.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6103, 3042, 26, 4201, 2620, 3011, 2720, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 \coth(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6103}$$

$$\frac{\int (e+fx)^2\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}$$

$$\Big\downarrow \; 4201$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{a}$$

$$\Big\downarrow \; 2620$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int (e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{a}$$

$$\Big\downarrow \; 3011$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int \text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{a}$$

$$\Big\downarrow \; 2720$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+}{3}}{a}$$

$$\Big\downarrow \; 6095$$

$$-\frac{b\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+}{3}}{a}$$

$$\Big\downarrow \; 2620$$

3.421. $\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2+}}\right.}{bd}\cdots\right.}{a}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+}{3}\cdots\right.}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{b\left(-\frac{2f\left(\frac{f\int\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+}}{a+\sqrt{a}}\right.}{d}\cdots\right.}{bd}\cdots\right.}{a}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+}{3}\cdots\right.}{a}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{b\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\cdots\right.}{bd}\cdots\right.}{a}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+}{3}\cdots\right.}{a}$$

$$\Big\downarrow \text{7143}$$

3.421. $\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right.}{bd}\right.}{a}$$

$$\frac{i\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\,\text{PolyLog}(3,-e^{2c+2dx-i\pi})}{4d^2}-\frac{(e+fx)\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{a}$$

input
```
Int[((e + f*x)^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((b*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/d^2))/(b*d)))/a) - (I*(((-1/3*I)*(e + f*x)^3)/f + (2*I)*(((e + f*
x)^2*Log[1 + E^(2*c - I*Pi + 2*d*x)])/(2*d) - (f*(-1/2*((e + f*x)*PolyLog[
2, -E^(2*c - I*Pi + 2*d*x)])/d + (f*PolyLog[3, -E^(2*c - I*Pi + 2*d*x)])/(
4*d^2)))/d)))/a
```

3.421.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.421. $\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.421. $\int \frac{(e+fx)^2 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.421.4 Maple [F]

$$\int \frac{(fx+e)^2 \coth(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.421.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(302) = 604$.

Time = 0.27 (sec) , antiderivative size = 813, normalized size of antiderivative = 2.50

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2\,f^2\mathrm{polylog}\left(3,\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}}{b}\right)+2\,f^2\mathrm{polylog}\left(3,\frac{a\cosh(dx+c)+a\sinh(dx+}{}\right.}{}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*f^2*polylog(3, (a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2))/b) - 2*f^2*polylog(3, cosh(d*x + c) + sinh(d*x + c)) - 2*f^2*pol
ylog(3, -cosh(d*x + c) - sinh(d*x + c)) - 2*(d*f^2*x + d*e*f)*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b + 1) - 2*(d*f^2*x + d*e*f)*dilog((a*cosh(d*x + c) +
a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
) - b)/b + 1) + 2*(d*f^2*x + d*e*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) +
 2*(d*f^2*x + d*e*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - (d^2*e^2 - 2*
c*d*e*f + c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a
^2 + b^2)/b^2) + 2*a) - (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(2*b*cosh(d*x +
 c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (d^2*f^2*x^2
+ 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x +
c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (
d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*log(-(a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b) + (d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*log(cosh(d*x + c) + sinh(
d*x + c) + 1) + (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(cosh(d*x + c) + sinh(d
*x + c) - 1) + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*log(-c...
```

3.421.6 Sympy [F]

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^2\coth{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.421.7 Maxima [F]

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^2*(log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*d) - log(e^(-d*x
- c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d)) + 2*(d*x*log(e^(d*x + c) +
1) + dilog(-e^(d*x + c)))*e*f/(a*d^2) + 2*(d*x*log(-e^(d*x + c) + 1) + dil
og(e^(d*x + c)))*e*f/(a*d^2) + (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog
(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3) + (d^2*x^2*log(-e
^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^
2/(a*d^3) - 2/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2)/(a*d^3) + integrate(-2*(b*f^
2*x^2 + 2*b*e*f*x - (a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x))/(a*b*e^(2*d*x
 + 2*c) + 2*a^2*e^(d*x + c) - a*b), x)
```

3.421.8 Giac [F]

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*coth(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.421.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.422 $\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.422.1 Optimal result

Integrand size = 24, antiderivative size = 205

$$\begin{aligned}\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = &-\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad}-\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad}\\ &+\frac{(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad}-\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2}\\ &-\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2}+\frac{f\,\mathrm{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^2}\end{aligned}$$

output
```
(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d-(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/a/d-(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d+1/2*f*polylog
(2,exp(2*d*x+2*c))/a/d^2-f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/
d^2-f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^2
```

3.422.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 431 vs. 2(205) = 410.

Time = 2.24 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.10

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{d^2fx^2+4de(c+dx)-2cf(c+dx)+f(c+dx)^2+\frac{4a(a^2+b^2)^{5/2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\left(-(a^2+b^2)^2\right)^{3/2}}+\frac{4a\sqrt{-(a^2+b^2)^2}de\,\mathrm{arctanh}\left(\right.}{(-a^2-b^2)^{3/2}}}{}$$

input
```
Integrate[((e + f*x)*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(d^2*f*x^2 + 4*d*e*(c + d*x) - 2*c*f*(c + d*x) + f*(c + d*x)^2 + (4*a*(a^2
 + b^2)^(5/2)*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(-(a^2 + b
^2)^2)^(3/2) + (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/S
qrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*d*(e + f*x)*Log[1 - E^(-c - d*x)]
+ 2*d*(e + f*x)*Log[1 + E^(-c - d*x)] - 2*f*(c + d*x)*Log[1 + (b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2])] - 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2])] + 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] - 2*d
*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] - 2*f*PolyLog[2, -E^(-c
 - d*x)] - 2*f*PolyLog[2, E^(-c - d*x)] - 2*f*PolyLog[2, (b*E^(c + d*x))/(
-a + Sqrt[a^2 + b^2])] - 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/(2*a*d^2)
```

3.422.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.28, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6103, 3042, 26, 4201, 2620, 2715, 2838, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6103}$$

$$\frac{\int (e+fx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$\big\downarrow$ 2620

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int \log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$\big\downarrow$ 2715

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$\big\downarrow$ 2838

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$\big\downarrow$ 6095

$$-\frac{b\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a}-\frac{i\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$\big\downarrow$ 2620

$$-\frac{b\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right.}{a}-\frac{i\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$\big\downarrow$ 2715

$$-\frac{b\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be}{\sqrt{a^2}}\right.}{bd}\right.}{a}-\frac{i\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$\big\downarrow$ 2838

3.422. $\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a}$$

$$\frac{i\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

input
```
Int[((e + f*x)*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((b*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)
))/a) - (I*(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((e + f*x)*Log[1 + E^(2*c -
I*Pi + 2*d*x)])/(2*d) + (f*PolyLog[2, -E^(2*c - I*Pi + 2*d*x)])/(4*d^2))))
/a
```

3.422.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.422. $\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

3.422.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(191) = 382$.

Time = 1.93 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.20

method	result
risch	$-\frac{f\ln\left(\frac{b\,e^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)x}{da}-\frac{f\ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{da}-\frac{f\,\mathrm{dilog}\left(\frac{-b\,e^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)}{d^2a}-\frac{f\,\mathrm{dilog}\left(\frac{b\,e^{dx+c}+\sqrt{a^2+b^2}}{a+\sqrt{a^2+b^2}}\right.}{d^2a}$

input
```
int((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/d*f/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d*f/
a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2*f/a*d
ilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*f/a*dil
og((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2*f/a*dilog(e
xp(d*x+c))+1/d*f/a*ln(exp(d*x+c)+1)*x+1/d*e/a*ln(exp(d*x+c)-1)+1/d*e/a*ln(
exp(d*x+c)+1)-1/d*e/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2*f/a*ln((
b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2*f/a*ln((-b*ex
p(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2*c*f/a*ln(exp(d*x
+c)-1)+1/d^2*c*f/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d^2*f/a*dilog(e
xp(d*x+c)+1)
```

3.422.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(188) = 376$.

Time = 0.30 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.32

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{f\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)+f\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx}{b}\right.}{}$$

input
```
integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + f*dilog((a*cosh(d*x + c) + a*
sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b + 1) - f*dilog(cosh(d*x + c) + sinh(d*x + c)) - f*dilog(-cosh(d*x +
 c) - sinh(d*x + c)) + (d*e - c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (d*e - c*f)*log(2*b*cosh(d*x + c)
+ 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (d*f*x + c*f)*log
(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2) - b)/b) + (d*f*x + c*f)*log(-(a*cosh(d*x + c) + a*s
inh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
 b)/b) - (d*f*x + d*e)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (d*e - c*f
)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - (d*f*x + c*f)*log(-cosh(d*x + c
) - sinh(d*x + c) + 1))/(a*d^2)
```

3.422.6 Sympy [F]

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\coth{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.422.7 Maxima [F]

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\coth{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e*(log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*d) - log(e^(-d*x -
c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d)) + f*integrate(2*x*(e^(d*x + c
) + e^(-d*x - c))/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) - e
^(-d*x - c))), x)
```

3.422.8 Giac [F]

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\coth{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*coth(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.422.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.423 $\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.423.1 Optimal result

Integrand size = 19, antiderivative size = 34

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\log(\sinh(c+dx))}{ad} - \frac{\log(a+b\sinh(c+dx))}{ad}$$

output
```
ln(sinh(d*x+c))/a/d-ln(a+b*sinh(d*x+c))/a/d
```

3.423.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\log(\sinh(c+dx)) - \log(a+b\sinh(c+dx))}{ad}$$

input
```
Integrate[Coth[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
(Log[Sinh[c + d*x]] - Log[a + b*Sinh[c + d*x]])/(a*d)
```

3.423.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 26, 3200, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i}{\tan(ic+idx)(a-ib\sin(ic+idx))}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{1}{(a-ib\sin(ic+idx))\tan(ic+idx)}dx$$
$$\Big\downarrow \text{3200}$$
$$\frac{\int \frac{\text{csch}(c+dx)}{b(a+b\sinh(c+dx))}d(b\sinh(c+dx))}{d}$$
$$\Big\downarrow \text{47}$$
$$\frac{\frac{\int \frac{\text{csch}(c+dx)}{b}d(b\sinh(c+dx))}{a}-\frac{\int \frac{1}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{a}}{d}$$
$$\Big\downarrow \text{14}$$
$$\frac{\frac{\log(b\sinh(c+dx))}{a}-\frac{\int \frac{1}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{a}}{d}$$
$$\Big\downarrow \text{16}$$
$$\frac{\frac{\log(b\sinh(c+dx))}{a}-\frac{\log(a+b\sinh(c+dx))}{a}}{d}$$

input
```
Int[Coth[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

output
```
(Log[b*Sinh[c + d*x]]/a - Log[a + b*Sinh[c + d*x]]/a)/d
```

3.423. $\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}dx$

3.423.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.423.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

method	result	size
risch	$\frac{\ln\left({\mathrm e}^{2 d x +2 c}-1\right)}{d a}-\frac{\ln\left({\mathrm e}^{2 d x +2 c}+\frac{2 a \,{\mathrm e}^{d x +c}}{b}-1\right)}{d a}$	53
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a -2 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-a \right)}{a}}{d}$	55
default	$\frac{\frac{\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a -2 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-a \right)}{a}}{d}$	55

input
```
int(coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*ln(exp(2*d*x+2*c)-1)-1/d/a*ln(exp(2*d*x+2*c)+2*a/b*exp(d*x+c)-1)
```

3.423.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)-\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{ad}$$

input
```
integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c)) - log(2*sin
h(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/(a*d)
```

3.423.6 Sympy [F]

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.423.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(34) = 68.

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{ad} + \frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

input
```
integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*d) + log(e^(-d*x - c)
+ 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d)
```

3.423.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2a\right|\right)}{a}-\frac{\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a}}{d}$$

input
```
integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/a - log(abs(e^(d*x + c) -
 e^(-d*x - c)))/a)/d
```

3.423.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 254, normalized size of antiderivative = 7.47

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a\sqrt{-a^2d^2}+b\,e^{dx}\,e^{c}\sqrt{-a^2d^2}-2\,a\,e^{2c}\,e^{2dx}\sqrt{-a^2d^2}-b\,e^{3c}\,e^{3dx}\sqrt{-a^2d^2}}{a^2\,d}\right)}{\sqrt{-a^2d^2}} - \frac{2\,\mathrm{atan}\Big(\left(4\,a^4\,b\,d\sqrt{-a^2d^2}+4\,a^2\,b^3\,d\sqrt{-a^2d^2}\right)\Big(\frac{1}{8\,a\,b\,d^2\,(a^2+b^2)^2}-e^{dx}\,e^{c}\Big(\frac{1}{16\,b^2\,d^2\,(a^2+b^2)^2}-\frac{(a^2+2\,b^2)^2}{16\,a^4\,b^2\,d^2\,(a^2+b^2)^2}\Big)}{\sqrt{-a^2d^2}}$$

input
```
int(coth(c + d*x)/(a + b*sinh(c + d*x)),x)
```

output
```
(2*atan((a*(-a^2*d^2)^(1/2) + b*exp(d*x)*exp(c)*(-a^2*d^2)^(1/2) - 2*a*exp
(2*c)*exp(2*d*x)*(-a^2*d^2)^(1/2) - b*exp(3*c)*exp(3*d*x)*(-a^2*d^2)^(1/2)
)/(a^2*d)))/(-a^2*d^2)^(1/2) - (2*atan((4*a^4*b*d*(-a^2*d^2)^(1/2) + 4*a^2
*b^3*d*(-a^2*d^2)^(1/2))*(1/(8*a*b*d^2*(a^2 + b^2)^2) - exp(d*x)*exp(c)*(1
/(16*b^2*d^2*(a^2 + b^2)^2) - (a^2 + 2*b^2)^2/(16*a^4*b^2*d^2*(a^2 + b^2)^
2)) + (a^2 + 2*b^2)/(8*a^3*b*d^2*(a^2 + b^2)^2))))/(-a^2*d^2)^(1/2)
```

3.424 $\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.424.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.424.2 Mathematica [N/A]

Not integrable

Time = 10.62 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Coth[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Coth[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.424.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Coth[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.424.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.424.4 Maple [N/A] (verified)

Not integrable

Time = 0.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\coth\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.424.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(coth(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.424.6 Sympy [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.424.7 Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(coth(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.424.8 Giac [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(coth(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.424.9 Mupad [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(coth(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(coth(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.425 $\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.425.1 Optimal result

Integrand size = 32, antiderivative size = 638

$$\begin{aligned}
&\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\operatorname{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd} \\
&\quad + \frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd} - \frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} \\
&\quad + \frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} - \frac{3\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2} \\
&\quad + \frac{3\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^2} + \frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3} \\
&\quad - \frac{6f^2(e+fx)\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3} + \frac{6\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^3} \\
&\quad - \frac{6\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^3} \\
&\quad - \frac{6f^3\operatorname{PolyLog}\left(4,-e^{c+dx}\right)}{ad^4} + \frac{6f^3\operatorname{PolyLog}\left(4,e^{c+dx}\right)}{ad^4} \\
&\quad - \frac{6\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^4} + \frac{6\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^4}
\end{aligned}$$

output
```
1/4*(f*x+e)^4/b/f-2*(f*x+e)^3*arctanh(exp(d*x+c))/a/d-3*f*(f*x+e)^2*polylo
g(2,-exp(d*x+c))/a/d^2+3*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a/d^2+6*f^2*(f*
x+e)*polylog(3,-exp(d*x+c))/a/d^3-6*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a/d^
3-6*f^3*polylog(4,-exp(d*x+c))/a/d^4+6*f^3*polylog(4,exp(d*x+c))/a/d^4-(f*
x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d+(f*x+e
)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d-3*f*(f*x+
e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d^2+
3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)
/a/b/d^2+6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b
^2)^(1/2)/a/b/d^3-6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)
))*(a^2+b^2)^(1/2)/a/b/d^3-6*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2
)))*(a^2+b^2)^(1/2)/a/b/d^4+6*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))*(a^2+b^2)^(1/2)/a/b/d^4
```

3.425.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.22

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x(4e^3+6e^2fx+4ef^2x^2+f^3x^3)}{4b}$$
$$+\frac{(e+fx)^3\log\left(1-e^{c+dx}\right)-(e+fx)^3\log\left(1+e^{c+dx}\right)-\frac{3f\left(d^2(e+fx)^2\operatorname{PolyLog}(2,-e^{c+dx})-2df(e+fx)\operatorname{PolyLog}(3,-e^{c+\cdots}\right.}{d^3}}{ad}$$
$$-\frac{\sqrt{a^2+b^2}\left(-2d^3e^3\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+3d^3e^2fx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+3d^3ef^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d\cdots\right.}{\cdots}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(4*b) + ((e + f*x)^3*Log[1
 - E^(c + d*x)] - (e + f*x)^3*Log[1 + E^(c + d*x)] - (3*f*(d^2*(e + f*x)^2
*PolyLog[2, -E^(c + d*x)] - 2*d*f*(e + f*x)*PolyLog[3, -E^(c + d*x)] + 2*f
^2*PolyLog[4, -E^(c + d*x)]))/d^3 + (3*f*(d^2*(e + f*x)^2*PolyLog[2, E^(c
+ d*x)] - 2*d*f*(e + f*x)*PolyLog[3, E^(c + d*x)] + 2*f^2*PolyLog[4, E^(c
+ d*x)]))/d^3)/(a*d) - (Sqrt[a^2 + b^2]*(-2*d^3*e^3*ArcTanh[(a + b*E^(c +
d*x))/Sqrt[a^2 + b^2]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
)] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e^
2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*x^2*Log
[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 + (b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c +
d*x))/(-a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] - 6*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2
+ b^2])] + 6*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
+ 6*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*f^3*P
olyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*f^3*PolyLog[4, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a*b*d^4)
```

3.425.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6119}$$

$$\frac{\int (e+fx)^3\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5973}$$

$$\frac{\int (e+fx)^3\sinh(c+dx)dx+\int (e+fx)^3\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^3\sin(ic+idx)dx+\int i(e+fx)^3\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\int (e+fx)^3\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\cosh(c+dx)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

3.425. $\displaystyle\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \ 3777$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \ 3042$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \ 3117$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \ 4670$$

3.425. $\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\frac{i\left(\frac{3if\int(e+fx)^2\log(1-e^{c+dx})dx}{d}-\frac{3if\int(e+fx)^2\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^3\operatorname{arctanh}(e^{c+dx})}{d}\right)-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\right.}{}\right.}{a}$$

$$\downarrow \text{3011}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,e^{c+dx})}{d}\right)}{d}\right.}{a}$$

$$\downarrow \text{6099}$$

$$-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^3dx}{b^2}+\frac{\int(e+fx)^3\sinh(c+dx)dx}{b}\right)}{a}+$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,e^{c+dx})}{d}\right)}{d}\right.}{a}$$

$$\downarrow \text{17}$$

$$-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)^3\sinh(c+dx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}+$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}(2,e^{c+dx})}{d}\right)}{d}\right.}{a}$$

$$\downarrow \text{3042}$$

3.425. $\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int-i(e+fx)^3\sin(ic+idx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int(e+fx)^3\sin(ic+idx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int(e+fx)^2\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

3.425. $\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 3777$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$$\Big\downarrow \; 26$$

3.425. $\displaystyle\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a} \qquad \frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a} \qquad \frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

3.425. $\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}\right)}{b}-\frac{a(}{}\right.}{a}$$

$$\Big\downarrow \; 3117$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b}\right)}{a}$$

$$\Big\downarrow \; 3803$$

3.425. $\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(\frac{2(a^2+b^2)\int-\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+}{d^2}\right.}{d}\right.}{d}\right.}{b}\right.}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}$$

$$\frac{b\left(-\frac{2(a^2+b^2)\int\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^4}{4b^2f}-\frac{i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c-}{d^2}\right.}{d}\right.}{d}\right.}{b}\right.}{a}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.425. $\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.425.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.425. $\int \frac{(e+fx)^3 \cosh(c+dx) \coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.425.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c) \coth(dx+c)}{a+b\sinh(dx+c)} dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.425.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1470 vs. $2(585) = 1170$.

Time = 0.32 (sec) , antiderivative size = 1470, normalized size of antiderivative = 2.30

$$\int \frac{(e+fx)^3 \cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
1/4*(a*d^4*f^3*x^4 + 4*a*d^4*e*f^2*x^3 + 6*a*d^4*e^2*f*x^2 + 4*a*d^4*e^3*x
 - 24*b*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24
*b*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*b*f^
3*polylog(4, cosh(d*x + c) + sinh(d*x + c)) - 24*b*f^3*polylog(4, -cosh(d*
x + c) - sinh(d*x + c)) - 12*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*
f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*(b*d^
2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) + 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d
*e*f^2 - b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh
(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(b*d^3*e^3 - 3*b*c*d^2*e^
2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x
+ c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(b*d^3*f^3
*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e
*f^2 + b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b)
+ 4*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^...
```

3.425.6 Sympy [F]

$$\int \frac{(e+fx)^3 \cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3 \cosh{(c+dx)}\coth{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*cosh(c + d*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

3.425.7 Maxima [F]

$$\int \frac{(e+fx)^3 \cosh(c+dx) \coth(c+dx)}{a+b\sinh(c+dx)} dx = \int \frac{(fx+e)^3 \cosh(dx+c) \coth(dx+c)}{b\sinh(dx+c)+a} dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
e^3*((d*x + c)/(b*d) - log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)
/(a*d) - sqrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(
-d*x - c) - a + sqrt(a^2 + b^2)))/(a*b*d)) - 3*(d*x*log(e^(d*x + c) + 1) +
 dilog(-e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d*x*log(-e^(d*x + c) + 1) + dilog
(e^(d*x + c)))*e^2*f/(a*d^2) + 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2)/b
 - 3*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog
(3, -e^(d*x + c)))*e*f^2/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*
x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*e*f^2/(a*d^3) - (d^3*x^3
*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -
e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(
d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c
)) + 6*polylog(4, e^(d*x + c)))*f^3/(a*d^4) - integrate(2*((a^2*f^3*e^c +
b^2*f^3*e^c)*x^3 + 3*(a^2*e*f^2*e^c + b^2*e*f^2*e^c)*x^2 + 3*(a^2*e^2*f*e^
c + b^2*e^2*f*e^c)*x)*e^(d*x)/(a*b^2*e^(2*d*x + 2*c) + 2*a^2*b*e^(d*x + c)
 - a*b^2), x)
```

3.425.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \cosh(c+dx) \coth(c+dx)}{a+b\sinh(c+dx)} dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.425.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)\,(e+f\,x)^3}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)*coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.426 $\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.426.1 Optimal result

Integrand size = 32, antiderivative size = 462

$$\begin{aligned}\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{(e+fx)^3}{3bf}-\frac{2(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{ad}\\
&-\frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd}\\
&+\frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd}\\
&-\frac{2f(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}\\
&+\frac{2f(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}\\
&-\frac{2\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2}\\
&+\frac{2\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^2}\\
&+\frac{2f^2\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}\\
&-\frac{2f^2\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3}\\
&+\frac{2\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^3}\\
&-\frac{2\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^3}\end{aligned}$$

output
```
1/3*(f*x+e)^3/b/f-2*(f*x+e)^2*arctanh(exp(d*x+c))/a/d-2*f*(f*x+e)*polylog(
2,-exp(d*x+c))/a/d^2+2*f*(f*x+e)*polylog(2,exp(d*x+c))/a/d^2+2*f^2*polylog
(3,-exp(d*x+c))/a/d^3-2*f^2*polylog(3,exp(d*x+c))/a/d^3-(f*x+e)^2*ln(1+b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d+(f*x+e)^2*ln(1+b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d-2*f*(f*x+e)*polylog(2,-b
*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d^2+2*f*(f*x+e)*polyl
og(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d^2+2*f^2*poly
log(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d^3-2*f^2*pol
ylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d^3
```

3.426.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.06

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x(3e^2+3efx+f^2x^2)}{3b}$$
$$+\frac{(e+fx)^2\log\left(1-e^{c+dx}\right)-(e+fx)^2\log\left(1+e^{c+dx}\right)-\frac{2f\left(d(e+fx)\operatorname{PolyLog}(2,-e^{c+dx})-f\operatorname{PolyLog}(3,-e^{c+dx})\right)}{d^2}+\frac{2f}{}}{ad}$$
$$-\frac{\sqrt{a^2+b^2}\left(-2d^2e^2\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^2f^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2d^2e\right.}{}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) + ((e + f*x)^2*Log[1 - E^(c + d*x)]
- (e + f*x)^2*Log[1 + E^(c + d*x)] - (2*f*(d*(e + f*x)*PolyLog[2, -E^(c +
d*x)] - f*PolyLog[3, -E^(c + d*x)]))/d^2 + (2*f*(d*(e + f*x)*PolyLog[2, E^
(c + d*x)] - f*PolyLog[3, E^(c + d*x)]))/d^2)/(a*d) - (Sqrt[a^2 + b^2]*(-2
*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2])] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2
*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*f^
2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a*b*d^3)
```

3.426.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6119}$$

$$\frac{\int (e+fx)^2\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5973}$$

$$\frac{\int (e+fx)^2\sinh(c+dx)dx+\int (e+fx)^2\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^2\sin(ic+idx)dx+\int i(e+fx)^2\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\int (e+fx)^2\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{2if\int (e+fx)\log\left(1-e^{c+dx}\right)dx}{d}-\frac{2if\int (e+fx)\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+f}{}\right.}{}\right.}{a}$$

$$\Big\downarrow \text{3011}$$

3.426. $\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}+\frac{2i(e+fx)^2\text{arc}}{a}\right.}{a}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\Big\downarrow \text{6099}$$

$$-\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^2dx}{b^2}+\frac{\int (e+fx)^2\sinh(c+dx)dx}{b}\right)}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\Big\downarrow \text{17}$$

$$-\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)^2\sinh(c+dx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int -i(e+fx)^2\sin(ic+idx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{a}$$

3.426. $\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 26$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int(e+fx)^2\sin(ic+idx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{a}$$

3.426. $\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow 26$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{a}$$

$$\Big\downarrow 3042$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{a}$$

$$\Big\downarrow 26$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{d}\right)}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{a}$$

$$\Big\downarrow 3118$$

3.426. $\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\right.}{d}}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{a}$$

$$\Big\downarrow \text{3803}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\right.}{d}}{a}$$

$$\frac{b\left(\frac{2(a^2+b^2)\int-\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\right.}{d}}{a}$$

$$\frac{b\left(-\frac{2(a^2+b^2)\int\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{b}\right)}{a}$$

$$\Big\downarrow \text{2694}$$

3.426. $\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(-\frac{2(a^2+b^2)\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}\right.}{d}\right.}{b}\right.}{a}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(-\frac{2(a^2+b^2)\left(\frac{b\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^3}{3b^2f}-\frac{i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(}{d^2}\right.}{d}\right.}{b}\right.}{a}$$

$$\Big\downarrow \textcolor{blue}{2620}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\frac{b\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{b^2}\right.}{a}$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.426. $\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
$Aborted
```

3.426.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.426. $\int \frac{(e+fx)^2 \cosh(c+dx) \coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.426. $\int \frac{(e+fx)^2 \cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.426.4 Maple [F]

$$\int \frac{(fx+e)^2 \cosh(dx+c)\coth(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.426.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(421) = 842$.

Time = 0.28 (sec) , antiderivative size = 992, normalized size of antiderivative = 2.15

$$\int \frac{(e+fx)^2 \cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
1/3*(a*d^3*f^2*x^3 + 3*a*d^3*e*f*x^2 + 3*a*d^3*e^2*x + 6*b*f^2*sqrt((a^2 +
 b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*b*f^2*sqrt((a^2 + b^2)/
b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*b*f^2*polylog(3, cosh(d*x + c
) + sinh(d*x + c)) + 6*b*f^2*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) -
6*(b*d*f^2*x + b*d*e*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*s
inh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
 b)/b + 1) + 6*(b*d*f^2*x + b*d*e*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d
*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) + 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt((a^
2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 +
b^2)/b^2) + 2*a) - 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt((a^2 + b^2
)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^
2) + 2*a) - 3*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*sq
rt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 3*(b*d^2*f^2*x^2 +
 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*co
sh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
a^2 + b^2)/b^2) - b)/b) + 6*(b*d*f^2*x + b*d*e*f)*dilog(cosh(d*x + c) +...
```

3.426.6 Sympy [F]

$$\int \frac{(e+fx)^2 \cosh(c+dx) \coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2 \cosh{(c+dx)} \coth{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*cosh(c + d*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

3.426.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)\coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
e^2*((d*x + c)/(b*d) - log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)
/(a*d) - sqrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(
-d*x - c) - a + sqrt(a^2 + b^2)))/(a*b*d)) + 1/3*(f^2*x^3 + 3*e*f*x^2)/b -
 2*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*e*f/(a*d^2) + 2*(d*x*l
og(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*e*f/(a*d^2) - (d^2*x^2*log(e^(d
*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2
/(a*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*p
olylog(3, e^(d*x + c)))*f^2/(a*d^3) - integrate(2*((a^2*f^2*e^c + b^2*f^2*
e^c)*x^2 + 2*(a^2*e*f*e^c + b^2*e*f*e^c)*x)*e^(d*x)/(a*b^2*e^(2*d*x + 2*c)
 + 2*a^2*b*e^(d*x + c) - a*b^2), x)
```

3.426.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.426.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.427 $\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.427.1 Optimal result

Integrand size = 30, antiderivative size = 286

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{ex}{b}+\frac{fx^2}{2b}-\frac{2(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd} + \frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd} - \frac{f\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{f\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} - \frac{\sqrt{a^2+b^2}f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2} + \frac{\sqrt{a^2+b^2}f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^2}$$

output
```
e*x/b+1/2*f*x^2/b-2*(f*x+e)*arctanh(exp(d*x+c))/a/d-f*polylog(2,-exp(d*x+c
))/a/d^2+f*polylog(2,exp(d*x+c))/a/d^2-(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d+(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))*(a^2+b^2)^(1/2)/a/b/d-f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2))
)*(a^2+b^2)^(1/2)/a/b/d^2+f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(
a^2+b^2)^(1/2)/a/b/d^2
```

3.427.2 Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.04

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{-a(c+dx)(cf-d(2e+fx))+2b\left(d(e+fx)\left(\log\left(1-e^{c+dx}\right)-\log\left(1+e^{c+dx}\right)\right)-f\operatorname{PolyLog}\left(2,-e^{c+dx}\right.\right.}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(a*(c + d*x)*(c*f - d*(2*e + f*x))) + 2*b*(d*(e + f*x)*(Log[1 - E^(c + d
*x)] - Log[1 + E^(c + d*x)]) - f*PolyLog[2, -E^(c + d*x)] + f*PolyLog[2, E
^(c + d*x)]) + 2*Sqrt[a^2 + b^2]*(2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a
^2 + b^2]] - 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f*(c + d
*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*E^(c + d*x))/(-a +
 Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
]))/(2*a*b*d^2)
```

3.427.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.29, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6119, 5973, 3042, 26, 3777, 3042, 3117, 4670, 2715, 2838, 6099, 17, 3042, 26, 3777, 3042, 3117, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \; 6119$$
$$\frac{\int (e+fx)\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$
$$\Big\downarrow \; 5973$$

$$\frac{\int (e+fx)\sinh(c+dx)dx+\int (e+fx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)\sin(ic+idx)dx+\int i(e+fx)\csc(ic+idx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\int (e+fx)\sin(ic+idx)dx}{a}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{if\int \log(1-e^{c+dx})dx}{d}-\frac{if\int \log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{if\int e^{-c-dx}\log(1-e^{c+dx})de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log(1+e^{c+dx})de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}$$

$$\Big\downarrow \text{2838}$$

3.427. $\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}$$

$$\Big\downarrow \text{6099}$$

$$-\frac{b\left(\frac{\left(a^2+b^2\right)\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)dx}{b^2}+\frac{\int (e+fx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}$$

$$\Big\downarrow \text{17}$$

$$-\frac{b\left(\frac{\left(a^2+b^2\right)\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)\sinh(c+dx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{\left(a^2+b^2\right)\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}+\frac{\int -i(e+fx)\sin(ic+idx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{\left(a^2+b^2\right)\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\int (e+fx)\sin(ic+idx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

3.427. $\displaystyle\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{a}$$

$$\Big\downarrow \text{3117}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{(a^2+b^2)\int\frac{e+fx}{a-ib\sin(ic+idx)}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{a}$$

$$\Big\downarrow \text{3803}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{2(a^2+b^2)\int-\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2(a^2+b^2)\int\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{a}$$

3.427. $\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{2694}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2(a^2+b^2)\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2(a^2+b^2)\left(\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e+fx)^2}{2b^2f}-\frac{i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{b}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b^2}-\frac{a(e\ldots}{\ldots}\right)}{a}$$

$$\Big\downarrow \text{2715}$$

3.427. $\displaystyle\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+\cdots}}\cdots\right)}{bd^2}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{b^2}\cdots\right)}{a}$$

$$\Big\downarrow \text{2838}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2(a^2+b^2)\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}\right)}{b^2}-\cdots\right)}{a}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(I*(((2*I)*(e + f*x)*ArcTanh[E^(c + d*x)])/d + (I*f*PolyLog[2, -E^(c + d*x
)])/d^2 - (I*f*PolyLog[2, E^(c + d*x)])/d^2) - I*((I*(e + f*x)*Cosh[c + d*
x])/d - (I*f*Sinh[c + d*x])/d^2))/a - (b*(-1/2*(a*(e + f*x)^2)/(b^2*f) - (
2*(a^2 + b^2)*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/b^2 - (I*((I*(e + f*x)*Cosh[c + d*
x])/d - (I*f*Sinh[c + d*x])/d^2))/b))/a
```

3.427. $\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.427.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.427. $\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.427. $\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.427.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(261) = 522$.

Time = 1.95 (sec) , antiderivative size = 970, normalized size of antiderivative = 3.39

method	result
risch	$\frac{f x^2}{2b} + \frac{ex}{b} + \frac{2ae \operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}} + \frac{e\ln(\mathrm{e}^{dx+c}-1)}{da} - \frac{e\ln(\mathrm{e}^{dx+c}+1)}{da} - \frac{2bcf \operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d^2a\sqrt{a^2+b^2}} + \frac{bf\ln\left(\frac{b\,\mathrm{e}^{dx}}{}\right.}{d^2}$

input
```
int((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERB
OSE)
```

output
```
1/2*f*x^2/b+e*x/b+2/b/d*a*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))+1/d*e/a*ln(exp(d*x+c)-1)-1/d*e/a*ln(exp(d*x+c)+1)-2*b/
d^2*c*f/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)
)+b/d^2*f/a/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^
2)^(1/2)))*c-2/b/d^2*c*a*f/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a
)/(a^2+b^2)^(1/2))-b/d^2*f/a/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2*f/a*dilog(exp(d*x+c))-1/d*f/a*ln(exp
(d*x+c)+1)*x-1/d^2*f/a*dilog(exp(d*x+c)+1)+2*b/d*e/a/(a^2+b^2)^(1/2)*arcta
nh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2*c*f/a*ln(exp(d*x+c)-1)+
1/b/d^2*a*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^
2)^(1/2)))*c-1/b/d^2*a*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*c+b/d*f/a/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+
b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-b/d*f/a/(a^2+b^2)^(1/2)*ln((-b*exp(d*
x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-b/d^2*f/a/(a^2+b^2)^(1/2)*
dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+b/d^2*f/a/(a
^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+
1/b/d*a*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)
^(1/2)))*x-1/b/d*a*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))*x-1/b/d^2*a*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(
a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/b/d^2*a*f/(a^2+b^2)^(1/2)*dil...
```

3.427.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(257) = 514$.

Time = 0.31 (sec) , antiderivative size = 598, normalized size of antiderivative = 2.09

$$\int \frac{(e + fx)\cosh(c + dx)\coth(c + dx)}{a + b\sinh(c + dx)}\, dx$$
$$= \frac{ad^2fx^2 + 2\,ad^2ex - 2\,bf\sqrt{\frac{a^2+b^2}{b^2}}\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)+2\,bf\sqrt{\cdots}}{\cdots}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
1/2*(a*d^2*f*x^2 + 2*a*d^2*e*x - 2*b*f*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh
(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b + 1) + 2*b*f*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b + 1) + 2*b*f*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2*b*f*
dilog(-cosh(d*x + c) - sinh(d*x + c)) + 2*(b*d*e - b*c*f)*sqrt((a^2 + b^2)
/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2
) + 2*a) - 2*(b*d*e - b*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) +
 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b*d*f*x + b*c*f
)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(b*d*f*x + b
*c*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(b*d*f*x
 + b*d*e)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*(b*d*e - b*c*f)*log(c
osh(d*x + c) + sinh(d*x + c) - 1) + 2*(b*d*f*x + b*c*f)*log(-cosh(d*x + c)
 - sinh(d*x + c) + 1))/(a*b*d^2)
```

3.427.6 Sympy [F]

$$\int \frac{(e + fx)\cosh(c + dx)\coth(c + dx)}{a + b\sinh(c + dx)}\, dx = \int \frac{(e + fx)\cosh\left(c + dx\right)\coth\left(c + dx\right)}{a + b\sinh\left(c + dx\right)}\, dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*cosh(c + d*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.427.7 Maxima [F]

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)\coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
-1/2*(4*(a^2*e^c + b^2*e^c)*integrate(x*e^(d*x)/(a*b^2*e^(2*d*x + 2*c) + 2
*a^2*b*e^(d*x + c) - a*b^2), x) - x^2/b - 2*integrate(x/(a*e^(d*x + c) + a
), x) - 2*integrate(x/(a*e^(d*x + c) - a), x))*f + e*((d*x + c)/(b*d) - lo
g(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d) - sqrt(a^2 + b^2)*
log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2
+ b^2)))/(a*b*d))
```

3.427.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
Timed out
```

3.427.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

input
```
int((cosh(c + d*x)*coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.428 $\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.428.1 Optimal result

Integrand size = 25, antiderivative size = 71

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x}{b} - \frac{\operatorname{arctanh}(\cosh(c+dx))}{ad} + \frac{2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{abd}$$

output
```
x/b-arctanh(cosh(d*x+c))/a/d+2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)
^(1/2))*(a^2+b^2)^(1/2)/a/b/d
```

3.428.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{ac+adx+2\sqrt{-a^2-b^2}\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right) - b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) + b\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{abd}$$

input
```
Integrate[(Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(a*c + a*d*x + 2*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a
^2 - b^2]] - b*Log[Cosh[(c + d*x)/2]] + b*Log[Sinh[(c + d*x)/2]])/(a*b*d)
```

3.428.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3042, 26, 3368, 26, 3042, 26, 3537, 26, 3042, 26, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos(ic+idx)^2}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos(ic+idx)^2}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3368}$$

$$i\int -\frac{i\text{csch}(c+dx)\left(\sinh^2(c+dx)+1\right)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\left(\sinh^2(c+dx)+1\right)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(1-\sin(ic+idx)^2\right)}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1-\sin(ic+idx)^2}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3537}$$

$$i\left(\frac{i\left(a^2+b^2\right)\int \frac{1}{a+b\sinh(c+dx)}dx}{ab}+\frac{\int -i\text{csch}(c+dx)dx}{a}-\frac{ix}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i(a^2+b^2)\int \frac{1}{a+b\sinh(c+dx)}dx}{ab}-\frac{i\int \operatorname{csch}(c+dx)dx}{a}-\frac{ix}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i(a^2+b^2)\int \frac{1}{a-ib\sin(ic+idx)}dx}{ab}-\frac{i\int i\csc(ic+idx)dx}{a}-\frac{ix}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i(a^2+b^2)\int \frac{1}{a-ib\sin(ic+idx)}dx}{ab}+\frac{\int \csc(ic+idx)dx}{a}-\frac{ix}{b}\right)$$

$$\Big\downarrow \text{3139}$$

$$i\left(\frac{2(a^2+b^2)\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{abd}+\frac{\int \csc(ic+idx)dx}{a}-\frac{ix}{b}\right)$$

$$\Big\downarrow \text{1083}$$

$$i\left(-\frac{4(a^2+b^2)\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{abd}+\frac{\int \csc(ic+idx)dx}{a}-\frac{ix}{b}\right)$$

$$\Big\downarrow \text{217}$$

$$i\left(\frac{\int \csc(ic+idx)dx}{a}+\frac{2i\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{abd}-\frac{ix}{b}\right)$$

$$\Big\downarrow \text{4257}$$

$$i\left(\frac{2i\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{abd}+\frac{i\text{arctanh}(\cosh(c+dx))}{ad}-\frac{ix}{b}\right)$$

input
```
Int[(Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
I*(((-I)*x)/b + (I*ArcTanh[Cosh[c + d*x]])/(a*d) + ((2*I)*Sqrt[a^2 + b^2]*
ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(a*b*d))
```

3.428. $\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.428.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3537
```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[C*(x
/(b*d)), x] + (Simp[(A*b^2 + a^2*C)/(b*(b*c - a*d))   Int[1/(a + b*Sin[e +
f*x]), x], x] - Simp[(c^2*C + A*d^2)/(d*(b*c - a*d))   Int[1/(c + d*Sin[e +
 f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.428. $\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.428.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{\left(2a^2+2b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{\left(2a^2+2b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
risch	$\frac{x}{b}+\frac{\ln\left(e^{dx+c}-1\right)}{da}-\frac{\ln\left(e^{dx+c}+1\right)}{da}+\frac{\sqrt{a^2+b^2}\,\ln\left(e^{dx+c}+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{dba}-\frac{\sqrt{a^2+b^2}\,\ln\left(e^{dx+c}-\frac{-a+\sqrt{a^2+b^2}}{b}\right)}{dba}$

input
```
int(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b*ln(tanh(1/2*d*x+1/2*c)-1)-(2*a^2+2*b^2)/a/b/(a^2+b^2)^(1/2)*arct
anh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+1/b*ln(tanh(1/2*d*x
+1/2*c)+1)+1/a*ln(tanh(1/2*d*x+1/2*c)))
```

3.428.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(68) = 136$.

Time = 0.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.94

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{adx-b\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)+b\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)+\sqrt{a^2+b^2}\log}{ab}$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(a*d*x - b*log(cosh(d*x + c) + sinh(d*x + c) + 1) + b*log(cosh(d*x + c) +
sinh(d*x + c) - 1) + sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d
*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b
)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a
))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(
d*x + c) + a)*sinh(d*x + c) - b)))/(a*b*d)
```

3.428.6 Sympy [F]

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh{(c+dx)}\coth{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(cosh(c + d*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.428.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.77

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{dx+c}{bd} - \frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad} - \frac{\sqrt{a^2+b^2}\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{abd}$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
(d*x + c)/(b*d) - log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d
) - sqrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x
- c) - a + sqrt(a^2 + b^2)))/(a*b*d)
```

3.428.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.59

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{dx+c}{b} - \frac{\log\left(e^{(dx+c)}+1\right)}{a} + \frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{a} - \frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{ab}}{d}$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
((d*x + c)/b - log(e^(d*x + c) + 1)/a + log(abs(e^(d*x + c) - 1))/a - sqrt
(a^2 + b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(
d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(a*b))/d
```

3.428.9 Mupad [B] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 384, normalized size of antiderivative = 5.41

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x}{b} + \frac{\ln\left(32\,a\,b^2+32\,a^3-32\,a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-32\,a\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{a\,d}$$
$$- \frac{\ln\left(32\,a\,b^2+32\,a^3+32\,a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+32\,a\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{a\,d}$$
$$- \frac{\ln\left(128\,a^5\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-64\,a^2\,b^3-64\,a^4\,b-32\,a\,b^3\,\sqrt{a^2+b^2}-64\,a^3\,b\,\sqrt{a^2+b^2}+160\,a^3\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+128\,a^4\,\mathrm{e}^{d\,x}}{a\,b\,d}$$
$$+ \frac{\ln\left(128\,a^5\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-64\,a^2\,b^3-64\,a^4\,b+32\,a\,b^3\,\sqrt{a^2+b^2}+64\,a^3\,b\,\sqrt{a^2+b^2}+160\,a^3\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-128\,a^4\,\mathrm{e}^{d\,x}}{a\,b\,d}$$

input
```
int((cosh(c + d*x)*coth(c + d*x))/(a + b*sinh(c + d*x)),x)
```

output
```
x/b + log(32*a*b^2 + 32*a^3 - 32*a^3*exp(d*x)*exp(c) - 32*a*b^2*exp(d*x)*e
xp(c))/(a*d) - log(32*a*b^2 + 32*a^3 + 32*a^3*exp(d*x)*exp(c) + 32*a*b^2*e
xp(d*x)*exp(c))/(a*d) - (log(128*a^5*exp(d*x)*exp(c) - 64*a^2*b^3 - 64*a^4
*b - 32*a*b^3*(a^2 + b^2)^(1/2) - 64*a^3*b*(a^2 + b^2)^(1/2) + 160*a^3*b^2
*exp(d*x)*exp(c) + 128*a^4*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 32*a*b^4*ex
p(d*x)*exp(c) + 96*a^2*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^
(1/2))/(a*b*d) + (log(128*a^5*exp(d*x)*exp(c) - 64*a^2*b^3 - 64*a^4*b + 32
*a*b^3*(a^2 + b^2)^(1/2) + 64*a^3*b*(a^2 + b^2)^(1/2) + 160*a^3*b^2*exp(d*
x)*exp(c) - 128*a^4*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 32*a*b^4*exp(d*x)*
exp(c) - 96*a^2*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/
(a*b*d)
```

3.429 $\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.429.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.429.2 Mathematica [N/A]

Not integrable

Time = 8.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

output
```
Integrate[(Cosh[c + d*x]*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

3.429.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

$$\Big\downarrow \; 6125$$

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.429.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.429.4 Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(dx+c)\coth(dx+c)}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.429.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)\coth(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
integral(cosh(d*x + c)*coth(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
 + c)), x)
```

3.429.6 Sympy [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\coth(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(cosh(c + d*x)*coth(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.429.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 167, normalized size of antiderivative = 5.22

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)\coth(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
-2*(a^2*e^c + b^2*e^c)*integrate(-e^(d*x)/(a*b^2*f*x + a*b^2*e - (a*b^2*f*
x*e^(2*c) + a*b^2*e*e^(2*c))*e^(2*d*x) - 2*(a^2*b*f*x*e^c + a^2*b*e*e^c)*e
^(d*x)), x) + log(f*x + e)/(b*f) + integrate(1/(a*f*x + a*e + (a*f*x*e^c +
 a*e*e^c)*e^(d*x)), x) + integrate(-1/(a*f*x + a*e - (a*f*x*e^c + a*e*e^c)
*e^(d*x)), x)
```

3.429.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
Timed out
```

3.429.9 Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((cosh(c + d*x)*coth(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)*coth(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.430 $\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.430.1 Optimal result

Integrand size = 34, antiderivative size = 656

$$\begin{aligned}
&\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{(e+fx)^4}{4af}+\frac{(a^2+b^2)(e+fx)^4}{4ab^2f}-\frac{6f^3\cosh(c+dx)}{bd^4}-\frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} \\
&\quad-\frac{(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d}-\frac{(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d} \\
&\quad+\frac{(e+fx)^3\log\left(1-e^{2(c+dx)}\right)}{ad}-\frac{3(a^2+b^2)f(e+fx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^2} \\
&\quad-\frac{3(a^2+b^2)f(e+fx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^2} \\
&\quad+\frac{3f(e+fx)^2\,\mathrm{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^2}+\frac{6(a^2+b^2)f^2(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^3} \\
&\quad+\frac{6(a^2+b^2)f^2(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^3}-\frac{3f^2(e+fx)\,\mathrm{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^3} \\
&\quad-\frac{6(a^2+b^2)f^3\,\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^4}-\frac{6(a^2+b^2)f^3\,\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^4} \\
&\quad+\frac{3f^3\,\mathrm{PolyLog}\left(4,e^{2(c+dx)}\right)}{4ad^4}+\frac{6f^2(e+fx)\sinh(c+dx)}{bd^3}+\frac{(e+fx)^3\sinh(c+dx)}{bd}
\end{aligned}$$

output
```
-1/4*(f*x+e)^4/a/f+1/4*(a^2+b^2)*(f*x+e)^4/a/b^2/f-6*f^3*cosh(d*x+c)/b/d^4
-3*f*(f*x+e)^2*cosh(d*x+c)/b/d^2+(f*x+e)^3*ln(1-exp(2*d*x+2*c))/a/d-(a^2+b
^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b^2/d-(a^2+b^2)*(f*
x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b^2/d+3/2*f*(f*x+e)^2*poly
log(2,exp(2*d*x+2*c))/a/d^2-3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c
)/(a-(a^2+b^2)^(1/2)))/a/b^2/d^2-3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b^2/d^2-3/2*f^2*(f*x+e)*polylog(3,exp(2*d*x+
2*c))/a/d^3+6*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/a/b^2/d^3+6*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/a/b^2/d^3+3/4*f^3*polylog(4,exp(2*d*x+2*c))/a/d^4-6*(a^2+b^2)*
f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b^2/d^4-6*(a^2+b^2)*f^3
*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b^2/d^4+6*f^2*(f*x+e)*sinh
(d*x+c)/b/d^3+(f*x+e)^3*sinh(d*x+c)/b/d
```

3.430.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3089 vs. $2(656) = 1312$.

Time = 10.26 (sec) , antiderivative size = 3089, normalized size of antiderivative = 4.71

$$\int \frac{(e+fx)^3 \cosh^2(c+dx) \coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
-1/2*(E^(2*c)*((e + f*x)^4/(E^(2*c)*f) - (2*(1 - E^(-2*c))*(e + f*x)^3*Log
[1 - E^(-c - d*x)])/d - (2*(1 - E^(-2*c))*(e + f*x)^3*Log[1 + E^(-c - d*x)
])/d + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2, -E^(-c - d*x)] + 2*
f*(d*(e + f*x)*PolyLog[3, -E^(-c - d*x)] + f*PolyLog[4, -E^(-c - d*x)])))/
(d^4*E^(2*c)) + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2, E^(-c - d*
x)] + 2*f*(d*(e + f*x)*PolyLog[3, E^(-c - d*x)] + f*PolyLog[4, E^(-c - d*x
)])))/(d^4*E^(2*c))))/(a*(-1 + E^(2*c))) + ((a^2 + b^2)*(4*e^3*E^(2*c)*x +
 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)*f^3*x^4 + (4*a*Sqrt[a
^2 + b^2]*e^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 +
b^2)^2]*d) + (4*a*Sqrt[-a^2 - b^2]*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^
2 + b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) - (2*e^3*E^(2*c)*Log[b - 2*a*E^(c + d*
x) - b*E^(2*(c + d*x))])/d + (2*e^3*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c
+ d*x)))])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b
^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c -
Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (b*E^
(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[1 +
(b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*c)*f^3
*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (
6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])...
```

3.430.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6119$$

$$\frac{\int (e+fx)^3\cosh^2(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 5973$$

$$\frac{\int (e+fx)^3\coth(c+dx)dx+\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx+\int-i(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx-i\int(e+fx)^3\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^3}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^4}{4f}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\int(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^4}{4f}\right.}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)}{}\right.}{2d}\right.\right.}{a}$$

$$\Big\downarrow \text{5969}$$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)-}{a}$$

$$\Big\downarrow \text{3042}$$

3.430. $\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)-}{a}$$

$$\Big\downarrow 25$$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$\Big\downarrow 3792$$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3f\left(\frac{f^2\int-\sinh^2(c+dx)dx}{2d^2}+\frac{1}{2}\int(e+fx)^2dx+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\right.}{}\right.\right.}{a}$$

$$\Big\downarrow 17$$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3f\left(\frac{f^2\int-\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+\right.}{}\right.\right.}{a}$$

$$\Big\downarrow 25$$

$$-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3f\left(-\frac{f^2\int\sinh^2(c+dx)dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+\right.}{}\right.\right.}{a}$$

$$\Big\downarrow 3042$$

3.430. $\displaystyle\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$\frac{\frac{3f\left(-\frac{f^2\int -\sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}-i\Bigg(2i\Bigg(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\Big(\frac{f\int(\cdots}{}}{}}{a}$$

$$\Big\downarrow \; 25$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$\frac{\frac{3f\left(\frac{f^2\int \sin(ic+idx)^2dx}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}-i\Bigg(2i\Bigg(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\Big(\frac{f\int(e+f\cdots}{}}{}}{a}$$

$$\Big\downarrow \; 3115$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$\frac{\frac{3f\left(\frac{f^2\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d^2}+\frac{f(e+fx)\sinh^2(c+dx)}{2d^2}-\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{(e+fx)^3}{6f}\right)}{2d}-i\Bigg(2i\Bigg(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}\cdots}{a}$$

$$\Big\downarrow \; 24$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+\cdots}{a}$$

$$\Big\downarrow \; 6099$$

$$-\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^3\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+$$
$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+\cdots}{a}$$

$$\Big\downarrow \; 3042$$

3.430. $\displaystyle\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3if\int-i(e+fx)^2\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int(e+fx)^2\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{3f\int-i(e+fx)^2\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$$\Big\downarrow \text{26}$$

3.430. $\displaystyle\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\int(e+fx)^2\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$\downarrow$ 3777

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)}{b}\right.}{a}$$

$\downarrow$ 3042

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^3\cosh(c+}{}\right.}{a}$$

$\downarrow$ 3777

3.430. $\displaystyle\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+\cdots}{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int(\cdots}{\cdots}\right)}\ a$$

$$\Big\downarrow 26$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+\cdots}{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+\cdots}{\cdots}\right)}\ a$$

$$\Big\downarrow 3042$$

3.430. $\displaystyle\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int(\ldots}{}\right.$$

$$\Big\downarrow \; 26$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a}$$

$$b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int(e-\ldots}{}\right.$$

$$\Big\downarrow \; 3118$$

3.430. $\displaystyle\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)+}{a} \quad b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\left(\frac{(e+fx)^3\sinh(c+dx)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sin}{d}\right.}{}\right.}{d}\right.}{b^2}\right.}{a}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.430.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.430. $\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.430. $\int \frac{(e+fx)^3 \cosh^2(c+dx) \coth(c+dx)}{a+b\sinh(c+dx)} \, dx$

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.430.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh(dx+c)^2 \coth(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.430.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3344 vs. $2(619) = 1238$.

Time = 0.34 (sec) , antiderivative size = 3344, normalized size of antiderivative = 5.10

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
-1/4*(2*a*b*d^3*f^3*x^3 + 2*a*b*d^3*e^3 + 6*a*b*d^2*e^2*f + 12*a*b*d*e*f^2
 + 12*a*b*f^3 + 6*(a*b*d^3*e*f^2 + a*b*d^2*f^3)*x^2 - 2*(a*b*d^3*f^3*x^3 +
 a*b*d^3*e^3 - 3*a*b*d^2*e^2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*
f^2 - a*b*d^2*f^3)*x^2 + 3*(a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)
*x)*cosh(d*x + c)^2 - 2*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*b*d^2*e^2*f +
 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(a*b*
d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*sinh(d*x + c)^2 + 6*(a*b*d^3
*e^2*f + 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x - (a^2*d^4*f^3*x^4 + 4*a^2*d^4*e
*f^2*x^3 + 6*a^2*d^4*e^2*f*x^2 + 4*a^2*d^4*e^3*x + 8*a^2*c*d^3*e^3 - 12*a^
2*c^2*d^2*e^2*f + 8*a^2*c^3*d*e*f^2 - 2*a^2*c^4*f^3)*cosh(d*x + c) + 12*((
(a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*
f)*cosh(d*x + c) + ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x +
(a^2 + b^2)*d^2*e^2*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x
+ c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
1) + 12*(((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2
)*d^2*e^2*f)*cosh(d*x + c) + ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*
e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b + 1) - 12*((b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*c
osh(d*x + c) + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*si...
```

3.430.6 Sympy [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*cosh(c + d*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x))
, x)
```

3.430.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^3\cosh{(dx+c)}^2\coth{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/2*e^3*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) -
 2*log(e^(-d*x - c) + 1)/(a*d) - 2*log(e^(-d*x - c) - 1)/(a*d) + 2*(a^2 +
b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*b^2*d)) + 3*(d*x*l
og(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d*x*log(-e^(
d*x + c) + 1) + dilog(e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d^2*x^2*log(e^(d*x
+ c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*e*f^2/
(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*
polylog(3, e^(d*x + c)))*e*f^2/(a*d^3) + (d^3*x^3*log(e^(d*x + c) + 1) + 3
*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(
4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2
*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x
+ c)))*f^3/(a*d^4) - 1/2*(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2)
/(a*d^4) - 1/4*(a*d^4*f^3*x^4*e^c + 4*a*d^4*e*f^2*x^3*e^c + 6*a*d^4*e^2*f*
x^2*e^c - 2*(b*d^3*f^3*x^3*e^(2*c) + 3*(d^3*e*f^2 - d^2*f^3)*b*x^2*e^(2*c)
 + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b*x*e^(2*c) - 3*(d^2*e^2*f - 2*d*
e*f^2 + 2*f^3)*b*e^(2*c))*e^(d*x) + 2*(b*d^3*f^3*x^3 + 3*(d^3*e*f^2 + d^2*
f^3)*b*x^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*b*x + 3*(d^2*e^2*f + 2*
d*e*f^2 + 2*f^3)*b)*e^(-d*x))*e^(-c)/(b^2*d^4) + integrate(-2*((a^2*b*f^3
+ b^3*f^3)*x^3 + 3*(a^2*b*e*f^2 + b^3*e*f^2)*x^2 + 3*(a^2*b*e^2*f + b^3*e^
2*f)*x - ((a^3*f^3*e^c + a*b^2*f^3*e^c)*x^3 + 3*(a^3*e*f^2*e^c + a*b^2*...
```

3.430.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument
Value
```

3.430.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\coth(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.431 $\int \frac{(e+fx)^2 \cosh^2(c+dx) \coth(c+dx)}{a+b\sinh(c+dx)} dx$

3.431.1 Optimal result

Integrand size = 34, antiderivative size = 486

$$
\begin{aligned}
&\int \frac{(e+fx)^2 \cosh^2(c+dx) \coth(c+dx)}{a+b\sinh(c+dx)} dx \\
&= -\frac{(e+fx)^3}{3af} + \frac{(a^2+b^2)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cosh(c+dx)}{bd^2} \\
&\quad - \frac{(a^2+b^2)(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d} - \frac{(a^2+b^2)(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d} \\
&\quad + \frac{(e+fx)^2 \log\left(1-e^{2(c+dx)}\right)}{ad} - \frac{2(a^2+b^2)f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^2} \\
&\quad - \frac{2(a^2+b^2)f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^2} + \frac{f(e+fx)\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^2} \\
&\quad + \frac{2(a^2+b^2)f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^3} + \frac{2(a^2+b^2)f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^3} \\
&\quad - \frac{f^2\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^3} + \frac{2f^2\sinh(c+dx)}{bd^3} + \frac{(e+fx)^2\sinh(c+dx)}{bd}
\end{aligned}
$$

output
```
-1/3*(f*x+e)^3/a/f+1/3*(a^2+b^2)*(f*x+e)^3/a/b^2/f-2*f*(f*x+e)*cosh(d*x+c)
/b/d^2+(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a/d-(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/a/b^2/d-(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(
a+(a^2+b^2)^(1/2)))/a/b^2/d+f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^2-2*(a
^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b^2/d^2-2
*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b^2/d^
2-1/2*f^2*polylog(3,exp(2*d*x+2*c))/a/d^3+2*(a^2+b^2)*f^2*polylog(3,-b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b^2/d^3+2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))/a/b^2/d^3+2*f^2*sinh(d*x+c)/b/d^3+(f*x+e)^2*sinh
(d*x+c)/b/d
```

3.431.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1462 vs. $2(486) = 972$.

Time = 8.33 (sec) , antiderivative size = 1462, normalized size of antiderivative = 3.01

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(-((a*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Coth[c])/b^2) - (E^(2*c)*((2*(e + f*x)
^3)/(E^(2*c)*f) - (3*(1 - E^(-2*c))*(e + f*x)^2*Log[1 - E^(-c - d*x)])/d -
 (3*(1 - E^(-2*c))*(e + f*x)^2*Log[1 + E^(-c - d*x)])/d + (6*(-1 + E^(2*c)
)*f*(d*(e + f*x)*PolyLog[2, -E^(-c - d*x)] + f*PolyLog[3, -E^(-c - d*x)]))
/(d^3*E^(2*c)) + (6*(-1 + E^(2*c))*f*(d*(e + f*x)*PolyLog[2, E^(-c - d*x)]
 + f*PolyLog[3, E^(-c - d*x)]))/(d^3*E^(2*c))))/(a*(-1 + E^(2*c))) + ((a^2
 + b^2)*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sq
rt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^
2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c
+ d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^
2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d
) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt
[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1
+ E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2
*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
 b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c -
Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E
^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(
2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*...
```

3.431.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6119}$$

$$\frac{\int (e+fx)^2\cosh^2(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5973}$$

$$\frac{\int (e+fx)^2\coth(c+dx)dx+\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx+\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}$$

$$\Big\downarrow \; 4201$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{a}$$

$$\Big\downarrow \; 2620$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int (e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{a}$$

$$\Big\downarrow \; 3011$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int \text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\,\text{PolyLo}}{}\right.}{d}\right.\right.}{a}$$

$$\Big\downarrow \; 2720$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c}}{4d^2}\right.}{d}\right.\right.}{a}$$

$$\Big\downarrow \; 5969$$

3.431. $\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}\right.}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}\right.}{a}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}\right.}{a}$$

$$\Big\downarrow \text{ 3791}$$

$$-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}\right.}{a}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}\right.}{a}$$

$$\Big\downarrow \text{ 6099}$$

3.431. $\displaystyle\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^2\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-i(\cdots}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-i(\cdots}{a}$$
$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-i(\cdots}{a}$$
$$\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(\frac{(a^2+b^2)\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^2\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-i(\cdots}{a}$$

$$\Big\downarrow \text{3042}$$

3.431. $\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$$\downarrow \color{blue}{26}$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$$\downarrow \color{blue}{3777}$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+d}{b}\right.}{a}$$

$$\downarrow \color{blue}{3042}$$

3.431. $\displaystyle\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh}{b}\right.}{a}$$

$$\Big\downarrow \; 3117$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}\right.}{a}$$

$$\Big\downarrow \; 5969$$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}\right.}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int(e+fx)\sinh^2(c+dx)dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(}{d^2}\right.}{d}\right.}{b^2}\right.}{a}$$

$$\Big\downarrow \; 3042$$

3.431. $\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-\frac{i(\cdots}{a}$$

$$b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}-\frac{f\int-\left((e+fx)\sin(ic+idx)^2\right)dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\cdots}{}\right.}{d}\right.}{b^2}\cdots\right.$$

$$a$$

$$\Big\downarrow \ 25$$

$$-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-\frac{i(\cdots}{a}$$

$$b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)^2\sinh^2(c+dx)}{2d}+\frac{f\int(e+fx)\sin(ic+idx)^2dx}{d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(\cdots}{d^2}\right.}{d}\right.}{b^2}\cdots\right.$$

$$a$$

$$\Big\downarrow \ 3791$$

$$-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)\right)-\frac{i(\cdots}{a}$$

$$b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{f\left(\frac{1}{2}\int(e+fx)dx+\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)^2\sinh(c+\cdots}{d}\right.}{}\cdots\right.$$

$$a$$

$$\Big\downarrow \ 17$$

3.431. $\displaystyle\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{f\left(\frac{f\sinh^2(c+dx)}{4d^2}-\frac{(e+fx)\sinh(c+dx)}{2d}\right.}{d}\right.}{a}$$

$\downarrow$ 6095

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{b^2}-\frac{a\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}\right)}{b^2}\right.}{a}$$

$\downarrow$ 2620

$$\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(}{}}{a}$$

$$b\left(\frac{(a^2+b^2)\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\right.}{b^2}\right.$$

$\downarrow$ 3011

3.431. $\displaystyle\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(\cdots}{a}\right.$$

$$b\left(\frac{(a^2+b^2)\left(-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\cdots}\right.}{d}\right.}{bd}\cdots\right.}{b^2}\right.$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.431.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.431. $\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.431. $\int \frac{(e+fx)^2 \cosh^2(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} \, dx$

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.431. $\int \frac{(e+fx)^2 \cosh^2(c+dx) \coth(c+dx)}{a+b\sinh(c+dx)} \, dx$

3.431.4 Maple [F]

$$\int \frac{(fx+e)^2 \cosh (dx+c)^2 \coth (dx+c)}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.431.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2101 vs. $2(459) = 918$.

Time = 0.32 (sec) , antiderivative size = 2101, normalized size of antiderivative = 4.32

$$\int \frac{(e+fx)^2 \cosh^2(c+dx) \coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
-1/6*(3*a*b*d^2*f^2*x^2 + 3*a*b*d^2*e^2 + 6*a*b*d*e*f + 6*a*b*f^2 - 3*(a*b
*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*
b*d*f^2)*x)*cosh(d*x + c)^2 - 3*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e
*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*sinh(d*x + c)^2 + 6*(a*b*d
^2*e*f + a*b*d*f^2)*x - 2*(a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^2 + 3*a^2*d^3
*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2)*cosh(d*x + c)
+ 12*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c) + ((a^2 + b^
2)*d*f^2*x + (a^2 + b^2)*d*e*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*
sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b + 1) + 12*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)
+ ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*sinh(d*x + c))*dilog((a*cosh(d
*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) - 12*((b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c) + (b
^2*d*f^2*x + b^2*d*e*f)*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)
) - 12*((b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c) + (b^2*d*f^2*x + b^2*d*e*f
)*sinh(d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x + c)) + 6*(((a^2 + b^2)*d
^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c) + ((a^
2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*sinh(d*x +
 c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2)
 + 2*a) + 6*(((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)...
```

3.431.6 Sympy [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*cosh(c + d*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x))
, x)
```

3.431.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\cosh(dx+c)^2\coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/2*e^2*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) -
 2*log(e^(-d*x - c) + 1)/(a*d) - 2*log(e^(-d*x - c) - 1)/(a*d) + 2*(a^2 +
b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*b^2*d)) + 2*(d*x*l
og(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*e*f/(a*d^2) + 2*(d*x*log(-e^(d*
x + c) + 1) + dilog(e^(d*x + c)))*e*f/(a*d^2) + (d^2*x^2*log(e^(d*x + c) +
 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3)
+ (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3,
 e^(d*x + c)))*f^2/(a*d^3) - 2/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2)/(a*d^3) - 1
/6*(2*a*d^3*f^2*x^3*e^c + 6*a*d^3*e*f*x^2*e^c - 3*(b*d^2*f^2*x^2*e^(2*c) +
 2*(d^2*e*f - d*f^2)*b*x*e^(2*c) - 2*(d*e*f - f^2)*b*e^(2*c))*e^(d*x) + 3*
(b*d^2*f^2*x^2 + 2*(d^2*e*f + d*f^2)*b*x + 2*(d*e*f + f^2)*b)*e^(-d*x))*e^
(-c)/(b^2*d^3) + integrate(-2*((a^2*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*e*f +
b^3*e*f)*x - ((a^3*f^2*e^c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e
*f*e^c)*x)*e^(d*x))/(a*b^3*e^(2*d*x + 2*c) + 2*a^2*b^2*e^(d*x + c) - a*b^3
), x)
```

3.431.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.431.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\coth(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.432 $\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.432.1 Optimal result

Integrand size = 32, antiderivative size = 322

$$\begin{aligned}\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{(e+fx)^2}{2af}+\frac{(a^2+b^2)(e+fx)^2}{2ab^2f}-\frac{f\cosh(c+dx)}{bd^2}\\ &\quad-\frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d}-\frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d}\\ &\quad+\frac{(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad}-\frac{(a^2+b^2)f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^2}\\ &\quad-\frac{(a^2+b^2)f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^2}+\frac{f\,\mathrm{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^2}+\frac{(e+fx)\sinh(c+dx)}{bd}\end{aligned}$$

output

```
-1/2*(f*x+e)^2/a/f+1/2*(a^2+b^2)*(f*x+e)^2/a/b^2/f-f*cosh(d*x+c)/b/d^2+(f*
x+e)*ln(1-exp(2*d*x+2*c))/a/d-(a^2+b^2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/a/b^2/d-(a^2+b^2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))/a/b^2/d+1/2*f*polylog(2,exp(2*d*x+2*c))/a/d^2-(a^2+b^2)*f*polylog(2,-b
*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b^2/d^2-(a^2+b^2)*f*polylog(2,-b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))/a/b^2/d^2+(f*x+e)*sinh(d*x+c)/b/d
```

3.432.2 Mathematica [A] (verified)

Time = 3.95 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.51

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{-\frac{2f\cosh(c+dx)}{b} + \frac{d\left(2ce+2dex+dfx^2+2(e+fx)\log\left(1-e^{-c-dx}\right)+2(e+fx)\log\left(1+e^{-c-dx}\right)\right)-2f\,\mathrm{PolyLog}\left(2,-e^{-c-dx}\right)-2f\,\mathrm{PolyLog}\left(2,e^{-c-d}\right.}{a}}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
((-2*f*Cosh[c + d*x])/b + (d*(2*c*e + 2*d*e*x + d*f*x^2 + 2*(e + f*x)*Log[
1 - E^(-c - d*x)] + 2*(e + f*x)*Log[1 + E^(-c - d*x)]) - 2*f*PolyLog[2, -E
^(-c - d*x)] - 2*f*PolyLog[2, E^(-c - d*x)])/a - ((a^2 + b^2)*(-2*d*e*(c +
 d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[
(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(
a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^
2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] +
2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b
 - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-
1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a*b^2)
+ (2*d*(e + f*x)*Sinh[c + d*x])/b)/(2*d^2)
```

3.432.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.91 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.31, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.906$, Rules used = {6119, 5973, 3042, 26, 4201, 2620, 2715, 2838, 5969, 3042, 25, 3115, 24, 6099, 3042, 3777, 26, 3042, 26, 3118, 5969, 3042, 25, 3115, 24, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6119$$

$$\frac{\int (e+fx)\cosh^2(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 5973$$

$$\frac{\int (e+fx)\coth(c+dx)dx+\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx+\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}$$

$$\Big\downarrow \; 4201$$

$$-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \; 2620$$

$$-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int \log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \; 2715$$

$$-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)\right.}{a}$$

$$\Big\downarrow \; 2838$$

3.432. $\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \; 5969$$

$$\frac{-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+-\frac{f\int \sinh^2(c+dx)dx}{2d}-i\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+-\frac{f\int -\sin(ic+idx)^2dx}{2d}-i\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{f\int \sin(ic+idx)^2dx}{2d}-i\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{a}$$

$$\Big\downarrow \; 3115$$

$$\frac{-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{f\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}-i\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{a}$$

$$\Big\downarrow \; 24$$

$$\frac{-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+-i\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow \; 6099$$

3.432. $\displaystyle\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int-i\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int-i\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$$\Big\downarrow \text{26}$$

3.432. $\displaystyle\int\frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int\sin(ic+idx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{a}+$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow \text{5969}$$

$$-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int\sinh^2(c+dx)dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{a}+$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}-\frac{f\int-\sin(ic+idx)^2dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\int\sin(ic+idx)^2dx}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{a}$$

$$\Big\downarrow \text{3115}$$

3.432. $\displaystyle\int\frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}+\frac{f\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{(e+fx)\sinh^2(c+dx)}{2d}}{b}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}\right)}{a}+\frac{-i\left(2i\left(\frac{f\,\mathrm{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow 24$$

$$-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{f\,\mathrm{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow 6095$$

$$-\frac{b\left(\frac{(a^2+b^2)\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\right.}{}}{}\right.}{a}$$

$$\frac{-i\left(2i\left(\frac{f\,\mathrm{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow 2620$$

$$-\frac{b\left(\frac{(a^2+b^2)\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{b^2}-\frac{a}{}\right.}{a}$$

$$\frac{-i\left(2i\left(\frac{f\,\mathrm{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow 2715$$

3.432. $\int\frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{(a^2+b^2)\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}\ldots}\right)}{bd}\right.}{b^2}\right.}{a}$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

$$\Big\downarrow \color{blue}{2838}$$

$$-\frac{b\left(\frac{(a^2+b^2)\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{b^2}-\ldots\right.}{a}$$

$$\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}(2,-e^{2c+2dx-i\pi})}{4d^2}+\frac{(e+fx)\log(1+e^{2c+2dx-i\pi})}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((-I)*(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((e + f*x)*Log[1 + E^(2*c - I*Pi
 + 2*d*x)])/(2*d) + (f*PolyLog[2, -E^(2*c - I*Pi + 2*d*x)])/(4*d^2))) + ((e
 + f*x)*Sinh[c + d*x]^2)/(2*d) + (f*(x/2 - (Cosh[c + d*x]*Sinh[c + d*x])/(
2*d)))/(2*d))/a - (b*(((a^2 + b^2)*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))])/(b*d^2)))/b^2 - (a*(-((f*Cosh[c + d*x])/d^2) + ((e
 + f*x)*Sinh[c + d*x])/d))/b^2 + (((e + f*x)*Sinh[c + d*x]^2)/(2*d) + (f*(x
/2 - (Cosh[c + d*x]*Sinh[c + d*x])/(2*d)))/(2*d))/b))/a
```

3.432. $\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.432.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

3.432. $\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

3.432. $\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.432.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(304) = 608$.

Time = 3.24 (sec) , antiderivative size = 932, normalized size of antiderivative = 2.89

method	result
risch	$\frac{2ea\ln(\mathrm{e}^{dx+c})}{d\,b^2}+\frac{af\,x^2}{2b^2}-\frac{f\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)x}{da}-\frac{f\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{da}+\frac{f\ln(\mathrm{e}^{dx+c}+1)x}{da}-\frac{f\ln\left(\frac{b\,\mathrm{e}^{dx+c}+}{a+\sqrt{}}\right.}{d^2}$

input
```
int((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
2/d/b^2*e*a*ln(exp(d*x+c))+1/2*a*f*x^2/b^2-1/d*f/a*ln((b*exp(d*x+c)+(a^2+b
^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d*f/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1
/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d*f/a*ln(exp(d*x+c)+1)*x-1/d^2*f/a*ln((b*
exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2*f/a*ln((-b*exp(
d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2*c*f/a*ln(exp(d*x+c
)-1)+1/d^2*c*f/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/2*(d*f*x+d*e-f)/b
/d^2*exp(d*x+c)-a*e*x/b^2-1/2*(d*f*x+d*e+f)/b/d^2*exp(-d*x-c)-2/d^2/b^2*c*
a*f*ln(exp(d*x+c))-1/d^2*f/a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(
a^2+b^2)^(1/2)))-1/d^2*f/a*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+
b^2)^(1/2)))-1/d^2*f/a*dilog(exp(d*x+c))+1/d*e/a*ln(exp(d*x+c)-1)+1/d*e/a*
ln(exp(d*x+c)+1)-1/d*e/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d^2*f/a*d
ilog(exp(d*x+c)+1)+1/d^2/b^2*a*f*c^2-1/d/b^2*a*e*ln(b*exp(2*d*x+2*c)+2*a*e
xp(d*x+c)-b)-1/d^2/b^2*a*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^
2+b^2)^(1/2)))-1/d^2/b^2*a*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^
2+b^2)^(1/2)))+2/d/b^2*a*f*c*x-1/d/b^2*a*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2
)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d/b^2*a*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-
a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2/b^2*a*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2/b^2*a*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)
+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2/b^2*c*a*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x
+c)-b)
```

3.432.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1108 vs. $2(301) = 602$.

Time = 0.29 (sec) , antiderivative size = 1108, normalized size of antiderivative = 3.44

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-1/2*(a*b*d*f*x + a*b*d*e + a*b*f - (a*b*d*f*x + a*b*d*e - a*b*f)*cosh(d*x
 + c)^2 - (a*b*d*f*x + a*b*d*e - a*b*f)*sinh(d*x + c)^2 - (a^2*d^2*f*x^2 +
 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f)*cosh(d*x + c) + 2*((a^2 + b^2)
*f*cosh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a
*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b + 1) + 2*((a^2 + b^2)*f*cosh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c
))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^2*f*cosh(d*x + c) + b^2*f
*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2*(b^2*f*cosh(d*x +
 c) + b^2*f*sinh(d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x + c)) + 2*(((a^
2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c) + ((a^2 + b^2)*d*e - (a^2 +
b^2)*c*f)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*s
qrt((a^2 + b^2)/b^2) + 2*a) + 2*(((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(
d*x + c) + ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*sinh(d*x + c))*log(2*b*cosh
(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(((a^
2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c) + ((a^2 + b^2)*d*f*x + (a^
2 + b^2)*c*f)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(((a^2
+ b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c) + ((a^2 + b^2)*d*f*x + (a^2
+ b^2)*c*f)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b...
```

3.432.6 Sympy [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*cosh(c + d*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

3.432.7 Maxima [F]

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)^2\coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/2*e*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) - 2
*log(e^(-d*x - c) + 1)/(a*d) - 2*log(e^(-d*x - c) - 1)/(a*d) + 2*(a^2 + b^
2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*b^2*d)) - 1/4*f*(2*(
a*d^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*e^(d*x) + (b*d*x + b)*e^(-d*x)
)*e^(-c)/(b^2*d^2) - integrate(8*((a^3*e^c + a*b^2*e^c)*x*e^(d*x) - (a^2*b
 + b^3)*x)/(a*b^3*e^(2*d*x + 2*c) + 2*a^2*b^2*e^(d*x + c) - a*b^3), x) + 4
*integrate(x/(a*e^(d*x + c) + a), x) - 4*integrate(x/(a*e^(d*x + c) - a),
x))
```

3.432.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.432.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^2\coth(c+d\,x)\,\left(e+f\,x\right)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.433 $\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.433.1 Optimal result

Integrand size = 27, antiderivative size = 57

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\log(\sinh(c+dx))}{ad} - \frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{ab^2d} + \frac{\sinh(c+dx)}{bd}$$

output
```
ln(sinh(d*x+c))/a/d-(a^2+b^2)*ln(a+b*sinh(d*x+c))/a/b^2/d+sinh(d*x+c)/b/d
```

3.433.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{\log(\sinh(c+dx))}{a} - \left(\frac{1}{a}+\frac{a}{b^2}\right)\log(a+b\sinh(c+dx)) + \frac{\sinh(c+dx)}{b}}{d}$$

input
```
Integrate[(Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(Log[Sinh[c + d*x]]/a - (a^(-1) + a/b^2)*Log[a + b*Sinh[c + d*x]] + Sinh[c
 + d*x]/b)/d
```

3.433.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3042, 26, 3316, 26, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos(ic+idx)^3}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos(ic+idx)^3}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$-\frac{i\int \frac{i\text{csch}(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\text{csch}(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\text{csch}(c+dx)\left(\sinh^2(c+dx)b^2+b^2\right)}{b(a+b\sinh(c+dx))}d(b\sinh(c+dx))}{b^2d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{-a^2-b^2}{a(a+b\sinh(c+dx))}+\frac{b\text{csch}(c+dx)}{a}+1\right)d(b\sinh(c+dx))}{b^2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a}+\frac{b^2\log(b\sinh(c+dx))}{a}+b\sinh(c+dx)}{b^2d}$$

input
```
Int[(Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((b^2*Log[b*Sinh[c + d*x]])/a - ((a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/a +
 b*Sinh[c + d*x])/(b^2*d)
```

3.433.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

3.433.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(57) = 114$.

Time = 2.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.33

method	result
risch	$\frac{ax}{b^2}+\frac{e^{dx+c}}{2bd}-\frac{e^{-dx-c}}{2bd}+\frac{2ac}{b^2d}+\frac{\ln(e^{2dx+2c}-1)}{da}-\frac{a\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{b^2d}-\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{da}$
derivativedivides	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{\left(-a^2-b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a\,b^2}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+}{d}$
default	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{\left(-a^2-b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a\,b^2}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+}{d}$

input
```
int(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a*x/b^2+1/2/b/d*exp(d*x+c)-1/2/b/d*exp(-d*x-c)+2*a/b^2/d*c+1/d/a*ln(exp(2*
d*x+2*c)-1)-a/b^2/d*ln(exp(2*d*x+2*c)+2*a/b*exp(d*x+c)-1)-1/d/a*ln(exp(2*d
*x+2*c)+2*a/b*exp(d*x+c)-1)
```

3.433.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(57) = 114$.

Time = 0.27 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.56

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{2\,a^2dx\cosh(dx+c)+ab\cosh(dx+c)^2+ab\sinh(dx+c)^2-ab-2\left((a^2+b^2)\cosh(dx+c)+(a^2+b^2)\mathrm{s}\right.}{}$$

input
```
integrate(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
1/2*(2*a^2*d*x*cosh(d*x + c) + a*b*cosh(d*x + c)^2 + a*b*sinh(d*x + c)^2 -
 a*b - 2*((a^2 + b^2)*cosh(d*x + c) + (a^2 + b^2)*sinh(d*x + c))*log(2*(b*
sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(b^2*cosh(d*x + c)
 + b^2*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))
 + 2*(a^2*d*x + a*b*cosh(d*x + c))*sinh(d*x + c))/(a*b^2*d*cosh(d*x + c) +
 a*b^2*d*sinh(d*x + c))
```

3.433.6 Sympy [F]

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(cosh(d*x+c)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(cosh(c + d*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.433.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(57) = 114$.

Time = 0.23 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.28

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(dx+c)a}{b^2d} + \frac{e^{(dx+c)}}{2\,bd} - \frac{e^{(-dx-c)}}{2\,bd} + \frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad} - \frac{(a^2+b^2)\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{ab^2d}$$

input
```
integrate(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-(d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) - 1/2*e^(-d*x - c)/(b*d) + lo
g(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d) - (a^2 + b^2)*log(
-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*b^2*d)
```

3.433.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.65

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{e^{(dx+c)}-e^{(-dx-c)}}{b} + \frac{2\,\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a} - \frac{2\,(a^2+b^2)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{ab^2}}{2\,d}$$

input
```
integrate(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*((e^(d*x + c) - e^(-d*x - c))/b + 2*log(abs(e^(d*x + c) - e^(-d*x - c)
))/a - 2*(a^2 + b^2)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a*b^2
))/d
```

3.433.9 Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 360, normalized size of antiderivative = 6.32

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{c+dx}}{2\,b\,d} - \frac{e^{-c-dx}}{2\,b\,d} - \frac{\ln\left(8\,a^5\,e^{d\,x}\,e^{c} - 16\,b^5 - 16\,a^2\,b^3 - 4\,a^4\,b + 16\,b^5\,e^{2\,c}\,e^{2\,d\,x} + 4\,a^4\,b\,e^{2\,c}\,e^{2\,d\,x} + 32\,a^3\,b^2\,e^{d\,x}\,e^{c} + 16\,a^2\,b^3\,e^{2\,c}\,e\right.}{a\,d} + \frac{\ln\left(4\,a^6 + 16\,b^6 + 32\,a^2\,b^4 + 20\,a^4\,b^2 - 4\,a^6\,e^{2\,c}\,e^{2\,d\,x} - 16\,b^6\,e^{2\,c}\,e^{2\,d\,x} - 32\,a^2\,b^4\,e^{2\,c}\,e^{2\,d\,x} - 20\,a^4\,b^2\,e^{2\,c}\,e^{2\,d\,x}\right.}{a\,d} + \frac{a\,x}{b^2} - \frac{a\,\ln\left(8\,a^5\,e^{d\,x}\,e^{c} - 16\,b^5 - 16\,a^2\,b^3 - 4\,a^4\,b + 16\,b^5\,e^{2\,c}\,e^{2\,d\,x} + 4\,a^4\,b\,e^{2\,c}\,e^{2\,d\,x} + 32\,a^3\,b^2\,e^{d\,x}\,e^{c} + 16\,a^2\,b^3\,e^{2\,c}\right.}{b^2\,d}$$

input
```
int((cosh(c + d*x)^2*coth(c + d*x))/(a + b*sinh(c + d*x)),x)
```

output
```
exp(c + d*x)/(2*b*d) - exp(- c - d*x)/(2*b*d) - log(8*a^5*exp(d*x)*exp(c)
- 16*b^5 - 16*a^2*b^3 - 4*a^4*b + 16*b^5*exp(2*c)*exp(2*d*x) + 4*a^4*b*exp
(2*c)*exp(2*d*x) + 32*a^3*b^2*exp(d*x)*exp(c) + 16*a^2*b^3*exp(2*c)*exp(2*
d*x) + 32*a*b^4*exp(d*x)*exp(c))/(a*d) + log(4*a^6 + 16*b^6 + 32*a^2*b^4 +
 20*a^4*b^2 - 4*a^6*exp(2*c)*exp(2*d*x) - 16*b^6*exp(2*c)*exp(2*d*x) - 32*
a^2*b^4*exp(2*c)*exp(2*d*x) - 20*a^4*b^2*exp(2*c)*exp(2*d*x))/(a*d) + (a*x
)/b^2 - (a*log(8*a^5*exp(d*x)*exp(c) - 16*b^5 - 16*a^2*b^3 - 4*a^4*b + 16*
b^5*exp(2*c)*exp(2*d*x) + 4*a^4*b*exp(2*c)*exp(2*d*x) + 32*a^3*b^2*exp(d*x
)*exp(c) + 16*a^2*b^3*exp(2*c)*exp(2*d*x) + 32*a*b^4*exp(d*x)*exp(c)))/(b^
2*d)
```

3.434 $\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.434.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.434.2 Mathematica [N/A]

Not integrable

Time = 44.85 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]^2*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Cosh[c + d*x]^2*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

3.434.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]^2*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.434.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.434.4 Maple [N/A] (verified)

Not integrable

Time = 1.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(dx+c\right)^{2}\coth\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.434.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh\left(dx+c\right)^2\coth\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(cosh(d*x + c)^2*coth(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

3.434.6 Sympy [N/A]

Not integrable

Time = 5.53 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh^2\left(c+dx\right)\coth\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(cosh(d*x+c)**2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(cosh(c + d*x)**2*coth(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)),
 x)
```

3.434.7 Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 249, normalized size of antiderivative = 7.32

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh\left(dx+c\right)^2\coth\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-1/2*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d*
e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f) + 1/
4*integrate(8*(a^2*b + b^3 - (a^3*e^c + a*b^2*e^c)*e^(d*x))/(a*b^3*f*x + a
*b^3*e - (a*b^3*f*x*e^(2*c) + a*b^3*e*e^(2*c))*e^(2*d*x) - 2*(a^2*b^2*f*x*
e^c + a^2*b^2*e*e^c)*e^(d*x)), x) - integrate(1/(a*f*x + a*e + (a*f*x*e^c
+ a*e*e^c)*e^(d*x)), x) + integrate(-1/(a*f*x + a*e - (a*f*x*e^c + a*e*e^c
)*e^(d*x)), x)
```

3.434.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.434.9 Mupad [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)^2\coth(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((cosh(c + d*x)^2*coth(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)^2*coth(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.435 $\int \frac{(e+fx)^3\mathbf{csch}(c+dx)\mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.435.1 Optimal result

Integrand size = 32, antiderivative size = 1049

$$
\begin{aligned}
&\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{2b(e+fx)^3\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{2(e+fx)^3\operatorname{arctanh}\left(e^{2c+2dx}\right)}{ad} \\
&\quad - \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d} - \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d} \\
&\quad + \frac{b^2(e+fx)^3\log\left(1+e^{2(c+dx)}\right)}{a\,(a^2+b^2)\,d} + \frac{3ibf(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^2} \\
&\quad - \frac{3ibf(e+fx)^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^2} - \frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^2} \\
&\quad - \frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^2} + \frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2a\,(a^2+b^2)\,d^2} \\
&\quad - \frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2ad^2} + \frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2ad^2} \\
&\quad - \frac{6ibf^2(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{(a^2+b^2)\,d^3} + \frac{6ibf^2(e+fx)\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{(a^2+b^2)\,d^3} \\
&\quad + \frac{6b^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^3} + \frac{6b^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^3} \\
&\quad - \frac{3b^2f^2(e+fx)\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2a\,(a^2+b^2)\,d^3} + \frac{3f^2(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{2ad^3} \\
&\quad - \frac{3f^2(e+fx)\operatorname{PolyLog}\left(3,e^{2c+2dx}\right)}{2ad^3} + \frac{6ibf^3\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)}{(a^2+b^2)\,d^4} \\
&\quad - \frac{6ibf^3\operatorname{PolyLog}\left(4,ie^{c+dx}\right)}{(a^2+b^2)\,d^4} - \frac{6b^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^4} \\
&\quad - \frac{6b^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^4} + \frac{3b^2f^3\operatorname{PolyLog}\left(4,-e^{2(c+dx)}\right)}{4a\,(a^2+b^2)\,d^4} \\
&\quad - \frac{3f^3\operatorname{PolyLog}\left(4,-e^{2c+2dx}\right)}{4ad^4} + \frac{3f^3\operatorname{PolyLog}\left(4,e^{2c+2dx}\right)}{4ad^4}
\end{aligned}
$$

output
```
3/4*f^3*polylog(4,exp(2*d*x+2*c))/a/d^4-2*(f*x+e)^3*arctanh(exp(2*d*x+2*c)
)/a/d-3/4*f^3*polylog(4,-exp(2*d*x+2*c))/a/d^4+3*I*b*f*(f*x+e)^2*polylog(2
,-I*exp(d*x+c))/(a^2+b^2)/d^2+6*I*b*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/(a
^2+b^2)/d^3+6*I*b*f^3*polylog(4,-I*exp(d*x+c))/(a^2+b^2)/d^4+3/2*f*(f*x+e)
^2*polylog(2,exp(2*d*x+2*c))/a/d^2-3/2*f^2*(f*x+e)*polylog(3,exp(2*d*x+2*c
))/a/d^3-2*b*(f*x+e)^3*arctan(exp(d*x+c))/(a^2+b^2)/d-3/2*f*(f*x+e)^2*poly
log(2,-exp(2*d*x+2*c))/a/d^2+3/4*b^2*f^3*polylog(4,-exp(2*d*x+2*c))/a/(a^2
+b^2)/d^4-6*b^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^
2)/d^4-6*b^2*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/
d^4-6*I*b*f^3*polylog(4,I*exp(d*x+c))/(a^2+b^2)/d^4+b^2*(f*x+e)^3*ln(1+exp
(2*d*x+2*c))/a/(a^2+b^2)/d-b^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/a/(a^2+b^2)/d-b^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a
/(a^2+b^2)/d+3/2*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/a/d^3+3/2*b^2*f*(f
*x+e)^2*polylog(2,-exp(2*d*x+2*c))/a/(a^2+b^2)/d^2-3/2*b^2*f^2*(f*x+e)*pol
ylog(3,-exp(2*d*x+2*c))/a/(a^2+b^2)/d^3-3*b^2*f*(f*x+e)^2*polylog(2,-b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^2-3*b^2*f*(f*x+e)^2*polylog(2,-
b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^2+6*b^2*f^2*(f*x+e)*polylo
g(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^3+6*b^2*f^2*(f*x+e)*p
olylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^3-3*I*b*f*(f*x+e
)^2*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2-6*I*b*f^2*(f*x+e)*polylog(3,-...
```

3.435.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3862 vs. $2(1049) = 2098$.

Time = 12.01 (sec) , antiderivative size = 3862, normalized size of antiderivative = 3.68

$$\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
2*((a*E^c*((e + f*x)^4/(4*E^c*f) + ((1 + E^(-c))*(e + f*x)^3*Log[1 + E^(-c
 - d*x)])/d - (3*(1 + E^c)*f*(d^2*(e + f*x)^2*PolyLog[2, -E^(-c - d*x)] +
2*f*(d*(e + f*x)*PolyLog[3, -E^(-c - d*x)] + f*PolyLog[4, -E^(-c - d*x)]))
)/(d^4*E^c)))/(2*(a^2 + b^2)*(1 + E^c)) + ((I/2)*a*E^c*((e + f*x)^4/(4*E^c
*f) + ((I + E^(-c))*(e + f*x)^3*Log[1 - I*E^(-c - d*x)])/d - (3*(1 + I*E^c
)*f*(d^2*(e + f*x)^2*PolyLog[2, I*E^(-c - d*x)] + 2*f*(d*(e + f*x)*PolyLog
[3, I*E^(-c - d*x)] + f*PolyLog[4, I*E^(-c - d*x)])))/(d^4*E^c)))/((a^2 +
b^2)*(-I + E^c)) - (b^2*E^(2*c)*((e + f*x)^4/(E^(2*c)*f) - (2*(1 - E^(-2*c
))*(e + f*x)^3*Log[1 - E^(-c - d*x)])/d - (2*(1 - E^(-2*c))*(e + f*x)^3*Lo
g[1 + E^(-c - d*x)])/d + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2, -
E^(-c - d*x)] + 2*f*(d*(e + f*x)*PolyLog[3, -E^(-c - d*x)] + f*PolyLog[4,
-E^(-c - d*x)])))/(d^4*E^(2*c)) + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*Pol
yLog[2, E^(-c - d*x)] + 2*f*(d*(e + f*x)*PolyLog[3, E^(-c - d*x)] + f*Poly
Log[4, E^(-c - d*x)])))/(d^4*E^(2*c))))/(4*a*(a^2 + b^2)*(-1 + E^(2*c))) -
 ((I/2)*b*((-2*I)*d^3*e^3*ArcTan[E^(c + d*x)] + 3*d^3*e^2*f*x*Log[1 - I*E^
(c + d*x)] + 3*d^3*e*f^2*x^2*Log[1 - I*E^(c + d*x)] + d^3*f^3*x^3*Log[1 -
I*E^(c + d*x)] - 3*d^3*e^2*f*x*Log[1 + I*E^(c + d*x)] - 3*d^3*e*f^2*x^2*Lo
g[1 + I*E^(c + d*x)] - d^3*f^3*x^3*Log[1 + I*E^(c + d*x)] - 3*d^2*f*(e + f
*x)^2*PolyLog[2, (-I)*E^(c + d*x)] + 3*d^2*f*(e + f*x)^2*PolyLog[2, I*E^(c
 + d*x)] + 6*d*e*f^2*PolyLog[3, (-I)*E^(c + d*x)] + 6*d*f^3*x*PolyLog[3...
```

3.435.3 Rubi [A] (verified)

Time = 4.03 (sec) , antiderivative size = 910, normalized size of antiderivative = 0.87, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {6123, 5984, 3042, 26, 4670, 3011, 6107, 6095, 2620, 3011, 7163, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5984}$$

$$\frac{2\int (e+fx)^3\text{csch}(2c+2dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2\int i(e+fx)^3\csc(2ic+2idx)dx}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\int (e+fx)^3\csc(2ic+2idx)dx}{a}$$

$$\Big\downarrow \; 4670$$

$$-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{3if\int (e+fx)^2\log\left(1-e^{2c+2dx}\right)dx}{2d}-\frac{3if\int (e+fx)^2\log\left(1+e^{2c+2dx}\right)dx}{2d}+\frac{i(e+fx)^3\text{arctanh}\left(e^{2c+2dx}\right)}{d}\right)}{a}$$

$$\Big\downarrow \; 3011$$

$$-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(-\frac{3if\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{2c+2\right.}}{2d}\right.}{2d}\right.}{a}$$

$$\Big\downarrow \; 6107$$

$$-\frac{b\left(\frac{b^2\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(-\frac{3if\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{2c+2\right.}}{2d}\right.}{2d}\right.}{a}$$

$$\Big\downarrow \; 6095$$

$$-\frac{b\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(-\frac{3if\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,e^{2c+2\right.}}{2d}\right.}{2d}\right.}{a}$$

3.435. $$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 2620$$

$$-\frac{b\left(\frac{b^2\left(-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int (e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+}{}\right.}{a^2+b^2}\right.}{a}$$

$$\frac{2i\left(-\frac{3if\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{2c+2}\right.}{2d}\right.}{2d}\right.}{a}$$

$$\Big\downarrow \; 3011$$

$$-\frac{b\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{I}}{}\right.}{bd}\right.}{a^2+b^2}\right.}{}$$

$$\frac{2i\left(-\frac{3if\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{2c+2}\right.}{2d}\right.}{2d}\right.}{a}$$

$$\Big\downarrow \; 7163$$

3.435. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyL}}{}\right.}{}\right.}{}\right.}{}\right.}{2i\left(-\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{2d}-\frac{f\int \operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)dx}{2d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,e^{2c+2dx}\right.}{2d}\right.}{}\right.}{}\right.}{a}$$

$\downarrow$ 2720

3.435. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{2i\left(-\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{2d}-\frac{f\int e^{-2c-2dx}\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{P}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{a\cdots}$$

$$\Big\downarrow \text{7143}$$

3.435. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{\int (e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyL}}{}\cdots\right)}{bd}\cdots\right)}{}\right)}{2i\left(\frac{i(e+fx)^3\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(4,-e^{2c+2dx}\right)}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\cdots\right.}{}\cdots\right)}{a}$$

$\downarrow$ 7293

3.435. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{\int \left(a(e+fx)^3\operatorname{sech}(c+dx)-b(e+fx)^3\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+\cdots}{\cdots}\cdots\right)}{bd}\cdots\right)}{\cdots}\right)}{\cdots}$$

$$\frac{2i\left(\frac{i(e+fx)^3\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(4,-e^{2c+2dx}\right)}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\cdots\right)}{\cdots}\cdots\right)}{a}$$

$$\Big\downarrow \; 2009$$

3.435. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^3}{d}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(4,-e^{2c+2dx}\right)}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\cdots\right.}{\cdots}\right.}{a}$$

$$b\left(\frac{\left(-\frac{(e+fx)^4}{4bf}+\frac{\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+fx)^3}{bd}+\frac{\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)(e+fx)^3}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\cdots\right.}{d}\cdots\right.}{bd}\cdots\right.}{\cdots}\cdots\right.$$

input
```
Int[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.435. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
-((b*((b^2*(-1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^4)/
(4*f) + (2*a*(e + f*x)^3*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^3*Log[1 + E
^(2*(c + d*x))])/d - ((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/
d^2 + ((3*I)*a*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/d^2 - (3*b*f*(e +
f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*d^2) + ((6*I)*a*f^2*(e + f*x)*Poly
Log[3, (-I)*E^(c + d*x)])/d^3 - ((6*I)*a*f^2*(e + f*x)*PolyLog[3, I*E^(c +
 d*x)])/d^3 + (3*b*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*d^3) - (
(6*I)*a*f^3*PolyLog[4, (-I)*E^(c + d*x)])/d^4 + ((6*I)*a*f^3*PolyLog[4, I*
E^(c + d*x)])/d^4 - (3*b*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*d^4))/(a^2 +
 b^2)))/a) + ((2*I)*((I*(e + f*x)^3*ArcTanh[E^(2*c + 2*d*x)])/d - (((3*I)/
2)*f*(-1/2*((e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/d + (f*(((e + f*x)*P
olyLog[3, -E^(2*c + 2*d*x)])/(2*d) - (f*PolyLog[4, -E^(2*c + 2*d*x)])/(...
```

3.435.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.435. $\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

3.435. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.435.4 Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c) \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.435.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2448 vs. $2(962) = 1924$.

Time = 0.36 (sec) , antiderivative size = 2448, normalized size of antiderivative = 2.33

$$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-(6*b^2*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*b^2*f^3*polylog(4, (a*
cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2))/b) - 6*(a^2 + b^2)*f^3*polylog(4, cosh(d*x + c) + sinh(
d*x + c)) - 6*(a^2 + b^2)*f^3*polylog(4, -cosh(d*x + c) - sinh(d*x + c)) +
 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*dilog((a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) + 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e
^2*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*((a^2 + b^2)*d^2*f^3*x^2
+ 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*dilog(cosh(d*x + c) +
 sinh(d*x + c)) + 3*(a^2*d^2*f^3*x^2 + I*a*b*d^2*f^3*x^2 + 2*a^2*d^2*e*f^2
*x + 2*I*a*b*d^2*e*f^2*x + a^2*d^2*e^2*f + I*a*b*d^2*e^2*f)*dilog(I*cosh(d
*x + c) + I*sinh(d*x + c)) + 3*(a^2*d^2*f^3*x^2 - I*a*b*d^2*f^3*x^2 + 2*a^
2*d^2*e*f^2*x - 2*I*a*b*d^2*e*f^2*x + a^2*d^2*e^2*f - I*a*b*d^2*e^2*f)*dil
og(-I*cosh(d*x + c) - I*sinh(d*x + c)) - 3*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a
^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*dilog(-cosh(d*x + c) - sinh
(d*x + c)) + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^
3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^
2) + 2*a) + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*...
```

3.435.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.435.7 Maxima [F]

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\text{csch}\,(dx+c)\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-e^3*(b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^3 + a*b^2)*d
) - 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)
/((a^2 + b^2)*d) - log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*
d)) + 3*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*e^2*f/(a*d^2) + 3
*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d^2*x
^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x
 + c)))*e*f^2/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(
d*x + c)) - 2*polylog(3, e^(d*x + c)))*e*f^2/(a*d^3) + (d^3*x^3*log(e^(d*x
 + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)
) + 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) +
1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*poly
log(4, e^(d*x + c)))*f^3/(a*d^4) - 1/2*(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*
d^4*e^2*f*x^2)/(a*d^4) + integrate(2*(b^3*f^3*x^3 + 3*b^3*e*f^2*x^2 + 3*b^
3*e^2*f*x - (a*b^2*f^3*x^3*e^c + 3*a*b^2*e*f^2*x^2*e^c + 3*a*b^2*e^2*f*x*e
^c)*e^(d*x))/(a^3*b + a*b^3 - (a^3*b*e^(2*c) + a*b^3*e^(2*c))*e^(2*d*x) -
2*(a^4*e^c + a^2*b^2*e^c)*e^(d*x)), x) - integrate(-2*(a*f^3*x^3 + 3*a*e*f
^2*x^2 + 3*a*e^2*f*x - (b*f^3*x^3*e^c + 3*b*e*f^2*x^2*e^c + 3*b*e^2*f*x*e^
c)*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

3.435.8 Giac [F]

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\text{csch}\,(dx+c)\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate((f*x + e)^3*csch(d*x + c)*sech(d*x + c)/(b*sinh(d*x + c) + a), x
)
```

3.435.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\cosh\,(c+d\,x)\,\sinh\,(c+d\,x)\,(a+b\sinh\,(c+d\,x))}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.436 $\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.436.1 Optimal result

Integrand size = 32, antiderivative size = 734

$$\begin{aligned}
&\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{2b(e+fx)^2\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{2(e+fx)^2\text{arctanh}\left(e^{2c+2dx}\right)}{ad} \\
&\quad - \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d} - \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d} \\
&\quad + \frac{b^2(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{a\,(a^2+b^2)\,d} + \frac{2ibf(e+fx)\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^2} \\
&\quad - \frac{2ibf(e+fx)\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^2} - \frac{2b^2f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^2} \\
&\quad - \frac{2b^2f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^2} + \frac{b^2f(e+fx)\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{a\,(a^2+b^2)\,d^2} \\
&\quad - \frac{f(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{ad^2} + \frac{f(e+fx)\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{ad^2} \\
&\quad - \frac{2ibf^2\,\text{PolyLog}\left(3,-ie^{c+dx}\right)}{(a^2+b^2)\,d^3} + \frac{2ibf^2\,\text{PolyLog}\left(3,ie^{c+dx}\right)}{(a^2+b^2)\,d^3} \\
&\quad + \frac{2b^2f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^3} + \frac{2b^2f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^3} \\
&\quad - \frac{b^2f^2\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2a\,(a^2+b^2)\,d^3} + \frac{f^2\,\text{PolyLog}\left(3,-e^{2c+2dx}\right)}{2ad^3} - \frac{f^2\,\text{PolyLog}\left(3,e^{2c+2dx}\right)}{2ad^3}
\end{aligned}$$

output
```
-2*b*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)/d-2*(f*x+e)^2*arctanh(exp(2*d*
x+2*c))/a/d+b^2*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/a/(a^2+b^2)/d-b^2*(f*x+e)^2
*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d-b^2*(f*x+e)^2*ln(1+b
*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d-2*I*b*f*(f*x+e)*polylog(2,I
*exp(d*x+c))/(a^2+b^2)/d^2+2*I*b*f^2*polylog(3,I*exp(d*x+c))/(a^2+b^2)/d^3
+b^2*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/a/(a^2+b^2)/d^2-f*(f*x+e)*polylo
g(2,-exp(2*d*x+2*c))/a/d^2+f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^2-2*b^2
*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^2-2*
b^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^2
+2*I*b*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^2-2*I*b*f^2*polylog(
3,-I*exp(d*x+c))/(a^2+b^2)/d^3-1/2*b^2*f^2*polylog(3,-exp(2*d*x+2*c))/a/(a
^2+b^2)/d^3+1/2*f^2*polylog(3,-exp(2*d*x+2*c))/a/d^3-1/2*f^2*polylog(3,exp
(2*d*x+2*c))/a/d^3+2*b^2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/
a/(a^2+b^2)/d^3+2*b^2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(
a^2+b^2)/d^3
```

3.436.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3730 vs. $2(734) = 1468$.

Time = 13.22 (sec) , antiderivative size = 3730, normalized size of antiderivative = 5.08

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
2*((a*E^c*((e + f*x)^3/(3*E^c*f) + ((1 + E^(-c))*(e + f*x)^2*Log[1 + E^(-c
 - d*x)])/d - (2*(1 + E^c)*f*(d*(e + f*x)*PolyLog[2, -E^(-c - d*x)] + f*Po
lyLog[3, -E^(-c - d*x)]))/(d^3*E^c)))/(2*(a^2 + b^2)*(1 + E^c)) + (d^2*(d*
x*((-3*I)*b*e*f*x + a*((-3*I)*e^2*E^c + 3*e*f*x + f^2*x^2)) + 3*(1 + I*E^c
)*f*x*(2*a*e - (2*I)*b*e + a*f*x)*Log[1 - I*E^(-c - d*x)] + 3*a*e^2*(1 + I
*E^c)*Log[I - E^(c + d*x)]) - (6*I)*d*(-I + E^c)*f*((-I)*b*e + a*(e + f*x)
)*PolyLog[2, I*E^(-c - d*x)] - (6*I)*a*(-I + E^c)*f^2*PolyLog[3, I*E^(-c -
 d*x)])/(6*(a - I*b)*((-I)*a + b)*d^3*(-I + E^c)) - (b^2*E^(2*c)*((2*(e +
f*x)^3)/(E^(2*c)*f) - (3*(1 - E^(-2*c))*(e + f*x)^2*Log[1 - E^(-c - d*x)])
/d - (3*(1 - E^(-2*c))*(e + f*x)^2*Log[1 + E^(-c - d*x)])/d + (6*(-1 + E^(
2*c))*f*(d*(e + f*x)*PolyLog[2, -E^(-c - d*x)] + f*PolyLog[3, -E^(-c - d*x
)]))/(d^3*E^(2*c)) + (6*(-1 + E^(2*c))*f*(d*(e + f*x)*PolyLog[2, E^(-c - d
*x)] + f*PolyLog[3, E^(-c - d*x)]))/(d^3*E^(2*c))))/(6*a*(a^2 + b^2)*(-1 +
 E^(2*c))) - ((I/2)*b*((-2*I)*d^2*e^2*ArcTan[E^(c + d*x)] + d^2*f^2*x^2*Lo
g[1 - I*E^(c + d*x)] - d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*f^2*x*Poly
Log[2, (-I)*E^(c + d*x)] + 2*d*f^2*x*PolyLog[2, I*E^(c + d*x)] + 2*f^2*Pol
yLog[3, (-I)*E^(c + d*x)] - 2*f^2*PolyLog[3, I*E^(c + d*x)]))/((a^2 + b^2)
*d^3) - ((-I)*b*d^3*e*E^(2*c)*f*x^2 + 2*a*d^2*e^2*ArcTan[1 - (1 + I)*E^(c
+ d*x)] + (2*I)*a*d^2*e^2*E^(2*c)*ArcTan[1 - (1 + I)*E^(c + d*x)] + (2*I)*
a*d^2*e*f*x*Log[1 - E^(c + d*x)] - 2*a*d^2*e*E^(2*c)*f*x*Log[1 - E^(c +...
```

3.436.3 Rubi [A] (verified)

Time = 3.13 (sec) , antiderivative size = 651, normalized size of antiderivative = 0.89, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {6123, 5984, 3042, 26, 4670, 3011, 2720, 6107, 6095, 2620, 3011, 2720, 7143, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5984}$$

$$\frac{2\int (e+fx)^2\text{csch}(2c+2dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2\int i(e+fx)^2\csc(2ic+2idx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\int (e+fx)^2\csc(2ic+2idx)dx}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{if\int (e+fx)\log\left(1-e^{2c+2dx}\right)dx}{d}-\frac{if\int (e+fx)\log\left(1+e^{2c+2dx}\right)dx}{d}+\frac{i(e+fx)^2\text{arctanh}\left(e^{2c+2dx}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(-\frac{if\left(\frac{f\int \text{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int \text{PolyLog}\left(2,e^{2c+2dx}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{i(e+fx}{}\right.}{a}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)}{}\right.}{d}\right.}{a}$$

$$\Big\downarrow \text{6107}$$

$$-\frac{b\left(\frac{b^2\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)}{}\right.}{d}\right.}{a}$$

$$\Big\downarrow \text{6095}$$

3.436. $\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+$$

$$\frac{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)}{}\right.}{d}\right.}{a}$$

$$\Big\downarrow \; 2620$$

$$-\frac{b\left(\frac{b^2\left(-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+f}{3bf}\right.}{a^2+b^2}\right.}{a}$$

$$\frac{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)}{}\right.}{d}\right.}{a}$$

$$\Big\downarrow \; 3011$$

$$-\frac{b\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+}}{a+\sqrt{a}}\right.}{d}\right.}{bd}\right.}{a^2+b^2}\right.}{a}$$

$$\frac{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)}{}\right.}{d}\right.}{a}$$

$$\Big\downarrow \; 2720$$

3.436. $\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$

$$-\frac{b\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\right.}{bd}\right.}{a^2+b^2}\right.}{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int e^{-2c-2dx}\,\text{PolyLog}\left(2,e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)}{}\right.}{d}\right.}{a}$$

$$\Big\downarrow \; 7143$$

$$-\frac{b\left(\frac{\int(e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\right.}{d}\right.}{}\right.}{}\right.}{a}$$

$$\frac{2i\left(\frac{i(e+fx)^2\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{if\left(\frac{f\,\text{PolyLog}\left(3,-e^{2c+2dx}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\,\text{PolyLog}\left(3,e^{2c+2dx}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2\right.}{2d}\right.}{d}\right.}{a}$$

$$\Big\downarrow \; 7293$$

3.436. $\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{\int\left(a(e+fx)^2\text{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{Po}}{}\right.}{}\right.}{}\right.}{a}$$

$$\frac{2i\left(\frac{i(e+fx)^2\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{if\left(\frac{f\,\text{PolyLog}\left(3,-e^{2c+2dx}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\,\text{PolyLog}\left(3,e^{2c+2dx}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2\right.}{2d}\right.}{d}\right.}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2i\left(\frac{i(e+fx)^2\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{if\left(\frac{f\,\text{PolyLog}\left(3,-e^{2c+2dx}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\,\text{PolyLog}\left(3,e^{2c+2dx}\right)}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2\right.}{2d}\right.}{d}\right.}{a}$$

$$\frac{b\left(\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}+\right.}{a^2+b^2}\right.}{}$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.436. $\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
-((b*((b^2*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))])/d^2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3)/(3*f) + (2*a*(
e + f*x)^2*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))
])/d - ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((2*I)*a*f
*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*(e + f*x)*PolyLog[2, -E^(
2*(c + d*x))])/d^2 + ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/d^3 - ((2*
I)*a*f^2*PolyLog[3, I*E^(c + d*x)])/d^3 + (b*f^2*PolyLog[3, -E^(2*(c + d*x
))])/(2*d^3))/(a^2 + b^2)))/a) + ((2*I)*((I*(e + f*x)^2*ArcTanh[E^(2*c + 2
*d*x)])/d - (I*f*(-1/2*((e + f*x)*PolyLog[2, -E^(2*c + 2*d*x)])/d + (f*Pol
yLog[3, -E^(2*c + 2*d*x)])/(4*d^2)))/d + (I*f*(-1/2*((e + f*x)*PolyLog[2,
E^(2*c + 2*d*x)])/d + (f*PolyLog[3, E^(2*c + 2*d*x)])/(4*d^2)))/d))/a
```

3.436.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.436. $\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

3.436. $\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.436.4 Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.436.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1535 vs. 2(677) = 1354.

Time = 0.31 (sec) , antiderivative size = 1535, normalized size of antiderivative = 2.09

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
(2*b^2*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*b^2*f^2*polylog(3, (a*c
osh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2))/b) - 2*(a^2 + b^2)*f^2*polylog(3, cosh(d*x + c) + sinh(d
*x + c)) - 2*(a^2 + b^2)*f^2*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) -
2*(b^2*d*f^2*x + b^2*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^
2*d*f^2*x + b^2*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*((a^2 +
b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2
*(a^2*d*f^2*x + I*a*b*d*f^2*x + a^2*d*e*f + I*a*b*d*e*f)*dilog(I*cosh(d*x
+ c) + I*sinh(d*x + c)) - 2*(a^2*d*f^2*x - I*a*b*d*f^2*x + a^2*d*e*f - I*a
*b*d*e*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + 2*((a^2 + b^2)*d*f^2
*x + (a^2 + b^2)*d*e*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - (b^2*d^2*e
^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c
) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*
c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/
b^2) + 2*a) - (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2
*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^2*d^2*f^2*x^2 + 2*b^2*d^2*...
```

3.436.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.436.7 Maxima [F]

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-e^2*(b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^3 + a*b^2)*d
) - 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)
/((a^2 + b^2)*d) - log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*
d)) + 2*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*e*f/(a*d^2) + 2*(
d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*e*f/(a*d^2) + (d^2*x^2*log
(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c))
)*f^2/(a*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c))
- 2*polylog(3, e^(d*x + c)))*f^2/(a*d^3) - 2/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^
2)/(a*d^3) + integrate(2*(b^3*f^2*x^2 + 2*b^3*e*f*x - (a*b^2*f^2*x^2*e^c +
 2*a*b^2*e*f*x*e^c)*e^(d*x))/(a^3*b + a*b^3 - (a^3*b*e^(2*c) + a*b^3*e^(2*
c))*e^(2*d*x) - 2*(a^4*e^c + a^2*b^2*e^c)*e^(d*x)), x) - integrate(-2*(a*f
^2*x^2 + 2*a*e*f*x - (b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^(d*x))/(a^2 + b^2 +
 (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

3.436.8 Giac [F]

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
integrate((f*x + e)^2*csch(d*x + c)*sech(d*x + c)/(b*sinh(d*x + c) + a), x
)
```

3.436.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.437 $\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.437.1 Optimal result

Integrand size = 30, antiderivative size = 439

$$\begin{aligned}
&\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{2b(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{2(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{ad} \\
&\quad - \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d} - \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d} \\
&\quad + \frac{b^2(e+fx)\log\left(1+e^{2(c+dx)}\right)}{a\,(a^2+b^2)\,d} + \frac{ibf\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^2} - \frac{ibf\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^2} \\
&\quad - \frac{b^2f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^2} - \frac{b^2f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d^2} \\
&\quad + \frac{b^2f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2a\,(a^2+b^2)\,d^2} - \frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2ad^2} + \frac{f\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2ad^2}
\end{aligned}$$

output
```
-2*b*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)/d-2*(f*x+e)*arctanh(exp(2*d*x+2*
c))/a/d+b^2*(f*x+e)*ln(1+exp(2*d*x+2*c))/a/(a^2+b^2)/d-b^2*(f*x+e)*ln(1+b*
exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d-b^2*(f*x+e)*ln(1+b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d+I*b*f*polylog(2,-I*exp(d*x+c))/(a^2+b
^2)/d^2-I*b*f*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2+1/2*b^2*f*polylog(2,-e
xp(2*d*x+2*c))/a/(a^2+b^2)/d^2-1/2*f*polylog(2,-exp(2*d*x+2*c))/a/d^2+1/2*
f*polylog(2,exp(2*d*x+2*c))/a/d^2-b^2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/a/(a^2+b^2)/d^2-b^2*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))/a/(a^2+b^2)/d^2
```

3.437.2 Mathematica [A] (verified)

Time = 8.37 (sec) , antiderivative size = 788, normalized size of antiderivative = 1.79

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{-\frac{2i(a^2-b^2)(de-cf)(c+dx)}{a(a^2+b^2)} - \frac{i(a^2-b^2)f(c+dx)^2}{a(a^2+b^2)} + \frac{2\left(\frac{1}{2}d^2fx^2+de(c+dx)-2(de-cf)(c+dx)+2f(c+dx)\log\left(1+e^{-c-dx}\right)+2(de-cf)\log(1+\cdots}{a}}{\cdots}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(((-2*I)*(a^2 - b^2)*(d*e - c*f)*(c + d*x))/(a*(a^2 + b^2)) - (I*(a^2 - b^
2)*f*(c + d*x)^2)/(a*(a^2 + b^2)) + (2*((d^2*f*x^2)/2 + d*e*(c + d*x) - 2*
(d*e - c*f)*(c + d*x) + 2*f*(c + d*x)*Log[1 + E^(-c - d*x)] + 2*(d*e - c*f
)*Log[1 + E^(c + d*x)] - 2*f*PolyLog[2, -E^(-c - d*x)]))/a + ((2 + 2*I)*((
d^2*f*x^2)/2 + d*e*(c + d*x) - (1 + I)*(d*e - c*f)*(c + d*x) + (1 + I)*f*(
c + d*x)*Log[1 - I*E^(-c - d*x)] + (1 + I)*(d*e - c*f)*Log[I - E^(c + d*x)
] - (1 + I)*f*PolyLog[2, I*E^(-c - d*x)]))/((-I)*a + b) + ((-1 - I)*d*(e +
 f*x)*(b*d*(e + f*x) + (2 + 2*I)*(I*a + b)*f*Log[1 - E^(-c - d*x)] + (2 -
2*I)*a*f*Log[1 + I*E^(-c - d*x)]) + 4*a*f^2*PolyLog[2, (-I)*E^(-c - d*x)]
- 4*(a - I*b)*f^2*PolyLog[2, E^(-c - d*x)])/(a*(a - I*b)*f) - (2*b^2*(-2*d
*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*
ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*
Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a
^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*
f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x)
 + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a
^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(
a*(a^2 + b^2)))/(4*d^2)
```

3.437.3 Rubi [A] (verified)

Time = 1.94 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6123, 5984, 3042, 26, 4670, 2715, 2838, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)\text{csch}(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5984}$$

$$\frac{2\int (e+fx)\text{csch}(2c+2dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2\int i(e+fx)\csc(2ic+2idx)dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\int (e+fx)\csc(2ic+2idx)dx}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{if\int \log\left(1-e^{2c+2dx}\right)dx}{2d}-\frac{if\int \log\left(1+e^{2c+2dx}\right)dx}{2d}+\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{if\int e^{-2c-2dx}\log\left(1-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{if\int e^{-2c-2dx}\log\left(1+e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}+\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)}{a}$$

$$\Big\downarrow \text{6107}$$

$$-\frac{b\left(\frac{b^2\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)}{a}$$

$$\Big\downarrow \text{6095}$$

$$-\frac{b\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{b\left(\frac{b^2\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int (e+fx)\ldots}{\ldots}\right)}{a}+\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\ldots\right)}{a^2+b^2}\ldots\right)}{a}+\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)}{a}$$

$$\Big\downarrow \text{2838}$$

3.437. $\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+\cdots}{\cdots}\right.}{a^2+b^2}\right.}{a}$$

$$\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)}{a}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\left(\frac{\int (a(e+fx)\text{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+\cdots\right.}{bd}\right.}{a^2+b^2}\right.}{a}$$

$$\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)}{a}-$$

$$\frac{b\left(\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{2a(e+fx)\cdots}{\cdots}\right.}{a}$$

input
```
Int[((e + f*x)*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.437. $\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
-((b*((b^2*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
b*d^2)))/(a^2 + b^2) + ((b*(e + f*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c
 + d*x)])/d - (b*(e + f*x)*Log[1 + E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2,
 (-I)*E^(c + d*x)])/d^2 + (I*a*f*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*Pol
yLog[2, -E^(2*(c + d*x))])/(2*d^2))/(a^2 + b^2)))/a) + ((2*I)*((I*(e + f*x
)*ArcTanh[E^(2*c + 2*d*x)])/d + ((I/4)*f*PolyLog[2, -E^(2*c + 2*d*x)])/d^2
 - ((I/4)*f*PolyLog[2, E^(2*c + 2*d*x)])/d^2))/a
```

3.437.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.437. $\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.437. $\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.437.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1064 vs. $2(411) = 822$.

Time = 3.67 (sec) , antiderivative size = 1065, normalized size of antiderivative = 2.43

method	result	size
risch	Expression too large to display	1065

input
```
int((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERB
OSE)
```

output
```
1/d*f/a*ln(exp(d*x+c)+1)*x-1/d^2*c*f/a*ln(exp(d*x+c)-1)-1/d*f*b^2/(a^2+b^2
)/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d*f*b^2/(
a^2+b^2)/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/
d^2*f*b^2/(a^2+b^2)/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(
1/2)))*c+1/d^2*c*f*b^2/(a^2+b^2)/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1
/d^2*f*b^2/(a^2+b^2)/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1
/2)))*c+4*I/d*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*x-1/d^2*f/a*dilog(exp(d
*x+c))+1/d*e/a*ln(exp(d*x+c)-1)-1/d^2*f*b^2/(a^2+b^2)/a*dilog((b*exp(d*x+c
)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+4/d^2*c*f/(4*a^2+4*b^2)*a*ln(1+e
xp(2*d*x+2*c))+8/d^2*c*f/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))-1/d*e*b^2/(a^2
+b^2)/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-4/d*f/(4*a^2+4*b^2)*ln(1+I*e
xp(d*x+c))*a*x-4/d^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*a*c-4*I/d^2*f/(4*a
^2+4*b^2)*dilog(1-I*exp(d*x+c))*b+4*I/d^2*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*
x+c))*b+1/d*e/a*ln(exp(d*x+c)+1)+1/d^2*f/a*dilog(exp(d*x+c)+1)-4/d*f/(4*a^
2+4*b^2)*ln(1-I*exp(d*x+c))*a*x-4/d^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*a
*c-1/d^2*f*b^2/(a^2+b^2)/a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^
2+b^2)^(1/2)))-4*I/d^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*c+4*I/d^2*f/(4
*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*c-4*I/d*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c)
)*b*x-4/d^2*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))*a-4/d^2*f/(4*a^2+4*b^2)*
dilog(1-I*exp(d*x+c))*a-4/d*e/(4*a^2+4*b^2)*a*ln(1+exp(2*d*x+2*c))-8/d*...
```

3.437.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(400) = 800$.

Time = 0.31 (sec) , antiderivative size = 808, normalized size of antiderivative = 1.84

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{b^2 f\operatorname{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)+b^2 f\operatorname{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\,\mathrm{co}}{}\right.}{}$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
-(b^2*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + b^2*f*dilog((a*cosh(d*x +
 c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b + 1) - (a^2 + b^2)*f*dilog(cosh(d*x + c) + sinh(d*x + c)) -
 (a^2 + b^2)*f*dilog(-cosh(d*x + c) - sinh(d*x + c)) + (a^2*f + I*a*b*f)*d
ilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + (a^2*f - I*a*b*f)*dilog(-I*cosh(
d*x + c) - I*sinh(d*x + c)) + (b^2*d*e - b^2*c*f)*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^2*d*e - b^2*c*f)
*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) + (b^2*d*f*x + b^2*c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^2*d*f*x
 + b^2*c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - ((a^2 + b^2)*d*f*x + (a^2
+ b^2)*d*e)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a^2*d*e + I*a*b*d*e
- a^2*c*f - I*a*b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) + I) + (a^2*d*e -
 I*a*b*d*e - a^2*c*f + I*a*b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) - I) -
 ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*log(cosh(d*x + c) + sinh(d*x + c) - 1
) + (a^2*d*f*x - I*a*b*d*f*x + a^2*c*f - I*a*b*c*f)*log(I*cosh(d*x + c) +
I*sinh(d*x + c) + 1) + (a^2*d*f*x + I*a*b*d*f*x + a^2*c*f + I*a*b*c*f)*log
(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) - ((a^2 + b^2)*d*f*x + (a^2 + ...
```

3.437.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.437.7 Maxima [F]

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
-e*(b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^3 + a*b^2)*d)
- 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)/(
(a^2 + b^2)*d) - log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d)
) + 4*f*integrate(2*x/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c)
 + e^(-d*x - c))*(e^(d*x + c) - e^(-d*x - c))), x)
```

3.437.8 Giac [F]

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
integrate((f*x + e)*csch(d*x + c)*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.437.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{e+f\,x}{\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.438 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.438.1 Optimal result

Integrand size = 25, antiderivative size = 90

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b\arctan(\sinh(c+dx))}{(a^2+b^2)\,d} - \frac{a\log(\cosh(c+dx))}{(a^2+b^2)\,d} + \frac{\log(\sinh(c+dx))}{ad} - \frac{b^2\log(a+b\sinh(c+dx))}{a\,(a^2+b^2)\,d}$$

output
```
-b*arctan(sinh(d*x+c))/(a^2+b^2)/d-a*ln(cosh(d*x+c))/(a^2+b^2)/d+ln(sinh(d
*x+c))/a/d-b^2*ln(a+b*sinh(d*x+c))/a/(a^2+b^2)/d
```

3.438.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{\log(i-\sinh(c+dx))}{a+ib} - \frac{2\log(\sinh(c+dx))}{a} + \frac{\log(i+\sinh(c+dx))}{a-ib} + \frac{2b^2\log(a+b\sinh(c+dx))}{a(a^2+b^2)}}{2d}$$

input
```
Integrate[(Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(Log[I - Sinh[c + d*x]]/(a + I*b) - (2*Log[Sinh[c + d*x]])/a + Log[I
+ Sinh[c + d*x]]/(a - I*b) + (2*b^2*Log[a + b*Sinh[c + d*x]])/(a*(a^2 + b^
2)))/d
```

3.438.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 26, 3316, 26, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)\cos(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\cos(ic+idx)\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$-\frac{ib\int \frac{i\text{csch}(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{b\int \frac{\text{csch}(c+dx)}{(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^2\int \frac{\text{csch}(c+dx)}{b(a+b\sinh(c+dx))(\sinh^2(c+dx)b^2+b^2)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{615}$$

$$\frac{b^2\int \left(\frac{\text{csch}(c+dx)}{ab^3}-\frac{1}{a(a^2+b^2)(a+b\sinh(c+dx))}+\frac{-b^2-a\sinh(c+dx)b}{b^2(a^2+b^2)(\sinh^2(c+dx)b^2+b^2)}\right)d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\left(-\frac{\arctan(\sinh(c+dx))}{b(a^2+b^2)}-\frac{a\log(b^2\sinh^2(c+dx)+b^2)}{2b^2(a^2+b^2)}-\frac{\log(a+b\sinh(c+dx))}{a(a^2+b^2)}+\frac{\log(b\sinh(c+dx))}{ab^2}\right)}{d}$$

input
```
Int[(Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^2*(-(ArcTan[Sinh[c + d*x]]/(b*(a^2 + b^2))) + Log[b*Sinh[c + d*x]]/(a*b
^2) - Log[a + b*Sinh[c + d*x]]/(a*(a^2 + b^2)) - (a*Log[b^2 + b^2*Sinh[c +
 d*x]^2])/(2*b^2*(a^2 + b^2))))/d
```

3.438.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

3.438. $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.438.4 Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{-a\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-2b\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2+b^2}-\frac{b^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)a}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{-a\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-2b\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2+b^2}-\frac{b^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)a}}{d}$
risch	$\frac{2a\,d^2x}{a^2d^2+b^2d^2}+\frac{2adc}{a^2d^2+b^2d^2}-\frac{2x}{a}-\frac{2c}{da}+\frac{2b^2x}{a(a^2+b^2)}+\frac{2b^2c}{ad(a^2+b^2)}+\frac{i\ln(\mathrm{e}^{dx+c}-i)b}{(a^2+b^2)d}-\frac{\ln(\mathrm{e}^{dx+c}-i)a}{(a^2+b^2)d}-\frac{i\ln(\mathrm{e}}{(a}$

input
```
int(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a*ln(tanh(1/2*d*x+1/2*c))+1/(a^2+b^2)*(-a*ln(1+tanh(1/2*d*x+1/2*c)^
2)-2*b*arctan(tanh(1/2*d*x+1/2*c)))-b^2/(a^2+b^2)/a*ln(tanh(1/2*d*x+1/2*c)
^2*a-2*b*tanh(1/2*d*x+1/2*c)-a))
```

3.438.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.49

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{2\,ab\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)+b^2\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)+a^2\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^3+ab^2)d}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*a*b*arctan(cosh(d*x + c) + sinh(d*x + c)) + b^2*log(2*(b*sinh(d*x + c)
 + a)/(cosh(d*x + c) - sinh(d*x + c))) + a^2*log(2*cosh(d*x + c)/(cosh(d*x
 + c) - sinh(d*x + c))) - (a^2 + b^2)*log(2*sinh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))))/((a^3 + a*b^2)*d)
```

3.438.6 Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.438.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.53

$$\begin{aligned}\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = &-\frac{b^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^3+ab^2)d}\\&+\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d}-\frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d}\\&+\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}\end{aligned}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^3 + a*b^2)*d) + 2
*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a^
2 + b^2)*d) + log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d)
```

3.438.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.63

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2\,b^3\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^3b+ab^3}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)b}{a^2+b^2}+\frac{a\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2}-\frac{2\,\log\left(\left|e^{(dx+c}\right.\right.}{}}{2\,d}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*b^3*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^3*b + a*b^3)
 + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*b/(a^2 + b^2) +
 a*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2) - 2*log(abs(e^(d*x
+ c) - e^(-d*x - c)))/a)/d
```

3.438.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{1}{\cosh(c+d\,x)\,\sinh(c+d\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int(1/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.439 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.439.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.439.2 Mathematica [N/A]

Not integrable

Time = 19.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

output
```
Integrate[(Csch[c + d*x]*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

3.439.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

$$\Big\downarrow \; 6125$$

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.439.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.439.4 Maple [N/A] (verified)

Not integrable

Time = 0.94 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(dx+c\right)\operatorname{sech}\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.439.5 Fricas [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
integral(csch(d*x + c)*sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
 + c)), x)
```

3.439.6 Sympy [N/A]

Not integrable

Time = 67.65 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)*sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.439.7 Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
integrate(csch(d*x + c)*sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x
)
```

3.439.8 Giac [N/A]

Not integrable

Time = 6.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
integrate(csch(d*x + c)*sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x
)
```

3.439.9 Mupad [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \int \frac{1}{\cosh(c+dx)\,\sinh(c+dx)\,(e+fx)\,(a+b\sinh(c+dx))}\,dx$$

input
```
int(1/(cosh(c + d*x)*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.440 $\int \frac{(e+fx)^3\mathrm{csch}(c+dx)\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.440.1 Optimal result

Integrand size = 34, antiderivative size = 1164

$$\int \frac{(e+fx)^3\mathrm{csch}(c+dx)\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{b(e+fx)^3}{(a^2+b^2)\,d} - \frac{6f(e+fx)^2\arctan\left(e^{c+dx}\right)}{ad^2} + \frac{6b^2f(e+fx)^2\arctan\left(e^{c+dx}\right)}{a\,(a^2+b^2)\,d^2}$$
$$- \frac{2(e+fx)^3\mathrm{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{b^3(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d}$$
$$+ \frac{b^3(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d} + \frac{3bf(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)\,d^2}$$
$$- \frac{3f(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} + \frac{6if^2(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}$$
$$- \frac{6ib^2f^2(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{a\,(a^2+b^2)\,d^3} - \frac{6if^2(e+fx)\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{ad^3}$$
$$+ \frac{6ib^2f^2(e+fx)\,\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{a\,(a^2+b^2)\,d^3} + \frac{3f(e+fx)^2\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}$$
$$- \frac{3b^3f(e+fx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d^2} + \frac{3b^3f(e+fx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d^2}$$
$$+ \frac{3bf^2(e+fx)\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)}{(a^2+b^2)\,d^3} + \frac{6f^2(e+fx)\,\mathrm{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}$$
$$- \frac{6if^3\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4} + \frac{6ib^2f^3\,\mathrm{PolyLog}\left(3,-ie^{c+dx}\right)}{a\,(a^2+b^2)\,d^4} + \frac{6if^3\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{ad^4}$$
$$- \frac{6ib^2f^3\,\mathrm{PolyLog}\left(3,ie^{c+dx}\right)}{a\,(a^2+b^2)\,d^4} - \frac{6f^2(e+fx)\,\mathrm{PolyLog}\left(3,e^{c+dx}\right)}{ad^3}$$
$$+ \frac{6b^3f^2(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d^3} - \frac{6b^3f^2(e+fx)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d^3}$$
$$- \frac{3bf^3\,\mathrm{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2\,(a^2+b^2)\,d^4} - \frac{6f^3\,\mathrm{PolyLog}\left(4,-e^{c+dx}\right)}{ad^4} + \frac{6f^3\,\mathrm{PolyLog}\left(4,e^{c+dx}\right)}{ad^4}$$
$$- \frac{6b^3f^3\,\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d^4} + \frac{6b^3f^3\,\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d^4}$$
$$+ \frac{(e+fx)^3\mathrm{sech}(c+dx)}{ad} - \frac{b^2(e+fx)^3\mathrm{sech}(c+dx)}{a\,(a^2+b^2)\,d} - \frac{b(e+fx)^3\tanh(c+dx)}{(a^2+b^2)\,d}$$

output
```
-6*I*b^2*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/(a^2+b^2)/d^3-b*(f*x+e)^3/
(a^2+b^2)/d+(f*x+e)^3*sech(d*x+c)/a/d-6*f^3*polylog(4,-exp(d*x+c))/a/d^4+6
*f^3*polylog(4,exp(d*x+c))/a/d^4-6*b^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)
/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^3-6*I*b^2*f^3*polylog(3,I*exp(d*
x+c))/a/(a^2+b^2)/d^4+6*b^2*f*(f*x+e)^2*arctan(exp(d*x+c))/a/(a^2+b^2)/d^2
-3*b^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2
)^(3/2)/d^2+3*b^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))
/a/(a^2+b^2)^(3/2)/d^2+6*b^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^3+6*I*b^2*f^3*polylog(3,-I*exp(d*x+c))/a/(
a^2+b^2)/d^4+6*I*b^2*f^2*(f*x+e)*polylog(2,I*exp(d*x+c))/a/(a^2+b^2)/d^3+6
*I*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^3-3*f*(f*x+e)^2*polylog(2,-exp
(d*x+c))/a/d^2+3*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a/d^2+6*f^2*(f*x+e)*pol
ylog(3,-exp(d*x+c))/a/d^3-6*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a/d^3-6*f*(f
*x+e)^2*arctan(exp(d*x+c))/a/d^2-b*(f*x+e)^3*tanh(d*x+c)/(a^2+b^2)/d+6*I*f
^3*polylog(3,I*exp(d*x+c))/a/d^4-b^2*(f*x+e)^3*sech(d*x+c)/a/(a^2+b^2)/d-b
^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d+b^
3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d-3/2
*b*f^3*polylog(3,-exp(2*d*x+2*c))/(a^2+b^2)/d^4-6*I*f^3*polylog(3,-I*exp(d
*x+c))/a/d^4+3*b*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^3+3*b*
f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d^2-6*b^3*f^3*polylog(4,-b*e...
```

3.440.2 Mathematica [A] (verified)

Time = 9.07 (sec) , antiderivative size = 1441, normalized size of antiderivative = 1.24

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
4*(-1/8*(f*Csch[c + d*x]*(12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*
c))*x - 12*b*d^3*e*f*x^2 - 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*Ar
cTan[E^(c + d*x)] + 6*b*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d
*x))]) + (12*I)*a*d*e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1
 + I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*
x)]) + 6*b*d*e*(1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - P
olyLog[2, -E^(2*(c + d*x))]) + (6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 -
I*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^
(c + d*x)] + 2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*
x)] - 2*PolyLog[3, I*E^(c + d*x)]) + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x
 - 3*Log[1 + E^(2*(c + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*Po
lyLog[3, -E^(2*(c + d*x))]))*(a + b*Sinh[c + d*x]))/((a^2 + b^2)*d^4*(1 +
E^(2*c))*(b + a*Csch[c + d*x])) + (Csch[c + d*x]*((e + f*x)^3*Log[1 - E^(c
 + d*x)] - (e + f*x)^3*Log[1 + E^(c + d*x)] - (3*f*(d^2*(e + f*x)^2*PolyLo
g[2, -E^(c + d*x)] - 2*d*f*(e + f*x)*PolyLog[3, -E^(c + d*x)] + 2*f^2*Poly
Log[4, -E^(c + d*x)]))/d^3 + (3*f*(d^2*(e + f*x)^2*PolyLog[2, E^(c + d*x)]
 - 2*d*f*(e + f*x)*PolyLog[3, E^(c + d*x)] + 2*f^2*PolyLog[4, E^(c + d*x)]
))/d^3)*(a + b*Sinh[c + d*x]))/(4*a*d*(b + a*Csch[c + d*x])) - (b^3*Csch[c
 + d*x]*(-2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 3*d^3*e
^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2...
```

3.440.3 Rubi [A] (verified)

Time = 4.93 (sec) , antiderivative size = 991, normalized size of antiderivative = 0.85, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {6123, 5985, 25, 6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 7163, 2720, 7143, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^3\text{csch}(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-3f\int -(e+fx)^2\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{3f\int (e+fx)^2\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 6107$$

$$\frac{3f\int (e+fx)^2\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{b^2\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{3f\int (e+fx)^2\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)^3}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \; 3803$$

$$\frac{3f\int (e+fx)^2\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{2b^2\int -\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \; 25$$

3.440. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3f\int(e+fx)^2\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int\frac{e^{c+dx}(e+fx)^3}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \textbf{2694}$$

$$\frac{3f\int(e+fx)^2\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int-\frac{e^{c+dx}(e+fx)^3}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{3f\int(e+fx)^2\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \textbf{2620}$$

$$\frac{3f\int(e+fx)^2\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^3\log\left(\frac{-b\ldots}{a-\ldots}\right)}{bd}\ldots\right.}{\ldots}\right.}{a^2+b^2}\right.}{a}$$

3.440. $\displaystyle\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \color{blue}{3011}$$

$$\frac{3f\int(e+fx)^2\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\text{sech}(c+dx)}{d}}{a}-$$

$$\frac{b\left(\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{Poly}}{}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.}{a}$$

$$\Big\downarrow \color{blue}{7163}$$

3.440. $\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3f\int (e+fx)^2\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-$$

$$b\left(\frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int \operatorname{PolyLog}\left(3,\right.}{}\right.}{d}\right.}{b}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.$$

$\downarrow$ 2720

3.440. $\displaystyle\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3f\int(e+fx)^2\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-$$

$$b\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\operatorname{Po}}{}\right.}{d}\right.}{}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.$$

$\downarrow$ 7143

3.440. $\int\frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3f\int(e+fx)^2\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-\cdots$$

$$b\left(\frac{\int(e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\cdots\right)}{d^{\cdots}}\cdots\right)}{d}\cdots\right)}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\cdots\right.$$

$$\Big\downarrow \; 7292$$

3.440. $\displaystyle\int\frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3f\int \frac{(e+fx)^2(\text{arctanh}(\cosh(c+dx))-\text{sech}(c+dx))}{d}dx-\frac{(e+fx)^3\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\text{sech}(c+dx)}{d}}{}-$$

$$b\left(\frac{\int(e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{a\left(b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\,\text{PolyLog}\left(4,-\cdots\right)}{d^{\cdots}}\right)}{d}\cdots\right)}{bd}\cdots\right)\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\cdots\right)$$

$$\Big\downarrow \; 27$$

3.440. $\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3f\int (e+fx)^2(\text{arctanh}(\cosh(c+dx))-\text{sech}(c+dx))dx}{d}-\frac{(e+fx)^3\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\text{sech}(c+dx)}{d}-$$

$$b\left(\frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{a\left(b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}-\frac{f\,\text{PolyLog}\left(4,-\cdots\right.}{d^{\cdots}}\cdots\right)}{d}\cdots\right)}{bd}\cdots\right)\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$\downarrow$ 7293

3.440. $\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f\int\left((e+fx)^2\operatorname{arctanh}(\cosh(c+dx))-(e+fx)^2\operatorname{sech}(c+dx)\right)dx}{d}-\frac{(e+fx)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^3\operatorname{sech}(c+dx)}{d}}{a}-$$

$$b\left(\frac{\int\left(a(e+fx)^3\operatorname{sech}^2(c+dx)-b(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(b\left(\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{3f\left(2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c\cdots}}{a+\sqrt{\cdots}}\right)}{d}\cdots\right.\right.}{\cdots}\right.\right.}{\cdots}\right.$$

$\downarrow$ 2009

3.440. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{\operatorname{arctanh}(\cosh(c+dx))(e+fx)^3}{d}+\frac{\operatorname{sech}(c+dx)(e+fx)^3}{d}+\frac{3f\left(-\frac{2\operatorname{arctanh}\left(e^{c+dx}\right)(e+fx)^3}{3f}+\frac{\operatorname{arctanh}(\cosh(c+dx))(e+fx)^3}{3f}-\frac{2\arctan\left(e^{c+d}\right.}{d}\right.}{}$$

$$b\left(-\frac{6ib\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)f^3}{d^4}+\frac{6ib\operatorname{PolyLog}\left(3,ie^{c+dx}\right)f^3}{d^4}+\frac{3a\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)f^3}{2d^4}+\frac{6ib(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)f^2}{d^3}-\frac{6ib(e+fx)\operatorname{PolyLog}\left(2,ie^{c+d}\right.}{d^3}\right.$$

input
```
Int[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

3.440. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(-(((e + f*x)^3*ArcTanh[Cosh[c + d*x]])/d) + (3*f*((-2*(e + f*x)^2*ArcTan[
E^(c + d*x)])/d - (2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(3*f) + ((e + f*x)^
3*ArcTanh[Cosh[c + d*x]])/(3*f) - ((e + f*x)^2*PolyLog[2, -E^(c + d*x)])/d
 + ((2*I)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 - ((2*I)*f*(e + f*
x)*PolyLog[2, I*E^(c + d*x)])/d^2 + ((e + f*x)^2*PolyLog[2, E^(c + d*x)])/
d + (2*f*(e + f*x)*PolyLog[3, -E^(c + d*x)])/d^2 - ((2*I)*f^2*PolyLog[3, (
-I)*E^(c + d*x)])/d^3 + ((2*I)*f^2*PolyLog[3, I*E^(c + d*x)])/d^3 - (2*f*(
e + f*x)*PolyLog[3, E^(c + d*x)])/d^2 - (2*f^2*PolyLog[4, -E^(c + d*x)])/d
^3 + (2*f^2*PolyLog[4, E^(c + d*x)])/d^3))/d + ((e + f*x)^3*Sech[c + d*x])
/d)/a - (b*((-2*b^2*(-1/2*(b*(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2]))])/d^2))/d))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^3*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*P
olyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(2*Sqrt[a^2 + b^
2])))/(a^2 + b^2) + ((a*(e + f*x)^3)/d - (6*b*f*(e + f*x)^2*ArcTan[E^(c +
d*x)])/d^2 - (3*a*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/d^2 + ((6*I)*...
```

3.440.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.440. $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.440. $\int \frac{(e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.440.4 Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c) \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.440.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 9707 vs. $2(1064) = 2128$.

Time = 0.51 (sec) , antiderivative size = 9707, normalized size of antiderivative = 8.34

$$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.440.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.440.7 Maxima [F]

$$\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-3*b*e^2*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2
 + b^2)*d^2)) - 6*a*f^3*integrate(x^2*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) +
 b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 6*b*f^3*integrate(x^2/(a^2*d
*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 12*a*e*f^2
*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) +
a^2*d + b^2*d), x) - 12*b*e*f^2*integrate(x/(a^2*d*e^(2*d*x + 2*c) + b^2*d
*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - (b^3*log((b*e^(-d*x - c) - a - sqr
t(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^3 + a*b^2)*sqrt(
a^2 + b^2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x
 - 2*c))*d) + log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))*e
^3 - 6*a*e^2*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) - 3*(d*x*log(e^(d*x +
 c) + 1) + dilog(-e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d*x*log(-e^(d*x + c) +
1) + dilog(e^(d*x + c)))*e^2*f/(a*d^2) + 2*(b*f^3*x^3 + 3*b*e*f^2*x^2 + 3*
b*e^2*f*x + (a*f^3*x^3*e^c + 3*a*e*f^2*x^2*e^c + 3*a*e^2*f*x*e^c)*e^(d*x))
/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) - 3*(d^2*x^2*
log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x +
c)))*e*f^2/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x
 + c)) - 2*polylog(3, e^(d*x + c)))*e*f^2/(a*d^3) - (d^3*x^3*log(e^(d*x +
c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) +
 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) + ...
```

3.440.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.440.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{(e+f\,x)^3}{\cosh\left(c+d\,x\right)^2\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.441 $\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.441.1 Optimal result

Integrand size = 34, antiderivative size = 795

$$
\begin{aligned}
&\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{b(e+fx)^2}{(a^2+b^2)\,d} - \frac{4f(e+fx)\arctan\left(e^{c+dx}\right)}{ad^2} + \frac{4b^2f(e+fx)\arctan\left(e^{c+dx}\right)}{a\,(a^2+b^2)\,d^2} \\
&\quad - \frac{2(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{b^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d} \\
&\quad + \frac{b^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d} + \frac{2bf(e+fx)\log\left(1+e^{2(c+dx)}\right)}{(a^2+b^2)\,d^2} \\
&\quad - \frac{2f(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} + \frac{2if^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3} \\
&\quad - \frac{2ib^2f^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{a\,(a^2+b^2)\,d^3} - \frac{2if^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{ad^3} \\
&\quad + \frac{2ib^2f^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{a\,(a^2+b^2)\,d^3} + \frac{2f(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} \\
&\quad - \frac{2b^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d^2} + \frac{2b^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d^2} \\
&\quad + \frac{bf^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{(a^2+b^2)\,d^3} + \frac{2f^2\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3} - \frac{2f^2\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3} \\
&\quad + \frac{2b^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d^3} - \frac{2b^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{3/2}\,d^3} \\
&\quad + \frac{(e+fx)^2\operatorname{sech}(c+dx)}{ad} - \frac{b^2(e+fx)^2\operatorname{sech}(c+dx)}{a\,(a^2+b^2)\,d} - \frac{b(e+fx)^2\tanh(c+dx)}{(a^2+b^2)\,d}
\end{aligned}
$$

output
```
-b*(f*x+e)^2/(a^2+b^2)/d-4*f*(f*x+e)*arctan(exp(d*x+c))/a/d^2+4*b^2*f*(f*x
+e)*arctan(exp(d*x+c))/a/(a^2+b^2)/d^2-2*(f*x+e)^2*arctanh(exp(d*x+c))/a/d
+2*b*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d^2-b^3*(f*x+e)^2*ln(1+b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d+b^3*(f*x+e)^2*ln(1+b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d-2*f*(f*x+e)*polylog(2,-exp
(d*x+c))/a/d^2+2*I*b^2*f^2*polylog(2,I*exp(d*x+c))/a/(a^2+b^2)/d^3-2*I*f^2
*polylog(2,I*exp(d*x+c))/a/d^3-2*I*b^2*f^2*polylog(2,-I*exp(d*x+c))/a/(a^2
+b^2)/d^3+2*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3+2*f*(f*x+e)*polylog(2,exp
(d*x+c))/a/d^2+b*f^2*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^3-2*b^3*f*(f*x
+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^2+2*b
^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2
)/d^2+2*f^2*polylog(3,-exp(d*x+c))/a/d^3-2*f^2*polylog(3,exp(d*x+c))/a/d^3
+2*b^3*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/
d^3-2*b^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/
2)/d^3+(f*x+e)^2*sech(d*x+c)/a/d-b^2*(f*x+e)^2*sech(d*x+c)/a/(a^2+b^2)/d-b
*(f*x+e)^2*tanh(d*x+c)/(a^2+b^2)/d
```

3.441.2 Mathematica [A] (verified)

Time = 8.30 (sec) , antiderivative size = 928, normalized size of antiderivative = 1.17

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= 4\Bigg(-\frac{f\text{csch}(c+dx)\left(4bd^2ee^{2c}x-4bd^2e\left(1+e^{2c}\right)x+2bd^2e^{2c}fx^2-2bd^2\left(1+e^{2c}\right)fx^2+4ade\left(1+e^{2c}\right)\text{arcta}\right.}{}$$

$$+\frac{\text{csch}(c+dx)\left(\left((e+fx)^2\log\left(1-e^{c+dx}\right)-(e+fx)^2\log\left(1+e^{c+dx}\right)-\frac{2f\left(d(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)-f\,\text{PolyLog}\right.}{d^2}\right.\right.}{4ad(b+a\text{csch}(c+dx))}$$

$$-\frac{b^3\text{csch}(c+dx)\left(-2d^2e^2\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^2f^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)\right.}{}$$

$$+\frac{\text{csch}(c+dx)\text{sech}(c)\text{sech}(c+dx)\left(ae^2\cosh(c)+2aefx\cosh(c)+af^2x^2\cosh(c)-be^2\sinh(dx)-2befx\right.}{4\left(a^2+b^2\right)d(b+a\text{csch}(c+dx))}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
4*(-1/4*(f*Csch[c + d*x]*(4*b*d^2*e*E^(2*c)*x - 4*b*d^2*e*(1 + E^(2*c))*x
+ 2*b*d^2*E^(2*c)*f*x^2 - 2*b*d^2*(1 + E^(2*c))*f*x^2 + 4*a*d*e*(1 + E^(2*
c))*ArcTan[E^(c + d*x)] + 2*b*d*e*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c +
 d*x))]) + (2*I)*a*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 +
I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]
) + b*(1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2,
 -E^(2*(c + d*x))])))*(a + b*Sinh[c + d*x]))/((a^2 + b^2)*d^3*(1 + E^(2*c))
*(b + a*Csch[c + d*x])) + (Csch[c + d*x]*((e + f*x)^2*Log[1 - E^(c + d*x)]
 - (e + f*x)^2*Log[1 + E^(c + d*x)] - (2*f*(d*(e + f*x)*PolyLog[2, -E^(c +
 d*x)] - f*PolyLog[3, -E^(c + d*x)]))/d^2 + (2*f*(d*(e + f*x)*PolyLog[2, E
^(c + d*x)] - f*PolyLog[3, E^(c + d*x)]))/d^2)*(a + b*Sinh[c + d*x]))/(4*a
*d*(b + a*Csch[c + d*x])) - (b^3*Csch[c + d*x]*(-2*d^2*e^2*ArcTanh[(a + b*
E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^2*f^2
*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*Poly
Log[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*f^2*PolyLog[3, (b*E^(c + d
*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])*(a + b*Sinh[c + d*x]))/(4*a*(a^2 + b^2)^(3/2)*d^3*(b +...
```

3.441.3 Rubi [A] (verified)

Time = 3.74 (sec) , antiderivative size = 695, normalized size of antiderivative = 0.87, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6123, 5985, 25, 6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-2f\int -\left((e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 6107$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{b^2\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)^2}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \; 3803$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{2b^2\int -\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \; 25$$

3.441. $\displaystyle\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int \frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \; 2694$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)^2}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \; 27$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \; 2620$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\int (e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be\cdots}{a-\sqrt{\cdots}}\right.}{bd}\cdots\right.}{\cdots}\cdots\right.}{a^2+b^2}\cdots\right.}{a}$$

3.441. $\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\Big\downarrow \ 3011$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-$$

$$\frac{b\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{\ }\right.}{a}$$

$$\Big\downarrow \ 2720$$

$$\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-$$

$$b\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)}{\ }\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{\ }\right.$$

3.441. $\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 7143$$

$$\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-$$

$$\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be}{a+\sqrt{}}\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.}{a}$$

$$\Big\downarrow \; 7292$$

$$\frac{2f\int\frac{(e+fx)(\text{arctanh}(\cosh(c+dx))-\text{sech}(c+dx))}{d}dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-$$

$$\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be}{a+\sqrt{}}\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2+b^2}}\right.}{}\right.}{a}$$

3.441. $\displaystyle\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$\downarrow$ 27

$$\frac{\frac{2f\int (e+fx)(\text{arctanh}(\cosh(c+dx))-\text{sech}(c+dx))dx}{d}-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{\cdots}}{a+\sqrt{\cdots}}\right)}{d}\cdots\right)}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\cdots\right)}{a}$$

$\downarrow$ 7293

$$\frac{\frac{2f\int ((e+fx)\text{arctanh}(\cosh(c+dx))-(e+fx)\text{sech}(c+dx))dx}{d}-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int \left(a(e+fx)^2\text{sech}^2(c+dx)-b(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\cdots\right)}{b\cdots}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{\cdots}\cdots\right)}{a}$$

$\downarrow$ 2009

3.441. $\displaystyle\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{2f\left(-\frac{2(e+fx)\arctan\left(e^{c+dx}\right)}{d}-\frac{(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{f}+\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2f}+\frac{if\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}+\frac{f\,\text{PolyLo}}{}\right.}{d}$$

$$b\left(-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}-\right.}{}\right)}{a^2+b^2}\right.$$

input `Int[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]`

3.441. $\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(-(((e + f*x)^2*ArcTanh[Cosh[c + d*x]])/d) + (2*f*((-2*(e + f*x)*ArcTan[E^
(c + d*x)])/d - ((e + f*x)^2*ArcTanh[E^(c + d*x)])/f + ((e + f*x)^2*ArcTan
h[Cosh[c + d*x]])/(2*f) - ((e + f*x)*PolyLog[2, -E^(c + d*x)])/d + (I*f*Po
lyLog[2, (-I)*E^(c + d*x)])/d^2 - (I*f*PolyLog[2, I*E^(c + d*x)])/d^2 + ((
e + f*x)*PolyLog[2, E^(c + d*x)])/d + (f*PolyLog[3, -E^(c + d*x)])/d^2 - (
f*PolyLog[3, E^(c + d*x)])/d^2))/d + ((e + f*x)^2*Sech[c + d*x])/d)/a - (b
*((-2*b^2*(-1/2*(b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/d^2))/(b*d)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + ((a*
(e + f*x)^2)/d - (4*b*f*(e + f*x)*ArcTan[E^(c + d*x)])/d^2 - (2*a*f*(e + f
*x)*Log[1 + E^(2*(c + d*x))])/d^2 + ((2*I)*b*f^2*PolyLog[2, (-I)*E^(c + d*
x)])/d^3 - ((2*I)*b*f^2*PolyLog[2, I*E^(c + d*x)])/d^3 - (a*f^2*PolyLog[2,
 -E^(2*(c + d*x))])/d^3 + (b*(e + f*x)^2*Sech[c + d*x])/d + (a*(e + f*x)^2
*Tanh[c + d*x])/d)/(a^2 + b^2)))/a
```

3.441.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.441. $\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.441. $\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.441.4 Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c) \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.441.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5562 vs. 2(729) = 1458.

Time = 0.40 (sec) , antiderivative size = 5562, normalized size of antiderivative = 7.00

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.441.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.441.7 Maxima [F]

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-2*b*e*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 +
 b^2)*d^2)) - 4*a*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2
*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 4*b*f^2*integrate(x/(a^2*d*e^(2*
d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - (b^3*log((b*e^(-
d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((
a^3 + a*b^2)*sqrt(a^2 + b^2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^
2 + b^2)*e^(-2*d*x - 2*c))*d) + log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x
- c) - 1)/(a*d))*e^2 - 4*a*e*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) + 2*(
b*f^2*x^2 + 2*b*e*f*x + (a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x))/(a^2*d +
b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) - 2*(d*x*log(e^(d*x + c
) + 1) + dilog(-e^(d*x + c)))*e*f/(a*d^2) + 2*(d*x*log(-e^(d*x + c) + 1) +
 dilog(e^(d*x + c)))*e*f/(a*d^2) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*d
ilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3) + (d^2*x^2*lo
g(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c))
)*f^2/(a*d^3) - integrate(-2*(b^3*f^2*x^2*e^c + 2*b^3*e*f*x*e^c)*e^(d*x)/(
a^3*b + a*b^3 - (a^3*b*e^(2*c) + a*b^3*e^(2*c))*e^(2*d*x) - 2*(a^4*e^c + a
^2*b^2*e^c)*e^(d*x)), x)
```

3.441.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.441.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\cosh(c+d\,x)^2\,\sinh(c+d\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.441. $\int \frac{(e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.442 $\int \frac{(e+fx)\mathrm{csch}(c+dx)\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.442.1 Optimal result

Integrand size = 32, antiderivative size = 442

$$\begin{aligned}
&\int \frac{(e+fx)\mathrm{csch}(c+dx)\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{f\arctan(\sinh(c+dx))}{ad^2}+\frac{b^2f\arctan(\sinh(c+dx))}{a(a^2+b^2)d^2}-\frac{2fx\text{arctanh}\left(e^{c+dx}\right)}{ad} \\
&\quad+\frac{fx\text{arctanh}(\cosh(c+dx))}{ad}-\frac{(e+fx)\text{arctanh}(\cosh(c+dx))}{ad} \\
&\quad-\frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{3/2}d}+\frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{3/2}d} \\
&\quad+\frac{bf\log(\cosh(c+dx))}{(a^2+b^2)d^2}-\frac{f\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{f\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} \\
&\quad-\frac{b^3f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{3/2}d^2}+\frac{b^3f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{3/2}d^2} \\
&\quad+\frac{(e+fx)\mathrm{sech}(c+dx)}{ad}-\frac{b^2(e+fx)\mathrm{sech}(c+dx)}{a(a^2+b^2)d}-\frac{b(e+fx)\tanh(c+dx)}{(a^2+b^2)d}
\end{aligned}$$

output
```
-f*arctan(sinh(d*x+c))/a/d^2+b^2*f*arctan(sinh(d*x+c))/a/(a^2+b^2)/d^2-2*f
*x*arctanh(exp(d*x+c))/a/d+f*x*arctanh(cosh(d*x+c))/a/d-(f*x+e)*arctanh(co
sh(d*x+c))/a/d+b*f*ln(cosh(d*x+c))/(a^2+b^2)/d^2-b^3*(f*x+e)*ln(1+b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d+b^3*(f*x+e)*ln(1+b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d-f*polylog(2,-exp(d*x+c))/a/d^2+
f*polylog(2,exp(d*x+c))/a/d^2-b^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/a/(a^2+b^2)^(3/2)/d^2+b^3*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/a/(a^2+b^2)^(3/2)/d^2+(f*x+e)*sech(d*x+c)/a/d-b^2*(f*x+e)*sech(d*x+c
)/a/(a^2+b^2)/d-b*(f*x+e)*tanh(d*x+c)/(a^2+b^2)/d
```

3.442.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.94 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.97

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\text{csch}(c+dx)(a+b\sinh(c+dx))\left(-\frac{2f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a-ib}-\frac{2f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a+ib}-\frac{if\log(\cosh(c+dx))}{a-ib}+\frac{if\log}{}\right.}{}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),
x]
```

output
```
(Csch[c + d*x]*(a + b*Sinh[c + d*x])*((-2*f*ArcTan[Tanh[(c + d*x)/2]])/(a
- I*b) - (2*f*ArcTan[Tanh[(c + d*x)/2]])/(a + I*b) - (I*f*Log[Cosh[c + d*x
]])/(a - I*b) + (I*f*Log[Cosh[c + d*x]])/(a + I*b) + (2*(d*(e + f*x)*(Log[
1 - E^(c + d*x)] - Log[1 + E^(c + d*x)]) - f*PolyLog[2, -E^(c + d*x)] + f*
PolyLog[2, E^(c + d*x)]))/a - (2*b^3*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/S
qrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(
c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/
(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))]))/(a*(a^2 + b^2)^(3/2)) + (2*d*(e + f*x)*Sech[c + d*x]*(a - b*Sinh[
c + d*x]))/(a^2 + b^2)))/(2*d^2*(b + a*Csch[c + d*x]))
```

3.442.3 Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.88, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6123, 5985, 2009, 6107, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6123$$

$$\frac{\int (e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 5985$$

$$\frac{-f\int \left(\frac{\text{sech}(c+dx)}{d}-\frac{\text{arctanh}(\cosh(c+dx))}{d}\right)dx-\frac{(e+fx)\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)\text{sech}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 2009$$

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}(e^{c+dx})}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{arc}}{}}{a}\;\frac{b\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 6107$$

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}(e^{c+dx})}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{arc}}{}}{a}\;\frac{b\left(\frac{b^2\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arc}}{a}$$

$$\frac{b\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{e+fx}{a-ib\sin(ic+idx)}dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \; 3803$$

$$-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arc}}{a}$$

$$\frac{b\left(\frac{2b^2\int -\frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}+\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \; 25$$

$$-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arc}}{a}$$

$$\frac{b\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\int \frac{e^{c+dx}(e+fx)}{-2e^{c+dx}a-be^{2(c+dx)}+b}dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \; 2694$$

$$-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arc}}{a}$$

$$\frac{b\left(\frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}-\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}-\frac{b\int -\frac{e^{c+dx}(e+fx)}{2\left(a+be^{c+dx}+\sqrt{a^2+b^2}\right)}dx}{\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \; 27$$

3.442. $$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}(e^{c+dx})}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{arc}}{}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}-\frac{b\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx}{2\sqrt{a^2+b^2}}\right)}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}(e^{c+dx})}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{arc}}{}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{2\sqrt{a}}\right.}{a^2+b^2}\right.}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}(e^{c+dx})}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{arc}}{}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{(e+fx)\log\left(\frac{}{a-}\right.}{b}\right.}{}\right.}{a^2+b^2}\right.}{a}$$

$$\Big\downarrow \text{2838}$$

3.442. $\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arc}\cdots}{}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}\cdots\right)}{2\sqrt{\cdots}}\cdots\right)}{a^2+b^2}\cdots\right)}{a}$$

$\downarrow$ 7293

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arc}\cdots}{}}{a}$$

$$\frac{b\left(\frac{\int\left(a(e+fx)\text{sech}^2(c+dx)-b(e+fx)\text{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2+b^2}}-\frac{b\left(\frac{f\,\text{P}\cdots}{}\right)}{}\cdots\right)}{a^2+b^2}\cdots\right)}{a}$$

$\downarrow$ 2009

$$\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arc}\cdots}{}}{a}$$

$$\frac{b\left(\frac{-\frac{af\log(\cosh(c+dx))}{d^2}+\frac{a(e+fx)\tanh(c+dx)}{d}-\frac{bf\arctan(\sinh(c+dx))}{d^2}+\frac{b(e+fx)\text{sech}(c+dx)}{d}}{a^2+b^2}-\frac{2b^2\left(\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c\cdots}}{\sqrt{a^2+\cdots}}\right)}{bd}\cdots\right)}{2\sqrt{a^2+b^2}}\cdots\right)}{}\cdots\right)}{a}$$

3.442. $\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(((e + f*x)*ArcTanh[Cosh[c + d*x]])/d) - f*(ArcTan[Sinh[c + d*x]]/d^2 +
(2*x*ArcTanh[E^(c + d*x)])/d - (x*ArcTanh[Cosh[c + d*x]])/d + PolyLog[2, -
E^(c + d*x)]/d^2 - PolyLog[2, E^(c + d*x)]/d^2) + ((e + f*x)*Sech[c + d*x]
)/d)/a - (b*((-2*b^2*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + (-((b*f*ArcTa
n[Sinh[c + d*x]])/d^2) - (a*f*Log[Cosh[c + d*x]])/d^2 + (b*(e + f*x)*Sech[
c + d*x])/d + (a*(e + f*x)*Tanh[c + d*x])/d)/(a^2 + b^2)))/a
```

3.442.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

3.442. $\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.442.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1814 vs. $2(419) = 838$.

Time = 10.70 (sec) , antiderivative size = 1815, normalized size of antiderivative = 4.11

method	result	size
risch	Expression too large to display	1815

input
```
int((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
-1/(a^2+b^2)/d*f*b^2/a*ln(exp(d*x+c)+1)*x+1/(a^2+b^2)^(3/2)/d^2*a*f*b*arct
anh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)/d^2*b^2*c*f/a*ln
(exp(d*x+c)-1)+1/(a^2+b^2)^(3/2)/d^2*f*b^3/a*arctanh(1/2*(2*b*exp(d*x+c)+2
*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(5/2)/d*f*b^5/a*ln((-b*exp(d*x+c)+(a^2+b^
2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(5/2)/d*f*b^5/a*ln((b*exp(
d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(5/2)/d*a*b^3
*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2
)^(5/2)/d*a*b^3*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))
*x-1/(a^2+b^2)^(5/2)/d^2*f*b^5/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+
(a^2+b^2)^(1/2)))*c+1/(a^2+b^2)^(5/2)/d^2*f*b^5/a*ln((b*exp(d*x+c)+(a^2+b^
2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/(a^2+b^2)^(5/2)/d^2*a*b^3*f*ln((-b*ex
p(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/(a^2+b^2)^(5/2)/d^2*
a*b^3*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/(a^2+
b^2)^(3/2)/d^2*b^3*c*f/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
-1/(a^2+b^2)^(5/2)/d^2*b^5*c*f/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2
)^(1/2))+1/(a^2+b^2)^(5/2)/d^2*c*a^3*f*b*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/
(a^2+b^2)^(1/2))-1/(a^2+b^2)^(3/2)/d^2*c*a*f*b*arctanh(1/2*(2*b*exp(d*x+c)
+2*a)/(a^2+b^2)^(1/2))-1/d^2/(a^2+b^2)^(5/2)*b*f*arctanh(1/2*(2*b*exp(d*x+
c)+2*a)/(a^2+b^2)^(1/2))*a^3-2/d^2/(a^2+b^2)^(5/2)*b^3*f*arctanh(1/2*(2*b*
exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a+2*(f*x+e)*(a*exp(d*x+c)+b)/d/(a^2+b^...
```

3.442.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2176 vs. 2(415) = 830.

Time = 0.32 (sec) , antiderivative size = 2176, normalized size of antiderivative = 4.92

$$\int \frac{(e+fx)\mathrm{csch}(c+dx)\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-(2*(a^3*b + a*b^3)*d*f*x*cosh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d*f*x*sinh(d
*x + c)^2 - 2*(a^3*b + a*b^3)*d*e + (b^4*f*cosh(d*x + c)^2 + 2*b^4*f*cosh(
d*x + c)*sinh(d*x + c) + b^4*f*sinh(d*x + c)^2 + b^4*f)*sqrt((a^2 + b^2)/b
^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^4*f*cosh(d*x + c)^2 + 2*b^
4*f*cosh(d*x + c)*sinh(d*x + c) + b^4*f*sinh(d*x + c)^2 + b^4*f)*sqrt((a^2
 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^4*d*e - b^4*c*f +
 (b^4*d*e - b^4*c*f)*cosh(d*x + c)^2 + 2*(b^4*d*e - b^4*c*f)*cosh(d*x + c)
*sinh(d*x + c) + (b^4*d*e - b^4*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2
)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) +
2*a) + (b^4*d*e - b^4*c*f + (b^4*d*e - b^4*c*f)*cosh(d*x + c)^2 + 2*(b^4*d
*e - b^4*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d*e - b^4*c*f)*sinh(d*x +
 c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2
*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^4*d*f*x + b^4*c*f + (b^4*d*f*x + b^4*
c*f)*cosh(d*x + c)^2 + 2*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*x + c)
 + (b^4*d*f*x + b^4*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*co
sh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
a^2 + b^2)/b^2) - b)/b) - (b^4*d*f*x + b^4*c*f + (b^4*d*f*x + b^4*c*f)*cos
h(d*x + c)^2 + 2*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b...
```

3.442.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\mathrm{csch}(c+dx)\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.442.7 Maxima [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)\,\text{sech}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(b^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqr
t(a^2 + b^2)))/((a^3 + a*b^2)*sqrt(a^2 + b^2)*d) - 2*(a*e^(-d*x - c) - b)/
((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d) + log(e^(-d*x - c) + 1)/(a*
d) - log(e^(-d*x - c) - 1)/(a*d))*e - (8*b^3*integrate(-1/4*x*e^(d*x + c)/
(a^3*b + a*b^3 - (a^3*b*e^(2*c) + a*b^3*e^(2*c))*e^(2*d*x) - 2*(a^4*e^c +
a^2*b^2*e^c)*e^(d*x)), x) - 2*(a*x*e^(d*x + c) + b*x)/(a^2*d + b^2*d + (a^
2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) + 2*b*x/((a^2 + b^2)*d) + 2*a*arct
an(e^(d*x + c))/((a^2 + b^2)*d^2) - b*log(e^(2*d*x + 2*c) + 1)/((a^2 + b^2
)*d^2) - 8*integrate(1/8*x/(a*e^(d*x + c) + a), x) - 8*integrate(1/8*x/(a*
e^(d*x + c) - a), x))*f
```

3.442.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.442.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\left(c+d\,x\right)^2\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.443 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.443.1 Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\cosh(c+dx))}{ad} + \frac{2b^3\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{3/2}d} + \frac{\operatorname{sech}(c+dx)}{ad} - \frac{b\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a\left(a^2+b^2\right)d}$$

output
```
-arctanh(cosh(d*x+c))/a/d+2*b^3*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2
)^(1/2))/a/(a^2+b^2)^(3/2)/d+sech(d*x+c)/a/d-b*sech(d*x+c)*(b+a*sinh(d*x+c
))/a/(a^2+b^2)/d
```

3.443.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(113) = 226.

Time = 1.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.06

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{-2b^3\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right) - a^2\sqrt{-a^2-b^2}\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) - b^2\sqrt{-a^2-b^2}\log\left(\cosh\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[(Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-2*b^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] - a^2*Sqrt[-a
^2 - b^2]*Log[Cosh[(c + d*x)/2]] - b^2*Sqrt[-a^2 - b^2]*Log[Cosh[(c + d*x)
/2]] + a^2*Sqrt[-a^2 - b^2]*Log[Sinh[(c + d*x)/2]] + b^2*Sqrt[-a^2 - b^2]*
Log[Sinh[(c + d*x)/2]] + a^2*Sqrt[-a^2 - b^2]*Sech[c + d*x] - a*b*Sqrt[-a^
2 - b^2]*Tanh[c + d*x])/(a*(-a^2 - b^2)^(3/2)*d))
```

3.443.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3042, 26, 3377, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i}{\sin(ic+idx)\cos(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{1}{\cos(ic+idx)^2\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \ 3377$$

$$i\int \left(\frac{ib\operatorname{sech}^2(c+dx)}{a(a+b\sinh(c+dx))}-\frac{i\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a}\right)dx$$

$$\Big\downarrow \ 2009$$

$$i\left(-\frac{2ib^3\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\left(a^2+b^2\right)^{3/2}}+\frac{ib\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{ad\left(a^2+b^2\right)}+\frac{i\operatorname{arctanh}(\cosh(c+dx))}{ad}-\frac{i\operatorname{sech}(c+\right.$$

input
```
Int[(Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
I*((I*ArcTanh[Cosh[c + d*x]])/(a*d) - ((2*I)*b^3*ArcTanh[(b - a*Tanh[(c +
d*x)/2])/Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(3/2)*d) - (I*Sech[c + d*x])/(a*
d) + (I*b*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a*(a^2 + b^2)*d))
```

3.443.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3377
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

3.443.4 Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{2b^3 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2\left(b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}{d}$
default	$\frac{-\frac{2b^3 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2\left(b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}{d}$
risch	$\frac{2a\,e^{dx+c}+2b}{d(a^2+b^2)(1+e^{2dx+2c})}+\frac{\ln(e^{dx+c}-1)}{da}-\frac{\ln(e^{dx+c}+1)}{da}+\frac{b^3\ln\left(e^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}da}-\frac{b^3\ln\left(e^{d\cdot}\right.}{}$

input
```
int(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/a*b^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a
^2+b^2)^(1/2))+1/a*ln(tanh(1/2*d*x+1/2*c))-2/(a^2+b^2)*(b*tanh(1/2*d*x+1/2
*c)-a)/(1+tanh(1/2*d*x+1/2*c)^2))
```

3.443.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(110) = 220$.

Time = 0.33 (sec) , antiderivative size = 581, normalized size of antiderivative = 5.14

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{2a^3b + 2ab^3 + \left(b^3\cosh(dx+c)^2 + 2b^3\cosh(dx+c)\sinh(dx+c) + b^3\sinh(dx+c)^2 + b^3\right)\sqrt{a^2+b^2}\log(}{}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
(2*a^3*b + 2*a*b^3 + (b^3*cosh(d*x + c)^2 + 2*b^3*cosh(d*x + c)*sinh(d*x +
 c) + b^3*sinh(d*x + c)^2 + b^3)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2
+ b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*
x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(
d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c)
+ 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a^4 + a^2*b^2)*cosh(d*x
 + c) - (a^4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^2 +
 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2
+ b^4)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a^4 + 2*
a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b
^2 + b^4)*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*sinh(d*x +
 c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(a^4 + a^2*b^2)*sinh(d*x
 + c))/((a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)^2 + 2*(a^5 + 2*a^3*b^2 +
 a*b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d*sinh(d
*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d)
```

3.443.6 Sympy [F]

$$\int \frac{\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\text{csch}\,(c+dx)\,\text{sech}^2\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.443.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.49

$$\int \frac{\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b^3\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^3+ab^2)\sqrt{a^2+b^2}d} + \frac{2\left(ae^{(-dx-c)}-b\right)}{(a^2+b^2+(a^2+b^2)e^{(-2\,dx-2\,c)})d} - \frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-b^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/((a^3 + a*b^2)*sqrt(a^2 + b^2)*d) + 2*(a*e^(-d*x - c) - b)/(
(a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d) - log(e^(-d*x - c) + 1)/(a*d
) + log(e^(-d*x - c) - 1)/(a*d)
```

3.443.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.29

$$\int \frac{\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{b^3\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^3+ab^2)\sqrt{a^2+b^2}} + \frac{\log\left(e^{(dx+c)}+1\right)}{a} - \frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{a} - \frac{2\left(ae^{(dx+c)}+b\right)}{(a^2+b^2)\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(b^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x +
c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^3 + a*b^2)*sqrt(a^2 + b^2)) + log(e^(d*
x + c) + 1)/a - log(abs(e^(d*x + c) - 1))/a - 2*(a*e^(d*x + c) + b)/((a^2
+ b^2)*(e^(2*d*x + 2*c) + 1)))/d
```

3.443.9 Mupad [B] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 668, normalized size of antiderivative = 5.91

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,b}{d\,(a^2+b^2)}+\frac{2\,a\,e^{c+d\,x}}{d\,(a^2+b^2)}}{e^{2\,c+2\,d\,x}+1}+\frac{\ln\left(e^{c+d\,x}-1\right)}{a\,d}-\frac{\ln\left(e^{c+d\,x}+1\right)}{a\,d}$$
$$-\frac{b^3\,\ln\left(\frac{32\,\left(-4\,e^{c+d\,x}\,a^3+2\,a^2\,b-5\,e^{c+d\,x}\,a\,b^2+2\,b^3\right)}{b^2\,{\left(a^2+b^2\right)}^2}-\frac{128\,a^{10}\,e^{c+d\,x}-64\,a^9\,b-96\,a\,b^9+64\,b^7\,\sqrt{{\left(a^2+b^2\right)}^3}-384\,a^3\,b^7-512\,a^5\,b^5-288\,a^7\,b^3+}{}\right.}{d\,a^7+3\,d\,a^5\,b^2}$$
$$+\frac{b^3\,\ln\left(\frac{32\,\left(-4\,e^{c+d\,x}\,a^3+2\,a^2\,b-5\,e^{c+d\,x}\,a\,b^2+2\,b^3\right)}{b^2\,{\left(a^2+b^2\right)}^2}-\frac{96\,a\,b^9+64\,a^9\,b-128\,a^{10}\,e^{c+d\,x}+64\,b^7\,\sqrt{{\left(a^2+b^2\right)}^3}+384\,a^3\,b^7+512\,a^5\,b^5+288\,a^7\,b^3-}{}\right.}{d\,a^7+3\,d\,a^5\,b^2}$$

input
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(2*c + 2*
d*x) + 1) + log(exp(c + d*x) - 1)/(a*d) - log(exp(c + d*x) + 1)/(a*d) - (b
^3*log((32*(2*a^2*b - 4*a^3*exp(c + d*x) + 2*b^3 - 5*a*b^2*exp(c + d*x)))/
(b^2*(a^2 + b^2)^2) - (128*a^10*exp(c + d*x) - 64*a^9*b - 96*a*b^9 + 64*b^
7*((a^2 + b^2)^3)^(1/2) - 384*a^3*b^7 - 512*a^5*b^5 - 288*a^7*b^3 + 288*a^
2*b^8*exp(c + d*x) + 960*a^4*b^6*exp(c + d*x) + 1152*a^6*b^4*exp(c + d*x)
+ 608*a^8*b^2*exp(c + d*x) - 64*a*b^6*exp(c + d*x)*((a^2 + b^2)^3)^(1/2) +
 32*a^3*b^4*exp(c + d*x)*((a^2 + b^2)^3)^(1/2))/(b^2*((a^2 + b^2)^3)^(3/2)
*(a^2 + b^2)))*((a^2 + b^2)^3)^(1/2))/(a^7*d + 3*a^3*b^4*d + 3*a^5*b^2*d +
 a*b^6*d) + (b^3*log((32*(2*a^2*b - 4*a^3*exp(c + d*x) + 2*b^3 - 5*a*b^2*e
xp(c + d*x)))/(b^2*(a^2 + b^2)^2) - (96*a*b^9 + 64*a^9*b - 128*a^10*exp(c
+ d*x) + 64*b^7*((a^2 + b^2)^3)^(1/2) + 384*a^3*b^7 + 512*a^5*b^5 + 288*a^
7*b^3 - 288*a^2*b^8*exp(c + d*x) - 960*a^4*b^6*exp(c + d*x) - 1152*a^6*b^4
*exp(c + d*x) - 608*a^8*b^2*exp(c + d*x) - 64*a*b^6*exp(c + d*x)*((a^2 + b
^2)^3)^(1/2) + 32*a^3*b^4*exp(c + d*x)*((a^2 + b^2)^3)^(1/2))/(b^2*((a^2 +
 b^2)^3)^(3/2)*(a^2 + b^2)))*((a^2 + b^2)^3)^(1/2))/(a^7*d + 3*a^3*b^4*d +
 3*a^5*b^2*d + a*b^6*d)
```

3.443. $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.444 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.444.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.444.2 Mathematica [N/A]

Not integrable

Time = 49.81 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Csch[c + d*x]*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

3.444.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.444.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.444.4 Maple [N/A] (verified)

Not integrable

Time = 0.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)^2}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.444.5 Fricas [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(csch(d*x + c)*sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

3.444.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.444.7 Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 463, normalized size of antiderivative = 13.62

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-8*b^3*integrate(-1/4*e^(d*x + c)/(a^3*b*e + a*b^3*e + (a^3*b*f + a*b^3*f)
*x - (a^3*b*e*e^(2*c) + a*b^3*e*e^(2*c) + (a^3*b*f*e^(2*c) + a*b^3*f*e^(2*
c))*x)*e^(2*d*x) - 2*(a^4*e*e^c + a^2*b^2*e*e^c + (a^4*f*e^c + a^2*b^2*f*e
^c)*x)*e^(d*x)), x) + 2*(a*e^(d*x + c) + b)/(a^2*d*e + b^2*d*e + (a^2*d*f
+ b^2*d*f)*x + (a^2*d*e*e^(2*c) + b^2*d*e*e^(2*c) + (a^2*d*f*e^(2*c) + b^2
*d*f*e^(2*c))*x)*e^(2*d*x)) + 8*integrate(1/4*(a*f*e^(d*x + c) + b*f)/(a^2
*d*e^2 + b^2*d*e^2 + (a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*e*f + b^2*d*e*
f)*x + (a^2*d*e^2*e^(2*c) + b^2*d*e^2*e^(2*c) + (a^2*d*f^2*e^(2*c) + b^2*d
*f^2*e^(2*c))*x^2 + 2*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x)*e^(2*d*x)
), x) + 8*integrate(1/8/(a*f*x + a*e + (a*f*x*e^c + a*e*e^c)*e^(d*x)), x)
+ 8*integrate(-1/8/(a*f*x + a*e - (a*f*x*e^c + a*e*e^c)*e^(d*x)), x)
```

3.444.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.444.9 Mupad [N/A]

Not integrable

Time = 5.66 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\begin{aligned}&\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx\\&=\int \frac{1}{\cosh\left(c+d\,x\right)^2\sinh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx\end{aligned}$$

input
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.445 $\int \frac{(e+fx)^2\mathbf{csch}(c+dx)\mathbf{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.445.1 Optimal result

Integrand size = 34, antiderivative size = 1185

$$
\begin{aligned}
\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = {} & \frac{efx}{ad}+\frac{f^2x^2}{2ad}-\frac{2b^3(e+fx)^2\arctan\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} \\
& -\frac{b(e+fx)^2\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d} \\
& +\frac{bf^2\arctan(\sinh(c+dx))}{(a^2+b^2)\,d^3} \\
& -\frac{2(e+fx)^2\operatorname{arctanh}\left(e^{2c+2dx}\right)}{ad} \\
& -\frac{b^4(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^2 d} \\
& -\frac{b^4(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^2 d} \\
& +\frac{b^4(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{a(a^2+b^2)^2 d} \\
& +\frac{f^2\log(\cosh(c+dx))}{ad^3} \\
& -\frac{b^2f^2\log(\cosh(c+dx))}{a(a^2+b^2)\,d^3} \\
& +\frac{2ib^3f(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)^2 d^2} \\
& +\frac{ibf(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)\,d^2} \\
& -\frac{2ib^3f(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)^2 d^2} \\
& -\frac{ibf(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)\,d^2} \\
& -\frac{2b^4f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^2 d^2} \\
& -\frac{2b^4f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^2 d^2} \\
& +\frac{b^4f(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{a(a^2+b^2)^2 d^2} \\
& -\frac{f(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{ad^2} \\
& +\frac{f(e+fx)\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{ad^2} \\
& -\frac{2ib^3f^2\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{(a^2+b^2)^2 d^3}
\end{aligned}
$$

output
```
-1/2*f^2*polylog(3,exp(2*d*x+2*c))/a/d^3-2*(f*x+e)^2*arctanh(exp(2*d*x+2*c
))/a/d+1/2*f^2*polylog(3,-exp(2*d*x+2*c))/a/d^3-1/2*(f*x+e)^2*tanh(d*x+c)^
2/a/d-2*b^4*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+
b^2)^2/d^2-2*b^4*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/
(a^2+b^2)^2/d^2-2*I*b^3*f*(f*x+e)*polylog(2,I*exp(d*x+c))/(a^2+b^2)^2/d^2-
I*b*f*(f*x+e)*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2+b*f^2*arctan(sinh(d*x+
c))/(a^2+b^2)/d^3+1/2*f^2*x^2/a/d+e*f*x/a/d-f*(f*x+e)*tanh(d*x+c)/a/d^2+I*
b*f^2*polylog(3,I*exp(d*x+c))/(a^2+b^2)/d^3-b*f*(f*x+e)*sech(d*x+c)/(a^2+b
^2)/d^2+b^4*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/a/(a^2+b^2)^2/d-b^2*f^2*ln(cosh
(d*x+c))/a/(a^2+b^2)/d^3-b^4*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2
)))/a/(a^2+b^2)^2/d-b^4*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a
/(a^2+b^2)^2/d+2*I*b^3*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b^2)^2/d^2+
I*b*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^2+b^4*f*(f*x+e)*polylog
(2,-exp(2*d*x+2*c))/a/(a^2+b^2)^2/d^2+2*I*b^3*f^2*polylog(3,I*exp(d*x+c))/
(a^2+b^2)^2/d^3+b^2*f*(f*x+e)*tanh(d*x+c)/a/(a^2+b^2)/d^2-f*(f*x+e)*polylo
g(2,-exp(2*d*x+2*c))/a/d^2+f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^2-1/2*b
^2*(f*x+e)^2*sech(d*x+c)^2/a/(a^2+b^2)/d-1/2*b*(f*x+e)^2*sech(d*x+c)*tanh(
d*x+c)/(a^2+b^2)/d+2*b^4*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/
a/(a^2+b^2)^2/d^3+2*b^4*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a
/(a^2+b^2)^2/d^3-2*I*b^3*f^2*polylog(3,-I*exp(d*x+c))/(a^2+b^2)^2/d^3-I...
```

3.445.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4072 vs. $2(1185) = 2370$.

Time = 13.16 (sec) , antiderivative size = 4072, normalized size of antiderivative = 3.44

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]
),x]
```

output
```
-1/3*(E^(2*c)*((2*(e + f*x)^3)/(E^(2*c)*f) - (3*(1 - E^(-2*c))*(e + f*x)^2
*Log[1 - E^(-c - d*x)])/d - (3*(1 - E^(-2*c))*(e + f*x)^2*Log[1 + E^(-c -
d*x)])/d + (6*(-1 + E^(2*c))*f*(d*(e + f*x)*PolyLog[2, -E^(-c - d*x)] + f*
PolyLog[3, -E^(-c - d*x)]))/(d^3*E^(2*c)) + (6*(-1 + E^(2*c))*f*(d*(e + f*
x)*PolyLog[2, E^(-c - d*x)] + f*PolyLog[3, E^(-c - d*x)]))/(d^3*E^(2*c))))
/(a*(-1 + E^(2*c))) - (-12*a^3*d^3*e^2*E^(2*c)*x - 24*a*b^2*d^3*e^2*E^(2*c
)*x + 12*a^3*d*E^(2*c)*f^2*x + 12*a*b^2*d*E^(2*c)*f^2*x - 12*a^3*d^3*e*E^(
2*c)*f*x^2 - 24*a*b^2*d^3*e*E^(2*c)*f*x^2 - 4*a^3*d^3*E^(2*c)*f^2*x^3 - 8*
a*b^2*d^3*E^(2*c)*f^2*x^3 + 6*a^2*b*d^2*e^2*ArcTan[E^(c + d*x)] + 18*b^3*d
^2*e^2*ArcTan[E^(c + d*x)] + 6*a^2*b*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] +
 18*b^3*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] - 12*a^2*b*f^2*ArcTan[E^(c + d
*x)] - 12*b^3*f^2*ArcTan[E^(c + d*x)] - 12*a^2*b*E^(2*c)*f^2*ArcTan[E^(c +
 d*x)] - 12*b^3*E^(2*c)*f^2*ArcTan[E^(c + d*x)] + (6*I)*a^2*b*d^2*e*f*x*Lo
g[1 - I*E^(c + d*x)] + (18*I)*b^3*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)
*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (18*I)*b^3*d^2*e*E^(2*c)
*f*x*Log[1 - I*E^(c + d*x)] + (3*I)*a^2*b*d^2*f^2*x^2*Log[1 - I*E^(c + d*x
)] + (9*I)*b^3*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*a^2*b*d^2*E^(2*c
)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (9*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I
*E^(c + d*x)] - (6*I)*a^2*b*d^2*e*f*x*Log[1 + I*E^(c + d*x)] - (18*I)*b^3*
d^2*e*f*x*Log[1 + I*E^(c + d*x)] - (6*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 ...
```

3.445.3 Rubi [A] (verified)

Time = 4.52 (sec) , antiderivative size = 993, normalized size of antiderivative = 0.84, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6123, 5985, 27, 6107, 6107, 6095, 2620, 3011, 2720, 7143, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^2\text{csch}(c+dx)\text{sech}^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-2f\int \frac{1}{2}(e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-f\int (e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 6107}$$

$$\frac{-f\int (e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \text{ 6107}$$

$$\frac{-f\int (e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \text{ 6095}$$

$$\frac{-f\int (e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a\cdots}{a^2+b^2}\right)}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \text{ 2620}$$

3.445. $\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\int(e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{\int(e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\cdots\right.}{bd}\right.}{a^2+b^2}}{a}\right.}{a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{-f\int(e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-\frac{be^{c}\cdots}{a+\sqrt{\cdots}}\right.}{d}\right.}{bd}\right.}{a^2+b^2}\right.}{a^2+b^2}\right.}{}$$

$$\Big\downarrow \text{2720}$$

3.445. $\displaystyle \int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\int(e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{b^2\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right.}{bd}\right.}{a^2+b^2}}{}\right.$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{-f\int(e+fx)\left(\frac{2\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{d}\right)dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{b^2\left(\frac{\int(e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c}}{a+\sqrt{}}\right)}{d^2}\right.}{}\right.}{}\right)}{a^2+b^2}\right.$$

$$\Big\downarrow \text{ 7292}$$

3.445. $\displaystyle\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\int \frac{(e+fx)(2\log(\tanh(c+dx))-\tanh^2(c+dx))}{d}dx-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{b^2\left(\frac{\int(e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c}}{a+\sqrt{}}\right)}{d^2}\right.}{}\right.}{}\right.}{a^2+b^2}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{f\int(e+fx)(2\log(\tanh(c+dx))-\tanh^2(c+dx))dx}{d}-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{b^2\left(\frac{\int(e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c}}{a+\sqrt{}}\right)}{d^2}\right.}{}\right.}{}\right.}{a^2+b^2}\right.$$

$$\Big\downarrow \; 7293$$

3.445. $\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{f\int \left(2(e+fx)\log(\tanh(c+dx))-(e+fx)\tanh^2(c+dx)\right)dx}{d}-\frac{(e+fx)^2\tanh^2(c+dx)}{2d}+\frac{(e+fx)^2\log(\tanh(c+dx))}{d}-$$

$$a$$

$$b\left(\frac{b^2\left(\frac{\int\left(a(e+fx)^2\operatorname{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}}{}\right.}{}\right.}{}\right.}{a^2+b^2}\right.$$

$\downarrow$ 2009

$$-\frac{\tanh^2(c+dx)(e+fx)^2}{2d}+\frac{\log(\tanh(c+dx))(e+fx)^2}{d}-\frac{f\left(\frac{2\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^2}{f}+\frac{\log(\tanh(c+dx))(e+fx)^2}{f}-\frac{(e+fx)^2}{2f}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2d}\right.}{d}\right.}{}$$

$$a$$

$$b\left(\frac{\left(\frac{\left(-\frac{(e+fx)^3}{3bf}+\frac{\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+fx)^2}{bd}+\frac{\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)(e+fx)^2}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\right.}{a^2+b^2}\right.}{}\right.$$

3.445. $\displaystyle\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(((e + f*x)^2*Log[Tanh[c + d*x]])/d - ((e + f*x)^2*Tanh[c + d*x]^2)/(2*d)
- (f*(-1/2*(e + f*x)^2/f + (2*(e + f*x)^2*ArcTanh[E^(2*c + 2*d*x)])/f - (f
*Log[Cosh[c + d*x]])/d^2 + ((e + f*x)^2*Log[Tanh[c + d*x]])/f + ((e + f*x)
*PolyLog[2, -E^(2*c + 2*d*x)])/d - ((e + f*x)*PolyLog[2, E^(2*c + 2*d*x)])
/d - (f*PolyLog[3, -E^(2*c + 2*d*x)])/(2*d^2) + (f*PolyLog[3, E^(2*c + 2*d
*x)])/(2*d^2) + ((e + f*x)*Tanh[c + d*x])/d))/d)/a - (b*((b^2*((b^2*(-1/3*
(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2
))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^
2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3)/(3*f) + (2*a*(e + f*x)^2*ArcTan
[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/d - ((2*I)*a*f
*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((2*I)*a*f*(e + f*x)*PolyLo
g[2, I*E^(c + d*x)])/d^2 - (b*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/d^
2 + ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/d^3 - ((2*I)*a*f^2*PolyLog[
3, I*E^(c + d*x)])/d^3 + (b*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*d^3))/(a^
2 + b^2)))/(a^2 + b^2) + ((a*(e + f*x)^2*ArcTan[E^(c + d*x)])/d - (a*f^2*A
rcTan[Sinh[c + d*x]])/d^3 + (b*f^2*Log[Cosh[c + d*x]])/d^3 - (I*a*f*(e ...
```

3.445.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.445. $\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

3.445. $\int \frac{(e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.445.4 Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c) \operatorname{sech}(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.445.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 16670 vs. 2(1097) = 2194.

Time = 0.56 (sec) , antiderivative size = 16670, normalized size of antiderivative = 14.07

$$\int \frac{(e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.445.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.445.7 Maxima [F]

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-a^2*b*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*
b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^
2 + b^4*d^2), x) - 3*b^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d
*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*
d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^3*d^2*f^2*integrate(x^2/(a^4*d^2*
e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c)
+ a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 4*a*b^2*d^2*f^2*integrate(x^2/(
a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x
 + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a^2*b*d^2*e*f*integra
te(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c)
+ b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 6*b^3
*d^2*e*f*integrate(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*
e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*
d^2), x) + 4*a^3*d^2*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*
d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 +
b^4*d^2), x) + 8*a*b^2*d^2*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^
2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*
d^2 + b^4*d^2), x) - a^3*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) -
log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) - a*b^2*f^2*(2*(d*
x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 +...
```

3.445.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.445.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)^3\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.446 $\int \frac{(e+fx)\mathbf{csch}(c+dx)\mathbf{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.446.1 Optimal result

Integrand size = 32, antiderivative size = 746

$$
\begin{aligned}
\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx
&= \frac{fx}{2ad} - \frac{2b^3(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} - \frac{b(e+fx)\arctan\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{2fx\text{arctanh}\left(e^{2c+2dx}\right)}{ad} \\
&- \frac{b^4(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^2\,d} - \frac{b^4(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^2\,d} \\
&+ \frac{b^4(e+fx)\log\left(1+e^{2(c+dx)}\right)}{a\,(a^2+b^2)^2\,d} - \frac{fx\log(\tanh(c+dx))}{ad} + \frac{(e+fx)\log(\tanh(c+dx))}{ad} \\
&+ \frac{ib^3f\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{(a^2+b^2)^2\,d^2} + \frac{ibf\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2\,(a^2+b^2)\,d^2} - \frac{ib^3f\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{(a^2+b^2)^2\,d^2} \\
&- \frac{ibf\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2\,(a^2+b^2)\,d^2} - \frac{b^4f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^2\,d^2} - \frac{b^4f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^2\,d^2} \\
&+ \frac{b^4f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2a\,(a^2+b^2)^2\,d^2} - \frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2ad^2} + \frac{f\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2ad^2} \\
&- \frac{bf\text{sech}(c+dx)}{2\,(a^2+b^2)\,d^2} - \frac{b^2(e+fx)\text{sech}^2(c+dx)}{2a\,(a^2+b^2)\,d} - \frac{f\tanh(c+dx)}{2ad^2} + \frac{b^2f\tanh(c+dx)}{2a\,(a^2+b^2)\,d^2} \\
&- \frac{b(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{2\,(a^2+b^2)\,d} - \frac{(e+fx)\tanh^2(c+dx)}{2ad}
\end{aligned}
$$

output
```
1/2*f*x/a/d-2*b^3*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)^2/d-b*(f*x+e)*arcta
n(exp(d*x+c))/(a^2+b^2)/d-2*f*x*arctanh(exp(2*d*x+2*c))/a/d+b^4*(f*x+e)*ln
(1+exp(2*d*x+2*c))/a/(a^2+b^2)^2/d-b^4*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/a/(a^2+b^2)^2/d-b^4*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))/a/(a^2+b^2)^2/d-f*x*ln(tanh(d*x+c))/a/d+(f*x+e)*ln(tanh(d*x+c))/a/d-1
/2*I*b*f*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2-I*b^3*f*polylog(2,I*exp(d*x
+c))/(a^2+b^2)^2/d^2+I*b^3*f*polylog(2,-I*exp(d*x+c))/(a^2+b^2)^2/d^2+1/2*
I*b*f*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^2+1/2*b^4*f*polylog(2,-exp(2*d*
x+2*c))/a/(a^2+b^2)^2/d^2-1/2*f*polylog(2,-exp(2*d*x+2*c))/a/d^2+1/2*f*pol
ylog(2,exp(2*d*x+2*c))/a/d^2-b^4*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/a/(a^2+b^2)^2/d^2-b^4*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))
/a/(a^2+b^2)^2/d^2-1/2*b*f*sech(d*x+c)/(a^2+b^2)/d^2-1/2*b^2*(f*x+e)*sech(
d*x+c)^2/a/(a^2+b^2)/d-1/2*f*tanh(d*x+c)/a/d^2+1/2*b^2*f*tanh(d*x+c)/a/(a^
2+b^2)/d^2-1/2*b*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a^2+b^2)/d-1/2*(f*x+e)*t
anh(d*x+c)^2/a/d
```

3.446.2 Mathematica [A] (verified)

Time = 10.91 (sec) , antiderivative size = 1080, normalized size of antiderivative = 1.45

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{\frac{1}{2}d^2fx^2+de(c+dx)+(de-cf+f(c+dx))\log\left(1-e^{-c-dx}\right)+(de-cf+f(c+dx))\log\left(1+e^{-c-dx}\right)}{ad^2}$$

$$-\frac{b^4\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\operatorname{arctanh}\left(\frac{a+be}{\sqrt{a}}\right.}{(-a^2-b^2)^{3/2}}\right.}{}$$

$$-\frac{-2a^3de(c+dx)-4ab^2de(c+dx)+2a^3cf(c+dx)+4ab^2cf(c+dx)-a^3f(c+dx)^2-2ab^2f(c+dx)^2}{}$$

$$+\frac{\operatorname{sech}(c+dx)(-bf-af\sinh(c+dx))}{2(a^2+b^2)d^2}$$

$$+\frac{\operatorname{sech}^2(c+dx)(ade-acf+af(c+dx)-bde\sinh(c+dx)+bcf\sinh(c+dx)-bf(c+dx)\sinh(c+dx}{2(a^2+b^2)d^2}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),
x]
```

output
```
((d^2*f*x^2)/2 + d*e*(c + d*x) + (d*e - c*f + f*(c + d*x))*Log[1 - E^(-c -
 d*x)] + (d*e - c*f + f*(c + d*x))*Log[1 + E^(-c - d*x)] - f*PolyLog[2, -E
^(-c - d*x)] - f*PolyLog[2, E^(-c - d*x)])/(a*d^2) - (b^4*(-2*d*e*(c + d*x
) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a +
 b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2
+ b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(
3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*
(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2
*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-1 +
E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(2*a*(a^2 +
b^2)^2*d^2) - (-2*a^3*d*e*(c + d*x) - 4*a*b^2*d*e*(c + d*x) + 2*a^3*c*f*(c
 + d*x) + 4*a*b^2*c*f*(c + d*x) - a^3*f*(c + d*x)^2 - 2*a*b^2*f*(c + d*x)^
2 + 2*a^2*b*d*e*ArcTan[E^(c + d*x)] + 6*b^3*d*e*ArcTan[E^(c + d*x)] - 2*a^
2*b*c*f*ArcTan[E^(c + d*x)] - 6*b^3*c*f*ArcTan[E^(c + d*x)] + I*a^2*b*f*(c
 + d*x)*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*f*(c + d*x)*Log[1 - I*E^(c + d*
x)] - I*a^2*b*f*(c + d*x)*Log[1 + I*E^(c + d*x)] - (3*I)*b^3*f*(c + d*x)*L
og[1 + I*E^(c + d*x)] + 2*a^3*d*e*Log[1 + E^(2*(c + d*x))] + 4*a*b^2*d*e*L
og[1 + E^(2*(c + d*x))] - 2*a^3*c*f*Log[1 + E^(2*(c + d*x))] - 4*a*b^2*c*f
*Log[1 + E^(2*(c + d*x))] + 2*a^3*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] ...
```

3.446.3 Rubi [A] (verified)

Time = 2.60 (sec) , antiderivative size = 627, normalized size of antiderivative = 0.84, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {6123, 5985, 2009, 6107, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6123}$$

$$\frac{\int (e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 5985}$$

$$\frac{-f\int\left(\frac{\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{2d}\right)dx-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+fx)\log(\tanh(c+dx))}{d}}{a}-\frac{b\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 2009$$

$$\frac{-f\left(\frac{2x\text{arctanh}(e^{2c+2dx})}{d}+\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh}{2d}}{a}$$
$$\frac{b\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 6107$$

$$\frac{-f\left(\frac{2x\text{arctanh}(e^{2c+2dx})}{d}+\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh}{2d}}{a}$$
$$\frac{b\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \; 6107$$

$$\frac{-f\left(\frac{2x\text{arctanh}(e^{2c+2dx})}{d}+\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh}{2d}}{a}$$
$$\frac{b\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a^2+b^2}\right)}{a}$$

$$\Big\downarrow \; 6095$$

$$\frac{-f\left(\frac{2x\text{arctanh}(e^{2c+2dx})}{d}+\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh}{2d}}{a}$$
$$\frac{b\left(\frac{\int(e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a-b}{a^2+b^2}\right.}{a^2+b^2}\right.}{a}$$

3.446. $\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$

$\downarrow$ 2620

$$\frac{-f\left(\frac{2x\operatorname{arctanh}(e^{2c+2dx})}{d}+\frac{\operatorname{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\operatorname{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh}{2d}}{a}$$

$$\frac{b\left(\frac{\int(e+fx)\operatorname{sech}^3(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{b^2\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\cdots\right)}{a^2+b^2}}{a^2+b^2}\right)}{a}$$

$\downarrow$ 2715

$$\frac{-f\left(\frac{2x\operatorname{arctanh}(e^{2c+2dx})}{d}+\frac{\operatorname{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\operatorname{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh}{2d}}{a}$$

$$\frac{b\left(\frac{b^2\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\cdots\right)}{a^2+b^2}\right)}{a^2+b^2}\right)}{a}$$

$\downarrow$ 2838

$$\frac{-f\left(\frac{2x\operatorname{arctanh}(e^{2c+2dx})}{d}+\frac{\operatorname{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\operatorname{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh}{2d}}{a}$$

$$\frac{b\left(\frac{b^2\left(\frac{\int(e+fx)\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\cdots}{}\right)}{a^2+b^2}\right)}{a^2+b^2}\right)}{a}$$

3.446. $\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$\Big\downarrow$ 7293

$$\frac{-f\left(\frac{2x\text{arctanh}(e^{2c+2dx})}{d}+\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh}{2d}}{a}$$

$$b\left(\frac{b^2\left(\frac{\int(a(e+fx)\text{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e}{}\right.}{a^2+b^2}}{a^2+b^2}\right.$$

$$\overline{a}$$

$\Big\downarrow$ 2009

$$\frac{-f\left(\frac{2x\text{arctanh}(e^{2c+2dx})}{d}+\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh}{2d}}{a}$$

$$b\left(\frac{b^2\left(\frac{b^2\left(\frac{f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{2a(e+fx}{}\right.}{a^2+b^2}\right.$$

input
```
Int[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

3.446. $\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(((e + f*x)*Log[Tanh[c + d*x]])/d - ((e + f*x)*Tanh[c + d*x]^2)/(2*d) - f*
(-1/2*x/d + (2*x*ArcTanh[E^(2*c + 2*d*x)])/d + (x*Log[Tanh[c + d*x]])/d +
PolyLog[2, -E^(2*c + 2*d*x)]/(2*d^2) - PolyLog[2, E^(2*c + 2*d*x)]/(2*d^2)
 + Tanh[c + d*x]/(2*d^2)))/a - (b*((b^2*((b^2*(-1/2*(e + f*x)^2/(b*f) + ((
e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x
)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2 + b^2) + ((b*(e + f*x)^2)
/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)*Log[1 + E^(2
*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*a*f*PolyLo
g[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*d^2))/(a^
2 + b^2)))/(a^2 + b^2) + ((a*(e + f*x)*ArcTan[E^(c + d*x)])/d - ((I/2)*a*f
*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + ((I/2)*a*f*PolyLog[2, I*E^(c + d*x)])
/d^2 + (a*f*Sech[c + d*x])/(2*d^2) + (b*(e + f*x)*Sech[c + d*x]^2)/(2*d) -
 (b*f*Tanh[c + d*x])/(2*d^2) + (a*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(
2*d))/(a^2 + b^2)))/a
```

3.446.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.446. $\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.446.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2579 vs. 2(693) = 1386.

Time = 30.79 (sec) , antiderivative size = 2580, normalized size of antiderivative = 3.46

method	result	size
risch	Expression too large to display	2580

input
```
int((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
1/(a^2+b^2)/d*f*b^2/a*ln(exp(d*x+c)+1)*x-1/(a^2+b^2)/d^2*b^2*c*f/a*ln(exp(
d*x+c)-1)-4/d/(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x-4/d^2/(a^
2+b^2)*a^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c+1/d^2/(a^2+b^2)^2*c*f*b^4/
a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/2/d^2/(a^2+b^2)^(3/2)*c*b^2*f*ar
ctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d/(a^2+b^2)^2*f*b^4/a*ln
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+2*I/d^2/(a^2+b^
2)*a^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*c-2*I/d^2/(a^2+b^2)*a^2*f/(4*a
^2+4*b^2)*ln(1-I*exp(d*x+c))*b*c+1/2/d/(a^2+b^2)^(5/2)*b^4*e*arctanh(1/2*(
2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/2/d/(a^2+b^2)^(3/2)*e*b^2*arctanh(1
/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-4/d/(a^2+b^2)*a^3*e/(4*a^2+4*b^2)
*ln(1+exp(2*d*x+2*c))-1/d/(a^2+b^2)^2*b^4*e/a*ln(b*exp(2*d*x+2*c)+2*a*exp(
d*x+c)-b)-4/d^2/(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))-4/d^2/
(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))-1/d^2/(a^2+b^2)^2*b^4*
f/a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/(a^2+b
^2)/d^2*a*f*dilog(exp(d*x+c))+1/(a^2+b^2)/d^2*a*f*dilog(exp(d*x+c)+1)+1/(a
^2+b^2)/d*e*a*ln(exp(d*x+c)-1)+1/(a^2+b^2)/d*e*a*ln(exp(d*x+c)+1)+(-b*d*f*
x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2*c)-b*d*e*exp(3*d*x+3*c)+2*a*d*e*exp
(2*d*x+2*c)+b*d*f*x*exp(d*x+c)-b*f*exp(3*d*x+3*c)+a*f*exp(2*d*x+2*c)+b*d*e
*exp(d*x+c)-f*b*exp(d*x+c)+a*f)/d^2/(a^2+b^2)/(1+exp(2*d*x+2*c))^2+4/d^2/(
a^2+b^2)*c*a^2*f/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))+8/d^2/(a^2+b^2)*c*f...
```

3.446.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7645 vs. 2(676) = 1352.

Time = 0.49 (sec) , antiderivative size = 7645, normalized size of antiderivative = 10.25

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
Too large to include
```

3.446.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.446.7 Maxima [F]

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)\,\text{sech}\,(dx+c)^3}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + 2*a^3*b^2 +
a*b^4)*d) - (a^2*b + 3*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*
d) + (a^3 + 2*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d)
 + (b*e^(-d*x - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^
2 + 2*(a^2 + b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d) - lo
g(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))*e - f*(((b*d*x*e^
(3*c) + b*e^(3*c))*e^(3*d*x) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d*x) - (
b*d*x*e^c - b*e^c)*e^(d*x) - a)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^
2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x)
) - 16*integrate(-1/8*(a*b^4*x*e^(d*x + c) - b^5*x)/(a^5*b + 2*a^3*b^3 + a
*b^5 - (a^5*b*e^(2*c) + 2*a^3*b^3*e^(2*c) + a*b^5*e^(2*c))*e^(2*d*x) - 2*(
a^6*e^c + 2*a^4*b^2*e^c + a^2*b^4*e^c)*e^(d*x)), x) + 16*integrate(1/16*((
a^2*b*e^c + 3*b^3*e^c)*x*e^(d*x) - 2*(a^3 + 2*a*b^2)*x)/(a^4 + 2*a^2*b^2 +
 b^4 + (a^4*e^(2*c) + 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x) + 16
*integrate(1/16*x/(a*e^(d*x + c) + a), x) - 16*integrate(1/16*x/(a*e^(d*x
+ c) - a), x))
```

3.446.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.446.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\left(c+d\,x\right)^3\,\sinh\left(c+d\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.447 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.447.1 Optimal result

Integrand size = 27, antiderivative size = 160

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b^3\arctan(\sinh(c+dx))}{(a^2+b^2)^2 d} - \frac{b\arctan(\sinh(c+dx))}{2(a^2+b^2)d} - \frac{a(a^2+2b^2)\log(\cosh(c+dx))}{(a^2+b^2)^2 d} + \frac{\log(\sinh(c+dx))}{ad} - \frac{b^4\log(a+b\sinh(c+dx))}{a(a^2+b^2)^2 d} + \frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d}$$

output
```
-b^3*arctan(sinh(d*x+c))/(a^2+b^2)^2/d-1/2*b*arctan(sinh(d*x+c))/(a^2+b^2)
/d-a*(a^2+2*b^2)*ln(cosh(d*x+c))/(a^2+b^2)^2/d+ln(sinh(d*x+c))/a/d-b^4*ln(
a+b*sinh(d*x+c))/a/(a^2+b^2)^2/d+1/2*sech(d*x+c)^2*(a-b*sinh(d*x+c))/(a^2+
b^2)/d
```

3.447.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{ab(a^2+b^2)\arctan(\sinh(c+dx)) - 2(a^2+b^2)^2\log(\sinh(c+dx)) + a\left(a^3+2ab^2+(-b^2)^{3/2}\right)\log\left(\sqrt{-b^2}\right.}{}$$

input
```
Integrate[(Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(a*b*(a^2 + b^2)*ArcTan[Sinh[c + d*x]] - 2*(a^2 + b^2)^2*Log[Sinh[c +
 d*x]] + a*(a^3 + 2*a*b^2 + (-b^2)^(3/2))*Log[Sqrt[-b^2] - b*Sinh[c + d*x]
] + 2*b^4*Log[a + b*Sinh[c + d*x]] + a*(a^3 + 2*a*b^2 - (-b^2)^(3/2))*Log[
Sqrt[-b^2] + b*Sinh[c + d*x]] - a^2*(a^2 + b^2)*Sech[c + d*x]^2 + a*b*(a^2
 + b^2)*Sech[c + d*x]*Tanh[c + d*x])/(a*(a^2 + b^2)^2*d)
```

3.447.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3042, 26, 3316, 26, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)\cos(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\cos(ic+idx)^3\sin(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{ib^3\int -\frac{i\operatorname{csch}(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \; 26$$

$$\frac{b^3 \int \frac{\operatorname{csch}(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{b^4 \int \frac{\operatorname{csch}(c+dx)}{b(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \; 615$$

$$\frac{b^4 \int \left(\frac{\operatorname{csch}(c+dx)}{ab^5} - \frac{1}{a(a^2+b^2)^2(a+b\sinh(c+dx))} + \frac{-b^4-a(a^2+2b^2)\sinh(c+dx)b}{b^4(a^2+b^2)^2\left(\sinh^2(c+dx)b^2+b^2\right)} + \frac{-b^2-a\sinh(c+dx)b}{b^2(a^2+b^2)\left(\sinh^2(c+dx)b^2+b^2\right)^2}\right)d(b\sinh(c+$$

$$\Big\downarrow \; 2009$$

$$\frac{b^4 \left(-\frac{\arctan(\sinh(c+dx))}{b(a^2+b^2)^2} - \frac{\arctan(\sinh(c+dx))}{2b^3(a^2+b^2)} + \frac{a-b\sinh(c+dx)}{2b^2(a^2+b^2)\left(b^2\sinh^2(c+dx)+b^2\right)} - \frac{\log(a+b\sinh(c+dx))}{a(a^2+b^2)^2} - \frac{a(a^2+2b^2)\log\left(b^2\sinh^2(c+d\right.}{2b^4(a^2+b^2)^2}\right.}{d}$$

input
```
Int[(Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^4*(-(ArcTan[Sinh[c + d*x]]/(b*(a^2 + b^2)^2)) - ArcTan[Sinh[c + d*x]]/(
2*b^3*(a^2 + b^2)) + Log[b*Sinh[c + d*x]]/(a*b^4) - Log[a + b*Sinh[c + d*x
]]/(a*(a^2 + b^2)^2) - (a*(a^2 + 2*b^2)*Log[b^2 + b^2*Sinh[c + d*x]^2])/(2
*b^4*(a^2 + b^2)^2) + (a - b*Sinh[c + d*x])/(2*b^2*(a^2 + b^2)*(b^2 + b^2*
Sinh[c + d*x]^2))))/d
```

3.447.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.447. $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

3.447.4 Maple [A] (verified)

Time = 15.01 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.44

method	result
derivativedivides	$-\frac{\frac{2\left(\frac{\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(a^3+ab^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{\left(2a^3+4ab^2\right)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}{4}\right.}{a^4+2a^2b^2+b^4}}{d}$
default	$-\frac{\frac{2\left(\frac{\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(a^3+ab^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{\left(2a^3+4ab^2\right)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}{4}\right.}{a^4+2a^2b^2+b^4}}{d}$
risch	$\frac{2d^2a^3x}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2d\,a^3c}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{4a\,b^2d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{4a\,b^2dc}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2x}{a}-\frac{2c}{da}+$

input
```
int(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(a^4+2*a^2*b^2+b^4)*(((-1/2*a^2*b-1/2*b^3)*tanh(1/2*d*x+1/2*c)^3+(
a^3+a*b^2)*tanh(1/2*d*x+1/2*c)^2+(1/2*a^2*b+1/2*b^3)*tanh(1/2*d*x+1/2*c))/
(1+tanh(1/2*d*x+1/2*c)^2)^2+1/4*(2*a^3+4*a*b^2)*ln(1+tanh(1/2*d*x+1/2*c)^2
)+1/2*(a^2*b+3*b^3)*arctan(tanh(1/2*d*x+1/2*c)))-b^4/a/(a^4+2*a^2*b^2+b^4)
*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a)+1/a*ln(tanh(1/2*d*x
+1/2*c)))
```

3.447.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1279 vs. $2(157) = 314$.

Time = 0.44 (sec) , antiderivative size = 1279, normalized size of antiderivative = 7.99

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-((a^3*b + a*b^3)*cosh(d*x + c)^3 + (a^3*b + a*b^3)*sinh(d*x + c)^3 - 2*(a
^4 + a^2*b^2)*cosh(d*x + c)^2 - (2*a^4 + 2*a^2*b^2 - 3*(a^3*b + a*b^3)*cos
h(d*x + c))*sinh(d*x + c)^2 + ((a^3*b + 3*a*b^3)*cosh(d*x + c)^4 + 4*(a^3*
b + 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b + 3*a*b^3)*sinh(d*x +
c)^4 + a^3*b + 3*a*b^3 + 2*(a^3*b + 3*a*b^3)*cosh(d*x + c)^2 + 2*(a^3*b +
3*a*b^3 + 3*(a^3*b + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3*b
 + 3*a*b^3)*cosh(d*x + c)^3 + (a^3*b + 3*a*b^3)*cosh(d*x + c))*sinh(d*x +
c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (a^3*b + a*b^3)*cosh(d*x + c)
+ (b^4*cosh(d*x + c)^4 + 4*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*sinh(d*
x + c)^4 + 2*b^4*cosh(d*x + c)^2 + b^4 + 2*(3*b^4*cosh(d*x + c)^2 + b^4)*s
inh(d*x + c)^2 + 4*(b^4*cosh(d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c)
)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + ((a^4 + 2
*a^2*b^2)*cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c
)^3 + (a^4 + 2*a^2*b^2)*sinh(d*x + c)^4 + a^4 + 2*a^2*b^2 + 2*(a^4 + 2*a^2
*b^2)*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + 3*(a^4 + 2*a^2*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 4*((a^4 + 2*a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 2*
a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c)
- sinh(d*x + c))) - ((a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^4 + 4*(a^4 + 2*
a^2*b^2 + b^4)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*sin
h(d*x + c)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(d...
```

3.447.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.447.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.66

$$\begin{aligned}\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx=&-\frac{b^4\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^5+2\,a^3b^2+ab^4)d}\\&+\frac{(a^2b+3\,b^3)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d}\\&-\frac{(a^3+2\,ab^2)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d}\\&-\frac{be^{(-dx-c)}-2\,ae^{(-2\,dx-2\,c)}-be^{(-3\,dx-3\,c)}}{(a^2+b^2+2\,(a^2+b^2)e^{(-2\,dx-2\,c)}+(a^2+b^2)e^{(-4\,dx-4\,c)})d}\\&+\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}\end{aligned}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + 2*a^3*b^2 + a
*b^4)*d) + (a^2*b + 3*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d
) - (a^3 + 2*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d)
- (b*e^(-d*x - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2
 + 2*(a^2 + b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d) + log
(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d)
```

3.447.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(157) = 314$.

Time = 0.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.14

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{4\,b^5\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^5b+2\,a^3b^3+ab^5} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(a^2b+3\,b^3\right)}{a^4+2\,a^2b^2+b^4} + \frac{2\,\left(a^3+2\,ab^2\right)\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2 \cdots\right.}{a^4+2\,a^2b^2+b^4}}{}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(4*b^5*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^5*b + 2*a^3*
b^3 + a*b^5) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a^
2*b + 3*b^3)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^3 + 2*a*b^2)*log((e^(d*x + c)
- e^(-d*x - c))^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) - 4*log(abs(e^(d*x + c) - e
^(-d*x - c)))/a - 2*(a^3*(e^(d*x + c) - e^(-d*x - c))^2 + 2*a*b^2*(e^(d*x
+ c) - e^(-d*x - c))^2 - 2*a^2*b*(e^(d*x + c) - e^(-d*x - c)) - 2*b^3*(e^(
d*x + c) - e^(-d*x - c)) + 8*a^3 + 12*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^
(d*x + c) - e^(-d*x - c))^2 + 4)))/d
```

3.447.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\,\sinh\left(c+d\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.448 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.448.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.448.2 Mathematica [N/A]

Not integrable

Time = 102.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Csch[c + d*x]*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

3.448.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{ 6125}$$
$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.448.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.448.4 Maple [N/A] (verified)

Not integrable

Time = 0.91 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(dx+c\right)\operatorname{sech}\left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.448.5 Fricas [N/A]

Not integrable

Time = 19.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)\operatorname{sech}\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(csch(d*x + c)*sech(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

3.448.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.448.7 Maxima [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 1214, normalized size of antiderivative = 35.71

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)\operatorname{sech}\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(a*f + (b*d*f*x*e^(3*c) + (d*e - f)*b*e^(3*c))*e^(3*d*x) - (2*a*d*f*x*e^(
2*c) + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) - (b*d*f*x*e^c + (d*e + f)*b*e^c)*
e^(d*x))/(a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*
(a^2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^(4*c) + b^2*d^2*e^2*e^(4*c)
 + (a^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*e*f*e^(4*c
) + b^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + 2*(a^2*d^2*e^2*e^(2*c) + b^2*d^2*e
^2*e^(2*c) + (a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*
e*f*e^(2*c) + b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) + 16*integrate(-1/8*(a*b^
4*e^(d*x + c) - b^5)/(a^5*b*e + 2*a^3*b^3*e + a*b^5*e + (a^5*b*f + 2*a^3*b
^3*f + a*b^5*f)*x - (a^5*b*e*e^(2*c) + 2*a^3*b^3*e*e^(2*c) + a*b^5*e*e^(2*
c) + (a^5*b*f*e^(2*c) + 2*a^3*b^3*f*e^(2*c) + a*b^5*f*e^(2*c))*x)*e^(2*d*x
) - 2*(a^6*e*e^c + 2*a^4*b^2*e*e^c + a^2*b^4*e*e^c + (a^6*f*e^c + 2*a^4*b^
2*f*e^c + a^2*b^4*f*e^c)*x)*e^(d*x)), x) - 16*integrate(-1/16*(2*(d^2*e^2
- f^2)*a^3 + 2*(2*d^2*e^2 - f^2)*a*b^2 + 2*(a^3*d^2*f^2 + 2*a*b^2*d^2*f^2)
*x^2 + 4*(a^3*d^2*e*f + 2*a*b^2*d^2*e*f)*x - ((d^2*e^2 - 2*f^2)*a^2*b*e^c
+ (3*d^2*e^2 - 2*f^2)*b^3*e^c + (a^2*b*d^2*f^2*e^c + 3*b^3*d^2*f^2*e^c)*x^
2 + 2*(a^2*b*d^2*e*f*e^c + 3*b^3*d^2*e*f*e^c)*x)*e^(d*x))/(a^4*d^2*e^3 + 2
*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^
2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2 + b^4*d^2*e*f^2)*x^2 +
 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^2*f)*x + (a^4*d^2*e...
```

3.448.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.448.9 Mupad [N/A]

Not integrable

Time = 12.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\,\sinh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.449 $\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.449.1 Optimal result

Integrand size = 32, antiderivative size = 601

$$
\begin{aligned}
&\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{6f(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^3\operatorname{csch}(c+dx)}{ad} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d} \\
&\quad + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d} - \frac{b(e+fx)^3\log\left(1-e^{2(c+dx)}\right)}{a^2d} \\
&\quad - \frac{6f^2(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^3} + \frac{6f^2(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^3} \\
&\quad + \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2} + \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2} \\
&\quad - \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^2d^2} + \frac{6f^3\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^4} - \frac{6f^3\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^4} \\
&\quad - \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3} - \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^3} \\
&\quad + \frac{3bf^2(e+fx)\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^2d^3} + \frac{6bf^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^4} \\
&\quad + \frac{6bf^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^4} - \frac{3bf^3\operatorname{PolyLog}\left(4,e^{2(c+dx)}\right)}{4a^2d^4}
\end{aligned}
$$

output
```
-6*f*(f*x+e)^2*arctanh(exp(d*x+c))/a/d^2-(f*x+e)^3*csch(d*x+c)/a/d-b*(f*x+
e)^3*ln(1-exp(2*d*x+2*c))/a^2/d+b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/a^2/d+b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d-6*
f^2*(f*x+e)*polylog(2,-exp(d*x+c))/a/d^3+6*f^2*(f*x+e)*polylog(2,exp(d*x+c
))/a/d^3-3/2*b*f*(f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a^2/d^2+3*b*f*(f*x+e)
^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^2+3*b*f*(f*x+e)^2*po
lylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^2+6*f^3*polylog(3,-exp(d*
x+c))/a/d^4-6*f^3*polylog(3,exp(d*x+c))/a/d^4+3/2*b*f^2*(f*x+e)*polylog(3,
exp(2*d*x+2*c))/a^2/d^3-6*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/a^2/d^3-6*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/a^2/d^3-3/4*b*f^3*polylog(4,exp(2*d*x+2*c))/a^2/d^4+6*b*f^3*polylog(
4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^4+6*b*f^3*polylog(4,-b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))/a^2/d^4
```

3.449.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2598 vs. $2(601) = 1202$.

Time = 11.11 (sec) , antiderivative size = 2598, normalized size of antiderivative = 4.32

$$\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
-(((e + f*x)^3*Csch[c])/(a*d)) + (2*d^3*e^2*(-1 + E^(2*c))*f*(b*d*e - 3*a*
f)*x + 2*d^3*e^2*(-1 + E^(2*c))*f*(b*d*e + 3*a*f)*x + b*d^4*(e + f*x)^4 -
6*d^2*e*(-1 + E^(2*c))*f^2*(b*d*e - 2*a*f)*x*Log[1 - E^(-c - d*x)] - 6*d^2
*(-1 + E^(2*c))*f^3*(b*d*e - a*f)*x^2*Log[1 - E^(-c - d*x)] - 2*b*d^3*(-1
+ E^(2*c))*f^4*x^3*Log[1 - E^(-c - d*x)] - 6*d^2*e*(-1 + E^(2*c))*f^2*(b*d
*e + 2*a*f)*x*Log[1 + E^(-c - d*x)] - 6*d^2*(-1 + E^(2*c))*f^3*(b*d*e + a*
f)*x^2*Log[1 + E^(-c - d*x)] - 2*b*d^3*(-1 + E^(2*c))*f^4*x^3*Log[1 + E^(-
c - d*x)] - 2*d^2*e^2*(-1 + E^(2*c))*f*(b*d*e - 3*a*f)*Log[1 - E^(c + d*x)
] - 2*d^2*e^2*(-1 + E^(2*c))*f*(b*d*e + 3*a*f)*Log[1 + E^(c + d*x)] + 6*d*
e*(-1 + E^(2*c))*f^2*(b*d*e + 2*a*f)*PolyLog[2, -E^(-c - d*x)] + 12*d*(-1
+ E^(2*c))*f^3*(b*d*e + a*f)*x*PolyLog[2, -E^(-c - d*x)] + 6*b*d^2*(-1 + E
^(2*c))*f^4*x^2*PolyLog[2, -E^(-c - d*x)] + 6*d*e*(-1 + E^(2*c))*f^2*(b*d*
e - 2*a*f)*PolyLog[2, E^(-c - d*x)] + 12*d*(-1 + E^(2*c))*f^3*(b*d*e - a*f
)*x*PolyLog[2, E^(-c - d*x)] + 6*b*d^2*(-1 + E^(2*c))*f^4*x^2*PolyLog[2, E
^(-c - d*x)] + 12*(-1 + E^(2*c))*f^3*(b*d*e + a*f)*PolyLog[3, -E^(-c - d*x
)] + 12*b*d*(-1 + E^(2*c))*f^4*x*PolyLog[3, -E^(-c - d*x)] - 12*(-1 + E^(2
*c))*f^3*(-(b*d*e) + a*f)*PolyLog[3, E^(-c - d*x)] + 12*b*d*(-1 + E^(2*c))
*f^4*x*PolyLog[3, E^(-c - d*x)] + 12*b*(-1 + E^(2*c))*f^4*PolyLog[4, -E^(-
c - d*x)] + 12*b*(-1 + E^(2*c))*f^4*PolyLog[4, E^(-c - d*x)])/(2*a^2*d^4*(
-1 + E^(2*c))*f) - (b*(4*e^3*E^(2*c)*x + 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2...
```

3.449.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.98 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.13, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6121, 5975, 3042, 26, 4670, 3011, 2720, 6103, 3042, 26, 4201, 2620, 3011, 6095, 2620, 3011, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6121}$$

$$\frac{\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5975}$$

$$\frac{\frac{3f\int (e+fx)^2\text{csch}(c+dx)dx}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3f\int i(e+fx)^2\csc(ic+idx)dx}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\int (e+fx)^2\csc(ic+idx)dx}{d}}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(\frac{2if\int (e+fx)\log\left(1-e^{c+dx}\right)dx}{d}-\frac{2if\int (e+fx)\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c-}\right.}{d}\right.}{d}\right.}{d}}{a}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c-}}{d^2}\right.}{d}\right.}{d}}{a}$$

$$\Big\downarrow \text{6103}$$

3.449. $\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{\int (e+fx)^3\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+$$

$$\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c-}}{d^2}\right.}{d}\right.}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c-}}{d^2}\right.}{d}\right.}{d}}{a}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c-}}{d^2}\right.}{d}\right.}{d}}{a}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int (e+fx)^3\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{4201}$$

$$\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c-}}{d^2}\right.}{d}\right.}{d}}{a}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)^3}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^4}{4f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

3.449. $\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3ifb\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c-}}{d^2}\right.}{d}\right.}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\int(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^4}{4f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 3011$$

$$-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3ifb\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c-}}{d^2}\right.}{d}\right.}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right.}{2d}\right.\right.}{a}\right.}{a}$$

$$\Big\downarrow \; 6095$$

$$-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3ifb\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c-}}{d^2}\right.}{d}\right.}{d}$$

$$\frac{b\left(-\frac{b\left(\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{a}-\frac{i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-}\right.}{d}\right.}{}\right.\right.}{a}\right.}{a}$$

$$\Big\downarrow \; 2620$$

3.449. $\displaystyle\int\frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c-}}{d^2}\right.}{d}\right.}{d}$$
$$a$$

$$b\left(-\frac{b\left(-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+}{4b}\right.}{a}\right.$$
$$a$$

$$\downarrow 3011$$

$$-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c-}}{d^2}\right.}{d}\right.}{d}$$
$$a$$

$$b\left(-\frac{b\left(-\frac{3f\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\text{Po}}{}\right.}{bd}\right.}{a}\right.$$

$$\downarrow 7143$$

3.449. $\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(\frac{2i(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,e^{c+dx}\right)}{d^2}-\cdots\right.}{\cdots}\right.}{d}$$

$$a$$

$$b\left(-\frac{b\left(-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\,\text{Po}\cdots}{\cdots}\right.}{bd}\right.}{a}\right.$$

$\downarrow$ 7163

$$-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(\frac{2i(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,e^{c+dx}\right)}{d^2}-\cdots\right.}{\cdots}\right.}{d}$$

$$a$$

$$b\left(-\frac{b\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLo}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 2720

3.449. $\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(\frac{2i(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\,\text{PolyLog}\left(3,e^{c+dx}\right)}{d^2}-\cdots\right.}{\cdots}\right.}{d}$$

$$\frac{b\left(-\frac{b\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{a}$$

$\downarrow$ 7143

3.449. $\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(\frac{2i(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{d^2}-\right.}{}\right.}{d}}{a}$$

$$b\left(-\frac{b\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3\right.}{d}\right.}{}\right.}{}\right.}{}\right.$$

input
```
Int[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.449. $\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(-(((e + f*x)^3*Csch[c + d*x])/d) + ((3*I)*f*(((2*I)*(e + f*x)^2*ArcTanh[E
^(c + d*x)])/d - ((2*I)*f*(-(((e + f*x)*PolyLog[2, -E^(c + d*x)])/d) + (f*
PolyLog[3, -E^(c + d*x)])/d^2))/d + ((2*I)*f*(-(((e + f*x)*PolyLog[2, E^(c
 + d*x)])/d) + (f*PolyLog[3, E^(c + d*x)])/d^2))/d))/d)/a - (b*(-((b*(-1/4
*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])
/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/d^2))/d))/(b*d)))/a) - (I*(((-1/4*I)*(e + f*x)^4)/f + (2*I)*(((e
+ f*x)^3*Log[1 + E^(2*c - I*Pi + 2*d*x)])/(2*d) - (3*f*(-1/2*((e + f*x)^2*
PolyLog[2, -E^(2*c - I*Pi + 2*d*x)])/d + (f*(((e + f*x)*PolyLog[3, -E^(2*c
 - I*Pi + 2*d*x)])/(2*d) - (f*PolyLog[4, -E^(2*c - I*Pi + 2*d*x)])/(4*d^2)
))/d))/(2*d))))/a))/a
```

3.449.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.449. $\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

3.449. $\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6121
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.449.4 Maple [F]

$$\int \frac{(fx+e)^3 \coth(dx+c)\operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.449.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4313 vs. $2(561) = 1122$.

Time = 0.36 (sec) , antiderivative size = 4313, normalized size of antiderivative = 7.18

$$\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-(2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cosh
(d*x + c) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f - (b*d^2*f^3*
x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)^2 - 2*(b*d^2*f^3*x^2 +
2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^3*x^
2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c)
 + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b + 1) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f - (b*d
^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)^2 - 2*(b*d^2*f^3
*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2
*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sinh(d*x + c)^2)*dilog((a*cosh(d
*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) - 3*(b*d^2*f^3*x^2 + b*d^2*e^2*f - 2*a*d*e*f^2 - (
b*d^2*f^3*x^2 + b*d^2*e^2*f - 2*a*d*e*f^2 + 2*(b*d^2*e*f^2 - a*d*f^3)*x)*c
osh(d*x + c)^2 - 2*(b*d^2*f^3*x^2 + b*d^2*e^2*f - 2*a*d*e*f^2 + 2*(b*d^2*e
*f^2 - a*d*f^3)*x)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^3*x^2 + b*d^2*e^
2*f - 2*a*d*e*f^2 + 2*(b*d^2*e*f^2 - a*d*f^3)*x)*sinh(d*x + c)^2 + 2*(b*d^
2*e*f^2 - a*d*f^3)*x)*dilog(cosh(d*x + c) + sinh(d*x + c)) - 3*(b*d^2*f^3*
x^2 + b*d^2*e^2*f + 2*a*d*e*f^2 - (b*d^2*f^3*x^2 + b*d^2*e^2*f + 2*a*d*e*f
^2 + 2*(b*d^2*e*f^2 + a*d*f^3)*x)*cosh(d*x + c)^2 - 2*(b*d^2*f^3*x^2 + b*d
^2*e^2*f + 2*a*d*e*f^2 + 2*(b*d^2*e*f^2 + a*d*f^3)*x)*cosh(d*x + c)*sin...
```

3.449.6 Sympy [F]

$$\int \frac{(e+fx)^3\coth(c+dx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3\coth{(c+dx)}\,\mathrm{csch}\,(c+dx)}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*coth(c + d*x)*csch(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

3.449.7 Maxima [F]

$$\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\coth{(dx+c)}\,\text{csch}{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
e^3*(2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*x - c)
 + b*e^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*l
og(e^(-d*x - c) - 1)/(a^2*d)) - 2*(f^3*x^3*e^c + 3*e*f^2*x^2*e^c + 3*e^2*f
*x*e^c)*e^(d*x)/(a*d*e^(2*d*x + 2*c) - a*d) - 3*e^2*f*log(e^(d*x + c) + 1)
/(a*d^2) + 3*e^2*f*log(e^(d*x + c) - 1)/(a*d^2) - (d^3*x^3*log(e^(d*x + c)
 + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6
*polylog(4, -e^(d*x + c)))*b*f^3/(a^2*d^4) - (d^3*x^3*log(-e^(d*x + c) + 1
) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polyl
og(4, e^(d*x + c)))*b*f^3/(a^2*d^4) - 3*(b*d*e^2*f + 2*a*e*f^2)*(d*x*log(e
^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 3*(b*d*e^2*f - 2*a*e*f^
2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) - 3*(b*d*e*f
^2 + a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*
polylog(3, -e^(d*x + c)))/(a^2*d^4) - 3*(b*d*e*f^2 - a*f^3)*(d^2*x^2*log(-
e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))/(
a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 + a*f^3)*d^3*x^3 + 6*(b*d^2*e
^2*f + 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2
 - a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f - 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) - int
egrate(-2*(b^2*f^3*x^3 + 3*b^2*e*f^2*x^2 + 3*b^2*e^2*f*x - (a*b*f^3*x^3*e^
c + 3*a*b*e*f^2*x^2*e^c + 3*a*b*e^2*f*x*e^c)*e^(d*x))/(a^2*b*e^(2*d*x + 2*
c) + 2*a^3*e^(d*x + c) - a^2*b), x)
```

3.449.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.449.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)^3}{\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x)^3)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)*(e + f*x)^3)/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.450 $\int \frac{(e+fx)^2 \coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.450.1 Optimal result

Integrand size = 32, antiderivative size = 419

$$\begin{aligned}
&\int \frac{(e+fx)^2 \coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{4f(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^2\operatorname{csch}(c+dx)}{ad} \\
&\quad + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d} \\
&\quad - \frac{b(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{a^2d} - \frac{2f^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^3} + \frac{2f^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^3} \\
&\quad + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2} \\
&\quad - \frac{bf(e+fx)\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{a^2d^2} - \frac{2bf^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3} \\
&\quad - \frac{2bf^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^3} + \frac{bf^2\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^2d^3}
\end{aligned}$$

output
```
-4*f*(f*x+e)*arctanh(exp(d*x+c))/a/d^2-(f*x+e)^2*csch(d*x+c)/a/d-b*(f*x+e)
^2*ln(1-exp(2*d*x+2*c))/a^2/d+b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/a^2/d+b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d-2*f^
2*polylog(2,-exp(d*x+c))/a/d^3+2*f^2*polylog(2,exp(d*x+c))/a/d^3-b*f*(f*x+
e)*polylog(2,exp(2*d*x+2*c))/a^2/d^2+2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)
/(a-(a^2+b^2)^(1/2)))/a^2/d^2+2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))/a^2/d^2+1/2*b*f^2*polylog(3,exp(2*d*x+2*c))/a^2/d^3-2*b*f^2
*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^3-2*b*f^2*polylog(3,-b
*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^3
```

3.450.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1735 vs. $2(419) = 838$.

Time = 10.38 (sec) , antiderivative size = 1735, normalized size of antiderivative = 4.14

$$\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
-(((e + f*x)^2*Csch[c])/(a*d)) + (3*d^2*e*(-1 + E^(2*c))*f*(b*d*e - 2*a*f)
*x + 3*d^2*e*(-1 + E^(2*c))*f*(b*d*e + 2*a*f)*x + 2*b*d^3*(e + f*x)^3 - 6*
d*(-1 + E^(2*c))*f^2*(b*d*e - a*f)*x*Log[1 - E^(-c - d*x)] - 3*b*d^2*(-1 +
 E^(2*c))*f^3*x^2*Log[1 - E^(-c - d*x)] - 6*d*(-1 + E^(2*c))*f^2*(b*d*e +
a*f)*x*Log[1 + E^(-c - d*x)] - 3*b*d^2*(-1 + E^(2*c))*f^3*x^2*Log[1 + E^(-
c - d*x)] - 3*d*e*(-1 + E^(2*c))*f*(b*d*e - 2*a*f)*Log[1 - E^(c + d*x)] -
3*d*e*(-1 + E^(2*c))*f*(b*d*e + 2*a*f)*Log[1 + E^(c + d*x)] + 6*(-1 + E^(2
*c))*f^2*(b*d*e + a*f)*PolyLog[2, -E^(-c - d*x)] + 6*b*d*(-1 + E^(2*c))*f^
3*x*PolyLog[2, -E^(-c - d*x)] - 6*(-1 + E^(2*c))*f^2*(-(b*d*e) + a*f)*Poly
Log[2, E^(-c - d*x)] + 6*b*d*(-1 + E^(2*c))*f^3*x*PolyLog[2, E^(-c - d*x)]
 + 6*b*(-1 + E^(2*c))*f^3*PolyLog[3, -E^(-c - d*x)] + 6*b*(-1 + E^(2*c))*f
^3*PolyLog[3, E^(-c - d*x)])/(3*a^2*d^3*(-1 + E^(2*c))*f) - (b*(6*e^2*E^(2
*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*A
rcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6
*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 -
 b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a +
 b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2
 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^
2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]
)/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d...
```

3.450.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.71 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.17, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.594$, Rules used = {6121, 5975, 3042, 26, 4670, 2715, 2838, 6103, 3042, 26, 4201, 2620, 3011, 2720, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6121}$$

$$\frac{\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5975}$$

$$\frac{\frac{2f\int (e+fx)\text{csch}(c+dx)dx}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2f\int i(e+fx)\csc(ic+idx)dx}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\int (e+fx)\csc(ic+idx)dx}{d}}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{if\int \log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{6103}$$

$$-\frac{b\left(\frac{\int (e+fx)^2\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

3.450. $\int \frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int-i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int(e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 4201}$$

$$\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{ 3011}$$

3.450. $\int\frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i}{}\right.}{a}\right.}{a}$$

$\downarrow$ 2720

$$\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}(}{2}\right.}{d}\right.\right.}{a}\right.}{a}$$

$\downarrow$ 6095

$$\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a}-\frac{i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c}\right.}{4d^2}\right.}{}\right.\right.}{}\right.}{a}$$

$\downarrow$ 2620

3.450. $\displaystyle\int\frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-$$

$$b\left(-\frac{b\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)}{3bf}\right.}{a}\right.$$

$$\Big\downarrow \text{3011}$$

$$\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-$$

$$b\left(-\frac{b\left(-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2}}\right.}{d}\right.}{bd}\right.}{a}\right.$$

$$\Big\downarrow \text{2720}$$

$$\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-$$

$$b\left(-\frac{b\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\right.}{bd}\right.}{a}\right.$$

$$\Big\downarrow \text{7143}$$

3.450. $\int \frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-$$

$$b\left(-\frac{b\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}\right.}{a}\right.$$

input
```
Int[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(((e + f*x)^2*Csch[c + d*x])/d) + ((2*I)*f*(((2*I)*(e + f*x)*ArcTanh[E^(
c + d*x)])/d + (I*f*PolyLog[2, -E^(c + d*x)])/d^2 - (I*f*PolyLog[2, E^(c +
 d*x)])/d^2))/d)/a - (b*(-((b*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2
, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[
2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/a) - (I*(((-1/3*I)*(e + f*
x)^3)/f + (2*I)*(((e + f*x)^2*Log[1 + E^(2*c - I*Pi + 2*d*x)])/(2*d) - (f*
(-1/2*((e + f*x)*PolyLog[2, -E^(2*c - I*Pi + 2*d*x)])/d + (f*PolyLog[3, -E
^(2*c - I*Pi + 2*d*x)])/(4*d^2)))/d)))/a))/a
```

3.450.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.450. $\int \frac{(e+fx)^2 \coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6121
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.450.4 Maple [F]

$$\int \frac{(fx+e)^2 \coth(dx+c) \operatorname{csch}(dx+c)}{a+b\sinh(dx+c)} dx$$

input
```
int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.450.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2528 vs. $2(391) = 782$.

Time = 0.32 (sec) , antiderivative size = 2528, normalized size of antiderivative = 6.03

$$\int \frac{(e+fx)^2 \coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-(2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cosh(d*x + c) + 2*(b*d*f^2
*x + b*d*e*f - (b*d*f^2*x + b*d*e*f)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*
e*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^2*x + b*d*e*f)*sinh(d*x + c)^2)*
dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x +
 c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b*d*f^2*x + b*d*e*f - (b*d*f^2
*x + b*d*e*f)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*e*f)*cosh(d*x + c)*sinh
(d*x + c) - (b*d*f^2*x + b*d*e*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b + 1) - 2*(b*d*f^2*x + b*d*e*f - a*f^2 - (b*d*f^2*x + b*d*e*f -
a*f^2)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*e*f - a*f^2)*cosh(d*x + c)*sin
h(d*x + c) - (b*d*f^2*x + b*d*e*f - a*f^2)*sinh(d*x + c)^2)*dilog(cosh(d*x
 + c) + sinh(d*x + c)) - 2*(b*d*f^2*x + b*d*e*f + a*f^2 - (b*d*f^2*x + b*d
*e*f + a*f^2)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*e*f + a*f^2)*cosh(d*x +
 c)*sinh(d*x + c) - (b*d*f^2*x + b*d*e*f + a*f^2)*sinh(d*x + c)^2)*dilog(-
cosh(d*x + c) - sinh(d*x + c)) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2 - (b
*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c)^2 - 2*(b*d^2*e^2 - 2*b*c
*d*e*f + b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*e^2 - 2*b*c*d*e*f
 + b*c^2*f^2)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2 -
 (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c)^2 - 2*(b*d^2*e^2 -...
```

3.450.6 Sympy [F]

$$\int \frac{(e+fx)^2 \coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2 \coth(c+dx)\,\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*coth(c + d*x)*csch(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

3.450.7 Maxima [F]

$$\int \frac{(e+fx)^2 \coth(c+dx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \coth(dx+c)\,\mathrm{csch}\,(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
e^2*(2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*x - c)
 + b*e^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*l
og(e^(-d*x - c) - 1)/(a^2*d)) - 2*(f^2*x^2*e^c + 2*e*f*x*e^c)*e^(d*x)/(a*d
*e^(2*d*x + 2*c) - a*d) - 2*e*f*log(e^(d*x + c) + 1)/(a*d^2) + 2*e*f*log(e
^(d*x + c) - 1)/(a*d^2) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(
d*x + c)) - 2*polylog(3, -e^(d*x + c)))*b*f^2/(a^2*d^3) - (d^2*x^2*log(-e^
(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*b*f
^2/(a^2*d^3) - 2*(b*d*e*f + a*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d
*x + c)))/(a^2*d^3) - 2*(b*d*e*f - a*f^2)*(d*x*log(-e^(d*x + c) + 1) + dil
og(e^(d*x + c)))/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f + a*f^2)*d^2*
x^2)/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f - a*f^2)*d^2*x^2)/(a^2*d^
3) - integrate(-2*(b^2*f^2*x^2 + 2*b^2*e*f*x - (a*b*f^2*x^2*e^c + 2*a*b*e*
f*x*e^c)*e^(d*x))/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x + c) - a^2*b), x)
```

3.450.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \coth(c+dx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.450.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)^2}{\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x)^2)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)*(e + f*x)^2)/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.451 $\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.451.1 Optimal result

Integrand size = 30, antiderivative size = 243

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{f\operatorname{arctanh}(\cosh(c+dx))}{ad^2}-\frac{(e+fx)\operatorname{csch}(c+dx)}{ad}+\frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d}+\frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d}-\frac{b(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^2d}+\frac{bf\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}+\frac{bf\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2}-\frac{bf\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^2d^2}$$

output
```
-f*arctanh(cosh(d*x+c))/a/d^2-(f*x+e)*csch(d*x+c)/a/d-b*(f*x+e)*ln(1-exp(2
*d*x+2*c))/a^2/d+b*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d+b*
(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d-1/2*b*f*polylog(2,exp
(2*d*x+2*c))/a^2/d^2+b*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/
d^2+b*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^2
```

3.451.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 621 vs. $2(243) = 486$.

Time = 8.15 (sec) , antiderivative size = 621, normalized size of antiderivative = 2.56

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\left(-de\cosh\left(\frac{1}{2}(c+dx)\right)+cf\cosh\left(\frac{1}{2}(c+dx)\right)-f(c+dx)\cosh\left(\frac{1}{2}(c+dx)\right)\right)\operatorname{csch}\left(\frac{1}{2}(c+dx)\right)}{2ad^2}$$
$$+\frac{-\frac{b(de-cf+f(c+dx))^2}{2f}+(-bde+af+bcf-bf(c+dx))\log\left(1-e^{-c-dx}\right)+(-bde-af+bcf-bf(c+dx}{a^2d^2}$$
$$+\frac{b\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\operatorname{arctanh}\left(\frac{a+be^{c}}{\sqrt{a^2-}}\right.}{(-a^2-b^2)^{3/2}}\right.}{}$$
$$+\frac{\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(de\sinh\left(\frac{1}{2}(c+dx)\right)-cf\sinh\left(\frac{1}{2}(c+dx)\right)+f(c+dx)\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2ad^2}$$

input
```
Integrate[((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((-(d*e*Cosh[(c + d*x)/2]) + c*f*Cosh[(c + d*x)/2] - f*(c + d*x)*Cosh[(c +
 d*x)/2])*Csch[(c + d*x)/2])/(2*a*d^2) + (-1/2*(b*(d*e - c*f + f*(c + d*x)
)^2)/f + (-(b*d*e) + a*f + b*c*f - b*f*(c + d*x))*Log[1 - E^(-c - d*x)] +
(-(b*d*e) - a*f + b*c*f - b*f*(c + d*x))*Log[1 + E^(-c - d*x)] + b*f*PolyL
og[2, -E^(-c - d*x)] + b*f*PolyLog[2, E^(-c - d*x)])/(a^2*d^2) + (b*(-2*d*
e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*A
rcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*S
qrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^
2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f
*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x)
+ b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^
2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(2
*a^2*d^2) + (Sech[(c + d*x)/2]*(d*e*Sinh[(c + d*x)/2] - c*f*Sinh[(c + d*x)
/2] + f*(c + d*x)*Sinh[(c + d*x)/2]))/(2*a*d^2)
```

3.451.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.25, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6121, 5975, 3042, 26, 4257, 6103, 3042, 26, 4201, 2620, 2715, 2838, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6121}$$

$$\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5975}$$

$$\frac{\frac{f\int \text{csch}(c+dx)dx}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)\text{csch}(c+dx)}{d}+\frac{f\int i\csc(ic+idx)dx}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)\text{csch}(c+dx)}{d}+\frac{if\int \csc(ic+idx)dx}{d}}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{6103}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int(e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{4201}$$

$$\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{2838}$$

$$\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{6095}$$

3.451. $\int\frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int \frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a}-\frac{i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+f}{2f}\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{f\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a}-\frac{i\left(2i\left(\frac{f}{}\right.\right.}{}\right.}{a}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a}-\frac{i\left(2i\left(\frac{f}{}\right.\right.}{}\right.}{a}$$

input
```
Int[((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.451. $\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(-((f*ArcTanh[Cosh[c + d*x]])/d^2) - ((e + f*x)*Csch[c + d*x])/d)/a - (b*(
-((b*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)
))/a) - (I*(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((e + f*x)*Log[1 + E^(2*c -
I*Pi + 2*d*x)])/(2*d) + (f*PolyLog[2, -E^(2*c - I*Pi + 2*d*x)])/(4*d^2)))
/a))/a
```

3.451.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.451. $\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6121
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

3.451.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(229) = 458$.

Time = 2.33 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.17

method	result
risch	$-\frac{2(fx+e)\mathrm{e}^{dx+c}}{da\left(\mathrm{e}^{2dx+2c}-1\right)}-\frac{be\ln\left(\mathrm{e}^{dx+c}-1\right)}{a^{2}d}-\frac{be\ln\left(\mathrm{e}^{dx+c}+1\right)}{a^{2}d}+\frac{be\ln\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)}{a^{2}d}+\frac{fb\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^{2}+b^{2}}+a}{a+\sqrt{a^{2}+b^{2}}}\right)c}{a^{2}d^{2}}+$

3.451. $\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
int((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERB
OSE)
```

output
```
-2/d*(f*x+e)/a*exp(d*x+c)/(exp(2*d*x+2*c)-1)-1/a^2/d*b*e*ln(exp(d*x+c)-1)-
1/a^2/d*b*e*ln(exp(d*x+c)+1)+1/a^2/d*b*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c
)-b)+1/a^2/d^2*f*b*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))
)*c+1/a^2/d^2*f*b*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)
))*c+1/a^2/d*f*b*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*
x+1/a^2/d*f*b*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x
-1/a^2/d*f*b*ln(exp(d*x+c)+1)*x+1/a^2/d^2*b*c*f*ln(exp(d*x+c)-1)-1/a^2/d^2
*b*c*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/a/d^2*f*ln(exp(d*x+c)-1)-1/
a/d^2*f*ln(exp(d*x+c)+1)-1/a^2/d^2*f*b*dilog(exp(d*x+c)+1)+1/a^2/d^2*f*b*d
ilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/a^2/d^2*f*b
*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/a^2/d^2*f*b
*dilog(exp(d*x+c))
```

3.451.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1221 vs. $2(226) = 452$.

Time = 0.29 (sec) , antiderivative size = 1221, normalized size of antiderivative = 5.02

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
-(2*(a*d*f*x + a*d*e)*cosh(d*x + c) - (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*
x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)*dilog((a*cosh(d*x + c) +
 a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b + 1) - (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c)
+ b*f*sinh(d*x + c)^2 - b*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b*f
*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2
 - b*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) + (b*f*cosh(d*x + c)^2 + 2*b*
f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)*dilog(-cosh(d*x
 + c) - sinh(d*x + c)) + (b*d*e - b*c*f - (b*d*e - b*c*f)*cosh(d*x + c)^2
- 2*(b*d*e - b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*e - b*c*f)*sinh(d*x
 + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/
b^2) + 2*a) + (b*d*e - b*c*f - (b*d*e - b*c*f)*cosh(d*x + c)^2 - 2*(b*d*e
- b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*e - b*c*f)*sinh(d*x + c)^2)*lo
g(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
 + (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*c
*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x + b*c*f)*sinh(d*x + c)^2)*log(-
(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2) - b)/b) + (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d
*x + c)^2 - 2*(b*d*f*x + b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x ...
```

3.451.6 Sympy [F]

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*coth(c + d*x)*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.451.7 Maxima [F]

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\coth(dx+c)\operatorname{csch}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
(2*b*d*integrate(1/2*x/(a^2*d*e^(d*x + c) + a^2*d), x) - 2*b*d*integrate(1
/2*x/(a^2*d*e^(d*x + c) - a^2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x
 + c) + 1)/(a^2*d^2)) - a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2
*d^2)) - 2*x*e^(d*x + c)/(a*d*e^(2*d*x + 2*c) - a*d) - 2*integrate((a*b*x*
e^(d*x + c) - b^2*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x + c) - a^2*b),
x))*f + e*(2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*
x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d)
 - b*log(e^(-d*x - c) - 1)/(a^2*d))
```

3.451.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
Timed out
```

3.451.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)}{\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x))/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)*(e + f*x))/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.452 $\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.452.1 Optimal result

Integrand size = 25, antiderivative size = 50

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\operatorname{csch}(c+dx)}{ad} - \frac{b\log(\sinh(c+dx))}{a^2d} + \frac{b\log(a+b\sinh(c+dx))}{a^2d}$$

output
```
-csch(d*x+c)/a/d-b*ln(sinh(d*x+c))/a^2/d+b*ln(a+b*sinh(d*x+c))/a^2/d
```

3.452.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\operatorname{csch}(c+dx)}{ad} - \frac{b\log(\sinh(c+dx))}{a^2d} + \frac{b\log(a+b\sinh(c+dx))}{a^2d}$$

input
```
Integrate[(Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-(Csch[c + d*x]/(a*d)) - (b*Log[Sinh[c + d*x]])/(a^2*d) + (b*Log[a + b*Sin
h[c + d*x]])/(a^2*d)
```

3.452.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 25, 3312, 25, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos(ic+idx)}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos(ic+idx)}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3312}$$

$$-\frac{\int -\frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{\operatorname{csch}^2(c+dx)}{b^2(a+b\sinh(c+dx))}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$\frac{b\int \left(\frac{\operatorname{csch}^2(c+dx)}{ab^2}-\frac{\operatorname{csch}(c+dx)}{a^2b}+\frac{1}{a^2(a+b\sinh(c+dx))}\right)d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{\log(b\sinh(c+dx))}{a^2}+\frac{\log(a+b\sinh(c+dx))}{a^2}-\frac{\operatorname{csch}(c+dx)}{ab}\right)}{d}$$

input
```
Int[(Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b*(-(Csch[c + d*x]/(a*b)) - Log[b*Sinh[c + d*x]]/a^2 + Log[a + b*Sinh[c +
 d*x]]/a^2))/d
```

3.452.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

3.452.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{\operatorname{csch}(dx+c)}{ad}+\frac{b\ln(a\ \operatorname{csch}(dx+c)+b)}{d\,a^2}$	35
default	$-\frac{\operatorname{csch}(dx+c)}{ad}+\frac{b\ln(a\ \operatorname{csch}(dx+c)+b)}{d\,a^2}$	35
risch	$-\frac{2\,\mathrm{e}^{dx+c}}{da(\mathrm{e}^{2dx+2c}-1)}+\frac{b\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\,\mathrm{e}^{dx+c}}{b}-1\right)}{a^2d}-\frac{b\ln(\mathrm{e}^{2dx+2c}-1)}{a^2d}$	82

input
```
int(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-csch(d*x+c)/a/d+1/d/a^2*b*ln(a*csch(d*x+c)+b)
```

3.452.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(50) = 100$.

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 4.22

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{2\,a\cosh\left(dx+c\right)-\left(b\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^{2}-b\right)\log\left(\frac{2\,(b\,s}{\cosh(dx}\right.}{a^{2}d\cosh\left(dx+c\right)^{2}+2\,a^{2}}$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*a*cosh(d*x + c) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c)
 + b*sinh(d*x + c)^2 - b)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sin
h(d*x + c))) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*si
nh(d*x + c)^2 - b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) +
2*a*sinh(d*x + c))/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x
 + c) + a^2*d*sinh(d*x + c)^2 - a^2*d)
```

3.452.6 Sympy [F]

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth\left(c+dx\right)\operatorname{csch}\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.452.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(50) = 100.

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.20

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2\,e^{(-dx-c)}}{(ae^{(-2\,dx-2\,c)}-a)d} + \frac{b\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{a^2d} - \frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d} - \frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*x - c) + b*
e^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^
(-d*x - c) - 1)/(a^2*d)
```

3.452.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(50) = 100.

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.20

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{b\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^2} - \frac{b\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a^2} + \frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)-2\,a}{a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}}{d}$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
(b*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/a^2 - b*log(abs(e^(d*x +
 c) - e^(-d*x - c)))/a^2 + (b*(e^(d*x + c) - e^(-d*x - c)) - 2*a)/(a^2*(e^
(d*x + c) - e^(-d*x - c))))/d
```

3.452.9 Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 409, normalized size of antiderivative = 8.18

$$\int \frac{\coth(c+dx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\left(2\,\mathrm{atan}\left(\left(\left(4\,a^3\,b\,d\,(b^2)^{5/2}\,\sqrt{-a^4\,d^2}+4\,a^5\,b\,d\,(b^2)^{3/2}\,\sqrt{-a^4\,d^2}\right)\,\left(\frac{1}{8\,a^3\,b^4\,d^2\,(a^2+b^2)^2}-\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(\frac{1}{16\,a^2\,b^5\,d^2\,(a^2+b^2)^2}-\right.\right.\right.\right.\right.}{}$$
$$-\frac{1}{a\,d\sinh\,(c+d\,x)}$$

input
```
int(coth(c + d*x)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
((2*atan((4*a^3*b*d*(b^2)^(5/2)*(-a^4*d^2)^(1/2) + 4*a^5*b*d*(b^2)^(3/2)*(
-a^4*d^2)^(1/2))*(1/(8*a^3*b^4*d^2*(a^2 + b^2)^2) - exp(d*x)*exp(c)*(1/(16
*a^2*b^5*d^2*(a^2 + b^2)^2) - (a^2 + 2*b^2)^2/(16*a^6*b^5*d^2*(a^2 + b^2)^
2)) + (a^2 + 2*b^2)/(8*a^5*b^4*d^2*(a^2 + b^2)^2))) + 2*atan(-(4*a^3*b^5*(
-a^4*d^2)^(1/2) + 4*a*b^7*(-a^4*d^2)^(1/2) - 4*b^8*exp(3*c)*exp(3*d*x)*(-a
^4*d^2)^(1/2) + 4*b^8*exp(d*x)*exp(c)*(-a^4*d^2)^(1/2) - 8*a*b^7*exp(2*c)*
exp(2*d*x)*(-a^4*d^2)^(1/2) + 4*a^2*b^6*exp(d*x)*exp(c)*(-a^4*d^2)^(1/2) -
 8*a^3*b^5*exp(2*c)*exp(2*d*x)*(-a^4*d^2)^(1/2) - 4*a^2*b^6*exp(3*c)*exp(3
*d*x)*(-a^4*d^2)^(1/2))/(b^4*(4*a^3*d*(b^2)^(3/2) + 4*a^5*d*(b^2)^(1/2))))
)*(b^2)^(1/2))/(-a^4*d^2)^(1/2) - 1/(a*d*sinh(c + d*x))
```

3.453 $\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.453.1 Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.453.2 Mathematica [N/A]

Not integrable

Time = 76.91 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Coth[c + d*x]*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

output
```
Integrate[(Coth[c + d*x]*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

3.453.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$
$$\Big\downarrow \text{ 6125}$$
$$\int \frac{\coth(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Coth[c + d*x]*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.453.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.453.4 Maple [N/A] (verified)

Not integrable

Time = 0.69 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\coth(dx+c)\,\text{csch}(dx+c)}{(fx+e)\,(a+b\sinh(dx+c))}dx$$

input
```
int(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.453.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth(dx+c)\operatorname{csch}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
fricas")
```

output
```
integral(coth(d*x + c)*csch(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
 + c)), x)
```

3.453.6 Sympy [N/A]

Not integrable

Time = 5.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(a+b\sinh(c+dx))\,(e+fx)}\,dx$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)*csch(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.453.7 Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 316, normalized size of antiderivative = 9.88

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth(dx+c)\operatorname{csch}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
maxima")
```

output
```
2*e^(d*x + c)/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*
x)) - 2*integrate(-1/2*(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*
f*x + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*
e^(d*x)), x) + 2*integrate(1/2*(b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*
a^2*d*e*f*x + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e
^2*e^c)*e^(d*x)), x) - 2*integrate(-(a*b*e^(d*x + c) - b^2)/(a^2*b*f*x + a
^2*b*e - (a^2*b*f*x*e^(2*c) + a^2*b*e*e^(2*c))*e^(2*d*x) - 2*(a^3*f*x*e^c
+ a^3*e*e^c)*e^(d*x)), x)
```

3.453.8 Giac [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="
giac")
```

output
```
Timed out
```

3.453.9 Mupad [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth(c+d\,x)}{\sinh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(coth(c + d*x)/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(coth(c + d*x)/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.454 $\int \frac{(e+fx)^3 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.454.1 Optimal result

Integrand size = 28, antiderivative size = 721

$$\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^3}{ad}+\frac{2b(e+fx)^3\operatorname{arctanh}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d} - \frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d} + \frac{3f(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad^2} + \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2} - \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2} + \frac{3\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2} - \frac{3\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2} + \frac{3f^2(e+fx)\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3} - \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{a^2d^3} + \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{a^2d^3} - \frac{6\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3} + \frac{6\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^3} - \frac{3f^3\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^4} + \frac{6bf^3\operatorname{PolyLog}\left(4,-e^{c+dx}\right)}{a^2d^4} - \frac{6bf^3\operatorname{PolyLog}\left(4,e^{c+dx}\right)}{a^2d^4} + \frac{6\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^4} - \frac{6\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^4}$$

output
```
-(f*x+e)^3/a/d+2*b*(f*x+e)^3*arctanh(exp(d*x+c))/a^2/d-(f*x+e)^3*coth(d*x+
c)/a/d+3*f*(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a/d^2+3*b*f*(f*x+e)^2*polylog(2,
-exp(d*x+c))/a^2/d^2-3*b*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a^2/d^2+3*f^2*(
f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^3-6*b*f^2*(f*x+e)*polylog(3,-exp(d*x+
c))/a^2/d^3+6*b*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a^2/d^3-3/2*f^3*polylog(
3,exp(2*d*x+2*c))/a/d^4+6*b*f^3*polylog(4,-exp(d*x+c))/a^2/d^4-6*b*f^3*pol
ylog(4,exp(d*x+c))/a^2/d^4+(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2))
)*(a^2+b^2)^(1/2)/a^2/d-(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(
a^2+b^2)^(1/2)/a^2/d+3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))*(a^2+b^2)^(1/2)/a^2/d^2-3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d^2-6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d^3+6*f^2*(f*x+e)*polylog(3,-b
*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d^3+6*f^3*polylog(4,-
b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d^4-6*f^3*polylog(4,
-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d^4
```

3.454.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1490 vs. $2(721) = 1442$.

Time = 7.17 (sec) , antiderivative size = 1490, normalized size of antiderivative = 2.07

$$\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^3*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-(d^3*e^2*(-1 + E^(2*c))*(b*d*e - 3*a*f)*x) + d^3*e^2*(-1 + E^(2*c))*(b
*d*e + 3*a*f)*x + 2*a*d^3*(e + f*x)^3 + 3*d^2*e*(-1 + E^(2*c))*f*(b*d*e -
2*a*f)*x*Log[1 - E^(-c - d*x)] + 3*d^2*(-1 + E^(2*c))*f^2*(b*d*e - a*f)*x^
2*Log[1 - E^(-c - d*x)] + b*d^3*(-1 + E^(2*c))*f^3*x^3*Log[1 - E^(-c - d*x
)] - 3*d^2*e*(-1 + E^(2*c))*f*(b*d*e + 2*a*f)*x*Log[1 + E^(-c - d*x)] - 3*
d^2*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*x^2*Log[1 + E^(-c - d*x)] - b*d^3*(-1
 + E^(2*c))*f^3*x^3*Log[1 + E^(-c - d*x)] + d^2*e^2*(-1 + E^(2*c))*(b*d*e
- 3*a*f)*Log[1 - E^(c + d*x)] - d^2*e^2*(-1 + E^(2*c))*(b*d*e + 3*a*f)*Log
[1 + E^(c + d*x)] + 3*d*e*(-1 + E^(2*c))*f*(b*d*e + 2*a*f)*PolyLog[2, -E^(
-c - d*x)] + 6*d*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*x*PolyLog[2, -E^(-c - d*
x)] + 3*b*d^2*(-1 + E^(2*c))*f^3*x^2*PolyLog[2, -E^(-c - d*x)] - 3*d*e*(-1
 + E^(2*c))*f*(b*d*e - 2*a*f)*PolyLog[2, E^(-c - d*x)] - 6*d*(-1 + E^(2*c)
)*f^2*(b*d*e - a*f)*x*PolyLog[2, E^(-c - d*x)] - 3*b*d^2*(-1 + E^(2*c))*f^
3*x^2*PolyLog[2, E^(-c - d*x)] + 6*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*PolyLo
g[3, -E^(-c - d*x)] + 6*b*d*(-1 + E^(2*c))*f^3*x*PolyLog[3, -E^(-c - d*x)]
 + 6*(-1 + E^(2*c))*f^2*(-(b*d*e) + a*f)*PolyLog[3, E^(-c - d*x)] - 6*b*d*
(-1 + E^(2*c))*f^3*x*PolyLog[3, E^(-c - d*x)] + 6*b*(-1 + E^(2*c))*f^3*Pol
yLog[4, -E^(-c - d*x)] - 6*b*(-1 + E^(2*c))*f^3*PolyLog[4, E^(-c - d*x)])/
(a^2*d^4*(-1 + E^(2*c)))) + (Sqrt[a^2 + b^2]*(-2*d^3*e^3*ArcTanh[(a + b*E^
(c + d*x))/Sqrt[a^2 + b^2]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a ...
```

3.454.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6103}$$

$$\frac{\int (e+fx)^3\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int (e+fx)^3\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{a}$$

$$\Big\downarrow \text{4203}$$

$$-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\int i(e+fx)^2\coth(c+dx)dx}{d}-\int(e+fx)^3dx+\frac{(e+fx)^3\coth(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{17}$$

$$-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\int i(e+fx)^2\coth(c+dx)dx}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{3f\int(e+fx)^2\coth(c+dx)dx}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{3f\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\int(e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\coth(c+d}{d}}{a}$$

3.454. $\int\frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{2720}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \text{6119}$$

$$-\frac{b\left(\frac{\int(e+fx)^3\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \text{5973}$$

$$-\frac{b\left(\frac{\int(e+fx)^3\sinh(c+dx)dx+\int(e+fx)^3\text{csch}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int-i(e+fx)^3\sin(ic+idx)dx+\int i(e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

3.454. $\displaystyle\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\int (e+fx)^3\sin(ic+idx)dx}{a}\right)}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \; 3777$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\cosh(c+dx)dx}{d}\right)}{a}\right)}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}\right)}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \; 3777$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{a}\right)}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

3.454. $\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}\right)}{d}\right)}{a}\right)}{a}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{a}\right)}{a}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}\right)}{d}\right)}{a}\right)}{a}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \text{3777}$$

3.454. $\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int c}{d}\right.}{d}\right.}{d}\right.}{a}\right.}{a}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int s}{}\right.}{d}\right.}{d}\right.}{a}\right.}{a}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \; 3117$$

3.454. $\displaystyle\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\left(\frac{i(e+fx)^3\cosh(c+dx)}{d}-\frac{3if\left(\frac{(e+fx)^2\sinh(c+dx)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sin}{}\right.}{d}\right.}{d}\right.}{a}\right.}{a}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \; 4670$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{3if\int (e+fx)^2\log\left(1-e^{c+dx}\right)dx}{d}-\frac{3if\int (e+fx)^2\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^3\operatorname{arctanh}\left(e^{c+dx}\right)}{d}\right)-i\left(\frac{i(e+fx)^3}{}\right.}{a}\right.}{a}$$

$$\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}$$

$$\Big\downarrow \; 3011$$

3.454. $\displaystyle\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)\cdots}{d}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)\cdots}{d}\cdots}{a}\cdots$$

$$\Big\downarrow \text{6099}$$

$$-\frac{b\left(-\frac{b\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^3dx}{b^2}+\frac{\int(e+fx)^3\sinh(c+dx)dx}{b}\right)}{a}+\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2\cdots\right)}{d}\cdots\right)}{d}\cdots\right)}{\cdots}\cdots\right)}{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)\cdots}{d}\cdots}{a}\cdots$$

$$\Big\downarrow \text{17}$$

3.454. $\displaystyle\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\left(\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int (e+fx)^3\sinh(c+dx)dx}{b}-\frac{a(e+fx)^4}{4b^2f}\right)}{a}+\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right.}{d}\right.}{}\right.}{\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}}$$

$\Big\downarrow$ 3042

$$-\frac{b\left(\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.}{a}\right.}{\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}}{a}}$$

input
```
Int[((e + f*x)^3*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.454. $\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.454.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.454. $\int \frac{(e+fx)^3 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

3.454. $\int \frac{(e+fx)^3 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.454.4 Maple [F]

$$\int \frac{(fx+e)^3 \coth (dx+c)^2}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.454.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4612 vs. $2(667) = 1334$.

Time = 0.37 (sec) , antiderivative size = 4612, normalized size of antiderivative = 6.40

$$\int \frac{(e+fx)^3 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*a*d^3*e^3 - 6*a*c*d^2*e^2*f + 6*a*c^2*d*e*f^2 - 2*a*c^3*f^3 + 2*(a*d^3
*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2
*d*e*f^2 + a*c^3*f^3)*cosh(d*x + c)^2 + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x
^2 + 3*a*d^3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*cosh
(d*x + c)*sinh(d*x + c) + 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e
^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*sinh(d*x + c)^2 +
3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f - (b*d^2*f^3*x^2 + 2*b*d^
2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)^2 - 2*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^
2*x + b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^3*x^2 + 2*b*d^2*
e*f^2*x + b*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b + 1) - 3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^
2*f - (b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)^2 - 2*
(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c
) - (b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt(
(a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b*d^3*e^3 - 3*b
*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3 - (b*d^3*e^3 - 3*b*c*d^2*e^2*f
+ 3*b*c^2*d*e*f^2 - b*c^3*f^3)*cosh(d*x + c)^2 - 2*(b*d^3*e^3 - 3*b*c*d^2*
e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) - (b*d...
```

3.454.6 Sympy [F]

$$\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^3\coth^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.454.7 Maxima [F]

$$\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\coth{(dx+c)}^2}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
e^3*(b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + s
qrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
- a + sqrt(a^2 + b^2)))/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d)) - 6*e^2*
f*x/(a*d) + 3*e^2*f*log(e^(d*x + c) + 1)/(a*d^2) + 3*e^2*f*log(e^(d*x + c)
 - 1)/(a*d^2) - 2*(f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x)/(a*d*e^(2*d*x + 2*c)
 - a*d) + (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) -
6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b*f^3/(a^2*d^
4) - (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x
*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b*f^3/(a^2*d^4) + 3*
(b*d*e^2*f + 2*a*e*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(
a^2*d^3) - 3*(b*d*e^2*f - 2*a*e*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^
(d*x + c)))/(a^2*d^3) + 3*(b*d*e*f^2 + a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1
) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^2*d^4) - 3*
(b*d*e*f^2 - a*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x +
c)) - 2*polylog(3, e^(d*x + c)))/(a^2*d^4) - 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e
*f^2 + a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f + 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) +
 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 - a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f - 2*a
*d*e*f^2)*d^2*x^2)/(a^2*d^4) + integrate(2*((a^2*f^3*e^c + b^2*f^3*e^c)*x^
3 + 3*(a^2*e*f^2*e^c + b^2*e*f^2*e^c)*x^2 + 3*(a^2*e^2*f*e^c + b^2*e^2*f*e
^c)*x)*e^(d*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x + c) - a^2*b), x)
```

3.454.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.454.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((coth(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.455 $\int \frac{(e+fx)^2 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.455.1 Optimal result

Integrand size = 28, antiderivative size = 517

$$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^2}{ad}+\frac{2b(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d} - \frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d} + \frac{2f(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad^2} + \frac{2bf(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2} - \frac{2bf(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2} + \frac{2\sqrt{a^2+b^2}f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2} - \frac{2\sqrt{a^2+b^2}f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2} + \frac{f^2\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3} - \frac{2bf^2\,\text{PolyLog}\left(3,-e^{c+dx}\right)}{a^2d^3} + \frac{2bf^2\,\text{PolyLog}\left(3,e^{c+dx}\right)}{a^2d^3} - \frac{2\sqrt{a^2+b^2}f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3} + \frac{2\sqrt{a^2+b^2}f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^3}$$

output
```
-(f*x+e)^2/a/d+2*b*(f*x+e)^2*arctanh(exp(d*x+c))/a^2/d-(f*x+e)^2*coth(d*x+
c)/a/d+2*f*(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d^2+2*b*f*(f*x+e)*polylog(2,-exp
(d*x+c))/a^2/d^2-2*b*f*(f*x+e)*polylog(2,exp(d*x+c))/a^2/d^2+f^2*polylog(2
,exp(2*d*x+2*c))/a/d^3-2*b*f^2*polylog(3,-exp(d*x+c))/a^2/d^3+2*b*f^2*poly
log(3,exp(d*x+c))/a^2/d^3+(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
*(a^2+b^2)^(1/2)/a^2/d-(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a
^2+b^2)^(1/2)/a^2/d+2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)
))*(a^2+b^2)^(1/2)/a^2/d^2-2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d^2-2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d^3+2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d^3
```

3.455.2 Mathematica [A] (verified)

Time = 6.91 (sec) , antiderivative size = 917, normalized size of antiderivative = 1.77

$$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{-d^2e(-1+e^{2c})(bde-2af)x+d^2e(-1+e^{2c})(bde+2af)x+2ad^2(e+fx)^2+2d(-1+e^{2c})f(bde-a}{}$$

$$+\frac{\sqrt{a^2+b^2}\left(-2d^2e^2\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2d^2efx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+d^2f^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2d^2e\right.}{}$$

$$+\frac{\operatorname{sech}\left(\frac{c}{2}\right)\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-e^2\sinh\left(\frac{dx}{2}\right)-2efx\sinh\left(\frac{dx}{2}\right)-f^2x^2\sinh\left(\frac{dx}{2}\right)\right)}{2ad}$$

$$+\frac{\operatorname{csch}\left(\frac{c}{2}\right)\operatorname{csch}\left(\frac{c}{2}+\frac{dx}{2}\right)\left(e^2\sinh\left(\frac{dx}{2}\right)+2efx\sinh\left(\frac{dx}{2}\right)+f^2x^2\sinh\left(\frac{dx}{2}\right)\right)}{2ad}$$

input
```
Integrate[((e + f*x)^2*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-(d^2*e*(-1 + E^(2*c))*(b*d*e - 2*a*f)*x) + d^2*e*(-1 + E^(2*c))*(b*d*e
 + 2*a*f)*x + 2*a*d^2*(e + f*x)^2 + 2*d*(-1 + E^(2*c))*f*(b*d*e - a*f)*x*L
og[1 - E^(-c - d*x)] + b*d^2*(-1 + E^(2*c))*f^2*x^2*Log[1 - E^(-c - d*x)]
- 2*d*(-1 + E^(2*c))*f*(b*d*e + a*f)*x*Log[1 + E^(-c - d*x)] - b*d^2*(-1 +
 E^(2*c))*f^2*x^2*Log[1 + E^(-c - d*x)] + d*e*(-1 + E^(2*c))*(b*d*e - 2*a*
f)*Log[1 - E^(c + d*x)] - d*e*(-1 + E^(2*c))*(b*d*e + 2*a*f)*Log[1 + E^(c
+ d*x)] + 2*(-1 + E^(2*c))*f*(b*d*e + a*f)*PolyLog[2, -E^(-c - d*x)] + 2*b
*d*(-1 + E^(2*c))*f^2*x*PolyLog[2, -E^(-c - d*x)] + 2*(-1 + E^(2*c))*f*(-(
b*d*e) + a*f)*PolyLog[2, E^(-c - d*x)] - 2*b*d*(-1 + E^(2*c))*f^2*x*PolyLo
g[2, E^(-c - d*x)] + 2*b*(-1 + E^(2*c))*f^2*PolyLog[3, -E^(-c - d*x)] - 2*
b*(-1 + E^(2*c))*f^2*PolyLog[3, E^(-c - d*x)])/(a^2*d^3*(-1 + E^(2*c)))) +
 (Sqrt[a^2 + b^2]*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]]
 + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^2*f^2*x^
2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*d^2*e*f*x*Log[1 + (b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a +
Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))] - 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
+ 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2*d^3) +
 (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(e^2*Sinh[(d*x)/2]) - 2*e*f*x*Sinh[(d...
```

3.455.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \ 6103$$

$$\frac{\int (e+fx)^2\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \ 25$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{a}$$

$$\Big\downarrow \text{4203}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\int i(e+fx)\coth(c+dx)dx}{d}-\int (e+fx)^2dx+\frac{(e+fx)^2\coth(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{17}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\int i(e+fx)\coth(c+dx)dx}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{2f\int (e+fx)\coth(c+dx)dx}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{2f\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int \log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

3.455. $\displaystyle\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{2838}$$

$$\frac{-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{6119}$$

$$\frac{-\frac{b\left(\frac{\int(e+fx)^2\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{5973}$$

$$\frac{-\frac{b\left(\frac{\int(e+fx)^2\sinh(c+dx)dx+\int(e+fx)^2\text{csch}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)^2\sin(ic+idx)dx+\int i(e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)^2\csc(ic+idx)dx-i\int(e+fx)^2\sin(ic+idx)dx}{a}\right)}{a}-\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{3777}$$

3.455. $\displaystyle\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \; 3777$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \; 3042$$

3.455. $\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{2if\int (e+fx)\log\left(1-e^{c+dx}\right)dx}{d}-\frac{2if\int (e+fx)\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-i\left(\frac{i(e+fx)^2\cdots}{d}\cdots\right.}{a}\cdots\right.}{a}$$

$$-\frac{\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \text{3011}$$

3.455. $\displaystyle\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyL}}{}\right.}{d}\right.}{}\right)}{a}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \; 2720$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c}\right.}{d^2}\right.}{}\right.}{}\right)}{a}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \; 6099$$

$$-\frac{b\left(-\frac{b\left(\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int (e+fx)^2dx}{b^2}+\frac{\int (e+fx)^2\sinh(c+dx)dx}{b}\right)}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyL}}{}\right.}{d}\right.}{}\right)}{}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \; 17$$

3.455. $\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)^2\sinh(c+dx)dx}{b}-\frac{a(e+fx)^3}{3b^2f}\right)}{a}+\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2\right.}{d}\right.}{d}\right.}{}\right.}{}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.}{a}\right.}{}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\left(\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c}\right.}{d}\right.}{d}\right.}{a}\right.}{}$$

$$\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}$$

input
```
Int[((e + f*x)^2*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.455. $\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.455.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.455. $\int \frac{(e+fx)^2 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.455. $\int \frac{(e+fx)^2 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.455.4 Maple [F]

$$\int \frac{(fx+e)^2 \coth(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.455.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2729 vs. $2(477) = 954$.

Time = 0.33 (sec) , antiderivative size = 2729, normalized size of antiderivative = 5.28

$$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*a*d^2*e^2 - 4*a*c*d*e*f + 2*a*c^2*f^2 + 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f
*x + 2*a*c*d*e*f - a*c^2*f^2)*cosh(d*x + c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2
*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + 2*(a*d^2*f
^2*x^2 + 2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*sinh(d*x + c)^2 + 2*(b*d
*f^2*x + b*d*e*f - (b*d*f^2*x + b*d*e*f)*cosh(d*x + c)^2 - 2*(b*d*f^2*x +
b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^2*x + b*d*e*f)*sinh(d*x + c)
^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b*d*
f^2*x + b*d*e*f - (b*d*f^2*x + b*d*e*f)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b
*d*e*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^2*x + b*d*e*f)*sinh(d*x + c)^
2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b*d^2*e
^2 - 2*b*c*d*e*f + b*c^2*f^2 - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(
d*x + c)^2 - 2*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c)*sinh(d*
x + c) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2
+ b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^
2)/b^2) + 2*a) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2 - (b*d^2*e^2 - 2*b*c
*d*e*f + b*c^2*f^2)*cosh(d*x + c)^2 - 2*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f
^2)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*si
nh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d...
```

3.455.6 Sympy [F]

$$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)^2\coth^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**2*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.455.7 Maxima [F]

$$\int \frac{(e+fx)^2 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \coth(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
e^2*(b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + s
qrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
- a + sqrt(a^2 + b^2)))/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d)) - 4*e*f*
x/(a*d) - 2*(f^2*x^2 + 2*e*f*x)/(a*d*e^(2*d*x + 2*c) - a*d) + 2*e*f*log(e^
(d*x + c) + 1)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/(a*d^2) + (d^2*x^2*log
(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c))
)*b*f^2/(a^2*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x +
c)) - 2*polylog(3, e^(d*x + c)))*b*f^2/(a^2*d^3) + 2*(b*d*e*f + a*f^2)*(d*
x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 2*(b*d*e*f - a*f
^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) - 1/3*(b*d^
3*f^2*x^3 + 3*(b*d*e*f + a*f^2)*d^2*x^2)/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 +
3*(b*d*e*f - a*f^2)*d^2*x^2)/(a^2*d^3) + integrate(2*((a^2*f^2*e^c + b^2*f
^2*e^c)*x^2 + 2*(a^2*e*f*e^c + b^2*e*f*e^c)*x)*e^(d*x)/(a^2*b*e^(2*d*x + 2
*c) + 2*a^3*e^(d*x + c) - a^2*b), x)
```

3.455.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.455.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\mathrm{coth}(c+d\,x)^2\,{\left(e+f\,x\right)}^2}{a+b\,\mathrm{sinh}\left(c+d\,x\right)}\,dx$$

input
```
int((coth(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.456 $\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.456.1 Optimal result

Integrand size = 26, antiderivative size = 294

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2b(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d} - \frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d} + \frac{f\log(\sinh(c+dx))}{ad^2} + \frac{bf\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2} - \frac{bf\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2} + \frac{\sqrt{a^2+b^2}f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2} - \frac{\sqrt{a^2+b^2}f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2}$$

output
```
2*b*(f*x+e)*arctanh(exp(d*x+c))/a^2/d-(f*x+e)*coth(d*x+c)/a/d+f*ln(sinh(d*
x+c))/a/d^2+b*f*polylog(2,-exp(d*x+c))/a^2/d^2-b*f*polylog(2,exp(d*x+c))/a
^2/d^2+(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/
d-(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d+f*p
olylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d^2-f*poly
log(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d^2
```

3.456.2 Mathematica [A] (verified)

Time = 3.28 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.18

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{ad(e+fx)\coth\left(\frac{1}{2}(c+dx)\right)-2\left(af(c+dx)+(af-bd(e+fx))\log\left(1-e^{-c-dx}\right)+(af+bd(e+fx)\right.}{}$$

input
```
Integrate[((e + f*x)*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(a*d*(e + f*x)*Coth[(c + d*x)/2] - 2*(a*f*(c + d*x) + (a*f - b*d*(e +
 f*x))*Log[1 - E^(-c - d*x)] + (a*f + b*d*(e + f*x))*Log[1 + E^(-c - d*x)]
 - b*f*PolyLog[2, -E^(-c - d*x)] + b*f*PolyLog[2, E^(-c - d*x)]) - 2*Sqrt[
a^2 + b^2]*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*Ar
cTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] -
f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + a*d*(e + f*x)*Ta
nh[(c + d*x)/2])/(a^2*d^2)
```

3.456.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \ 6103$$
$$\frac{\int (e+fx)\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$
$$\Big\downarrow \ 3042$$
$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -\left((e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)^2\right)dx}{a}$$
$$\Big\downarrow \ 25$$
$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{a}$$

$$\Big\downarrow \; 4203$$

$$-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{if\int i\coth(c+dx)dx}{d}-\int(e+fx)dx+\frac{(e+fx)\coth(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 17$$

$$-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{if\int i\coth(c+dx)dx}{d}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{f\int\coth(c+dx)dx}{d}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{f\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{if\int\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \; 3956$$

$$-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \; 6119$$

$$-\frac{b\left(\frac{\int(e+fx)\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \; 5973$$

$$-\frac{b\left(\frac{\int(e+fx)\sinh(c+dx)dx+\int(e+fx)\text{csch}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \; 3042$$

3.456. $\displaystyle\int\frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)\sin(ic+idx)dx+\int i(e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\int (e+fx)\sin(ic+idx)dx}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{ 3117}$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{ 4670}$$

3.456. $\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{if\int\log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c}{d^2}\right.}{a}}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-i\left(\frac{i(e}{}\right.}{a}}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(}{d^2}\right.}{a}}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{6099}$$

$$-\frac{b\left(-\frac{b\left(\frac{\left(a^2+b^2\right)\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)dx}{b^2}+\frac{\int(e+fx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLo}}{}\right.}{a}}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{17}$$

$$-\frac{b\left(-\frac{b\left(\frac{\left(a^2+b^2\right)\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{b^2}+\frac{\int(e+fx)\sinh(c+dx)dx}{b}-\frac{a(e+fx)^2}{2b^2f}\right)}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}(}{d^2}\right.}{a}}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

3.456. $\int\frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \; 3042$$

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{\left(a^2+b^2\right)\int\frac{e}{a-ib\,\text{si}}}{b^2}\right.}{}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{\left(a^2+b^2\right)\int\frac{e}{a-ib\,\text{si}}}{b^2}\right.}{}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \; 3777$$

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{\left(a^2+b^2\right)\int\frac{e}{a-ib\,\text{si}}}{b^2}\right.}{}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{\left(a^2+b^2\right)\int\frac{e}{a-ib\,\text{si}}}{b^2}\right.}{}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \; 3117$$

3.456. $\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{\left(a^2+b^2\right)\int\frac{e}{a-ib\,\mathrm{si}}}{b^2}\right.}{}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{3803}$$

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(\frac{2\left(a^2+b^2\right)\int-\frac{}{-2e}}{}\right.}{}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2\left(a^2+b^2\right)\int\frac{}{-2e}}{}\right.}{}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$$\Big\downarrow \text{2694}$$

3.456. $\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\int\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

$\downarrow$ 27

$$-\frac{b\left(\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}-\frac{b\left(-\frac{2\left(a^2+b^2\right)\left(\frac{b\int\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{a}$$

$$\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}$$

input
```
Int[((e + f*x)*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.456. $\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.456.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.456. $\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

3.456. $\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.456.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1016 vs. $2(271) = 542$.

Time = 2.44 (sec) , antiderivative size = 1017, normalized size of antiderivative = 3.46

method	result	size
risch	Expression too large to display	1017

input
```
int((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/a^2/d*f*b^2/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^
2+b^2)^(1/2)))*x-1/a^2/d*f*b^2/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^
(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/a^2/d^2*f*b^2/(a^2+b^2)^(1/2)*ln((-b*exp
(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/a^2/d^2*f*b^2/(a^2+b^
2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-2/a^2/
d*b^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+
1/a^2/d^2*f*b^2/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-
a+(a^2+b^2)^(1/2)))+1/a^2/d*f*b*ln(exp(d*x+c)+1)*x+1/a^2/d^2*b*c*f*ln(exp(
d*x+c)-1)+2/a^2/d^2*b^2*c*f/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))-1/a^2/d^2*f*b^2/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/d*(f*x+e)/a/(exp(2*d*x+2*c)-1)-1/d
^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))*c-1/d*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^
2)^(1/2)))*x+1/d*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-
a+(a^2+b^2)^(1/2)))*x+2/d^2*c*f/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c
)+2*a)/(a^2+b^2)^(1/2))+1/d^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2
)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/a^2/d*b*e*ln(exp(d*x+c)-1)+1/a^2/d*b*
e*ln(exp(d*x+c)+1)+1/a^2/d^2*f*b*dilog(exp(d*x+c)+1)+1/a^2/d^2*f*b*dilog(e
xp(d*x+c))+1/a/d^2*f*ln(exp(d*x+c)-1)+1/a/d^2*f*ln(exp(d*x+c)+1)-2/d*e/(a^
2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d^2*f/...
```

3.456.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1338 vs. $2(267) = 534$.

Time = 0.29 (sec) , antiderivative size = 1338, normalized size of antiderivative = 4.55

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*a*d*e - 2*a*c*f + 2*(a*d*f*x + a*c*f)*cosh(d*x + c)^2 + 4*(a*d*f*x + a
*c*f)*cosh(d*x + c)*sinh(d*x + c) + 2*(a*d*f*x + a*c*f)*sinh(d*x + c)^2 -
(b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x +
c)^2 - b*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
 + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
 (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x +
 c)^2 - b*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c
) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
- (b*d*e - b*c*f - (b*d*e - b*c*f)*cosh(d*x + c)^2 - 2*(b*d*e - b*c*f)*cos
h(d*x + c)*sinh(d*x + c) - (b*d*e - b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^
2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) + (b*d*e - b*c*f - (b*d*e - b*c*f)*cosh(d*x + c)^2 - 2*(b*d*e -
 b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*e - b*c*f)*sinh(d*x + c)^2)*sqr
t((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a
^2 + b^2)/b^2) + 2*a) + (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)
^2 - 2*(b*d*f*x + b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x + b*c*f)*s
inh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) -
(b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*c*f)
*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x + b*c*f)*sinh(d*x + c)^2)*sqrt(...
```

3.456.6 Sympy [F]

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\coth^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.456.7 Maxima [F]

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\coth\left(dx+c\right)^2}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(b*d*integrate(x/(a^2*d*e^(d*x + c) + a^2*d), x) + b*d*integrate(x/(a^2*d
*e^(d*x + c) - a^2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) + 1)/
(a^2*d^2)) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^2)) - 2*
(a^2*e^c + b^2*e^c)*integrate(x*e^(d*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(
d*x + c) - a^2*b), x) + 2*x/(a*d*e^(2*d*x + 2*c) - a*d))*f + e*(b*log(e^(-
d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + sqrt(a^2 + b^2)*
log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2
+ b^2)))/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d))
```

3.456.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.456.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\mathrm{coth}(c+d\,x)^2\,(e+f\,x)}{a+b\,\mathrm{sinh}\left(c+d\,x\right)}\,dx$$

input
```
int((coth(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.457 $\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.457.1 Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\text{arctanh}(\cosh(c+dx))}{a^2d} - \frac{2\sqrt{a^2+b^2}\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d} - \frac{\coth(c+dx)}{ad}$$

output
```
b*arctanh(cosh(d*x+c))/a^2/d-coth(d*x+c)/a/d-2*arctanh((b-a*tanh(1/2*d*x+1
/2*c))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/a^2/d
```

3.457.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.45

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{4\sqrt{-a^2-b^2}\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right) + a\coth\left(\frac{1}{2}(c+dx)\right) - 2b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) + 2b\log\left(\sinh\left(\right.\right.}{2a^2d}$$

input
```
Integrate[Coth[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(4*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]
] + a*Coth[(c + d*x)/2] - 2*b*Log[Cosh[(c + d*x)/2]] + 2*b*Log[Sinh[(c + d
*x)/2]] + a*Tanh[(c + d*x)/2])/(a^2*d)
```

3.457.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.810, Rules used = {3042, 25, 3202, 25, 3042, 25, 3535, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{(a-ib\sin(ic+idx))\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{3202}$$

$$-\int -\frac{\operatorname{csch}^2(c+dx)\left(\sinh^2(c+dx)+1\right)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\left(\sinh^2(c+dx)+1\right)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1-\sin(ic+idx)^2}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1-\sin(ic+idx)^2}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3535}$$

$$-\frac{\int \frac{\operatorname{csch}(c+dx)(b-a\sinh(c+dx))}{a+b\sinh(c+dx)}dx}{a}-\frac{\coth(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{\int \frac{i(b+ia\sin(ic+idx))}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\int \frac{b+ia\sin(ic+idx)}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx}{a}$$

$$\Big\downarrow \text{3480}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\left(\frac{i(a^2+b^2)\int \frac{1}{a+b\sinh(c+dx)}dx}{a}+\frac{b\int -i\operatorname{csch}(c+dx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\left(\frac{i(a^2+b^2)\int \frac{1}{a+b\sinh(c+dx)}dx}{a}-\frac{ib\int \operatorname{csch}(c+dx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\left(\frac{i(a^2+b^2)\int \frac{1}{a-ib\sin(ic+idx)}dx}{a}-\frac{ib\int i\csc(ic+idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\left(\frac{i(a^2+b^2)\int \frac{1}{a-ib\sin(ic+idx)}dx}{a}+\frac{b\int \csc(ic+idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\left(\frac{2(a^2+b^2)\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}+\frac{b\int \csc(ic+idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\left(\frac{b\int \csc(ic+idx)dx}{a}-\frac{4(a^2+b^2)\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{ad}\right)}{a}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\left(\frac{b\int \csc(ic+idx)dx}{a}+\frac{2i\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad}\right)}{a}$$

$$\Big\downarrow \text{4257}$$

3.457. $\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{\coth(c+dx)}{ad}-\frac{i\left(\frac{2i\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad}+\frac{ib\text{arctanh}(\cosh(c+dx))}{ad}\right)}{a}$$

input
```
Int[Coth[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

output
```
((-I)*((I*b*ArcTanh[Cosh[c + d*x]])/(a*d) + ((2*I)*Sqrt[a^2 + b^2]*ArcTanh
[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(a*d)))/a - Coth[c + d*x]/(a*d)
```

3.457.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

3.457. $\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3202
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.457.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{\left(-4a^2-4b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{\left(-4a^2-4b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$
risch	$-\frac{2}{da\left(e^{2dx+2c}-1\right)}+\frac{b\ln\left(e^{dx+c}+1\right)}{a^2d}-\frac{b\ln\left(e^{dx+c}-1\right)}{a^2d}+\frac{\sqrt{a^2+b^2}\,\ln\left(e^{dx+c}-\frac{-a+\sqrt{a^2+b^2}}{b}\right)}{d\,a^2}-\frac{\sqrt{a^2+b^2}\,\ln\left(e^{dx+c}\right.}{d\,a^2}$

input
```
int(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)-1/2/a^2*(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arc
tanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1/2/a/tanh(1/2*d*x
+1/2*c)-1/a^2*b*ln(tanh(1/2*d*x+1/2*c)))
```

3.457.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(74) = 148$.

Time = 0.27 (sec) , antiderivative size = 360, normalized size of antiderivative = 4.68

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\sqrt{a^2+b^2}\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-1\right)\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c}{b}\right.}{}$$

input
```
integrate(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(sqrt(a^2 + b^2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d
*x + c)^2 - 1)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh
(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sq
rt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2
+ b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x
 + c) - b)) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sin
h(d*x + c)^2 - b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*log(cosh(d*
x + c) + sinh(d*x + c) - 1) - 2*a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d
*x + c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2 - a^2*d)
```

3.457.6 Sympy [F]

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\coth^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate(coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.457.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.74

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d}-\frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}+\frac{\sqrt{a^2+b^2}\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{a^2d}+\frac{2}{(ae^{(-2\,dx-2\,c)}-a)d}$$

input
```
integrate(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + sqrt(a
^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a +
 sqrt(a^2 + b^2)))/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d)
```

3.457.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.56

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{\frac{b\log\left(e^{(dx+c)}+1\right)}{a^2}-\frac{b\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^2}+\frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{a^2}-\frac{2}{a\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

input
```
integrate(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
(b*log(e^(d*x + c) + 1)/a^2 - b*log(abs(e^(d*x + c) - 1))/a^2 + sqrt(a^2 +
 b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x +
c) + 2*a + 2*sqrt(a^2 + b^2)))/a^2 - 2/(a*(e^(2*d*x + 2*c) - 1)))/d
```

3.457.9 Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 380, normalized size of antiderivative = 4.94

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{2}{a\,d-a\,d\,\mathrm{e}^{2\,c+2\,d\,x}}-\frac{b\,\ln\left(32\,a^2+32\,b^2-32\,a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-32\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\right)}{a^2\,d}$$
$$+\frac{b\,\ln\left(32\,a^2+32\,b^2+32\,a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+32\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\right)}{a^2\,d}$$
$$+\frac{\ln\left(128\,a^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-64\,a\,b^3-64\,a^3\,b-32\,b^3\,\sqrt{a^2+b^2}+32\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-64\,a^2\,b\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+}{a^2\,d}$$
$$-\frac{\ln\left(32\,b^3\,\sqrt{a^2+b^2}-64\,a\,b^3-64\,a^3\,b+128\,a^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+32\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+64\,a^2\,b\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-}{a^2\,d}$$

input
```
int(coth(c + d*x)^2/(a + b*sinh(c + d*x)),x)
```

output
```
2/(a*d - a*d*exp(2*c + 2*d*x)) - (b*log(32*a^2 + 32*b^2 - 32*a^2*exp(d*x)*
exp(c) - 32*b^2*exp(d*x)*exp(c)))/(a^2*d) + (b*log(32*a^2 + 32*b^2 + 32*a^
2*exp(d*x)*exp(c) + 32*b^2*exp(d*x)*exp(c)))/(a^2*d) + (log(128*a^4*exp(d*
x)*exp(c) - 64*a*b^3 - 64*a^3*b - 32*b^3*(a^2 + b^2)^(1/2) + 32*b^4*exp(d*
x)*exp(c) - 64*a^2*b*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(d*x)*exp(c) + 128
*a^3*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 96*a*b^2*exp(d*x)*exp(c)*(a^2 + b
^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^2*d) - (log(32*b^3*(a^2 + b^2)^(1/2) - 64
*a*b^3 - 64*a^3*b + 128*a^4*exp(d*x)*exp(c) + 32*b^4*exp(d*x)*exp(c) + 64*
a^2*b*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(d*x)*exp(c) - 128*a^3*exp(d*x)*e
xp(c)*(a^2 + b^2)^(1/2) - 96*a*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2
 + b^2)^(1/2))/(a^2*d)
```

3.457. $\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.458 $\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.458.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\mathrm{Int}\left(\frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.458.2 Mathematica [N/A]

Not integrable

Time = 67.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Coth[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Coth[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.458.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$\Big\downarrow \text{6111}$$
$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Coth[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.458.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.458.4 Maple [N/A] (verified)

Not integrable

Time = 0.68 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\coth\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.458.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(coth(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.458.6 Sympy [N/A]

Not integrable

Time = 2.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth^2(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(coth(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.458.7 Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 311, normalized size of antiderivative = 11.11

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2*(a^2*e^c + b^2*e^c)*integrate(-e^(d*x)/(a^2*b*f*x + a^2*b*e - (a^2*b*f*x
*e^(2*c) + a^2*b*e*e^(2*c))*e^(2*d*x) - 2*(a^3*f*x*e^c + a^3*e*e^c)*e^(d*x
)), x) + 2/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*x))
 - integrate(-(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2
*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)),
 x) - integrate((b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a
^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)
), x)
```

3.458.8 Giac [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.458.9 Mupad [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(coth(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(coth(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.459 $\int \frac{(e+fx)^3 \cosh(c+dx) \coth^2(c+dx)}{a+b \sinh(c+dx)}\,dx$

3.459.1 Optimal result

Integrand size = 34, antiderivative size = 718

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{b(e+fx)^4}{4a^2f}-\frac{(a^2+b^2)(e+fx)^4}{4a^2bf}-\frac{6f(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{ad^2}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{ad}$$
$$+\frac{(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd}+\frac{(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd}$$
$$-\frac{b(e+fx)^3\log\left(1-e^{2(c+dx)}\right)}{a^2d}-\frac{6f^2(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^3}$$
$$+\frac{6f^2(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^3}+\frac{3(a^2+b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^2}$$
$$+\frac{3(a^2+b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd^2}$$
$$-\frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^2d^2}+\frac{6f^3\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^4}$$
$$-\frac{6f^3\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^4}-\frac{6(a^2+b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^3}$$
$$-\frac{6(a^2+b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd^3}$$
$$+\frac{3bf^2(e+fx)\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^2d^3}+\frac{6(a^2+b^2)f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^4}$$
$$+\frac{6(a^2+b^2)f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd^4}-\frac{3bf^3\operatorname{PolyLog}\left(4,e^{2(c+dx)}\right)}{4a^2d^4}$$

output
```
1/4*b*(f*x+e)^4/a^2/f-1/4*(a^2+b^2)*(f*x+e)^4/a^2/b/f-6*f*(f*x+e)^2*arctan
h(exp(d*x+c))/a/d^2-(f*x+e)^3*csch(d*x+c)/a/d-b*(f*x+e)^3*ln(1-exp(2*d*x+2
*c))/a^2/d+(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/
b/d+(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d-6*f
^2*(f*x+e)*polylog(2,-exp(d*x+c))/a/d^3+6*f^2*(f*x+e)*polylog(2,exp(d*x+c)
)/a/d^3-3/2*b*f*(f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a^2/d^2+3*(a^2+b^2)*f*
(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d^2+3*(a^2+b^
2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d^2+6*f^
3*polylog(3,-exp(d*x+c))/a/d^4-6*f^3*polylog(3,exp(d*x+c))/a/d^4+3/2*b*f^2
*(f*x+e)*polylog(3,exp(2*d*x+2*c))/a^2/d^3-6*(a^2+b^2)*f^2*(f*x+e)*polylog
(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d^3-6*(a^2+b^2)*f^2*(f*x+e)*po
lylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d^3-3/4*b*f^3*polylog(4,e
xp(2*d*x+2*c))/a^2/d^4+6*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/a^2/b/d^4+6*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/a^2/b/d^4
```

3.459.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2696 vs. $2(718) = 1436$.

Time = 10.57 (sec) , antiderivative size = 2696, normalized size of antiderivative = 3.75

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(2*d^3*e^2*(-1 + E^(2*c))*f*(b*d*e - 3*a*f)*x + 2*d^3*e^2*(-1 + E^(2*c))*f
*(b*d*e + 3*a*f)*x + b*d^4*(e + f*x)^4 - 6*d^2*e*(-1 + E^(2*c))*f^2*(b*d*e
 - 2*a*f)*x*Log[1 - E^(-c - d*x)] - 6*d^2*(-1 + E^(2*c))*f^3*(b*d*e - a*f)
*x^2*Log[1 - E^(-c - d*x)] - 2*b*d^3*(-1 + E^(2*c))*f^4*x^3*Log[1 - E^(-c
- d*x)] - 6*d^2*e*(-1 + E^(2*c))*f^2*(b*d*e + 2*a*f)*x*Log[1 + E^(-c - d*x
)] - 6*d^2*(-1 + E^(2*c))*f^3*(b*d*e + a*f)*x^2*Log[1 + E^(-c - d*x)] - 2*
b*d^3*(-1 + E^(2*c))*f^4*x^3*Log[1 + E^(-c - d*x)] - 2*d^2*e^2*(-1 + E^(2*
c))*f*(b*d*e - 3*a*f)*Log[1 - E^(c + d*x)] - 2*d^2*e^2*(-1 + E^(2*c))*f*(b
*d*e + 3*a*f)*Log[1 + E^(c + d*x)] + 6*d*e*(-1 + E^(2*c))*f^2*(b*d*e + 2*a
*f)*PolyLog[2, -E^(-c - d*x)] + 12*d*(-1 + E^(2*c))*f^3*(b*d*e + a*f)*x*Po
lyLog[2, -E^(-c - d*x)] + 6*b*d^2*(-1 + E^(2*c))*f^4*x^2*PolyLog[2, -E^(-c
 - d*x)] + 6*d*e*(-1 + E^(2*c))*f^2*(b*d*e - 2*a*f)*PolyLog[2, E^(-c - d*x
)] + 12*d*(-1 + E^(2*c))*f^3*(b*d*e - a*f)*x*PolyLog[2, E^(-c - d*x)] + 6*
b*d^2*(-1 + E^(2*c))*f^4*x^2*PolyLog[2, E^(-c - d*x)] + 12*(-1 + E^(2*c))*
f^3*(b*d*e + a*f)*PolyLog[3, -E^(-c - d*x)] + 12*b*d*(-1 + E^(2*c))*f^4*x*
PolyLog[3, -E^(-c - d*x)] - 12*(-1 + E^(2*c))*f^3*(-(b*d*e) + a*f)*PolyLog
[3, E^(-c - d*x)] + 12*b*d*(-1 + E^(2*c))*f^4*x*PolyLog[3, E^(-c - d*x)] +
 12*b*(-1 + E^(2*c))*f^4*PolyLog[4, -E^(-c - d*x)] + 12*b*(-1 + E^(2*c))*f
^4*PolyLog[4, E^(-c - d*x)])/(2*a^2*d^4*(-1 + E^(2*c))*f) - ((a^2 + b^2)*(
4*e^3*E^(2*c)*x + 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)*f...
```

3.459.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6119}$$

$$\frac{\int (e+fx)^3\cosh(c+dx)\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5973}$$

$$\frac{\int (e+fx)^3\cosh(c+dx)dx+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{3if\int -i(e+fx)^2\sinh(c+dx)dx}{d}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{-\frac{3f\int (e+fx)^2\sinh(c+dx)dx}{d}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{3f\int -i(e+fx)^2\sin(ic+idx)dx}{d}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3if\int (e+fx)^2\sin(ic+idx)dx}{d}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\cosh(c+dx)dx}{d}\right)}{d}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}$$

3.459. $\displaystyle\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3777}$$

$$\frac{-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{if\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{d}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}+\frac{if\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c}{d}}{a}$$

$$\Big\downarrow \text{3118}$$

$$\frac{-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{5975}$$

3.459. $\displaystyle\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3f\int (e+fx)^2\text{csch}(c+dx)dx}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{(e+fx)^3\text{csch}(c+}{d}}{a}}{}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3f\int i(e+fx)^2\csc(ic+idx)dx}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{(e+fx)^3\text{csch}(c-}{d}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3if\int (e+fx)^2\csc(ic+idx)dx}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}+\frac{(e+fx)^3\sinh(c+dx)}{d}-\frac{(e+fx)^3\text{csch}(c-}{d}}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3if\left(\frac{2if\int (e+fx)\log\left(1-e^{c+dx}\right)dx}{d}-\frac{2if\int (e+fx)\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\left(\frac{(e+fx)\sinh(c+d}{d}\right.}{d}\right.}{d}}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3if\left(-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}+\frac{2i(e+fx)^2\text{arctanh}}{d}\right.}{d}}{a}$$

$$\Big\downarrow \text{2720}$$

3.459. $\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$3if\Bigg(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,e^{c+dx}\right.}{d}\right.}{d}$$
$$\overline{\qquad d \qquad}$$

$$\Big\downarrow \text{ 6119}$$

$$-\frac{b\left(\frac{\int(e+fx)^3\cosh^2(c+dx)\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+$$

$$3if\Bigg(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,e^{c+dx}\right.}{d}\right.}{d}$$
$$\overline{\qquad d \qquad}$$

$$\Big\downarrow \text{ 5973}$$

$$-\frac{b\left(\frac{\int(e+fx)^3\coth(c+dx)dx+\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+$$

$$3if\Bigg(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,e^{c+dx}\right.}{d}\right.}{d}$$
$$\overline{\qquad d \qquad}$$

$$\Big\downarrow \text{ 3042}$$

$$3if\Bigg(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\mathrm{PolyLog}\left(2,e^{c+dx}\right.}{d}\right.}{d}$$
$$\overline{\qquad d \qquad}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx+\int -i(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 26}$$

3.459. $\displaystyle\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx-i\int(e+fx)^3\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{4201}$$

$$\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^3}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^4}{4f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\int(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{3011}$$

3.459. $\displaystyle\int\frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3 i f\left(-\frac{2 i f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2 i f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right)}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^3\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{d}\right)}{}\right)\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 5969$$

$$\frac{3 i f\left(-\frac{2 i f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2 i f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right)}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}\right)\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{3 i f\left(-\frac{2 i f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2 i f\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right)}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}\right)\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 25$$

3.459. $\displaystyle\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right.}{d}\right.}{d}\right.}{d}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-}\right.}{2d}\right.}{2d}\right.\right.}{a}\right.}{a}$$

input
```
Int[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.459.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.459. $\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.459. $\int \frac{(e+fx)^3 \cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.459.4 Maple [F]

$$\int \frac{(fx+e)^3 \cosh (dx+c) \coth (dx+c)^2}{a+b\sinh (dx+c)}dx$$

input
```
int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.459.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5829 vs. $2(674) = 1348$.

Time = 0.36 (sec) , antiderivative size = 5829, normalized size of antiderivative = 8.12

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.459.6 Sympy [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3\cosh\left(c+dx\right)\coth^2\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate((f*x+e)**3*cosh(d*x+c)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*cosh(c + d*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x))
, x)
```

3.459.7 Maxima [F]

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\cosh\left(dx+c\right)\coth\left(dx+c\right)^2}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
e^3*((d*x + c)/(b*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log
(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + (a^2 + b^2)
*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*b*d)) - 3*e^2*f*log(
e^(d*x + c) + 1)/(a*d^2) + 3*e^2*f*log(e^(d*x + c) - 1)/(a*d^2) - 1/4*(a*d
*f^3*x^4 + 4*a*d*e*f^2*x^3 + 6*a*d*e^2*f*x^2 - (a*d*f^3*x^4*e^(2*c) + 4*a*
d*e*f^2*x^3*e^(2*c) + 6*a*d*e^2*f*x^2*e^(2*c))*e^(2*d*x) + 8*(b*f^3*x^3*e^
c + 3*b*e*f^2*x^2*e^c + 3*b*e^2*f*x*e^c)*e^(d*x))/(a*b*d*e^(2*d*x + 2*c) -
 a*b*d) - (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) -
6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b*f^3/(a^2*d^
4) - (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x
*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b*f^3/(a^2*d^4) - 3*
(b*d*e^2*f + 2*a*e*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(
a^2*d^3) - 3*(b*d*e^2*f - 2*a*e*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^
(d*x + c)))/(a^2*d^3) - 3*(b*d*e*f^2 + a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1
) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^2*d^4) - 3*
(b*d*e*f^2 - a*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x +
c)) - 2*polylog(3, e^(d*x + c)))/(a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e
*f^2 + a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f + 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) +
 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 - a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f - 2*a
*d*e*f^2)*d^2*x^2)/(a^2*d^4) - integrate(-2*((a^2*b*f^3 + b^3*f^3)*x^3 ...
```

3.459.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.459.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.460 $\int \frac{(e+fx)^2 \cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.460.1 Optimal result

Integrand size = 34, antiderivative size = 518

$$\begin{aligned}
&\int \frac{(e+fx)^2 \cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{b(e+fx)^3}{3a^2f}-\frac{(a^2+b^2)(e+fx)^3}{3a^2bf}-\frac{4f(e+fx)\operatorname{arctanh}(e^{c+dx})}{ad^2}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{ad} \\
&+\frac{(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd}+\frac{(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd} \\
&-\frac{b(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{a^2d}-\frac{2f^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^3} \\
&+\frac{2f^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^3}+\frac{2(a^2+b^2)f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^2} \\
&+\frac{2(a^2+b^2)f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd^2} \\
&-\frac{bf(e+fx)\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{a^2d^2}-\frac{2(a^2+b^2)f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^3} \\
&-\frac{2(a^2+b^2)f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd^3}+\frac{bf^2\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^2d^3}
\end{aligned}$$

output
```
1/3*b*(f*x+e)^3/a^2/f-1/3*(a^2+b^2)*(f*x+e)^3/a^2/b/f-4*f*(f*x+e)*arctanh(
exp(d*x+c))/a/d^2-(f*x+e)^2*csch(d*x+c)/a/d-b*(f*x+e)^2*ln(1-exp(2*d*x+2*c
))/a^2/d+(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/
d+(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d-2*f^2
*polylog(2,-exp(d*x+c))/a/d^3+2*f^2*polylog(2,exp(d*x+c))/a/d^3-b*f*(f*x+e
)*polylog(2,exp(2*d*x+2*c))/a^2/d^2+2*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d^2+2*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*
exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d^2+1/2*b*f^2*polylog(3,exp(2*d*x+2*
c))/a^2/d^3-2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a
^2/b/d^3-2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/
b/d^3
```

3.460.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1806 vs. $2(518) = 1036$.

Time = 10.06 (sec) , antiderivative size = 1806, normalized size of antiderivative = 3.49

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(3*d^2*e*(-1 + E^(2*c))*f*(b*d*e - 2*a*f)*x + 3*d^2*e*(-1 + E^(2*c))*f*(b*
d*e + 2*a*f)*x + 2*b*d^3*(e + f*x)^3 - 6*d*(-1 + E^(2*c))*f^2*(b*d*e - a*f
)*x*Log[1 - E^(-c - d*x)] - 3*b*d^2*(-1 + E^(2*c))*f^3*x^2*Log[1 - E^(-c -
 d*x)] - 6*d*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*x*Log[1 + E^(-c - d*x)] - 3*
b*d^2*(-1 + E^(2*c))*f^3*x^2*Log[1 + E^(-c - d*x)] - 3*d*e*(-1 + E^(2*c))*
f*(b*d*e - 2*a*f)*Log[1 - E^(c + d*x)] - 3*d*e*(-1 + E^(2*c))*f*(b*d*e + 2
*a*f)*Log[1 + E^(c + d*x)] + 6*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*PolyLog[2,
 -E^(-c - d*x)] + 6*b*d*(-1 + E^(2*c))*f^3*x*PolyLog[2, -E^(-c - d*x)] - 6
*(-1 + E^(2*c))*f^2*(-(b*d*e) + a*f)*PolyLog[2, E^(-c - d*x)] + 6*b*d*(-1
+ E^(2*c))*f^3*x*PolyLog[2, E^(-c - d*x)] + 6*b*(-1 + E^(2*c))*f^3*PolyLog
[3, -E^(-c - d*x)] + 6*b*(-1 + E^(2*c))*f^3*PolyLog[3, E^(-c - d*x)])/(3*a
^2*d^3*(-1 + E^(2*c))*f) - ((a^2 + b^2)*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x
^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x
))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]
*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3
/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^
2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*
ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*
e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Lo
g[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b*...
```

3.460.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6119}$$

$$\frac{\int (e+fx)^2\cosh(c+dx)\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5973}$$

$$\frac{\int (e+fx)^2\cosh(c+dx)dx+\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \; 3777$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}+\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{2f\int (e+fx)\sinh(c+dx)dx}{d}+\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}+\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2if\int (e+fx)\sin(ic+idx)dx}{d}+\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 3777$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \cosh(c+dx)dx}{d}\right)}{d}+\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}+\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}$$

3.460. $\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\Big\downarrow \; 3117$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 5975$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2f\int (e+fx)\text{csch}(c+dx)dx}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2f\int i(e+fx)\csc(ic+idx)dx}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2if\int (e+fx)\csc(ic+idx)dx}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 4670$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2if\left(\frac{if\int \log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx}{d}}{a}$$

$$\Big\downarrow \; 2715$$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2if\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right.}{d}}{a}$$

$$\Big\downarrow \; 2838$$

3.460. $\displaystyle \int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+d}{d}}{a}$$

$$\Big\downarrow \text{6119}$$

$$-\frac{b\left(\frac{\int (e+fx)^2\cosh^2(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+d}{d}}{a}$$

$$\Big\downarrow \text{5973}$$

$$-\frac{b\left(\frac{\int (e+fx)^2\coth(c+dx)dx+\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+d}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+d}{d}}{a}$$
$$\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx+\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+d}{d}}{a}$$
$$\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{4201}$$

3.460. $\displaystyle\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+d}{d}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+d}{d}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+d}{d}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\right.}{}\right.\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+d}{d}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c}\right.}{4d^2}\right.}{}\right.\right.}{a}\right.}{a}$$

3.460. $\displaystyle\int\frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\downarrow \; 5969$$

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+d}{d}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyL}}{}\right.}{d}\right.\right.}{a}\right.}{a}$$

$$\downarrow \; 3042$$

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+d}{d}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyL}}{}\right.}{d}\right.\right.}{a}\right.}{a}$$

$$\downarrow \; 25$$

$$\frac{\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+d}{d}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyL}}{}\right.}{d}\right.\right.}{a}\right.}{a}$$

$$\downarrow \; 3791$$

3.460. $\displaystyle\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+d}{d}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyL}}{}\right.}{d}\right.\right.}{}\right.}{a}$$

$$\Big\downarrow \; 17$$

$$\frac{\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+d}{d}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyL}}{}\right.}{d}\right.\right.}{}\right.}{a}$$

$$\Big\downarrow \; 6099$$

$$\frac{\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+d}{d}}{a}$$

$$b\left(-\frac{b\left(\frac{\left(a^2+b^2\right)\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)^2\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\right.\right.}{}\right.$$

input
```
Int[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.460. $\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.460.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.460. $\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.460. $\int \frac{(e+fx)^2 \cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.460.4 Maple [F]

$$\int \frac{(fx+e)^2 \cosh(dx+c) \coth(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.460.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3506 vs. $2(486) = 972$.

Time = 0.32 (sec) , antiderivative size = 3506, normalized size of antiderivative = 6.77

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
1/3*(a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^2 + 3*a^2*d^3*e^2*x + 6*a^2*c*d^2*e
^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2 - (a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^
2 + 3*a^2*d^3*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2)*c
osh(d*x + c)^2 - (a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^2 + 3*a^2*d^3*e^2*x +
6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2)*sinh(d*x + c)^2 - 6*(a*
b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + a*b*d^2*e^2)*cosh(d*x + c) - 6*((a^2 + b
^2)*d*f^2*x + (a^2 + b^2)*d*e*f - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f
)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x +
 c)*sinh(d*x + c) - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*sinh(d*x + c
)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 6*((a^2 + b^2)*d*f^2*x + (a^
2 + b^2)*d*e*f - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)^2
 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)*sinh(d*x + c)
 - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*sinh(d*x + c)^2)*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b + 1) + 6*(b^2*d*f^2*x + b^2*d*e*f - a*b*f^2 - (b^2*d
*f^2*x + b^2*d*e*f - a*b*f^2)*cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*e*f
 - a*b*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b^2*d*f^2*x + b^2*d*e*f - a*b*f
^2)*sinh(d*x + c)^2)*dilog(cosh(d*x + c) + sinh(d*x + c)) + 6*(b^2*d*f^2*x
 + b^2*d*e*f + a*b*f^2 - (b^2*d*f^2*x + b^2*d*e*f + a*b*f^2)*cosh(d*x +...
```

3.460.6 Sympy [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*cosh(d*x+c)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*cosh(c + d*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x))
, x)
```

3.460.7 Maxima [F]

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)^2\cosh(dx+c)\coth(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
e^2*((d*x + c)/(b*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log
(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + (a^2 + b^2)
*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*b*d)) - 1/3*(a*d*f^2
*x^3 + 3*a*d*e*f*x^2 - (a*d*f^2*x^3*e^(2*c) + 3*a*d*e*f*x^2*e^(2*c))*e^(2*
d*x) + 6*(b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^(d*x))/(a*b*d*e^(2*d*x + 2*c) -
 a*b*d) - 2*e*f*log(e^(d*x + c) + 1)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/
(a*d^2) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*po
lylog(3, -e^(d*x + c)))*b*f^2/(a^2*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) +
 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*b*f^2/(a^2*d^3) - 2
*(b*d*e*f + a*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d
^3) - 2*(b*d*e*f - a*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))
/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f + a*f^2)*d^2*x^2)/(a^2*d^3) +
 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f - a*f^2)*d^2*x^2)/(a^2*d^3) - integrate(-
2*((a^2*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*e*f + b^3*e*f)*x - ((a^3*f^2*e^c +
 a*b^2*f^2*e^c)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e*f*e^c)*x)*e^(d*x))/(a^2*b^2
*e^(2*d*x + 2*c) + 2*a^3*b*e^(d*x + c) - a^2*b^2), x)
```

3.460.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.460.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.461 $\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.461.1 Optimal result

Integrand size = 32, antiderivative size = 324

$$\begin{aligned}
&\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{b(e+fx)^2}{2a^2f} - \frac{(a^2+b^2)(e+fx)^2}{2a^2bf} - \frac{f\operatorname{arctanh}(\cosh(c+dx))}{ad^2} - \frac{(e+fx)\operatorname{csch}(c+dx)}{ad} \\
&\quad + \frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd} + \frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd} \\
&\quad - \frac{b(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^2d} + \frac{(a^2+b^2)f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^2} \\
&\quad + \frac{(a^2+b^2)f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd^2} - \frac{bf\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^2d^2}
\end{aligned}$$

output
```
1/2*b*(f*x+e)^2/a^2/f-1/2*(a^2+b^2)*(f*x+e)^2/a^2/b/f-f*arctanh(cosh(d*x+c
))/a/d^2-(f*x+e)*csch(d*x+c)/a/d-b*(f*x+e)*ln(1-exp(2*d*x+2*c))/a^2/d+(a^2
+b^2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d+(a^2+b^2)*(f*
x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d-1/2*b*f*polylog(2,exp(
2*d*x+2*c))/a^2/d^2+(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)
))/a^2/b/d^2+(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/
b/d^2
```

3.461.2 Mathematica [A] (verified)

Time = 7.16 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.56

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{-\frac{bd^2(e+fx)^2}{f} - ad(e+fx)\coth\left(\frac{1}{2}(c+dx)\right) - 2(-af+bd(e+fx))\log\left(1-e^{-c-dx}\right) - 2(af+bd(e+fx))}{}$$

input
```
Integrate[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),
x]
```

output
```
(-((b*d^2*(e + f*x)^2)/f) - a*d*(e + f*x)*Coth[(c + d*x)/2] - 2*(-(a*f) +
b*d*(e + f*x))*Log[1 - E^(-c - d*x)] - 2*(a*f + b*d*(e + f*x))*Log[1 + E^(
-c - d*x)] + 2*b*f*PolyLog[2, -E^(-c - d*x)] + 2*b*f*PolyLog[2, E^(-c - d*
x)] + ((a^2 + b^2)*(-2*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (
4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt
[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x
))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2
*d*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E
^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))]))/b + a*d*(e + f*x)*Tanh[(c + d*x)/2])/(2*a^2*d^2)
```

3.461.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6119}$$

$$\frac{\int (e+fx)\cosh(c+dx)\coth^2(c+dx)dx}{a} - \frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 5973}$$

$$\frac{\int (e+fx)\cosh(c+dx)dx+\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx+\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \; 3777$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx-\frac{if\int -i\sinh(c+dx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx-\frac{f\int \sinh(c+dx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx-\frac{f\int -i\sin(ic+idx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx+\frac{if\int \sin(ic+idx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 3118$$

$$\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 5975$$

3.461. $\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\int \text{csch}(c+dx)dx}{d}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{f\int i\csc(ic+idx)dx}{d}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{if\int \csc(ic+idx)dx}{d}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}$$

$$\Big\downarrow \; 4257$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 6119$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)\cosh^2(c+dx)\coth(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 5973$$

$$\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)\coth(c+dx)dx+\int (e+fx)\cosh(c+dx)\sinh(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 3042$$

3.461. $\displaystyle\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx+\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\int(e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 4201}$$

$$\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2\ldots}}{4d^2}\right.\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{ 2838}$$

3.461. $\displaystyle\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right.}{a}\right.}{a}$$

$$\Big\downarrow \; 5969$$

$$\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{f\int\sinh^2(c+dx)dx}{2d}-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2}{2d}}{a}\right.}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{f\int-\sin(ic+idx)^2dx}{2d}-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sin}{2}}{a}\right.}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{f\int\sin(ic+idx)^2dx}{2d}-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(}{2d}}{a}\right.}{a}$$

$$\Big\downarrow \; 3115$$

$$\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{f\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{a}\right.}{a}$$

$$\Big\downarrow \; 24$$

3.461. $\displaystyle\int\frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-$$
$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh}{}\right.}{}}{a}\right.}{a}$$

$$\Big\downarrow \color{blue}{6099}$$

$$\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-$$
$$\frac{b\left(-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\int(e+fx)\cosh(c+dx)dx}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+f}{}\right.\right.}{}\right.}{a}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-$$
$$\frac{b\left(\frac{-i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}-\frac{b\left(\frac{(a^2+}{}\right.}{}\right.}{a}$$

$$\Big\downarrow \color{blue}{3777}$$

$$\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-$$
$$\frac{b\left(\frac{-i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}-\frac{b\left(\frac{(a^2+}{}\right.}{}\right.}{a}$$

$$\Big\downarrow \color{blue}{26}$$

$$\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-$$
$$\frac{b\left(-\frac{b\left(\frac{(a^2+b^2)\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{b^2}-\frac{a\left(\frac{(e+fx)\sinh(c+dx)}{d}-\frac{f\int\sinh(c+dx)dx}{d}\right)}{b^2}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)dx}{b}\right)}{a}+\frac{-i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c}\right.}{4d^2}\right.\right.}{}\right.}{a}$$

3.461. $\int\frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{-i\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)+\frac{(e+fx)\sinh^2(c+dx)}{2d}+\frac{f\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}}{a}-\frac{b\left(\frac{\left(a^2+\right.}{}\right.}{}\right.}{a}$$

input
```
Int[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.461.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.461. $\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6099
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Cos
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Cosh[c + d*x]^(n -
 2)*Sinh[c + d*x], x], x] + Simp[(a^2 + b^2)/b^2   Int[(e + f*x)^m*(Cosh[c
+ d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.461.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(306) = 612$.

Time = 3.02 (sec) , antiderivative size = 938, normalized size of antiderivative = 2.90

method	result
risch	$\frac{fb\ln\left(\frac{b\,e^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)c}{a^2d^2}+\frac{fb\ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)c}{a^2d^2}+\frac{fb\ln\left(\frac{b\,e^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)x}{a^2d}+\frac{fb\ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)}{a^2d}$

input
```
int((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
1/a^2/d^2*f*b*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1
/a^2/d^2*f*b*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+
1/a^2/d*f*b*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/a
^2/d*f*b*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/a^
2/d*f*b*ln(exp(d*x+c)+1)*x+1/a^2/d^2*b*c*f*ln(exp(d*x+c)-1)-1/a^2/d^2*b*c*
f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d*(f*x+e)/a*exp(d*x+c)/(exp(2*d*
x+2*c)-1)-1/d^2/b*f*c^2-2/d/b*e*ln(exp(d*x+c))+1/d/b*e*ln(b*exp(2*d*x+2*c)
+2*a*exp(d*x+c)-b)+1/d^2/b*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(
a^2+b^2)^(1/2)))+1/d^2/b*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+
b^2)^(1/2)))-1/2*f*x^2/b+e*x/b-1/d^2/b*c*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x
+c)-b)+1/d/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+
1/d/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2
/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+2/d^2/b*
c*f*ln(exp(d*x+c))-2/d/b*c*f*x+1/d^2/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+
a)/(a+(a^2+b^2)^(1/2)))*c-1/a^2/d*b*e*ln(exp(d*x+c)-1)-1/a^2/d*b*e*ln(exp(
d*x+c)+1)+1/a^2/d*b*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/a^2/d^2*f*b*
dilog(exp(d*x+c)+1)+1/a^2/d^2*f*b*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))+1/a^2/d^2*f*b*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)
/(a+(a^2+b^2)^(1/2)))+1/a^2/d^2*f*b*dilog(exp(d*x+c))+1/a/d^2*f*ln(exp(d*x
+c)-1)-1/a/d^2*f*ln(exp(d*x+c)+1)
```

3.461.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. $2(303) = 606$.

Time = 0.29 (sec) , antiderivative size = 1735, normalized size of antiderivative = 5.35

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
1/2*(a^2*d^2*f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f - (a^2*d^2*
f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f)*cosh(d*x + c)^2 - (a^2*
d^2*f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f)*sinh(d*x + c)^2 - 4
*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c) + 2*((a^2 + b^2)*f*cosh(d*x + c)^2 +
2*(a^2 + b^2)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c)^
2 - (a^2 + b^2)*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*((a^2 + b^2)
*f*cosh(d*x + c)^2 + 2*(a^2 + b^2)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2 +
b^2)*f*sinh(d*x + c)^2 - (a^2 + b^2)*f)*dilog((a*cosh(d*x + c) + a*sinh(d*
x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
+ 1) - 2*(b^2*f*cosh(d*x + c)^2 + 2*b^2*f*cosh(d*x + c)*sinh(d*x + c) + b^
2*f*sinh(d*x + c)^2 - b^2*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2*(b^2
*f*cosh(d*x + c)^2 + 2*b^2*f*cosh(d*x + c)*sinh(d*x + c) + b^2*f*sinh(d*x
+ c)^2 - b^2*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - 2*((a^2 + b^2)*d*e
 - (a^2 + b^2)*c*f - ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)^2 -
 2*((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2
 + b^2)*d*e - (a^2 + b^2)*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*
b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*((a^2 + b^2)*d*e -
(a^2 + b^2)*c*f - ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)^2 - 2*
((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2...
```

3.461.6 Sympy [F]

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)\cosh\left(c+dx\right)\coth^2\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*cosh(c + d*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x
)
```

3.461.7 Maxima [F]

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\cosh(dx+c)\coth(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
1/2*(2*b*d*integrate(x/(a^2*d*e^(d*x + c) + a^2*d), x) - 2*b*d*integrate(x
/(a^2*d*e^(d*x + c) - a^2*d), x) + 2*a*((d*x + c)/(a^2*d^2) - log(e^(d*x +
 c) + 1)/(a^2*d^2)) - 2*a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2
*d^2)) + (a*d*x^2*e^(2*d*x + 2*c) - a*d*x^2 - 4*b*x*e^(d*x + c))/(a*b*d*e^
(2*d*x + 2*c) - a*b*d) - integrate(4*((a^3*e^c + a*b^2*e^c)*x*e^(d*x) - (a
^2*b + b^3)*x)/(a^2*b^2*e^(2*d*x + 2*c) + 2*a^3*b*e^(d*x + c) - a^2*b^2),
x))*f + e*((d*x + c)/(b*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) -
 b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + (a^2
+ b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*b*d))
```

3.461.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.461.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)^2\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.462 $\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.462.1 Optimal result

Integrand size = 27, antiderivative size = 59

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\operatorname{csch}(c+dx)}{ad} - \frac{b\log(\sinh(c+dx))}{a^2d} + \frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^2bd}$$

output
```
-csch(d*x+c)/a/d-b*ln(sinh(d*x+c))/a^2/d+(a^2+b^2)*ln(a+b*sinh(d*x+c))/a^2
/b/d
```

3.462.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{-ab\operatorname{csch}(c+dx) - b^2\log(\sinh(c+dx)) + (a^2+b^2)\log(a+b\sinh(c+dx))}{a^2bd}$$

input
```
Integrate[(Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(a*b*Csch[c + d*x]) - b^2*Log[Sinh[c + d*x]] + (a^2 + b^2)*Log[a + b*Sin
h[c + d*x]])/(a^2*b*d)
```

3.462.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos(ic+idx)^3}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos(ic+idx)^3}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{\int \frac{\text{csch}^2(c+dx)(\sinh^2(c+dx)b^2+b^2)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\text{csch}^2(c+dx)(\sinh^2(c+dx)b^2+b^2)}{b^2(a+b\sinh(c+dx))}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{\text{csch}^2(c+dx)}{a}-\frac{b\text{csch}(c+dx)}{a^2}+\frac{a^2+b^2}{a^2(a+b\sinh(c+dx))}\right)d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^2\log(b\sinh(c+dx))}{a^2}+\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^2}-\frac{b\text{csch}(c+dx)}{a}}{bd}$$

input
```
Int[(Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-((b*Csch[c + d*x])/a) - (b^2*Log[b*Sinh[c + d*x]])/a^2 + ((a^2 + b^2)*Lo
g[a + b*Sinh[c + d*x]])/a^2)/(b*d)
```

3.462.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

3.462.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(59) = 118$.

Time = 1.69 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.17

method	result
risch	$-\frac{x}{b}-\frac{2c}{bd}-\frac{2\,\mathrm{e}^{dx+c}}{da\left(\mathrm{e}^{2dx+2c}-1\right)}-\frac{b\ln\left(\mathrm{e}^{2dx+2c}-1\right)}{a^{2}d}+\frac{\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\,\mathrm{e}^{dx+c}}{b}-1\right)}{bd}+\frac{b\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\,\mathrm{e}^{dx+c}}{b}-1\right)}{a^{2}d}$
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{\left(2a^{2}+2b^{2}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{2a^{2}b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-}{d}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{\left(2a^{2}+2b^{2}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{2a^{2}b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-}{d}$

input
```
int(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-x/b-2/b/d*c-2/d/a*exp(d*x+c)/(exp(2*d*x+2*c)-1)-b/a^2/d*ln(exp(2*d*x+2*c)
-1)+1/b/d*ln(exp(2*d*x+2*c)+2*a/b*exp(d*x+c)-1)+b/a^2/d*ln(exp(2*d*x+2*c)+
2*a/b*exp(d*x+c)-1)
```

3.462.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(59) = 118$.

Time = 0.26 (sec) , antiderivative size = 299, normalized size of antiderivative = 5.07

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{a^2dx\cosh(dx+c)^2 + a^2dx\sinh(dx+c)^2 - a^2dx + 2\,ab\cosh(dx+c) - \left((a^2+b^2)\cosh(dx+c)^2 + 2\,(\right.}{}$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-(a^2*d*x*cosh(d*x + c)^2 + a^2*d*x*sinh(d*x + c)^2 - a^2*d*x + 2*a*b*cosh
(d*x + c) - ((a^2 + b^2)*cosh(d*x + c)^2 + 2*(a^2 + b^2)*cosh(d*x + c)*sin
h(d*x + c) + (a^2 + b^2)*sinh(d*x + c)^2 - a^2 - b^2)*log(2*(b*sinh(d*x +
c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + (b^2*cosh(d*x + c)^2 + 2*b^2*co
sh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - b^2)*log(2*sinh(d*x + c)
/(cosh(d*x + c) - sinh(d*x + c))) + 2*(a^2*d*x*cosh(d*x + c) + a*b)*sinh(d
*x + c))/(a^2*b*d*cosh(d*x + c)^2 + 2*a^2*b*d*cosh(d*x + c)*sinh(d*x + c)
+ a^2*b*d*sinh(d*x + c)^2 - a^2*b*d)
```

3.462.6 Sympy [F]

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(cosh(c + d*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.462.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(59) = 118$.

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.22

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{dx+c}{bd} + \frac{2\,e^{(-dx-c)}}{(ae^{(-2\,dx-2\,c)}-a)d} - \frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d} - \frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d} + \frac{(a^2+b^2)\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{a^2bd}$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
(d*x + c)/(b*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-
d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + (a^2 + b^2)*log(
-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*b*d)
```

3.462.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(59) = 118$.

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.05

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{b\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a^2} - \frac{(a^2+b^2)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^2b} - \frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)-2\,a}{a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}}{d}$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-(b*log(abs(e^(d*x + c) - e^(-d*x - c)))/a^2 - (a^2 + b^2)*log(abs(b*(e^(d
*x + c) - e^(-d*x - c)) + 2*a))/(a^2*b) - (b*(e^(d*x + c) - e^(-d*x - c))
- 2*a)/(a^2*(e^(d*x + c) - e^(-d*x - c))))/d
```

3.462.9 Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 356, normalized size of antiderivative = 6.03

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2\,\mathrm{e}^{c+d\,x}}{a\,d-a\,d\,\mathrm{e}^{2\,c+2\,d\,x}} - \frac{x}{b}$$
$$+\frac{\ln\left(8\,a^5\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-16\,b^5-16\,a^2\,b^3-4\,a^4\,b+16\,b^5\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+4\,a^4\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+32\,a^3\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+16\,a^2\,b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}\right.}{b\,d}$$
$$+\frac{b\,\ln\left(8\,a^5\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-16\,b^5-16\,a^2\,b^3-4\,a^4\,b+16\,b^5\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+4\,a^4\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+32\,a^3\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+16\,a^2\,b^3\,\mathrm{e}^{2\,c}\right.}{a^2\,d}$$
$$-\frac{b\,\ln\left(4\,a^6+16\,b^6+32\,a^2\,b^4+20\,a^4\,b^2-4\,a^6\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-16\,b^6\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-32\,a^2\,b^4\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-20\,a^4\,b^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2}\right.}{a^2\,d}$$

input
```
int((cosh(c + d*x)*coth(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
(2*exp(c + d*x))/(a*d - a*d*exp(2*c + 2*d*x)) - x/b + log(8*a^5*exp(d*x)*e
xp(c) - 16*b^5 - 16*a^2*b^3 - 4*a^4*b + 16*b^5*exp(2*c)*exp(2*d*x) + 4*a^4
*b*exp(2*c)*exp(2*d*x) + 32*a^3*b^2*exp(d*x)*exp(c) + 16*a^2*b^3*exp(2*c)*
exp(2*d*x) + 32*a*b^4*exp(d*x)*exp(c))/(b*d) + (b*log(8*a^5*exp(d*x)*exp(c
) - 16*b^5 - 16*a^2*b^3 - 4*a^4*b + 16*b^5*exp(2*c)*exp(2*d*x) + 4*a^4*b*e
xp(2*c)*exp(2*d*x) + 32*a^3*b^2*exp(d*x)*exp(c) + 16*a^2*b^3*exp(2*c)*exp(
2*d*x) + 32*a*b^4*exp(d*x)*exp(c)))/(a^2*d) - (b*log(4*a^6 + 16*b^6 + 32*a
^2*b^4 + 20*a^4*b^2 - 4*a^6*exp(2*c)*exp(2*d*x) - 16*b^6*exp(2*c)*exp(2*d*
x) - 32*a^2*b^4*exp(2*c)*exp(2*d*x) - 20*a^4*b^2*exp(2*c)*exp(2*d*x)))/(a^
2*d)
```

3.463 $\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.463.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \mathrm{Int}\left(\frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.463.2 Mathematica [N/A]

Not integrable

Time = 54.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Cosh[c + d*x]*Coth[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Cosh[c + d*x]*Coth[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

3.463.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Cosh[c + d*x]*Coth[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.463.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.463.4 Maple [N/A] (verified)

Not integrable

Time = 0.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(dx+c)\coth(dx+c)^2}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.463.5 Fricas [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)\coth(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(cosh(d*x + c)*coth(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

3.463.6 Sympy [N/A]

Not integrable

Time = 5.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\coth^2(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(cosh(c + d*x)*coth(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)),
 x)
```

3.463.7 Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 353, normalized size of antiderivative = 10.38

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(dx+c)\coth(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
2*e^(d*x + c)/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*
x)) + log(f*x + e)/(b*f) - 1/2*integrate(-2*(b*d*f*x + b*d*e + a*f)/(a^2*d
*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*
e^c + a^2*d*e^2*e^c)*e^(d*x)), x) + 1/2*integrate(2*(b*d*f*x + b*d*e - a*f
)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*
d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) - 1/2*integrate(4*(a^2*b + b^3 -
 (a^3*e^c + a*b^2*e^c)*e^(d*x))/(a^2*b^2*f*x + a^2*b^2*e - (a^2*b^2*f*x*e^
(2*c) + a^2*b^2*e*e^(2*c))*e^(2*d*x) - 2*(a^3*b*f*x*e^c + a^3*b*e*e^c)*e^(
d*x)), x)
```

3.463.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.463.9 Mupad [N/A]

Not integrable

Time = 3.91 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((cosh(c + d*x)*coth(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((cosh(c + d*x)*coth(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.464 $\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.464.1 Optimal result

Integrand size = 34, antiderivative size = 1428

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
2*b^2*(f*x+e)^3*arctan(exp(d*x+c))/a/(a^2+b^2)/d+3/2*b*f*(f*x+e)^2*polylog
(2,-exp(2*d*x+2*c))/a^2/d^2-3/2*b*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/a
^2/d^3-3/4*b^3*f^3*polylog(4,-exp(2*d*x+2*c))/a^2/(a^2+b^2)/d^4+6*b^3*f^3*
polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^4+6*b^3*f^3*p
olylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^4-3*I*f*(f*x+e
)^2*polylog(2,I*exp(d*x+c))/a/d^2-6*I*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))
/a/d^3+6*I*b^2*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/a/(a^2+b^2)/d^3+3*I*f*
(f*x+e)^2*polylog(2,-I*exp(d*x+c))/a/d^2+6*I*f^2*(f*x+e)*polylog(3,I*exp(d
*x+c))/a/d^3+6*f^3*polylog(3,-exp(d*x+c))/a/d^4-6*f^3*polylog(3,exp(d*x+c)
)/a/d^4-3*I*b^2*f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/a/(a^2+b^2)/d^2-6*I*b
^2*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/a/(a^2+b^2)/d^3-(f*x+e)^3*csch(d*x+
c)/a/d-3/2*b^3*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/a^2/(a^2+b^2)/d^2+3/
2*b^3*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/a^2/(a^2+b^2)/d^3+3*b^3*f*(f*
x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2+3*b^
3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d
^2-6*b^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2
+b^2)/d^3-6*b^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a
^2/(a^2+b^2)/d^3-6*I*b^2*f^3*polylog(4,-I*exp(d*x+c))/a/(a^2+b^2)/d^4+6*I*
f^3*polylog(4,-I*exp(d*x+c))/a/d^4-b^3*(f*x+e)^3*ln(1+exp(2*d*x+2*c))/a^2/
(a^2+b^2)/d+b^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a...
```

3.464.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4139 vs. 2(1428) = 2856.

Time = 11.13 (sec) , antiderivative size = 4139, normalized size of antiderivative = 2.90

$$\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(2*d^3*e^2*(-1 + E^(2*c))*f*(b*d*e - 3*a*f)*x + 2*d^3*e^2*(-1 + E^(2*c))*f
*(b*d*e + 3*a*f)*x + b*d^4*(e + f*x)^4 - 6*d^2*e*(-1 + E^(2*c))*f^2*(b*d*e
 - 2*a*f)*x*Log[1 - E^(-c - d*x)] - 6*d^2*(-1 + E^(2*c))*f^3*(b*d*e - a*f)
*x^2*Log[1 - E^(-c - d*x)] - 2*b*d^3*(-1 + E^(2*c))*f^4*x^3*Log[1 - E^(-c
- d*x)] - 6*d^2*e*(-1 + E^(2*c))*f^2*(b*d*e + 2*a*f)*x*Log[1 + E^(-c - d*x
)] - 6*d^2*(-1 + E^(2*c))*f^3*(b*d*e + a*f)*x^2*Log[1 + E^(-c - d*x)] - 2*
b*d^3*(-1 + E^(2*c))*f^4*x^3*Log[1 + E^(-c - d*x)] - 2*d^2*e^2*(-1 + E^(2*
c))*f*(b*d*e - 3*a*f)*Log[1 - E^(c + d*x)] - 2*d^2*e^2*(-1 + E^(2*c))*f*(b
*d*e + 3*a*f)*Log[1 + E^(c + d*x)] + 6*d*e*(-1 + E^(2*c))*f^2*(b*d*e + 2*a
*f)*PolyLog[2, -E^(-c - d*x)] + 12*d*(-1 + E^(2*c))*f^3*(b*d*e + a*f)*x*Po
lyLog[2, -E^(-c - d*x)] + 6*b*d^2*(-1 + E^(2*c))*f^4*x^2*PolyLog[2, -E^(-c
 - d*x)] + 6*d*e*(-1 + E^(2*c))*f^2*(b*d*e - 2*a*f)*PolyLog[2, E^(-c - d*x
)] + 12*d*(-1 + E^(2*c))*f^3*(b*d*e - a*f)*x*PolyLog[2, E^(-c - d*x)] + 6*
b*d^2*(-1 + E^(2*c))*f^4*x^2*PolyLog[2, E^(-c - d*x)] + 12*(-1 + E^(2*c))*
f^3*(b*d*e + a*f)*PolyLog[3, -E^(-c - d*x)] + 12*b*d*(-1 + E^(2*c))*f^4*x*
PolyLog[3, -E^(-c - d*x)] - 12*(-1 + E^(2*c))*f^3*(-(b*d*e) + a*f)*PolyLog
[3, E^(-c - d*x)] + 12*b*d*(-1 + E^(2*c))*f^4*x*PolyLog[3, E^(-c - d*x)] +
 12*b*(-1 + E^(2*c))*f^4*PolyLog[4, -E^(-c - d*x)] + 12*b*(-1 + E^(2*c))*f
^4*PolyLog[4, E^(-c - d*x)])/(2*a^2*d^4*(-1 + E^(2*c))*f) - (8*b*d^4*e^3*E
^(2*c)*x + 12*b*d^4*e^2*E^(2*c)*f*x^2 + 8*b*d^4*e*E^(2*c)*f^2*x^3 + 2*b...
```

3.464.3 Rubi [A] (verified)

Time = 6.19 (sec) , antiderivative size = 1259, normalized size of antiderivative = 0.88, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {6123, 5985, 25, 6123, 5984, 3042, 26, 4670, 3011, 6107, 6095, 2620, 3011, 7163, 2720, 7143, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6123}$$

$$\frac{\int (e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 5985}$$

$$\frac{-3f\int -(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 6123}$$

$$\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 5984}$$

$$\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{2\int (e+fx)^3\text{csch}(2c+2dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2\int i(e+fx)^3\csc(2ic+2idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\int (e+fx)^3\csc(2ic+2idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \; 4670$$

$$\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{3if\int (e+fx)^2\log\left(1-e^{2c+2dx}\right)dx}{2d}-\frac{3if\int (e+fx)^2\log\left(1+e^{2c+2dx}\right)dx}{2d}+\frac{i(e+fx)^3\text{arctanh}\left(e^{2c+2dx}\right)}{d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 3011$$

$$\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(-\frac{3if\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,e^{2c+2\cdots}\right.}{d}\cdots\right.}{\cdots}\cdots\right)}{a}\cdots\right)}{a}$$

$$\Big\downarrow \; 6107$$

3.464. $\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{b^2\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+2i\left(-\frac{3if\left(\frac{f\int(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2\cdots\right.}{2d}\right.}{2d}\cdots\right.\right.$$

$$\cdots\quad a$$

$\downarrow$ 6095

$$\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^4}{4bf}\right)}{a^2+b^2}+\frac{\int(e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+2i\left(-\frac{3if\left(f\int(e+fx)\text{PolyL}\cdots\right.}{\cdots}\right.\right.$$

$\downarrow$ 2620

$$\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{b^2\left(-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\cdots\right.}{a^2+b^2}\right.}{a}\right.$$

$\downarrow$ 3011

3.464. $\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{b^2\left(-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)}{}\right.}{bd}\right.}{a^2+b^2}\right.}{}\right.$$

$\downarrow$ 7163

3.464. $\int\frac{(e+fx)^3\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\int\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{Po}}{}\right.}{}\right.}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 2720

3.464. $\int\frac{(e+fx)^3\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^3}{d}+3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx}{a}-$$

$$b\left(\frac{2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^3}{d}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{2d}-\frac{f\int e^{-2c-2dx}\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2}\right.}{2d}}{2d}}{a}\right.$$

$\downarrow$ 7143

3.464. $\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{\int(e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2 \ldots\right.}{d}\right.}{bd}\right.}{}\right.}{}\right.$$

$\downarrow$ 7292

3.464. $\int\frac{(e+fx)^3\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3f\int \frac{(e+fx)^2(\arctan(\sinh(c+dx))+\operatorname{csch}(c+dx))}{d}dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{\int (e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2\right.}{d}\right.}{bd}\right.}{}\right.}{}\right.$$

$$\Big\downarrow 27$$

3.464. $\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{3f\int (e+fx)^2(\arctan(\sinh(c+dx))+\operatorname{csch}(c+dx))dx}{d}-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}-$$

$$a$$

$$b\left(-\frac{b\left(\frac{\int (e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2\right.}{d}\right.}{bd}\right.}{}\right.}{}\right.$$

$\downarrow$ 7293

3.464. $\displaystyle \int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3f\int\left(\arctan(\sinh(c+dx))(e+fx)^2+\operatorname{csch}(c+dx)(e+fx)^2\right)dx}{d}-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{\int\left(a(e+fx)^3\operatorname{sech}(c+dx)-b(e+fx)^3\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{3f\left(\frac{2f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}-\frac{f\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}\right)}{d}-\frac{(e+fx)^2}{}\right.}{bd}\right.}{}\right.}{}\right.$$

$\downarrow$ 2009

3.464. $\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^3}{d}+\frac{3f\left(-\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^3}{3f}+\frac{\arctan(\sinh(c+dx))(e+fx)^3}{3f}-\frac{2\operatorname{arctanh}\left(e^{c+dx}\right)(e+fx)}{d}\right.}{}$$

$$b\left(\frac{2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^3}{d}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{2d}-\frac{f\operatorname{PolyLog}\left(4,-e^{2c+2dx}\right)}{4d^2}\right)}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{2d}+\frac{3if\left(\frac{f\left(\right.}{}\right.}{}\right.}{a}\right.$$

input
```
Int[((e + f*x)^3*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.464. $\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(-(((e + f*x)^3*ArcTan[Sinh[c + d*x]])/d) - ((e + f*x)^3*Csch[c + d*x])/d
+ (3*f*((-2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(3*f) + ((e + f*x)^3*ArcTan[S
inh[c + d*x]])/(3*f) - (2*(e + f*x)^2*ArcTanh[E^(c + d*x)])/d - (2*f*(e +
f*x)*PolyLog[2, -E^(c + d*x)])/d^2 + (I*(e + f*x)^2*PolyLog[2, (-I)*E^(c +
 d*x)])/d - (I*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/d + (2*f*(e + f*x)*P
olyLog[2, E^(c + d*x)])/d^2 + (2*f^2*PolyLog[3, -E^(c + d*x)])/d^3 - ((2*I
)*f*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/d^2 + ((2*I)*f*(e + f*x)*PolyL
og[3, I*E^(c + d*x)])/d^2 - (2*f^2*PolyLog[3, E^(c + d*x)])/d^3 + ((2*I)*f
^2*PolyLog[4, (-I)*E^(c + d*x)])/d^3 - ((2*I)*f^2*PolyLog[4, I*E^(c + d*x)
])/d^3))/d)/a - (b*(-((b*((b^2*(-1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (3*f*(-(((e + f*x)^2*PolyLo
g[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f*x)*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d) - (3*f*(-(((e + f*x)^
2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (2*f*(((e + f
*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d - (f*PolyLog[4
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d^2))/d))/(b*d)))/(a^2 + b^2)
 + ((b*(e + f*x)^4)/(4*f) + (2*a*(e + f*x)^3*ArcTan[E^(c + d*x)])/d - (b*(
e + f*x)^3*Log[1 + E^(2*(c + d*x))])/d - ((3*I)*a*f*(e + f*x)^2*PolyLog...
```

3.464.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.464. $\int \frac{(e+fx)^3 \text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.464. $\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.464. $\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.464.4 Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)^2 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.464.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 9763 vs. $2(1301) = 2602$.

Time = 0.49 (sec) , antiderivative size = 9763, normalized size of antiderivative = 6.84

$$\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.464.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.464.7 Maxima [F]

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\text{csch}\,(dx+c)^2\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
(b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) +
 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((
a^2 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*
x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e^3 - 2*(f^3*x^3*e^
c + 3*e*f^2*x^2*e^c + 3*e^2*f*x*e^c)*e^(d*x)/(a*d*e^(2*d*x + 2*c) - a*d) -
 3*e^2*f*log(e^(d*x + c) + 1)/(a*d^2) + 3*e^2*f*log(e^(d*x + c) - 1)/(a*d^
2) - (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x
*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b*f^3/(a^2*d^4) -
(d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*poly
log(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b*f^3/(a^2*d^4) - 3*(b*d*
e^2*f + 2*a*e*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d
^3) - 3*(b*d*e^2*f - 2*a*e*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x
+ c)))/(a^2*d^3) - 3*(b*d*e*f^2 + a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2
*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^2*d^4) - 3*(b*d*
e*f^2 - a*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) -
 2*polylog(3, e^(d*x + c)))/(a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2
+ a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f + 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) + 1/4*
(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 - a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f - 2*a*d*e*
f^2)*d^2*x^2)/(a^2*d^4) - integrate(2*(b^4*f^3*x^3 + 3*b^4*e*f^2*x^2 + 3*b
^4*e^2*f*x - (a*b^3*f^3*x^3*e^c + 3*a*b^3*e*f^2*x^2*e^c + 3*a*b^3*e^2*f...
```

3.464.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.464.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{(e+f\,x)^3}{\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.465 $\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} \, dx$

3.465.1 Optimal result

Integrand size = 34, antiderivative size = 982

$$
\begin{aligned}
&\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= -\frac{2(e+fx)^2\arctan\left(e^{c+dx}\right)}{ad}+\frac{2b^2(e+fx)^2\arctan\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} \\
&\quad -\frac{4f(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{ad^2}+\frac{2b(e+fx)^2\operatorname{arctanh}\left(e^{2c+2dx}\right)}{a^2d} \\
&\quad -\frac{(e+fx)^2\operatorname{csch}(c+dx)}{ad}+\frac{b^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d} \\
&\quad +\frac{b^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d}-\frac{b^3(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{a^2\left(a^2+b^2\right)d} \\
&\quad -\frac{2f^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^3}+\frac{2if(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2} \\
&\quad -\frac{2ib^2f(e+fx)\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{a\left(a^2+b^2\right)d^2}-\frac{2if(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{ad^2} \\
&\quad +\frac{2ib^2f(e+fx)\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{a\left(a^2+b^2\right)d^2}+\frac{2f^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^3} \\
&\quad +\frac{2b^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2}+\frac{2b^3f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2} \\
&\quad -\frac{b^3f(e+fx)\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{a^2\left(a^2+b^2\right)d^2}+\frac{bf(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{a^2d^2} \\
&\quad -\frac{bf(e+fx)\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{a^2d^2}-\frac{2if^2\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3} \\
&\quad +\frac{2ib^2f^2\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{a\left(a^2+b^2\right)d^3}+\frac{2if^2\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{ad^3}-\frac{2ib^2f^2\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{a\left(a^2+b^2\right)d^3} \\
&\quad -\frac{2b^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^3}-\frac{2b^3f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^3} \\
&\quad +\frac{b^3f^2\operatorname{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2a^2\left(a^2+b^2\right)d^3}-\frac{bf^2\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{2a^2d^3}+\frac{bf^2\operatorname{PolyLog}\left(3,e^{2c+2dx}\right)}{2a^2d^3}
\end{aligned}
$$

output
```
2*b^2*(f*x+e)^2*arctan(exp(d*x+c))/a/(a^2+b^2)/d+1/2*b^3*f^2*polylog(3,-ex
p(2*d*x+2*c))/a^2/(a^2+b^2)/d^3-2*b^3*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/a^2/(a^2+b^2)/d^3-2*b^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/a^2/(a^2+b^2)/d^3+2*I*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^2
+2*I*b^2*f^2*polylog(3,-I*exp(d*x+c))/a/(a^2+b^2)/d^3+2*I*b^2*f*(f*x+e)*po
lylog(2,I*exp(d*x+c))/a/(a^2+b^2)/d^2-b^3*f*(f*x+e)*polylog(2,-exp(2*d*x+2
*c))/a^2/(a^2+b^2)/d^2-2*f^2*polylog(2,-exp(d*x+c))/a/d^3+2*f^2*polylog(2,
exp(d*x+c))/a/d^3-2*I*b^2*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/(a^2+b^2)/d
^2-(f*x+e)^2*csch(d*x+c)/a/d-2*I*f*(f*x+e)*polylog(2,I*exp(d*x+c))/a/d^2+2
*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/
d^2+2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+
b^2)/d^2-2*I*b^2*f^2*polylog(3,I*exp(d*x+c))/a/(a^2+b^2)/d^3+b*f*(f*x+e)*p
olylog(2,-exp(2*d*x+2*c))/a^2/d^2+2*I*f^2*polylog(3,I*exp(d*x+c))/a/d^3-b^
3*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/a^2/(a^2+b^2)/d+b^3*(f*x+e)^2*ln(1+b*exp(
d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d+b^3*(f*x+e)^2*ln(1+b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d-4*f*(f*x+e)*arctanh(exp(d*x+c))/a/d
^2+1/2*b*f^2*polylog(3,exp(2*d*x+2*c))/a^2/d^3-b*f*(f*x+e)*polylog(2,exp(2
*d*x+2*c))/a^2/d^2+2*b*(f*x+e)^2*arctanh(exp(2*d*x+2*c))/a^2/d-1/2*b*f^2*p
olylog(3,-exp(2*d*x+2*c))/a^2/d^3-2*I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3-2
*(f*x+e)^2*arctan(exp(d*x+c))/a/d
```

3.465.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2323 vs. $2(982) = 1964$.

Time = 10.20 (sec) , antiderivative size = 2323, normalized size of antiderivative = 2.37

$$\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(3*d^2*e*(-1 + E^(2*c))*f*(b*d*e - 2*a*f)*x + 3*d^2*e*(-1 + E^(2*c))*f*(b*
d*e + 2*a*f)*x + 2*b*d^3*(e + f*x)^3 - 6*d*(-1 + E^(2*c))*f^2*(b*d*e - a*f
)*x*Log[1 - E^(-c - d*x)] - 3*b*d^2*(-1 + E^(2*c))*f^3*x^2*Log[1 - E^(-c -
 d*x)] - 6*d*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*x*Log[1 + E^(-c - d*x)] - 3*
b*d^2*(-1 + E^(2*c))*f^3*x^2*Log[1 + E^(-c - d*x)] - 3*d*e*(-1 + E^(2*c))*
f*(b*d*e - 2*a*f)*Log[1 - E^(c + d*x)] - 3*d*e*(-1 + E^(2*c))*f*(b*d*e + 2
*a*f)*Log[1 + E^(c + d*x)] + 6*(-1 + E^(2*c))*f^2*(b*d*e + a*f)*PolyLog[2,
 -E^(-c - d*x)] + 6*b*d*(-1 + E^(2*c))*f^3*x*PolyLog[2, -E^(-c - d*x)] - 6
*(-1 + E^(2*c))*f^2*(-(b*d*e) + a*f)*PolyLog[2, E^(-c - d*x)] + 6*b*d*(-1
+ E^(2*c))*f^3*x*PolyLog[2, E^(-c - d*x)] + 6*b*(-1 + E^(2*c))*f^3*PolyLog
[3, -E^(-c - d*x)] + 6*b*(-1 + E^(2*c))*f^3*PolyLog[3, E^(-c - d*x)])/(3*a
^2*d^3*(-1 + E^(2*c))*f) - (12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(
2*c))*x - 12*b*d^3*e*f*x^2 - 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*
ArcTan[E^(c + d*x)] + 6*b*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c +
 d*x))]) + (12*I)*a*d*e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log
[1 + I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c +
d*x)]) + 6*b*d*e*(1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) -
 PolyLog[2, -E^(2*(c + d*x))]) + (6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1
- I*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*
E^(c + d*x)] + 2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c...
```

3.465.3 Rubi [A] (verified)

Time = 4.69 (sec) , antiderivative size = 889, normalized size of antiderivative = 0.91, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {6123, 5985, 25, 6123, 5984, 3042, 26, 4670, 3011, 2720, 6107, 6095, 2620, 3011, 2720, 7143, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-2f\int -\left((e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 6123$$

$$\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 5984$$

$$\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{2\int(e+fx)^2\operatorname{csch}(2c+2dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2\int i(e+fx)^2\csc(2ic+2idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\int(e+fx)^2\csc(2ic+2idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \; 4670$$

3.465. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{if\int (e+fx)\log\left(1-e^{2c+2dx}\right)dx}{d}-\frac{if\int (e+fx)\log\left(1+e^{2c+2dx}\right)dx}{d}+\frac{i(e+fx)^2\text{arctanh}\left(e^{2c+2dx}\right)}{d}\right)}{a}\right)}{a}}{}$$

$$\downarrow \text{3011}$$

$$\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(-\frac{if\left(\frac{f\int \text{PolyLog}\left(2,-e^{2c+2dx}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int \text{PolyLog}\left(2,e^{2c+2dx}\right)dx}{2d}-\frac{(e+fx)\text{ P}}{}\right.}{d}\right.}{a}\right.}{a}$$

$$\downarrow \text{2720}$$

$$\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\int e^{-2c-2dx}\text{ PolyL}}{}\right.}{}\right.}{a}\right.}{a}$$

$$\downarrow \text{6107}$$

$$\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{b^2\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\text{PolyLog}\left(2,-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{(e+fx)\text{ P}}{}\right.}{d}\right.}{}\right.}{a}$$

$$\downarrow \text{6095}$$

3.465. $\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^3}{3bf}\right)}{a^2+b^2}+\frac{\int(e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(-\frac{if\left(\frac{f\int e^{-2c-2dx}\mathrm{Po}\dots}{\dots}\right.}{\dots}\right.}{\dots}\right.$$

$$\Big\downarrow \; 2620$$

$$\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{b^2\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+\dots}{3\dots}\right.}{a^2+b^2}\dots\right.}{a}\dots\right.$$

$$\Big\downarrow \; 3011$$

$$\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{b^2\left(-\frac{2f\left(\frac{f\int\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\dots}\right)}{d}\right.}{bd}\dots\right.}{a^2+b^2}\dots\right.}{a}\dots\right.$$

3.465. $\displaystyle\int\frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{2720}$$

$$\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)de^{c+dx}}{d^2}\cdots\right.}{bd}\cdots\right)}{a^2+b^2}\cdots\right)}{\cdots}\right.}{\cdots}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{\cdots}}{a+\sqrt{\cdots}}\right)}{d^2}\cdots\right.}{\cdots}\cdots\right)}{\cdots}\right)}{a}\cdots\right.}{\cdots}$$

3.465. $\displaystyle\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$\Big\downarrow$ 7292

$$\frac{2f\int \frac{(e+fx)(\arctan(\sinh(c+dx))+\text{csch}(c+dx))}{d}dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{\int(e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be}{a+\sqrt{}}\right)}{d^2}\right.}{}\right.}{}\right)}{a}\right.$$

$\Big\downarrow$ 27

$$\frac{2f\int(e+fx)(\arctan(\sinh(c+dx))+\text{csch}(c+dx))dx}{d}-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{\int(e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\,\text{PolyLog}\left(3,-\frac{be}{a+\sqrt{}}\right)}{d^2}\right.}{}\right.}{}\right)}{a}\right.$$

$\Big\downarrow$ 7293

3.465. $\displaystyle\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{2f\int((e+fx)\arctan(\sinh(c+dx))+(e+fx)\operatorname{csch}(c+dx))dx}{d}-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-$$

$$b\left(-\frac{b\left(\frac{\int\left(a(e+fx)^2\operatorname{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2}+\frac{b^2\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}\right)}{bd}-\frac{2f\left(\frac{f\operatorname{PolyLo}\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.}{a}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan(\sinh(c+dx))(e+fx)^2}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^2}{d}+\frac{2f\left(-\frac{\arctan\left(e^{c+dx}\right)(e+fx)^2}{f}+\frac{\arctan(\sinh(c+dx))(e+fx)^2}{2f}-\frac{2\operatorname{arctanh}\left(e^{c+dx}\right)(e+fx)}{d}+\ldots\right.}{\ldots}$$

$$b\left(\frac{2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^2}{d}-\frac{if\left(\frac{f\operatorname{PolyLog}\left(3,-e^{2c+2dx}\right)}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d}\right)}{d}+\frac{if\left(\frac{f\operatorname{PolyLog}\left(3,e^{2c+2dx}\right)}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{2c+2dx}\ldots\right.\right)}{2d}\right.}{d}\right.}{a}\right.$$

3.465. $\displaystyle\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-(((e + f*x)^2*ArcTan[Sinh[c + d*x]])/d) - ((e + f*x)^2*Csch[c + d*x])/d
+ (2*f*(-(((e + f*x)^2*ArcTan[E^(c + d*x)])/f) + ((e + f*x)^2*ArcTan[Sinh[
c + d*x]])/(2*f) - (2*(e + f*x)*ArcTanh[E^(c + d*x)])/d - (f*PolyLog[2, -E
^(c + d*x)])/d^2 + (I*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d - (I*(e +
f*x)*PolyLog[2, I*E^(c + d*x)])/d + (f*PolyLog[2, E^(c + d*x)])/d^2 - (I*f
*PolyLog[3, (-I)*E^(c + d*x)])/d^2 + (I*f*PolyLog[3, I*E^(c + d*x)])/d^2))
/d)/a - (b*(-((b*((b^2*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3)/(3
*f) + (2*a*(e + f*x)^2*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 + E^(
2*(c + d*x))])/d - ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d^2
+ ((2*I)*a*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*(e + f*x)*Pol
yLog[2, -E^(2*(c + d*x))])/d^2 + ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)]
)/d^3 - ((2*I)*a*f^2*PolyLog[3, I*E^(c + d*x)])/d^3 + (b*f^2*PolyLog[3, -E
^(2*(c + d*x))])/(2*d^3))/(a^2 + b^2)))/a) + ((2*I)*((I*(e + f*x)^2*ArcTan
h[E^(2*c + 2*d*x)])/d - (I*f*(-1/2*((e + f*x)*PolyLog[2, -E^(2*c + 2*d*...
```

3.465.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.465. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.465. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6095

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7292

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.465. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.465.4 Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^2 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.465.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5664 vs. $2(897) = 1794$.

Time = 0.38 (sec) , antiderivative size = 5664, normalized size of antiderivative = 5.77

$$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.465.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.465.7 Maxima [F]

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{csch}\,(dx+c)^2\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
(b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) +
 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((
a^2 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*
x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e^2 - 2*(f^2*x^2*e^
c + 2*e*f*x*e^c)*e^(d*x)/(a*d*e^(2*d*x + 2*c) - a*d) - 2*e*f*log(e^(d*x +
c) + 1)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/(a*d^2) - (d^2*x^2*log(e^(d*x
 + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*b*f^2
/(a^2*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2
*polylog(3, e^(d*x + c)))*b*f^2/(a^2*d^3) - 2*(b*d*e*f + a*f^2)*(d*x*log(e
^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 2*(b*d*e*f - a*f^2)*(d*
x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) + 1/3*(b*d^3*f^2*x
^3 + 3*(b*d*e*f + a*f^2)*d^2*x^2)/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*
e*f - a*f^2)*d^2*x^2)/(a^2*d^3) - integrate(2*(b^4*f^2*x^2 + 2*b^4*e*f*x -
 (a*b^3*f^2*x^2*e^c + 2*a*b^3*e*f*x*e^c)*e^(d*x))/(a^4*b + a^2*b^3 - (a^4*
b*e^(2*c) + a^2*b^3*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + a^3*b^2*e^c)*e^(d*x)
), x) - integrate(2*(b*f^2*x^2 + 2*b*e*f*x + (a*f^2*x^2*e^c + 2*a*e*f*x*e^
c)*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

3.465.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.465.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.466 $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.466.1 Optimal result

Integrand size = 32, antiderivative size = 591

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2fx\arctan\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)\arctan\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} + \frac{fx\arctan(\sinh(c+dx))}{ad} - \frac{(e+fx)\arctan(\sinh(c+dx))}{ad} + \frac{2b(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{a^2d} - \frac{f\operatorname{arctanh}(\cosh(c+dx))}{ad^2} - \frac{(e+fx)\operatorname{csch}(c+dx)}{ad} + \frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d} + \frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d} - \frac{b^3(e+fx)\log\left(1+e^{2(c+dx)}\right)}{a^2\left(a^2+b^2\right)d} + \frac{if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2} - \frac{ib^2f\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{a\left(a^2+b^2\right)d^2} - \frac{if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{ad^2} + \frac{ib^2f\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{a\left(a^2+b^2\right)d^2} + \frac{b^3f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2} + \frac{b^3f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)d^2} - \frac{b^3f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2a^2\left(a^2+b^2\right)d^2} + \frac{bf\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2a^2d^2} - \frac{bf\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2a^2d^2}$$

output
```
-2*f*x*arctan(exp(d*x+c))/a/d+2*b^2*(f*x+e)*arctan(exp(d*x+c))/a/(a^2+b^2)
/d+f*x*arctan(sinh(d*x+c))/a/d-(f*x+e)*arctan(sinh(d*x+c))/a/d+2*b*(f*x+e)
*arctanh(exp(2*d*x+2*c))/a^2/d-f*arctanh(cosh(d*x+c))/a/d^2-(f*x+e)*csch(d
*x+c)/a/d-b^3*(f*x+e)*ln(1+exp(2*d*x+2*c))/a^2/(a^2+b^2)/d+b^3*(f*x+e)*ln(
1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d+b^3*(f*x+e)*ln(1+b*exp
(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d+I*b^2*f*polylog(2,I*exp(d*x+c
))/a/(a^2+b^2)/d^2-I*f*polylog(2,I*exp(d*x+c))/a/d^2+I*f*polylog(2,-I*exp(
d*x+c))/a/d^2-I*b^2*f*polylog(2,-I*exp(d*x+c))/a/(a^2+b^2)/d^2-1/2*b^3*f*p
olylog(2,-exp(2*d*x+2*c))/a^2/(a^2+b^2)/d^2+1/2*b*f*polylog(2,-exp(2*d*x+2
*c))/a^2/d^2-1/2*b*f*polylog(2,exp(2*d*x+2*c))/a^2/d^2+b^3*f*polylog(2,-b*
exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2+b^3*f*polylog(2,-b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2
```

3.466.2 Mathematica [A] (verified)

Time = 8.95 (sec) , antiderivative size = 864, normalized size of antiderivative = 1.46

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\left(-de\cosh\left(\frac{1}{2}(c+dx)\right)+cf\cosh\left(\frac{1}{2}(c+dx)\right)-f(c+dx)\cosh\left(\frac{1}{2}(c+dx)\right)\right)\operatorname{csch}\left(\frac{1}{2}(c+dx)\right)}{2ad^2}$$
$$+\frac{-\frac{b(de-cf+f(c+dx))^2}{2f}+(-bde+af+bcf-bf(c+dx))\log\left(1-e^{-c-dx}\right)+(-bde-af+bcf-bf(c+dx}{a^2d^2}$$
$$+\frac{b^3\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\operatorname{arctanh}\left(\frac{a+be}{\sqrt{a^2}}\right.}{(-a^2-b^2)^{3/2}}\right.}{}$$
$$+\frac{-bde(c+dx)+bcf(c+dx)-\frac{1}{2}bf(c+dx)^2-2ade\arctan\left(e^{c+dx}\right)+2acf\arctan\left(e^{c+dx}\right)-iaf(c+dx}{}$$
$$+\frac{\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(de\sinh\left(\frac{1}{2}(c+dx)\right)-cf\sinh\left(\frac{1}{2}(c+dx)\right)+f(c+dx)\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2ad^2}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
((-(d*e*Cosh[(c + d*x)/2]) + c*f*Cosh[(c + d*x)/2] - f*(c + d*x)*Cosh[(c +
 d*x)/2])*Csch[(c + d*x)/2])/(2*a*d^2) + (-1/2*(b*(d*e - c*f + f*(c + d*x)
)^2)/f + (-(b*d*e) + a*f + b*c*f - b*f*(c + d*x))*Log[1 - E^(-c - d*x)] +
(-(b*d*e) - a*f + b*c*f - b*f*(c + d*x))*Log[1 + E^(-c - d*x)] + b*f*PolyL
og[2, -E^(-c - d*x)] + b*f*PolyLog[2, E^(-c - d*x)])/(a^2*d^2) + (b^3*(-2*
d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e
*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a
*Sqrt[-(a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-
a^2 - b^2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])] + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c
*f*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x
) + b*(-1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[
a^2 + b^2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/
(2*a^2*(a^2 + b^2)*d^2) + (-(b*d*e*(c + d*x)) + b*c*f*(c + d*x) - (b*f*(c
+ d*x)^2)/2 - 2*a*d*e*ArcTan[E^(c + d*x)] + 2*a*c*f*ArcTan[E^(c + d*x)] -
I*a*f*(c + d*x)*Log[1 - I*E^(c + d*x)] + I*a*f*(c + d*x)*Log[1 + I*E^(c +
d*x)] + b*d*e*Log[1 + E^(2*(c + d*x))] - b*c*f*Log[1 + E^(2*(c + d*x))] +
b*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] + I*a*f*PolyLog[2, (-I)*E^(c + d*x)
] - I*a*f*PolyLog[2, I*E^(c + d*x)] + (b*f*PolyLog[2, -E^(2*(c + d*x))])/2
)/((a^2 + b^2)*d^2) + (Sech[(c + d*x)/2]*(d*e*Sinh[(c + d*x)/2] - c*f*S...
```

3.466.3 Rubi [A] (verified)

Time = 2.84 (sec) , antiderivative size = 529, normalized size of antiderivative = 0.90, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.531$, Rules used = {6123, 5985, 2009, 6123, 5984, 3042, 26, 4670, 2715, 2838, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-f\int\left(-\frac{\arctan(\sinh(c+dx))}{d}-\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\,\text{arc}}{}}{a}$$

$$\Big\downarrow \text{6123}$$

$$-\frac{b\left(\frac{\int(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\,\text{arc}}{}}{a}$$

$$\Big\downarrow \text{5984}$$

$$-\frac{b\left(\frac{2\int(e+fx)\text{csch}(2c+2dx)dx}{a}-\frac{b\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\,\text{arc}}{}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\,\text{arc}}{}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2\int i(e+fx)\csc(2ic+2idx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\,\text{arc}}{}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\int(e+fx)\csc(2ic+2idx)dx}{a}\right)}{a}$$

3.466. $\int\frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$

$$\Big\downarrow \text{4670}$$

$$\frac{-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{arc}}{}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{if\int\log\left(1-e^{2c+2dx}\right)dx}{2d}-\frac{if\int\log\left(1+e^{2c+2dx}\right)dx}{2d}+\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{arc}}{}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{if\int e^{-2c-2dx}\log\left(1-e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}-\frac{if\int e^{-2c-2dx}\log\left(1+e^{2c+2dx}\right)de^{2c+2dx}}{4d^2}+\frac{i(e+fx)\text{arctanh}\left(e^{2c+2d}\right.}{d}\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{2838}$$

$$\frac{-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{arc}}{}}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{6107}$$

$$\frac{-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\text{arc}}{}}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{b^2\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{Poly}}{}\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{6095}$$

3.466. $\displaystyle\int\frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\,\text{arc}}{}$$

$$a$$

$$b\left(-\frac{b\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}\right)}{a}+\frac{2i\left(\frac{i(e+fx)\text{arctanh}(e^{2c+}}{d}\right.}{}\right.$$

$$a$$

$$\Big\downarrow \ 2620$$

$$-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\,\text{arc}}{}$$

$$a$$

$$b\left(-\frac{b\left(\frac{b^2\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}\right)}{a^2+b^2}+\frac{\int(e+fx)\text{S}}{}\right.}{a}\right.$$

$$a$$

$$\Big\downarrow \ 2715$$

$$-f\left(\frac{2x\arctan(e^{c+dx})}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}(2,-ie^{c+dx})}{d^2}+\frac{i\,\text{PolyLog}(2,ie^{c+dx})}{d^2}\right)-\frac{(e+fx)\,\text{arc}}{}$$

$$a$$

$$b\left(-\frac{b\left(\frac{b^2\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+\right.}{bd}\right.}{a^2+b^2}\right.}{a}\right.$$

$$\Big\downarrow \ 2838$$

3.466. $\displaystyle\int\frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arc}\cdots}{\cdots}$$

$$a$$

$$b\left(-\frac{b\left(\frac{\int(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\,\text{l}\cdots}{\cdots}\right.}{a^2+b^2}\right.}{a}\right.$$

$$a$$

$\big\downarrow$ 7293

$$-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arc}\cdots}{\cdots}$$

$$a$$

$$b\left(-\frac{b\left(\frac{\int(a(e+fx)\text{sech}(c+dx)-b(e+fx)\tanh(c+dx))dx}{a^2+b^2}+\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\cdots\right.}{a^2+b^2}\right.}{a}\right.$$

$$a$$

$\big\downarrow$ 2009

$$-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arc}\cdots}{\cdots}$$

$$a$$

$$b\left(\frac{2i\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)}{a}-\frac{b\left(\frac{b^2\left(\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c\cdots}}{a+\sqrt{\cdots}}\right)}{bd^2}\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.466. $\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(-(((e + f*x)*ArcTan[Sinh[c + d*x]])/d) - ((e + f*x)*Csch[c + d*x])/d - f*
((2*x*ArcTan[E^(c + d*x)])/d - (x*ArcTan[Sinh[c + d*x]])/d + ArcTanh[Cosh[
c + d*x]]/d^2 - (I*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*PolyLog[2, I*E^(
c + d*x)])/d^2))/a - (b*(-((b*((b^2*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 +
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2 + b^2) + ((b*(e + f*x)^2)/(2*f) + (
2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)*Log[1 + E^(2*(c + d*x)
)])/d - (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*a*f*PolyLog[2, I*E^(
c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*d^2))/(a^2 + b^2)))
/a) + ((2*I)*((I*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/d + ((I/4)*f*PolyLog[
2, -E^(2*c + 2*d*x)])/d^2 - ((I/4)*f*PolyLog[2, E^(2*c + 2*d*x)])/d^2))/a)
)/a
```

3.466.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.466. $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

3.466. $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.466.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1528 vs. 2(556) = 1112.

Time = 8.33 (sec) , antiderivative size = 1529, normalized size of antiderivative = 2.59

method	result	size
risch	Expression too large to display	1529

input
```
int((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
-1/a^2/d*f*b*ln(exp(d*x+c)+1)*x+1/a^2/d^2*b*c*f*ln(exp(d*x+c)-1)-2/d*(f*x+
e)/a*exp(d*x+c)/(exp(2*d*x+2*c)-1)-1/a^2/d*b*e*ln(exp(d*x+c)-1)-1/a^2/d*b*
e*ln(exp(d*x+c)+1)-1/a^2/d^2*f*b*dilog(exp(d*x+c)+1)+1/a^2/d^2*f*b*dilog(e
xp(d*x+c))-b/d^2*c*f/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))+4*I*a/d*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x-4*I*a/d^2*f/(4
*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c+4*I*a/d^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+
c))*c-1/a/d*e*b^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))-a/d*e*b/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))+1/a^2/d^2*f*b^3/(a^2+b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a
)/(-a+(a^2+b^2)^(1/2)))+1/a^2/d^2*f*b^3/(a^2+b^2)*dilog((b*exp(d*x+c)+(a^2
+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/a/d^2*f*b^3/(a^2+b^2)^(3/2)*arctanh(
1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+8*a/d^2*c*f/(4*a^2+4*b^2)*arctan
(exp(d*x+c))+1/a/d*e*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))+4*I*a/d^2*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))-4*I*a/d^2*f
/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))+1/a/d^2*b^3*c*f/(a^2+b^2)^(3/2)*arcta
nh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-4*I*a/d*f/(4*a^2+4*b^2)*ln(1-
I*exp(d*x+c))*x+1/a^2/d*f*b^3/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-
a)/(-a+(a^2+b^2)^(1/2)))*x+1/a^2/d*f*b^3/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b
^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/a^2/d^2*b^3*c*f/(a^2+b^2)*ln(b*exp(2
*d*x+2*c)+2*a*exp(d*x+c)-b)+a/d^2*b*c*f/(a^2+b^2)^(3/2)*arctanh(1/2*(2*...
```

3.466.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2593 vs. $2(539) = 1078$.

Time = 0.35 (sec) , antiderivative size = 2593, normalized size of antiderivative = 4.39

$$\int \frac{(e+fx)\mathrm{csch}^2(c+dx)\mathrm{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
-(2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e)*cosh(d*x + c) - (b^3*f*cosh(
d*x + c)^2 + 2*b^3*f*cosh(d*x + c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 -
 b^3*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*f*cosh(d*x + c)^2 +
2*b^3*f*cosh(d*x + c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 - b^3*f)*dilog
((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) + ((a^2*b + b^3)*f*cosh(d*x + c)^2 + 2*(
a^2*b + b^3)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^3)*f*sinh(d*x + c)
^2 - (a^2*b + b^3)*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) - (I*a^3*f - a^
2*b*f + (-I*a^3*f + a^2*b*f)*cosh(d*x + c)^2 - 2*(I*a^3*f - a^2*b*f)*cosh(
d*x + c)*sinh(d*x + c) + (-I*a^3*f + a^2*b*f)*sinh(d*x + c)^2)*dilog(I*cos
h(d*x + c) + I*sinh(d*x + c)) - (-I*a^3*f - a^2*b*f + (I*a^3*f + a^2*b*f)*
cosh(d*x + c)^2 - 2*(-I*a^3*f - a^2*b*f)*cosh(d*x + c)*sinh(d*x + c) + (I*
a^3*f + a^2*b*f)*sinh(d*x + c)^2)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)
) + ((a^2*b + b^3)*f*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sin
h(d*x + c) + (a^2*b + b^3)*f*sinh(d*x + c)^2 - (a^2*b + b^3)*f)*dilog(-cos
h(d*x + c) - sinh(d*x + c)) + (b^3*d*e - b^3*c*f - (b^3*d*e - b^3*c*f)*cos
h(d*x + c)^2 - 2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*
e - b^3*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d*e - b^3*c*f - (b^3*d*e - b^3*...
```

3.466.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.466.7 Maxima [F]

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\,\text{csch}\,(dx+c)^2\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
(b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) +
 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((
a^2 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*
x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e + (8*b*d*integrat
e(1/8*x/(a^2*d*e^(d*x + c) + a^2*d), x) - 8*b*d*integrate(1/8*x/(a^2*d*e^(
d*x + c) - a^2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) + 1)/(a^2
*d^2)) - a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^2)) - 2*x*e^
(d*x + c)/(a*d*e^(2*d*x + 2*c) - a*d) - 8*integrate(-1/4*(a*b^3*x*e^(d*x +
 c) - b^4*x)/(a^4*b + a^2*b^3 - (a^4*b*e^(2*c) + a^2*b^3*e^(2*c))*e^(2*d*x
) - 2*(a^5*e^c + a^3*b^2*e^c)*e^(d*x)), x) - 8*integrate(1/4*(a*x*e^(d*x +
 c) + b*x)/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x))*f
```

3.466.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.466.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{e+f\,x}{\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.467 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.467.1 Optimal result

Integrand size = 27, antiderivative size = 104

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\arctan(\sinh(c+dx))}{(a^2+b^2)\,d} - \frac{\operatorname{csch}(c+dx)}{ad} + \frac{b\log(\cosh(c+dx))}{(a^2+b^2)\,d} - \frac{b\log(\sinh(c+dx))}{a^2d} + \frac{b^3\log(a+b\sinh(c+dx))}{a^2\,(a^2+b^2)\,d}$$

output
```
-a*arctan(sinh(d*x+c))/(a^2+b^2)/d-csch(d*x+c)/a/d+b*ln(cosh(d*x+c))/(a^2+
b^2)/d-b*ln(sinh(d*x+c))/a^2/d+b^3*ln(a+b*sinh(d*x+c))/a^2/(a^2+b^2)/d
```

3.467.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.54

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b^3\left(\frac{\operatorname{csch}(c+dx)}{ab^3} + \frac{\log(\sinh(c+dx))}{a^2b^2} - \frac{\left(b^2+a\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\sinh(c+dx)\right)}{2b^4(a^2+b^2)} - \frac{\log(a+b\sinh(c+dx))}{a^2(a^2+b^2)} - \frac{\left(1+\frac{a}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\right.}{2b^2(a^2+b^2)}\right.}{d}$$

input
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
-((b^3*(Csch[c + d*x]/(a*b^3) + Log[Sinh[c + d*x]]/(a^2*b^2) - ((b^2 + a*S
qrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[c + d*x]])/(2*b^4*(a^2 + b^2)) - Log[a
+ b*Sinh[c + d*x]]/(a^2*(a^2 + b^2)) - ((1 + a/Sqrt[-b^2])*Log[Sqrt[-b^2]
+ b*Sinh[c + d*x]])/(2*b^2*(a^2 + b^2))))/d)
```

3.467.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 25, 3316, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ic+idx)^2\cos(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cos(ic+idx)\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{b\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^3\int \frac{\operatorname{csch}^2(c+dx)}{b^2(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{615}$$

$$\frac{b^3\int \left(\frac{\operatorname{csch}^2(c+dx)}{ab^4}-\frac{\operatorname{csch}(c+dx)}{a^2b^3}+\frac{1}{a^2(a^2+b^2)(a+b\sinh(c+dx))}+\frac{b\sinh(c+dx)-a}{b^2(a^2+b^2)\left(\sinh^2(c+dx)b^2+b^2\right)}\right)d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^3\left(-\frac{a\arctan(\sinh(c+dx))}{b^3(a^2+b^2)}+\frac{\log(b^2\sinh^2(c+dx)+b^2)}{2b^2(a^2+b^2)}-\frac{\log(b\sinh(c+dx))}{a^2b^2}+\frac{\log(a+b\sinh(c+dx))}{a^2(a^2+b^2)}-\frac{\operatorname{csch}(c+dx)}{ab^3}\right)}{d}$$

input
```
Int[(Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^3*(-((a*ArcTan[Sinh[c + d*x]])/(b^3*(a^2 + b^2))) - Csch[c + d*x]/(a*b^
3) - Log[b*Sinh[c + d*x]]/(a^2*b^2) + Log[a + b*Sinh[c + d*x]]/(a^2*(a^2 +
 b^2)) + Log[b^2 + b^2*Sinh[c + d*x]^2]/(2*b^2*(a^2 + b^2))))/d
```

3.467.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

3.467. $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.467.4 Maple [A] (verified)

Time = 3.76 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{2b\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)-4a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^{2}+2b^{2}}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{2}}+\frac{b^{3}\ln\left(\tanh\left(\frac{dx}{2}+\right.\right.}{a}}{d}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{2b\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)-4a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^{2}+2b^{2}}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{2}}+\frac{b^{3}\ln\left(\tanh\left(\frac{dx}{2}+\right.\right.}{a}}{d}$
risch	$-\frac{2b\,d^{2}x}{a^{2}d^{2}+b^{2}d^{2}}-\frac{2bdc}{a^{2}d^{2}+b^{2}d^{2}}-\frac{2b^{3}x}{a^{2}(a^{2}+b^{2})}-\frac{2b^{3}c}{a^{2}d(a^{2}+b^{2})}+\frac{2bx}{a^{2}}+\frac{2bc}{a^{2}d}-\frac{2\,\mathrm{e}^{dx+c}}{da\left(\mathrm{e}^{2dx+2c}-1\right)}+\frac{i\ln\left(\mathrm{e}^{dx+c}-i\right)a}{(a^{2}+b^{2})d}$

input
```
int(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/a*tanh(1/2*d*x+1/2*c)+1/2/(a^2+b^2)*(2*b*ln(1+tanh(1/2*d*x+1/2*c)
^2)-4*a*arctan(tanh(1/2*d*x+1/2*c)))-1/2/a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(
tanh(1/2*d*x+1/2*c))+b^3/a^2/(a^2+b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh
(1/2*d*x+1/2*c)-a))
```

3.467.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(104) = 208.

Time = 0.32 (sec) , antiderivative size = 441, normalized size of antiderivative = 4.24

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2\left(a^3\cosh\left(dx+c\right)^2+2\,a^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a^3\sinh\left(dx+c\right)^2-a^3\right)\arctan\left(\cosh\left(dx+c\right)+\right.}{}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-(2*(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*
x + c)^2 - a^3)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(a^3 + a*b^2)*co
sh(d*x + c) - (b^3*cosh(d*x + c)^2 + 2*b^3*cosh(d*x + c)*sinh(d*x + c) + b
^3*sinh(d*x + c)^2 - b^3)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sin
h(d*x + c))) - (a^2*b*cosh(d*x + c)^2 + 2*a^2*b*cosh(d*x + c)*sinh(d*x + c
) + a^2*b*sinh(d*x + c)^2 - a^2*b)*log(2*cosh(d*x + c)/(cosh(d*x + c) - si
nh(d*x + c))) - (a^2*b + b^3 - (a^2*b + b^3)*cosh(d*x + c)^2 - 2*(a^2*b +
b^3)*cosh(d*x + c)*sinh(d*x + c) - (a^2*b + b^3)*sinh(d*x + c)^2)*log(2*si
nh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(a^3 + a*b^2)*sinh(d*x +
c))/((a^4 + a^2*b^2)*d*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*d*cosh(d*x + c)
*sinh(d*x + c) + (a^4 + a^2*b^2)*d*sinh(d*x + c)^2 - (a^4 + a^2*b^2)*d)
```

3.467.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(csch(c + d*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.467.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.66

$$\begin{aligned}\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{b^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^4+a^2b^2)d} \\ &\quad+\frac{2\,a\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d} \\ &\quad+\frac{b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d}+\frac{2\,e^{(-dx-c)}}{(ae^{(-2\,dx-2\,c)}-a)d} \\ &\quad-\frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d}-\frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}\end{aligned}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) +
2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a
^2 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*x
 - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d)
```

3.467.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.92

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\frac{2\,b^4\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^4b+a^2b^3} - \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)a}{a^2+b^2} + \frac{b\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2} - \frac{2\,b\log\left(\left|e^{(dx+c)}-\right.\right.}{a^2}}{2\,d}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*b^4*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b + a^2*b^3
) - (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*a/(a^2 + b^2)
+ b*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2) - 2*b*log(abs(e^(d
*x + c) - e^(-d*x - c)))/a^2 + 2*(b*(e^(d*x + c) - e^(-d*x - c)) - 2*a)/(a
^2*(e^(d*x + c) - e^(-d*x - c))))/d
```

3.467.9 Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.37

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\ln\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)}{b\,d+a\,d\,1\mathrm{i}} + \frac{b^3\,\ln\left(2\,a\,\mathrm{e}^{c+d\,x}-b+b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{d\,a^4+d\,a^2\,b^2} - \frac{2\,\mathrm{e}^{c+d\,x}}{a\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)} - \frac{b\,\ln\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}{a^2\,d} + \frac{\ln\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)\,1\mathrm{i}}{a\,d+b\,d\,1\mathrm{i}}$$

input
```
int(1/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
log(exp(c + d*x) + 1i)/(a*d*1i + b*d) + (log(exp(c + d*x)*1i + 1)*1i)/(a*d
 + b*d*1i) + (b^3*log(2*a*exp(c + d*x) - b + b*exp(2*c + 2*d*x)))/(a^4*d +
 a^2*b^2*d) - (2*exp(c + d*x))/(a*d*(exp(2*c + 2*d*x) - 1)) - (b*log(exp(2
*c + 2*d*x) - 1))/(a^2*d)
```

3.467. $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.468 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.468.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.468.2 Mathematica [N/A]

Not integrable

Time = 41.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

3.468.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{ 6125}$$
$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]^2*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.468.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.468.4 Maple [N/A] (verified)

Not integrable

Time = 0.87 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(dx+c\right)^{2}\operatorname{sech}\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.468.5 Fricas [N/A]

Not integrable

Time = 6.70 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^2\operatorname{sech}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(csch(d*x + c)^2*sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

3.468.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.468.7 Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 472, normalized size of antiderivative = 13.88

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^2\operatorname{sech}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
2*e^(d*x + c)/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*
x)) - 8*integrate(-1/4*(a*b^3*e^(d*x + c) - b^4)/(a^4*b*e + a^2*b^3*e + (a
^4*b*f + a^2*b^3*f)*x - (a^4*b*e*e^(2*c) + a^2*b^3*e*e^(2*c) + (a^4*b*f*e^
(2*c) + a^2*b^3*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^5*e*e^c + a^3*b^2*e*e^c + (
a^5*f*e^c + a^3*b^2*f*e^c)*x)*e^(d*x)), x) - 8*integrate(-1/8*(b*d*f*x + b
*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 - (a^2*d*f^2*x^2*e^
c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) + 8*integrate(1/8*(b*d
*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 + (a^2*d*f^
2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) - 8*integrate(
1/4*(a*e^(d*x + c) + b)/(a^2*e + b^2*e + (a^2*f + b^2*f)*x + (a^2*e*e^(2*c
) + b^2*e*e^(2*c) + (a^2*f*e^(2*c) + b^2*f*e^(2*c))*x)*e^(2*d*x)), x)
```

3.468.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.468.9 Mupad [N/A]

Not integrable

Time = 3.66 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\cosh(c+d\,x)\,\sinh(c+d\,x)^2\,(e+f\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int(1/(cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.469 $\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.469.1 Optimal result

Integrand size = 36, antiderivative size = 914

$$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{2(e+fx)^2}{ad}+\frac{b^2(e+fx)^2}{a(a^2+b^2)d}+\frac{4bf(e+fx)\arctan\left(e^{c+dx}\right)}{a^2d^2}-\frac{4b^3f(e+fx)\arctan\left(e^{c+dx}\right)}{a^2(a^2+b^2)d^2}$$
$$+\frac{2b(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{a^2d}-\frac{2(e+fx)^2\coth(2c+2dx)}{ad}$$
$$+\frac{b^4(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d}-\frac{b^4(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d}$$
$$-\frac{2b^2f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{a(a^2+b^2)d^2}+\frac{2f(e+fx)\log\left(1-e^{4(c+dx)}\right)}{ad^2}$$
$$+\frac{2bf(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2}-\frac{2ibf^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{a^2d^3}$$
$$+\frac{2ib^3f^2\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{a^2(a^2+b^2)d^3}+\frac{2ibf^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{a^2d^3}-\frac{2ib^3f^2\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{a^2(a^2+b^2)d^3}$$
$$-\frac{2bf(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2}+\frac{2b^4f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2}$$
$$-\frac{2b^4f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2}-\frac{b^2f^2\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{a(a^2+b^2)d^3}$$
$$+\frac{f^2\operatorname{PolyLog}\left(2,e^{4(c+dx)}\right)}{2ad^3}-\frac{2bf^2\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{a^2d^3}+\frac{2bf^2\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{a^2d^3}$$
$$-\frac{2b^4f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^3}+\frac{2b^4f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^3}$$
$$-\frac{b(e+fx)^2\operatorname{sech}(c+dx)}{a^2d}+\frac{b^3(e+fx)^2\operatorname{sech}(c+dx)}{a^2(a^2+b^2)d}+\frac{b^2(e+fx)^2\tanh(c+dx)}{a(a^2+b^2)d}$$

output
```
2*I*b^3*f^2*polylog(2,-I*exp(d*x+c))/a^2/(a^2+b^2)/d^3+2*I*b*f^2*polylog(2
,I*exp(d*x+c))/a^2/d^3+4*b*f*(f*x+e)*arctan(exp(d*x+c))/a^2/d^2-2*b^4*f^2*
polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^3+2*b^4
*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^3-
2*I*b*f^2*polylog(2,-I*exp(d*x+c))/a^2/d^3-4*b^3*f*(f*x+e)*arctan(exp(d*x+
c))/a^2/(a^2+b^2)/d^2-2*b^2*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/a/(a^2+b^2)/d^2
+2*b^4*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2
)^(3/2)/d^2-2*b^4*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a
^2/(a^2+b^2)^(3/2)/d^2-2*I*b^3*f^2*polylog(2,I*exp(d*x+c))/a^2/(a^2+b^2)/d
^3+2*b*f*(f*x+e)*polylog(2,-exp(d*x+c))/a^2/d^2-2*b*f*(f*x+e)*polylog(2,ex
p(d*x+c))/a^2/d^2-2*(f*x+e)^2/a/d+2*b*(f*x+e)^2*arctanh(exp(d*x+c))/a^2/d-
2*b*f^2*polylog(3,-exp(d*x+c))/a^2/d^3+2*b*f^2*polylog(3,exp(d*x+c))/a^2/d
^3+2*f*(f*x+e)*ln(1-exp(4*d*x+4*c))/a/d^2-b^2*f^2*polylog(2,-exp(2*d*x+2*c
))/a/(a^2+b^2)/d^3+b^3*(f*x+e)^2*sech(d*x+c)/a^2/(a^2+b^2)/d+b^2*(f*x+e)^2
*tanh(d*x+c)/a/(a^2+b^2)/d+b^2*(f*x+e)^2/a/(a^2+b^2)/d-b*(f*x+e)^2*sech(d*
x+c)/a^2/d+b^4*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b
^2)^(3/2)/d-b^4*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+
b^2)^(3/2)/d-2*(f*x+e)^2*coth(2*d*x+2*c)/a/d+1/2*f^2*polylog(2,exp(4*d*x+4
*c))/a/d^3
```

3.469.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2029 vs. 2(914) = 1828.

Time = 9.33 (sec) , antiderivative size = 2029, normalized size of antiderivative = 2.22

$$\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*
x]),x]
```

output
```
4*((a*(d*(e + f*x)*(d*(e + f*x) + 2*(1 + E^c)*f*Log[1 + E^(-c - d*x)]) - 2
*(1 + E^c)*f^2*PolyLog[2, -E^(-c - d*x)]))/(4*(a^2 + b^2)*d^3*(1 + E^c)) +
 (a*(d*(e + f*x)*((-I)*d*(e + f*x) + 2*(-I + E^c)*f*Log[1 - I*E^(-c - d*x)
]) - 2*(-I + E^c)*f^2*PolyLog[2, I*E^(-c - d*x)]))/(4*(a^2 + b^2)*d^3*(-I
+ E^c)) + ((I/2)*a*(-(d*(e + f*x)*(d*(e + f*x) + (1 + I*E^(2*c))*f*Log[1 -
 E^(-c - d*x)] + (1 + I*E^(2*c))*f*Log[1 + I*E^(-c - d*x)])) + (1 + I*E^(2
*c))*f^2*PolyLog[2, (-I)*E^(-c - d*x)] + (1 + I*E^(2*c))*f^2*PolyLog[2, E^
(-c - d*x)]))/((a^2 + b^2)*d^3*(-I + E^(2*c))) + ((I/2)*b*f*(d*((-2*I)*e*A
rcTan[E^(c + d*x)] + f*x*Log[1 - I*E^(c + d*x)] - f*x*Log[1 + I*E^(c + d*x
)]) - f*PolyLog[2, (-I)*E^(c + d*x)] + f*PolyLog[2, I*E^(c + d*x)]))/((a^2
 + b^2)*d^3) - (b*(4*a*b*d^2*e*E^(2*c)*f*x + 2*a*b*d^2*E^(2*c)*f^2*x^2 + 2
*a^2*d^2*e^2*ArcTanh[E^(c + d*x)] + 2*b^2*d^2*e^2*ArcTanh[E^(c + d*x)] - 2
*a^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] - 2*b^2*d^2*e^2*E^(2*c)*ArcTanh[
E^(c + d*x)] - 2*a^2*d^2*e*f*x*Log[1 - E^(c + d*x)] - 2*b^2*d^2*e*f*x*Log[
1 - E^(c + d*x)] + 2*a^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 2*b^2*d^
2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] - a^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)
] - b^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)] + a^2*d^2*E^(2*c)*f^2*x^2*Log[1 -
 E^(c + d*x)] + b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 2*a^2*d^2*e
*f*x*Log[1 + E^(c + d*x)] + 2*b^2*d^2*e*f*x*Log[1 + E^(c + d*x)] - 2*a^2*d
^2*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - 2*b^2*d^2*e*E^(2*c)*f*x*Log[1 +...
```

3.469.3 Rubi [A] (verified)

Time = 5.68 (sec) , antiderivative size = 818, normalized size of antiderivative = 0.89, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.806$, Rules used = {6123, 5984, 3042, 25, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838, 6123, 5985, 25, 6107, 3042, 3803, 25, 2694, 27, 2620, 3011, 2720, 7143, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{6123}$$
$$\frac{\int (e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$
$$\Big\downarrow \text{5984}$$
$$\frac{4\int (e+fx)^2\text{csch}^2(2c+2dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{4\int -(e+fx)^2\csc(2ic+2idx)^2dx}{a}$$

$$\Big\downarrow \; 25$$

$$-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\int (e+fx)^2\csc(2ic+2idx)^2dx}{a}$$

$$\Big\downarrow \; 4672$$

$$-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}-\frac{if\int -i(e+fx)\coth(2c+2dx)dx}{d}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}-\frac{f\int (e+fx)\coth(2c+2dx)dx}{d}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}-\frac{f\int -i(e+fx)\tan\left(2ic+2idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\int (e+fx)\tan\left(\frac{1}{2}(4ic+\pi)+2idx\right)dx}{d}\right)}{a}$$

$$\Big\downarrow \; 4201$$

$$-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\int \frac{e^{4c+4dx-i\pi}(e+fx)}{1+e^{4c+4dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \; 2620$$

$$-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}-\frac{f\int \log\left(1+e^{4c+4dx-i\pi}\right)dx}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \; 2715$$

3.469. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}-\frac{f\int e^{-4c-4dx+i\pi}\log\left(1+e^{4c+4dx-i\pi}\right)de^{4c+4dx-i\pi}}{16d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \; 2838$$

$$-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \; 6123$$

$$-\frac{b\left(\frac{\int (e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \; 5985$$

$$-\frac{b\left(\frac{-2f\int -\left((e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \; 25$$

3.469. $\displaystyle\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{6107}$$

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{b^2\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\right.}{}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(}{a^2+b^2}\right.}{}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{3803}$$

3.469. $\int\frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{2b^2\int-\frac{e^{c+dx}(e+fx}{-2e^{c+dx}a-be^{2(c}}}{a^2+b^2}\right.}{a}\right.}{\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}}$$

$$\downarrow 25$$

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(}{a^2+b^2}\right.}{a}\right.}{\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}}$$

$$\downarrow 2694$$

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(}{a^2+b^2}\right.}{}\right.}{\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}\quad a}$$

$$\downarrow 27$$

3.469. $\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(\cdots}{a^2+b^2}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\operatorname{PolyLog}(2,-e^{4c+4dx-i\pi})}{16d^2}+\frac{(e+fx)\log(1+e^{4c+4dx-i\pi})}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$\downarrow$ 2620

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(\cdots}{a^2+b^2}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\operatorname{PolyLog}(2,-e^{4c+4dx-i\pi})}{16d^2}+\frac{(e+fx)\log(1+e^{4c+4dx-i\pi})}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$\downarrow$ 3011

3.469. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(\cdots}{a^2+b^2}\cdots\right)}{\cdots}\right)}{\cdots}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{2720}$$

3.469. $\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(\ldots}{a^2+b^2}\ldots\right)}{\ldots}\right)}{\ldots}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \; 7143$$

3.469. $\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(\cdots}{a^2+b^2}\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow\ 7292$$

3.469. $\int\frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{2f\int \frac{(e+fx)(\text{arctanh}(\cosh(c+dx))-\text{sech}(c+dx))}{d}dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b}{a^2+b^2}\right.}{}\right.}{}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{27}$$

3.469. $\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{\frac{2f\int(e+fx)(\operatorname{arctanh}(\cosh(c+dx))-\operatorname{sech}(c+dx))dx}{d}-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-b\,\mathrm{s}}{a^2+b^2}\right.}{}\right.}{}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \; 7293$$

3.469. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{\frac{2f\int((e+fx)\text{arctanh}(\cosh(c+dx))-(e+fx)\text{sech}(c+dx))dx}{d}-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int\left(a(e+fx)^2\text{sech}^2(c+\cdots\right.}{\cdots}\right.}{\cdots}\right)}{\cdots}$$

$$\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}$$

$$\Big\downarrow \text{2009}$$

3.469. $\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{4\left(\frac{\coth(2c+2dx)(e+fx)^2}{2d}+\frac{if\left(2i\left(\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}+\frac{f\,\mathrm{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}-$$

$$b\left(\frac{-\frac{\operatorname{arctanh}(\cosh(c+dx))(e+fx)^2}{d}+\frac{\operatorname{sech}(c+dx)(e+fx)^2}{d}+\frac{2f\left(-\frac{\operatorname{arctanh}\left(e^{c+dx}\right)(e+fx)^2}{f}+\frac{\operatorname{arctanh}(\cosh(c+dx))(e+fx)^2}{2f}-\frac{2\arctan\left(e^{c+dx}\right)(e+f\right.}{d}}{}\right.$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

3.469. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(-4*(((e + f*x)^2*Coth[2*c + 2*d*x])/(2*d) + (I*f*(((-1/2*I)*(e + f*x)^2)/
f + (2*I)*(((e + f*x)*Log[1 + E^(4*c - I*Pi + 4*d*x)])/(4*d) + (f*PolyLog[
2, -E^(4*c - I*Pi + 4*d*x)])/(16*d^2))))/d))/a - (b*((-(((e + f*x)^2*ArcTa
nh[Cosh[c + d*x]])/d) + (2*f*((-2*(e + f*x)*ArcTan[E^(c + d*x)])/d - ((e +
 f*x)^2*ArcTanh[E^(c + d*x)])/f + ((e + f*x)^2*ArcTanh[Cosh[c + d*x]])/(2*
f) - ((e + f*x)*PolyLog[2, -E^(c + d*x)])/d + (I*f*PolyLog[2, (-I)*E^(c +
d*x)])/d^2 - (I*f*PolyLog[2, I*E^(c + d*x)])/d^2 + ((e + f*x)*PolyLog[2, E
^(c + d*x)])/d + (f*PolyLog[3, -E^(c + d*x)])/d^2 - (f*PolyLog[3, E^(c + d
*x)])/d^2))/d + ((e + f*x)^2*Sech[c + d*x])/d)/a - (b*((-2*b^2*(-1/2*(b*((
(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) - (2*f*(
-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d) + (f
*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/d^2))/(b*d)))/Sqrt[
a^2 + b^2] + (b*(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/d
^2))/(b*d)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + ((a*(e + f*x)^2)/d - (4*b
*f*(e + f*x)*ArcTan[E^(c + d*x)])/d^2 - (2*a*f*(e + f*x)*Log[1 + E^(2*(c +
 d*x))])/d^2 + ((2*I)*b*f^2*PolyLog[2, (-I)*E^(c + d*x)])/d^3 - ((2*I)*b*f
^2*PolyLog[2, I*E^(c + d*x)])/d^3 - (a*f^2*PolyLog[2, -E^(2*(c + d*x))])/d
^3 + (b*(e + f*x)^2*Sech[c + d*x])/d + (a*(e + f*x)^2*Tanh[c + d*x])/d)...
```

3.469.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.469. $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.469. $\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

3.469. $\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.469.4 Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^2 \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.469.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10432 vs. $2(845) = 1690$.

Time = 0.51 (sec) , antiderivative size = 10432, normalized size of antiderivative = 11.41

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

3.469.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\mathrm{csch}^2(c+dx)\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.469.7 Maxima [F]

$$\int \frac{(e+fx)^2\mathrm{csch}^2(c+dx)\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\mathrm{csch}\left(dx+c\right)^2\,\mathrm{sech}\left(dx+c\right)^2}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
-2*a*e*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 +
 b^2)*d^2)) + 4*b*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2
*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 4*a*f^2*integrate(x/(a^2*d*e^(2*
d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + (b^4*log((b*e^(-
d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((
a^4 + a^2*b^2)*sqrt(a^2 + b^2)*d) - 2*(a*b*e^(-d*x - c) + b^2*e^(-2*d*x -
2*c) - a*b*e^(-3*d*x - 3*c) + 2*a^2 + b^2)/((a^3 + a*b^2 - (a^3 + a*b^2)*e
^(-4*d*x - 4*c))*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c)
 - 1)/(a^2*d))*e^2 - 4*e*f*x/(a*d) + 4*b*e*f*arctan(e^(d*x + c))/((a^2 + b
^2)*d^2) + 2*((2*a^2*f^2 + b^2*f^2)*x^2 + 2*(2*a^2*e*f + b^2*e*f)*x + (a*b
*f^2*x^2*e^(3*c) + 2*a*b*e*f*x*e^(3*c))*e^(3*d*x) + (b^2*f^2*x^2*e^(2*c) +
 2*b^2*e*f*x*e^(2*c))*e^(2*d*x) - (a*b*f^2*x^2*e^c + 2*a*b*e*f*x*e^c)*e^(d
*x))/(a^3*d + a*b^2*d - (a^3*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x)) + 2*e
*f*log(e^(d*x + c) + 1)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/(a*d^2) + (d^
2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(
d*x + c)))*b*f^2/(a^2*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(
e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*b*f^2/(a^2*d^3) + 2*(b*d*e*f + a
*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 2*(b*d*
e*f - a*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) -
1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f + a*f^2)*d^2*x^2)/(a^2*d^3) + 1/3*(b*d^...
```

3.469.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
Timed out
```

3.469.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)^2\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x
)
```

3.470 $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.470.1 Optimal result

Integrand size = 34, antiderivative size = 499

$$
\begin{aligned}
&\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{bf\arctan(\sinh(c+dx))}{a^2d^2} - \frac{b^3f\arctan(\sinh(c+dx))}{a^2(a^2+b^2)d^2} + \frac{2bfx\operatorname{arctanh}(e^{c+dx})}{a^2d} \\
&\quad - \frac{bfx\operatorname{arctanh}(\cosh(c+dx))}{a^2d} + \frac{b(e+fx)\operatorname{arctanh}(\cosh(c+dx))}{a^2d} \\
&\quad - \frac{2(e+fx)\coth(2c+2dx)}{ad} + \frac{b^4(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d} \\
&\quad - \frac{b^4(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d} - \frac{b^2f\log(\cosh(c+dx))}{a(a^2+b^2)d^2} \\
&\quad + \frac{f\log(\sinh(2c+2dx))}{ad^2} + \frac{bf\operatorname{PolyLog}(2,-e^{c+dx})}{a^2d^2} - \frac{bf\operatorname{PolyLog}(2,e^{c+dx})}{a^2d^2} \\
&\quad + \frac{b^4f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2} - \frac{b^4f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d^2} \\
&\quad - \frac{b(e+fx)\operatorname{sech}(c+dx)}{a^2d} + \frac{b^3(e+fx)\operatorname{sech}(c+dx)}{a^2(a^2+b^2)d} + \frac{b^2(e+fx)\tanh(c+dx)}{a(a^2+b^2)d}
\end{aligned}
$$

output
```
b*f*arctan(sinh(d*x+c))/a^2/d^2-b^3*f*arctan(sinh(d*x+c))/a^2/(a^2+b^2)/d^
2+2*b*f*x*arctanh(exp(d*x+c))/a^2/d-b*f*x*arctanh(cosh(d*x+c))/a^2/d+b*(f*
x+e)*arctanh(cosh(d*x+c))/a^2/d-2*(f*x+e)*coth(2*d*x+2*c)/a/d-b^2*f*ln(cos
h(d*x+c))/a/(a^2+b^2)/d^2+f*ln(sinh(2*d*x+2*c))/a/d^2+b^4*(f*x+e)*ln(1+b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d-b^4*(f*x+e)*ln(1+b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d+b*f*polylog(2,-exp(d*x
+c))/a^2/d^2-b*f*polylog(2,exp(d*x+c))/a^2/d^2+b^4*f*polylog(2,-b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2-b^4*f*polylog(2,-b*exp(d*x
+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2-b*(f*x+e)*sech(d*x+c)/a^2
/d+b^3*(f*x+e)*sech(d*x+c)/a^2/(a^2+b^2)/d+b^2*(f*x+e)*tanh(d*x+c)/a/(a^2+
b^2)/d
```

3.470.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.79 (sec) , antiderivative size = 1295, normalized size of antiderivative = 2.60

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
4*(-1/8*(f*(c + d*x))/((a + I*b)*d^2) + ((I/8)*((2 - I)*a^3*d*f + (3*I)*a^
2*b*d*f - I*a*b^2*d*f + I*b^3*d*f + a^2*b*c*d*f + I*a*b^2*c*d*f)*(c + d*x)
)/(a*(a + I*b)*(a^2 + b^2)*d^3) - ((I/16)*b*f*(c + d*x)^2)/((a^2 + b^2)*d^
2) + ((I/4)*f*ArcTan[(a*Cosh[(c + d*x)/2] - b*Cosh[(c + d*x)/2] + a*Sinh[(
c + d*x)/2] + b*Sinh[(c + d*x)/2])/(a*Cosh[(c + d*x)/2] + b*Cosh[(c + d*x)
/2] - a*Sinh[(c + d*x)/2] + b*Sinh[(c + d*x)/2])])/((a + I*b)*d^2) + ((-(d
*e*Cosh[(c + d*x)/2]) + c*f*Cosh[(c + d*x)/2] - f*(c + d*x)*Cosh[(c + d*x)
/2])*Csch[(c + d*x)/2])/(8*a*d^2) + (f*Log[Cosh[c + d*x]])/(8*(a + I*b)*d^
2) + (f*((a*b*d^2*x^2)/2 + (a^2 + b^2)*(c + d*x) - 2*(a^2 + b^2 - a*b*c)*(
c + d*x) + 2*a*b*(c + d*x)*Log[1 + E^(-c - d*x)] + 2*(a^2 + b^2 - a*b*c)*L
og[1 + E^(c + d*x)] - 2*a*b*PolyLog[2, -E^(-c - d*x)]))/(8*a*(a^2 + b^2)*d
^2) + ((1/16 + I/16)*f*(-4*a^2*(c + d*x) - (2*I)*a*b*(c + d*x) - 2*b^2*(c
+ d*x) - 2*a*b*c*(c + d*x) + a*b*(c + d*x)^2 - 2*(2*a^2 + b^2 + a*b*c)*Arc
Tan[E^(c + d*x)] - (4 - 4*I)*a^2*ArcTan[1 - (1 + I)*E^(c + d*x)] - 4*a*b*A
rcTan[1 - (1 + I)*E^(c + d*x)] - 4*b^2*ArcTan[1 - (1 + I)*E^(c + d*x)] - 4
*a*b*c*ArcTan[1 - (1 + I)*E^(c + d*x)] + 4*a^2*Log[1 - E^(c + d*x)] + (2*I
)*a*b*Log[1 - E^(c + d*x)] + 2*b^2*Log[1 - E^(c + d*x)] + 2*a*b*c*Log[1 -
E^(c + d*x)] - (2 - 2*I)*a*b*(c + d*x)*Log[1 - E^(c + d*x)] + 2*a*b*Log[I
+ E^(c + d*x)] - (2*I)*a^2*Log[1 + E^(2*(c + d*x))] - I*b^2*Log[1 + E^(2*(
c + d*x))] - I*a*b*c*Log[1 + E^(2*(c + d*x))] - (2 - 2*I)*a*b*PolyLog[2...
```

3.470.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.02 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.89, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.676, Rules used = {6123, 5984, 3042, 25, 4672, 26, 3042, 26, 3956, 6123, 5985, 2009, 6107, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5984}$$

$$\frac{4\int(e+fx)\text{csch}^2(2c+2dx)dx}{a}-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \color{blue}{3042}$$

$$-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{4\int-\left((e+fx)\csc(2ic+2idx)^2\right)dx}{a}$$

$$\Big\downarrow \color{blue}{25}$$

$$-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\int(e+fx)\csc(2ic+2idx)^2dx}{a}$$

$$\Big\downarrow \color{blue}{4672}$$

$$-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{if\int-i\coth(2c+2dx)dx}{2d}\right)}{a}$$

$$\Big\downarrow \color{blue}{26}$$

$$-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\int\coth(2c+2dx)dx}{2d}\right)}{a}$$

$$\Big\downarrow \color{blue}{3042}$$

$$-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\int-i\tan\left(2ic+2idx+\frac{\pi}{2}\right)dx}{2d}\right)}{a}$$

$$\Big\downarrow \color{blue}{26}$$

$$-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}+\frac{if\int\tan\left(\frac{1}{2}(4ic+\pi)+2idx\right)dx}{2d}\right)}{a}$$

$$\Big\downarrow \color{blue}{3956}$$

$$-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$$\Big\downarrow \color{blue}{6123}$$

$$-\frac{b\left(\frac{\int(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$$\Big\downarrow \color{blue}{5985}$$

3.470. $\int\frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{-f\int\left(\frac{\operatorname{sech}(c+dx)}{d}-\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}\right)dx-\frac{(e+fx)\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(c-}{d}}{a}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$$\Big\downarrow \textbf{6107}$$

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(c-}{d}}{a}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(c-}{d}}{a}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$$\Big\downarrow \textbf{3803}$$

3.470. $\displaystyle\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cosh(c\cdot}{d}}{a}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cosh(c\cdot}{d}}{a}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cosh(c\cdot}{d}}{a}\right.}{}$$

$$\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$$\Big\downarrow \text{27}$$

3.470. $\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(c\cdots}{d}}{a}\right.}{a}$$

$$\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(c\cdots}{d}}{a}\right.}{}$$

$$\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}$$

$$\Big\downarrow \text{2715}$$

3.470. $\displaystyle\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(c\cdots}{d}}{a}\right.}{\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}}$$

$\Big\downarrow$ 2838

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(c\cdots}{d}}{a}\right.}{\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}}$$

$\Big\downarrow$ 7293

3.470. $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(c\cdots}{d}}{a}\right.}{\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(c\cdots}{d}}{a}\right.}{\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}}$$

3.470. $\displaystyle\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-4*(((e + f*x)*Coth[2*c + 2*d*x])/(2*d) - (f*Log[(-I)*Sinh[2*c + 2*d*x]])
/(4*d^2)))/a - (b*((-(((e + f*x)*ArcTanh[Cosh[c + d*x]])/d) - f*(ArcTan[Si
nh[c + d*x]]/d^2 + (2*x*ArcTanh[E^(c + d*x)])/d - (x*ArcTanh[Cosh[c + d*x]
])/d + PolyLog[2, -E^(c + d*x)]/d^2 - PolyLog[2, E^(c + d*x)]/d^2) + ((e +
 f*x)*Sech[c + d*x])/d)/a - (b*((-2*b^2*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/(b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1
 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b
^2) + (-((b*f*ArcTan[Sinh[c + d*x]])/d^2) - (a*f*Log[Cosh[c + d*x]])/d^2 +
 (b*(e + f*x)*Sech[c + d*x])/d + (a*(e + f*x)*Tanh[c + d*x])/d)/(a^2 + b^2
)))/a))/a
```

3.470.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

3.470. $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.470.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3716 vs. $2(476) = 952$.

Time = 22.31 (sec) , antiderivative size = 3717, normalized size of antiderivative = 7.45

method	result	size
risch	Expression too large to display	3717

input
```
int((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURN
VERBOSE)
```

output
```
1/2*(-2*(a^2+b^2)^(3/2)*ln(exp(d*x+c)+1)*b^3*d*f*x-2*ln((-b*exp(d*x+c)+(a^
2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2*b^4*d*f*x+2*ln((b*exp(d*x+c)+(a^
2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2*b^4*d*f*x-2*(a^2+b^2)^(3/2)*ln(ex
p(d*x+c)-1)*a^2*b*c*f+2*(a^2+b^2)^(3/2)*ln(exp(d*x+c)-1)*a^2*b*d*e-2*(a^2+
b^2)^(3/2)*ln(exp(d*x+c)+1)*a^2*b*d*e+2*(a^2+b^2)*arctanh(1/2*(2*b*exp(d*x
+c)+2*a)/(a^2+b^2)^(1/2))*a^2*b^2*c*f-2*(a^2+b^2)*arctanh(1/2*(2*b*exp(d*x
+c)+2*a)/(a^2+b^2)^(1/2))*a^2*b^2*d*e-8*exp(4*d*x+4*c)*arctanh(1/2*(2*b*ex
p(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^6*f-exp(4*d*x+4*c)*arctanh(1/2*(2*b*exp(d
*x+c)+2*a)/(a^2+b^2)^(1/2))*b^6*f+2*exp(4*d*x+4*c)*dilog((-b*exp(d*x+c)+(a
^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*b^6*f+2*(a^2+b^2)^(3/2)*exp(4*d*x+4
*c)*ln(exp(d*x+c)+1)*a^2*b*d*f*x+4*(a^2+b^2)^(3/2)*exp(d*x+c)*a^2*b*d*f*x-
4*(a^2+b^2)^(3/2)*exp(3*d*x+3*c)*a^2*b*d*f*x-4*(a^2+b^2)^(3/2)*exp(2*d*x+2
*c)*a*b^2*d*f*x+2*(a^2+b^2)^(3/2)*exp(4*d*x+4*c)*ln(exp(d*x+c)-1)*a^2*b*c*
f+2*exp(4*d*x+4*c)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))*b^6*d*f*x-2*exp(4*d*x+4*c)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))*b^6*d*f*x+2*(a^2+b^2)^2*exp(4*d*x+4*c)*arctanh(1/2*(2*b*exp(
d*x+c)+2*a)/(a^2+b^2)^(1/2))*b^2*c*f-2*(a^2+b^2)^2*exp(4*d*x+4*c)*arctanh(
1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*b^2*d*e+2*(a^2+b^2)^(3/2)*exp(4*
d*x+4*c)*ln(exp(d*x+c)+1)*b^3*d*f*x+2*exp(4*d*x+4*c)*ln((-b*exp(d*x+c)+(a^
2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2*b^4*d*f*x-2*exp(4*d*x+4*c)*ln...
```

3.470.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4086 vs. $2(472) = 944$.

Time = 0.37 (sec) , antiderivative size = 4086, normalized size of antiderivative = 8.19

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
-(2*((2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2*a^3*b^2 + a*b^4)*c*f)*co
sh(d*x + c)^4 + 2*((2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2*a^3*b^2 +
a*b^4)*c*f)*sinh(d*x + c)^4 + 2*((a^4*b + a^2*b^3)*d*f*x + (a^4*b + a^2*b^
3)*d*e)*cosh(d*x + c)^3 + 2*((a^4*b + a^2*b^3)*d*f*x + (a^4*b + a^2*b^3)*d
*e + 4*((2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2*a^3*b^2 + a*b^4)*c*f)
*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*d*e - 2*(a
^5 + 2*a^3*b^2 + a*b^4)*c*f + 2*((a^3*b^2 + a*b^4)*d*f*x + (a^3*b^2 + a*b^
4)*d*e)*cosh(d*x + c)^2 + 2*((a^3*b^2 + a*b^4)*d*f*x + (a^3*b^2 + a*b^4)*d
*e + 6*((2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2*a^3*b^2 + a*b^4)*c*f)
*cosh(d*x + c)^2 + 3*((a^4*b + a^2*b^3)*d*f*x + (a^4*b + a^2*b^3)*d*e)*cos
h(d*x + c))*sinh(d*x + c)^2 - (b^5*f*cosh(d*x + c)^4 + 4*b^5*f*cosh(d*x +
c)^3*sinh(d*x + c) + 6*b^5*f*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^5*f*cos
h(d*x + c)*sinh(d*x + c)^3 + b^5*f*sinh(d*x + c)^4 - b^5*f)*sqrt((a^2 + b^
2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^5*f*cosh(d*x + c)^4 +
4*b^5*f*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^5*f*cosh(d*x + c)^2*sinh(d*x +
 c)^2 + 4*b^5*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^5*f*sinh(d*x + c)^4 - b^
5*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^5*d
*e - b^5*c*f - (b^5*d*e - b^5*c*f)*cosh(d*x + c)^4 - 4*(b^5*d*e - b^5*c...
```

3.470.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.470.7 Maxima [F]

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\operatorname{csch}(dx+c)^2\operatorname{sech}(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
(b^4*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)*d) - 2*(a*b*e^(-d*x - c) +
b^2*e^(-2*d*x - 2*c) - a*b*e^(-3*d*x - 3*c) + 2*a^2 + b^2)/((a^3 + a*b^2 -
 (a^3 + a*b^2)*e^(-4*d*x - 4*c))*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b*
log(e^(-d*x - c) - 1)/(a^2*d))*e + (16*b^4*integrate(-1/8*x*e^(d*x + c)/(a
^4*b + a^2*b^3 - (a^4*b*e^(2*c) + a^2*b^3*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c
+ a^3*b^2*e^c)*e^(d*x)), x) - 16*b*d*integrate(1/16*x/(a^2*d*e^(d*x + c) +
 a^2*d), x) - 16*b*d*integrate(1/16*x/(a^2*d*e^(d*x + c) - a^2*d), x) - a*
((d*x + c)/(a^2*d^2) - log(e^(d*x + c) + 1)/(a^2*d^2)) - a*((d*x + c)/(a^2
*d^2) - log(e^(d*x + c) - 1)/(a^2*d^2)) + 2*(a*b*x*e^(3*d*x + 3*c) + b^2*x
*e^(2*d*x + 2*c) - a*b*x*e^(d*x + c) + (2*a^2 + b^2)*x)/(a^3*d + a*b^2*d -
 (a^3*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x)) - 2*a*x/((a^2 + b^2)*d) + 2*
b*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) + a*log(e^(2*d*x + 2*c) + 1)/((a^2
 + b^2)*d^2))*f
```

3.470.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.470.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{e+f\,x}{\cosh\left(c+d\,x\right)^2\,\sinh\left(c+d\,x\right)^2\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.471 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.471.1 Optimal result

Integrand size = 29, antiderivative size = 144

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\operatorname{arctanh}(\cosh(c+dx))}{a^2d} - \frac{2b^4\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d} - \frac{\coth(c+dx)}{ad} - \frac{b\operatorname{sech}(c+dx)}{a^2d} + \frac{b^2\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a^2\left(a^2+b^2\right)d} - \frac{\tanh(c+dx)}{ad}$$

output
```
b*arctanh(cosh(d*x+c))/a^2/d-2*b^4*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+
b^2)^(1/2))/a^2/(a^2+b^2)^(3/2)/d-coth(d*x+c)/a/d-b*sech(d*x+c)/a^2/d+b^2*
sech(d*x+c)*(b+a*sinh(d*x+c))/a^2/(a^2+b^2)/d-tanh(d*x+c)/a/d
```

3.471.2 Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{4b^4\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{a^2(-a^2-b^2)^{3/2}} + \frac{\coth\left(\frac{1}{2}(c+dx)\right)}{a} - \frac{2b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{a^2} + \frac{2b\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{a^2} + \frac{2\operatorname{sech}(c+dx)(b+a\sinh(c+}{a^2+b^2}}{2d}$$

input
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*((4*b^4*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(a^2*(-a^
2 - b^2)^(3/2)) + Coth[(c + d*x)/2]/a - (2*b*Log[Cosh[(c + d*x)/2]])/a^2 +
 (2*b*Log[Sinh[(c + d*x)/2]])/a^2 + (2*Sech[c + d*x]*(b + a*Sinh[c + d*x])
)/(a^2 + b^2) + Tanh[(c + d*x)/2]/a)/d
```

3.471.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {3042, 25, 3377, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ic+idx)^2\cos(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cos(ic+idx)^2\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3377}$$

$$-\int \left(-\frac{\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a}+\frac{b\text{csch}(c+dx)\text{sech}^2(c+dx)}{a^2}-\frac{b^2\text{sech}^2(c+dx)}{a^2(a+b\sinh(c+dx))}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2b^4\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\left(a^2+b^2\right)^{3/2}}+\frac{b\text{arctanh}(\cosh(c+dx))}{a^2d}+\frac{b^2\text{sech}(c+dx)(a\sinh(c+dx)+b)}{a^2d\left(a^2+b^2\right)}-\frac{b\text{sech}(c+dx)}{a^2d}-\frac{\tanh(c+dx)}{ad}-\frac{\coth(c+dx)}{ad}$$

input
```
Int[(Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b*ArcTanh[Cosh[c + d*x]])/(a^2*d) - (2*b^4*ArcTanh[(b - a*Tanh[(c + d*x)/
2])/Sqrt[a^2 + b^2]])/(a^2*(a^2 + b^2)^(3/2)*d) - Coth[c + d*x]/(a*d) - (b
*Sech[c + d*x])/(a^2*d) + (b^2*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2*(
a^2 + b^2)*d) - Tanh[c + d*x]/(a*d)
```

3.471.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3377
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

3.471.4 Maple [A] (verified)

Time = 10.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{2b^4 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{2b^4 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}{d}$
risch	$-\frac{2\left(e^{3dx+3c}ab+b^2e^{2dx+2c}-e^{dx+c}ab+2a^2+b^2\right)}{da\left(e^{2dx+2c}-1\right)\left(a^2+b^2\right)\left(1+e^{2dx+2c}\right)}-\frac{b\ln\left(e^{dx+c}-1\right)}{a^2d}+\frac{b^4\ln\left(e^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d\,a^2}-$

input
```
int(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)+2/a^2*b^4/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a
*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1/2/a/tanh(1/2*d*x+1/2*c)-1/a^2
*b*ln(tanh(1/2*d*x+1/2*c))+2/(a^2+b^2)*(-a*tanh(1/2*d*x+1/2*c)-b)/(1+tanh(
1/2*d*x+1/2*c)^2))
```

3.471.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1040 vs. $2(141) = 282$.

Time = 0.33 (sec) , antiderivative size = 1040, normalized size of antiderivative = 7.22

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fric
as")
```

output
```
-(4*a^5 + 6*a^3*b^2 + 2*a*b^4 + 2*(a^4*b + a^2*b^3)*cosh(d*x + c)^3 + 2*(a
^4*b + a^2*b^3)*sinh(d*x + c)^3 + 2*(a^3*b^2 + a*b^4)*cosh(d*x + c)^2 + 2*
(a^3*b^2 + a*b^4 + 3*(a^4*b + a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - (b
^4*cosh(d*x + c)^4 + 4*b^4*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^4*cosh(d*x
+ c)^2*sinh(d*x + c)^2 + 4*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*sinh(d*
x + c)^4 - b^4)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x +
c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sin
h(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b
*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x +
 c) + a)*sinh(d*x + c) - b)) - 2*(a^4*b + a^2*b^3)*cosh(d*x + c) + (a^4*b
+ 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^4 - 4*(a^4*b +
 2*a^2*b^3 + b^5)*cosh(d*x + c)^3*sinh(d*x + c) - 6*(a^4*b + 2*a^2*b^3 + b
^5)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x
 + c)*sinh(d*x + c)^3 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(d*x + c)^4)*log(cos
h(d*x + c) + sinh(d*x + c) + 1) - (a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^
2*b^3 + b^5)*cosh(d*x + c)^4 - 4*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^3
*sinh(d*x + c) - 6*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^2*sinh(d*x + c)
^2 - 4*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)*sinh(d*x + c)^3 - (a^4*b +
2*a^2*b^3 + b^5)*sinh(d*x + c)^4)*log(cosh(d*x + c) + sinh(d*x + c) - 1) -
 2*(a^4*b + a^2*b^3 - 3*(a^4*b + a^2*b^3)*cosh(d*x + c)^2 - 2*(a^3*b^2 ...
```

3.471.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.471.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.44

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^4\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}d} - \frac{2\left(abe^{(-dx-c)}+b^2e^{(-2\,dx-2\,c)}-abe^{(-3\,dx-3\,c)}+2\,a^2+b^2\right)}{(a^3+ab^2-(a^3+ab^2)e^{(-4\,dx-4\,c)})d} + \frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d} - \frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxi
ma")
```

output
```
b^4*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)*d) - 2*(a*b*e^(-d*x - c) + b
^2*e^(-2*d*x - 2*c) - a*b*e^(-3*d*x - 3*c) + 2*a^2 + b^2)/((a^3 + a*b^2 -
(a^3 + a*b^2)*e^(-4*d*x - 4*c))*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b*l
og(e^(-d*x - c) - 1)/(a^2*d)
```

3.471.8 Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.28

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{\frac{b^4\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}}+\frac{b\log\left(e^{(dx+c)}+1\right)}{a^2}-\frac{b\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^2}-\frac{2\,\left(abe^{(3\,dx+3\,c)}+b^2e^{(2\,dx+2\,c)}-abe^{(dx+c)}+2\,a^2+b^2\right)}{(a^3+ab^2)\left(e^{(4\,dx+4\,c)}-1\right)}}{d}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac
")
```

output
```
(b^4*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c
) + 2*a + 2*sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)) + b*log(e^
(d*x + c) + 1)/a^2 - b*log(abs(e^(d*x + c) - 1))/a^2 - 2*(a*b*e^(3*d*x + 3
*c) + b^2*e^(2*d*x + 2*c) - a*b*e^(d*x + c) + 2*a^2 + b^2)/((a^3 + a*b^2)*
(e^(4*d*x + 4*c) - 1)))/d
```

3.471.9 Mupad [B] (verification not implemented)

Time = 6.55 (sec) , antiderivative size = 768, normalized size of antiderivative = 5.33

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\frac{b^4\,\ln\left(\frac{64\,b^8\,\sqrt{{\left(a^2+b^2\right)}^3}-96\,a\,b^{10}-384\,a^3\,b^8-512\,a^5\,b^6-288\,a^7\,b^4-64\,a^9\,b^2+288\,a^2\,b^9\,\mathrm{e}^{c+d\,x}+960\,a^4\,b^7\,\mathrm{e}^{c+d\,x}+1152\,a^6\,b^5\,\mathrm{e}^{c+d\,x}+608\,a^8\,b^3\,\mathrm{e}^{c}\cdots}{a^3\,{\left({\left(a^2+b^2\right)}^3\right)}^{3/2}\,\left(a^2+b^2\right)}\cdots\right.}{d\,a^8+3\,d\,a^6\,b^2+\cdots}$$
$$-\frac{\frac{2\,b^4\,\mathrm{e}^{3\,c+3\,d\,x}}{d\,\left(a^2\,b^3+b^5\right)}-\frac{2\,b^4\,\mathrm{e}^{c+d\,x}}{d\,\left(a^2\,b^3+b^5\right)}+\frac{2\,b^3\,\left(2\,a^2+b^2\right)}{a\,d\,\left(a^2\,b^3+b^5\right)}+\frac{2\,b^5\,\mathrm{e}^{2\,c+2\,d\,x}}{a\,d\,\left(a^2\,b^3+b^5\right)}}{\mathrm{e}^{4\,c+4\,d\,x}-1}$$
$$-\frac{b^4\,\ln\left(\frac{96\,a\,b^{10}+64\,b^8\,\sqrt{{\left(a^2+b^2\right)}^3}+384\,a^3\,b^8+512\,a^5\,b^6+288\,a^7\,b^4+64\,a^9\,b^2-288\,a^2\,b^9\,\mathrm{e}^{c+d\,x}-960\,a^4\,b^7\,\mathrm{e}^{c+d\,x}-1152\,a^6\,b^5\,\mathrm{e}^{c+d\,x}-608\,a^8\,b^3\cdots}{a^3\,{\left({\left(a^2+b^2\right)}^3\right)}^{3/2}\,\left(a^2+b^2\right)}\cdots\right.}{d\,a^8+3\,d\,a^6\,b\cdots}$$
$$-\frac{b\,\ln\left(\mathrm{e}^{c+d\,x}-1\right)}{a^2\,d}+\frac{b\,\ln\left(\mathrm{e}^{c+d\,x}+1\right)}{a^2\,d}$$

input
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
(b^4*log((64*b^8*((a^2 + b^2)^3)^(1/2) - 96*a*b^10 - 384*a^3*b^8 - 512*a^5
*b^6 - 288*a^7*b^4 - 64*a^9*b^2 + 288*a^2*b^9*exp(c + d*x) + 960*a^4*b^7*e
xp(c + d*x) + 1152*a^6*b^5*exp(c + d*x) + 608*a^8*b^3*exp(c + d*x) + 128*a
^10*b*exp(c + d*x) - 64*a*b^7*exp(c + d*x)*((a^2 + b^2)^3)^(1/2) + 32*a^3*
b^5*exp(c + d*x)*((a^2 + b^2)^3)^(1/2))/(a^3*((a^2 + b^2)^3)^(3/2)*(a^2 +
b^2)) - (32*b*(2*a^2*b - 4*a^3*exp(c + d*x) + 2*b^3 - 5*a*b^2*exp(c + d*x)
))/(a^3*(a^2 + b^2)^2))*((a^2 + b^2)^3)^(1/2))/(a^8*d + a^2*b^6*d + 3*a^4*
b^4*d + 3*a^6*b^2*d) - ((2*b^4*exp(3*c + 3*d*x))/(d*(b^5 + a^2*b^3)) - (2*
b^4*exp(c + d*x))/(d*(b^5 + a^2*b^3)) + (2*b^3*(2*a^2 + b^2))/(a*d*(b^5 +
a^2*b^3)) + (2*b^5*exp(2*c + 2*d*x))/(a*d*(b^5 + a^2*b^3)))/(exp(4*c + 4*d
*x) - 1) - (b^4*log((96*a*b^10 + 64*b^8*((a^2 + b^2)^3)^(1/2) + 384*a^3*b^
8 + 512*a^5*b^6 + 288*a^7*b^4 + 64*a^9*b^2 - 288*a^2*b^9*exp(c + d*x) - 96
0*a^4*b^7*exp(c + d*x) - 1152*a^6*b^5*exp(c + d*x) - 608*a^8*b^3*exp(c + d
*x) - 128*a^10*b*exp(c + d*x) - 64*a*b^7*exp(c + d*x)*((a^2 + b^2)^3)^(1/2
) + 32*a^3*b^5*exp(c + d*x)*((a^2 + b^2)^3)^(1/2))/(a^3*((a^2 + b^2)^3)^(3
/2)*(a^2 + b^2)) - (32*b*(2*a^2*b - 4*a^3*exp(c + d*x) + 2*b^3 - 5*a*b^2*e
xp(c + d*x)))/(a^3*(a^2 + b^2)^2))*((a^2 + b^2)^3)^(1/2))/(a^8*d + a^2*b^6
*d + 3*a^4*b^4*d + 3*a^6*b^2*d) - (b*log(exp(c + d*x) - 1))/(a^2*d) + (b*l
og(exp(c + d*x) + 1))/(a^2*d)
```

3.471. $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.472 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.472.1 Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.472.2 Mathematica [N/A]

Not integrable

Time = 52.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])),x]
```

output
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

3.472.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]^2*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.472.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.472.4 Maple [N/A] (verified)

Not integrable

Time = 0.82 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(dx+c\right)^{2}\operatorname{sech}\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.472.5 Fricas [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^2\operatorname{sech}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
integral(csch(d*x + c)^2*sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh
(d*x + c)), x)
```

3.472.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.472.7 Maxima [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 626, normalized size of antiderivative = 17.39

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^2\operatorname{sech}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
16*b^4*integrate(-1/8*e^(d*x + c)/(a^4*b*e + a^2*b^3*e + (a^4*b*f + a^2*b^
3*f)*x - (a^4*b*e*e^(2*c) + a^2*b^3*e*e^(2*c) + (a^4*b*f*e^(2*c) + a^2*b^3
*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^5*e*e^c + a^3*b^2*e*e^c + (a^5*f*e^c + a^3
*b^2*f*e^c)*x)*e^(d*x)), x) + 2*(a*b*e^(3*d*x + 3*c) + b^2*e^(2*d*x + 2*c)
 - a*b*e^(d*x + c) + 2*a^2 + b^2)/(a^3*d*e + a*b^2*d*e + (a^3*d*f + a*b^2*
d*f)*x - (a^3*d*e*e^(4*c) + a*b^2*d*e*e^(4*c) + (a^3*d*f*e^(4*c) + a*b^2*d
*f*e^(4*c))*x)*e^(4*d*x)) - 16*integrate(-1/16*(b*d*f*x + b*d*e + a*f)/(a^
2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f
*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) - 16*integrate(1/16*(b*d*f*x + b*d*e
- a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2
*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) - 16*integrate(1/8*(b*f*e^(
d*x + c) - a*f)/(a^2*d*e^2 + b^2*d*e^2 + (a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(
a^2*d*e*f + b^2*d*e*f)*x + (a^2*d*e^2*e^(2*c) + b^2*d*e^2*e^(2*c) + (a^2*d
*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 + 2*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e
^(2*c))*x)*e^(2*d*x)), x)
```

3.472.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.472.9 Mupad [N/A]

Not integrable

Time = 6.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\cosh(c+d\,x)^2\,\sinh(c+d\,x)^2\,\left(e+f\,x\right)\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x
)
```

3.472. $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.473 $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.473.1 Optimal result

Integrand size = 34, antiderivative size = 978

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\begin{aligned}
&= -\frac{bfx}{2a^2d} - \frac{3fx\arctan\left(e^{c+dx}\right)}{ad} + \frac{2b^4(e+fx)\arctan\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)^2d} + \frac{b^2(e+fx)\arctan\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} \\
&+ \frac{3fx\arctan(\sinh(c+dx))}{2ad} - \frac{3(e+fx)\arctan(\sinh(c+dx))}{2ad} + \frac{2bfx\operatorname{arctanh}\left(e^{2c+2dx}\right)}{a^2d} \\
&- \frac{f\operatorname{arctanh}(\cosh(c+dx))}{ad^2} - \frac{3(e+fx)\operatorname{csch}(c+dx)}{2ad} + \frac{b^5(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^2d} \\
&+ \frac{b^5(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^2d} - \frac{b^5(e+fx)\log\left(1+e^{2(c+dx)}\right)}{a^2\left(a^2+b^2\right)^2d} \\
&+ \frac{bfx\log(\tanh(c+dx))}{a^2d} - \frac{b(e+fx)\log(\tanh(c+dx))}{a^2d} + \frac{3if\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{2ad^2} \\
&- \frac{ib^4f\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{a\left(a^2+b^2\right)^2d^2} - \frac{ib^2f\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{2a\left(a^2+b^2\right)d^2} - \frac{3if\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{2ad^2} \\
&+ \frac{ib^4f\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{a\left(a^2+b^2\right)^2d^2} + \frac{ib^2f\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{2a\left(a^2+b^2\right)d^2} + \frac{b^5f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^2d^2} \\
&+ \frac{b^5f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^2d^2} - \frac{b^5f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2a^2\left(a^2+b^2\right)^2d^2} + \frac{bf\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2a^2d^2} \\
&- \frac{bf\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2a^2d^2} - \frac{f\operatorname{sech}(c+dx)}{2ad^2} + \frac{b^2f\operatorname{sech}(c+dx)}{2a\left(a^2+b^2\right)d^2} \\
&+ \frac{b^3(e+fx)\operatorname{sech}^2(c+dx)}{2a^2\left(a^2+b^2\right)d} + \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{2ad} + \frac{bf\tanh(c+dx)}{2a^2d^2} \\
&- \frac{b^3f\tanh(c+dx)}{2a^2\left(a^2+b^2\right)d^2} + \frac{b^2(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{2a\left(a^2+b^2\right)d} + \frac{b(e+fx)\tanh^2(c+dx)}{2a^2d}
\end{aligned}$$

output
```
1/2*I*b^2*f*polylog(2,I*exp(d*x+c))/a/(a^2+b^2)/d^2+I*b^4*f*polylog(2,I*ex
p(d*x+c))/a/(a^2+b^2)^2/d^2+3/2*I*f*polylog(2,-I*exp(d*x+c))/a/d^2-f*arcta
nh(cosh(d*x+c))/a/d^2-3/2*(f*x+e)*csch(d*x+c)/a/d+b^2*(f*x+e)*arctan(exp(d
*x+c))/a/(a^2+b^2)/d+2*b^4*(f*x+e)*arctan(exp(d*x+c))/a/(a^2+b^2)^2/d+2*b*
f*x*arctanh(exp(2*d*x+2*c))/a^2/d-1/2*b^5*f*polylog(2,-exp(2*d*x+2*c))/a^2
/(a^2+b^2)^2/d^2+1/2*b^2*f*sech(d*x+c)/a/(a^2+b^2)/d^2-3/2*(f*x+e)*arctan(
sinh(d*x+c))/a/d+1/2*b^2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/a/(a^2+b^2)/d-I*b
^4*f*polylog(2,-I*exp(d*x+c))/a/(a^2+b^2)^2/d^2-1/2*I*b^2*f*polylog(2,-I*e
xp(d*x+c))/a/(a^2+b^2)/d^2+3/2*f*x*arctan(sinh(d*x+c))/a/d-1/2*f*sech(d*x+
c)/a/d^2-1/2*b*f*polylog(2,exp(2*d*x+2*c))/a^2/d^2-3*f*x*arctan(exp(d*x+c)
)/a/d+1/2*b*f*polylog(2,-exp(2*d*x+2*c))/a^2/d^2-1/2*b*f*x/a^2/d+1/2*(f*x+
e)*csch(d*x+c)*sech(d*x+c)^2/a/d+1/2*b*f*tanh(d*x+c)/a^2/d^2+1/2*b*(f*x+e)
*tanh(d*x+c)^2/a^2/d-3/2*I*f*polylog(2,I*exp(d*x+c))/a/d^2-b^5*(f*x+e)*ln(
1+exp(2*d*x+2*c))/a^2/(a^2+b^2)^2/d+b^5*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/a^2/(a^2+b^2)^2/d+b^5*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))/a^2/(a^2+b^2)^2/d+b*f*x*ln(tanh(d*x+c))/a^2/d+b^5*f*polylog(2,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^2/d^2+b^5*f*polylog(2,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^2/d^2-b*(f*x+e)*ln(tanh(d*x+c))/
a^2/d+1/2*b^3*(f*x+e)*sech(d*x+c)^2/a^2/(a^2+b^2)/d-1/2*b^3*f*tanh(d*x+c)/
a^2/(a^2+b^2)/d^2
```

3.473.2 Mathematica [A] (warning: unable to verify)

Time = 11.64 (sec) , antiderivative size = 1437, normalized size of antiderivative = 1.47

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]
),x]
```

output
```
8*(((-(d*e*Cosh[(c + d*x)/2]) + c*f*Cosh[(c + d*x)/2] - f*(c + d*x)*Cosh[(
c + d*x)/2])*Csch[(c + d*x)/2]*Csch[c + d*x]*(a + b*Sinh[c + d*x]))/(16*a*
d^2*(b + a*Csch[c + d*x])) + (Csch[c + d*x]*(-1/2*(b*(d*e - c*f + f*(c + d
*x))^2)/f + (-(b*d*e) + a*f + b*c*f - b*f*(c + d*x))*Log[1 - E^(-c - d*x)]
 + (-(b*d*e) - a*f + b*c*f - b*f*(c + d*x))*Log[1 + E^(-c - d*x)] + b*f*Po
lyLog[2, -E^(-c - d*x)] + b*f*PolyLog[2, E^(-c - d*x)])*(a + b*Sinh[c + d*
x]))/(8*a^2*d^2*(b + a*Csch[c + d*x])) + (b^5*Csch[c + d*x]*(-2*d*e*(c + d
*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a
 + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^
2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)
^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*
f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b -
 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-1
+ E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]
)] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])*(a + b*Sinh
[c + d*x]))/(16*a^2*(a^2 + b^2)^2*d^2*(b + a*Csch[c + d*x])) + (Csch[c + d
*x]*(-2*a^2*b*d*e*(c + d*x) - 4*b^3*d*e*(c + d*x) + 2*a^2*b*c*f*(c + d*x)
+ 4*b^3*c*f*(c + d*x) - a^2*b*f*(c + d*x)^2 - 2*b^3*f*(c + d*x)^2 - 6*a^3*
d*e*ArcTan[E^(c + d*x)] - 10*a*b^2*d*e*ArcTan[E^(c + d*x)] + 6*a^3*c*f*Arc
Tan[E^(c + d*x)] + 10*a*b^2*c*f*ArcTan[E^(c + d*x)] - (3*I)*a^3*f*(c + ...
```

3.473.3 Rubi [A] (verified)

Time = 3.82 (sec) , antiderivative size = 803, normalized size of antiderivative = 0.82, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6123, 5985, 2009, 6123, 5985, 2009, 6107, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-f\int\left(\frac{\text{csch}(c+dx)\text{sech}^2(c+dx)}{2d}-\frac{3\arctan(\sinh(c+dx))}{2d}-\frac{3\text{csch}(c+dx)}{2d}\right)dx-\frac{3(e+fx)\arctan(\sinh(c+dx))}{2d}-\frac{3(e+fx)\text{csch}(c+dx)}{2d}}{a}$$

$$\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$\frac{-f\left(\frac{3x\arctan(e^{c+dx})}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\,\text{PolyLog}(2,-ie^{c+dx})}{2d^2}+\frac{3i\,\text{PolyLog}(2,ie^{c+dx})}{2d^2}+\frac{\text{sech}(c+}{2d^2}\right.}{a}$$

$$\Big\downarrow \text{6123}$$

$$-\frac{b\left(\frac{\int(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+$$
$$\frac{-f\left(\frac{3x\arctan(e^{c+dx})}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\,\text{PolyLog}(2,-ie^{c+dx})}{2d^2}+\frac{3i\,\text{PolyLog}(2,ie^{c+dx})}{2d^2}+\frac{\text{sech}(c+}{2d^2}\right.}{a}$$

$$\Big\downarrow \text{5985}$$

$$-\frac{b\left(\frac{-f\int\left(\frac{\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{2d}\right)dx-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+fx)\log(\tanh(c+dx))}{d}}{a}-\frac{b\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+$$
$$\frac{-f\left(\frac{3x\arctan(e^{c+dx})}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\,\text{PolyLog}(2,-ie^{c+dx})}{2d^2}+\frac{3i\,\text{PolyLog}(2,ie^{c+dx})}{2d^2}+\frac{\text{sech}(c+}{2d^2}\right.}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{-f\left(\frac{2x\text{arctanh}(e^{2c+2dx})}{d}+\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}-\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+fx}{}}{a}\right.}{a}$$
$$\frac{-f\left(\frac{3x\arctan(e^{c+dx})}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\,\text{PolyLog}(2,-ie^{c+dx})}{2d^2}+\frac{3i\,\text{PolyLog}(2,ie^{c+dx})}{2d^2}+\frac{\text{sech}(c+}{2d^2}\right.}{a}$$

$$\Big\downarrow \text{6107}$$

3.473. $\int\frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$

$$-\frac{b\left(\frac{-f\left(\frac{2x\,\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+fx}{}}{a}\right.}{a}$$

$$\frac{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\frac{3i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2d^2}+\frac{\text{sech}(c+}{2d^2}\right.}{a}$$

$$\Big\downarrow \; 6107$$

$$-\frac{b\left(\frac{-f\left(\frac{2x\,\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+fx}{}}{a}\right.}{a}$$

$$\frac{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\frac{3i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2d^2}+\frac{\text{sech}(c+}{2d^2}\right.}{a}$$

$$\Big\downarrow \; 6095$$

$$-\frac{b\left(\frac{-f\left(\frac{2x\,\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+fx}{}}{a}\right.}{}$$

$$\frac{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\frac{3i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2d^2}+\frac{\text{sech}(c+}{2d^2}\right.}{a}$$

$$\Big\downarrow \; 2620$$

3.473. $\displaystyle\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{-f\left(\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+fx}{}}{a}\right.}{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\frac{3i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2d^2}+\frac{\text{sech}(c+}{2d^2}\right.}$$

$$a$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\left(\frac{-f\left(\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+fx}{}}{a}\right.}{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\frac{3i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{2d^2}+\frac{\text{sech}(c+}{2d^2}\right.}$$

$$a$$

3.473. $\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$\downarrow$ 2838

$$-\frac{b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+fx}{}}{a}\right.}{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\frac{3i\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{2d^2}+\frac{\operatorname{sech}(c+}{2d^2}\right.}$$

$$a$$

$\downarrow$ 7293

3.473. $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+fx}{}}{a}\right.}{}$$

$$\frac{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\frac{3i\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{2d^2}+\frac{\operatorname{sech}(c+}{2d^2}\right.}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{2d}-\frac{3(e+fx)\arctan(\sinh(c+dx))}{2d}-\frac{3(e+fx)\operatorname{csch}(c+dx)}{2d}-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}\right.}{a}$$

$$\frac{b\left(\frac{-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+fx)\log(\tanh(c+dx))}{d}-f\left(\frac{2\operatorname{arctanh}\left(e^{2c+2dx}\right)x}{d}+\frac{\log(\tanh(c+dx))x}{d}-\frac{x}{2d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\right.}{a}\right.}{}$$

3.473. $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
((-3*(e + f*x)*ArcTan[Sinh[c + d*x]])/(2*d) - (3*(e + f*x)*Csch[c + d*x])/
(2*d) + ((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(2*d) - f*((3*x*ArcTan[E
^(c + d*x)])/d - (3*x*ArcTan[Sinh[c + d*x]])/(2*d) + ArcTanh[Cosh[c + d*x]
]/d^2 - (((3*I)/2)*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (((3*I)/2)*PolyLog[
2, I*E^(c + d*x)])/d^2 + Sech[c + d*x]/(2*d^2)))/a - (b*((((e + f*x)*Log[T
anh[c + d*x]])/d - ((e + f*x)*Tanh[c + d*x]^2)/(2*d) - f*(-1/2*x/d + (2*x*
ArcTanh[E^(2*c + 2*d*x)])/d + (x*Log[Tanh[c + d*x]])/d + PolyLog[2, -E^(2*
c + 2*d*x)]/(2*d^2) - PolyLog[2, E^(2*c + 2*d*x)]/(2*d^2) + Tanh[c + d*x]/
(2*d^2)))/a - (b*((b^2*((b^2*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b*d^2)))/(a^2 + b^2) + ((b*(e + f*x)^2)/(2*f) + (2*a*(e
+ f*x)*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)*Log[1 + E^(2*(c + d*x))])/d -
 (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*a*f*PolyLog[2, I*E^(c + d*x
)])/d^2 - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*d^2))/(a^2 + b^2)))/(a^2 +
 b^2) + ((a*(e + f*x)*ArcTan[E^(c + d*x)])/d - ((I/2)*a*f*PolyLog[2, (-I)*
E^(c + d*x)])/d^2 + ((I/2)*a*f*PolyLog[2, I*E^(c + d*x)])/d^2 + (a*f*Sech[
c + d*x])/(2*d^2) + (b*(e + f*x)*Sech[c + d*x]^2)/(2*d) - (b*f*Tanh[c + d*
x])/(2*d^2) + (a*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d))/(a^2 + b...
```

3.473.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.473. $\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.473. $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.473.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3279 vs. $2(904) = 1808$.

Time = 60.27 (sec) , antiderivative size = 3280, normalized size of antiderivative = 3.35

method	result	size
risch	Expression too large to display	3280

input
```
int((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURN
VERBOSE)
```

output
```
4/d*a^2/(a^2+b^2)*b*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x+4/d*a^2/(a^2+b^2)
*b*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x+20/d^2*a/(a^2+b^2)*c*b^2*f/(4*a^2+
4*b^2)*arctan(exp(d*x+c))+2/d^2/a/(a^2+b^2)^(5/2)*c*b^5*f*arctanh(1/2*(2*b
*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+7/2/d^2*a/(a^2+b^2)^(5/2)*c*b^3*f*arctan
h(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-4/d^2*a^2/(a^2+b^2)*c*b*f/(4*a
^2+4*b^2)*ln(1+exp(2*d*x+2*c))+3/2/d^2*a^3/(a^2+b^2)^(5/2)*c*b*f*arctanh(1
/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+4/d^2*a^2/(a^2+b^2)*b*f/(4*a^2+4*
b^2)*ln(1+I*exp(d*x+c))*c+4/d^2*a^2/(a^2+b^2)*b*f/(4*a^2+4*b^2)*ln(1-I*exp
(d*x+c))*c+10*I/d^2*a/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))-
10*I/d^2*a/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))+6*I/d*a^3/(
a^2+b^2)*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x-6*I/d*a^3/(a^2+b^2)*f/(4*a^2
+4*b^2)*ln(1-I*exp(d*x+c))*x-6*I/d^2*a^3/(a^2+b^2)*f/(4*a^2+4*b^2)*ln(1-I*
exp(d*x+c))*c+6*I/d^2*a^3/(a^2+b^2)*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c-1
/d^2/(a^2+b^2)^(5/2)*b*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
*a^3-2/d^2/(a^2+b^2)^(5/2)*b^3*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2
)^(1/2))*a-2/d/a/(a^2+b^2)^(5/2)*e*b^5*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))+8/d/(a^2+b^2)*f*b^3/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x-8/d^
2/(a^2+b^2)*c*b^3*f/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))+8/d^2/(a^2+b^2)*f*b
^3/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c+8/d^2/(a^2+b^2)*f*b^3/(4*a^2+4*b^2)*
ln(1-I*exp(d*x+c))*c+8/d/(a^2+b^2)*f*b^3/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c...
```

3.473.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 15223 vs. 2(881) = 1762.

Time = 0.62 (sec) , antiderivative size = 15223, normalized size of antiderivative = 15.57

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.473.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)**2*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.473.7 Maxima [F]

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)^2\,\text{sech}\,(dx+c)^3}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
(b^5*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^6 + 2*a^4*b^2 + a
^2*b^4)*d) + (3*a^3 + 5*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^
4)*d) + (a^2*b + 2*b^3)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)
*d) - (2*a*b*e^(-2*d*x - 2*c) - 2*a*b*e^(-4*d*x - 4*c) + (3*a^2 + 2*b^2)*e
^(-d*x - c) + 2*(a^2 + 2*b^2)*e^(-3*d*x - 3*c) + (3*a^2 + 2*b^2)*e^(-5*d*x
 - 5*c))/((a^3 + a*b^2 + (a^3 + a*b^2)*e^(-2*d*x - 2*c) - (a^3 + a*b^2)*e^
(-4*d*x - 4*c) - (a^3 + a*b^2)*e^(-6*d*x - 6*c))*d) - b*log(e^(-d*x - c) +
 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e + (32*b*d*integrate(1/32*
x/(a^2*d*e^(d*x + c) + a^2*d), x) - 32*b*d*integrate(1/32*x/(a^2*d*e^(d*x
+ c) - a^2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) + 1)/(a^2*d^2
)) - a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^2)) - (2*a*b*d*x
*e^(2*d*x + 2*c) - 2*(a^2*d*e^(3*c) + 2*b^2*d*e^(3*c))*x*e^(3*d*x) + a*b -
 (a^2*e^(5*c) + (3*a^2*d*e^(5*c) + 2*b^2*d*e^(5*c))*x)*e^(5*d*x) - (2*a*b*
d*x*e^(4*c) + a*b*e^(4*c))*e^(4*d*x) + (a^2*e^c - (3*a^2*d*e^c + 2*b^2*d*e
^c)*x)*e^(d*x))/(a^3*d^2 + a*b^2*d^2 - (a^3*d^2*e^(6*c) + a*b^2*d^2*e^(6*c
))*e^(6*d*x) - (a^3*d^2*e^(4*c) + a*b^2*d^2*e^(4*c))*e^(4*d*x) + (a^3*d^2*
e^(2*c) + a*b^2*d^2*e^(2*c))*e^(2*d*x)) - 32*integrate(-1/16*(a*b^5*x*e^(d
*x + c) - b^6*x)/(a^6*b + 2*a^4*b^3 + a^2*b^5 - (a^6*b*e^(2*c) + 2*a^4*b^3
*e^(2*c) + a^2*b^5*e^(2*c))*e^(2*d*x) - 2*(a^7*e^c + 2*a^5*b^2*e^c + a^3*b
^4*e^c)*e^(d*x)), x) - 32*integrate(1/32*((3*a^3*e^c + 5*a*b^2*e^c)*x*e...
```

3.473.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.473.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$=\int \frac{e+f\,x}{\cosh\left(c+d\,x\right)^3\,\sinh\left(c+d\,x\right)^2\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.474 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.474.1 Optimal result

Integrand size = 29, antiderivative size = 180

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\arctan(\sinh(c+dx))}{2(a^2+b^2)\,d} - \frac{a(a^2+2b^2)\arctan(\sinh(c+dx))}{(a^2+b^2)^2\,d} - \frac{\operatorname{csch}(c+dx)}{ad} + \frac{b(a^2+2b^2)\log(\cosh(c+dx))}{(a^2+b^2)^2\,d} - \frac{b\log(\sinh(c+dx))}{a^2d} + \frac{b^5\log(a+b\sinh(c+dx))}{a^2(a^2+b^2)^2\,d} - \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)\,d}$$

output
```
-1/2*a*arctan(sinh(d*x+c))/(a^2+b^2)/d-a*(a^2+2*b^2)*arctan(sinh(d*x+c))/(
a^2+b^2)^2/d-csch(d*x+c)/a/d+b*(a^2+2*b^2)*ln(cosh(d*x+c))/(a^2+b^2)^2/d-b
*ln(sinh(d*x+c))/a^2/d+b^5*ln(a+b*sinh(d*x+c))/a^2/(a^2+b^2)^2/d-1/2*sech(
d*x+c)^2*(b+a*sinh(d*x+c))/(a^2+b^2)/d
```

3.474.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{\operatorname{csch}(c+dx)(a+b\sinh(c+dx))\left(\frac{a\arctan(\sinh(c+dx))}{a^2+b^2}+\frac{2\operatorname{csch}(c+dx)}{a}-\frac{(ia+b)(a^2+2b^2)\log(i-\sinh(c+dx))}{(a^2+b^2)^2}+\frac{2b\log(\mathrm{s}}{}\right.}{2d(b+a\operatorname{csch}(}$$

input
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
-1/2*(Csch[c + d*x]*(a + b*Sinh[c + d*x])*((a*ArcTan[Sinh[c + d*x]])/(a^2
+ b^2) + (2*Csch[c + d*x])/a - ((I*a + b)*(a^2 + 2*b^2)*Log[I - Sinh[c + d
*x]])/(a^2 + b^2)^2 + (2*b*Log[Sinh[c + d*x]])/a^2 + ((I*a - b)*(a^2 + 2*b
^2)*Log[I + Sinh[c + d*x]])/(a^2 + b^2)^2 - (2*b^5*Log[a + b*Sinh[c + d*x]
])/(a^2*(a^2 + b^2)^2) + (b*Sech[c + d*x]^2)/(a^2 + b^2) + (a*Sech[c + d*x
]*Tanh[c + d*x])/(a^2 + b^2)))/(d*(b + a*Csch[c + d*x]))
```

3.474.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {3042, 25, 3316, 25, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{1}{\sin(ic+idx)^2\cos(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{1}{\cos(ic+idx)^3\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \; 3316$$

$$-\frac{b^3\int -\frac{\text{csch}^2(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{b^3\int \frac{\text{csch}^2(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{b^5\int \frac{\text{csch}^2(c+dx)}{b^2(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \; 615$$

$$\frac{b^5\int \left(\frac{\text{csch}^2(c+dx)}{ab^6}-\frac{\text{csch}(c+dx)}{a^2b^5}+\frac{1}{a^2(a^2+b^2)^2(a+b\sinh(c+dx))}-\frac{(a^2+2b^2)(a-b\sinh(c+dx))}{b^4(a^2+b^2)^2(\sinh^2(c+dx)b^2+b^2)}+\frac{b\sinh(c+dx)-a}{b^2(a^2+b^2)(\sinh^2(c+dx)b^2+b^2)^2}\right.}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b^5\left(-\frac{a(a^2+2b^2)\arctan(\sinh(c+dx))}{b^5(a^2+b^2)^2}-\frac{a\arctan(\sinh(c+dx))}{2b^5(a^2+b^2)}-\frac{\log(b\sinh(c+dx))}{a^2b^4}+\frac{\log(a+b\sinh(c+dx))}{a^2(a^2+b^2)^2}-\frac{ab\sinh(c+dx)+b^2}{2b^4(a^2+b^2)(b^2\sinh^2(c+dx)+b}\right.}{d}$$

input
```
Int[(Csch[c + d*x]^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^5*(-1/2*(a*ArcTan[Sinh[c + d*x]])/(b^5*(a^2 + b^2)) - (a*(a^2 + 2*b^2)*
ArcTan[Sinh[c + d*x]])/(b^5*(a^2 + b^2)^2) - Csch[c + d*x]/(a*b^5) - Log[b
*Sinh[c + d*x]]/(a^2*b^4) + Log[a + b*Sinh[c + d*x]]/(a^2*(a^2 + b^2)^2) +
 ((a^2 + 2*b^2)*Log[b^2 + b^2*Sinh[c + d*x]^2])/(2*b^4*(a^2 + b^2)^2) - (b
^2 + a*b*Sinh[c + d*x])/(2*b^4*(a^2 + b^2)*(b^2 + b^2*Sinh[c + d*x]^2))))/
d
```

3.474.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.474. $\int \frac{\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

3.474.4 Maple [A] (verified)

Time = 28.20 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{b^5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)^2a^2}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{2\left(\frac{\left(-\frac{1}{2}a^3-\frac{1}{2}ab^2\right)\tanh(}{}\right.}{}}{}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{b^5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)^2a^2}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{2\left(\frac{\left(-\frac{1}{2}a^3-\frac{1}{2}ab^2\right)\tanh(}{}\right.}{}}{}$
risch	$-\frac{2a^2b\,d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2a^2bdc}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{4b^3d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{4b^3dc}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2bx}{a^2}+\frac{2bc}{a^2d}$

input
```
int(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/a*tanh(1/2*d*x+1/2*c)+b^5/(a^2+b^2)^2/a^2*ln(tanh(1/2*d*x+1/2*c)^
2*a-2*b*tanh(1/2*d*x+1/2*c)-a)-1/2/a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(tanh(1
/2*d*x+1/2*c))-2/(a^2+b^2)^2*(((-1/2*a^3-1/2*a*b^2)*tanh(1/2*d*x+1/2*c)^3+
(-a^2*b-b^3)*tanh(1/2*d*x+1/2*c)^2+(1/2*a^3+1/2*a*b^2)*tanh(1/2*d*x+1/2*c)
)/(1+tanh(1/2*d*x+1/2*c)^2)^2+1/4*(-2*a^2*b-4*b^3)*ln(1+tanh(1/2*d*x+1/2*c
)^2)+1/2*(3*a^3+5*a*b^2)*arctan(tanh(1/2*d*x+1/2*c))))
```

3.474.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2568 vs. $2(176) = 352$.

Time = 0.49 (sec) , antiderivative size = 2568, normalized size of antiderivative = 14.27

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fric
as")
```

output
```
-((3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^5 + (3*a^5 + 5*a^3*b^2 + 2*a
*b^4)*sinh(d*x + c)^5 + 2*(a^4*b + a^2*b^3)*cosh(d*x + c)^4 + (2*a^4*b + 2
*a^2*b^3 + 5*(3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4
+ 2*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^3 + 2*(a^5 + 3*a^3*b^2 + 2*a
*b^4 + 5*(3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^2 + 4*(a^4*b + a^2*b^
3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(a^4*b + a^2*b^3)*cosh(d*x + c)^2 -
2*(a^4*b + a^2*b^3 - 5*(3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^3 - 6*(
a^4*b + a^2*b^3)*cosh(d*x + c)^2 - 3*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(d*x
+ c))*sinh(d*x + c)^2 + ((3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^6 + 6*(3*a^5 +
5*a^3*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^5 + 5*a^3*b^2)*sinh(d*x +
c)^6 - 3*a^5 - 5*a^3*b^2 + (3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^4 + (3*a^5 +
5*a^3*b^2 + 15*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5
*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^3 + (3*a^5 + 5*a^3*b^2)*cosh(d*x + c))*
sinh(d*x + c)^3 - (3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^2 - (3*a^5 + 5*a^3*b^2
 - 15*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^4 - 6*(3*a^5 + 5*a^3*b^2)*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^5 + 2*(3
*a^5 + 5*a^3*b^2)*cosh(d*x + c)^3 - (3*a^5 + 5*a^3*b^2)*cosh(d*x + c))*sin
h(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + (3*a^5 + 5*a^3*b^2 + 2
*a*b^4)*cosh(d*x + c) - (b^5*cosh(d*x + c)^6 + 6*b^5*cosh(d*x + c)*sinh(d*
x + c)^5 + b^5*sinh(d*x + c)^6 + b^5*cosh(d*x + c)^4 - b^5*cosh(d*x + c...
```

3.474.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.474.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.94

$$\begin{aligned}\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{b^5\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^6+2\,a^4b^2+a^2b^4)d} \\ &+ \frac{(3\,a^3+5\,ab^2)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d} + \frac{(a^2b+2\,b^3)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d} \\ &- \frac{2\,abe^{(-2\,dx-2\,c)}-2\,abe^{(-4\,dx-4\,c)}+(3\,a^2+2\,b^2)e^{(-dx-c)}+2\,(a^2+2\,b^2)e^{(-3\,dx-3\,c)}+(3\,a^2+2\,b^2)e^{(-5\,dx-5\,c)}}{(a^3+ab^2+(a^3+ab^2)e^{(-2\,dx-2\,c)}-(a^3+ab^2)e^{(-4\,dx-4\,c)}-(a^3+ab^2)e^{(-6\,dx-6\,c)})d} \\ &- \frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d} - \frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}\end{aligned}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxi
ma")
```

output
```
b^5*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^6 + 2*a^4*b^2 + a^
2*b^4)*d) + (3*a^3 + 5*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4
)*d) + (a^2*b + 2*b^3)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*
d) - (2*a*b*e^(-2*d*x - 2*c) - 2*a*b*e^(-4*d*x - 4*c) + (3*a^2 + 2*b^2)*e^
(-d*x - c) + 2*(a^2 + 2*b^2)*e^(-3*d*x - 3*c) + (3*a^2 + 2*b^2)*e^(-5*d*x
- 5*c))/((a^3 + a*b^2 + (a^3 + a*b^2)*e^(-2*d*x - 2*c) - (a^3 + a*b^2)*e^(
-4*d*x - 4*c) - (a^3 + a*b^2)*e^(-6*d*x - 6*c))*d) - b*log(e^(-d*x - c) +
1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d)
```

3.474.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(176) = 352$.

Time = 0.30 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.54

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\frac{12\,b^6\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^6b+2\,a^4b^3+a^2b^5} - \frac{3\,\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(3\,a^3+5\,ab^2)}{a^4+2\,a^2b^2+b^4} + \frac{6\,\left(a^2b+2\,b^3\right)\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2\right.}{a^4+2\,a^2b^2+b^4}}{}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac
")
```

output
```
1/12*(12*b^6*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^6*b + 2*a^4
*b^3 + a^2*b^5) - 3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c))
)*(3*a^3 + 5*a*b^2)/(a^4 + 2*a^2*b^2 + b^4) + 6*(a^2*b + 2*b^3)*log((e^(d*
x + c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) - 12*b*log(abs(e^(d*
x + c) - e^(-d*x - c)))/a^2 + 4*(b^5*(e^(d*x + c) - e^(-d*x - c))^3 - 9*a^
5*(e^(d*x + c) - e^(-d*x - c))^2 - 15*a^3*b^2*(e^(d*x + c) - e^(-d*x - c))
^2 - 6*a*b^4*(e^(d*x + c) - e^(-d*x - c))^2 - 6*a^4*b*(e^(d*x + c) - e^(-d
*x - c)) - 6*a^2*b^3*(e^(d*x + c) - e^(-d*x - c)) + 4*b^5*(e^(d*x + c) - e
^(-d*x - c)) - 24*a^5 - 48*a^3*b^2 - 24*a*b^4)/((a^6 + 2*a^4*b^2 + a^2*b^4
)*((e^(d*x + c) - e^(-d*x - c))^3 + 4*e^(d*x + c) - 4*e^(-d*x - c))))/d
```

3.474.9 Mupad [B] (verification not implemented)

Time = 8.67 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.21

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2\,b}{d\,{\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}^2\,\left(a^2+b^2\right)}-\frac{b\,\ln\left({\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}-1\right)}{a^2\,d}+\frac{2\,b\,\ln\left(1+{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,1{}\mathrm{i}\right)}{d\,{\left(-b+a\,1{}\mathrm{i}\right)}^2}-\frac{2\,b^3}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)\,{\left(a^2+b^2\right)}^2}-\frac{2\,{\mathrm{e}}^{c+d\,x}}{a\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}+\frac{2\,b\,\ln\left({\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c+1{}\mathrm{i}\right)}{d\,{\left(b+a\,1{}\mathrm{i}\right)}^2}-\frac{2\,a^2\,b}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)\,{\left(a^2+b^2\right)}^2}-\frac{a^3\,{\mathrm{e}}^{c+d\,x}}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)\,{\left(a^2+b^2\right)}^2}+\frac{b^5\,\ln\left(2\,a\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c-b+b\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\right)}{a^2\,d\,{\left(a^2+b^2\right)}^2}+\frac{2\,a\,{\mathrm{e}}^{c+d\,x}}{d\,{\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}^2\,\left(a^2+b^2\right)}-\frac{a\,b^2\,{\mathrm{e}}^{c+d\,x}}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)\,{\left(a^2+b^2\right)}^2}-\frac{a\,\ln\left(1+{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{2\,d\,{\left(-b+a\,1{}\mathrm{i}\right)}^2}+\frac{a\,\ln\left({\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c+1{}\mathrm{i}\right)\,3{}\mathrm{i}}{2\,d\,{\left(b+a\,1{}\mathrm{i}\right)}^2}$$

input
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
(2*b)/(d*(exp(2*c + 2*d*x) + 1)^2*(a^2 + b^2)) - (b*log(exp(2*c)*exp(2*d*x
) - 1))/(a^2*d) - (a*log(exp(d*x)*exp(c)*1i + 1)*3i)/(2*d*(a*1i - b)^2) +
(2*b*log(exp(d*x)*exp(c)*1i + 1))/(d*(a*1i - b)^2) - (2*b^3)/(d*(exp(2*c +
 2*d*x) + 1)*(a^2 + b^2)^2) - (2*exp(c + d*x))/(a*d*(exp(2*c + 2*d*x) - 1)
) + (a*log(exp(d*x)*exp(c) + 1i)*3i)/(2*d*(a*1i + b)^2) + (2*b*log(exp(d*x
)*exp(c) + 1i))/(d*(a*1i + b)^2) - (2*a^2*b)/(d*(exp(2*c + 2*d*x) + 1)*(a^
2 + b^2)^2) - (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)*(a^2 + b^2)^2)
+ (b^5*log(2*a*exp(d*x)*exp(c) - b + b*exp(2*c)*exp(2*d*x)))/(a^2*d*(a^2 +
 b^2)^2) + (2*a*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)^2*(a^2 + b^2)) - (
a*b^2*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)*(a^2 + b^2)^2)
```

3.475 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.475.1 Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.475.2 Mathematica [N/A]

Not integrable

Time = 109.98 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])),x]
```

output
```
Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

3.475.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

$$\Big\downarrow \text{6125}$$

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]^2*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.475.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.475.4 Maple [N/A] (verified)

Not integrable

Time = 0.80 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(dx+c)^2\operatorname{sech}(dx+c)^3}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.475.5 Fricas [N/A]

Not integrable

Time = 157.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^2\operatorname{sech}(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
integral(csch(d*x + c)^2*sech(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh
(d*x + c)), x)
```

3.475.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.475.7 Maxima [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 1586, normalized size of antiderivative = 44.06

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^2\operatorname{sech}(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
(a*b*f + (2*b^2*d*e*e^(5*c) + (3*d*e - f)*a^2*e^(5*c) + (3*a^2*d*f*e^(5*c)
 + 2*b^2*d*f*e^(5*c))*x)*e^(5*d*x) + (2*a*b*d*f*x*e^(4*c) + (2*d*e - f)*a*
b*e^(4*c))*e^(4*d*x) + 2*(a^2*d*e*e^(3*c) + 2*b^2*d*e*e^(3*c) + (a^2*d*f*e
^(3*c) + 2*b^2*d*f*e^(3*c))*x)*e^(3*d*x) - 2*(a*b*d*f*x*e^(2*c) + a*b*d*e*
e^(2*c))*e^(2*d*x) + (2*b^2*d*e*e^c + (3*d*e + f)*a^2*e^c + (3*a^2*d*f*e^c
 + 2*b^2*d*f*e^c)*x)*e^(d*x))/(a^3*d^2*e^2 + a*b^2*d^2*e^2 + (a^3*d^2*f^2
+ a*b^2*d^2*f^2)*x^2 + 2*(a^3*d^2*e*f + a*b^2*d^2*e*f)*x - (a^3*d^2*e^2*e^
(6*c) + a*b^2*d^2*e^2*e^(6*c) + (a^3*d^2*f^2*e^(6*c) + a*b^2*d^2*f^2*e^(6*
c))*x^2 + 2*(a^3*d^2*e*f*e^(6*c) + a*b^2*d^2*e*f*e^(6*c))*x)*e^(6*d*x) - (
a^3*d^2*e^2*e^(4*c) + a*b^2*d^2*e^2*e^(4*c) + (a^3*d^2*f^2*e^(4*c) + a*b^2
*d^2*f^2*e^(4*c))*x^2 + 2*(a^3*d^2*e*f*e^(4*c) + a*b^2*d^2*e*f*e^(4*c))*x)
*e^(4*d*x) + (a^3*d^2*e^2*e^(2*c) + a*b^2*d^2*e^2*e^(2*c) + (a^3*d^2*f^2*e
^(2*c) + a*b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^3*d^2*e*f*e^(2*c) + a*b^2*d^2*e
*f*e^(2*c))*x)*e^(2*d*x)) - 32*integrate(-1/16*(a*b^5*e^(d*x + c) - b^6)/(
a^6*b*e + 2*a^4*b^3*e + a^2*b^5*e + (a^6*b*f + 2*a^4*b^3*f + a^2*b^5*f)*x
- (a^6*b*e*e^(2*c) + 2*a^4*b^3*e*e^(2*c) + a^2*b^5*e*e^(2*c) + (a^6*b*f*e^
(2*c) + 2*a^4*b^3*f*e^(2*c) + a^2*b^5*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^7*e*e
^c + 2*a^5*b^2*e*e^c + a^3*b^4*e*e^c + (a^7*f*e^c + 2*a^5*b^2*f*e^c + a^3*
b^4*f*e^c)*x)*e^(d*x)), x) - 32*integrate(1/32*(2*(d^2*e^2 - f^2)*a^2*b +
2*(2*d^2*e^2 - f^2)*b^3 + 2*(a^2*b*d^2*f^2 + 2*b^3*d^2*f^2)*x^2 + 4*(a^...
```

3.475.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.475.9 Mupad [N/A]

Not integrable

Time = 20.55 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\sinh\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x
)
```

3.476 $\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.476.1 Optimal result

Integrand size $= 34$, antiderivative size $= 752$

$$\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{3f(e+fx)^2}{2ad^2}+\frac{6bf(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{a^2d^2}-\frac{3f(e+fx)^2\coth(c+dx)}{2ad^2}$$
$$+\frac{b(e+fx)^3\operatorname{csch}(c+dx)}{a^2d}-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2ad}$$
$$-\frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d}-\frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d}$$
$$+\frac{3f^2(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad^3}+\frac{b^2(e+fx)^3\log\left(1-e^{2(c+dx)}\right)}{a^3d}$$
$$+\frac{6bf^2(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^3}-\frac{6bf^2(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^3}$$
$$-\frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2}$$
$$+\frac{3f^3\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^4}+\frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^3d^2}$$
$$-\frac{6bf^3\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{a^2d^4}+\frac{6bf^3\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{a^2d^4}$$
$$+\frac{6b^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3}+\frac{6b^2f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^3}$$
$$-\frac{3b^2f^2(e+fx)\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^3d^3}-\frac{6b^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^4}$$
$$-\frac{6b^2f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^4}+\frac{3b^2f^3\operatorname{PolyLog}\left(4,e^{2(c+dx)}\right)}{4a^3d^4}$$

output
```
-3/2*f*(f*x+e)^2/a/d^2+6*b*f*(f*x+e)^2*arctanh(exp(d*x+c))/a^2/d^2-3/2*f*(
f*x+e)^2*coth(d*x+c)/a/d^2+b*(f*x+e)^3*csch(d*x+c)/a^2/d-1/2*(f*x+e)^3*csc
h(d*x+c)^2/a/d+3*f^2*(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d^3+b^2*(f*x+e)^3*ln(1
-exp(2*d*x+2*c))/a^3/d-b^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2))
)/a^3/d-b^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d+6*b*f^2
*(f*x+e)*polylog(2,-exp(d*x+c))/a^2/d^3-6*b*f^2*(f*x+e)*polylog(2,exp(d*x+
c))/a^2/d^3+3/2*f^3*polylog(2,exp(2*d*x+2*c))/a/d^4+3/2*b^2*f*(f*x+e)^2*po
lylog(2,exp(2*d*x+2*c))/a^3/d^2-3*b^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/a^3/d^2-3*b^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/a^3/d^2-6*b*f^3*polylog(3,-exp(d*x+c))/a^2/d^4+6*b*f^3*p
olylog(3,exp(d*x+c))/a^2/d^4-3/2*b^2*f^2*(f*x+e)*polylog(3,exp(2*d*x+2*c))
/a^3/d^3+6*b^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^
3/d^3+6*b^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d
^3+3/4*b^2*f^3*polylog(4,exp(2*d*x+2*c))/a^3/d^4-6*b^2*f^3*polylog(4,-b*ex
p(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^4-6*b^2*f^3*polylog(4,-b*exp(d*x+c)/(a
+(a^2+b^2)^(1/2)))/a^3/d^4
```

3.476.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3254 vs. 2(752) = 1504.

Time = 11.72 (sec) , antiderivative size = 3254, normalized size of antiderivative = 4.33

$$\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(b*(e + f*x)^3*Csch[c])/(a^2*d) + ((-e^3 - 3*e^2*f*x - 3*e*f^2*x^2 - f^3*x
^3)*Csch[c/2 + (d*x)/2]^2)/(8*a*d) - (8*b^2*d^4*e^3*E^(2*c)*x + 24*a^2*d^2
*e*E^(2*c)*f^2*x + 12*b^2*d^4*e^2*E^(2*c)*f*x^2 + 12*a^2*d^2*E^(2*c)*f^3*x
^2 + 8*b^2*d^4*e*E^(2*c)*f^2*x^3 + 2*b^2*d^4*E^(2*c)*f^3*x^4 + 24*a*b*d^2*
e^2*f*ArcTanh[E^(c + d*x)] - 24*a*b*d^2*e^2*E^(2*c)*f*ArcTanh[E^(c + d*x)]
 - 24*a*b*d^2*e*f^2*x*Log[1 - E^(c + d*x)] + 24*a*b*d^2*e*E^(2*c)*f^2*x*Lo
g[1 - E^(c + d*x)] - 12*a*b*d^2*f^3*x^2*Log[1 - E^(c + d*x)] + 12*a*b*d^2*
E^(2*c)*f^3*x^2*Log[1 - E^(c + d*x)] + 24*a*b*d^2*e*f^2*x*Log[1 + E^(c + d
*x)] - 24*a*b*d^2*e*E^(2*c)*f^2*x*Log[1 + E^(c + d*x)] + 12*a*b*d^2*f^3*x^
2*Log[1 + E^(c + d*x)] - 12*a*b*d^2*E^(2*c)*f^3*x^2*Log[1 + E^(c + d*x)] +
 4*b^2*d^3*e^3*Log[1 - E^(2*(c + d*x))] - 4*b^2*d^3*e^3*E^(2*c)*Log[1 - E^
(2*(c + d*x))] + 12*a^2*d*e*f^2*Log[1 - E^(2*(c + d*x))] - 12*a^2*d*e*E^(2
*c)*f^2*Log[1 - E^(2*(c + d*x))] + 12*b^2*d^3*e^2*f*x*Log[1 - E^(2*(c + d*
x))] - 12*b^2*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(2*(c + d*x))] + 12*a^2*d*f^3*
x*Log[1 - E^(2*(c + d*x))] - 12*a^2*d*E^(2*c)*f^3*x*Log[1 - E^(2*(c + d*x)
)] + 12*b^2*d^3*e*f^2*x^2*Log[1 - E^(2*(c + d*x))] - 12*b^2*d^3*e*E^(2*c)*
f^2*x^2*Log[1 - E^(2*(c + d*x))] + 4*b^2*d^3*f^3*x^3*Log[1 - E^(2*(c + d*x
))] - 4*b^2*d^3*E^(2*c)*f^3*x^3*Log[1 - E^(2*(c + d*x))] - 24*a*b*d*(-1 +
E^(2*c))*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)] + 24*a*b*d*(-1 + E^(2*c))*
f^2*(e + f*x)*PolyLog[2, E^(c + d*x)] + 6*b^2*d^2*e^2*f*PolyLog[2, E^(2...
```

3.476.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6121}$$

$$\frac{\int (e+fx)^3\coth(c+dx)\operatorname{csch}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5975}$$

$$\frac{\frac{3f\int (e+fx)^2\operatorname{csch}^2(c+dx)dx}{2d}-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}}{a}-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}+\frac{3f\int -(e+fx)^2\csc(ic+idx)^2dx}{2d}}{a}$$

$$\Big\downarrow \ 25$$

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\int (e+fx)^2\csc(ic+idx)^2dx}{2d}}{a}$$

$$\Big\downarrow \ 4672$$

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2if\int -i(e+fx)\coth(c+dx)dx}{d}\right)}{2d}}{a}$$

$$\Big\downarrow \ 26$$

$$\frac{-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int (e+fx)\coth(c+dx)dx}{d}\right)}{2d}-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{2f\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{2d}}{a}$$

$$\Big\downarrow \ 26$$

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}\right)}{2d}}{a}$$

$$\Big\downarrow \ 4201$$

$$-\frac{b\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\int \frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}$$

$$\Big\downarrow \ 2620$$

3.476. $\int \frac{(e+fx)^3\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{b\int\frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}}{}$$

$$\downarrow \text{2715}$$

$$-\frac{b\int\frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e}{}\right.}{d}\right.}{2d}}{a}$$

$$\downarrow \text{2838}$$

$$-\frac{b\int\frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}$$

$$\downarrow \text{6121}$$

$$-\frac{b\left(\frac{\int(e+fx)^3\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}$$

$$\downarrow \text{5975}$$

$$-\frac{b\left(\frac{\frac{3f\int(e+fx)^2\text{csch}(c+dx)dx}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}$$

3.476. $\int\frac{(e+fx)^3\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3f\int i(e+fx)^2\csc(ic+idx)dx}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\int(e+fx)^2\csc(ic+idx)dx}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{4670}$$

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(\frac{2if\int(e+fx)\log\left(1-e^{c+dx}\right)dx}{d}-\frac{2if\int(e+fx)\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{3011}$$

3.476. $\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int\operatorname{PolyLog}\left(2,e\right.}{d}\right.}{}\right.}{d}}{a}\right.}{a}$$

$$\Big\downarrow \color{blue}{2720}$$

$$-\frac{\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(f\right.}{}\right.}{}}{a}\right.}{a}$$

$$\Big\downarrow \color{blue}{6103}$$

$$-\frac{\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-\frac{b\left(-\frac{b\left(\frac{\int(e+fx)^3\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)}{}\right.}{d}\right.}{}}{}\right.}{a}$$

3.476. $\int\frac{(e+fx)^3\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$\downarrow$ 3042

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-\frac{b\left(\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+\cdots}}{d^2}\cdots\right)}{d}\cdots\right)}{d}}{a}\cdots\right)}{a}$$

$\downarrow$ 26

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-\frac{b\left(\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+\cdots}}{d^2}\cdots\right)}{d}\cdots\right)}{d}}{a}\cdots\right)}{a}$$

$\downarrow$ 4201

3.476. $\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-\frac{b\left(\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+\cdots}}{d^2}\cdots\right)}{d}\cdots\right)}{d}\cdots}{a}\cdots\right)}{a}\cdots$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{-\frac{(e+fx)^3\text{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-\frac{b\left(\frac{-\frac{(e+fx)^3\text{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+\cdots}}{d^2}\cdots\right)}{d}\cdots\right)}{d}\cdots}{a}\cdots\right)}{\cdots}$$

$$\Big\downarrow \text{ 3011}$$

3.476. $\int \frac{(e+fx)^3\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-$$

$$\frac{b\left(\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+\cdots}}{d^2}\cdots\right)}{d}\cdots\right)}{d}}{a}\cdots\right)}{\cdots}$$

$$\Big\downarrow \text{6095}$$

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-$$

$$\frac{b\left(\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+\cdots}}{d^2}\cdots\right)}{d}\cdots\right)}{d}}{a}\cdots\right)}{\cdots}$$

$$\Big\downarrow \text{2620}$$

3.476. $\displaystyle\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+}}{d^2}\right.}{d}\right.}{d}}{a}\right.$$

$\downarrow$ 3011

$$\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+}}{d^2}\right.}{d}\right.}{d}}{a}\right.$$

3.476. $\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{7143}$$

$$-\frac{-\frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{2d}-\frac{3f\left(\frac{(e+fx)^2\coth(c+dx)}{d}+\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}+\frac{3if\left(\frac{2i(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{d^2}-\cdots\right.}{d}\right.}{d}}{a}\right.$$

$$\Big\downarrow \text{7163}$$

3.476. $\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{-\frac{\operatorname{csch}^2(c+dx)(e+fx)^3}{2d}-\frac{3f\left(\frac{\coth(c+dx)(e+fx)^2}{d}+\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}+\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{2d}}{a}-$$

$$b\left(\frac{\frac{3if\left(\frac{2i\operatorname{arctanh}\left(e^{c+dx}\right)(e+fx)^2}{d}-\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right.}{d}\right.}{d}}{a}\right.$$

input
```
Int[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

3.476. $\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
$Aborted
```

3.476.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.476. $\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

3.476. $\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6121
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.476.4 Maple [F]

$$\int \frac{(fx+e)^3 \coth(dx+c) \operatorname{csch}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.476.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11595 vs. $2(703) = 1406$.

Time = 0.41 (sec) , antiderivative size = 11595, normalized size of antiderivative = 15.42

$$\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.476.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.476.7 Maxima [F]

$$\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \coth(dx+c)\operatorname{csch}(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-e^3*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2
*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + b^2*log(-2*a*e^(-d*x
- c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - b^2*log(e^(-d*x - c) + 1)/(a^3*d)
 - b^2*log(e^(-d*x - c) - 1)/(a^3*d)) + (3*a*f^3*x^2 + 6*a*e*f^2*x + 3*a*e
^2*f + 2*(b*d*f^3*x^3*e^(3*c) + 3*b*d*e*f^2*x^2*e^(3*c) + 3*b*d*e^2*f*x*e^
(3*c))*e^(3*d*x) - (2*a*d*f^3*x^3*e^(2*c) + 3*a*e^2*f*e^(2*c) + 3*(2*d*e*f
^2 + f^3)*a*x^2*e^(2*c) + 6*(d*e^2*f + e*f^2)*a*x*e^(2*c))*e^(2*d*x) - 2*(
b*d*f^3*x^3*e^c + 3*b*d*e*f^2*x^2*e^c + 3*b*d*e^2*f*x*e^c)*e^(d*x))/(a^2*d
^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) + (d^3*x^3*log(e
^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x
 + c)) + 6*polylog(4, -e^(d*x + c)))*b^2*f^3/(a^3*d^4) + (d^3*x^3*log(-e^(
d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c
)) + 6*polylog(4, e^(d*x + c)))*b^2*f^3/(a^3*d^4) - 3*(b*d*e^2*f + a*e*f^2
)*x/(a^2*d^2) + 3*(b*d*e^2*f - a*e*f^2)*x/(a^2*d^2) + 3*(b*d*e^2*f + a*e*f
^2)*log(e^(d*x + c) + 1)/(a^2*d^3) - 3*(b*d*e^2*f - a*e*f^2)*log(e^(d*x +
c) - 1)/(a^2*d^3) + 3*(b^2*d*e*f^2 + a*b*f^3)*(d^2*x^2*log(e^(d*x + c) + 1
) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^3*d^4) + 3*
(b^2*d*e*f^2 - a*b*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*
x + c)) - 2*polylog(3, e^(d*x + c)))/(a^3*d^4) + 3*(b^2*d^2*e^2*f + 2*a*b*
d*e*f^2 + a^2*f^3)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^...
```

3.476.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.476.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)^3}{{\sinh\left(c+d\,x\right)}^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x)^3)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)*(e + f*x)^3)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x
)
```

3.477 $\int \frac{(e+fx)^2 \coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.477.1 Optimal result

Integrand size = 34, antiderivative size = 502

$$\begin{aligned}
&\int \frac{(e+fx)^2 \coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{4bf(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{a^2d^2} - \frac{f(e+fx)\coth(c+dx)}{ad^2} + \frac{b(e+fx)^2\text{csch}(c+dx)}{a^2d} \\
&\quad - \frac{(e+fx)^2\text{csch}^2(c+dx)}{2ad} - \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d} \\
&\quad - \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d} + \frac{b^2(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{a^3d} \\
&\quad + \frac{f^2\log(\sinh(c+dx))}{ad^3} + \frac{2bf^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^3} - \frac{2bf^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^3} \\
&\quad - \frac{2b^2f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2} - \frac{2b^2f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2} \\
&\quad + \frac{b^2f(e+fx)\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{a^3d^2} + \frac{2b^2f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3} \\
&\quad + \frac{2b^2f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^3} - \frac{b^2f^2\,\text{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^3d^3}
\end{aligned}$$

output
```
4*b*f*(f*x+e)*arctanh(exp(d*x+c))/a^2/d^2-f*(f*x+e)*coth(d*x+c)/a/d^2+b*(f
*x+e)^2*csch(d*x+c)/a^2/d-1/2*(f*x+e)^2*csch(d*x+c)^2/a/d+b^2*(f*x+e)^2*ln
(1-exp(2*d*x+2*c))/a^3/d+f^2*ln(sinh(d*x+c))/a/d^3-b^2*(f*x+e)^2*ln(1+b*ex
p(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d-b^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))/a^3/d+2*b*f^2*polylog(2,-exp(d*x+c))/a^2/d^3-2*b*f^2*polylo
g(2,exp(d*x+c))/a^2/d^3+b^2*f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a^3/d^2-2*
b^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^2-2*b^2*f
*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^2-1/2*b^2*f^2*
polylog(3,exp(2*d*x+2*c))/a^3/d^3+2*b^2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/a^3/d^3+2*b^2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))/a^3/d^3
```

3.477.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1816 vs. $2(502) = 1004$.

Time = 10.32 (sec) , antiderivative size = 1816, normalized size of antiderivative = 3.62

$$\int \frac{(e+fx)^2 \coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(b*(e + f*x)^2*Csch[c])/(a^2*d) + ((-e^2 - 2*e*f*x - f^2*x^2)*Csch[c/2 + (
d*x)/2]^2)/(8*a*d) - (12*d*E^(2*c)*(b^2*d^2*e^2 + a^2*f^2)*x - 12*d*(-1 +
E^(2*c))*(b^2*d^2*e^2 + a^2*f^2)*x + 12*b^2*d^3*e*f*x^2 + 4*b^2*d^3*f^2*x^
3 - 24*a*b*d*e*(-1 + E^(2*c))*f*ArcTanh[E^(c + d*x)] + 6*b^2*d^2*e^2*(-1 +
 E^(2*c))*(2*d*x - Log[1 - E^(2*(c + d*x))]) + 6*a^2*(-1 + E^(2*c))*f^2*(2
*d*x - Log[1 - E^(2*(c + d*x))]) + 12*a*b*(-1 + E^(2*c))*f^2*(d*x*(Log[1 -
 E^(c + d*x)] - Log[1 + E^(c + d*x)]) - PolyLog[2, -E^(c + d*x)] + PolyLog
[2, E^(c + d*x)]) + 6*b^2*d*e*(-1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 - E^(2*
(c + d*x))]) - PolyLog[2, E^(2*(c + d*x))]) + b^2*(-1 + E^(2*c))*f^2*(2*d^
2*x^2*(2*d*x - 3*Log[1 - E^(2*(c + d*x))]) - 6*d*x*PolyLog[2, E^(2*(c + d*
x))] + 3*PolyLog[3, E^(2*(c + d*x))]))/(6*a^3*d^3*(-1 + E^(2*c))) + (b^2*(
6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 +
b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^
2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/S
qrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*Ar
cTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*
Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^
2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c
 + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x
)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*...
```

3.477.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.73 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.13, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.824, Rules used = {6121, 5975, 3042, 25, 4672, 26, 3042, 26, 3956, 6121, 5975, 3042, 26, 4670, 2715, 2838, 6103, 3042, 26, 4201, 2620, 3011, 2720, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6121}$$

$$\frac{\int (e+fx)^2\coth(c+dx)\text{csch}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 5975}$$

$$\frac{\frac{f\int (e+fx)\operatorname{csch}^2(c+dx)dx}{d}-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}}{a}-\frac{b\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}+\frac{f\int -\left((e+fx)\csc(ic+idx)^2\right)dx}{d}}{a}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{b\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\int (e+fx)\csc(ic+idx)^2dx}{d}}{a}$$

$$\Big\downarrow \text{ 4672}$$

$$-\frac{b\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{if\int -i\coth(c+dx)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int \coth(c+dx)dx}{d}\right)}{d}-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}}{a}-\frac{b\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{b\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}+\frac{if\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 3956}$$

$$-\frac{b\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 6121}$$

$$-\frac{b\left(\frac{\int (e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{ 5975}$$

3.477. $$\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$-\frac{b\left(\frac{\frac{2f\int(e+fx)\text{csch}(c+dx)dx}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2f\int i(e+fx)\csc(ic+idx)dx}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\int(e+fx)\csc(ic+idx)dx}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{4670}$$

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{if\int\log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arct}}{}\right.}{d}}{a}\right.}{a}$$

$$\Big\downarrow \text{2838}$$

3.477. $\int \frac{(e+fx)^2\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{ 6103}$$

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{\int(e+fx)^2\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\cdots\right)}{d}}{a}\cdots\right)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-\frac{b\left(\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\int-\cdots\right)}{a}\cdots\right)}{a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{-\frac{(e+fx)^2\text{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-\frac{b\left(\frac{-\frac{(e+fx)^2\text{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-i\int(\cdots\right)}{a}\cdots\right)}{a}$$

$$\Big\downarrow \text{ 4201}$$

3.477. $\int\frac{(e+fx)^2\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$\frac{b\left(\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2\right.}{}\right.}{}\right.}{a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$\frac{b\left(\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2\right.}{}\right.}{}\right.}{a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$\frac{b\left(\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2\right.}{}\right.}{}\right.}{a}$$

$$\Big\downarrow \text{2720}$$

3.477. $\displaystyle\int\frac{(e+fx)^2\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2\right.}{}\right.}{}\right.$$

$$a$$

$\downarrow$ 6095

$$\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\right.}{}\right.}{}\right.$$

$\downarrow$ 2620

3.477. $\displaystyle\int\frac{(e+fx)^2\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}\right.}{bd}\right.}{}\right.}{}\right.$$

$\downarrow$ 3011

$$\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{2f\left(\frac{f\int\operatorname{PolyLog}\left(2,-\frac{be^{c+}}{a-\sqrt{a}}\right.}{d}\right.}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 2720

3.477. $\int\frac{(e+fx)^2\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,\ldots\right.}{d}\ldots\right.}{\ldots}\ldots\right.}{\ldots}\ldots\right.}{\ldots}\ldots\right.$$

$$\Big\downarrow \; 7143$$

$$\frac{-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{f\left(\frac{(e+fx)\coth(c+dx)}{d}-\frac{f\log(-i\sinh(c+dx))}{d^2}\right)}{d}}{a}-$$

$$b\left(\frac{-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}+\frac{2if\left(\frac{2i(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{2f\left(\frac{f\operatorname{PolyLog}\left(3,-\frac{be^{c+d\ldots}}{a-\sqrt{a^2\ldots}}\right.}{d^2}\ldots\right.}{\ldots}\ldots\right.}{\ldots}\ldots\right.}{\ldots}\ldots\right.$$

3.477. $\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-1/2*((e + f*x)^2*Csch[c + d*x]^2)/d - (f*(((e + f*x)*Coth[c + d*x])/d -
(f*Log[(-I)*Sinh[c + d*x]])/d^2))/d)/a - (b*((-(((e + f*x)^2*Csch[c + d*x]
)/d) + ((2*I)*f*(((2*I)*(e + f*x)*ArcTanh[E^(c + d*x)])/d + (I*f*PolyLog[2
, -E^(c + d*x)])/d^2 - (I*f*PolyLog[2, E^(c + d*x)])/d^2))/d)/a - (b*(-((b
*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
)])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
))])/d^2))/(b*d)))/a) - (I*(((-1/3*I)*(e + f*x)^3)/f + (2*I)*(((e + f*x)^2
*Log[1 + E^(2*c - I*Pi + 2*d*x)])/(2*d) - (f*(-1/2*((e + f*x)*PolyLog[2, -
E^(2*c - I*Pi + 2*d*x)])/d + (f*PolyLog[3, -E^(2*c - I*Pi + 2*d*x)])/(4*d^
2)))/d)))/a))/a))/a
```

3.477.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.477. $\int \frac{(e+fx)^2 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.477. $\int \frac{(e+fx)^2 \coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6121
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.477.4 Maple [F]

$$\int \frac{(fx+e)^2 \coth(dx+c) \operatorname{csch}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.477.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6479 vs. $2(472) = 944$.

Time $= 0.34$ (sec) , antiderivative size $= 6479$, normalized size of antiderivative $= 12.91$

$$\int \frac{(e+fx)^2 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.477.6 Sympy [F]

$$\int \frac{(e+fx)^2 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*coth(c + d*x)*csch(c + d*x)**2/(a + b*sinh(c + d*x))
, x)
```

3.477.7 Maxima [F]

$$\int \frac{(e+fx)^2 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \coth(dx+c)\operatorname{csch}(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-e^2*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2
*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + b^2*log(-2*a*e^(-d*x
- c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - b^2*log(e^(-d*x - c) + 1)/(a^3*d)
 - b^2*log(e^(-d*x - c) - 1)/(a^3*d)) + 2*(a*f^2*x + a*e*f + (b*d*f^2*x^2*
e^(3*c) + 2*b*d*e*f*x*e^(3*c))*e^(3*d*x) - (a*d*f^2*x^2*e^(2*c) + a*e*f*e^
(2*c) + (2*d*e*f + f^2)*a*x*e^(2*c))*e^(2*d*x) - (b*d*f^2*x^2*e^c + 2*b*d*
e*f*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) +
 a^2*d^2) + (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*
polylog(3, -e^(d*x + c)))*b^2*f^2/(a^3*d^3) + (d^2*x^2*log(-e^(d*x + c) +
1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*b^2*f^2/(a^3*d^
3) - (2*b*d*e*f + a*f^2)*x/(a^2*d^2) + (2*b*d*e*f - a*f^2)*x/(a^2*d^2) + (
2*b*d*e*f + a*f^2)*log(e^(d*x + c) + 1)/(a^2*d^3) - (2*b*d*e*f - a*f^2)*lo
g(e^(d*x + c) - 1)/(a^2*d^3) + 2*(b^2*d*e*f + a*b*f^2)*(d*x*log(e^(d*x + c
) + 1) + dilog(-e^(d*x + c)))/(a^3*d^3) + 2*(b^2*d*e*f - a*b*f^2)*(d*x*log
(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^3*d^3) - 1/3*(b^2*d^3*f^2*x^3
+ 3*(b^2*d*e*f + a*b*f^2)*d^2*x^2)/(a^3*d^3) - 1/3*(b^2*d^3*f^2*x^3 + 3*(b
^2*d*e*f - a*b*f^2)*d^2*x^2)/(a^3*d^3) + integrate(-2*(b^3*f^2*x^2 + 2*b^3
*e*f*x - (a*b^2*f^2*x^2*e^c + 2*a*b^2*e*f*x*e^c)*e^(d*x))/(a^3*b*e^(2*d*x
+ 2*c) + 2*a^4*e^(d*x + c) - a^3*b), x)
```

3.477.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.477.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)^2}{\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x)^2)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)*(e + f*x)^2)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x
)
```

3.478 $\int \frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.478.1 Optimal result

Integrand size = 32, antiderivative size = 298

$$\begin{aligned}
&\int \frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{bf\text{arctanh}(\cosh(c+dx))}{a^2d^2} - \frac{f\coth(c+dx)}{2ad^2} \\
&\quad + \frac{b(e+fx)\text{csch}(c+dx)}{a^2d} - \frac{(e+fx)\text{csch}^2(c+dx)}{2ad} \\
&\quad - \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d} - \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d} \\
&\quad + \frac{b^2(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^3d} - \frac{b^2f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2} \\
&\quad - \frac{b^2f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2} + \frac{b^2f\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^3d^2}
\end{aligned}$$

output
```
b*f*arctanh(cosh(d*x+c))/a^2/d^2-1/2*f*coth(d*x+c)/a/d^2+b*(f*x+e)*csch(d*
x+c)/a^2/d-1/2*(f*x+e)*csch(d*x+c)^2/a/d+b^2*(f*x+e)*ln(1-exp(2*d*x+2*c))/
a^3/d-b^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d-b^2*(f*x+e)
*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d+1/2*b^2*f*polylog(2,exp(2*d*
x+2*c))/a^3/d^2-b^2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^2
-b^2*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^2
```

3.478.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 713 vs. $2(298) = 596$.

Time = 8.40 (sec) , antiderivative size = 713, normalized size of antiderivative = 2.39

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\left(2bde\cosh\left(\frac{1}{2}(c+dx)\right)-af\cosh\left(\frac{1}{2}(c+dx)\right)-2bcf\cosh\left(\frac{1}{2}(c+dx)\right)+2bf(c+dx)\cosh\left(\frac{1}{2}(c+dx)\right)\right)\text{c}}{4a^2d^2}$$
$$+\frac{(-de+cf-f(c+dx))\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{b\left(\frac{b(de+dfx)^2}{2f}+(bde-af+bdfx)\log\left(1-e^{-c-dx}\right)+(bde+af+bdfx)\log\left(1+e^{-c-dx}\right)-bf\,\text{PolyLog}\left(\right.\right.}{a^3d^2}$$
$$-\frac{b^2\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\,\text{arctanh}\left(\frac{a+be}{\sqrt{a}}\right.}{(-a^2-b^2)^{3/2}}\right.}{}$$
$$+\frac{(de-cf+f(c+dx))\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{\text{sech}\left(\frac{1}{2}(c+dx)\right)\left(-2bde\sinh\left(\frac{1}{2}(c+dx)\right)-af\sinh\left(\frac{1}{2}(c+dx)\right)+2bcf\sinh\left(\frac{1}{2}(c+dx)\right)-2bf(c+dx)\right.}{4a^2d^2}$$

input
```
Integrate[((e + f*x)*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),
x]
```

output
```
((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*
x)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2)
+ ((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) + (b*((b*(d
*e + d*f*x)^2)/(2*f) + (b*d*e - a*f + b*d*f*x)*Log[1 - E^(-c - d*x)] + (b*
d*e + a*f + b*d*f*x)*Log[1 + E^(-c - d*x)] - b*f*PolyLog[2, -E^(-c - d*x)]
 - b*f*PolyLog[2, E^(-c - d*x)]))/(a^3*d^2) - (b^2*(-2*d*e*(c + d*x) + 2*c
*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a + b*E^(c
 + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)^
2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) +
2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*(c + d*
x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2*a*E^(c
 + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c
 + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f*
PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(2*a^3*d^2) + ((d*e
 - c*f + f*(c + d*x))*Sech[(c + d*x)/2]^2)/(8*a*d^2) + (Sech[(c + d*x)/2]*
(-2*b*d*e*Sinh[(c + d*x)/2] - a*f*Sinh[(c + d*x)/2] + 2*b*c*f*Sinh[(c + d*
x)/2] - 2*b*f*(c + d*x)*Sinh[(c + d*x)/2]))/(4*a^2*d^2)
```

3.478.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.17, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {6121, 5975, 3042, 25, 4254, 24, 6121, 5975, 3042, 26, 4257, 6103, 3042, 26, 4201, 2620, 2715, 2838, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6121}$$

$$\frac{\int (e+fx)\coth(c+dx)\text{csch}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5975}$$

$$\frac{\frac{f\int \text{csch}^2(c+dx)dx}{2d}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}+\frac{f\int -\csc(ic+idx)^2dx}{2d}}{a}$$

$$\Big\downarrow \; 25$$

$$-\frac{b\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}-\frac{f\int \csc(ic+idx)^2dx}{2d}}{a}$$

$$\Big\downarrow \; 4254$$

$$-\frac{b\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}-\frac{if\int 1d(-i\coth(c+dx))}{2d^2}}{a}$$

$$\Big\downarrow \; 24$$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 6121$$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 5975$$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{\frac{f\int \text{csch}(c+dx)dx}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)\text{csch}(c+dx)}{d}+\frac{f\int i\csc(ic+idx)dx}{d}}{a}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{(e+fx)\text{csch}(c+dx)}{d}+\frac{if\int \csc(ic+idx)dx}{d}}{a}\right)}{a}$$

$$\Big\downarrow \; 4257$$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 6103$$

3.478. $\displaystyle\int \frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int-i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\int(e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{a}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{4201}$$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\text{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{a}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

3.478. $\displaystyle\int\frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{csch}^2(c+dx)}{2d}}{a}-b\left(\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)\right.}{a}\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{csch}^2(c+dx)}{2d}}{a}-b\left(\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c-\cdots}\right)}{4d^2}\right.\right.}{a}\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{csch}^2(c+dx)}{2d}}{a}-b\left(\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)\right.}{a}\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{ 6095}$$

$$\frac{\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{csch}^2(c+dx)}{2d}}{a}-b\left(\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\frac{e^{c+dx}(e+fx)}{a+be^{c+dx}+\sqrt{a^2+b^2}}dx-\frac{(e+fx)^2}{2bf}\right)}{a}-\frac{i\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2\cdots}\right)}{4d^2}\right.\right.}{}\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{ 2620}$$

3.478. $\displaystyle\int\frac{(e+fx)\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{f\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bd}-\frac{f\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\right.}{a}\right.}{}\right.}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{f\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{(e+}{}\right.}{a}\right.}{}\right.}{a}$$

$$\Big\downarrow \text{2838}$$

$$\frac{-\frac{f\coth(c+dx)}{2d^2}-\frac{(e+fx)\operatorname{csch}^2(c+dx)}{2d}}{a}-\frac{b\left(\frac{-\frac{f\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{(e+fx)\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}\right.}{a}\right.}{}\right.}{a}$$

input
```
Int[((e + f*x)*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

3.478. $\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(-1/2*(f*Coth[c + d*x])/d^2 - ((e + f*x)*Csch[c + d*x]^2)/(2*d))/a - (b*((
-((f*ArcTanh[Cosh[c + d*x]])/d^2) - ((e + f*x)*Csch[c + d*x])/d)/a - (b*(-
((b*(-1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(
b*d^2) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2))
)/a) - (I*(((-1/2*I)*(e + f*x)^2)/f + (2*I)*(((e + f*x)*Log[1 + E^(2*c - I
*Pi + 2*d*x)])/(2*d) + (f*PolyLog[2, -E^(2*c - I*Pi + 2*d*x)])/(4*d^2))))/
a))/a))/a
```

3.478.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.478. $\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6121
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

3.478. $\int \frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.478.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(280) = 560$.

Time = 2.16 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.18

method	result
risch	$-\frac{-2bdfx\,e^{3dx+3c}+2adfx\,e^{2dx+2c}-2bde\,e^{3dx+3c}+2ade\,e^{2dx+2c}+2bdfx\,e^{dx+c}+af\,e^{2dx+2c}+2bde\,e^{dx+c}-af}{a^2d^2\left(e^{2dx+2c}-1\right)^2}-\frac{b^2 f\,\mathrm{dilog}\left(e^{dx+c}\right)}{d^2a^3}+$

input
```
int((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
-(-2*b*d*f*x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2*c)-2*b*d*e*exp(3*d*x+3*c
)+2*a*d*e*exp(2*d*x+2*c)+2*b*d*f*x*exp(d*x+c)+a*f*exp(2*d*x+2*c)+2*b*d*e*e
xp(d*x+c)-a*f)/a^2/d^2/(exp(2*d*x+2*c)-1)^2-1/d^2*b^2/a^3*f*dilog(exp(d*x+
c))+1/d^2*b^2/a^3*f*dilog(exp(d*x+c)+1)-1/d^2*b^2/a^3*f*ln((b*exp(d*x+c)+(
a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2*b^2/a^3*f*ln((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d*b^2/a^3*f*ln((b*exp(d*x+c)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d*b^2/a^3*f*ln((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d*b^2/a^3*f*ln(exp(d*x+c)+1)
*x-1/d^2*b^2/a^3*c*f*ln(exp(d*x+c)-1)+1/d^2*b^2/a^3*c*f*ln(b*exp(2*d*x+2*c
)+2*a*exp(d*x+c)-b)+1/d*b^2/a^3*e*ln(exp(d*x+c)-1)+1/d*b^2/a^3*e*ln(exp(d*
x+c)+1)-1/d*b^2/a^3*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2*b^2/a^3*
f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*b^2/
a^3*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2*b/
a^2*f*ln(exp(d*x+c)-1)+1/d^2*b/a^2*f*ln(exp(d*x+c)+1)
```

3.478.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2899 vs. $2(277) = 554$.

Time = 0.32 (sec) , antiderivative size = 2899, normalized size of antiderivative = 9.73

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
(2*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c)^3 + 2*(a*b*d*f*x + a*b*d*e)*sinh(d*
x + c)^3 + a^2*f - (2*a^2*d*f*x + 2*a^2*d*e + a^2*f)*cosh(d*x + c)^2 - (2*
a^2*d*f*x + 2*a^2*d*e + a^2*f - 6*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c))*sin
h(d*x + c)^2 - 2*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c) - (b^2*f*cosh(d*x + c
)^4 + 4*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*f*sinh(d*x + c)^4 - 2*b^
2*f*cosh(d*x + c)^2 + b^2*f + 2*(3*b^2*f*cosh(d*x + c)^2 - b^2*f)*sinh(d*x
 + c)^2 + 4*(b^2*f*cosh(d*x + c)^3 - b^2*f*cosh(d*x + c))*sinh(d*x + c))*d
ilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^2*f*cosh(d*x + c)^4 + 4*b^2*f*c
osh(d*x + c)*sinh(d*x + c)^3 + b^2*f*sinh(d*x + c)^4 - 2*b^2*f*cosh(d*x +
c)^2 + b^2*f + 2*(3*b^2*f*cosh(d*x + c)^2 - b^2*f)*sinh(d*x + c)^2 + 4*(b^
2*f*cosh(d*x + c)^3 - b^2*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2) - b)/b + 1) + (b^2*f*cosh(d*x + c)^4 + 4*b^2*f*cosh(d*x + c)*si
nh(d*x + c)^3 + b^2*f*sinh(d*x + c)^4 - 2*b^2*f*cosh(d*x + c)^2 + b^2*f +
2*(3*b^2*f*cosh(d*x + c)^2 - b^2*f)*sinh(d*x + c)^2 + 4*(b^2*f*cosh(d*x +
c)^3 - b^2*f*cosh(d*x + c))*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x
+ c)) + (b^2*f*cosh(d*x + c)^4 + 4*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + b
^2*f*sinh(d*x + c)^4 - 2*b^2*f*cosh(d*x + c)^2 + b^2*f + 2*(3*b^2*f*cosh(d
*x + c)^2 - b^2*f)*sinh(d*x + c)^2 + 4*(b^2*f*cosh(d*x + c)^3 - b^2*f*c...
```

3.478.6 Sympy [F]

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)\coth(c+dx)\,\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*coth(c + d*x)*csch(c + d*x)**2/(a + b*sinh(c + d*x)), x
)
```

3.478.7 Maxima [F]

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\coth(dx+c)\operatorname{csch}(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(4*b^2*d*integrate(1/4*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 4*b^2*d*integr
ate(1/4*x/(a^3*d*e^(d*x + c) - a^3*d), x) + a*b*((d*x + c)/(a^3*d^2) - log
(e^(d*x + c) + 1)/(a^3*d^2)) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c)
- 1)/(a^3*d^2)) - (2*b*d*x*e^(3*d*x + 3*c) - 2*b*d*x*e^(d*x + c) - (2*a*d*
x*e^(2*c) + a*e^(2*c))*e^(2*d*x) + a)/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2
*e^(2*d*x + 2*c) + a^2*d^2) - 4*integrate(1/2*(a*b^2*x*e^(d*x + c) - b^3*x
)/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) - a^3*b), x))*f - e*(2*(b*e^(
-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-2*d*x - 2
*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + b^2*log(-2*a*e^(-d*x - c) + b*e^(-2
*d*x - 2*c) - b)/(a^3*d) - b^2*log(e^(-d*x - c) + 1)/(a^3*d) - b^2*log(e^(
-d*x - c) - 1)/(a^3*d))
```

3.478.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.478.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)}{\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((coth(c + d*x)*(e + f*x))/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)*(e + f*x))/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.479 $\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.479.1 Optimal result

Integrand size = 27, antiderivative size = 72

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\text{csch}(c+dx)}{a^2d} - \frac{\text{csch}^2(c+dx)}{2ad} + \frac{b^2\log(\sinh(c+dx))}{a^3d} - \frac{b^2\log(a+b\sinh(c+dx))}{a^3d}$$

output
```
b*csch(d*x+c)/a^2/d-1/2*csch(d*x+c)^2/a/d+b^2*ln(sinh(d*x+c))/a^3/d-b^2*ln
(a+b*sinh(d*x+c))/a^3/d
```

3.479.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2ab\text{csch}(c+dx) - a^2\text{csch}^2(c+dx) + 2b^2(\log(\sinh(c+dx)) - \log(a+b\sinh(c+dx)))}{2a^3d}$$

input
```
Integrate[(Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(2*a*b*Csch[c + d*x] - a^2*Csch[c + d*x]^2 + 2*b^2*(Log[Sinh[c + d*x]] - L
og[a + b*Sinh[c + d*x]]))/(2*a^3*d)
```

3.479.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3042, 26, 3312, 26, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos(ic+idx)}{\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ic+idx)}{\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3312}$$

$$-\frac{i\int \frac{i\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}d(b\sinh(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^2\int \frac{\text{csch}^3(c+dx)}{b^3(a+b\sinh(c+dx))}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$\frac{b^2\int \left(\frac{\text{csch}^3(c+dx)}{ab^3}-\frac{\text{csch}^2(c+dx)}{a^2b^2}+\frac{\text{csch}(c+dx)}{a^3b}-\frac{1}{a^3(a+b\sinh(c+dx))}\right)d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\left(\frac{\log(b\sinh(c+dx))}{a^3}-\frac{\log(a+b\sinh(c+dx))}{a^3}+\frac{\text{csch}(c+dx)}{a^2b}-\frac{\text{csch}^2(c+dx)}{2ab^2}\right)}{d}$$

input
```
Int[(Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

3.479. $\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx$

output
```
(b^2*(Csch[c + d*x]/(a^2*b) - Csch[c + d*x]^2/(2*a*b^2) + Log[b*Sinh[c + d
*x]]/a^3 - Log[a + b*Sinh[c + d*x]]/a^3))/d
```

3.479.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

3.479. $\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.479.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{\operatorname{csch}(dx+c)^2}{2ad}+\frac{b\ \operatorname{csch}(dx+c)}{a^2d}-\frac{b^2\ln(a\ \operatorname{csch}(dx+c)+b)}{d\,a^3}$	54
default	$-\frac{\operatorname{csch}(dx+c)^2}{2ad}+\frac{b\ \operatorname{csch}(dx+c)}{a^2d}-\frac{b^2\ln(a\ \operatorname{csch}(dx+c)+b)}{d\,a^3}$	54
risch	$-\frac{2\,\mathrm{e}^{dx+c}\left(-b\,\mathrm{e}^{2dx+2c}+a\,\mathrm{e}^{dx+c}+b\right)}{a^2d\left(\mathrm{e}^{2dx+2c}-1\right)^2}-\frac{b^2\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\,\mathrm{e}^{dx+c}}{b}-1\right)}{d\,a^3}+\frac{b^2\ln\left(\mathrm{e}^{2dx+2c}-1\right)}{d\,a^3}$	108

input
```
int(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*csch(d*x+c)^2/a/d+b*csch(d*x+c)/a^2/d-1/d*b^2/a^3*ln(a*csch(d*x+c)+b)
```

3.479.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. 2(70) = 140.

Time = 0.26 (sec) , antiderivative size = 545, normalized size of antiderivative = 7.57

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{2\,ab\cosh\left(dx+c\right)^3+2\,ab\sinh\left(dx+c\right)^3-2\,a^2\cosh\left(dx+c\right)^2-2\,ab\cosh\left(dx+c\right)+2\left(3\,ab\cosh\left(dx+c\right)\right.}{}$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
(2*a*b*cosh(d*x + c)^3 + 2*a*b*sinh(d*x + c)^3 - 2*a^2*cosh(d*x + c)^2 - 2
*a*b*cosh(d*x + c) + 2*(3*a*b*cosh(d*x + c) - a^2)*sinh(d*x + c)^2 - (b^2*
cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^
4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^
2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*log(2
*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + (b^2*cosh(d*x +
c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 - 2*b^2*c
osh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^2 + b^2 + 4
*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x +
 c)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(3*a*b*cosh(d*x + c)^2 - 2*a^2*co
sh(d*x + c) - a*b)*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*
x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4 - 2*a^3*d*cosh(d*x + c)^2 +
 a^3*d + 2*(3*a^3*d*cosh(d*x + c)^2 - a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*co
sh(d*x + c)^3 - a^3*d*cosh(d*x + c))*sinh(d*x + c))
```

3.479.6 Sympy [F]

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth\left(c+dx\right)\operatorname{csch}^2\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)*csch(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

3.479.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(70) = 140$.

Time = 0.21 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.24

$$\begin{aligned}\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = &-\frac{2\left(be^{(-dx-c)}-ae^{(-2\,dx-2\,c)}-be^{(-3\,dx-3\,c)}\right)}{\left(2\,a^2e^{(-2\,dx-2\,c)}-a^2e^{(-4\,dx-4\,c)}-a^2\right)d}\\&-\frac{b^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{a^3d}\\&+\frac{b^2\log\left(e^{(-dx-c)}+1\right)}{a^3d}+\frac{b^2\log\left(e^{(-dx-c)}-1\right)}{a^3d}\end{aligned}$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-
2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - b^2*log(-2*a*e^(-d*x - c)
+ b*e^(-2*d*x - 2*c) - b)/(a^3*d) + b^2*log(e^(-d*x - c) + 1)/(a^3*d) + b^
2*log(e^(-d*x - c) - 1)/(a^3*d)
```

3.479.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.01

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2\,b^2\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^3} - \frac{2\,b^2\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a^3} + \frac{3\,b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-4\,ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)+4\,a^2}{a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2}}{2\,d}$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*b^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/a^3 - 2*b^2*log
(abs(e^(d*x + c) - e^(-d*x - c)))/a^3 + (3*b^2*(e^(d*x + c) - e^(-d*x - c)
)^2 - 4*a*b*(e^(d*x + c) - e^(-d*x - c)) + 4*a^2)/(a^3*(e^(d*x + c) - e^(-
d*x - c))^2))/d
```

3.479.9 Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 470, normalized size of antiderivative = 6.53

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2}{a\,d}-\frac{2\,b\,e^{c+d\,x}}{a^2\,d}}{e^{2\,c+2\,d\,x}-1} - \frac{\left(2\,\mathrm{atan}\left(-\frac{4\,a^3\,b^5\,\sqrt{-a^6\,d^2}+4\,a\,b^7\,\sqrt{-a^6\,d^2}-4\,b^8\,e^{3\,c}\,e^{3\,d\,x}\,\sqrt{-a^6\,d^2}+4\,b^8\,e^{d\,x}\,e^{c}\,\sqrt{-a^6\,d^2}-8\,a\,b^7\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{-a^6\,d^2}+4\,a^2\,b^6\,e^{d\,x}\,e^{c}\,\sqrt{-a^6}}{4\,a^4\,b\,d\,\left(b^4\right)^{3/2}+4\,a^6\,b^3\,d\,\sqrt{b^4}}\right.\right.}{} - \frac{2}{a\,d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int(coth(c + d*x)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

output
```
- (2/(a*d) - (2*b*exp(c + d*x))/(a^2*d))/(exp(2*c + 2*d*x) - 1) - ((2*atan
(-(4*a^3*b^5*(-a^6*d^2)^(1/2) + 4*a*b^7*(-a^6*d^2)^(1/2) - 4*b^8*exp(3*c)*
exp(3*d*x)*(-a^6*d^2)^(1/2) + 4*b^8*exp(d*x)*exp(c)*(-a^6*d^2)^(1/2) - 8*a
*b^7*exp(2*c)*exp(2*d*x)*(-a^6*d^2)^(1/2) + 4*a^2*b^6*exp(d*x)*exp(c)*(-a^
6*d^2)^(1/2) - 8*a^3*b^5*exp(2*c)*exp(2*d*x)*(-a^6*d^2)^(1/2) - 4*a^2*b^6*
exp(3*c)*exp(3*d*x)*(-a^6*d^2)^(1/2))/(4*a^4*b*d*(b^4)^(3/2) + 4*a^6*b^3*d
*(b^4)^(1/2))) + 2*atan((4*a^4*b^5*d*(b^4)^(1/2)*(-a^6*d^2)^(1/2) + 4*a^6*
b^3*d*(b^4)^(1/2)*(-a^6*d^2)^(1/2))*(1/(8*a^5*b^5*d^2*(a^2 + b^2)^2) - exp
(d*x)*exp(c)*(1/(16*a^4*b^6*d^2*(a^2 + b^2)^2) - (a^2 + 2*b^2)^2/(16*a^8*b
^6*d^2*(a^2 + b^2)^2)) + (a^2 + 2*b^2)/(8*a^7*b^5*d^2*(a^2 + b^2)^2))))*(b
^4)^(1/2))/(-a^6*d^2)^(1/2) - 2/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x
) + 1))
```

3.479. $\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.480 $\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.480.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.480.2 Mathematica [N/A]

Not integrable

Time = 142.62 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Coth[c + d*x]*Csch[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Coth[c + d*x]*Csch[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

3.480.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Coth[c + d*x]*Csch[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.480.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.480.4 Maple [N/A] (verified)

Not integrable

Time = 0.59 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\coth(dx+c)\,\text{csch}(dx+c)^2}{(fx+e)\,(a+b\sinh(dx+c))}dx$$

input
```
int(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.480.5 Fricas [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth\left(dx+c\right)\text{csch}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(coth(d*x + c)*csch(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

3.480.6 Sympy [N/A]

Not integrable

Time = 16.47 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth\left(c+dx\right)\text{csch}^2\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)*csch(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)),
 x)
```

3.480.7 Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 684, normalized size of antiderivative = 20.12

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth\left(dx+c\right)\text{csch}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(a*f - 2*(b*d*f*x*e^(3*c) + b*d*e*e^(3*c))*e^(3*d*x) + (2*a*d*f*x*e^(2*c)
 + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) + 2*(b*d*f*x*e^c + b*d*e*e^c)*e^(d*x))
/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*d^2*f^2*x^2*e^(4*
c) + 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e^(4*d*x) - 2*(a^2*d^2
*f^2*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e^2*e^(2*c))*e^(2*d*x
)) + 4*integrate(-1/4*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + a*b*d*e*f + a^2*f^2
 + (2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 +
 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*
x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) - 4*integr
ate(1/4*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 - a*b*d*e*f + a^2*f^2 + (2*b^2*d^2*
e*f - a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2
*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^
3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + 4*integrate(-1/2*(a*b^
2*e^(d*x + c) - b^3)/(a^3*b*f*x + a^3*b*e - (a^3*b*f*x*e^(2*c) + a^3*b*e*e
^(2*c))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e*e^c)*e^(d*x)), x)
```

3.480.8 Giac [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.480.9 Mupad [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+d\,x)}{\sinh\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(coth(c + d*x)/(sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(coth(c + d*x)/(sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.481 $\int \frac{(e+fx)^3 \coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.481.1 Optimal result

Integrand size = 34, antiderivative size = 1038

$$\int \frac{(e+fx)^3\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\begin{aligned}
&= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\operatorname{arctanh}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+fx)^3\operatorname{arctanh}\left(e^{c+dx}\right)}{ad} \\
&- \frac{2b^2(e+fx)^3\operatorname{arctanh}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+fx)^3\coth(c+dx)}{a^2d} \\
&- \frac{3f(e+fx)^2\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} \\
&- \frac{b\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d} + \frac{b\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d} \\
&- \frac{3bf(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{a^2d^2} - \frac{3f^3\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^4} \\
&- \frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{2ad^2} - \frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2} \\
&+ \frac{3f^3\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^4} + \frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{2ad^2} \\
&+ \frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2} - \frac{3b\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2} \\
&+ \frac{3b\sqrt{a^2+b^2}f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2} \\
&- \frac{3bf^2(e+fx)\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{a^2d^3} + \frac{3f^2(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3} \\
&+ \frac{6b^2f^2(e+fx)\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{a^3d^3} - \frac{3f^2(e+fx)\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3} \\
&- \frac{6b^2f^2(e+fx)\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{a^3d^3} + \frac{6b\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3} \\
&- \frac{6b\sqrt{a^2+b^2}f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^3} + \frac{3bf^3\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^2d^4} \\
&- \frac{3f^3\operatorname{PolyLog}\left(4,-e^{c+dx}\right)}{ad^4} - \frac{6b^2f^3\operatorname{PolyLog}\left(4,-e^{c+dx}\right)}{a^3d^4} \\
&+ \frac{3f^3\operatorname{PolyLog}\left(4,e^{c+dx}\right)}{ad^4} + \frac{6b^2f^3\operatorname{PolyLog}\left(4,e^{c+dx}\right)}{a^3d^4} \\
&- \frac{6b\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^4} + \frac{6b\sqrt{a^2+b^2}f^3\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^4}
\end{aligned}$$

output
```
-3*b*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1
/2)/a^3/d^2+3*b*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(
a^2+b^2)^(1/2)/a^3/d^2+6*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))*(a^2+b^2)^(1/2)/a^3/d^3-3*b^2*f*(f*x+e)^2*polylog(2,-exp(d*x+c))
/a^3/d^2+3*b^2*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a^3/d^2-3*b*f^2*(f*x+e)*p
olylog(2,exp(2*d*x+2*c))/a^2/d^3+6*b^2*f^2*(f*x+e)*polylog(3,-exp(d*x+c))/
a^3/d^3-6*b^2*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a^3/d^3-3*b*f*(f*x+e)^2*ln
(1-exp(2*d*x+2*c))/a^2/d^2-3*f^3*polylog(4,-exp(d*x+c))/a/d^4+3*f^3*polylo
g(4,exp(d*x+c))/a/d^4-6*f^2*(f*x+e)*arctanh(exp(d*x+c))/a/d^3-3/2*f*(f*x+e
)^2*csch(d*x+c)/a/d^2-1/2*(f*x+e)^3*coth(d*x+c)*csch(d*x+c)/a/d-3/2*f*(f*x
+e)^2*polylog(2,-exp(d*x+c))/a/d^2+3/2*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a
/d^2+3*f^2*(f*x+e)*polylog(3,-exp(d*x+c))/a/d^3-3*f^2*(f*x+e)*polylog(3,ex
p(d*x+c))/a/d^3-6*b*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+
b^2)^(1/2)/a^3/d^4+6*b*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a
^2+b^2)^(1/2)/a^3/d^4-b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(
a^2+b^2)^(1/2)/a^3/d+b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a
^2+b^2)^(1/2)/a^3/d+b*(f*x+e)^3*coth(d*x+c)/a^2/d+b*(f*x+e)^3/a^2/d-3*f^3*
polylog(2,-exp(d*x+c))/a/d^4+3*f^3*polylog(2,exp(d*x+c))/a/d^4-2*b^2*(f*x+
e)^3*arctanh(exp(d*x+c))/a^3/d+3/2*b*f^3*polylog(3,exp(2*d*x+2*c))/a^2/d^4
-6*b^2*f^3*polylog(4,-exp(d*x+c))/a^3/d^4+6*b^2*f^3*polylog(4,exp(d*x+c...
```

3.481.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2799 vs. $2(1038) = 2076$.

Time = 9.49 (sec) , antiderivative size = 2799, normalized size of antiderivative = 2.70

$$\int \frac{(e+fx)^3 \coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(12*a*b*d^3*e^2*E^(2*c)*f*x + 12*a*b*d^3*e*E^(2*c)*f^2*x^2 + 4*a*b*d^3*E^(
2*c)*f^3*x^3 + 2*a^2*d^3*e^3*ArcTanh[E^(c + d*x)] + 4*b^2*d^3*e^3*ArcTanh[
E^(c + d*x)] - 2*a^2*d^3*e^3*E^(2*c)*ArcTanh[E^(c + d*x)] - 4*b^2*d^3*e^3*
E^(2*c)*ArcTanh[E^(c + d*x)] + 12*a^2*d*e*f^2*ArcTanh[E^(c + d*x)] - 12*a^
2*d*e*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] - 3*a^2*d^3*e^2*f*x*Log[1 - E^(c +
d*x)] - 6*b^2*d^3*e^2*f*x*Log[1 - E^(c + d*x)] + 3*a^2*d^3*e^2*E^(2*c)*f*x
*Log[1 - E^(c + d*x)] + 6*b^2*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(c + d*x)] - 6
*a^2*d*f^3*x*Log[1 - E^(c + d*x)] + 6*a^2*d*E^(2*c)*f^3*x*Log[1 - E^(c + d
*x)] - 3*a^2*d^3*e*f^2*x^2*Log[1 - E^(c + d*x)] - 6*b^2*d^3*e*f^2*x^2*Log[
1 - E^(c + d*x)] + 3*a^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 6*b^
2*d^3*e*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] - a^2*d^3*f^3*x^3*Log[1 - E^(
c + d*x)] - 2*b^2*d^3*f^3*x^3*Log[1 - E^(c + d*x)] + a^2*d^3*E^(2*c)*f^3*x
^3*Log[1 - E^(c + d*x)] + 2*b^2*d^3*E^(2*c)*f^3*x^3*Log[1 - E^(c + d*x)] +
 3*a^2*d^3*e^2*f*x*Log[1 + E^(c + d*x)] + 6*b^2*d^3*e^2*f*x*Log[1 + E^(c +
 d*x)] - 3*a^2*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - 6*b^2*d^3*e^2*E^
(2*c)*f*x*Log[1 + E^(c + d*x)] + 6*a^2*d*f^3*x*Log[1 + E^(c + d*x)] - 6*a^
2*d*E^(2*c)*f^3*x*Log[1 + E^(c + d*x)] + 3*a^2*d^3*e*f^2*x^2*Log[1 + E^(c
+ d*x)] + 6*b^2*d^3*e*f^2*x^2*Log[1 + E^(c + d*x)] - 3*a^2*d^3*e*E^(2*c)*f
^2*x^2*Log[1 + E^(c + d*x)] - 6*b^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E^(c + d
*x)] + a^2*d^3*f^3*x^3*Log[1 + E^(c + d*x)] + 2*b^2*d^3*f^3*x^3*Log[1 +...
```

3.481.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6121$$

$$\frac{\int (e+fx)^3 \coth^2(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 5980$$

$$\frac{\int (e+fx)^3\text{csch}^3(c+dx)dx+\int (e+fx)^3\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int i(e+fx)^3\csc(ic+idx)dx+\int -i(e+fx)^3\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\csc(ic+idx)dx-i\int (e+fx)^3\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$\frac{i\left(\frac{3if\int (e+fx)^2\log(1-e^{c+dx})dx}{d}-\frac{3if\int (e+fx)^2\log(1+e^{c+dx})dx}{d}+\frac{2i(e+fx)^3\text{arctanh}(e^{c+dx})}{d}\right)-i\int (e+fx)^3\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\text{PolyLog}(2,e^{c+dx})}{d}\right)}{d}\right.}{a}$$

$$\Big\downarrow \text{4674}$$

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\text{PolyLog}(2,e^{c+dx})}{d}\right)}{d}\right.}{}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}(2,-e^{c+dx})dx}{d}-\frac{(e+fx)^2\text{PolyLog}(2,-e^{c+dx})}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\text{PolyLog}(2,e^{c+dx})dx}{d}-\frac{(e+fx)^2\text{PolyLog}(2,e^{c+dx})}{d}\right)}{d}\right.}{}$$

$$\Big\downarrow \text{3042}$$

3.481. $\int \frac{(e+fx)^3\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.$$

$\downarrow$ 26

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.$$

$\downarrow$ 4670

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.$$

$\downarrow$ 2715

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.$$

$\downarrow$ 2838

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$i\left(-\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.$$

$\downarrow$ 3011

3.481. $\int \frac{(e+fx)^3\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.$$

$$\Big\downarrow \text{ 6103}$$

$$-\frac{b\left(\frac{\int(e+fx)^3\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+$$

$$i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 25}$$

$$i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}\right.$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int(e+fx)^3\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 4203}$$

3.481. $\displaystyle\int \frac{(e+fx)^3\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\int i(e+fx)^2\coth(c+dx)dx}{d}-\int (e+fx)^3dx+\frac{(e+fx)^3\coth(c+dx)}{d}}{a}\right)}{a}}$$

$$\Big\downarrow \; 17$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\int i(e+fx)^2\coth(c+dx)dx}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}\right)}{a}}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{3f\int (e+fx)^2\coth(c+dx)dx}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}\right)}{a}+$$
$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int (e+fx)\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int (e+fx)\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\,\mathrm{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{3f\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}\right)}{a}}$$

$$\Big\downarrow \; 26$$

3.481. $\int \frac{(e+fx)^3\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\int(e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}\right)}{a}}$$

$$\Big\downarrow \textcolor{blue}{4201}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\coth(c+dx)}{d}-\frac{(e+fx)^4}{4f}}{a}\right)}{a}}$$

$$\Big\downarrow \textcolor{blue}{2620}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)-\frac{i(e+fx)^3}{3f}\right)}{d}+\frac{(e+fx)^3\coth(c+}{d}}{a}\right.}{a}}$$

$$\Big\downarrow \textcolor{blue}{3011}$$

3.481. $\displaystyle\int \frac{(e+fx)^3\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-}\right.}{2d}\right.}{d}\right.\right.}{d}}{a}\right.}{a}$$

$$\Big\downarrow \text{2720}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+}{}\right.}{d}\right.\right.}{d}}{a}\right.}{a}$$

$$\Big\downarrow \text{6119}$$

$$\frac{i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.}{}$$

$$\frac{b\left(-\frac{b\left(\frac{\int(e+fx)^3\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-\right.}{4d^2}\right.}{}\right.\right.}{}\right.}{a}$$

$$\Big\downarrow \text{5973}$$

3.481. $\int\frac{(e+fx)^3\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx$

$$i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.$$

$$b\left(-\frac{b\left(\frac{\int(e+fx)^3\sinh(c+dx)dx+\int(e+fx)^3\operatorname{csch}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i}}{}\right.}{}\right.\right.}{}\right.$$

$$a$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.$$

$$b\left(-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int-i(e+fx)^3\sin(ic+idx)dx+\int i(e+fx)^3\csc(ic+idx)dx}{a}\right)}{a}-\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2}}{}\right.}{}\right.\right.}{}\right.$$

$$a$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{3if\left(\frac{2f\int(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}-\right.$$

$$b\left(-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)^3\csc(ic+idx)dx-i\int(e+fx)^3\sin(ic+idx)dx}{a}\right)}{a}-\frac{3if\left(2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2d}}{}\right.}{}\right.\right.}{}\right.$$

$$a$$

3.481. $\displaystyle\int\frac{(e+fx)^3\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}dx$

input
```
Int[((e + f*x)^3*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.481.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.481. $\int \frac{(e+fx)^3 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

3.481. $\int \frac{(e+fx)^3 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5980
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 6121
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

3.481. $\int \frac{(e+fx)^3 \coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.481.4 Maple [F]

$$\int \frac{(fx+e)^3 \coth(dx+c)^2 \operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.481.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13504 vs. $2(962) = 1924$.

Time = 0.52 (sec) , antiderivative size = 13504, normalized size of antiderivative = 13.01

$$\int \frac{(e+fx)^3 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.481.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.481.7 Maxima [F]

$$\int \frac{(e+fx)^3\coth^2(c+dx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\coth(dx+c)^2\operatorname{csch}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
1/2*e^3*(2*(a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2
*b)/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 + 2*b
^2)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2 + 2*b^2)*log(e^(-d*x - c) - 1)/(a
^3*d) - 2*(a^2*b + b^3)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-
d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d)) - (2*b*d*f^3*x^3
 + 6*b*d*e*f^2*x^2 + 6*b*d*e^2*f*x + (a*d*f^3*x^3*e^(3*c) + 3*a*e^2*f*e^(3
*c) + 3*(d*e*f^2 + f^3)*a*x^2*e^(3*c) + 3*(d*e^2*f + 2*e*f^2)*a*x*e^(3*c))
*e^(3*d*x) - 2*(b*d*f^3*x^3*e^(2*c) + 3*b*d*e*f^2*x^2*e^(2*c) + 3*b*d*e^2*
f*x*e^(2*c))*e^(2*d*x) + (a*d*f^3*x^3*e^c - 3*a*e^2*f*e^c + 3*(d*e*f^2 - f
^3)*a*x^2*e^c + 3*(d*e^2*f - 2*e*f^2)*a*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x
+ 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) + 3*(b*d*e^2*f + a*e*f^2)*x/
(a^2*d^2) + 3*(b*d*e^2*f - a*e*f^2)*x/(a^2*d^2) - 3*(b*d*e^2*f + a*e*f^2)*
log(e^(d*x + c) + 1)/(a^2*d^3) - 3*(b*d*e^2*f - a*e*f^2)*log(e^(d*x + c) -
 1)/(a^2*d^3) - 1/2*(d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*
x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*(a^
2*f^3 + 2*b^2*f^3)/(a^3*d^4) + 1/2*(d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*
x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d
*x + c)))*(a^2*f^3 + 2*b^2*f^3)/(a^3*d^4) - 3/2*(a^2*d*e*f^2 + 2*b^2*d*e*f
^2 + 2*a*b*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c))
- 2*polylog(3, -e^(d*x + c)))/(a^3*d^4) + 3/2*(a^2*d*e*f^2 + 2*b^2*d*e*...
```

3.481.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\coth^2(c+dx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.481.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\coth(c+d\,x)^2\,(e+f\,x)^3}{\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((coth(c + d*x)^2*(e + f*x)^3)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)^2*(e + f*x)^3)/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x
)
```

3.482 $\int \frac{(e+fx)^2 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.482.1 Optimal result

Integrand size $= 34$, antiderivative size $= 714$

$$\int \frac{(e+fx)^2\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\begin{aligned}
&= \frac{b(e+fx)^2}{a^2d} - \frac{(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\operatorname{arctanh}\left(e^{c+dx}\right)}{a^3d} \\
&- \frac{f^2\operatorname{arctanh}(\cosh(c+dx))}{ad^3} + \frac{b(e+fx)^2\coth(c+dx)}{a^2d} \\
&- \frac{f(e+fx)\operatorname{csch}(c+dx)}{ad^2} - \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} \\
&- \frac{b\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d} + \frac{b\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d} \\
&- \frac{2bf(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^2d^2} - \frac{f(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2} \\
&- \frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2} + \frac{f(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{ad^2} \\
&+ \frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2} - \frac{2b\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2} \\
&+ \frac{2b\sqrt{a^2+b^2}f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2} - \frac{bf^2\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{a^2d^3} \\
&+ \frac{f^2\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3} + \frac{2b^2f^2\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{a^3d^3} \\
&- \frac{f^2\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{ad^3} - \frac{2b^2f^2\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{a^3d^3} \\
&+ \frac{2b\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3} - \frac{2b\sqrt{a^2+b^2}f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^3}
\end{aligned}$$

output
```
b*(f*x+e)^2/a^2/d-(f*x+e)^2*arctanh(exp(d*x+c))/a/d-2*b^2*(f*x+e)^2*arctan
h(exp(d*x+c))/a^3/d-f^2*arctanh(cosh(d*x+c))/a/d^3+b*(f*x+e)^2*coth(d*x+c)
/a^2/d-f*(f*x+e)*csch(d*x+c)/a/d^2-1/2*(f*x+e)^2*coth(d*x+c)*csch(d*x+c)/a
/d-2*b*f*(f*x+e)*ln(1-exp(2*d*x+2*c))/a^2/d^2-f*(f*x+e)*polylog(2,-exp(d*x
+c))/a/d^2-2*b^2*f*(f*x+e)*polylog(2,-exp(d*x+c))/a^3/d^2+f*(f*x+e)*polylo
g(2,exp(d*x+c))/a/d^2+2*b^2*f*(f*x+e)*polylog(2,exp(d*x+c))/a^3/d^2-b*f^2*
polylog(2,exp(2*d*x+2*c))/a^2/d^3+f^2*polylog(3,-exp(d*x+c))/a/d^3+2*b^2*f
^2*polylog(3,-exp(d*x+c))/a^3/d^3-f^2*polylog(3,exp(d*x+c))/a/d^3-2*b^2*f^
2*polylog(3,exp(d*x+c))/a^3/d^3-b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))*(a^2+b^2)^(1/2)/a^3/d+b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))*(a^2+b^2)^(1/2)/a^3/d-2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^3/d^2+2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^3/d^2+2*b*f^2*polylog(3,-b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^3/d^3-2*b*f^2*polylog(3,-b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^3/d^3
```

3.482.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1530 vs. $2(714) = 1428$.

Time = 8.34 (sec) , antiderivative size = 1530, normalized size of antiderivative = 2.14

$$\int \frac{(e+fx)^2 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)^2*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(8*a*b*d^2*e*E^(2*c)*f*x + 4*a*b*d^2*E^(2*c)*f^2*x^2 + 2*a^2*d^2*e^2*ArcTa
nh[E^(c + d*x)] + 4*b^2*d^2*e^2*ArcTanh[E^(c + d*x)] - 2*a^2*d^2*e^2*E^(2*
c)*ArcTanh[E^(c + d*x)] - 4*b^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] + 4*a
^2*f^2*ArcTanh[E^(c + d*x)] - 4*a^2*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] - 2*a
^2*d^2*e*f*x*Log[1 - E^(c + d*x)] - 4*b^2*d^2*e*f*x*Log[1 - E^(c + d*x)] +
 2*a^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 4*b^2*d^2*e*E^(2*c)*f*x*Lo
g[1 - E^(c + d*x)] - a^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)] - 2*b^2*d^2*f^2*
x^2*Log[1 - E^(c + d*x)] + a^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] +
2*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 2*a^2*d^2*e*f*x*Log[1 + E
^(c + d*x)] + 4*b^2*d^2*e*f*x*Log[1 + E^(c + d*x)] - 2*a^2*d^2*e*E^(2*c)*f
*x*Log[1 + E^(c + d*x)] - 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] + a
^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)] + 2*b^2*d^2*f^2*x^2*Log[1 + E^(c + d*x
)] - a^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] - 2*b^2*d^2*E^(2*c)*f^2*
x^2*Log[1 + E^(c + d*x)] + 4*a*b*d*e*f*Log[1 - E^(2*(c + d*x))] - 4*a*b*d*
e*E^(2*c)*f*Log[1 - E^(2*(c + d*x))] + 4*a*b*d*f^2*x*Log[1 - E^(2*(c + d*x
))] - 4*a*b*d*E^(2*c)*f^2*x*Log[1 - E^(2*(c + d*x))] - 2*(a^2 + 2*b^2)*d*(
-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -E^(c + d*x)] + 2*(a^2 + 2*b^2)*d*(-1
 + E^(2*c))*f*(e + f*x)*PolyLog[2, E^(c + d*x)] + 2*a*b*f^2*PolyLog[2, E^(
2*(c + d*x))] - 2*a*b*E^(2*c)*f^2*PolyLog[2, E^(2*(c + d*x))] - 2*a^2*f^2*
PolyLog[3, -E^(c + d*x)] - 4*b^2*f^2*PolyLog[3, -E^(c + d*x)] + 2*a^2*E...
```

3.482.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \ 6121$$

$$\frac{\int (e+fx)^2\coth^2(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \ 5980$$

$$\frac{\int (e+fx)^2\text{csch}^3(c+dx)dx+\int (e+fx)^2\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int i(e+fx)^2\csc(ic+idx)dx+\int -i(e+fx)^2\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\csc(ic+idx)dx-i\int (e+fx)^2\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$\frac{i\left(\frac{2if\int (e+fx)\log\left(1-e^{c+dx}\right)dx}{d}-\frac{2if\int (e+fx)\log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)^2\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-i\int (e+fx)^2\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$\frac{i\left(-\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,-e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int \text{PolyLog}\left(2,e^{c+dx}\right)dx}{d}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)}{d}\right)}{d}+\frac{2i(e+fx)^2\text{arc}}{d}\right.}{a}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.}{a}$$

$$\Big\downarrow \text{4674}$$

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$
$$i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+}\right.}{d}\right.}{d}\right.$$

$$\Big\downarrow \text{26}$$

3.482. $\displaystyle\int \frac{(e+fx)^2\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\right.}{d}\right.$$
$$-\frac{b\int\frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\right.}{d}\right.$$
$$-\frac{b\int\frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\right.}{d}\right.$$
$$-\frac{b\int\frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\Big\downarrow \text{4257}$$

$$i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\right.}{d}\right.$$
$$-\frac{b\int\frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\Big\downarrow \text{4670}$$

$$i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+\cdots}\right)}{d}\right.}{d}\right.$$
$$-\frac{b\int\frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\Big\downarrow \text{3011}$$

3.482. $\int\frac{(e+fx)^2\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+\right.}{d}\right.}{d}\right.$$

$$\Big\downarrow \textbf{2720}$$

$$-\frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+\right.}{d}\right.}{d}\right.$$

$$\Big\downarrow \textbf{6103}$$

$$-\frac{b\left(\frac{\int (e+fx)^2\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+$$

$$i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+\right.}{d}\right.}{d}\right.$$

$$\Big\downarrow \textbf{3042}$$

$$i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\,\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,e^{c+\right.}{d}\right.}{d}\right.$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx}{a}\right)}{a}$$

$$\Big\downarrow \textbf{25}$$

3.482. $\int \frac{(e+fx)^2\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+\right.}}{d}\right.}{d}}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int(e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{a}\right)}{a}$$

$$\Big\downarrow \text{4203}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+\right.}}{d}\right.}{d}}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\int i(e+fx)\coth(c+dx)dx}{d}-\int(e+fx)^2dx+\frac{(e+fx)^2\coth(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{17}$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+\right.}}{d}\right.}{d}}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\int i(e+fx)\coth(c+dx)dx}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{2f\int(e+fx)\coth(c+dx)dx}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}\right)}{a}+$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+\right.}}{d}\right.}{d}}{}$$

$$\Big\downarrow \text{3042}$$

3.482. $\displaystyle\int\frac{(e+fx)^2\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{2f\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}\right)}{a}$$

$$\Big\downarrow \ 26$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\int(e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}\right)}{a}$$

$$\Big\downarrow \ 4201$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\left(2i\int\frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}}{a}\right)}{a}$$

$$\Big\downarrow \ 2620$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.}{}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e\cdots}{}}{a}\right.}{a}$$

3.482. $\int \frac{(e+fx)^2\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \ 2715$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.}{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}\cdots}{a}\right.}{a}$$

$$\Big\downarrow \ 2838$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.}{b\left(-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\cdots}{a}\right.}{a}$$

$$\Big\downarrow \ 6119$$

$$\frac{i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,e^{c+\cdots}\right.}{d}\right.}{d}\right.}{b\left(-\frac{b\left(\frac{\int(e+fx)^2\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)-\frac{i(e\cdots}{}\right.}{d}}{a}\right.}{a}$$

$$\Big\downarrow \ 5973$$

3.482. $\int\frac{(e+fx)^2\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+\right.}}{d}\right.}{d}\right.$$

$$b\left(-\frac{b\left(\frac{\int(e+fx)^2\sinh(c+dx)dx+\int(e+fx)^2\text{csch}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2\right.}}{2d}\right.\right.}{d}}{a}\right.$$

$$\overline{\qquad a \qquad}$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{d}\right)}{d}+\frac{2if\left(\frac{f\int e^{-c-dx}\text{PolyLog}\left(2,e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{(e+fx)\text{PolyLog}\left(2,e^{c+\right.}}{d}\right.}{d}\right.$$

$$b\left(-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int-i(e+fx)^2\sin(ic+idx)dx+\int i(e+fx)^2\csc(ic+idx)dx}{a}\right)}{a}-\frac{2if\left(2i\left(\frac{f\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e\right.}{2d}\right.\right.}{d}\right.$$

$$\overline{\qquad a \qquad}$$

input
```
Int[((e + f*x)^2*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.482.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

3.482. $\int \frac{(e+fx)^2\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.482. $\int \frac{(e+fx)^2 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5980
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

3.482. $\int \frac{(e+fx)^2 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 6121
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

3.482.4 Maple [F]

$$\int \frac{(fx+e)^2 \coth(dx+c)^2 \operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.482.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7726 vs. $2(665) = 1330$.

Time = 0.37 (sec) , antiderivative size = 7726, normalized size of antiderivative = 10.82

$$\int \frac{(e+fx)^2 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.482.6 Sympy [F]

$$\int \frac{(e+fx)^2\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2\coth^2(c+dx)\,\text{csch}\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*coth(c + d*x)**2*csch(c + d*x)/(a + b*sinh(c + d*x))
, x)
```

3.482.7 Maxima [F]

$$\int \frac{(e+fx)^2\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\coth\,(dx+c)^2\,\text{csch}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
1/2*e^2*(2*(a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2
*b)/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 + 2*b
^2)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2 + 2*b^2)*log(e^(-d*x - c) - 1)/(a
^3*d) - 2*(a^2*b + b^3)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-
d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d)) - (2*b*d*f^2*x^2
 + 4*b*d*e*f*x + (a*d*f^2*x^2*e^(3*c) + 2*a*e*f*e^(3*c) + 2*(d*e*f + f^2)*
a*x*e^(3*c))*e^(3*d*x) - 2*(b*d*f^2*x^2*e^(2*c) + 2*b*d*e*f*x*e^(2*c))*e^(
2*d*x) + (a*d*f^2*x^2*e^c - 2*a*e*f*e^c + 2*(d*e*f - f^2)*a*x*e^c)*e^(d*x)
)/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) + (2*b*d
*e*f + a*f^2)*x/(a^2*d^2) + (2*b*d*e*f - a*f^2)*x/(a^2*d^2) - (2*b*d*e*f +
 a*f^2)*log(e^(d*x + c) + 1)/(a^2*d^3) - (2*b*d*e*f - a*f^2)*log(e^(d*x +
c) - 1)/(a^2*d^3) - 1/2*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*
x + c)) - 2*polylog(3, -e^(d*x + c)))*(a^2*f^2 + 2*b^2*f^2)/(a^3*d^3) + 1/
2*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3,
 e^(d*x + c)))*(a^2*f^2 + 2*b^2*f^2)/(a^3*d^3) - (a^2*d*e*f + 2*b^2*d*e*f
+ 2*a*b*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^3*d^3) +
(a^2*d*e*f + 2*b^2*d*e*f - 2*a*b*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e
^(d*x + c)))/(a^3*d^3) + 1/6*((a^2*f^2 + 2*b^2*f^2)*d^3*x^3 + 3*(a^2*d*e*f
 + 2*b^2*d*e*f + 2*a*b*f^2)*d^2*x^2)/(a^3*d^3) - 1/6*((a^2*f^2 + 2*b^2*f^2
)*d^3*x^3 + 3*(a^2*d*e*f + 2*b^2*d*e*f - 2*a*b*f^2)*d^2*x^2)/(a^3*d^3) ...
```

3.482.8 Giac [F]

$$\int \frac{(e+fx)^2\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\coth\left(dx+c\right)^2\operatorname{csch}\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
sage0*x
```

3.482.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)^2\,(e+f\,x)^2}{\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((coth(c + d*x)^2*(e + f*x)^2)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)^2*(e + f*x)^2)/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x
)
```

3.483 $\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.483.1 Optimal result

Integrand size = 32, antiderivative size = 413

$$
\begin{aligned}
\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = & -\frac{(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{ad} \\
& -\frac{2b^2(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{a^3d} \\
& +\frac{b(e+fx)\coth(c+dx)}{a^2d}-\frac{f\text{csch}(c+dx)}{2ad^2} \\
& -\frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2ad} \\
& -\frac{b\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d} \\
& +\frac{b\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d} \\
& -\frac{bf\log(\sinh(c+dx))}{a^2d^2}-\frac{f\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2ad^2} \\
& -\frac{b^2f\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2} \\
& +\frac{f\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2ad^2}+\frac{b^2f\,\text{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2} \\
& -\frac{b\sqrt{a^2+b^2}f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2} \\
& +\frac{b\sqrt{a^2+b^2}f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2}
\end{aligned}
$$

output
```
-(f*x+e)*arctanh(exp(d*x+c))/a/d-2*b^2*(f*x+e)*arctanh(exp(d*x+c))/a^3/d+b
*(f*x+e)*coth(d*x+c)/a^2/d-1/2*f*csch(d*x+c)/a/d^2-1/2*(f*x+e)*coth(d*x+c)
*csch(d*x+c)/a/d-b*f*ln(sinh(d*x+c))/a^2/d^2-1/2*f*polylog(2,-exp(d*x+c))/
a/d^2-b^2*f*polylog(2,-exp(d*x+c))/a^3/d^2+1/2*f*polylog(2,exp(d*x+c))/a/d
^2+b^2*f*polylog(2,exp(d*x+c))/a^3/d^2-b*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^3/d+b*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))*(a^2+b^2)^(1/2)/a^3/d-b*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))*(a^2+b^2)^(1/2)/a^3/d^2+b*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))*(a^2+b^2)^(1/2)/a^3/d^2
```

3.483.2 Mathematica [A] (verified)

Time = 8.83 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.49

$$\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\left(2bde\cosh\left(\frac{1}{2}(c+dx)\right)-af\cosh\left(\frac{1}{2}(c+dx)\right)-2bcf\cosh\left(\frac{1}{2}(c+dx)\right)+2bf(c+dx)\cosh\left(\frac{1}{2}(c+dx)\right)\right)\text{c}}{4a^2d^2}$$
$$+\frac{(-de+cf-f(c+dx))\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{-2abf(c+dx)+(-2abf+a^2(de+dfx)+2b^2(de+dfx))\log\left(1-e^{-c-dx}\right)-\left(2abf+a^2(de+dfx)+\right.}{2a}$$
$$-\frac{b\sqrt{a^2+b^2}\left(-2de\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2cf\text{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-f(c+\right.}{a^3d^2}$$
$$+\frac{(-de+cf-f(c+dx))\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{\text{sech}\left(\frac{1}{2}(c+dx)\right)\left(2bde\sinh\left(\frac{1}{2}(c+dx)\right)+af\sinh\left(\frac{1}{2}(c+dx)\right)-2bcf\sinh\left(\frac{1}{2}(c+dx)\right)+2bf(c+dx)\text{si}\right.}{4a^2d^2}$$

input
```
Integrate[((e + f*x)*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*
x)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2)
+ ((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) + (-2*a*b*f
*(c + d*x) + (-2*a*b*f + a^2*(d*e + d*f*x) + 2*b^2*(d*e + d*f*x))*Log[1 -
E^(-c - d*x)] - (2*a*b*f + a^2*(d*e + d*f*x) + 2*b^2*(d*e + d*f*x))*Log[1
+ E^(-c - d*x)] + (a^2 + 2*b^2)*f*PolyLog[2, -E^(-c - d*x)] - (a^2 + 2*b^2
)*f*PolyLog[2, E^(-c - d*x)])/(2*a^3*d^2) - (b*Sqrt[a^2 + b^2]*(-2*d*e*Arc
Tanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*
x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + f*P
olyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^3*d^2) + ((-(d*e) + c*f - f*(c + d*x
))*Sech[(c + d*x)/2]^2)/(8*a*d^2) + (Sech[(c + d*x)/2]*(2*b*d*e*Sinh[(c +
d*x)/2] + a*f*Sinh[(c + d*x)/2] - 2*b*c*f*Sinh[(c + d*x)/2] + 2*b*f*(c + d
*x)*Sinh[(c + d*x)/2]))/(4*a^2*d^2)
```

3.483.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6121}$$

$$\frac{\int (e+fx)\coth^2(c+dx)\operatorname{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 5980}$$

$$\frac{\int (e+fx)\operatorname{csch}^3(c+dx)dx+\int (e+fx)\operatorname{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int i(e+fx)\csc(ic+idx)dx+\int -i(e+fx)\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\csc(ic+idx)dx-i\int (e+fx)\csc(ic+idx)^3dx}{a}$$

$$\Big\downarrow \text{ 4670}$$

$$\frac{i\left(\frac{if\int \log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+e^{c+dx}\right)dx}{d}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-i\int (e+fx)\csc(ic+idx)^3dx}{a}-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\Big\downarrow \text{2715}$$

$$\frac{i\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-dx}\log\left(1+e^{c+dx}\right)de^{c+dx}}{d^2}+\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}\right)-i\int (e+fx)\csc(ic+idx)^{3}}{a}-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\Big\downarrow \text{2838}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\int (e+fx)\csc(ic+idx)^3dx}{a}-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\Big\downarrow \text{4673}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\int -i(e+fx)\text{csch}(c+dx)dx-\frac{if\text{csch}(c-}{2d^2}\right.}{a}-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(-\frac{1}{2}i\int (e+fx)\text{csch}(c+dx)dx-\frac{if\text{csch}(c+}{2d^2}\right.}{a}-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(-\frac{1}{2}i\int i(e+fx)\csc(ic+idx)dx-\frac{if\text{csch}(}{2d^2}\right.}{a}-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\Big\downarrow \text{26}$$

3.483. $\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\int (e+fx)\csc(ic+idx)dx-\frac{if\text{csch}(c+dx}{2d^2}\right.}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{if\int \log\left(1-e^{c+dx}\right)dx}{d}-\frac{if\int \log\left(1+e^{c+dx}\right)dx}{d}\right.\right.}{a}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{if\int e^{-c-dx}\log\left(1-e^{c+dx}\right)de^{c+dx}}{d^2}-\frac{if\int e^{-c-}}{}\right.\right.}{a}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,-e}{d^2}\right.\right.}{a}$$

$$\Big\downarrow \text{6103}$$

$$-\frac{b\left(\frac{\int (e+fx)\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,-e}{d^2}\right.\right.}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,-e}{d^2}\right.\right.}{a}$$
$$\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -\left((e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)^2\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{25}$$

3.483. $\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,-e}{d^2}\right.\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\int(e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{a}\right)}{a}$$

$$\Big\downarrow \; 4203$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,-e}{d^2}\right.\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{if\int i\coth(c+dx)dx}{d}-\int(e+fx)dx+\frac{(e+fx)\coth(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \; 17$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,-e}{d^2}\right.\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{if\int i\coth(c+dx)dx}{d}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{f\int\coth(c+dx)dx}{d}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}+$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,-e}{d^2}\right.\right.}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}(2,-e}{d^2}\right.\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{f\int-i\tan\left(ic+idx+\frac{\pi}{2}\right)dx}{d}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}$$

$$\Big\downarrow \; 26$$

3.483. $\int\frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}\right.\right.}{a} \\ \frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{\frac{if\int\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx}{d}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}$$

$$\Big\downarrow \text{3956}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}\right.\right.}{a} \\ \frac{b\left(-\frac{b\int\frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}$$

$$\Big\downarrow \text{6119}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}\right.\right.}{a} \\ \frac{b\left(-\frac{b\left(\frac{\int(e+fx)\cosh(c+dx)\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}$$

$$\Big\downarrow \text{5973}$$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}\right.\right.}{a} \\ \frac{b\left(-\frac{b\left(\frac{\int(e+fx)\sinh(c+dx)dx+\int(e+fx)\text{csch}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

3.483. $\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e}{d^2}\right.\right.}{a}$$
$$\frac{b\left(-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int -i(e+fx)\sin(ic+idx)dx+\int i(e+fx)\csc(ic+idx)dx}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}$$

$\downarrow$ 26

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e}{d^2}\right.\right.}{a}$$
$$\frac{b\left(-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)\csc(ic+idx)dx-i\int(e+fx)\sin(ic+idx)dx}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-\frac{(e+fx)^2}{2f}}{a}\right)}{a}$$

$\downarrow$ 3777

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e}{d^2}\right.\right.}{a}$$
$$\frac{b\left(-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+}{d}}{a}\right.}{a}$$

$\downarrow$ 3042

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{if\,\text{PolyLog}(2,e^{c+dx})}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}(e^{c+dx})}{d}+\frac{if\,\text{PolyLog}(2,-e}{d^2}\right.\right.}{a}$$
$$\frac{b\left(-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\cot}{d}}{a}\right.}{a}$$

$\downarrow$ 3117

3.483. $\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{if\,\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-i\left(\frac{1}{2}\left(\frac{2i(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e\right.}{d^2}\right.\right.}{a}$$

$$\frac{b\left(-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int(e+fx)\csc(ic+idx)dx-i\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{a}\right)}{a}-\frac{-\frac{f\log(-i\sinh(c+dx))}{d^2}+\frac{(e+fx)\coth(c+dx)}{d}-}{a}\right.}{a}$$

input
```
Int[((e + f*x)*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.483.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.483. $\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.483. $\int \frac{(e+fx)\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5980
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

rule 6121
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

3.483.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1283 vs. $2(379) = 758$.

Time = 1.96 (sec) , antiderivative size = 1284, normalized size of antiderivative = 3.11

method	result	size
risch	Expression too large to display	1284

input
```
int((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
-1/2/d*f/a*ln(exp(d*x+c)+1)*x-1/2/d^2*c*f/a*ln(exp(d*x+c)-1)-2*b/d^2*c*f/a
/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2*b
^2/a^3*f*dilog(exp(d*x+c))-1/d^2*b^2/a^3*f*dilog(exp(d*x+c)+1)+1/d*b^2/a^3
*e*ln(exp(d*x+c)-1)-1/d*b^2/a^3*e*ln(exp(d*x+c)+1)-1/d^2*b/a^2*f*ln(exp(d*
x+c)-1)-1/d^2*b/a^2*f*ln(exp(d*x+c)+1)+2/a/d*e*b/(a^2+b^2)^(1/2)*arctanh(1
/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d^2/a^3*f*b^3/(a^2+b^2)^(1/2)*l
n((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2/a^3*f*b^3/
(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
*c+1/d^2/a*b*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))*c-1/2/d^2*f/a*dilog(exp(d*x+c))+1/2/d*e/a*ln(exp(d*x+c)-1)-1
/2/d*e/a*ln(exp(d*x+c)+1)-1/2/d^2*f/a*dilog(exp(d*x+c)+1)-1/d^2/a*b*f/(a^2
+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1
/d/a*b*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^
(1/2)))*x-1/d/a*b*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(
-a+(a^2+b^2)^(1/2)))*x+1/d/a^3*f*b^3/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2
+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d/a^3*f*b^3/(a^2+b^2)^(1/2)*ln((-b
*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2/d^2/a^3*b^3*c*f/(
a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d/a^3*b
^3*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d
^2/a^3*f*b^3/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-...
```

3.483.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3585 vs. $2(373) = 746$.

Time = 0.34 (sec) , antiderivative size = 3585, normalized size of antiderivative = 8.68

$$\int \frac{(e+fx)\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
1/2*(4*(a*b*d*f*x + a*b*c*f)*cosh(d*x + c)^4 + 4*(a*b*d*f*x + a*b*c*f)*sin
h(d*x + c)^4 - 4*a*b*d*e + 4*a*b*c*f - 2*(a^2*d*f*x + a^2*d*e + a^2*f)*cos
h(d*x + c)^3 - 2*(a^2*d*f*x + a^2*d*e + a^2*f - 8*(a*b*d*f*x + a*b*c*f)*co
sh(d*x + c))*sinh(d*x + c)^3 - 4*(a*b*d*f*x - a*b*d*e + 2*a*b*c*f)*cosh(d*
x + c)^2 - 2*(2*a*b*d*f*x - 2*a*b*d*e + 4*a*b*c*f - 12*(a*b*d*f*x + a*b*c*
f)*cosh(d*x + c)^2 + 3*(a^2*d*f*x + a^2*d*e + a^2*f)*cosh(d*x + c))*sinh(d
*x + c)^2 - 2*(b^2*f*cosh(d*x + c)^4 + 4*b^2*f*cosh(d*x + c)*sinh(d*x + c)
^3 + b^2*f*sinh(d*x + c)^4 - 2*b^2*f*cosh(d*x + c)^2 + b^2*f + 2*(3*b^2*f*
cosh(d*x + c)^2 - b^2*f)*sinh(d*x + c)^2 + 4*(b^2*f*cosh(d*x + c)^3 - b^2*
f*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x +
c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2) - b)/b + 1) + 2*(b^2*f*cosh(d*x + c)^4 + 4*b^2*f*cosh(d*x + c)*sinh
(d*x + c)^3 + b^2*f*sinh(d*x + c)^4 - 2*b^2*f*cosh(d*x + c)^2 + b^2*f + 2*
(3*b^2*f*cosh(d*x + c)^2 - b^2*f)*sinh(d*x + c)^2 + 4*(b^2*f*cosh(d*x + c)
^3 - b^2*f*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*co
sh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
a^2 + b^2)/b^2) - b)/b + 1) + 2*((b^2*d*e - b^2*c*f)*cosh(d*x + c)^4 + 4*(
b^2*d*e - b^2*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*e - b^2*c*f)*sin
h(d*x + c)^4 + b^2*d*e - b^2*c*f - 2*(b^2*d*e - b^2*c*f)*cosh(d*x + c)^2 -
 2*(b^2*d*e - b^2*c*f - 3*(b^2*d*e - b^2*c*f)*cosh(d*x + c)^2)*sinh(d*x...
```

3.483.6 Sympy [F]

$$\int \frac{(e+fx)\coth^2(c+dx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)\coth^2{(c+dx)}\,\mathrm{csch}\,(c+dx)}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*coth(c + d*x)**2*csch(c + d*x)/(a + b*sinh(c + d*x)), x
)
```

3.483.7 Maxima [F]

$$\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\coth{(dx+c)}^2\text{csch}{(dx+c)}}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
(2*a^2*d*integrate(1/4*x/(a^3*d*e^(d*x + c) + a^3*d), x) + 4*b^2*d*integra
te(1/4*x/(a^3*d*e^(d*x + c) + a^3*d), x) + 2*a^2*d*integrate(1/4*x/(a^3*d*
e^(d*x + c) - a^3*d), x) + 4*b^2*d*integrate(1/4*x/(a^3*d*e^(d*x + c) - a^
3*d), x) + a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) + 1)/(a^3*d^2)) + a*
b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^3*d^2)) - 2*(a^2*b*e^c +
b^3*e^c)*integrate(x*e^(d*x)/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) -
a^3*b), x) + (2*b*d*x*e^(2*d*x + 2*c) - 2*b*d*x - (a*d*x*e^(3*c) + a*e^(3*
c))*e^(3*d*x) - (a*d*x*e^c - a*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*
a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2))*f + 1/2*e*(2*(a*e^(-d*x - c) + 2*b*e^(
-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2*b)/((2*a^2*e^(-2*d*x - 2*c) - a^2*e
^(-4*d*x - 4*c) - a^2)*d) - (a^2 + 2*b^2)*log(e^(-d*x - c) + 1)/(a^3*d) +
(a^2 + 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d) - 2*(a^2*b + b^3)*log((b*e^(-d
*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(sq
rt(a^2 + b^2)*a^3*d))
```

3.483.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.483.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)^2\,(e+f\,x)}{\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((coth(c + d*x)^2*(e + f*x))/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int((coth(c + d*x)^2*(e + f*x))/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.484 $\int \frac{\coth^2(c+dx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.484.1 Optimal result

Integrand size = 27, antiderivative size = 111

$$\int \frac{\coth^2(c+dx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(a^2+2b^2)\,\mathrm{arctanh}(\cosh(c+dx))}{2a^3d} + \frac{2b\sqrt{a^2+b^2}\mathrm{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\mathrm{csch}(c+dx)}{2ad}$$

output
```
-1/2*(a^2+2*b^2)*arctanh(cosh(d*x+c))/a^3/d+b*coth(d*x+c)/a^2/d-1/2*coth(d
*x+c)*csch(d*x+c)/a/d+2*b*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2
))*(a^2+b^2)^(1/2)/a^3/d
```

3.484.2 Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.50

$$\int \frac{\coth^2(c+dx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{16b\sqrt{-a^2-b^2}\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right) + 4ab\coth\left(\frac{1}{2}(c+dx)\right) - a^2\mathrm{csch}^2\left(\frac{1}{2}(c+dx)\right) - 4(a^2+2b^2)\log\left(\right.}{8a^3d}$$

input
```
Integrate[(Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(16*b*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]]
+ 4*a*b*Coth[(c + d*x)/2] - a^2*Csch[(c + d*x)/2]^2 - 4*(a^2 + 2*b^2)*Log[
Cosh[(c + d*x)/2]] + 4*(a^2 + 2*b^2)*Log[Sinh[(c + d*x)/2]] - a^2*Sech[(c
+ d*x)/2]^2 + 4*a*b*Tanh[(c + d*x)/2])/(8*a^3*d)
```

3.484.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.23, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {3042, 26, 3368, 26, 3042, 26, 3535, 26, 3042, 25, 3534, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos(ic+idx)^2}{\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ic+idx)^2}{\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3368}$$

$$-i\int \frac{i\text{csch}^3(c+dx)\left(\sinh^2(c+dx)+1\right)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\left(\sinh^2(c+dx)+1\right)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(1-\sin(ic+idx)^2\right)}{\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{1-\sin(ic+idx)^2}{\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \; 3535$$

$$-i\left(\frac{\int -\frac{i\text{csch}^2(c+dx)(b\sinh^2(c+dx)-a\sinh(c+dx)+2b)}{a+b\sinh(c+dx)}dx}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(-\frac{i\int \frac{\text{csch}^2(c+dx)(b\sinh^2(c+dx)-a\sinh(c+dx)+2b)}{a+b\sinh(c+dx)}dx}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \; 3042$$

$$-i\left(-\frac{i\int -\frac{-b\sin(ic+idx)^2+ia\sin(ic+idx)+2b}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \; 25$$

$$-i\left(\frac{i\int \frac{-b\sin(ic+idx)^2+ia\sin(ic+idx)+2b}{\sin(ic+idx)^2(a-ib\sin(ic+idx))}dx}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \; 3534$$

$$-i\left(\frac{i\left(\frac{\int \frac{\text{csch}(c+dx)\left(a^2-b\sinh(c+dx)a+2b^2\right)}{a+b\sinh(c+dx)}dx}{a}+\frac{2b\coth(c+dx)}{ad}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \; 3042$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{\int \frac{i\left(a^2+ib\sin(ic+idx)a+2b^2\right)}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \; 26$$

3.484. $\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\int \frac{a^2+ib\sin(ic+idx)a+2b^2}{\sin(ic+idx)(a-ib\sin(ic+idx))}dx}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{3480}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\left(\frac{2ib\left(a^2+b^2\right)\int \frac{1}{a+b\sinh(c+dx)}dx}{a}+\frac{\left(a^2+2b^2\right)\int -i\text{csch}(c+dx)dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\left(\frac{2ib\left(a^2+b^2\right)\int \frac{1}{a+b\sinh(c+dx)}dx}{a}-\frac{i\left(a^2+2b^2\right)\int \text{csch}(c+dx)dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\left(\frac{2ib\left(a^2+b^2\right)\int \frac{1}{a-ib\sin(ic+idx)}dx}{a}-\frac{i\left(a^2+2b^2\right)\int i\csc(ic+idx)dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\left(\frac{2ib\left(a^2+b^2\right)\int \frac{1}{a-ib\sin(ic+idx)}dx}{a}+\frac{\left(a^2+2b^2\right)\int \csc(ic+idx)dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

3.484. $\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{3139}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\left(\frac{(a^2+2b^2)\int\csc(ic+idx)dx}{a}+\frac{4b(a^2+b^2)\int\frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\right)}{a}\right)}{2a}-\frac{i\coth\right.$$

$$\Big\downarrow \text{1083}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\left(\frac{(a^2+2b^2)\int\csc(ic+idx)dx}{a}-\frac{8b(a^2+b^2)\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{ad}\right)}{a}\right)}{2a}-\frac{i\coth(c+\right.$$

$$\Big\downarrow \text{217}$$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\left(\frac{(a^2+2b^2)\int\csc(ic+idx)dx}{a}+\frac{4ib\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{4257}$$

3.484. $\int\frac{\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-i\left(\frac{i\left(\frac{2b\coth(c+dx)}{ad}+\frac{i\left(\frac{i(a^2+2b^2)\text{arctanh}(\cosh(c+dx))}{ad}+\frac{4ib\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad}\right)}{a}\right)}{2a}-\frac{i\coth(c+dx)\text{csch}(c+d}{2ad}\right.$$

input
```
Int[(Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-I)*(((I/2)*((I*((I*(a^2 + 2*b^2)*ArcTanh[Cosh[c + d*x]])/(a*d) + ((4*I)*
b*Sqrt[a^2 + b^2]*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(a*d)))/
a + (2*b*Coth[c + d*x])/(a*d)))/a - ((I/2)*Coth[c + d*x]*Csch[c + d*x])/(a
*d))
```

3.484.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.484. $\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.484.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(2a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\sqrt{a^2+b^2}\ \mathrm{arctanh}}{}}{d}$
default	$\frac{\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(2a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\sqrt{a^2+b^2}\ \mathrm{arctanh}}{}}{d}$
risch	$-\frac{e^{3dx+3c}a-2b\,e^{2dx+2c}+a\,e^{dx+c}+2b}{a^2d\left(e^{2dx+2c}-1\right)^2}+\frac{\sqrt{a^2+b^2}\,b\ln\left(e^{dx+c}+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{d\,a^3}-\frac{\sqrt{a^2+b^2}\,b\ln\left(e^{dx+c}-\frac{-a+\sqrt{a^2+b^2}}{b}\right)}{d\,a^3}$

input
```
int(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4/a^2*(1/2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c))-1/8/a/t
anh(1/2*d*x+1/2*c)^2+1/4/a^3*(2*a^2+4*b^2)*ln(tanh(1/2*d*x+1/2*c))+1/2*b/a
^2/tanh(1/2*d*x+1/2*c)-2*b*(a^2+b^2)^(1/2)/a^3*arctanh(1/2*(2*a*tanh(1/2*d
*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

3.484.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(104) = 208$.

Time = 0.29 (sec) , antiderivative size = 892, normalized size of antiderivative = 8.04

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
-1/2*(2*a^2*cosh(d*x + c)^3 + 2*a^2*sinh(d*x + c)^3 - 4*a*b*cosh(d*x + c)^
2 + 2*a^2*cosh(d*x + c) + 2*(3*a^2*cosh(d*x + c) - 2*a*b)*sinh(d*x + c)^2
- 2*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x +
c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 +
 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(a^2 + b^2
)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2
*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)
*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x
+ c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) +
 4*a*b + ((a^2 + 2*b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*b^2)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2 + 2*b^2)*sinh(d*x + c)^4 - 2*(a^2 + 2*b^2)*cosh(d*x +
 c)^2 + 2*(3*(a^2 + 2*b^2)*cosh(d*x + c)^2 - a^2 - 2*b^2)*sinh(d*x + c)^2
+ a^2 + 2*b^2 + 4*((a^2 + 2*b^2)*cosh(d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x
+ c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - ((a^2 + 2*b^
2)*cosh(d*x + c)^4 + 4*(a^2 + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
+ 2*b^2)*sinh(d*x + c)^4 - 2*(a^2 + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2
*b^2)*cosh(d*x + c)^2 - a^2 - 2*b^2)*sinh(d*x + c)^2 + a^2 + 2*b^2 + 4*((a
^2 + 2*b^2)*cosh(d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*
log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*a^2*cosh(d*x + c)^2 - 4*a*b*
cosh(d*x + c) + a^2)*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*co...
```

3.484.6 Sympy [F]

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth^2(c+dx)\,\text{csch}\,(c+dx)}{a+b\sinh\,(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)**2*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

3.484.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(104) = 208$.

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.95

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{ae^{(-dx-c)}+2\,be^{(-2\,dx-2\,c)}+ae^{(-3\,dx-3\,c)}-2\,b}{(2\,a^2e^{(-2\,dx-2\,c)}-a^2e^{(-4\,dx-4\,c)}-a^2)d} - \frac{(a^2+2\,b^2)\log\left(e^{(-dx-c)}+1\right)}{2\,a^3d} + \frac{(a^2+2\,b^2)\log\left(e^{(-dx-c)}-1\right)}{2\,a^3d} - \frac{(a^2b+b^3)\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^3d}$$

input
```
integrate(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
(a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2*b)/((2*a^2
*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - 1/2*(a^2 + 2*b^2)*log
(e^(-d*x - c) + 1)/(a^3*d) + 1/2*(a^2 + 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*
d) - (a^2*b + b^3)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x -
 c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d)
```

3.484.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.64

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx =$$

$$-\frac{\frac{(a^2+2\,b^2)\log\left(e^{(dx+c)}+1\right)}{a^3} - \frac{(a^2+2\,b^2)\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^3} + \frac{2\,(a^2b+b^3)\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^3} + \frac{2\,(ae^{(3\,dx+3\,c)}-2\,be^{(2\,dx+2\,c}}{a^2\left(e^{(2\,dx+2\,c)}-\right.}}{2\,d}$$

input
```
integrate(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*((a^2 + 2*b^2)*log(e^(d*x + c) + 1)/a^3 - (a^2 + 2*b^2)*log(abs(e^(d*
x + c) - 1))/a^3 + 2*(a^2*b + b^3)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(
a^2 + b^2))/abs(2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^
2)*a^3) + 2*(a*e^(3*d*x + 3*c) - 2*b*e^(2*d*x + 2*c) + a*e^(d*x + c) + 2*b
)/(a^2*(e^(2*d*x + 2*c) - 1)^2))/d
```

3.484.9 Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 628, normalized size of antiderivative = 5.66

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{e^{c+dx}}{a\,d-a\,d\,e^{2\,c+2\,d\,x}} - \frac{2\,e^{c+d\,x}}{a\,d-2\,a\,d\,e^{2\,c+2\,d\,x}+a\,d\,e^{4\,c+4\,d\,x}}$$
$$-\frac{2\,b}{a^2\,d-a^2\,d\,e^{2\,c+2\,d\,x}}+\frac{\ln\left(4\,a^4+8\,b^4+12\,a^2\,b^2-4\,a^4\,e^{d\,x}\,e^c-8\,b^4\,e^{d\,x}\,e^c-12\,a^2\,b^2\,e^{d\,x}\,e^c\right)}{2\,a\,d}$$
$$-\frac{\ln\left(4\,a^4+8\,b^4+12\,a^2\,b^2+4\,a^4\,e^{d\,x}\,e^c+8\,b^4\,e^{d\,x}\,e^c+12\,a^2\,b^2\,e^{d\,x}\,e^c\right)}{2\,a\,d}$$
$$+\frac{b^2\,\ln\left(4\,a^4+8\,b^4+12\,a^2\,b^2-4\,a^4\,e^{d\,x}\,e^c-8\,b^4\,e^{d\,x}\,e^c-12\,a^2\,b^2\,e^{d\,x}\,e^c\right)}{a^3\,d}$$
$$-\frac{b^2\,\ln\left(4\,a^4+8\,b^4+12\,a^2\,b^2+4\,a^4\,e^{d\,x}\,e^c+8\,b^4\,e^{d\,x}\,e^c+12\,a^2\,b^2\,e^{d\,x}\,e^c\right)}{a^3\,d}$$
$$-\frac{b\,\ln\left(32\,a^4\,e^{d\,x}\,e^c-16\,a\,b^3-16\,a^3\,b-8\,b^3\,\sqrt{a^2+b^2}+8\,b^4\,e^{d\,x}\,e^c-16\,a^2\,b\,\sqrt{a^2+b^2}+40\,a^2\,b^2\,e^{d\,x}\,e^c+32}{a^3\,d}$$
$$+\frac{b\,\ln\left(8\,b^3\,\sqrt{a^2+b^2}-16\,a\,b^3-16\,a^3\,b+32\,a^4\,e^{d\,x}\,e^c+8\,b^4\,e^{d\,x}\,e^c+16\,a^2\,b\,\sqrt{a^2+b^2}+40\,a^2\,b^2\,e^{d\,x}\,e^c-32}{a^3\,d}$$

input
```
int(coth(c + d*x)^2/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

output
```
exp(c + d*x)/(a*d - a*d*exp(2*c + 2*d*x)) - (2*exp(c + d*x))/(a*d - 2*a*d*
exp(2*c + 2*d*x) + a*d*exp(4*c + 4*d*x)) - (2*b)/(a^2*d - a^2*d*exp(2*c +
2*d*x)) + log(4*a^4 + 8*b^4 + 12*a^2*b^2 - 4*a^4*exp(d*x)*exp(c) - 8*b^4*e
xp(d*x)*exp(c) - 12*a^2*b^2*exp(d*x)*exp(c))/(2*a*d) - log(4*a^4 + 8*b^4 +
 12*a^2*b^2 + 4*a^4*exp(d*x)*exp(c) + 8*b^4*exp(d*x)*exp(c) + 12*a^2*b^2*e
xp(d*x)*exp(c))/(2*a*d) + (b^2*log(4*a^4 + 8*b^4 + 12*a^2*b^2 - 4*a^4*exp(
d*x)*exp(c) - 8*b^4*exp(d*x)*exp(c) - 12*a^2*b^2*exp(d*x)*exp(c)))/(a^3*d)
 - (b^2*log(4*a^4 + 8*b^4 + 12*a^2*b^2 + 4*a^4*exp(d*x)*exp(c) + 8*b^4*exp
(d*x)*exp(c) + 12*a^2*b^2*exp(d*x)*exp(c)))/(a^3*d) - (b*log(32*a^4*exp(d*
x)*exp(c) - 16*a*b^3 - 16*a^3*b - 8*b^3*(a^2 + b^2)^(1/2) + 8*b^4*exp(d*x)
*exp(c) - 16*a^2*b*(a^2 + b^2)^(1/2) + 40*a^2*b^2*exp(d*x)*exp(c) + 32*a^3
*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 24*a*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^
(1/2))*(a^2 + b^2)^(1/2))/(a^3*d) + (b*log(8*b^3*(a^2 + b^2)^(1/2) - 16*a*
b^3 - 16*a^3*b + 32*a^4*exp(d*x)*exp(c) + 8*b^4*exp(d*x)*exp(c) + 16*a^2*b
*(a^2 + b^2)^(1/2) + 40*a^2*b^2*exp(d*x)*exp(c) - 32*a^3*exp(d*x)*exp(c)*(
a^2 + b^2)^(1/2) - 24*a*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)
^(1/2))/(a^3*d)
```

3.484. $\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.485 $\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.485.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\coth^2(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.485.2 Mathematica [N/A]

Not integrable

Time = 70.68 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Coth[c + d*x]^2*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Coth[c + d*x]^2*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

3.485.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \ 6125$$
$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Coth[c + d*x]^2*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.485.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.485.4 Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\coth\left(dx+c\right)^{2}\operatorname{csch}\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.485.5 Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth\left(dx+c\right)^2\text{csch}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(coth(d*x + c)^2*csch(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

3.485.6 Sympy [N/A]

Not integrable

Time = 15.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth^2\left(c+dx\right)\text{csch}\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

input
```
integrate(coth(d*x+c)**2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)**2*csch(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)),
 x)
```

3.485.7 Maxima [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 762, normalized size of antiderivative = 22.41

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth\left(dx+c\right)^2\text{csch}\left(dx+c\right)}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-2*(a^2*b*e^c + b^3*e^c)*integrate(-e^(d*x)/(a^3*b*f*x + a^3*b*e - (a^3*b*
f*x*e^(2*c) + a^3*b*e*e^(2*c))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e*e^c)*e^(
d*x)), x) - (2*b*d*f*x + 2*b*d*e + (a*d*f*x*e^(3*c) + (d*e - f)*a*e^(3*c))
*e^(3*d*x) - 2*(b*d*f*x*e^(2*c) + b*d*e*e^(2*c))*e^(2*d*x) + (a*d*f*x*e^c
+ (d*e + f)*a*e^c)*e^(d*x))/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e
^2 + (a^2*d^2*f^2*x^2*e^(4*c) + 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4
*c))*e^(4*d*x) - 2*(a^2*d^2*f^2*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^
2*d^2*e^2*e^(2*c))*e^(2*d*x)) + 2*integrate(-1/4*(2*b^2*d^2*e^2 + 2*a*b*d*
e*f + (d^2*e^2 + 2*f^2)*a^2 + (a^2*d^2*f^2 + 2*b^2*d^2*f^2)*x^2 + 2*(a^2*d
^2*e*f + 2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*
x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e
*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + 2*i
ntegrate(1/4*(2*b^2*d^2*e^2 - 2*a*b*d*e*f + (d^2*e^2 + 2*f^2)*a^2 + (a^2*d
^2*f^2 + 2*b^2*d^2*f^2)*x^2 + 2*(a^2*d^2*e*f + 2*b^2*d^2*e*f - a*b*d*f^2)*
x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^
3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c
 + a^3*d^2*e^3*e^c)*e^(d*x)), x)
```

3.485.8 Giac [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.485.9 Mupad [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+d\,x)^2}{\sinh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(coth(c + d*x)^2/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(coth(c + d*x)^2/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.486 $\int \frac{(e+fx)^3 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.486.1 Optimal result

Integrand size = 28, antiderivative size = 972

$$
\begin{aligned}
\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = & -\frac{3f(e+fx)^2}{2ad^2}+\frac{(e+fx)^3}{2ad}-\frac{(e+fx)^4}{4af} \\
& -\frac{b^2(e+fx)^4}{4a^3f}+\frac{(a^2+b^2)(e+fx)^4}{4a^3f} \\
& +\frac{6bf(e+fx)^2\operatorname{arctanh}(e^{c+dx})}{a^2d^2}-\frac{3f(e+fx)^2\coth(c+dx)}{2ad^2} \\
& -\frac{(e+fx)^3\coth^2(c+dx)}{2ad}+\frac{b(e+fx)^3\operatorname{csch}(c+dx)}{a^2d} \\
& -\frac{(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d} \\
& -\frac{(a^2+b^2)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d} \\
& +\frac{3f^2(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad^3} \\
& +\frac{(e+fx)^3\log\left(1-e^{2(c+dx)}\right)}{ad} \\
& +\frac{b^2(e+fx)^3\log\left(1-e^{2(c+dx)}\right)}{a^3d} \\
& +\frac{6bf^2(e+fx)\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^3} \\
& -\frac{6bf^2(e+fx)\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^3} \\
& -\frac{3(a^2+b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2} \\
& -\frac{3(a^2+b^2)f(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2} \\
& +\frac{3f^3\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^4} \\
& +\frac{3f(e+fx)^2\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^2} \\
& +\frac{3b^2f(e+fx)^2\operatorname{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^3d^2} \\
& -\frac{6bf^3\operatorname{PolyLog}\left(3,-e^{c+dx}\right)}{a^2d^4}+\frac{6bf^3\operatorname{PolyLog}\left(3,e^{c+dx}\right)}{a^2d^4} \\
& +\frac{6(a^2+b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3} \\
& +\frac{6(a^2+b^2)f^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^3} \\
& -\frac{3f^2(e+fx)\operatorname{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^3}
\end{aligned}
$$

output
```
3/4*f^3*polylog(4,exp(2*d*x+2*c))/a/d^4-6*b*f^3*polylog(3,-exp(d*x+c))/a^2
/d^4+6*b*f^3*polylog(3,exp(d*x+c))/a^2/d^4+3/4*b^2*f^3*polylog(4,exp(2*d*x
+2*c))/a^3/d^4-6*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2))
)/a^3/d^4-6*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3
/d^4+(f*x+e)^3*ln(1-exp(2*d*x+2*c))/a/d+6*b*f*(f*x+e)^2*arctanh(exp(d*x+c)
)/a^2/d^2+6*b*f^2*(f*x+e)*polylog(2,-exp(d*x+c))/a^2/d^3+3/2*f*(f*x+e)^2*p
olylog(2,exp(2*d*x+2*c))/a/d^2-3/2*f^2*(f*x+e)*polylog(3,exp(2*d*x+2*c))/a
/d^3-6*b*f^2*(f*x+e)*polylog(2,exp(d*x+c))/a^2/d^3+3/2*b^2*f*(f*x+e)^2*pol
ylog(2,exp(2*d*x+2*c))/a^3/d^2-(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/a^3/d-(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/a^3/d+1/2*(f*x+e)^3/a/d-1/4*(f*x+e)^4/a/f-3/2*b^2*f^2*(f*x+e)*polyl
og(3,exp(2*d*x+2*c))/a^3/d^3-3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/a^3/d^2-3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^2+6*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*ex
p(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^3+6*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b
*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^3-3/2*f*(f*x+e)^2*coth(d*x+c)/a/d^2
+3*f^2*(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d^3+b*(f*x+e)^3*csch(d*x+c)/a^2/d+b^
2*(f*x+e)^3*ln(1-exp(2*d*x+2*c))/a^3/d+3/2*f^3*polylog(2,exp(2*d*x+2*c))/a
/d^4-1/4*b^2*(f*x+e)^4/a^3/f+1/4*(a^2+b^2)*(f*x+e)^4/a^3/f-1/2*(f*x+e)^3*c
oth(d*x+c)^2/a/d-3/2*f*(f*x+e)^2/a/d^2
```

3.486.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3868 vs. $2(972) = 1944$.

Time = 12.73 (sec) , antiderivative size = 3868, normalized size of antiderivative = 3.98

$$\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b*(e + f*x)^3*Csch[c])/(a^2*d) + ((-e^3 - 3*e^2*f*x - 3*e*f^2*x^2 - f^3*x
^3)*Csch[c/2 + (d*x)/2]^2)/(8*a*d) - (8*a^2*d^4*e^3*E^(2*c)*x + 8*b^2*d^4*
e^3*E^(2*c)*x + 24*a^2*d^2*e*E^(2*c)*f^2*x + 12*a^2*d^4*e^2*E^(2*c)*f*x^2
+ 12*b^2*d^4*e^2*E^(2*c)*f*x^2 + 12*a^2*d^2*E^(2*c)*f^3*x^2 + 8*a^2*d^4*e*
E^(2*c)*f^2*x^3 + 8*b^2*d^4*e*E^(2*c)*f^2*x^3 + 2*a^2*d^4*E^(2*c)*f^3*x^4
+ 2*b^2*d^4*E^(2*c)*f^3*x^4 + 24*a*b*d^2*e^2*f*ArcTanh[E^(c + d*x)] - 24*a
*b*d^2*e^2*E^(2*c)*f*ArcTanh[E^(c + d*x)] - 24*a*b*d^2*e*f^2*x*Log[1 - E^(
c + d*x)] + 24*a*b*d^2*e*E^(2*c)*f^2*x*Log[1 - E^(c + d*x)] - 12*a*b*d^2*f
^3*x^2*Log[1 - E^(c + d*x)] + 12*a*b*d^2*E^(2*c)*f^3*x^2*Log[1 - E^(c + d*
x)] + 24*a*b*d^2*e*f^2*x*Log[1 + E^(c + d*x)] - 24*a*b*d^2*e*E^(2*c)*f^2*x
*Log[1 + E^(c + d*x)] + 12*a*b*d^2*f^3*x^2*Log[1 + E^(c + d*x)] - 12*a*b*d
^2*E^(2*c)*f^3*x^2*Log[1 + E^(c + d*x)] + 4*a^2*d^3*e^3*Log[1 - E^(2*(c +
d*x))] + 4*b^2*d^3*e^3*Log[1 - E^(2*(c + d*x))] - 4*a^2*d^3*e^3*E^(2*c)*Lo
g[1 - E^(2*(c + d*x))] - 4*b^2*d^3*e^3*E^(2*c)*Log[1 - E^(2*(c + d*x))] +
12*a^2*d*e*f^2*Log[1 - E^(2*(c + d*x))] - 12*a^2*d*e*E^(2*c)*f^2*Log[1 - E
^(2*(c + d*x))] + 12*a^2*d^3*e^2*f*x*Log[1 - E^(2*(c + d*x))] + 12*b^2*d^3
*e^2*f*x*Log[1 - E^(2*(c + d*x))] - 12*a^2*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(
2*(c + d*x))] - 12*b^2*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(2*(c + d*x))] + 12*a
^2*d*f^3*x*Log[1 - E^(2*(c + d*x))] - 12*a^2*d*E^(2*c)*f^3*x*Log[1 - E^(2*
(c + d*x))] + 12*a^2*d^3*e*f^2*x^2*Log[1 - E^(2*(c + d*x))] + 12*b^2*d^...
```

3.486.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6103$$

$$\frac{\int (e+fx)^3\coth^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int i(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)^3dx}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^3\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^3dx}{a}$$

$$\Big\downarrow \; 4203$$

$$\frac{i\left(\frac{3if\int -(e+fx)^2\coth^2(c+dx)dx}{2d}-\int i(e+fx)^3\coth(c+dx)dx+\frac{i(e+fx)^3\coth^2(c+dx)}{2d}\right)}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{i\left(-\frac{3if\int (e+fx)^2\coth^2(c+dx)dx}{2d}-\int i(e+fx)^3\coth(c+dx)dx+\frac{i(e+fx)^3\coth^2(c+dx)}{2d}\right)}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{i\left(-\frac{3if\int (e+fx)^2\coth^2(c+dx)dx}{2d}-i\int (e+fx)^3\coth(c+dx)dx+\frac{i(e+fx)^3\coth^2(c+dx)}{2d}\right)}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\left(-i\int -i(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{3if\int -(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d}+\frac{i(e+fx)^3\coth^2(c+dx)}{2d}\right)}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{i\left(-i\int -i(e+fx)^3\tan\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{3if\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{2d}+\frac{i(e+fx)^3\coth^2(c+dx)}{2d}\right)}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{i\left(-\int (e+fx)^3\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx+\frac{3if\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{2d}+\frac{i(e+fx)^3\coth^2(c+dx)}{2d}\right)}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 4201$$

3.486. $\displaystyle\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(-2i\int \frac{e^{2c+2dx-i\pi}(e+fx)^3}{1+e^{2c+2dx-i\pi}}dx+\frac{3if\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{2d}+\frac{i(e+fx)^3\coth^2(c+dx)}{2d}+\frac{i(e+fx)^4}{4f}\right)}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\Big\downarrow \text{2620}$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\int (e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)+\frac{3if\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{2d}+\frac{i(e+fx)^3\coth^2(c\right.}{2d}}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\Big\downarrow \text{3011}$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\int (e+fx)^2\text{ta}\right.}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\Big\downarrow \text{4203}$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\int i(e+fx\right.}{}\right.}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\Big\downarrow \text{17}$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\int i(e+fx\right.}{}\right.}{a}-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\Big\downarrow \text{26}$$

3.486. $\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)\right)+\frac{3if\left(-\frac{2f\int(e+fx}{}\right.}{}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)\right)+\frac{3if\left(-\frac{2f\int -i(e+}{}\right.}{}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)\right)+\frac{3if\left(\frac{2if\int(e+fx)}{}\right.}{}}{a}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)\right)+\frac{3if\left(\frac{2if\left(2i\int \frac{e^{2}}{}\right.}{}\right.}{}}{a}$$

$$\Big\downarrow \text{2620}$$

3.486. $\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(2if\left(2i\left((e-\right.\right.\right.}{}\right.}{a}$$

$$\Big\downarrow \; 2715$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\frac{i\left(\frac{3if\left(\frac{2if\left(2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)-\frac{i(e+fx)^2}{2f}\right)}{d}+\frac{(e+fx)^2\coth(c+dx)}{d}-\frac{(e+fx)^3}{3f}\right)}{2d}-2\right.}{}$$

$$\Big\downarrow \; 2838$$

$$-\frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(2if\left(2i\left(\frac{f\,\mathrm{F}}{}\right.\right.\right.}{}\right.}{a}$$

$$\Big\downarrow \; 6119$$

$$-\frac{b\left(\frac{\int(e+fx)^3\cosh(c+dx)\coth^2(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(2if\left(2i\left(\frac{f\,\mathrm{F}}{}\right.\right.\right.}{}\right.}{a}$$

3.486. $\displaystyle\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{5973}$$

$$-\frac{b\left(\frac{\int (e+fx)^3\cosh(c+dx)dx+\int (e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\left(2i\left(\frac{f\,\mathrm{F}}{}\right.\right.}{}\right.}{}\right.}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\left(2i\left(\frac{f\,\mathrm{F}}{}\right.\right.}{}\right.}{}\right.}{a}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)dx+\int (e+fx)^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int (e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(\frac{2if\left(2i\left(\frac{f\,\mathrm{F}}{}\right.\right.}{}\right.}{}\right.}{a}$$

$$\frac{b\left(-\frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{3if\int -i(e+fx)^2\sinh(c+dx)dx}{d}+\int (e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

3.486. $\displaystyle\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{-\frac{3f\int(e+fx)^2\sinh(c+dx)dx}{d}+\int(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(2if\left(2i\left(\frac{f\,\mathrm{F}}{}\right.\right.\right.}{}\right.}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(2if\left(2i\left(\frac{f\,\mathrm{F}}{}\right.\right.\right.}{}\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{3f\int-i(e+fx)^2\sin(ic+idx)dx}{d}+\int(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log(1+e^{2c+2dx-i\pi})}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(2if\left(2i\left(\frac{f\,\mathrm{F}}{}\right.\right.\right.}{}\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3if\int(e+fx)^2\sin(ic+idx)dx}{d}+\int(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)dx+\frac{(e+fx)^3\sinh(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \; 3777$$

3.486. $\int\frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(2if\left(2i\left(\frac{f\,\mathrm{P}}{}\right.\right.\right.}{}\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\cosh(c+dx)dx}{d}\right)}{d}+\int(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)dx+\frac{(e+f}{}}{a}\right.}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^3\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{3f\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{d}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{2d}\right)+\frac{3if\left(2if\left(2i\left(\frac{f\,\mathrm{P}}{}\right.\right.\right.}{}\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{3if\left(\frac{i(e+fx)^2\cosh(c+dx)}{d}-\frac{2if\int(e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}+\int(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)dx+}{a}\right.}{a}$$

input
```
Int[((e + f*x)^3*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.486.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

3.486. $\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.486. $\int \frac{(e+fx)^3 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.486.4 Maple [F]

$$\int \frac{(fx+e)^3 \coth(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.486.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13683 vs. $2(908) = 1816$.

Time = 0.48 (sec) , antiderivative size = 13683, normalized size of antiderivative = 14.08

$$\int \frac{(e+fx)^3 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

3.486.6 Sympy [F]

$$\int \frac{(e+fx)^3 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^3 \coth^3{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)**3*coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**3*coth(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

3.486.7 Maxima [F]

$$\int \frac{(e+fx)^3 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3 \coth{(dx+c)}^3}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^3*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2
*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 + b^2)*log(-2*a*
e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x -
 c) + 1)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c) - 1)/(a^3*d)) + (3*a*f^3*x
^2 + 6*a*e*f^2*x + 3*a*e^2*f + 2*(b*d*f^3*x^3*e^(3*c) + 3*b*d*e*f^2*x^2*e^
(3*c) + 3*b*d*e^2*f*x*e^(3*c))*e^(3*d*x) - (2*a*d*f^3*x^3*e^(2*c) + 3*a*e^
2*f*e^(2*c) + 3*(2*d*e*f^2 + f^3)*a*x^2*e^(2*c) + 6*(d*e^2*f + e*f^2)*a*x*
e^(2*c))*e^(2*d*x) - 2*(b*d*f^3*x^3*e^c + 3*b*d*e*f^2*x^2*e^c + 3*b*d*e^2*
f*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a
^2*d^2) - 3*(b*d*e^2*f + a*e*f^2)*x/(a^2*d^2) + 3*(b*d*e^2*f - a*e*f^2)*x/
(a^2*d^2) + 3*(b*d*e^2*f + a*e*f^2)*log(e^(d*x + c) + 1)/(a^2*d^3) - 3*(b*
d*e^2*f - a*e*f^2)*log(e^(d*x + c) - 1)/(a^2*d^3) + (d^3*x^3*log(e^(d*x +
c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) +
 6*polylog(4, -e^(d*x + c)))*(a^2*f^3 + b^2*f^3)/(a^3*d^4) + (d^3*x^3*log(
-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x
 + c)) + 6*polylog(4, e^(d*x + c)))*(a^2*f^3 + b^2*f^3)/(a^3*d^4) + 3*(a^2
*d*e*f^2 + b^2*d*e*f^2 + a*b*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*di
log(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^3*d^4) + 3*(a^2*d*e*f^2
 + b^2*d*e*f^2 - a*b*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(
d*x + c)) - 2*polylog(3, e^(d*x + c)))/(a^3*d^4) + 3*(b^2*d^2*e^2*f + 2...
```

3.486.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 \coth^3(c+dx)}{a+b\sinh(c+dx)}\, dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.486.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{\coth(c+d\,x)^3\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((coth(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.487 $\int \frac{(e+fx)^2 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.487.1 Optimal result

Integrand size = 28, antiderivative size = 689

$$
\begin{aligned}
\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{efx}{ad}+\frac{f^2x^2}{2ad}-\frac{(e+fx)^3}{3af}-\frac{b^2(e+fx)^3}{3a^3f}+\frac{(a^2+b^2)\,(e+fx)^3}{3a^3f}\\
&+\frac{4bf(e+fx)\text{arctanh}\left(e^{c+dx}\right)}{a^2d^2}-\frac{f(e+fx)\coth(c+dx)}{ad^2}\\
&-\frac{(e+fx)^2\coth^2(c+dx)}{2ad}+\frac{b(e+fx)^2\text{csch}(c+dx)}{a^2d}\\
&-\frac{(a^2+b^2)\,(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d}\\
&-\frac{(a^2+b^2)\,(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d}\\
&+\frac{(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad}\\
&+\frac{b^2(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{a^3d}+\frac{f^2\log(\sinh(c+dx))}{ad^3}\\
&+\frac{2bf^2\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^3}-\frac{2bf^2\,\text{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^3}\\
&-\frac{2(a^2+b^2)\,f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}\\
&-\frac{2(a^2+b^2)\,f(e+fx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2}\\
&+\frac{f(e+fx)\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^2}\\
&+\frac{b^2f(e+fx)\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{a^3d^2}\\
&+\frac{2(a^2+b^2)\,f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3}\\
&+\frac{2(a^2+b^2)\,f^2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^3}\\
&-\frac{f^2\,\text{PolyLog}\left(3,e^{2(c+dx)}\right)}{2ad^3}-\frac{b^2f^2\,\text{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^3d^3}
\end{aligned}
$$

output
```
e*f*x/a/d+1/2*f^2*x^2/a/d-1/3*(f*x+e)^3/a/f-1/3*b^2*(f*x+e)^3/a^3/f+1/3*(a
^2+b^2)*(f*x+e)^3/a^3/f+4*b*f*(f*x+e)*arctanh(exp(d*x+c))/a^2/d^2-f*(f*x+e
)*coth(d*x+c)/a/d^2-1/2*(f*x+e)^2*coth(d*x+c)^2/a/d+b*(f*x+e)^2*csch(d*x+c
)/a^2/d+(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a/d+b^2*(f*x+e)^2*ln(1-exp(2*d*x+2*
c))/a^3/d+f^2*ln(sinh(d*x+c))/a/d^3-(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/a^3/d-(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/a^3/d+2*b*f^2*polylog(2,-exp(d*x+c))/a^2/d^3-2*b*f^2*polylog(2
,exp(d*x+c))/a^2/d^3+f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^2+b^2*f*(f*x+
e)*polylog(2,exp(2*d*x+2*c))/a^3/d^2-2*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*ex
p(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^2-2*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*e
xp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^2-1/2*f^2*polylog(3,exp(2*d*x+2*c))/a
/d^3-1/2*b^2*f^2*polylog(3,exp(2*d*x+2*c))/a^3/d^3+2*(a^2+b^2)*f^2*polylog
(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^3+2*(a^2+b^2)*f^2*polylog(3,-b
*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^3
```

3.487.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2403 vs. 2(689) = 1378.

Time = 10.70 (sec) , antiderivative size = 2403, normalized size of antiderivative = 3.49

$$\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b*(e + f*x)^2*Csch[c])/(a^2*d) + ((-e^2 - 2*e*f*x - f^2*x^2)*Csch[c/2 + (
d*x)/2]^2)/(8*a*d) - (12*a^2*d^3*e^2*E^(2*c)*x + 12*b^2*d^3*e^2*E^(2*c)*x
+ 12*a^2*d*E^(2*c)*f^2*x + 12*a^2*d^3*e*E^(2*c)*f*x^2 + 12*b^2*d^3*e*E^(2*
c)*f*x^2 + 4*a^2*d^3*E^(2*c)*f^2*x^3 + 4*b^2*d^3*E^(2*c)*f^2*x^3 + 24*a*b*
d*e*f*ArcTanh[E^(c + d*x)] - 24*a*b*d*e*E^(2*c)*f*ArcTanh[E^(c + d*x)] - 1
2*a*b*d*f^2*x*Log[1 - E^(c + d*x)] + 12*a*b*d*E^(2*c)*f^2*x*Log[1 - E^(c +
 d*x)] + 12*a*b*d*f^2*x*Log[1 + E^(c + d*x)] - 12*a*b*d*E^(2*c)*f^2*x*Log[
1 + E^(c + d*x)] + 6*a^2*d^2*e^2*Log[1 - E^(2*(c + d*x))] + 6*b^2*d^2*e^2*
Log[1 - E^(2*(c + d*x))] - 6*a^2*d^2*e^2*E^(2*c)*Log[1 - E^(2*(c + d*x))]
- 6*b^2*d^2*e^2*E^(2*c)*Log[1 - E^(2*(c + d*x))] + 6*a^2*f^2*Log[1 - E^(2*
(c + d*x))] - 6*a^2*E^(2*c)*f^2*Log[1 - E^(2*(c + d*x))] + 12*a^2*d^2*e*f*
x*Log[1 - E^(2*(c + d*x))] + 12*b^2*d^2*e*f*x*Log[1 - E^(2*(c + d*x))] - 1
2*a^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(2*(c + d*x))] - 12*b^2*d^2*e*E^(2*c)*f*
x*Log[1 - E^(2*(c + d*x))] + 6*a^2*d^2*f^2*x^2*Log[1 - E^(2*(c + d*x))] +
6*b^2*d^2*f^2*x^2*Log[1 - E^(2*(c + d*x))] - 6*a^2*d^2*E^(2*c)*f^2*x^2*Log
[1 - E^(2*(c + d*x))] - 6*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(2*(c + d*x))]
 - 12*a*b*(-1 + E^(2*c))*f^2*PolyLog[2, -E^(c + d*x)] + 12*a*b*(-1 + E^(2*
c))*f^2*PolyLog[2, E^(c + d*x)] + 6*a^2*d*e*f*PolyLog[2, E^(2*(c + d*x))]
+ 6*b^2*d*e*f*PolyLog[2, E^(2*(c + d*x))] - 6*a^2*d*e*E^(2*c)*f*PolyLog[2,
 E^(2*(c + d*x))] - 6*b^2*d*e*E^(2*c)*f*PolyLog[2, E^(2*(c + d*x))] + 6...
```

3.487.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6103}$$

$$\frac{\int (e+fx)^2\coth^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)^3dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^3dx}{a}$$

$$\Big\downarrow \text{4203}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(\frac{if\int -((e+fx)\coth^2(c+dx))dx}{d}-\int i(e+fx)^2\coth(c+dx)dx+\frac{i(e+fx)^2\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{if\int (e+fx)\coth^2(c+dx)dx}{d}-\int i(e+fx)^2\coth(c+dx)dx+\frac{i(e+fx)^2\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\frac{if\int (e+fx)\coth^2(c+dx)dx}{d}-i\int (e+fx)^2\coth(c+dx)dx+\frac{i(e+fx)^2\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-i\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{if\int -\left((e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)^2\right)dx}{d}+\frac{i(e+fx)^2\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-i\int -i(e+fx)^2\tan\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{if\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{d}+\frac{i(e+fx)^2\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\int (e+fx)^2\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx+\frac{if\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{d}+\frac{i(e+fx)^2\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{4201}$$

3.487. $\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\int \frac{e^{2c+2dx-i\pi}(e+fx)^2}{1+e^{2c+2dx-i\pi}}dx+\frac{if\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{d}+\frac{i(e+fx)^2\coth^2(c+dx)}{2d}+\frac{i(e+fx)^3}{3f}\right)}{a}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int (e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)dx}{d}\right)+\frac{if\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{d}+\frac{i(e+fx)^2\coth^2(c+dx)}{2d}\right.}{a}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int \text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)dx}{2d}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi\right.}{d}\right.}{a}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{\ }\right.}{a}$$

$$\Big\downarrow \text{4203}$$

$$-\frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{\ }\right.}{a}$$

$$\Big\downarrow \text{17}$$

3.487. $\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{}\cdots\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{}\cdots\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{}\cdots\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{}\cdots\right)}{a}$$

$$\Big\downarrow \text{3956}$$

$$\frac{-\frac{b\int\frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{}\cdots\right)}{a}$$

$$\Big\downarrow \text{6119}$$

3.487. $\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{\int (e+fx)^2\cosh(c+dx)\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{\ldots}\right)}{a}$$

$$\Big\downarrow \; 5973$$

$$-\frac{b\left(\frac{\int (e+fx)^2\cosh(c+dx)dx+\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{\ldots}\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{\ldots}\right)}{a}$$
$$\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\int (e+fx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \; 3777$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{\ldots}\right)}{a}$$
$$\frac{b\left(-\frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{2if\int -i(e+fx)\sinh(c+dx)dx}{d}+\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \; 26$$

3.487. $\displaystyle\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{-\frac{2f\int(e+fx)\sinh(c+dx)dx}{d}+\int(e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{}\cdots\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{}\cdots\right)}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-\frac{2f\int -i(e+fx)\sin(ic+idx)dx}{d}+\int(e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{}\cdots\right)}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2if\int(e+fx)\sin(ic+idx)dx}{d}+\int(e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{}\cdots\right)}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\cosh(c+dx)dx}{d}\right)}{d}+\int(e+fx)^2\coth(c+dx)\text{csch}(c+dx)dx+\frac{(e+fx)^2\sinh(c-\cdots}{d}}{a}\cdots\right)}{a}$$

$$\Big\downarrow \text{3042}$$

3.487. $\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{\ldots}\right.$$

$$\frac{\qquad\qquad}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}+\int(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)dx+\frac{(e+fx)^2\sinh\ldots}{\ldots}}{a}\right.}{a}$$

$$\Big\downarrow \; 3117$$

$$i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{\ldots}\right.$$

$$\frac{\qquad\qquad}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)dx+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}\ldots}{a}\right.}{a}$$

$$\Big\downarrow \; 5975$$

$$i\left(-2i\left(\frac{(e+fx)^2\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\left(\frac{f\int e^{-2c-2dx+i\pi}\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}-\frac{(e+fx)\operatorname{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{2d}\right)}{d}\right)+\frac{if}{\ldots}\right.$$

$$\frac{\qquad\qquad}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{2f\int(e+fx)\operatorname{csch}(c+dx)dx}{d}+\frac{2if\left(\frac{i(e+fx)\cosh(c+dx)}{d}-\frac{if\sinh(c+dx)}{d^2}\right)}{d}+\frac{(e+fx)^2\sinh(c+dx)}{d}-\frac{(e+fx)^2\operatorname{cs}\ldots}{a\ldots}}{a}\right.}{a}$$

input
```
Int[((e + f*x)^2*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.487. $\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.487.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.487. $\int \frac{(e+fx)^2 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

3.487. $\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.487.4 Maple [F]

$$\int \frac{(fx+e)^2 \coth(dx+c)^3}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

3.487.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7775 vs. $2(648) = 1296$.

Time = 0.35 (sec) , antiderivative size = 7775, normalized size of antiderivative = 11.28

$$\int \frac{(e+fx)^2 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

3.487.6 Sympy [F]

$$\int \frac{(e+fx)^2 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+fx)^2 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

input
```
integrate((f*x+e)**2*coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)**2*coth(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

3.487.7 Maxima [F]

$$\int \frac{(e+fx)^2 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \coth(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-e^2*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2
*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 + b^2)*log(-2*a*
e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x -
 c) + 1)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c) - 1)/(a^3*d)) + 2*(a*f^2*x
 + a*e*f + (b*d*f^2*x^2*e^(3*c) + 2*b*d*e*f*x*e^(3*c))*e^(3*d*x) - (a*d*f^
2*x^2*e^(2*c) + a*e*f*e^(2*c) + (2*d*e*f + f^2)*a*x*e^(2*c))*e^(2*d*x) - (
b*d*f^2*x^2*e^c + 2*b*d*e*f*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a
^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) - (2*b*d*e*f + a*f^2)*x/(a^2*d^2) + (2*b
*d*e*f - a*f^2)*x/(a^2*d^2) + (2*b*d*e*f + a*f^2)*log(e^(d*x + c) + 1)/(a^
2*d^3) - (2*b*d*e*f - a*f^2)*log(e^(d*x + c) - 1)/(a^2*d^3) + (d^2*x^2*log
(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c))
)*(a^2*f^2 + b^2*f^2)/(a^3*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*d
ilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*(a^2*f^2 + b^2*f^2)/(a^3*d^
3) + 2*(a^2*d*e*f + b^2*d*e*f + a*b*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog
(-e^(d*x + c)))/(a^3*d^3) + 2*(a^2*d*e*f + b^2*d*e*f - a*b*f^2)*(d*x*log(-
e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^3*d^3) - 1/3*((a^2*f^2 + b^2*f^2
)*d^3*x^3 + 3*(a^2*d*e*f + b^2*d*e*f + a*b*f^2)*d^2*x^2)/(a^3*d^3) - 1/3*(
(a^2*f^2 + b^2*f^2)*d^3*x^3 + 3*(a^2*d*e*f + b^2*d*e*f - a*b*f^2)*d^2*x^2)
/(a^3*d^3) + integrate(-2*((a^2*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*e*f + b^3*
e*f)*x - ((a^3*f^2*e^c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e*...
```

3.487.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.487.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

input
```
int((coth(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.488 $\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.488.1 Optimal result

Integrand size = 26, antiderivative size = 435

$$\begin{aligned}\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{fx}{2ad}-\frac{(e+fx)^2}{2af}-\frac{b^2(e+fx)^2}{2a^3f}+\frac{(a^2+b^2)(e+fx)^2}{2a^3f}\\ &+\frac{bf\,\text{arctanh}(\cosh(c+dx))}{a^2d^2}-\frac{f\coth(c+dx)}{2ad^2}\\ &-\frac{(e+fx)\coth^2(c+dx)}{2ad}+\frac{b(e+fx)\text{csch}(c+dx)}{a^2d}\\ &-\frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d}\\ &-\frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d}\\ &+\frac{(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad}+\frac{b^2(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^3d}\\ &-\frac{(a^2+b^2)f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}\\ &-\frac{(a^2+b^2)f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2}\\ &+\frac{f\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^2}+\frac{b^2f\,\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^3d^2}\end{aligned}$$

output
```
1/2*f*x/a/d-1/2*(f*x+e)^2/a/f-1/2*b^2*(f*x+e)^2/a^3/f+1/2*(a^2+b^2)*(f*x+e
)^2/a^3/f+b*f*arctanh(cosh(d*x+c))/a^2/d^2-1/2*f*coth(d*x+c)/a/d^2-1/2*(f*
x+e)*coth(d*x+c)^2/a/d+b*(f*x+e)*csch(d*x+c)/a^2/d+(f*x+e)*ln(1-exp(2*d*x+
2*c))/a/d+b^2*(f*x+e)*ln(1-exp(2*d*x+2*c))/a^3/d-(a^2+b^2)*(f*x+e)*ln(1+b*
exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d-(a^2+b^2)*(f*x+e)*ln(1+b*exp(d*x+c)/
(a+(a^2+b^2)^(1/2)))/a^3/d+1/2*f*polylog(2,exp(2*d*x+2*c))/a/d^2+1/2*b^2*f
*polylog(2,exp(2*d*x+2*c))/a^3/d^2-(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/a^3/d^2-(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/a^3/d^2
```

3.488.2 Mathematica [A] (warning: unable to verify)

Time = 8.63 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.76

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\left(2bde\cosh\left(\frac{1}{2}(c+dx)\right)-af\cosh\left(\frac{1}{2}(c+dx)\right)-2bcf\cosh\left(\frac{1}{2}(c+dx)\right)+2bf(c+dx)\cosh\left(\frac{1}{2}(c+dx)\right)\right)\mathrm{c}}{4a^2d^2}$$
$$+\frac{(-de+cf-f(c+dx))\mathrm{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{\frac{(a^2+b^2)(de+dfx)^2}{2f}+(-abf+a^2(de+dfx)+b^2(de+dfx))\log\left(1-e^{-c-dx}\right)+(abf+a^2(de+dfx)+b^2(d}{a^3d^2}$$
$$-\frac{(a^2+b^2)\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\mathrm{arcta}}{(-a^2-b^2)^3}\right.}{}$$
$$+\frac{(de-cf+f(c+dx))\mathrm{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{\mathrm{sech}\left(\frac{1}{2}(c+dx)\right)\left(-2bde\sinh\left(\frac{1}{2}(c+dx)\right)-af\sinh\left(\frac{1}{2}(c+dx)\right)+2bcf\sinh\left(\frac{1}{2}(c+dx)\right)-2bf(c+dx)\right.}{4a^2d^2}$$

input
```
Integrate[((e + f*x)*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*
x)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2)
+ ((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) + (((a^2 +
b^2)*(d*e + d*f*x)^2)/(2*f) + (-(a*b*f) + a^2*(d*e + d*f*x) + b^2*(d*e + d
*f*x))*Log[1 - E^(-c - d*x)] + (a*b*f + a^2*(d*e + d*f*x) + b^2*(d*e + d*f
*x))*Log[1 + E^(-c - d*x)] - (a^2 + b^2)*f*PolyLog[2, -E^(-c - d*x)] - (a^
2 + b^2)*f*PolyLog[2, E^(-c - d*x)])/(a^3*d^2) - ((a^2 + b^2)*(-2*d*e*(c +
 d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[
(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(
a^2 + b^2)^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^
2)^(3/2) + 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] +
2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b
 - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-
1 + E^(2*(c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])] + 2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(2*a^3*d
^2) + ((d*e - c*f + f*(c + d*x))*Sech[(c + d*x)/2]^2)/(8*a*d^2) + (Sech[(c
 + d*x)/2]*(-2*b*d*e*Sinh[(c + d*x)/2] - a*f*Sinh[(c + d*x)/2] + 2*b*c*f*S
inh[(c + d*x)/2] - 2*b*f*(c + d*x)*Sinh[(c + d*x)/2]))/(4*a^2*d^2)
```

3.488.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 6103$$

$$\frac{\int (e+fx)\coth^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)^3dx}{a}$$

$$\Big\downarrow \; 26$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^3dx}{a}$$

$$\Big\downarrow \; 4203$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\int i(e+fx)\coth(c+dx)dx+\frac{if\int -\coth^2(c+dx)dx}{2d}+\frac{i(e+fx)\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\int i(e+fx)\coth(c+dx)dx-\frac{if\int \coth^2(c+dx)dx}{2d}+\frac{i(e+fx)\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-i\int (e+fx)\coth(c+dx)dx-\frac{if\int \coth^2(c+dx)dx}{2d}+\frac{i(e+fx)\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-i\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{if\int -\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx}{2d}+\frac{i(e+fx)\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-i\int -i(e+fx)\tan\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{if\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{2d}+\frac{i(e+fx)\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx+\frac{if\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx}{2d}+\frac{i(e+fx)\coth^2(c+dx)}{2d}\right)}{a}$$

$$\Big\downarrow \text{3954}$$

3.488. $\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx+\frac{if\left(\frac{\coth(c+dx)}{d}-\int 1dx\right)}{2d}+\frac{i(e+fx)\coth^2(c+dx)}{2d}\right)}{a}$$

$\Big\downarrow$ 24

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-\int (e+fx)\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}\right)}{a}$$

$\Big\downarrow$ 4201

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\int \frac{e^{2c+2dx-i\pi}(e+fx)}{1+e^{2c+2dx-i\pi}}dx+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$\Big\downarrow$ 2620

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int \log\left(1+e^{2c+2dx-i\pi}\right)dx}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$\Big\downarrow$ 2715

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}-\frac{f\int e^{-2c-2dx+i\pi}\log\left(1+e^{2c+2dx-i\pi}\right)de^{2c+2dx-i\pi}}{4d^2}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\right.}{a}$$

$\Big\downarrow$ 2838

$$-\frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$\Big\downarrow$ 6119

3.488. $\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(\frac{\int (e+fx)\cosh(c+dx)\coth^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \text{5973}$$

$$-\frac{b\left(\frac{\int (e+fx)\cosh(c+dx)dx+\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx+\int (e+fx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3777}$$

$$\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}-\frac{b\left(-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx-\frac{if\int -i\sinh(c+dx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\left(\frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)dx-\frac{f\int \sinh(c+dx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

3.488. $\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(-2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)\coth(c+dx)\mathrm{csch}(c+dx)dx-\frac{f\int-i\sin(ic+idx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \textbf{26}$$

$$\frac{i\left(-2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)\coth(c+dx)\mathrm{csch}(c+dx)dx+\frac{if\int\sin(ic+idx)dx}{d}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \textbf{3118}$$

$$-\frac{b\left(\frac{\int(e+fx)\coth(c+dx)\mathrm{csch}(c+dx)dx-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \textbf{5975}$$

$$-\frac{b\left(\frac{\frac{f\int\mathrm{csch}(c+dx)dx}{d}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\mathrm{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{i\left(-2i\left(\frac{f\,\mathrm{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{f\int i\csc(ic+idx)dx}{d}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\mathrm{csch}(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \textbf{26}$$

3.488. $\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\frac{if\int\csc(ic+idx)dx}{d}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}\right)}{a}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{b\left(\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \text{6119}$$

$$-\frac{b\left(\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)\cosh^2(c+dx)\coth(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\cosh^3(c}{a+b\sinh(c+d}}{a}\right.}{a}\right.}{a}+\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \text{5973}$$

$$-\frac{b\left(\frac{-\frac{f\,\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)\coth(c+dx)dx+\int(e+fx)\cosh(c+dx)\sinh(c+dx)d}{a}\right.}{a}\right.}{a}+\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

3.488. $\int\frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{i\left(-2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{2c+2dx-i\pi}\right)}{4d^2}+\frac{(e+fx)\log\left(1+e^{2c+2dx-i\pi}\right)}{2d}\right)+\frac{i(e+fx)\coth^2(c+dx)}{2d}+\frac{if\left(\frac{\coth(c+dx)}{d}-x\right)}{2d}+\frac{i(e+fx)^2}{2f}\right)}{a}-$$

$$\frac{b\left(\frac{-\frac{f\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{f\cosh(c+dx)}{d^2}+\frac{(e+fx)\sinh(c+dx)}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{\int(e+fx)\cosh(c+dx)\sinh(c+d}{}\right.}{a}\right.}{a}$$

input
```
Int[((e + f*x)*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.488.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5973
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

3.488. $\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6119
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sin
h[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[
n, 0] && IGtQ[p, 0]
```

3.488.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1097 vs. $2(407) = 814$.

Time = 1.98 (sec) , antiderivative size = 1098, normalized size of antiderivative = 2.52

method	result	size
risch	Expression too large to display	1098

input
```
int((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(-2*b*d*f*x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2*c)-2*b*d*e*exp(3*d*x+3*c
)+2*a*d*e*exp(2*d*x+2*c)+2*b*d*f*x*exp(d*x+c)+a*f*exp(2*d*x+2*c)+2*b*d*e*e
xp(d*x+c)-a*f)/a^2/d^2/(exp(2*d*x+2*c)-1)^2-1/d*f/a*ln((b*exp(d*x+c)+(a^2+
b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d*f/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d*f/a*ln(exp(d*x+c)+1)*x-1/d^2*f/a*ln((b
*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2*f/a*ln((-b*exp
(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2*c*f/a*ln(exp(d*x+
c)-1)+1/d^2*c*f/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2*b^2/a^3*f*di
log(exp(d*x+c))+1/d^2*b^2/a^3*f*dilog(exp(d*x+c)+1)+1/d*b^2/a^3*e*ln(exp(d
*x+c)-1)+1/d*b^2/a^3*e*ln(exp(d*x+c)+1)-1/d*b^2/a^3*e*ln(b*exp(2*d*x+2*c)+
2*a*exp(d*x+c)-b)-1/d^2*b^2/a^3*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))-1/d^2*b^2/a^3*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+
a)/(a+(a^2+b^2)^(1/2)))-1/d^2*b/a^2*f*ln(exp(d*x+c)-1)+1/d^2*b/a^2*f*ln(ex
p(d*x+c)+1)-1/d^2*f/a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2
)^(1/2)))-1/d^2*f/a*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1
/2)))-1/d^2*f/a*dilog(exp(d*x+c))+1/d*e/a*ln(exp(d*x+c)-1)-1/d^2*b^2/a^3*f
*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2*b^2/a^3*
f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d*b^2/a^3
*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d*b^2/a^3*
f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d*e/a*...
```

3.488.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3547 vs. $2(403) = 806$.

Time = 0.31 (sec) , antiderivative size = 3547, normalized size of antiderivative = 8.15

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c)^3 + 2*(a*b*d*f*x + a*b*d*e)*sinh(d*
x + c)^3 + a^2*f - (2*a^2*d*f*x + 2*a^2*d*e + a^2*f)*cosh(d*x + c)^2 - (2*
a^2*d*f*x + 2*a^2*d*e + a^2*f - 6*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c))*sin
h(d*x + c)^2 - 2*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c) - ((a^2 + b^2)*f*cosh
(d*x + c)^4 + 4*(a^2 + b^2)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + b^2)*
f*sinh(d*x + c)^4 - 2*(a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(3*(a^2 + b^2)*f*c
osh(d*x + c)^2 - (a^2 + b^2)*f)*sinh(d*x + c)^2 + (a^2 + b^2)*f + 4*((a^2
+ b^2)*f*cosh(d*x + c)^3 - (a^2 + b^2)*f*cosh(d*x + c))*sinh(d*x + c))*dil
og((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - ((a^2 + b^2)*f*cosh(d*x + c)^4 + 4*(
a^2 + b^2)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + b^2)*f*sinh(d*x + c)^4
 - 2*(a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(3*(a^2 + b^2)*f*cosh(d*x + c)^2 -
(a^2 + b^2)*f)*sinh(d*x + c)^2 + (a^2 + b^2)*f + 4*((a^2 + b^2)*f*cosh(d*x
 + c)^3 - (a^2 + b^2)*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2) - b)/b + 1) + ((a^2 + b^2)*f*cosh(d*x + c)^4 + 4*(a^2 + b^2)*f*cosh
(d*x + c)*sinh(d*x + c)^3 + (a^2 + b^2)*f*sinh(d*x + c)^4 - 2*(a^2 + b^2)*
f*cosh(d*x + c)^2 + 2*(3*(a^2 + b^2)*f*cosh(d*x + c)^2 - (a^2 + b^2)*f)*si
nh(d*x + c)^2 + (a^2 + b^2)*f + 4*((a^2 + b^2)*f*cosh(d*x + c)^3 - (a^2 +
b^2)*f*cosh(d*x + c))*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c...
```

3.488.6 Sympy [F]

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(e+fx)\coth^3{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral((e + f*x)*coth(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

3.488.7 Maxima [F]

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\int \frac{(fx+e)\coth{(dx+c)}^3}{b\sinh{(dx+c)}+a}\,dx$$

input
```
integrate((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(a^2*d*integrate(x/(a^3*d*e^(d*x + c) + a^3*d), x) + b^2*d*integrate(x/(a
^3*d*e^(d*x + c) + a^3*d), x) - a^2*d*integrate(x/(a^3*d*e^(d*x + c) - a^3
*d), x) - b^2*d*integrate(x/(a^3*d*e^(d*x + c) - a^3*d), x) + a*b*((d*x +
c)/(a^3*d^2) - log(e^(d*x + c) + 1)/(a^3*d^2)) - a*b*((d*x + c)/(a^3*d^2)
- log(e^(d*x + c) - 1)/(a^3*d^2)) - (2*b*d*x*e^(3*d*x + 3*c) - 2*b*d*x*e^(
d*x + c) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d*x) + a)/(a^2*d^2*e^(4*d*x
+ 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) - integrate(2*((a^3*e^c + a*
b^2*e^c)*x*e^(d*x) - (a^2*b + b^3)*x)/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*
x + c) - a^3*b), x))*f - e*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(
-3*d*x - 3*c))/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) +
 (a^2 + b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - (a^
2 + b^2)*log(e^(-d*x - c) + 1)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c) - 1)
/(a^3*d))
```

3.488.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.488.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth(c+d\,x)^3\,(e+f\,x)}{a+b\sinh\,(c+d\,x)}\,dx$$

input
```
int((coth(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

output
```
int((coth(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.489 $\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.489.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} + \frac{(a^2+b^2)\log(\sinh(c+dx))}{a^3d} - \frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^3d}$$

output
```
b*csch(d*x+c)/a^2/d-1/2*csch(d*x+c)^2/a/d+(a^2+b^2)*ln(sinh(d*x+c))/a^3/d-
(a^2+b^2)*ln(a+b*sinh(d*x+c))/a^3/d
```

3.489.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{2ab\operatorname{csch}(c+dx) - a^2\operatorname{csch}^2(c+dx) + 2(a^2+b^2)\left(\log(\sinh(c+dx)) - \log(a+b\sinh(c+dx))\right)}{2a^3d}$$

input
```
Integrate[Coth[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
(2*a*b*Csch[c + d*x] - a^2*Csch[c + d*x]^2 + 2*(a^2 + b^2)*(Log[Sinh[c + d
*x]] - Log[a + b*Sinh[c + d*x]]))/(2*a^3*d)
```

3.489.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 3200, 25, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{(a-ib\sin(ic+idx))\tan(ic+idx)^3}dx$$

$$\Big\downarrow \text{3200}$$

$$-\frac{\int -\frac{\text{csch}^3(c+dx)(\sinh^2(c+dx)b^2+b^2)}{b^3(a+b\sinh(c+dx))}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\text{csch}^3(c+dx)(\sinh^2(c+dx)b^2+b^2)}{b^3(a+b\sinh(c+dx))}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{\text{csch}^3(c+dx)}{ab}-\frac{\text{csch}^2(c+dx)}{a^2}+\frac{(a^2+b^2)\text{csch}(c+dx)}{a^3b}+\frac{-a^2-b^2}{a^3(a+b\sinh(c+dx))}\right)d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{b\text{csch}(c+dx)}{a^2}-\frac{(a^2+b^2)\log(b\sinh(c+dx))}{a^3}+\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^3}+\frac{\text{csch}^2(c+dx)}{2a}}{d}$$

input
```
Int[Coth[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

output
```
-((-((b*Csch[c + d*x])/a^2) + Csch[c + d*x]^2/(2*a) - ((a^2 + b^2)*Log[b*S
inh[c + d*x]])/a^3 + ((a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/a^3)/d)
```

3.489.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.489.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.79

method	result
derivativedivides	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(4a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{\left(-4a^2-4b^2\right)\ln\left(\right.}{}}{d}$
default	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(4a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{\left(-4a^2-4b^2\right)\ln\left(\right.}{}}{d}$
risch	$-\frac{2\,e^{dx+c}\left(-b\,e^{2dx+2c}+a\,e^{dx+c}+b\right)}{a^2 d\left(e^{2dx+2c}-1\right)^2}+\frac{\ln\left(e^{2dx+2c}-1\right)}{da}+\frac{b^2\ln\left(e^{2dx+2c}-1\right)}{d\,a^3}-\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{da}-\frac{\ln\left(e^{2\right.}{}$

3.489. $\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
int(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4/a^2*(1/2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c))-1/8/a/
tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(4*a^2+4*b^2)*ln(tanh(1/2*d*x+1/2*c))+1/2*b/
a^2/tanh(1/2*d*x+1/2*c)+1/4/a^3*(-4*a^2-4*b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-
2*b*tanh(1/2*d*x+1/2*c)-a))
```

3.489.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(78) = 156$.

Time = 0.27 (sec) , antiderivative size = 617, normalized size of antiderivative = 7.71

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{2\,ab\cosh\left(dx+c\right)^3+2\,ab\sinh\left(dx+c\right)^3-2\,a^2\cosh\left(dx+c\right)^2-2\,ab\cosh\left(dx+c\right)+2\left(3\,ab\cosh\left(dx+c\right)\right.}{}$$

input
```
integrate(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
(2*a*b*cosh(d*x + c)^3 + 2*a*b*sinh(d*x + c)^3 - 2*a^2*cosh(d*x + c)^2 - 2
*a*b*cosh(d*x + c) + 2*(3*a*b*cosh(d*x + c) - a^2)*sinh(d*x + c)^2 - ((a^2
 + b^2)*cosh(d*x + c)^4 + 4*(a^2 + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a
^2 + b^2)*sinh(d*x + c)^4 - 2*(a^2 + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + b^
2)*cosh(d*x + c)^2 - a^2 - b^2)*sinh(d*x + c)^2 + a^2 + b^2 + 4*((a^2 + b^
2)*cosh(d*x + c)^3 - (a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*si
nh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + ((a^2 + b^2)*cosh(d*x
+ c)^4 + 4*(a^2 + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + b^2)*sinh(d*
x + c)^4 - 2*(a^2 + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + b^2)*cosh(d*x + c)^
2 - a^2 - b^2)*sinh(d*x + c)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(d*x + c)^
3 - (a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*
x + c) - sinh(d*x + c))) + 2*(3*a*b*cosh(d*x + c)^2 - 2*a^2*cosh(d*x + c)
- a*b)*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)*sinh(
d*x + c)^3 + a^3*d*sinh(d*x + c)^4 - 2*a^3*d*cosh(d*x + c)^2 + a^3*d + 2*(
3*a^3*d*cosh(d*x + c)^2 - a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^
3 - a^3*d*cosh(d*x + c))*sinh(d*x + c))
```

3.489.6 Sympy [F]

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\coth^3\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

input
```
integrate(coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

3.489.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(78) = 156$.

Time = 0.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.16

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{2\left(be^{(-dx-c)} - ae^{(-2\,dx-2\,c)} - be^{(-3\,dx-3\,c)}\right)}{(2\,a^2e^{(-2\,dx-2\,c)} - a^2e^{(-4\,dx-4\,c)} - a^2)d} - \frac{(a^2+b^2)\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{a^3d} + \frac{(a^2+b^2)\log\left(e^{(-dx-c)}+1\right)}{a^3d} + \frac{(a^2+b^2)\log\left(e^{(-dx-c)}-1\right)}{a^3d}$$

input
```
integrate(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-
2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 + b^2)*log(-2*a*e^(-d
*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) + (a^2 + b^2)*log(e^(-d*x - c) +
 1)/(a^3*d) + (a^2 + b^2)*log(e^(-d*x - c) - 1)/(a^3*d)
```

3.489.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(78) = 156$.

Time = 0.34 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.30

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2\,(a^2+b^2)\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a^3} - \frac{2\,(a^2b+b^3)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^3b} - \frac{3\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+3\,b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2 -}{a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2}}{2\,d}$$

input
```
integrate(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*(a^2 + b^2)*log(abs(e^(d*x + c) - e^(-d*x - c)))/a^3 - 2*(a^2*b + b
^3)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^3*b) - (3*a^2*(e^(d*
x + c) - e^(-d*x - c))^2 + 3*b^2*(e^(d*x + c) - e^(-d*x - c))^2 - 4*a*b*(e
^(d*x + c) - e^(-d*x - c)) + 4*a^2)/(a^3*(e^(d*x + c) - e^(-d*x - c))^2))/
d
```

3.489.9 Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 1329, normalized size of antiderivative = 16.61

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(coth(c + d*x)^3/(a + b*sinh(c + d*x)),x)
```

output
```
((2*atan((a^2*(-a^6*d^2)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6
*d^2)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(2*a^3*d*(a^2 + b^2)^2) + ((a^7
*d + a^5*b^2*d)*(-a^6*d^2)^(1/2))/(2*a^6*d^2*((a^2 + b^2)^2)^(1/2)*(a^2 +
b^2)) - (a^6*b^2*exp(2*c)*exp(2*d*x)*(-a^6*d^2)^(1/2)*((4*(a^2 + 2*b^2)*(a
^4 + b^4 + 2*a^2*b^2))/(a^9*b^2*d*(a^2 + b^2)^2) + (2*(2*a^4*b^3*d + 2*a^6
*b*d)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^11*b^3*d^2*((a^2 + b^2)^2)^(1/2)*(
a^2 + b^2)) + (4*(a^2*(-a^6*d^2)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b
^2*(-a^6*d^2)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))*(a^4 + b^4 + 2*a^2*b^2)
^(1/2))/(a^9*b^2*d*(a^2 + b^2)^2*(-a^6*d^2)^(1/2)) + (4*(a^7*d + a^5*b^2*d
)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^12*b^2*d^2*((a^2 + b^2)^2)^(1/2)*(a^2
+ b^2))))/(8*(a^4 + b^4 + 2*a^2*b^2)^(1/2)) + (a^6*b^2*exp(3*c)*exp(3*d*x)
*((2*(a^7*d + a^5*b^2*d)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^11*b^3*d^2*((a^
2 + b^2)^2)^(1/2)*(a^2 + b^2)) - (2*(a^2 + 2*b^2)*(a^2*(-a^6*d^2)^(1/2)*(a
^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6*d^2)^(1/2)*(a^4 + b^4 + 2*a^2*b^
2)^(1/2))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^10*b^3*d*(a^2 + b^2)^2*(-a^6*d
^2)^(1/2)))*(-a^6*d^2)^(1/2))/(8*(a^4 + b^4 + 2*a^2*b^2)^(1/2)) - (a^6*b^2
*exp(d*x)*exp(c)*(-a^6*d^2)^(1/2)*((8*(a^4 + b^4 + 2*a^2*b^2))/(a^8*b*d*(a
^2 + b^2)^2) - (4*(2*a^4*b^3*d + 2*a^6*b*d)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))
/(a^12*b^2*d^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) + (2*(a^7*d + a^5*b^2*d)
*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^11*b^3*d^2*((a^2 + b^2)^2)^(1/2)*(a^...
```

3.490 $\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.490.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Int}\left(\frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.490.2 Mathematica [N/A]

Not integrable

Time = 139.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[Coth[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
Integrate[Coth[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

3.490.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{ 6111}$$
$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[Coth[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.490.3.1 Defintions of rubi rules used

rule 6111
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Sinh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.490.4 Maple [N/A] (verified)

Not integrable

Time = 0.65 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\coth\left(dx+c\right)^{3}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.490.5 Fricas [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(coth(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

3.490.6 Sympy [N/A]

Not integrable

Time = 4.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth^3(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

input
```
integrate(coth(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Integral(coth(c + d*x)**3/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

3.490.7 Maxima [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 751, normalized size of antiderivative = 26.82

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-(a*f - 2*(b*d*f*x*e^(3*c) + b*d*e*e^(3*c))*e^(3*d*x) + (2*a*d*f*x*e^(2*c)
 + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) + 2*(b*d*f*x*e^c + b*d*e*e^c)*e^(d*x))
/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*d^2*f^2*x^2*e^(4*
c) + 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e^(4*d*x) - 2*(a^2*d^2
*f^2*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e^2*e^(2*c))*e^(2*d*x
)) + integrate(-(b^2*d^2*e^2 + a*b*d*e*f + (d^2*e^2 + f^2)*a^2 + (a^2*d^2*
f^2 + b^2*d^2*f^2)*x^2 + (2*a^2*d^2*e*f + 2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a
^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 - (
a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^
3*d^2*e^3*e^c)*e^(d*x)), x) - integrate((b^2*d^2*e^2 - a*b*d*e*f + (d^2*e^
2 + f^2)*a^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + (2*a^2*d^2*e*f + 2*b^2*d^
2*e*f - a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e
^2*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*
a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + integrate(2*(a^2*b +
 b^3 - (a^3*e^c + a*b^2*e^c)*e^(d*x))/(a^3*b*f*x + a^3*b*e - (a^3*b*f*x*e^
(2*c) + a^3*b*e*e^(2*c))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e*e^c)*e^(d*x)),
 x)
```

3.490.8 Giac [F(-1)]

Timed out.

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

3.490.9 Mupad [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+d\,x)^3}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(coth(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(coth(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.490. $\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.491 $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.491.1 Optimal result

Integrand size = 34, antiderivative size = 1795

$$\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
-3/4*f^3*polylog(4,exp(2*d*x+2*c))/a/d^4+2*(f*x+e)^3*arctanh(exp(2*d*x+2*c
))/a/d+3/4*f^3*polylog(4,-exp(2*d*x+2*c))/a/d^4-6*b*f^3*polylog(3,-exp(d*x
+c))/a^2/d^4+6*b*f^3*polylog(3,exp(d*x+c))/a^2/d^4+3/4*b^2*f^3*polylog(4,e
xp(2*d*x+2*c))/a^3/d^4-6*I*b^3*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/a^2/(a
^2+b^2)/d^3-3*I*b^3*f*(f*x+e)^2*polylog(2,I*exp(d*x+c))/a^2/(a^2+b^2)/d^2+
b^4*(f*x+e)^3*ln(1+exp(2*d*x+2*c))/a^3/(a^2+b^2)/d+6*b*f*(f*x+e)^2*arctanh
(exp(d*x+c))/a^2/d^2+6*b*f^2*(f*x+e)*polylog(2,-exp(d*x+c))/a^2/d^3-3/2*f*
(f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a/d^2+3/2*f^2*(f*x+e)*polylog(3,exp(2*
d*x+2*c))/a/d^3+3/2*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/a/d^2-6*b*f^2*(
f*x+e)*polylog(2,exp(d*x+c))/a^2/d^3+3/2*b^2*f*(f*x+e)^2*polylog(2,exp(2*d
*x+2*c))/a^3/d^2-2*b^3*(f*x+e)^3*arctan(exp(d*x+c))/a^2/(a^2+b^2)/d-3/2*b^
2*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/a^3/d^2+3/2*b^2*f^2*(f*x+e)*polyl
og(3,-exp(2*d*x+2*c))/a^3/d^3+3/4*b^4*f^3*polylog(4,-exp(2*d*x+2*c))/a^3/(
a^2+b^2)/d^4-6*b^4*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a
^2+b^2)/d^4-6*b^4*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^
2+b^2)/d^4-6*I*b*f^3*polylog(4,-I*exp(d*x+c))/a^2/d^4+6*I*b*f^2*(f*x+e)*po
lylog(3,-I*exp(d*x+c))/a^2/d^3+6*I*b^3*f^3*polylog(4,-I*exp(d*x+c))/a^2/(a
^2+b^2)/d^4+3*I*b*f*(f*x+e)^2*polylog(2,I*exp(d*x+c))/a^2/d^2+3*I*b^3*f*(f
*x+e)^2*polylog(2,-I*exp(d*x+c))/a^2/(a^2+b^2)/d^2+6*I*b^3*f^2*(f*x+e)*pol
ylog(3,I*exp(d*x+c))/a^2/(a^2+b^2)/d^3+1/2*(f*x+e)^3/a/d-3/2*b^2*f^2*(f...
```

3.491.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5813 vs. 2(1795) = 3590.

Time = 12.84 (sec) , antiderivative size = 5813, normalized size of antiderivative = 3.24

$$\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^3*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
Result too large to show
```

3.491.3 Rubi [A] (verified)

Time = 8.16 (sec) , antiderivative size = 1596, normalized size of antiderivative = 0.89, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.676, Rules used = {6123, 5985, 27, 6123, 5985, 25, 6123, 5984, 3042, 26, 4670, 3011, 6107, 6095, 2620, 3011, 7163, 2720, 7143, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-3f\int -\frac{1}{2}(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-\frac{b\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3}{2}f\int (e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \ 6123$$

$$\frac{\frac{3}{2}f\int (e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \ 5985$$

$$\frac{\frac{3}{2}f\int (e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{-3f\int -(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3}{2}f\int (e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \ 6123$$

$$\frac{\frac{3}{2}f\int (e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)dx}{a}\cdots\right)}{a}\right)}{a}$$

$$\Big\downarrow \ 5984$$

3.491. $\displaystyle\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3}{2}f\int (e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{2\int (e+fx)^3\operatorname{csch}(2c+2dx)dx}{a}-\frac{b\int}{}\right.}{a}\right.}{a}$$

$$\downarrow \text{3042}$$

$$\frac{\frac{3}{2}f\int (e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2\int i}{}\right.}{a}\right.}{a}$$

$$\downarrow \text{26}$$

$$\frac{\frac{3}{2}f\int (e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\int}{}\right.}{a}\right.}{a}$$

$$\downarrow \text{4670}$$

$$\frac{\frac{3}{2}f\int (e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{3f\int (e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\right.}{}\right.}{}\right.}{a}$$

$$\downarrow \text{3011}$$

3.491. $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\right.}{}\right.}{}\right.$$

$\downarrow$ 6107

$$\frac{\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{b^2\int\frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\int\right.}{}\right.}{}\right.$$

$\downarrow$ 6095

3.491. $\int\frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(b^2\left(\int\frac{e^{c+dx}(e+fx)^3}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\right.\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(b^2\left(-\frac{3f\int(e+fx)^2\log\left(\frac{e^{c+d}}{a-\sqrt{a^2}}\right.}{bd}\right.\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{3011}$$

3.491. $\displaystyle\int\frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^3\coth^2(c+dx)}{2d}-\frac{(e+fx)^3\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^3\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(b^2\left(-\frac{3f\left(\frac{2f\int(e+fx)\operatorname{PolyLog}(\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 7163

3.491. $\int\frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{\coth^2(c+dx)(e+fx)^3}{2d}-\frac{\log(\tanh(c+dx))(e+fx)^3}{d}+\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx}{a}-$$

$$b\left(\frac{-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^3}{d}+3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx}{a}-\frac{b\left(2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^3}{d}\right.\right.}{}\right.$$

3.491. $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$\downarrow$ 2720

3.491. $\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{\coth^2(c+dx)(e+fx)^3}{2d}-\frac{\log(\tanh(c+dx))(e+fx)^3}{d}+\frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx}{a}-$$

$$b\left(\frac{-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^3}{d}+3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx}{a}-\frac{b\left(2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^3}{d}\right.\right.}{}\right.$$

3.491. $\int\frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\downarrow 7143$$

3.491. $\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx) \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{\coth^2(c+dx)(e+fx)^3}{2d} - \frac{\log(\tanh(c+dx))(e+fx)^3}{d} + \frac{3}{2}f\int(e+fx)^2\left(\frac{\coth^2(c+dx)}{d} + \frac{2\log(\tanh(c+dx))}{d}\right)dx}{a} -$$

$$b\left(\frac{-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d} - \frac{\operatorname{csch}(c+dx)(e+fx)^3}{d} + 3f\int(e+fx)^2\left(\frac{\arctan(\sinh(c+dx))}{d} + \frac{\operatorname{csch}(c+dx)}{d}\right)dx}{a} - \frac{b\left(2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^3}{d}\right.\right.}{}\right.$$

3.491. $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$\downarrow$ 7292

3.491. $\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx) \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{\coth^2(c+dx)(e+fx)^3}{2d}-\frac{\log(\tanh(c+dx))(e+fx)^3}{d}+\frac{3}{2}f\int\frac{(e+fx)^2(\coth^2(c+dx)+2\log(\tanh(c+dx)))}{d}dx}{a}-$$

$$b\left(\frac{-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^3}{d}+3f\int\frac{(e+fx)^2(\arctan(\sinh(c+dx))+\operatorname{csch}(c+dx))}{d}dx}{a}-\frac{b\left(\frac{2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^3}{d}-\cdots\right)}{\cdots}\right.}{\cdots}\right.$$

3.491. $\int\frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow 27$$

3.491. $\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx) \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-\frac{\coth^2(c+dx)(e+fx)^3}{2d}-\frac{\log(\tanh(c+dx))(e+fx)^3}{d}+\frac{3f\int(e+fx)^2\left(\coth^2(c+dx)+2\log(\tanh(c+dx))\right)dx}{2d}}{a}-$$

$$b\left(\frac{-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^3}{d}+\frac{3f\int(e+fx)^2(\arctan(\sinh(c+dx))+\operatorname{csch}(c+dx))dx}{d}}{a}-\frac{b\left(\frac{2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^3}{d}-\frac{3i}{}\right.}{}\right.}{}\right.$$

3.491. $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.491. $\downarrow$ 7293

3.491. $\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx) \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} \, dx$

$$\frac{-\frac{\coth^2(c+dx)(e+fx)^3}{2d}-\frac{\log(\tanh(c+dx))(e+fx)^3}{d}+\frac{3f\int\left(\coth^2(c+dx)(e+fx)^2+2\log(\tanh(c+dx))(e+fx)^2\right)dx}{2d}}{a}-$$

$$b\left(\frac{-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^3}{d}+\frac{3f\int\left(\arctan(\sinh(c+dx))(e+fx)^2+\operatorname{csch}(c+dx)(e+fx)^2\right)dx}{d}}{a}-\frac{b\left(2i\left(\frac{i\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+}{d}\right.\right.}{}\right.$$

3.491. $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.491.

$\downarrow$ 2009

3.491. $\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx) \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\, dx$

$$-\frac{\coth^2(c+dx)(e+fx)^3}{2d} - \frac{\log(\tanh(c+dx))(e+fx)^3}{d} + \frac{3f\left(\frac{4\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^3}{3f} + \frac{2\log(\tanh(c+dx))(e+fx)^3}{3f} + \frac{(e+fx)^3}{3f} - \frac{\coth(c+dx)(e+fx}{d}\right.}{}$$

$$b\left(\frac{-\frac{\arctan(\sinh(c+dx))(e+fx)^3}{d} - \frac{\operatorname{csch}(c+dx)(e+fx)^3}{d} + \frac{3f\left(-\frac{2\arctan\left(e^{c+dx}\right)(e+fx)^3}{3f} + \frac{\arctan(\sinh(c+dx))(e+fx)^3}{3f} - \frac{2\operatorname{arctanh}\left(e^{c+dx}\right)(e+fx)^2}{d} + \frac{i\,\mathrm{Po}}{}\right.}{}}{}\right.$$

3.491. $\displaystyle\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^3*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-1/2*((e + f*x)^3*Coth[c + d*x]^2)/d - ((e + f*x)^3*Log[Tanh[c + d*x]])/d
 + (3*f*(-((e + f*x)^2/d) + (e + f*x)^3/(3*f) + (4*(e + f*x)^3*ArcTanh[E^(
2*c + 2*d*x)])/(3*f) - ((e + f*x)^2*Coth[c + d*x])/d + (2*f*(e + f*x)*Log[
1 - E^(2*(c + d*x))])/d^2 + (2*(e + f*x)^3*Log[Tanh[c + d*x]])/(3*f) + (f^
2*PolyLog[2, E^(2*(c + d*x))])/d^3 + ((e + f*x)^2*PolyLog[2, -E^(2*c + 2*d
*x)])/d - ((e + f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/d - (f*(e + f*x)*PolyL
og[3, -E^(2*c + 2*d*x)])/d^2 + (f*(e + f*x)*PolyLog[3, E^(2*c + 2*d*x)])/d
^2 + (f^2*PolyLog[4, -E^(2*c + 2*d*x)])/(2*d^3) - (f^2*PolyLog[4, E^(2*c +
 2*d*x)])/(2*d^3)))/(2*d))/a - (b*((-(((e + f*x)^3*ArcTan[Sinh[c + d*x]])/
d) - ((e + f*x)^3*Csch[c + d*x])/d + (3*f*((-2*(e + f*x)^3*ArcTan[E^(c + d
*x)])/(3*f) + ((e + f*x)^3*ArcTan[Sinh[c + d*x]])/(3*f) - (2*(e + f*x)^2*A
rcTanh[E^(c + d*x)])/d - (2*f*(e + f*x)*PolyLog[2, -E^(c + d*x)])/d^2 + (I
*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/d - (I*(e + f*x)^2*PolyLog[2, I
*E^(c + d*x)])/d + (2*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/d^2 + (2*f^2*Po
lyLog[3, -E^(c + d*x)])/d^3 - ((2*I)*f*(e + f*x)*PolyLog[3, (-I)*E^(c + d*
x)])/d^2 + ((2*I)*f*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/d^2 - (2*f^2*Poly
Log[3, E^(c + d*x)])/d^3 + ((2*I)*f^2*PolyLog[4, (-I)*E^(c + d*x)])/d^3 -
((2*I)*f^2*PolyLog[4, I*E^(c + d*x)])/d^3))/d)/a - (b*(-((b*((b^2*(-1/4*(e
 + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])...
```

3.491.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.491. $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.491. $\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx) \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.491.4 Maple [F]

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)^3 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.491.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23903 vs. 2(1636) = 3272.

Time = 0.77 (sec) , antiderivative size = 23903, normalized size of antiderivative = 13.32

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.491.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.491.7 Maxima [F]

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^3\,\text{csch}\,(dx+c)^3\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-(b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + a^3*b^2)*d)
+ 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/(
(a^2 + b^2)*d) + 2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*
c))/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 - b^2
)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d
))*e^3 + (3*a*f^3*x^2 + 6*a*e*f^2*x + 3*a*e^2*f + 2*(b*d*f^3*x^3*e^(3*c) +
 3*b*d*e*f^2*x^2*e^(3*c) + 3*b*d*e^2*f*x*e^(3*c))*e^(3*d*x) - (2*a*d*f^3*x
^3*e^(2*c) + 3*a*e^2*f*e^(2*c) + 3*(2*d*e*f^2 + f^3)*a*x^2*e^(2*c) + 6*(d*
e^2*f + e*f^2)*a*x*e^(2*c))*e^(2*d*x) - 2*(b*d*f^3*x^3*e^c + 3*b*d*e*f^2*x
^2*e^c + 3*b*d*e^2*f*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*
e^(2*d*x + 2*c) + a^2*d^2) - 3*(b*d*e^2*f + a*e*f^2)*x/(a^2*d^2) + 3*(b*d*
e^2*f - a*e*f^2)*x/(a^2*d^2) + 3*(b*d*e^2*f + a*e*f^2)*log(e^(d*x + c) + 1
)/(a^2*d^3) - 3*(b*d*e^2*f - a*e*f^2)*log(e^(d*x + c) - 1)/(a^2*d^3) - (d^
3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog
(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*(a^2*f^3 - b^2*f^3)/(a^3*d
^4) - (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*
x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*(a^2*f^3 - b^2*f^3)
/(a^3*d^4) - 3*(a^2*d*e*f^2 - b^2*d*e*f^2 - a*b*f^3)*(d^2*x^2*log(e^(d*x +
 c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^3*d^
4) - 3*(a^2*d*e*f^2 - b^2*d*e*f^2 + a*b*f^3)*(d^2*x^2*log(-e^(d*x + c) ...
```

3.491.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.491.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^3}{\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)^3\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.492 $\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.492.1 Optimal result

Integrand size = 34, antiderivative size = 1219

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
1/2*f^2*polylog(3,exp(2*d*x+2*c))/a/d^3+2*(f*x+e)^2*arctanh(exp(2*d*x+2*c)
)/a/d-1/2*f^2*polylog(3,-exp(2*d*x+2*c))/a/d^3+2*b*f^2*polylog(2,-exp(d*x+
c))/a^2/d^3-2*b*f^2*polylog(2,exp(d*x+c))/a^2/d^3-1/2*b^2*f^2*polylog(3,ex
p(2*d*x+2*c))/a^3/d^3-2*I*b^3*f*(f*x+e)*polylog(2,I*exp(d*x+c))/a^2/(a^2+b
^2)/d^2+f^2*ln(sinh(d*x+c))/a/d^3+b^2*f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/
a^3/d^2-2*b^3*(f*x+e)^2*arctan(exp(d*x+c))/a^2/(a^2+b^2)/d-1/2*b^4*f^2*pol
ylog(3,-exp(2*d*x+2*c))/a^3/(a^2+b^2)/d^3+2*b^4*f^2*polylog(3,-b*exp(d*x+c
)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d^3+2*b^4*f^2*polylog(3,-b*exp(d*x+c)
/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d^3-2*I*b*f^2*polylog(3,I*exp(d*x+c))/
a^2/d^3+b^4*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/a^3/(a^2+b^2)/d^2+2*I*b*f
^2*polylog(3,-I*exp(d*x+c))/a^2/d^3+1/2*f^2*x^2/a/d+4*b*f*(f*x+e)*arctanh(
exp(d*x+c))/a^2/d^2-b^2*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/a^3/d^2+b^4*(
f*x+e)^2*ln(1+exp(2*d*x+2*c))/a^3/(a^2+b^2)/d-b^4*(f*x+e)^2*ln(1+b*exp(d*x
+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d-b^4*(f*x+e)^2*ln(1+b*exp(d*x+c)/(
a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d+2*I*b*f*(f*x+e)*polylog(2,I*exp(d*x+c)
)/a^2/d^2+2*I*b^3*f^2*polylog(3,I*exp(d*x+c))/a^2/(a^2+b^2)/d^3+2*I*b^3*f*
(f*x+e)*polylog(2,-I*exp(d*x+c))/a^2/(a^2+b^2)/d^2+e*f*x/a/d+f*(f*x+e)*pol
ylog(2,-exp(2*d*x+2*c))/a/d^2-f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^2-f*
(f*x+e)*coth(d*x+c)/a/d^2+b*(f*x+e)^2*csch(d*x+c)/a^2/d-1/2*(f*x+e)^2*coth
(d*x+c)^2/a/d-2*b^4*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2...
```

3.492.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2784 vs. 2(1219) = 2438.

Time = 10.38 (sec) , antiderivative size = 2784, normalized size of antiderivative = 2.28

$$\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]
),x]
```

output
```
(b*(e + f*x)^2*Csch[c])/(a^2*d) - ((e + f*x)^2*Csch[(c + d*x)/2]^2)/(8*a*d
) + (-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*f
*x^2 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] -
6*a*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d*
e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) -
 PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 +
E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c +
 d*x))]) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2
*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*P
olyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I
*E^(c + d*x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c
 + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c +
d*x))]))/(6*(a^2 + b^2)*d^3*(1 + E^(2*c))) + (12*a^2*d^3*e^2*E^(2*c)*x - 1
2*b^2*d^3*e^2*E^(2*c)*x - 12*a^2*d*E^(2*c)*f^2*x + 12*a^2*d^3*e*E^(2*c)*f*
x^2 - 12*b^2*d^3*e*E^(2*c)*f*x^2 + 4*a^2*d^3*E^(2*c)*f^2*x^3 - 4*b^2*d^3*E
^(2*c)*f^2*x^3 - 24*a*b*d*e*f*ArcTanh[E^(c + d*x)] + 24*a*b*d*e*E^(2*c)*f*
ArcTanh[E^(c + d*x)] + 12*a*b*d*f^2*x*Log[1 - E^(c + d*x)] - 12*a*b*d*E^(2
*c)*f^2*x*Log[1 - E^(c + d*x)] - 12*a*b*d*f^2*x*Log[1 + E^(c + d*x)] + 12*
a*b*d*E^(2*c)*f^2*x*Log[1 + E^(c + d*x)] + 6*a^2*d^2*e^2*Log[1 - E^(2*(c +
 d*x))] - 6*b^2*d^2*e^2*Log[1 - E^(2*(c + d*x))] - 6*a^2*d^2*e^2*E^(2*c...
```

3.492.3 Rubi [A] (verified)

Time = 6.16 (sec) , antiderivative size = 1118, normalized size of antiderivative = 0.92, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.676$, Rules used = {6123, 5985, 27, 6123, 5985, 25, 6123, 5984, 3042, 26, 4670, 3011, 2720, 6107, 6095, 2620, 3011, 2720, 7143, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6123}$$

$$\frac{\int (e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 5985}$$

$$\frac{-2f\int -\frac{1}{2}(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f\int (e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 6123}$$

$$\frac{f\int (e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{\int (e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{ 5985}$$

$$\frac{\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{-2f\int-\left((e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c}{a+b\sinh(c+dx)}}{a}\right.}{a}}{}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right.}{a}$$

$$\Big\downarrow \textbf{6123}$$

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)dx}{a}\right.}{a}\right.}{a}$$

$$\Big\downarrow \textbf{5984}$$

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(\frac{2\int(e+fx)^2\text{csch}(2c+2dx)dx}{a}-\frac{b\int\frac{(e}{}}{}\right.}{a}\right.}{a}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\text{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2\int i(}{}\right.}{a}\right.}{a}$$

3.492. $\displaystyle\int\frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$\downarrow$ 26

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\int(e\cdots}{\cdots}\right.}{a}\right.}{a}$$

$\downarrow$ 4670

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(\frac{i\cdots}{-\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{a}$$

$\downarrow$ 3011

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(-\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$\downarrow$ 2720

3.492. $\displaystyle\int\frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{2i\left(-\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\Big\downarrow \text{ 6107}$$

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{b^2\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\int(\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\Big\downarrow \text{ 6095}$$

3.492. $\displaystyle\int\frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{b^2\left(\int\frac{e^{c+dx}(e+fx)^2}{a+be^{c+dx}-\sqrt{a^2+b^2}}dx+\int\right.}{a}\right.}{}\right.}{}\right.$$

$\Big\downarrow$ 2620

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{b^2\left(-\frac{2f\int(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}\right.}{bd}\right.}{}\right.}{}\right.}{}\right.$$

$\Big\downarrow$ 3011

3.492. $\displaystyle\int\frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{f\int (e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(b^2\left(-\frac{2f\left(\frac{f\int \operatorname{PolyLog}\left(2,-\frac{be^{\cdots}}{a-\sqrt{\cdots}}\right)\cdots}{d}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 2720

3.492. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{f\int (e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{2f\int (e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(b^2\left(-\frac{2f\left(\frac{f\int e^{-c-dx}\operatorname{PolyLog}\left(\right.}{}\right.}{}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 7143

3.492. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{f\int(e+fx)\left(\frac{\coth^2(c+dx)}{d}+\frac{2\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{2f\int(e+fx)\left(\frac{\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{\left(b-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(}{a^2+b^2}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \ 7292$$

3.492. $\int\frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{f\int \frac{(e+fx)\left(\coth^2(c+dx)+2\log(\tanh(c+dx))\right)}{d}dx-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{2f\int \frac{(e+fx)(\arctan(\sinh(c+dx))+\operatorname{csch}(c+dx))}{d}dx-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{\int (e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+\cdots}{a^2+b^2}\right.}{}\right.}{}\right.$$

$$\downarrow 27$$

3.492. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\int(e+fx)\left(\coth^2(c+dx)+2\log(\tanh(c+dx))\right)dx}{d}-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{\frac{2f\int(e+fx)(\arctan(\sinh(c+dx))+\operatorname{csch}(c+dx))dx}{d}-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b\left(-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}(c+dx)(a-b\sinh(c+dx)}{a^2+b^2}\right.}{}\right.}{}\right.$$

$\downarrow$ 7293

3.492. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{f\int((e+fx)\coth^2(c+dx)+2(e+fx)\log(\tanh(c+dx)))dx}{d}-\frac{(e+fx)^2\coth^2(c+dx)}{2d}-\frac{(e+fx)^2\log(\tanh(c+dx))}{d}}{a}-$$

$$b\left(\frac{\frac{2f\int((e+fx)\arctan(\sinh(c+dx))+(e+fx)\operatorname{csch}(c+dx))dx}{d}-\frac{(e+fx)^2\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)^2\operatorname{csch}(c+dx)}{d}}{a}-\frac{b-\frac{b\left(\frac{\int\left(a(e+fx)^2\operatorname{sech}(c+dx)-b(e\right.}{a^2+b^2}\right.}{}}{}\right.$$

$\downarrow$ 2009

3.492. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{\coth^2(c+dx)(e+fx)^2}{2d}-\frac{\log(\tanh(c+dx))(e+fx)^2}{d}+\frac{f\left(\frac{2\operatorname{arctanh}\left(e^{2c+2dx}\right)(e+fx)^2}{f}+\frac{\log(\tanh(c+dx))(e+fx)^2}{f}+\frac{(e+fx)^2}{2f}-\frac{\coth(c+dx)(e+fx)}{d}+\right.}{a}$$

$$b\left(-\frac{\arctan(\sinh(c+dx))(e+fx)^2}{d}-\frac{\operatorname{csch}(c+dx)(e+fx)^2}{d}+\frac{2f\left(-\frac{\arctan\left(e^{c+dx}\right)(e+fx)^2}{f}+\frac{\arctan(\sinh(c+dx))(e+fx)^2}{2f}-\frac{2\operatorname{arctanh}\left(e^{c+dx}\right)(e+fx)}{d}+\frac{i\operatorname{PolyL}}{}\right.}{a}\right.$$

input
```
Int[((e + f*x)^2*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

3.492. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
(-1/2*((e + f*x)^2*Coth[c + d*x]^2)/d - ((e + f*x)^2*Log[Tanh[c + d*x]])/d
 + (f*((e + f*x)^2/(2*f) + (2*(e + f*x)^2*ArcTanh[E^(2*c + 2*d*x)])/f - ((
e + f*x)*Coth[c + d*x])/d + (f*Log[Sinh[c + d*x]])/d^2 + ((e + f*x)^2*Log[
Tanh[c + d*x]])/f + ((e + f*x)*PolyLog[2, -E^(2*c + 2*d*x)])/d - ((e + f*x
)*PolyLog[2, E^(2*c + 2*d*x)])/d - (f*PolyLog[3, -E^(2*c + 2*d*x)])/(2*d^2
) + (f*PolyLog[3, E^(2*c + 2*d*x)])/(2*d^2)))/d)/a - (b*((-(((e + f*x)^2*A
rcTan[Sinh[c + d*x]])/d) - ((e + f*x)^2*Csch[c + d*x])/d + (2*f*(-(((e + f
*x)^2*ArcTan[E^(c + d*x)])/f) + ((e + f*x)^2*ArcTan[Sinh[c + d*x]])/(2*f)
- (2*(e + f*x)*ArcTanh[E^(c + d*x)])/d - (f*PolyLog[2, -E^(c + d*x)])/d^2
+ (I*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/d - (I*(e + f*x)*PolyLog[2, I
*E^(c + d*x)])/d + (f*PolyLog[2, E^(c + d*x)])/d^2 - (I*f*PolyLog[3, (-I)*
E^(c + d*x)])/d^2 + (I*f*PolyLog[3, I*E^(c + d*x)])/d^2))/d)/a - (b*(-((b*
((b^2*(-1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2])])/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2]))])/d^2))/(b*d) - (2*f*(-(((e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/d) + (f*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/d^2))/(b*d)))/(a^2 + b^2) + ((b*(e + f*x)^3)/(3*f) + (2*a*(e + f
*x)^2*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])...
```

3.492.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.492. $\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

3.492. $\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx) \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.492. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.492.4 Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^3 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

3.492.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 13309 vs. 2(1119) = 2238.

Time = 0.50 (sec) , antiderivative size = 13309, normalized size of antiderivative = 10.92

$$\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.492.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.492.7 Maxima [F]

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2\,\text{csch}\,(dx+c)^3\,\text{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-(b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + a^3*b^2)*d)
+ 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/(
(a^2 + b^2)*d) + 2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*
c))/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 - b^2
)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d
))*e^2 + 2*(a*f^2*x + a*e*f + (b*d*f^2*x^2*e^(3*c) + 2*b*d*e*f*x*e^(3*c))*
e^(3*d*x) - (a*d*f^2*x^2*e^(2*c) + a*e*f*e^(2*c) + (2*d*e*f + f^2)*a*x*e^(
2*c))*e^(2*d*x) - (b*d*f^2*x^2*e^c + 2*b*d*e*f*x*e^c)*e^(d*x))/(a^2*d^2*e^
(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) - (2*b*d*e*f + a*f^2)
*x/(a^2*d^2) + (2*b*d*e*f - a*f^2)*x/(a^2*d^2) + (2*b*d*e*f + a*f^2)*log(e
^(d*x + c) + 1)/(a^2*d^3) - (2*b*d*e*f - a*f^2)*log(e^(d*x + c) - 1)/(a^2*
d^3) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polyl
og(3, -e^(d*x + c)))*(a^2*f^2 - b^2*f^2)/(a^3*d^3) - (d^2*x^2*log(-e^(d*x
+ c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*(a^2*f^2
 - b^2*f^2)/(a^3*d^3) - 2*(a^2*d*e*f - b^2*d*e*f - a*b*f^2)*(d*x*log(e^(d*
x + c) + 1) + dilog(-e^(d*x + c)))/(a^3*d^3) - 2*(a^2*d*e*f - b^2*d*e*f +
a*b*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^3*d^3) + 1/3*
((a^2*f^2 - b^2*f^2)*d^3*x^3 + 3*(a^2*d*e*f - b^2*d*e*f + a*b*f^2)*d^2*x^2
)/(a^3*d^3) + 1/3*((a^2*f^2 - b^2*f^2)*d^3*x^3 + 3*(a^2*d*e*f - b^2*d*e*f
- a*b*f^2)*d^2*x^2)/(a^3*d^3) + integrate(2*(b^5*f^2*x^2 + 2*b^5*e*f*x ...
```

3.492.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.492.9 Mupad [F(-1)]

Timed out.

$$
\begin{aligned}
&\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx \\
&= \int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)^3\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx
\end{aligned}
$$

input
```
int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.493 $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.493.1 Optimal result

Integrand size = 32, antiderivative size = 762

$$
\begin{aligned}
\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = {} & \frac{fx}{2ad}+\frac{2bfx\arctan\left(e^{c+dx}\right)}{a^2d} \\
& -\frac{2b^3(e+fx)\arctan\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} \\
& -\frac{bfx\arctan(\sinh(c+dx))}{a^2d} \\
& +\frac{b(e+fx)\arctan(\sinh(c+dx))}{a^2d} \\
& +\frac{2fx\operatorname{arctanh}\left(e^{2c+2dx}\right)}{ad} \\
& -\frac{2b^2(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{a^3d} \\
& +\frac{bf\operatorname{arctanh}(\cosh(c+dx))}{a^2d^2} \\
& -\frac{f\coth(c+dx)}{2ad^2}-\frac{(e+fx)\coth^2(c+dx)}{2ad} \\
& +\frac{b(e+fx)\operatorname{csch}(c+dx)}{a^2d} \\
& -\frac{b^4(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)d} \\
& -\frac{b^4(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)d} \\
& +\frac{b^4(e+fx)\log\left(1+e^{2(c+dx)}\right)}{a^3\left(a^2+b^2\right)d} \\
& +\frac{fx\log(\tanh(c+dx))}{ad} \\
& -\frac{(e+fx)\log(\tanh(c+dx))}{ad} \\
& -\frac{ibf\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{a^2d^2} \\
& +\frac{ib^3f\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{a^2\left(a^2+b^2\right)d^2} \\
& +\frac{ibf\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{a^2d^2} \\
& -\frac{ib^3f\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{a^2\left(a^2+b^2\right)d^2} \\
& -\frac{b^4f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)d^2} \\
& -\frac{b^4f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)d^2} \\
& b^4f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)
\end{aligned}
$$

output
```
-1/2*f*polylog(2,exp(2*d*x+2*c))/a/d^2+1/2*f*polylog(2,-exp(2*d*x+2*c))/a/
d^2+1/2*b^2*f*polylog(2,exp(2*d*x+2*c))/a^3/d^2-(f*x+e)*ln(tanh(d*x+c))/a/
d+b*f*arctanh(cosh(d*x+c))/a^2/d^2+b*(f*x+e)*csch(d*x+c)/a^2/d+I*b^3*f*pol
ylog(2,-I*exp(d*x+c))/a^2/(a^2+b^2)/d^2-b*f*x*arctan(sinh(d*x+c))/a^2/d+I*
b*f*polylog(2,I*exp(d*x+c))/a^2/d^2+b*(f*x+e)*arctan(sinh(d*x+c))/a^2/d+f*
x*ln(tanh(d*x+c))/a/d+b^4*(f*x+e)*ln(1+exp(2*d*x+2*c))/a^3/(a^2+b^2)/d-b^4
*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d-b^4*(f*x+e
)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d-b^4*f*polylog(2,-
b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d^2-b^4*f*polylog(2,-b*exp
(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d^2+1/2*b^4*f*polylog(2,-exp(2*
d*x+2*c))/a^3/(a^2+b^2)/d^2-I*b*f*polylog(2,-I*exp(d*x+c))/a^2/d^2+2*f*x*a
rctanh(exp(2*d*x+2*c))/a/d-1/2*f*coth(d*x+c)/a/d^2-1/2*(f*x+e)*coth(d*x+c)
^2/a/d+2*b*f*x*arctan(exp(d*x+c))/a^2/d-2*b^3*(f*x+e)*arctan(exp(d*x+c))/a
^2/(a^2+b^2)/d-I*b^3*f*polylog(2,I*exp(d*x+c))/a^2/(a^2+b^2)/d^2+1/2*f*x/a
/d-2*b^2*(f*x+e)*arctanh(exp(2*d*x+2*c))/a^3/d-1/2*b^2*f*polylog(2,-exp(2*
d*x+2*c))/a^3/d^2
```

3.493.2 Mathematica [A] (warning: unable to verify)

Time = 9.43 (sec) , antiderivative size = 1009, normalized size of antiderivative = 1.32

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\left(2bde\cosh\left(\frac{1}{2}(c+dx)\right)-af\cosh\left(\frac{1}{2}(c+dx)\right)-2bcf\cosh\left(\frac{1}{2}(c+dx)\right)+2bf(c+dx)\cosh\left(\frac{1}{2}(c+dx)\right)\right)\mathrm{c}}{4a^2d^2}$$
$$+\frac{(-de+cf-f(c+dx))\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{-\frac{(a^2-b^2)(de+dfx)^2}{2f}-(abf+a^2(de+dfx)-b^2(de+dfx))\log\left(1-e^{-c-dx}\right)+\left(abf-a^2(de+dfx)+b^2(d\right.}{a^3d^2}$$
$$-\frac{b^4\left(-2de(c+dx)+2cf(c+dx)-f(c+dx)^2+\frac{4a\sqrt{a^2+b^2}de\arctan\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-(a^2+b^2)^2}}-\frac{4a\sqrt{-(a^2+b^2)^2}de\operatorname{arctanh}\left(\frac{a+be}{\sqrt{a}}\right.}{(-a^2-b^2)^{3/2}}\right.}{}$$
$$+\frac{-ade(c+dx)+acf(c+dx)-\frac{1}{2}af(c+dx)^2+2bde\arctan\left(e^{c+dx}\right)-2bcf\arctan\left(e^{c+dx}\right)+ibf(c+dx}{}$$
$$+\frac{(de-cf+f(c+dx))\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+\frac{\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(-2bde\sinh\left(\frac{1}{2}(c+dx)\right)-af\sinh\left(\frac{1}{2}(c+dx)\right)+2bcf\sinh\left(\frac{1}{2}(c+dx)\right)-2bf(c+dx)\right.}{4a^2d^2}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),
x]
```

output
```
((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*
x)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2)
+ ((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) + (-1/2*((a
^2 - b^2)*(d*e + d*f*x)^2)/f - (a*b*f + a^2*(d*e + d*f*x) - b^2*(d*e + d*f
*x))*Log[1 - E^(-c - d*x)] + (a*b*f - a^2*(d*e + d*f*x) + b^2*(d*e + d*f*x
))*Log[1 + E^(-c - d*x)] + (a^2 - b^2)*f*PolyLog[2, -E^(-c - d*x)] + (a^2
- b^2)*f*PolyLog[2, E^(-c - d*x)])/(a^3*d^2) - (b^4*(-2*d*e*(c + d*x) + 2*
c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*e*ArcTan[(a + b*E^(
c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - (4*a*Sqrt[-(a^2 + b^2)
^2]*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(-a^2 - b^2)^(3/2) +
 2*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*f*(c + d
*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*c*f*Log[b - 2*a*E^(
c + d*x) - b*E^(2*(c + d*x))] + 2*d*e*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(
c + d*x)))] + 2*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f
*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(2*a^3*(a^2 + b^2)
*d^2) + (-(a*d*e*(c + d*x)) + a*c*f*(c + d*x) - (a*f*(c + d*x)^2)/2 + 2*b*
d*e*ArcTan[E^(c + d*x)] - 2*b*c*f*ArcTan[E^(c + d*x)] + I*b*f*(c + d*x)*Lo
g[1 - I*E^(c + d*x)] - I*b*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + a*d*e*Log[
1 + E^(2*(c + d*x))] - a*c*f*Log[1 + E^(2*(c + d*x))] + a*f*(c + d*x)*Log[
1 + E^(2*(c + d*x))] - I*b*f*PolyLog[2, (-I)*E^(c + d*x)] + I*b*f*PolyL...
```

3.493.3 Rubi [A] (verified)

Time = 4.09 (sec) , antiderivative size = 673, normalized size of antiderivative = 0.88, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6123, 5985, 2009, 6123, 5985, 2009, 6123, 5984, 3042, 26, 4670, 2715, 2838, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{ 6123}$$

$$\frac{\int (e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-f\int\left(-\frac{\coth^2(c+dx)}{2d}-\frac{\log(\tanh(c+dx))}{d}\right)dx-\frac{(e+fx)\coth^2(c+dx)}{2d}-\frac{(e+fx)\log(\tanh(c+dx))}{d}}{a}-\frac{b\int\frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\text{cot}}{2}}{a}$$
$$\frac{b\int\frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{6123}$$

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\text{cot}}{2}}{a}$$
$$\frac{b\left(\frac{\int(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\text{cot}}{2}}{a}$$
$$\frac{b\left(\frac{-f\int\left(-\frac{\arctan(\sinh(c+dx))}{d}-\frac{\text{csch}(c+dx)}{d}\right)dx-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}-\frac{(e+fx)\text{csch}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\text{cot}}{2}}{a}$$
$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyI}}{}\right.}{a}\right.}{a}$$

$$\Big\downarrow \text{6123}$$

3.493. $\int\frac{(e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{-f\left(-\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\cot}{2}}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{\int(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\right.}{}\right.}{a}$$

$$\downarrow 5984$$

$$\frac{-f\left(-\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\cot}{2}}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{2\int(e+fx)\operatorname{csch}(2c+2dx)dx}{a}-\frac{b\int\frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\operatorname{PolyLog}}{}\right.}{}\right.}{a}$$

$$\downarrow 3042$$

$$\frac{-f\left(-\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\cot}{2}}{a}$$

$$\frac{b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}-}{a}\right.}{a}$$

$$\downarrow 26$$

$$\frac{-f\left(-\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\cot}{2}}{a}$$

$$\frac{b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}-}{a}\right.}{a}$$

$$\downarrow 4670$$

3.493. $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{\frac{-f\left(-\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\cot\cdots}{2\cdots}}{a} \qquad b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}-\cdots}{a}\right.}{a}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{-f\left(-\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\cot\cdots}{2\cdots}}{a} \qquad b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}-\cdots}{a}\right.}{\cdots}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{-f\left(-\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\cot\cdots}{2\cdots}}{a} \qquad b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}-\cdots}{a}\right.}{a}$$

$$\Big\downarrow \text{6107}$$

3.493. $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\coth}{2d}}{a}$$

$$b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}-}{a}\right.$$

$$\Big\downarrow \text{6095}$$

$$\frac{-f\left(-\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\coth}{2d}}{a}$$

$$b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}-}{a}\right.$$

$$\Big\downarrow \text{2620}$$

3.493. $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\left(-\frac{2x\,\text{arctanh}(e^{2c+2dx})}{d}-\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}+\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,\text{cot}}{2}}{a}$$

$$b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}-}{a}\right.$$

$$\Big\downarrow \text{2715}$$

$$\frac{-f\left(-\frac{2x\,\text{arctanh}(e^{2c+2dx})}{d}-\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}+\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\,\text{cot}}{2}}{a}$$

$$b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}-}{a}\right.$$

3.493. $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{2838}$$

$$\frac{-f\left(-\frac{2x\text{arctanh}(e^{2c+2dx})}{d}-\frac{\text{PolyLog}(2,-e^{2c+2dx})}{2d^2}+\frac{\text{PolyLog}(2,e^{2c+2dx})}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\text{cot}}{2\ldots}}{a}$$

$$b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}-\ldots}{a}\right.$$

$$\Big\downarrow \text{7293}$$

3.493. $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\left(-\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\cot}{2}}{a}$$

$$b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}-}{a}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{-f\left(-\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}+\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\coth(c+dx)}{2d^2}-\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\cot}{2}}{a}$$

$$b\left(\frac{-f\left(\frac{2x\arctan\left(e^{c+dx}\right)}{d}-\frac{x\arctan(\sinh(c+dx))}{d}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{d^2}-\frac{i\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2}+\frac{i\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\arctan(\sinh(c+dx))}{d}-}{a}\right.$$

3.493. $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(-1/2*((e + f*x)*Coth[c + d*x]^2)/d - ((e + f*x)*Log[Tanh[c + d*x]])/d - f
*(-1/2*x/d - (2*x*ArcTanh[E^(2*c + 2*d*x)])/d + Coth[c + d*x]/(2*d^2) - (x
*Log[Tanh[c + d*x]])/d - PolyLog[2, -E^(2*c + 2*d*x)]/(2*d^2) + PolyLog[2,
 E^(2*c + 2*d*x)]/(2*d^2)))/a - (b*((-(((e + f*x)*ArcTan[Sinh[c + d*x]])/d
) - ((e + f*x)*Csch[c + d*x])/d - f*((2*x*ArcTan[E^(c + d*x)])/d - (x*ArcT
an[Sinh[c + d*x]])/d + ArcTanh[Cosh[c + d*x]]/d^2 - (I*PolyLog[2, (-I)*E^(
c + d*x)])/d^2 + (I*PolyLog[2, I*E^(c + d*x)])/d^2))/a - (b*(-((b*((b^2*(-
1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2)
 + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2
 + b^2) + ((b*(e + f*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d -
 (b*(e + f*x)*Log[1 + E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*E^(c +
d*x)])/d^2 + (I*a*f*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2
*(c + d*x))])/(2*d^2))/(a^2 + b^2)))/a) + ((2*I)*((I*(e + f*x)*ArcTanh[E^(
2*c + 2*d*x)])/d + ((I/4)*f*PolyLog[2, -E^(2*c + 2*d*x)])/d^2 - ((I/4)*f*P
olyLog[2, E^(2*c + 2*d*x)])/d^2))/a))/a))/a
```

3.493.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.493. $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

3.493. $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.493.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1477 vs. 2(714) = 1428.

Time = 15.11 (sec) , antiderivative size = 1478, normalized size of antiderivative = 1.94

method	result	size
risch	Expression too large to display	1478

input
```
int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVE
RBOSE)
```

output
```
-(-2*b*d*f*x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2*c)-2*b*d*e*exp(3*d*x+3*c
)+2*a*d*e*exp(2*d*x+2*c)+2*b*d*f*x*exp(d*x+c)+a*f*exp(2*d*x+2*c)+2*b*d*e*e
xp(d*x+c)-a*f)/a^2/d^2/(exp(2*d*x+2*c)-1)^2-1/d*f/a*ln(exp(d*x+c)+1)*x+1/d
^2*c*f/a*ln(exp(d*x+c)-1)-1/d/a^3*f*b^4/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^
2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*b^2/a^3*f*dilog(exp(d*x+c))+1/d^2
*b^2/a^3*f*dilog(exp(d*x+c)+1)+1/d*b^2/a^3*e*ln(exp(d*x+c)-1)+1/d*b^2/a^3*
e*ln(exp(d*x+c)+1)-1/d^2*b/a^2*f*ln(exp(d*x+c)-1)+1/d^2*b/a^2*f*ln(exp(d*x
+c)+1)+1/d^2*f/a*dilog(exp(d*x+c))-1/d*e/a*ln(exp(d*x+c)-1)-1/d*e/a*ln(exp
(d*x+c)+1)-1/d^2*f/a*dilog(exp(d*x+c)+1)-1/d^2/a^3*f*b^4/(a^2+b^2)*ln((-b*
exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2/a^3*f*b^4/(a^2
+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+4*I/d*f/(
4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*x-4*I/d*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c
))*b*x-4*I/d^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*c+4*I/d^2*f/(4*a^2+4*b
^2)*ln(1-I*exp(d*x+c))*b*c-1/d^2/a^3*f*b^4/(a^2+b^2)*dilog((b*exp(d*x+c)+(
a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2/a^3*f*b^4/(a^2+b^2)*dilog((-b
*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d/a^3*b^4*e/(a^2+b^
2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-4/d^2*a*c*f/(4*a^2+4*b^2)*ln(1+ex
p(2*d*x+2*c))+4/d^2*a*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c+4/d^2*a*f/(4*a^
2+4*b^2)*ln(1-I*exp(d*x+c))*c+4/d*a*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x+4
/d*a*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x-1/d^2/a^2*b^2*f/(a^2+b^2)^(1/...
```

3.493.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5731 vs. $2(695) = 1390$.

Time = 0.42 (sec) , antiderivative size = 5731, normalized size of antiderivative = 7.52

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
Too large to include
```

3.493.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\mathrm{csch}^3(c+dx)\mathrm{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.493.7 Maxima [F]

$$\int \frac{(e+fx)\mathrm{csch}^3(c+dx)\mathrm{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\mathrm{csch}\,(dx+c)^3\,\mathrm{sech}\,(dx+c)}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + a^3*b^2)*d)
+ 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/(
(a^2 + b^2)*d) + 2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*
c))/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 - b^2
)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d
))*e + (16*a^2*d*integrate(1/16*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 16*b^2
*d*integrate(1/16*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 16*a^2*d*integrate(1
/16*x/(a^3*d*e^(d*x + c) - a^3*d), x) + 16*b^2*d*integrate(1/16*x/(a^3*d*e
^(d*x + c) - a^3*d), x) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) + 1)/
(a^3*d^2)) + a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^3*d^2)) +
(2*b*d*x*e^(3*d*x + 3*c) - 2*b*d*x*e^(d*x + c) - (2*a*d*x*e^(2*c) + a*e^(2
*c))*e^(2*d*x) + a)/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) +
 a^2*d^2) + 16*integrate(-1/8*(a*b^4*x*e^(d*x + c) - b^5*x)/(a^5*b + a^3*b
^3 - (a^5*b*e^(2*c) + a^3*b^3*e^(2*c))*e^(2*d*x) - 2*(a^6*e^c + a^4*b^2*e^
c)*e^(d*x)), x) + 16*integrate(1/8*(b*x*e^(d*x + c) - a*x)/(a^2 + b^2 + (a
^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x))*f
```

3.493.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.493.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \int \frac{e+f\,x}{\cosh\left(c+d\,x\right)\,{\sinh\left(c+d\,x\right)}^3\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.494 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.494.1 Optimal result

Integrand size = 27, antiderivative size = 130

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b\arctan(\sinh(c+dx))}{(a^2+b^2)\,d} + \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} + \frac{a\log(\cosh(c+dx))}{(a^2+b^2)\,d} - \frac{(a^2-b^2)\log(\sinh(c+dx))}{a^3d} - \frac{b^4\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)\,d}$$

output
```
b*arctan(sinh(d*x+c))/(a^2+b^2)/d+b*csch(d*x+c)/a^2/d-1/2*csch(d*x+c)^2/a/
d+a*ln(cosh(d*x+c))/(a^2+b^2)/d-(a^2-b^2)*ln(sinh(d*x+c))/a^3/d-b^4*ln(a+b
*sinh(d*x+c))/a^3/(a^2+b^2)/d
```

3.494.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\frac{2b\operatorname{csch}(c+dx)}{a^2} - \frac{\operatorname{csch}^2(c+dx)}{a} - \frac{2(a-b)(a+b)\log(\sinh(c+dx))}{a^3} + \frac{\left(a-\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\sinh(c+dx)\right)}{a^2+b^2} - \frac{2b^4\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)} +}{2d}$$

input
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
((2*b*Csch[c + d*x])/a^2 - Csch[c + d*x]^2/a - (2*(a - b)*(a + b)*Log[Sinh
[c + d*x]])/a^3 + ((a - Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[c + d*x]])/(a^
2 + b^2) - (2*b^4*Log[a + b*Sinh[c + d*x]])/(a^3*(a^2 + b^2)) + ((a + Sqrt
[-b^2])*Log[Sqrt[-b^2] + b*Sinh[c + d*x]])/(a^2 + b^2))/(2*d)
```

3.494.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 26, 3316, 26, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\sin(ic+idx)^3\cos(ic+idx)(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\cos(ic+idx)\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{ib\int -\frac{i\operatorname{csch}^3(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{b\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^4\int \frac{\operatorname{csch}^3(c+dx)}{b^3(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{615}$$

$$\frac{b^4 \int \left(\frac{\operatorname{csch}^3(c+dx)}{ab^5} - \frac{\operatorname{csch}^2(c+dx)}{a^2b^4} + \frac{(b^2-a^2)\operatorname{csch}(c+dx)}{a^3b^5} - \frac{1}{a^3(a^2+b^2)(a+b\sinh(c+dx))} + \frac{b^2+a\sinh(c+dx)b}{b^4(a^2+b^2)(\sinh^2(c+dx)b^2+b^2)}\right) d(b\sinh}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^4\left(\frac{\arctan(\sinh(c+dx))}{b^3(a^2+b^2)} + \frac{\operatorname{csch}(c+dx)}{a^2b^3} + \frac{a\log(b^2\sinh^2(c+dx)+b^2)}{2b^4(a^2+b^2)} - \frac{\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)} - \frac{(a^2-b^2)\log(b\sinh(c+dx))}{a^3b^4} - \frac{\operatorname{csch}^2(c+dx}{2ab^4}\right.}{d}$$

input
```
Int[(Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^4*(ArcTan[Sinh[c + d*x]]/(b^3*(a^2 + b^2)) + Csch[c + d*x]/(a^2*b^3) -
Csch[c + d*x]^2/(2*a*b^4) - ((a^2 - b^2)*Log[b*Sinh[c + d*x]])/(a^3*b^4) -
 Log[a + b*Sinh[c + d*x]]/(a^3*(a^2 + b^2)) + (a*Log[b^2 + b^2*Sinh[c + d*
x]^2])/(2*b^4*(a^2 + b^2))))/d
```

3.494.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

3.494.4 Maple [A] (verified)

Time = 7.07 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{b^4\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^3\left(a^2+b^2\right)}+\frac{4a\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+8b\arctan\left(\tanh\left(\cdots\right.\right.}{4a^2+4b^2}}{d}$
default	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{b^4\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^3\left(a^2+b^2\right)}+\frac{4a\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+8b\arctan\left(\tanh\left(\cdots\right.\right.}{4a^2+4b^2}}{d}$
risch	$-\frac{2a\,d^2x}{a^2d^2+b^2d^2}-\frac{2adc}{a^2d^2+b^2d^2}+\frac{2x}{a}+\frac{2c}{da}-\frac{2b^2x}{a^3}-\frac{2b^2c}{d\,a^3}+\frac{2b^4x}{a^3(a^2+b^2)}+\frac{2b^4c}{d\,a^3(a^2+b^2)}-\frac{2\,e^{dx+c}\left(-b\,e^{2dx+2c}\cdots\right.}{a^2d\left(e^{2dx+2c}\cdots\right.}$

input
```
int(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4/a^2*(1/2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c))-b^4/a^
3/(a^2+b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a)+1/4/(a^2
+b^2)*(4*a*ln(1+tanh(1/2*d*x+1/2*c)^2)+8*b*arctan(tanh(1/2*d*x+1/2*c)))-1/
8/a/tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(-4*a^2+4*b^2)*ln(tanh(1/2*d*x+1/2*c))+1
/2*b/a^2/tanh(1/2*d*x+1/2*c))
```

3.494.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(128) = 256$.

Time = 0.35 (sec) , antiderivative size = 1035, normalized size of antiderivative = 7.96

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas
")
```

output
```
(2*(a^3*b + a*b^3)*cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*sinh(d*x + c)^3 - 2
*(a^4 + a^2*b^2)*cosh(d*x + c)^2 - 2*(a^4 + a^2*b^2 - 3*(a^3*b + a*b^3)*co
sh(d*x + c))*sinh(d*x + c)^2 + 2*(a^3*b*cosh(d*x + c)^4 + 4*a^3*b*cosh(d*x
 + c)*sinh(d*x + c)^3 + a^3*b*sinh(d*x + c)^4 - 2*a^3*b*cosh(d*x + c)^2 +
a^3*b + 2*(3*a^3*b*cosh(d*x + c)^2 - a^3*b)*sinh(d*x + c)^2 + 4*(a^3*b*cos
h(d*x + c)^3 - a^3*b*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) +
sinh(d*x + c)) - 2*(a^3*b + a*b^3)*cosh(d*x + c) - (b^4*cosh(d*x + c)^4 +
4*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*sinh(d*x + c)^4 - 2*b^4*cosh(d*x
 + c)^2 + b^4 + 2*(3*b^4*cosh(d*x + c)^2 - b^4)*sinh(d*x + c)^2 + 4*(b^4*c
osh(d*x + c)^3 - b^4*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c)
+ a)/(cosh(d*x + c) - sinh(d*x + c))) + (a^4*cosh(d*x + c)^4 + 4*a^4*cosh(
d*x + c)*sinh(d*x + c)^3 + a^4*sinh(d*x + c)^4 - 2*a^4*cosh(d*x + c)^2 + a
^4 + 2*(3*a^4*cosh(d*x + c)^2 - a^4)*sinh(d*x + c)^2 + 4*(a^4*cosh(d*x + c
)^3 - a^4*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c)
 - sinh(d*x + c))) - ((a^4 - b^4)*cosh(d*x + c)^4 + 4*(a^4 - b^4)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (a^4 - b^4)*sinh(d*x + c)^4 + a^4 - b^4 - 2*(a^4 -
 b^4)*cosh(d*x + c)^2 - 2*(a^4 - b^4 - 3*(a^4 - b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 4*((a^4 - b^4)*cosh(d*x + c)^3 - (a^4 - b^4)*cosh(d*x + c))*
sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - 2*(a
^3*b + a*b^3 - 3*(a^3*b + a*b^3)*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*co...
```

3.494.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.494.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.82

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b^4\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^5+a^3b^2)d} - \frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d} + \frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d} - \frac{2\left(be^{(-dx-c)}-ae^{(-2\,dx-2\,c)}-be^{(-3\,dx-3\,c)}\right)}{(2\,a^2e^{(-2\,dx-2\,c)}-a^2e^{(-4\,dx-4\,c)}-a^2)d} - \frac{(a^2-b^2)\log\left(e^{(-dx-c)}+1\right)}{a^3d} - \frac{(a^2-b^2)\log\left(e^{(-dx-c)}-1\right)}{a^3d}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima
")
```

output
```
-b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + a^3*b^2)*d) -
 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)/((
a^2 + b^2)*d) - 2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c
))/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 - b^2)
*log(e^(-d*x - c) + 1)/(a^3*d) - (a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d)
```

3.494.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(128) = 256$.

Time = 0.29 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.02

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2\,b^5\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^5b+a^3b^3} - \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)b}{a^2+b^2} - \frac{a\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2} + \frac{2\,(a^2-b^2)\log}{}}{2\,d}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*b^5*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^5*b + a^3*b^
3) - (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*b/(a^2 + b^2)
 - a*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2) + 2*(a^2 - b^2)*l
og(abs(e^(d*x + c) - e^(-d*x - c)))/a^3 - (3*a^2*(e^(d*x + c) - e^(-d*x -
c))^2 - 3*b^2*(e^(d*x + c) - e^(-d*x - c))^2 + 4*a*b*(e^(d*x + c) - e^(-d*
x - c)) - 4*a^2)/(a^3*(e^(d*x + c) - e^(-d*x - c))^2))/d
```

3.494.9 Mupad [B] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.51

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\ln\left(e^{c+d\,x}+1\mathrm{i}\right)}{a\,d-b\,d\,1\mathrm{i}} - \frac{\frac{2}{a\,d}-\frac{2\,b\,e^{c+d\,x}}{a^2\,d}}{e^{2\,c+2\,d\,x}-1} - \frac{2}{a\,d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)} - \frac{b^4\,\ln\left(2\,a\,e^{c+d\,x}-b+b\,e^{2\,c+2\,d\,x}\right)}{d\,a^5+d\,a^3\,b^2} - \frac{\ln\left(e^{2\,c+2\,d\,x}-1\right)\,\left(a^2-b^2\right)}{a^3\,d} + \frac{\ln\left(1+e^{c+d\,x}\,1\mathrm{i}\right)\,1\mathrm{i}}{-b\,d+a\,d\,1\mathrm{i}}$$

input
```
int(1/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
log(exp(c + d*x) + 1i)/(a*d - b*d*1i) - (2/(a*d) - (2*b*exp(c + d*x))/(a^2
*d))/(exp(2*c + 2*d*x) - 1) + (log(exp(c + d*x)*1i + 1)*1i)/(a*d*1i - b*d)
 - 2/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (b^4*log(2*a*exp(
c + d*x) - b + b*exp(2*c + 2*d*x)))/(a^5*d + a^3*b^2*d) - (log(exp(2*c + 2
*d*x) - 1)*(a^2 - b^2))/(a^3*d)
```

3.495 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.495.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.495.2 Mathematica [N/A]

Not integrable

Time = 112.92 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
),x]
```

output
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

3.495.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]^3*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.495.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.495.4 Maple [N/A] (verified)

Not integrable

Time = 0.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(dx+c\right)^{3}\operatorname{sech}\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.495.5 Fricas [N/A]

Not integrable

Time = 37.81 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^3\operatorname{sech}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="fricas")
```

output
```
integral(csch(d*x + c)^3*sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d
*x + c)), x)
```

3.495.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.495.7 Maxima [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 912, normalized size of antiderivative = 26.82

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(dx+c)^3\operatorname{sech}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="maxima")
```

output
```
-(a*f - 2*(b*d*f*x*e^(3*c) + b*d*e*e^(3*c))*e^(3*d*x) + (2*a*d*f*x*e^(2*c)
 + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) + 2*(b*d*f*x*e^c + b*d*e*e^c)*e^(d*x))
/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*d^2*f^2*x^2*e^(4*
c) + 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e^(4*d*x) - 2*(a^2*d^2
*f^2*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e^2*e^(2*c))*e^(2*d*x
)) - 16*integrate(1/16*(b^2*d^2*e^2 + a*b*d*e*f - (d^2*e^2 - f^2)*a^2 - (a
^2*d^2*f^2 - b^2*d^2*f^2)*x^2 - (2*a^2*d^2*e*f - 2*b^2*d^2*e*f - a*b*d*f^2
)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*
e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e
^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + 16*integrate(-1/16*(b^2*d^2*e^2 - a*b
*d*e*f - (d^2*e^2 - f^2)*a^2 - (a^2*d^2*f^2 - b^2*d^2*f^2)*x^2 - (2*a^2*d^
2*e*f - 2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x
^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*
f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + 16*i
ntegrate(-1/8*(a*b^4*e^(d*x + c) - b^5)/(a^5*b*e + a^3*b^3*e + (a^5*b*f +
a^3*b^3*f)*x - (a^5*b*e*e^(2*c) + a^3*b^3*e*e^(2*c) + (a^5*b*f*e^(2*c) + a
^3*b^3*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^6*e*e^c + a^4*b^2*e*e^c + (a^6*f*e^c
 + a^4*b^2*f*e^c)*x)*e^(d*x)), x) + 16*integrate(1/8*(b*e^(d*x + c) - a)/(
a^2*e + b^2*e + (a^2*f + b^2*f)*x + (a^2*e*e^(2*c) + b^2*e*e^(2*c) + (a^2*
f*e^(2*c) + b^2*f*e^(2*c))*x)*e^(2*d*x)), x)
```

3.495.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
="giac")
```

output
```
Timed out
```

3.495.9 Mupad [N/A]

Not integrable

Time = 11.51 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)\,{\sinh\left(c+d\,x\right)}^3\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)} \,d x$$

input
```
int(1/(cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.496 $\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.496.1 Optimal result

Integrand size = 36, antiderivative size = 1245

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

output
```
-2*b^2*f*(f*x+e)*polylog(2,-exp(d*x+c))/a^3/d^2+2*b^2*f*(f*x+e)*polylog(2,
exp(d*x+c))/a^3/d^2+3*(f*x+e)^2*arctanh(exp(d*x+c))/a/d-3*f^2*polylog(3,-e
xp(d*x+c))/a/d^3+3*f^2*polylog(3,exp(d*x+c))/a/d^3-f^2*arctanh(cosh(d*x+c)
)/a/d^3-2*b*f*(f*x+e)*ln(1-exp(4*d*x+4*c))/a^2/d^2+2*b^5*f^2*polylog(3,-b*
exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(3/2)/d^3+2*I*f^2*polylog(2,
I*exp(d*x+c))/a/d^3-2*I*b^2*f^2*polylog(2,I*exp(d*x+c))/a^3/d^3-2*b^5*f^2*
polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(3/2)/d^3+2*I*b
^4*f^2*polylog(2,I*exp(d*x+c))/a^3/(a^2+b^2)/d^3+2*I*b^2*f^2*polylog(2,-I*
exp(d*x+c))/a^3/d^3-3/2*(f*x+e)^2*sech(d*x+c)/a/d-4*b^2*f*(f*x+e)*arctan(e
xp(d*x+c))/a^3/d^2-b^5*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^
3/(a^2+b^2)^(3/2)/d+b^5*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a
^3/(a^2+b^2)^(3/2)/d+2*b*(f*x+e)^2/a^2/d-2*b^2*(f*x+e)^2*arctanh(exp(d*x+c
))/a^3/d+2*b^2*f^2*polylog(3,-exp(d*x+c))/a^3/d^3-2*b^2*f^2*polylog(3,exp(
d*x+c))/a^3/d^3+4*b^4*f*(f*x+e)*arctan(exp(d*x+c))/a^3/(a^2+b^2)/d^2+2*b^3
*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/a^2/(a^2+b^2)/d^2-2*b^5*f*(f*x+e)*polylog(
2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(3/2)/d^2+2*b^5*f*(f*x+
e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(3/2)/d^2-2*
I*b^4*f^2*polylog(2,-I*exp(d*x+c))/a^3/(a^2+b^2)/d^3+b^3*f^2*polylog(2,-ex
p(2*d*x+2*c))/a^2/(a^2+b^2)/d^3-b^4*(f*x+e)^2*sech(d*x+c)/a^3/(a^2+b^2)/d-
b^3*(f*x+e)^2*tanh(d*x+c)/a^2/(a^2+b^2)/d-b^3*(f*x+e)^2/a^2/(a^2+b^2)/d...
```

3.496.2 Mathematica [A] (warning: unable to verify)

Time = 9.79 (sec) , antiderivative size = 2346, normalized size of antiderivative = 1.88

$$\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^2*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*
x]),x]
```

output
```
(f*(4*b*d^2*e*E^(2*c)*x - 4*b*d^2*e*(1 + E^(2*c))*x + 2*b*d^2*E^(2*c)*f*x^
2 - 2*b*d^2*(1 + E^(2*c))*f*x^2 + 4*a*d*e*(1 + E^(2*c))*ArcTan[E^(c + d*x)
] + 2*b*d*e*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (2*I)*a*(1
+ E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - Poly
Log[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + b*(1 + E^(2*c))*f*
(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]))))/
((a^2 + b^2)*d^3*(1 + E^(2*c))) + (8*a*b*d^2*e*E^(2*c)*f*x + 4*a*b*d^2*E^(
2*c)*f^2*x^2 - 6*a^2*d^2*e^2*ArcTanh[E^(c + d*x)] + 4*b^2*d^2*e^2*ArcTanh[
E^(c + d*x)] + 6*a^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] - 4*b^2*d^2*e^2*
E^(2*c)*ArcTanh[E^(c + d*x)] + 4*a^2*f^2*ArcTanh[E^(c + d*x)] - 4*a^2*E^(2
*c)*f^2*ArcTanh[E^(c + d*x)] + 6*a^2*d^2*e*f*x*Log[1 - E^(c + d*x)] - 4*b^
2*d^2*e*f*x*Log[1 - E^(c + d*x)] - 6*a^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c +
d*x)] + 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 3*a^2*d^2*f^2*x^2*L
og[1 - E^(c + d*x)] - 2*b^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)] - 3*a^2*d^2*E
^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 2*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^
(c + d*x)] - 6*a^2*d^2*e*f*x*Log[1 + E^(c + d*x)] + 4*b^2*d^2*e*f*x*Log[1
+ E^(c + d*x)] + 6*a^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - 4*b^2*d^2*
e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - 3*a^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)
] + 2*b^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)] + 3*a^2*d^2*E^(2*c)*f^2*x^2*Log
[1 + E^(c + d*x)] - 2*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] + 4*...
```

3.496.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-2f\int \frac{1}{2}(e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}}{a}$$

$$\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{-f\int (e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$\frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{6123}$$

$$\frac{-f\int (e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$\frac{b\left(\frac{\int (e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{5984}$$

$$\frac{-f\int (e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$\frac{b\left(\frac{4\int (e+fx)^2\text{csch}^2(2c+2dx)dx}{a}-\frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

3.496. $\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\int (e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-3}{a} \quad \frac{b\left(-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{4\int -(e+fx)^2\csc(2ic+2idx)^2dx}{a}\right)}{a}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{-f\int (e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-3}{a} \quad \frac{b\left(-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\int (e+fx)^2\csc(2ic+2idx)^2dx}{a}\right)}{a}$$

$$\Big\downarrow \color{blue}{4672}$$

$$\frac{-f\int (e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-3}{a} \quad \frac{b\left(-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}-\frac{if\int -i(e+fx)\coth(2c+2dx)dx}{d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \color{blue}{26}$$

$$\frac{-f\int (e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-3}{a} \quad \frac{b\left(-\frac{b\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}-\frac{f\int (e+fx)\coth(2c+2dx)dx}{d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \color{blue}{3042}$$

3.496. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-3}{a} \\ \frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}-\frac{f\int-i(e+fx)\tan\left(2ic+2idx+\frac{\pi}{2}\right)dx}{d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-3}{a} \\ \frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\int(e+fx)\tan\left(\frac{1}{2}(4ic+\pi)+2idx\right)dx}{d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 4201$$

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-3}{a} \\ \frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\int\frac{e^{4c+4dx-i\pi}(e+fx)}{1+e^{4c+4dx-i\pi}}dx-\frac{i(e+fx)^2}{2f}\right)}{d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 2620$$

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-3}{a} \\ \frac{b\left(-\frac{b\int\frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}-\frac{f\int\log\left(1+e^{4c+4dx-i\pi}\right)dx}{4d}\right)-\frac{i(e+fx}{2f}\right.}{d}\right.}{a}\right.}{a}$$

$$\Big\downarrow \; 2715$$

3.496. $\int\frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\int(e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}-\frac{f\int e^{-4c-4dx+i\pi}\log\left(1+e^{4c+4dx-i\pi}\right.}{16d^2}\right.\right.}{d}\right.}{a}\right.}{a}$$

$\downarrow$ 2838

$$\frac{-f\int(e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+fx)\log\left(1+e^{4c+4dx-i\pi}\right)}{4d}\right)-\frac{i(e+f}{2f}\right.}{d}\right.}{a}\right.}{a}$$

$\downarrow$ 6123

$$\frac{-f\int(e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{\int(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{4\left(\frac{(e+fx)^2\coth(2c+2dx)}{2d}+\frac{if\left(2i\left(\frac{f\,\text{PolyLog}\left(2,-e^{4c+4dx-i\pi}\right)}{16d^2}+\frac{(e+}{}\right.\right.}{d}\right.}{a}\right.}{a}$$

$\downarrow$ 5985

3.496. $\int\frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx$

$$\frac{-f\int (e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{-2f\int -\left((e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}^2(c+d}{a+b\sinh(c+dx)}}{a}\right.}{a}\right.}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{-f\int (e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}\right.}{a}$$

$$\Big\downarrow \; 6107$$

$$\frac{-f\int (e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$\frac{b\left(-\frac{b\left(\frac{2f\int (e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{b^2\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}+\int (\right.}{}\right.}{a}\right.}{}$$

$$\Big\downarrow \; 3042$$

3.496. $\displaystyle\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-}{a^2+b^2}\right.}{}\right.}{a}\right.$$

$$\Big\downarrow 3803$$

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{2b^2\int-\frac{e^{c+dx}(e+fx)^2}{-2e^{c+dx}a-be^{2(c+d}}}{a^2+b^2}\right.}{}\right.}{a}\right.$$

$$\Big\downarrow 25$$

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-}{a^2+b^2}\right.}{}\right.}{a}\right.$$

3.496. $\displaystyle\int\frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow \text{2694}$$

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-}{a^2+b^2}\right.}{}\right.}{a}\right.$$

$$\Big\downarrow \text{27}$$

3.496. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\int(e+fx)\left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{d}+\frac{3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3\text{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\text{arctanh}(\cosh(c+dx))}{d}-\frac{\text{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-}{a^2+b^2}\right.}{}\right.}{a}\right.$$

$$\downarrow 2620$$

3.496. $\int\frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-}{a^2+b^2}\right.}{}\right.}{}\right.$$

$$\downarrow 3011$$

3.496. $\int\frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{\left(b\,\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-}{a^2+b^2}\right.}{}\right.}{}\right.$$

3.496. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\downarrow 2720$$

3.496. $\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-}{a^2+b^2}\right.}{}\right.}{}\right.$$

3.496. $\int\frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\downarrow 7143$$

3.496. $\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx) \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\int(e+fx)\left(-\frac{\operatorname{sech}(c+dx)\operatorname{csch}^2(c+dx)}{d}+\frac{3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3\operatorname{sech}(c+dx)}{d}\right)dx+\frac{3(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-3}{a}$$

$$b\left(-\frac{b\left(\frac{2f\int(e+fx)\left(\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{\operatorname{sech}(c+dx)}{d}\right)dx-\frac{(e+fx)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\operatorname{sech}^2(c+dx)(a-}{a^2+b^2}\right.}{}\right.}{}\right.$$

3.496. $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\downarrow 7292$$

3.496. $\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\int \frac{(e+fx)\left(-\text{sech}(c+dx)\text{csch}^2(c+dx)+3\text{arctanh}(\cosh(c+dx))-3\text{sech}(c+dx)\right)}{d}dx+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3(e+fx)^2\text{se}}{2a}}{a}$$

$$b\left(\frac{2f\int\frac{(e+fx)(\text{arctanh}(\cosh(c+dx))-\text{sech}(c+dx))}{d}dx-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b}{a^2+b^2}\right.}{}\right.$$

$$b\left(-\frac{}{}\right.$$

3.496. $\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\Big\downarrow 27$$

3.496. $\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{f\int(e+fx)\left(-\text{sech}(c+dx)\text{csch}^2(c+dx)+3\text{arctanh}(\cosh(c+dx))-3\text{sech}(c+dx)\right)dx}{d}+\frac{3(e+fx)^2\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3(e+fx)^2\text{sech}}{2d}$$

$$a$$

$$b\left(-\frac{b\left(\frac{\frac{2f\int(e+fx)(\text{arctanh}(\cosh(c+dx))-\text{sech}(c+dx))dx}{d}-\frac{(e+fx)^2\text{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)^2\text{sech}(c+dx)}{d}}{a}-\frac{b\left(\frac{\int(e+fx)^2\text{sech}^2(c+dx)(a-b\,\text{s}}{a^2+b^2}\right.}{}\right.}{}\right.$$

3.496. $\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

input
```
Int[((e + f*x)^2*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
$Aborted
```

3.496.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.496. $\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx) \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

3.496. $\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx) \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.496.4 Maple [F]

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^3 \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}dx$$

input
```
int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

output
```
int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

3.496.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 29722 vs. 2(1152) = 2304.

Time = 0.78 (sec) , antiderivative size = 29722, normalized size of antiderivative = 23.87

$$\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="fricas")
```

output
```
Too large to include
```

3.496.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*csch(d*x+c)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.496.7 Maxima [F]

$$\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^3 \operatorname{sech}(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="maxima")
```

output
```
2*b*e*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 +
b^2)*d^2)) + 4*a*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*
d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 4*b*f^2*integrate(x/(a^2*d*e^(2*d
*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 1/2*(2*b^5*log((b
*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)
))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)*d) + 2*(4*a^2*b*e^(-2*d*x - 2*c) + 2*b
^3*e^(-4*d*x - 4*c) - 4*a^2*b - 2*b^3 + (3*a^3 + a*b^2)*e^(-d*x - c) - 2*(
a^3 - a*b^2)*e^(-3*d*x - 3*c) + (3*a^3 + a*b^2)*e^(-5*d*x - 5*c))/((a^4 +
a^2*b^2 - (a^4 + a^2*b^2)*e^(-2*d*x - 2*c) - (a^4 + a^2*b^2)*e^(-4*d*x - 4
*c) + (a^4 + a^2*b^2)*e^(-6*d*x - 6*c))*d) - (3*a^2 - 2*b^2)*log(e^(-d*x -
 c) + 1)/(a^3*d) + (3*a^2 - 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e^2 + 4*
a*e*f*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) - (2*(2*a^2*b*d*f^2 + b^3*d*f^
2)*x^2 + 4*(2*a^2*b*d*e*f + b^3*d*e*f)*x + (2*a^3*e*f*e^(5*c) + 2*a*b^2*e*
f*e^(5*c) + (3*a^3*d*f^2*e^(5*c) + a*b^2*d*f^2*e^(5*c))*x^2 + 2*((3*d*e*f
+ f^2)*a^3*e^(5*c) + (d*e*f + f^2)*a*b^2*e^(5*c))*x)*e^(5*d*x) - 2*(b^3*d*
f^2*x^2*e^(4*c) + 2*b^3*d*e*f*x*e^(4*c))*e^(4*d*x) - 2*((a^3*d*f^2*e^(3*c)
 - a*b^2*d*f^2*e^(3*c))*x^2 + 2*(a^3*d*e*f*e^(3*c) - a*b^2*d*e*f*e^(3*c))*
x)*e^(3*d*x) - 4*(a^2*b*d*f^2*x^2*e^(2*c) + 2*a^2*b*d*e*f*x*e^(2*c))*e^(2*
d*x) - (2*a^3*e*f*e^c + 2*a*b^2*e*f*e^c - (3*a^3*d*f^2*e^c + a*b^2*d*f^2*e
^c)*x^2 - 2*((3*d*e*f - f^2)*a^3*e^c + (d*e*f - f^2)*a*b^2*e^c)*x)*e^(d...
```

3.496.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algor
ithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument
Value
```

3.496.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)^2\sinh\left(c+d\,x\right)^3\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x
)
```

3.497 $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.497.1 Optimal result

Integrand size = 34, antiderivative size = 699

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{f\arctan(\sinh(c+dx))}{ad^2} - \frac{b^2f\arctan(\sinh(c+dx))}{a^3d^2} + \frac{b^4f\arctan(\sinh(c+dx))}{a^3(a^2+b^2)d^2} + \frac{3fx\text{arctanh}(e^{c+dx})}{ad} - \frac{2b^2fx\text{arctanh}(e^{c+dx})}{a^3d} - \frac{3fx\text{arctanh}(\cosh(c+dx))}{2ad} + \frac{b^2fx\text{arctanh}(\cosh(c+dx))}{a^3d} + \frac{3(e+fx)\text{arctanh}(\cosh(c+dx))}{2ad} - \frac{b^2(e+fx)\text{arctanh}(\cosh(c+dx))}{a^3d} + \frac{2b(e+fx)\coth(2c+2dx)}{a^2d} - \frac{f\text{csch}(c+dx)}{2ad^2} - \frac{b^5(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}d} + \frac{b^5(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}d} + \frac{b^3f\log(\cosh(c+dx))}{a^2(a^2+b^2)d^2} - \frac{bf\log(\sinh(2c+2dx))}{a^2d^2} + \frac{3f\,\text{PolyLog}(2,-e^{c+dx})}{2ad^2} - \frac{b^2f\,\text{PolyLog}(2,-e^{c+dx})}{a^3d^2} - \frac{3f\,\text{PolyLog}(2,e^{c+dx})}{2ad^2} + \frac{b^2f\,\text{PolyLog}(2,e^{c+dx})}{a^3d^2} - \frac{b^5f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}d^2} + \frac{b^5f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}d^2} - \frac{3(e+fx)\text{sech}(c+dx)}{2ad}$$

$$\frac{b^2(e+fx)\text{sech}(c+dx)}{}$$

output
```
2*b*(f*x+e)*coth(2*d*x+2*c)/a^2/d-1/2*(f*x+e)*csch(d*x+c)^2*sech(d*x+c)/a/
d+3/2*f*polylog(2,-exp(d*x+c))/a/d^2-3/2*f*polylog(2,exp(d*x+c))/a/d^2+f*a
rctan(sinh(d*x+c))/a/d^2-3/2*f*x*arctanh(cosh(d*x+c))/a/d+3/2*(f*x+e)*arct
anh(cosh(d*x+c))/a/d-3/2*(f*x+e)*sech(d*x+c)/a/d-2*b^2*f*x*arctanh(exp(d*x
+c))/a^3/d+b^4*f*arctan(sinh(d*x+c))/a^3/(a^2+b^2)/d^2+b^2*f*x*arctanh(cos
h(d*x+c))/a^3/d+b^3*f*ln(cosh(d*x+c))/a^2/(a^2+b^2)/d^2-b^5*(f*x+e)*ln(1+b
*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(3/2)/d+b^5*(f*x+e)*ln(1+b*
exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(3/2)/d-b^5*f*polylog(2,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(3/2)/d^2+b^5*f*polylog(2,-b*
exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(3/2)/d^2-b^4*(f*x+e)*sech(d
*x+c)/a^3/(a^2+b^2)/d-b^3*(f*x+e)*tanh(d*x+c)/a^2/(a^2+b^2)/d-b^2*f*polylo
g(2,-exp(d*x+c))/a^3/d^2+b^2*f*polylog(2,exp(d*x+c))/a^3/d^2-b^2*f*arctan(
sinh(d*x+c))/a^3/d^2-b^2*(f*x+e)*arctanh(cosh(d*x+c))/a^3/d-b*f*ln(sinh(2*
d*x+2*c))/a^2/d^2+b^2*(f*x+e)*sech(d*x+c)/a^3/d+3*f*x*arctanh(exp(d*x+c))/
a/d-1/2*f*csch(d*x+c)/a/d^2
```

3.497.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.44 (sec) , antiderivative size = 798, normalized size of antiderivative = 1.14

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{(a-ib)d^2} + \frac{f\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{(a+ib)d^2}$$
$$+ \frac{\left(2bde\cosh\left(\frac{1}{2}(c+dx)\right) - af\cosh\left(\frac{1}{2}(c+dx)\right) - 2bcf\cosh\left(\frac{1}{2}(c+dx)\right) + 2bf(c+dx)\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{4a^2d^2}$$
$$+ \frac{(-de+cf-f(c+dx))\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2} + \frac{if\log(\cosh(c+dx))}{2(a-ib)d^2} - \frac{if\log(\cosh(c+dx))}{2(a+ib)d^2}$$
$$+ \frac{-2abf(c+dx) - (2abf+3a^2(de+dfx)-2b^2(de+dfx))\log\left(1-e^{-c-dx}\right) + (-2abf+3a^2(de+dfx)}{}$$
$$- \frac{b^5\left(-2de\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) + 2cf\operatorname{arctanh}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) + f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) - f(c+dx)\log\left(\right.\right.}{a^3\left(a^2+b^2\right)^{3/2}d^2}$$
$$+ \frac{(-de+cf-f(c+dx))\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8ad^2}$$
$$+ \frac{\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(2bde\sinh\left(\frac{1}{2}(c+dx)\right) + af\sinh\left(\frac{1}{2}(c+dx)\right) - 2bcf\sinh\left(\frac{1}{2}(c+dx)\right) + 2bf(c+dx)\operatorname{si}\right.}{4a^2d^2}$$
$$+ \frac{\operatorname{sech}(c+dx)(-ade+acf-af(c+dx)+bde\sinh(c+dx)-bcf\sinh(c+dx)+bf(c+dx)\sinh(c+d}{(a^2+b^2)\,d^2}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]
),x]
```

output
```
(f*ArcTan[Tanh[(c + d*x)/2]])/((a - I*b)*d^2) + (f*ArcTan[Tanh[(c + d*x)/2
]])/((a + I*b)*d^2) + ((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2]
- 2*b*c*f*Cosh[(c + d*x)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c +
 d*x)/2])/(4*a^2*d^2) + ((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)
/(8*a*d^2) + ((I/2)*f*Log[Cosh[c + d*x]])/((a - I*b)*d^2) - ((I/2)*f*Log[C
osh[c + d*x]])/((a + I*b)*d^2) + (-2*a*b*f*(c + d*x) - (2*a*b*f + 3*a^2*(d
*e + d*f*x) - 2*b^2*(d*e + d*f*x))*Log[1 - E^(-c - d*x)] + (-2*a*b*f + 3*a
^2*(d*e + d*f*x) - 2*b^2*(d*e + d*f*x))*Log[1 + E^(-c - d*x)] - (3*a^2 - 2
*b^2)*f*PolyLog[2, -E^(-c - d*x)] + (3*a^2 - 2*b^2)*f*PolyLog[2, E^(-c - d
*x)])/(2*a^3*d^2) - (b^5*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^
2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log
[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a
^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^
3*(a^2 + b^2)^(3/2)*d^2) + ((-(d*e) + c*f - f*(c + d*x))*Sech[(c + d*x)/2]
^2)/(8*a*d^2) + (Sech[(c + d*x)/2]*(2*b*d*e*Sinh[(c + d*x)/2] + a*f*Sinh[(
c + d*x)/2] - 2*b*c*f*Sinh[(c + d*x)/2] + 2*b*f*(c + d*x)*Sinh[(c + d*x)/2
]))/(4*a^2*d^2) + (Sech[c + d*x]*(-(a*d*e) + a*c*f - a*f*(c + d*x) + b*d*e
*Sinh[c + d*x] - b*c*f*Sinh[c + d*x] + b*f*(c + d*x)*Sinh[c + d*x]))/((a^2
 + b^2)*d^2)
```

3.497.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 4.39 (sec) , antiderivative size = 611, normalized size of antiderivative = 0.87, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {6123, 5985, 2009, 6123, 5984, 3042, 25, 4672, 26, 3042, 26, 3956, 6123, 5985, 2009, 6107, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5985}$$

$$\frac{-f\int \left(-\frac{\text{sech}(c+dx)\text{csch}^2(c+dx)}{2d}+\frac{3\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\text{sech}(c+dx)}{2d}\right)dx+\frac{3(e+fx)\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3(e+fx)\text{sech}}{2d}}{a}$$
$$\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}(e^{c+dx})}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}(2,-e^{c+dx})}{2d^2}+\frac{3\,\text{PolyLog}(2,e^{c+dx})}{2d^2}+\frac{\text{csch}(}{2d}\right.}{a}$$
$$\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{6123}$$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}(e^{c+dx})}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}(2,-e^{c+dx})}{2d^2}+\frac{3\,\text{PolyLog}(2,e^{c+dx})}{2d^2}+\frac{\text{csch}(}{2d}\right.}{a}$$
$$\frac{b\left(\frac{\int (e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{5984}$$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}(e^{c+dx})}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}(2,-e^{c+dx})}{2d^2}+\frac{3\,\text{PolyLog}(2,e^{c+dx})}{2d^2}+\frac{\text{csch}(}{2d}\right.}{a}$$
$$\frac{b\left(\frac{4\int (e+fx)\text{csch}^2(2c+2dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

3.497. $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\text{csch}(}{2d}\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{4\int-\left((e+fx)\csc(2ic+2idx)^2\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\text{csch}(}{2d}\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\int(e+fx)\csc(2ic+2idx)^2dx}{a}\right)}{a}$$

$$\Big\downarrow \; 4672$$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\text{csch}(}{2d}\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{if\int-i\coth(2c+2dx)dx}{2d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\text{csch}(}{2d}\right.}{a}$$

$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\int\coth(2c+2dx)dx}{2d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 3042$$

3.497. $\int\frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\text{csch}(}{2d}\right.$$
$$\frac{}{a}$$
$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\int-i\tan\left(2ic+2idx+\frac{\pi}{2}\right)dx}{2d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 26$$

$$-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\text{csch}(}{2d}\right.$$
$$\frac{}{a}$$
$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}+\frac{if\int\tan\left(\frac{1}{2}(4ic+\pi)+2idx\right)dx}{2d}\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 3956$$

$$-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\text{csch}(}{2d}\right.$$
$$\frac{}{a}$$
$$\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 6123$$

$$-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\text{csch}(}{2d}\right.$$
$$\frac{}{a}$$
$$\frac{b\left(-\frac{b\left(\frac{\int(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{4\left(\frac{(e+fx)\coth(2c+2dx)}{2d}-\frac{f\log(-i\sinh(2c+2dx))}{4d^2}\right)}{a}\right)}{a}$$

$$\Big\downarrow \; 5985$$

3.497. $\int\frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\operatorname{csch}(}{2d}\right.$$

$$a$$

$$b\left(-\frac{b\left(\frac{-f\int\left(\frac{\operatorname{sech}(c+dx)}{d}-\frac{\operatorname{arctanh}(\cosh(c+dx))}{d}\right)dx-\frac{(e+fx)\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(e+fx)\operatorname{sech}(c+dx)}{d}}{a}-\frac{b\int\frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-\frac{4\left(\right.}{}\right.$$

$$a$$

$$\Big\downarrow \ 2009$$

$$-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\operatorname{csch}(}{2d}\right.$$

$$a$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(}{d}}{a}\right.}{a}\right.$$

$$a$$

$$\Big\downarrow \ 6107$$

$$-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\operatorname{csch}(}{2d}\right.$$

$$a$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(}{d}}{a}\right.}{a}\right.$$

$$\Big\downarrow \ 3042$$

3.497. $\displaystyle\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}(e^{c+dx})}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}(2,-e^{c+dx})}{2d^2}+\frac{3\operatorname{PolyLog}(2,e^{c+dx})}{2d^2}+\frac{\operatorname{csch}(\cdots}{2d}\right.}{a}$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}(e^{c+dx})}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\operatorname{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(\cdots}{d}}{a}\right.}{a}\right.$$

$$\Big\downarrow \ 3803$$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}(e^{c+dx})}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}(2,-e^{c+dx})}{2d^2}+\frac{3\operatorname{PolyLog}(2,e^{c+dx})}{2d^2}+\frac{\operatorname{csch}(\cdots}{2d}\right.}{a}$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}(e^{c+dx})}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\operatorname{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(\cdots}{d}}{a}\right.}{a}\right.$$

$$\Big\downarrow \ 25$$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}(e^{c+dx})}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}(2,-e^{c+dx})}{2d^2}+\frac{3\operatorname{PolyLog}(2,e^{c+dx})}{2d^2}+\frac{\operatorname{csch}(\cdots}{2d}\right.}{a}$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}(e^{c+dx})}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\operatorname{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(\cdots}{d}}{a}\right.}{a}\right.$$

$$\Big\downarrow \ 2694$$

3.497. $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\operatorname{csch}(}{2d}\right.$$

$$a$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\operatorname{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(}{d}}{a}\right.}{}\right.$$

$$\Big\downarrow 27$$

3.497. $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}(e^{c+dx})}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}(2,-e^{c+dx})}{2d^2}+\frac{3\operatorname{PolyLog}(2,e^{c+dx})}{2d^2}+\frac{\operatorname{csch}(\cdots}{2d}\right.}{a}$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}(e^{c+dx})}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\operatorname{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(\cdots}{d}}{a}\right.}{a}\right.$$

$$\Big\downarrow \text{2620}$$

3.497. $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\text{csch}(\ldots}{2d}\right.$$

$$a$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cosh(\ldots}{d}}{a}\right.}{\ldots}\right.$$

$$\Big\downarrow \text{ 2715}$$

3.497. $\quad \int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}(e^{c+dx})}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}(2,-e^{c+dx})}{2d^2}+\frac{3\operatorname{PolyLog}(2,e^{c+dx})}{2d^2}+\frac{\operatorname{csch}(}{2d}\right.$$

$$a$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}(e^{c+dx})}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\operatorname{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(}{d}}{a}\right.}{}\right.$$

$$\Big\downarrow \text{ 2838}$$

3.497. $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\operatorname{arctanh}(e^{c+dx})}{d}+\frac{3x\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{3\operatorname{PolyLog}(2,-e^{c+dx})}{2d^2}+\frac{3\operatorname{PolyLog}(2,e^{c+dx})}{2d^2}+\frac{\operatorname{csch}(}{2d}\right.}{a}$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\operatorname{arctanh}(e^{c+dx})}{d}-\frac{x\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{\operatorname{PolyLog}(2,-e^{c+dx})}{d^2}-\frac{\operatorname{PolyLog}(2,e^{c+dx})}{d^2}\right)-\frac{(e+fx)\operatorname{arctanh}(\cosh(}{d}}{a}\right.}{}\right.$$

$$\downarrow \text{7293}$$

3.497. $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\text{csch}(\ldots}{2d}\right.$$

$$a$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)}{a}-\frac{(e+fx)\text{arctanh}(\cosh(\ldots}{d}\right.}{}\right.$$

$$\downarrow 2009$$

3.497. $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$\frac{-f\left(-\frac{\arctan(\sinh(c+dx))}{d^2}-\frac{3x\text{arctanh}\left(e^{c+dx}\right)}{d}+\frac{3x\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{2d^2}+\frac{3\,\text{PolyLog}\left(2,e^{c+dx}\right)}{2d^2}+\frac{\text{csch}(}{2d}\right.}{a}$$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{\arctan(\sinh(c+dx))}{d^2}+\frac{2x\text{arctanh}\left(e^{c+dx}\right)}{d}-\frac{x\text{arctanh}(\cosh(c+dx))}{d}+\frac{\text{PolyLog}\left(2,-e^{c+dx}\right)}{d^2}-\frac{\text{PolyLog}\left(2,e^{c+dx}\right)}{d^2}\right)-\frac{(e+fx)\text{arctanh}(\cosh(}{d}}{a}\right.}{}\right.$$

input `Int[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]`

3.497. $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
((3*(e + f*x)*ArcTanh[Cosh[c + d*x]])/(2*d) - f*(-(ArcTan[Sinh[c + d*x]]/d
^2) - (3*x*ArcTanh[E^(c + d*x)])/d + (3*x*ArcTanh[Cosh[c + d*x]])/(2*d) +
Csch[c + d*x]/(2*d^2) - (3*PolyLog[2, -E^(c + d*x)])/(2*d^2) + (3*PolyLog[
2, E^(c + d*x)])/(2*d^2)) - (3*(e + f*x)*Sech[c + d*x])/(2*d) - ((e + f*x)
*Csch[c + d*x]^2*Sech[c + d*x])/(2*d))/a - (b*((-4*(((e + f*x)*Coth[2*c +
2*d*x])/(2*d) - (f*Log[(-I)*Sinh[2*c + 2*d*x]])/(4*d^2)))/a - (b*((-(((e +
 f*x)*ArcTanh[Cosh[c + d*x]])/d) - f*(ArcTan[Sinh[c + d*x]]/d^2 + (2*x*Arc
Tanh[E^(c + d*x)])/d - (x*ArcTanh[Cosh[c + d*x]])/d + PolyLog[2, -E^(c + d
*x)]/d^2 - PolyLog[2, E^(c + d*x)]/d^2) + ((e + f*x)*Sech[c + d*x])/d)/a -
 (b*((-2*b^2*(-1/2*(b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(
b*d^2)))/Sqrt[a^2 + b^2] + (b*(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])])/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))])/(b*d^2)))/(2*Sqrt[a^2 + b^2])))/(a^2 + b^2) + (-((b*f*ArcTan[Sinh[c
 + d*x]])/d^2) - (a*f*Log[Cosh[c + d*x]])/d^2 + (b*(e + f*x)*Sech[c + d*x]
)/d + (a*(e + f*x)*Tanh[c + d*x])/d)/(a^2 + b^2)))/a))/a))/a
```

3.497.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.497. $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

3.497. $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.497.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2766 vs. 2(659) = 1318.

Time = 37.58 (sec) , antiderivative size = 2767, normalized size of antiderivative = 3.96

method	result	size
risch	Expression too large to display	2767

input
```
int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURN
VERBOSE)
```

output
```
-3/2/d^2/a/(a^2+b^2)^(5/2)*c*b^5*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b
^2)^(1/2))-2/d^2*a/(a^2+b^2)^(5/2)*c*b^3*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a
)/(a^2+b^2)^(1/2))-3/2/d^2*a^3/(a^2+b^2)^(5/2)*c*b*f*arctanh(1/2*(2*b*exp(
d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d^2/(a^2+b^2)^(5/2)*b*f*arctanh(1/2*(2*b*ex
p(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^3+2/d^2/(a^2+b^2)^(5/2)*b^3*f*arctanh(1/2
*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a+3/2/d/a/(a^2+b^2)^(5/2)*e*b^5*arc
tanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(5/2)/d/a*b^5*f
*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^
(5/2)/d/a*b^5*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x
-1/(a^2+b^2)^(5/2)/d/a^3*b^7*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a
^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(5/2)/d/a^3*b^7*f*ln((b*exp(d*x+c)+(a^2+b^2)
^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(5/2)/d^2/a*b^5*f*ln((b*exp(d
*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-(3*d*f*x*a^3*exp(5*d*x+5*c
)+a*d*f*x*b^2*exp(5*d*x+5*c)+3*d*e*a^3*exp(5*d*x+5*c)+a*b^2*d*e*exp(5*d*x+
5*c)-2*b^3*d*f*x*exp(4*d*x+4*c)-2*d*f*x*a^3*exp(3*d*x+3*c)+a^3*f*exp(5*d*x
+5*c)+2*a*d*f*x*b^2*exp(3*d*x+3*c)+a*b^2*f*exp(5*d*x+5*c)-2*b^3*d*e*exp(4*
d*x+4*c)-2*d*e*a^3*exp(3*d*x+3*c)-4*exp(2*d*x+2*c)*a^2*b*d*f*x+2*a*b^2*d*e
*exp(3*d*x+3*c)+3*a^3*d*f*x*exp(d*x+c)-4*a^2*b*d*e*exp(2*d*x+2*c)+exp(d*x+
c)*a*b^2*d*f*x+3*a^3*d*e*exp(d*x+c)+4*a^2*b*d*f*x+exp(d*x+c)*a*b^2*d*e+2*b
^3*d*f*x-a^3*f*exp(d*x+c)+4*a^2*b*d*e-a*b^2*f*exp(d*x+c)+2*b^3*d*e)/d^2...
```

3.497.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11126 vs. $2(653) = 1306$.

Time = 0.55 (sec) , antiderivative size = 11126, normalized size of antiderivative = 15.92

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.497.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.497.7 Maxima [F]

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)^3\,\text{sech}\,(dx+c)^2}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-1/2*(2*b^5*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a
 + sqrt(a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)*d) + 2*(4*a^2*b*e^(-
2*d*x - 2*c) + 2*b^3*e^(-4*d*x - 4*c) - 4*a^2*b - 2*b^3 + (3*a^3 + a*b^2)*
e^(-d*x - c) - 2*(a^3 - a*b^2)*e^(-3*d*x - 3*c) + (3*a^3 + a*b^2)*e^(-5*d*
x - 5*c))/((a^4 + a^2*b^2 - (a^4 + a^2*b^2)*e^(-2*d*x - 2*c) - (a^4 + a^2*
b^2)*e^(-4*d*x - 4*c) + (a^4 + a^2*b^2)*e^(-6*d*x - 6*c))*d) - (3*a^2 - 2*
b^2)*log(e^(-d*x - c) + 1)/(a^3*d) + (3*a^2 - 2*b^2)*log(e^(-d*x - c) - 1)
/(a^3*d))*e - (32*b^5*integrate(-1/16*x*e^(d*x + c)/(a^5*b + a^3*b^3 - (a^
5*b*e^(2*c) + a^3*b^3*e^(2*c))*e^(2*d*x) - 2*(a^6*e^c + a^4*b^2*e^c)*e^(d*
x)), x) + 96*a^2*d*integrate(1/64*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 64*b
^2*d*integrate(1/64*x/(a^3*d*e^(d*x + c) + a^3*d), x) + 96*a^2*d*integrate
(1/64*x/(a^3*d*e^(d*x + c) - a^3*d), x) - 64*b^2*d*integrate(1/64*x/(a^3*d
*e^(d*x + c) - a^3*d), x) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) + 1
)/(a^3*d^2)) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^3*d^2))
- (2*b^3*d*x*e^(4*d*x + 4*c) + 4*a^2*b*d*x*e^(2*d*x + 2*c) + 2*(a^3*d*e^(3
*c) - a*b^2*d*e^(3*c))*x*e^(3*d*x) - 2*(2*a^2*b*d + b^3*d)*x - (a^3*e^(5*c
) + a*b^2*e^(5*c) + (3*a^3*d*e^(5*c) + a*b^2*d*e^(5*c))*x)*e^(5*d*x) + (a^
3*e^c + a*b^2*e^c - (3*a^3*d*e^c + a*b^2*d*e^c)*x)*e^(d*x))/(a^4*d^2 + a^2
*b^2*d^2 + (a^4*d^2*e^(6*c) + a^2*b^2*d^2*e^(6*c))*e^(6*d*x) - (a^4*d^2*e^
(4*c) + a^2*b^2*d^2*e^(4*c))*e^(4*d*x) - (a^4*d^2*e^(2*c) + a^2*b^2*d^2...
```

3.497.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.497.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \int \frac{e+f\,x}{\cosh\left(c+d\,x\right)^2\,\sinh\left(c+d\,x\right)^3\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.498 $\int \frac{\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.498.1 Optimal result

Integrand size = 29, antiderivative size = 206

$$\int \frac{\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{3\text{arctanh}(\cosh(c+dx))}{2ad} - \frac{b^2\text{arctanh}(\cosh(c+dx))}{a^3d} + \frac{2b^5\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^{3/2}d} + \frac{b\coth(c+dx)}{a^2d} - \frac{3\text{sech}(c+dx)}{2ad} + \frac{b^2\text{sech}(c+dx)}{a^3d} - \frac{\text{csch}^2(c+dx)\text{sech}(c+dx)}{2ad} - \frac{b^3\text{sech}(c+dx)(b+a\sinh(c+dx))}{a^3\left(a^2+b^2\right)d} + \frac{b\tanh(c+dx)}{a^2d}$$

output
```
3/2*arctanh(cosh(d*x+c))/a/d-b^2*arctanh(cosh(d*x+c))/a^3/d+2*b^5*arctanh(
(b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/a^3/(a^2+b^2)^(3/2)/d+b*coth(d*
x+c)/a^2/d-3/2*sech(d*x+c)/a/d+b^2*sech(d*x+c)/a^3/d-1/2*csch(d*x+c)^2*sec
h(d*x+c)/a/d-b^3*sech(d*x+c)*(b+a*sinh(d*x+c))/a^3/(a^2+b^2)/d+b*tanh(d*x+
c)/a^2/d
```

3.498.2 Mathematica [A] (verified)

Time = 3.37 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\frac{16b^5\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{a^3(-a^2-b^2)^{3/2}} + \frac{4b\coth\left(\frac{1}{2}(c+dx)\right)}{a^2} - \frac{\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{a} + \frac{4(3a^2-2b^2)\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} + \frac{4(-3a^2+2b^2)\log(\sinh}{a^3}}{8d}$$

input
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
((16*b^5*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(a^3*(-a^2 -
b^2)^(3/2)) + (4*b*Coth[(c + d*x)/2])/a^2 - Csch[(c + d*x)/2]^2/a + (4*(3*
a^2 - 2*b^2)*Log[Cosh[(c + d*x)/2]])/a^3 + (4*(-3*a^2 + 2*b^2)*Log[Sinh[(c
 + d*x)/2]])/a^3 - Sech[(c + d*x)/2]^2/a + (8*Sech[c + d*x]*(-a + b*Sinh[c
 + d*x]))/(a^2 + b^2) + (4*b*Tanh[(c + d*x)/2])/a^2)/(8*d)
```

3.498.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {3042, 26, 3377, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i}{\sin(ic+idx)^3\cos(ic+idx)^2(a-ib\sin(ic+idx))}dx$$
$$\Big\downarrow \text{26}$$
$$-i\int \frac{1}{\cos(ic+idx)^2\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$
$$\Big\downarrow \text{3377}$$

$$-i\int\left(-\frac{i\text{sech}^2(c+dx)b^3}{a^3(a+b\sinh(c+dx))}+\frac{i\text{csch}(c+dx)\text{sech}^2(c+dx)b^2}{a^3}-\frac{i\text{csch}^2(c+dx)\text{sech}^2(c+dx)b}{a^2}+\frac{i\text{csch}^3(c+dx)}{a}\right.$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{ib^2\text{arctanh}(\cosh(c+dx))}{a^3d}+\frac{ib^2\text{sech}(c+dx)}{a^3d}+\frac{ib\tanh(c+dx)}{a^2d}+\frac{ib\coth(c+dx)}{a^2d}+\frac{2ib^5\text{arctanh}\left(\frac{b-a\tanh(\frac{1}{2}}{\sqrt{a^2+}}\right.}{a^3d\left(a^2+b^2\right)^{3/}}\right.$$

input
```
Int[(Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

output
```
(-I)*((((3*I)/2)*ArcTanh[Cosh[c + d*x]])/(a*d) - (I*b^2*ArcTanh[Cosh[c + d
*x]])/(a^3*d) + ((2*I)*b^5*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^
2]])/(a^3*(a^2 + b^2)^(3/2)*d) + (I*b*Coth[c + d*x])/(a^2*d) - (((3*I)/2)*
Sech[c + d*x])/(a*d) + (I*b^2*Sech[c + d*x])/(a^3*d) - ((I/2)*Csch[c + d*x
]^2*Sech[c + d*x])/(a*d) - (I*b^3*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a^
3*(a^2 + b^2)*d) + (I*b*Tanh[c + d*x])/(a^2*d))
```

3.498.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3377
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

3.498.4 Maple [A] (verified)

Time = 18.53 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^{2}}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{\left(-6a^{2}+4b^{2}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^{3}}+\frac{b}{2a^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b^{5}\operatorname{arctanh}\left(\frac{2a\,\mathrm{ta}}{}\right.}{a^{3}\left(a^{2}\right.}}{d}$
default	$\frac{\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^{2}}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{\left(-6a^{2}+4b^{2}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^{3}}+\frac{b}{2a^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b^{5}\operatorname{arctanh}\left(\frac{2a\,\mathrm{ta}}{}\right.}{a^{3}\left(a^{2}\right.}}{d}$
risch	$-\frac{3\,\mathrm{e}^{dx+c}a^{3}+b^{2}a\,\mathrm{e}^{dx+c}+3a^{3}\mathrm{e}^{5dx+5c}+\mathrm{e}^{5dx+5c}a\,b^{2}-2\,\mathrm{e}^{4dx+4c}b^{3}-2a^{3}\mathrm{e}^{3dx+3c}+2\,\mathrm{e}^{3dx+3c}a\,b^{2}-4\,\mathrm{e}^{2dx+2c}a^{2}b+4a^{2}b+2b^{3}}{a^{2}d\left(\mathrm{e}^{2dx+2c}-1\right)^{2}\left(a^{2}+b^{2}\right)\left(1+\mathrm{e}^{2dx+2c}\right)}$

input
```
int(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4/a^2*(1/2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c))-1/8/a/t
anh(1/2*d*x+1/2*c)^2+1/4/a^3*(-6*a^2+4*b^2)*ln(tanh(1/2*d*x+1/2*c))+1/2*b/
a^2/tanh(1/2*d*x+1/2*c)-2/a^3*b^5/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/
2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+2/(a^2+b^2)*(b*tanh(1/2*d*x+1/2*c)-a)/(
1+tanh(1/2*d*x+1/2*c)^2))
```

3.498.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2653 vs. 2(197) = 394.

Time = 0.45 (sec) , antiderivative size = 2653, normalized size of antiderivative = 12.88

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fric
as")
```

output
```
-1/2*(8*a^5*b + 12*a^3*b^3 + 4*a*b^5 + 2*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*cos
h(d*x + c)^5 + 2*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*sinh(d*x + c)^5 - 4*(a^3*b^
3 + a*b^5)*cosh(d*x + c)^4 - 2*(2*a^3*b^3 + 2*a*b^5 - 5*(3*a^6 + 4*a^4*b^2
 + a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(a^6 - a^2*b^4)*cosh(d*x +
c)^3 - 4*(a^6 - a^2*b^4 - 5*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*cosh(d*x + c)^2
+ 4*(a^3*b^3 + a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(a^5*b + a^3*b^3)
*cosh(d*x + c)^2 - 4*(2*a^5*b + 2*a^3*b^3 - 5*(3*a^6 + 4*a^4*b^2 + a^2*b^4
)*cosh(d*x + c)^3 + 6*(a^3*b^3 + a*b^5)*cosh(d*x + c)^2 + 3*(a^6 - a^2*b^4
)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b^5*cosh(d*x + c)^6 + 6*b^5*cosh(d*x
 + c)*sinh(d*x + c)^5 + b^5*sinh(d*x + c)^6 - b^5*cosh(d*x + c)^4 - b^5*co
sh(d*x + c)^2 + b^5 + (15*b^5*cosh(d*x + c)^2 - b^5)*sinh(d*x + c)^4 + 4*(
5*b^5*cosh(d*x + c)^3 - b^5*cosh(d*x + c))*sinh(d*x + c)^3 + (15*b^5*cosh(
d*x + c)^4 - 6*b^5*cosh(d*x + c)^2 - b^5)*sinh(d*x + c)^2 + 2*(3*b^5*cosh(
d*x + c)^5 - 2*b^5*cosh(d*x + c)^3 - b^5*cosh(d*x + c))*sinh(d*x + c))*sqr
t(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d
*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt
(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 +
b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x +
 c) - b)) + 2*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*cosh(d*x + c) - ((3*a^6 + 4*a^
4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^6 + 6*(3*a^6 + 4*a^4*b^2 - a^2*b...
```

3.498.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.498.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.62

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b^5\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^5+a^3b^2)\sqrt{a^2+b^2}d} - \frac{4\,a^2be^{(-2\,dx-2\,c)}+2\,b^3e^{(-4\,dx-4\,c)}-4\,a^2b-2\,b^3+(3\,a^3+ab^2)e^{(-dx-c)}-2\,(a^3-ab^2)e^{(-3\,dx-3\,c)}+(3\,a^3+}{(a^4+a^2b^2-(a^4+a^2b^2)e^{(-2\,dx-2\,c)}-(a^4+a^2b^2)e^{(-4\,dx-4\,c)}+(a^4+a^2b^2)e^{(-6\,dx-6\,c)})d} + \frac{(3\,a^2-2\,b^2)\log\left(e^{(-dx-c)}+1\right)}{2\,a^3d} - \frac{(3\,a^2-2\,b^2)\log\left(e^{(-dx-c)}-1\right)}{2\,a^3d}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxi
ma")
```

output
```
-b^5*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt
(a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)*d) - (4*a^2*b*e^(-2*d*x - 2
*c) + 2*b^3*e^(-4*d*x - 4*c) - 4*a^2*b - 2*b^3 + (3*a^3 + a*b^2)*e^(-d*x -
 c) - 2*(a^3 - a*b^2)*e^(-3*d*x - 3*c) + (3*a^3 + a*b^2)*e^(-5*d*x - 5*c))
/((a^4 + a^2*b^2 - (a^4 + a^2*b^2)*e^(-2*d*x - 2*c) - (a^4 + a^2*b^2)*e^(-
4*d*x - 4*c) + (a^4 + a^2*b^2)*e^(-6*d*x - 6*c))*d) + 1/2*(3*a^2 - 2*b^2)*
log(e^(-d*x - c) + 1)/(a^3*d) - 1/2*(3*a^2 - 2*b^2)*log(e^(-d*x - c) - 1)/
(a^3*d)
```

3.498.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\frac{2\,b^5\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^5+a^3b^2)\sqrt{a^2+b^2}} + \frac{4\,(ae^{(dx+c)}+b)}{(a^2+b^2)(e^{(2\,dx+2\,c)}+1)} - \frac{(3\,a^2-2\,b^2)\log(e^{(dx+c)}+1)}{a^3} + \frac{(3\,a^2-2\,b^2)\log(\left|e^{(dx+c)}-1\right|)}{a^3} + \frac{2\,(a}{}}{2\,d}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac
")
```

output
```
-1/2*(2*b^5*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(
d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)) + 4
*(a*e^(d*x + c) + b)/((a^2 + b^2)*(e^(2*d*x + 2*c) + 1)) - (3*a^2 - 2*b^2)
*log(e^(d*x + c) + 1)/a^3 + (3*a^2 - 2*b^2)*log(abs(e^(d*x + c) - 1))/a^3
+ 2*(a*e^(3*d*x + 3*c) - 2*b*e^(2*d*x + 2*c) + a*e^(d*x + c) + 2*b)/(a^2*(
e^(2*d*x + 2*c) - 1)^2))/d
```

3.498.9 Mupad [B] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.58

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{b^5\ln\left(2a^4b-4a^5e^{c+dx}+b^5+3a^2b^3+4a^2e^{c+dx}\sqrt{(a^2+b^2)^3}+b^2e^{c+dx}\sqrt{(a^2+b^2)^3}-7a^3b^2e^{c+dx}-2\right.}{d\,a^9+3\,d\,a^7b^2+3\,d\,a^5b^4+d\,a^3b^6}$$

$$-\frac{\frac{e^{c+dx}}{a\,d}-\frac{2\,(a^2b+b^3)}{a^2d\,(a^2+b^2)}}{e^{2c+2dx}-1}-\frac{\frac{2b}{d\,(a^2+b^2)}+\frac{2\,a\,e^{c+dx}}{d\,(a^2+b^2)}}{e^{2c+2dx}+1}$$

$$-\frac{\ln\left(e^{c+dx}-1\right)\,(3a^2-2b^2)}{2a^3d}+\frac{\ln\left(e^{c+dx}+1\right)\,(3a^2-2b^2)}{2a^3d}$$

$$-\frac{b^5\ln\left(4a^5e^{c+dx}-2a^4b-b^5-3a^2b^3+4a^2e^{c+dx}\sqrt{(a^2+b^2)^3}+b^2e^{c+dx}\sqrt{(a^2+b^2)^3}+7a^3b^2e^{c+dx}\right.}{d\,a^9+3\,d\,a^7b^2+3\,d\,a^5b^4+d\,a^3b^6}$$

$$-\frac{2\,e^{c+dx}}{a\,d\,(e^{4c+4dx}-2\,e^{2c+2dx}+1)}$$

input
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
(b^5*log(2*a^4*b - 4*a^5*exp(c + d*x) + b^5 + 3*a^2*b^3 + 4*a^2*exp(c + d*
x)*((a^2 + b^2)^3)^(1/2) + b^2*exp(c + d*x)*((a^2 + b^2)^3)^(1/2) - 7*a^3*
b^2*exp(c + d*x) - 2*a*b*((a^2 + b^2)^3)^(1/2) - 3*a*b^4*exp(c + d*x))*((a
^2 + b^2)^3)^(1/2))/(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d) - (exp
(c + d*x)/(a*d) - (2*(a^2*b + b^3))/(a^2*d*(a^2 + b^2)))/(exp(2*c + 2*d*x)
 - 1) - ((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(
2*c + 2*d*x) + 1) - (log(exp(c + d*x) - 1)*(3*a^2 - 2*b^2))/(2*a^3*d) + (l
og(exp(c + d*x) + 1)*(3*a^2 - 2*b^2))/(2*a^3*d) - (b^5*log(4*a^5*exp(c + d
*x) - 2*a^4*b - b^5 - 3*a^2*b^3 + 4*a^2*exp(c + d*x)*((a^2 + b^2)^3)^(1/2)
 + b^2*exp(c + d*x)*((a^2 + b^2)^3)^(1/2) + 7*a^3*b^2*exp(c + d*x) - 2*a*b
*((a^2 + b^2)^3)^(1/2) + 3*a*b^4*exp(c + d*x))*((a^2 + b^2)^3)^(1/2))/(a^9
*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d) - (2*exp(c + d*x))/(a*d*(exp(4
*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.498. $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.499 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.499.1 Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\operatorname{Int}\left(\frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.499.2 Mathematica [N/A]

Not integrable

Time = 79.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])),x]
```

output
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

3.499.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

$$\Big\downarrow \text{6125}$$

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]^3*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.499.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.499.4 Maple [N/A] (verified)

Not integrable

Time = 0.61 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(dx+c\right)^{3}\operatorname{sech}\left(dx+c\right)^{2}}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}dx$$

input
```
int(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.499.5 Fricas [N/A]

Not integrable

Time = 1.89 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^3\operatorname{sech}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
integral(csch(d*x + c)^3*sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh
(d*x + c)), x)
```

3.499.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.499.7 Maxima [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 1426, normalized size of antiderivative = 39.61

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^3\operatorname{sech}\left(dx+c\right)^2}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-32*b^5*integrate(-1/16*e^(d*x + c)/(a^5*b*e + a^3*b^3*e + (a^5*b*f + a^3*
b^3*f)*x - (a^5*b*e*e^(2*c) + a^3*b^3*e*e^(2*c) + (a^5*b*f*e^(2*c) + a^3*b
^3*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^6*e*e^c + a^4*b^2*e*e^c + (a^6*f*e^c + a
^4*b^2*f*e^c)*x)*e^(d*x)), x) - (4*a^2*b*d*e + 2*b^3*d*e + 2*(2*a^2*b*d*f
+ b^3*d*f)*x + ((3*d*e - f)*a^3*e^(5*c) + (d*e - f)*a*b^2*e^(5*c) + (3*a^3
*d*f*e^(5*c) + a*b^2*d*f*e^(5*c))*x)*e^(5*d*x) - 2*(b^3*d*f*x*e^(4*c) + b^
3*d*e*e^(4*c))*e^(4*d*x) - 2*(a^3*d*e*e^(3*c) - a*b^2*d*e*e^(3*c) + (a^3*d
*f*e^(3*c) - a*b^2*d*f*e^(3*c))*x)*e^(3*d*x) - 4*(a^2*b*d*f*x*e^(2*c) + a^
2*b*d*e*e^(2*c))*e^(2*d*x) + ((3*d*e + f)*a^3*e^c + (d*e + f)*a*b^2*e^c +
(3*a^3*d*f*e^c + a*b^2*d*f*e^c)*x)*e^(d*x))/(a^4*d^2*e^2 + a^2*b^2*d^2*e^2
 + (a^4*d^2*f^2 + a^2*b^2*d^2*f^2)*x^2 + 2*(a^4*d^2*e*f + a^2*b^2*d^2*e*f)
*x + (a^4*d^2*e^2*e^(6*c) + a^2*b^2*d^2*e^2*e^(6*c) + (a^4*d^2*f^2*e^(6*c)
 + a^2*b^2*d^2*f^2*e^(6*c))*x^2 + 2*(a^4*d^2*e*f*e^(6*c) + a^2*b^2*d^2*e*f
*e^(6*c))*x)*e^(6*d*x) - (a^4*d^2*e^2*e^(4*c) + a^2*b^2*d^2*e^2*e^(4*c) +
(a^4*d^2*f^2*e^(4*c) + a^2*b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^4*d^2*e*f*e^(4*
c) + a^2*b^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) - (a^4*d^2*e^2*e^(2*c) + a^2*b^
2*d^2*e^2*e^(2*c) + (a^4*d^2*f^2*e^(2*c) + a^2*b^2*d^2*f^2*e^(2*c))*x^2 +
2*(a^4*d^2*e*f*e^(2*c) + a^2*b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) - 32*integ
rate(1/64*(2*b^2*d^2*e^2 + 2*a*b*d*e*f - (3*d^2*e^2 - 2*f^2)*a^2 - (3*a^2*
d^2*f^2 - 2*b^2*d^2*f^2)*x^2 - 2*(3*a^2*d^2*e*f - 2*b^2*d^2*e*f - a*b*d...
```

3.499.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.499.9 Mupad [N/A]

Not integrable

Time = 18.74 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$= \int \frac{1}{\cosh(c+d\,x)^2\sinh(c+d\,x)^3\,(e+f\,x)\,(a+b\sinh(c+d\,x))}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))), x
)
```

3.500 $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.500.1 Optimal result

Integrand size = 34, antiderivative size = 1122

$$
\begin{aligned}
&\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx\\
&=\frac{b^2fx}{2a^3d}+\frac{3bfx\arctan\left(e^{c+dx}\right)}{a^2d}-\frac{2b^5(e+fx)\arctan\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)^2d}-\frac{b^3(e+fx)\arctan\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d}\\
&-\frac{3bfx\arctan(\sinh(c+dx))}{2a^2d}+\frac{3b(e+fx)\arctan(\sinh(c+dx))}{2a^2d}\\
&-\frac{2b^2fx\operatorname{arctanh}\left(e^{2c+2dx}\right)}{a^3d}+\frac{4(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{ad}+\frac{bf\operatorname{arctanh}(\cosh(c+dx))}{a^2d^2}\\
&+\frac{3b(e+fx)\operatorname{csch}(c+dx)}{2a^2d}-\frac{f\operatorname{csch}(2c+2dx)}{ad^2}-\frac{2(e+fx)\coth(2c+2dx)\operatorname{csch}(2c+2dx)}{ad}\\
&-\frac{b^6(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^2d}-\frac{b^6(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^2d}\\
&+\frac{b^6(e+fx)\log\left(1+e^{2(c+dx)}\right)}{a^3\left(a^2+b^2\right)^2d}-\frac{b^2fx\log(\tanh(c+dx))}{a^3d}+\frac{b^2(e+fx)\log(\tanh(c+dx))}{a^3d}\\
&-\frac{3ibf\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{2a^2d^2}+\frac{ib^5f\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{a^2\left(a^2+b^2\right)^2d^2}+\frac{ib^3f\operatorname{PolyLog}\left(2,-ie^{c+dx}\right)}{2a^2\left(a^2+b^2\right)d^2}\\
&+\frac{3ibf\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{2a^2d^2}-\frac{ib^5f\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{a^2\left(a^2+b^2\right)^2d^2}-\frac{ib^3f\operatorname{PolyLog}\left(2,ie^{c+dx}\right)}{2a^2\left(a^2+b^2\right)d^2}\\
&-\frac{b^6f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^2d^2}-\frac{b^6f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^2d^2}\\
&+\frac{b^6f\operatorname{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2a^3\left(a^2+b^2\right)^2d^2}+\frac{f\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{ad^2}-\frac{b^2f\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2a^3d^2}\\
&-\frac{f\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{ad^2}+\frac{b^2f\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2a^3d^2}+\frac{bf\operatorname{sech}(c+dx)}{2a^2d^2}-\frac{b^3f\operatorname{sech}(c+dx)}{2a^2\left(a^2+b^2\right)d^2}\\
&-\frac{b^4(e+fx)\operatorname{sech}^2(c+dx)}{2a^3\left(a^2+b^2\right)d}-\frac{b(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{2a^2d}-\frac{b^2f\tanh(c+dx)}{2a^3d^2}\\
&+\frac{b^4f\tanh(c+dx)}{2a^3\left(a^2+b^2\right)d^2}-\frac{b^3(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{2a^2\left(a^2+b^2\right)d}-\frac{b^2(e+fx)\tanh^2(c+dx)}{2a^3d}
\end{aligned}
$$

output
```
-f*polylog(2,exp(2*d*x+2*c))/a/d^2+4*(f*x+e)*arctanh(exp(2*d*x+2*c))/a/d+f
*polylog(2,-exp(2*d*x+2*c))/a/d^2+1/2*b^2*f*polylog(2,exp(2*d*x+2*c))/a^3/
d^2-2*b^5*(f*x+e)*arctan(exp(d*x+c))/a^2/(a^2+b^2)^2/d-2*b^2*f*x*arctanh(e
xp(2*d*x+2*c))/a^3/d+1/2*b^6*f*polylog(2,-exp(2*d*x+2*c))/a^3/(a^2+b^2)^2/
d^2-1/2*b^3*f*sech(d*x+c)/a^2/(a^2+b^2)/d^2-1/2*b^4*(f*x+e)*sech(d*x+c)^2/
a^3/(a^2+b^2)/d-1/2*b*(f*x+e)*csch(d*x+c)*sech(d*x+c)^2/a^2/d+1/2*b^4*f*ta
nh(d*x+c)/a^3/(a^2+b^2)/d^2-3/2*I*b*f*polylog(2,-I*exp(d*x+c))/a^2/d^2+1/2
*b^2*f*x/a^3/d-2*(f*x+e)*coth(2*d*x+2*c)*csch(2*d*x+2*c)/a/d+1/2*b*f*sech(
d*x+c)/a^2/d^2-1/2*b^2*f*tanh(d*x+c)/a^3/d^2-1/2*b^2*(f*x+e)*tanh(d*x+c)^2
/a^3/d+1/2*I*b^3*f*polylog(2,-I*exp(d*x+c))/a^2/(a^2+b^2)/d^2+b*f*arctanh(
cosh(d*x+c))/a^2/d^2+3/2*b*(f*x+e)*csch(d*x+c)/a^2/d-f*csch(2*d*x+2*c)/a/d
^2-3/2*b*f*x*arctan(sinh(d*x+c))/a^2/d+3/2*b*(f*x+e)*arctan(sinh(d*x+c))/a
^2/d+b^6*(f*x+e)*ln(1+exp(2*d*x+2*c))/a^3/(a^2+b^2)^2/d-b^6*(f*x+e)*ln(1+b
*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^2/d-b^6*(f*x+e)*ln(1+b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^2/d-b^2*f*x*ln(tanh(d*x+c))/a^3/
d-b^6*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^2/d^2-b
^6*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^2/d^2+b^2*
(f*x+e)*ln(tanh(d*x+c))/a^3/d+3*b*f*x*arctan(exp(d*x+c))/a^2/d-b^3*(f*x+e)
*arctan(exp(d*x+c))/a^2/(a^2+b^2)/d-1/2*b^3*(f*x+e)*sech(d*x+c)*tanh(d*x+c
)/a^2/(a^2+b^2)/d-I*b^5*f*polylog(2,I*exp(d*x+c))/a^2/(a^2+b^2)^2/d^2-1...
```

3.500.2 Mathematica [A] (warning: unable to verify)

Time = 10.27 (sec) , antiderivative size = 1552, normalized size of antiderivative = 1.38

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]
),x]
```

output
```
8*(((I/16)*(2*a^6 + 3*a^4*b^2 + b^6)*(d*e - c*f)*(c + d*x))/(a^3*(a^2 + b^
2)^2*d^2) + ((I/32)*(2*a^6 + 3*a^4*b^2 + b^6)*f*(c + d*x)^2)/(a^3*(a^2 + b
^2)^2*d^2) + (b*f*Log[Cosh[(c + d*x)/2]])/(8*a^2*d^2) - (b*f*Log[Sinh[(c +
 d*x)/2]])/(8*a^2*d^2) - ((2*a^2 - b^2)*((d^2*f*x^2)/2 + d*e*(c + d*x) - 2
*(d*e - c*f)*(c + d*x) + 2*f*(c + d*x)*Log[1 + E^(-c - d*x)] + 2*(d*e - c*
f)*Log[1 + E^(c + d*x)] - 2*f*PolyLog[2, -E^(-c - d*x)]))/(16*a^3*d^2) + (
(1/16 - I/16)*a*(2*a^2 + 3*b^2)*((d^2*f*x^2)/2 + d*e*(c + d*x) - (1 + I)*(
d*e - c*f)*(c + d*x) + (1 + I)*f*(c + d*x)*Log[1 - I*E^(-c - d*x)] + (1 +
I)*(d*e - c*f)*Log[I - E^(c + d*x)] - (1 + I)*f*PolyLog[2, I*E^(-c - d*x)]
))/((a^2 + b^2)^2*d^2) + ((1/16 + I/16)*(((-1/2*I)*b^6*(d*e - c*f + f*(c +
 d*x))^2)/f - (1 - I)*(2*a^2 - b^2)*(a^2 + b^2)^2*(d*e - c*f + f*(c + d*x)
)*Log[1 - E^(-c - d*x)] + (1 - I)*a^4*(2*a^2 + 3*b^2)*(d*e - c*f + f*(c +
d*x))*Log[1 + I*E^(-c - d*x)] - (1 - I)*a^4*(2*a^2 + 3*b^2)*f*PolyLog[2, (
-I)*E^(-c - d*x)] + (1 - I)*(2*a^2 - b^2)*(a^2 + b^2)^2*f*PolyLog[2, E^(-c
 - d*x)]))/(a^3*(a^2 + b^2)^2*d^2) + ((I/16)*b*(3*a^2 + 5*b^2)*((-2*I)*d*e
*ArcTan[E^(c + d*x)] + (2*I)*c*f*ArcTan[E^(c + d*x)] + f*(c + d*x)*Log[1 -
 I*E^(c + d*x)] - f*(c + d*x)*Log[1 + I*E^(c + d*x)] - f*PolyLog[2, (-I)*E
^(c + d*x)] + f*PolyLog[2, I*E^(c + d*x)]))/((a^2 + b^2)^2*d^2) - (b^6*(-2
*d*e*(c + d*x) + 2*c*f*(c + d*x) - f*(c + d*x)^2 + (4*a*Sqrt[a^2 + b^2]*d*
e*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2] - ...
```

3.500.3 Rubi [A] (verified)

Time = 5.60 (sec) , antiderivative size = 943, normalized size of antiderivative = 0.84, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.735$, Rules used = {6123, 5984, 3042, 26, 4673, 26, 3042, 26, 4670, 2715, 2838, 6123, 5985, 2009, 6123, 5985, 2009, 6107, 6107, 6095, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{6123}$$

$$\frac{\int (e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{5984}$$

$$\frac{8\int (e+fx)\text{csch}^3(2c+2dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{8\int -i(e+fx)\csc(2ic+2idx)^3dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{8i\int (e+fx)\csc(2ic+2idx)^3dx}{a}$$

$$\Big\downarrow \text{4673}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{8i\left(\frac{1}{2}\int -i(e+fx)\text{csch}(2c+2dx)dx-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{csch}(2c+2dx)}{4d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{8i\left(-\frac{1}{2}i\int (e+fx)\text{csch}(2c+2dx)dx-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{csch}(2c+2dx)}{4d}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{8i\left(-\frac{1}{2}i\int i(e+fx)\csc(2ic+2idx)dx-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{csch}(2c+2dx)}{4d}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{8i\left(\frac{1}{2}\int (e+fx)\csc(2ic+2idx)dx-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{csch}(2c+2dx)}{4d}\right)}{a}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}-\frac{8i\left(\frac{1}{2}\left(\frac{if\int \log\left(1-e^{2c+2dx}\right)dx}{2d}-\frac{if\int \log\left(1+e^{2c+2dx}\right)dx}{2d}+\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}\right)-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{cs}}{4d}\right.}{a}$$

$$\Big\downarrow \text{2715}$$

3.500. $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}-$$

$$\frac{8i\left(\frac{1}{2}\left(\frac{if\int e^{-2c-2dx}\log(1-e^{2c+2dx})de^{2c+2dx}}{4d^2}-\frac{if\int e^{-2c-2dx}\log(1+e^{2c+2dx})de^{2c+2dx}}{4d^2}+\frac{i(e+fx)\text{arctanh}(e^{2c+2dx})}{d}\right)-\frac{if\text{csch}(2c+2}{8d^2}\right.}{a}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}-$$

$$\frac{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\text{arctanh}(e^{2c+2dx})}{d}+\frac{if\text{PolyLog}(2,-e^{2c+2dx})}{4d^2}-\frac{if\text{PolyLog}(2,e^{2c+2dx})}{4d^2}\right)-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{C}}{4d}\right.}{a}$$

$$\Big\downarrow \text{6123}$$

$$-\frac{b\left(\frac{\int(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)dx}{a}-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}-$$

$$\frac{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\text{arctanh}(e^{2c+2dx})}{d}+\frac{if\text{PolyLog}(2,-e^{2c+2dx})}{4d^2}-\frac{if\text{PolyLog}(2,e^{2c+2dx})}{4d^2}\right)-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{C}}{4d}\right.}{a}$$

$$\Big\downarrow \text{5985}$$

$$-\frac{b\left(\frac{-f\int\left(\frac{\text{csch}(c+dx)\text{sech}^2(c+dx)}{2d}-\frac{3\arctan(\sinh(c+dx))}{2d}-\frac{3\text{csch}(c+dx)}{2d}\right)dx-\frac{3(e+fx)\arctan(\sinh(c+dx))}{2d}-\frac{3(e+fx)\text{csch}(c+dx)}{2d}+\frac{(e+fx)\text{csch}(c+}{}}{a}\right.}{a}$$

$$\frac{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\text{arctanh}(e^{2c+2dx})}{d}+\frac{if\text{PolyLog}(2,-e^{2c+2dx})}{4d^2}-\frac{if\text{PolyLog}(2,e^{2c+2dx})}{4d^2}\right)-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{C}}{4d}\right.}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{b\int\frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}+\frac{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+dx))}{d^2}-\frac{3i\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2d^2}+\right.}{a}\right.}{a}$$

$$\frac{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\text{arctanh}(e^{2c+2dx})}{d}+\frac{if\text{PolyLog}(2,-e^{2c+2dx})}{4d^2}-\frac{if\text{PolyLog}(2,e^{2c+2dx})}{4d^2}\right)-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{C}}{4d}\right.}{a}$$

$$\Big\downarrow \text{6123}$$

3.500. $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(-\frac{b\left(\frac{\int (e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)dx}{a}-\frac{b\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\text{arctanh}(\cosh(c+}{d^2}\right.}{}\right.}{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{C}}{4d}\right.}{a}$$

$$\Big\downarrow \; 5985$$

$$-\frac{b\left(-\frac{b\left(\frac{-f\int\left(\frac{\log(\tanh(c+dx))}{d}-\frac{\tanh^2(c+dx)}{2d}\right)dx-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+fx)\log(\tanh(c+dx))}{d}}{a}-\frac{b\int\frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx}{a}\right)}{a}+\frac{-f\left(\frac{3x\arctan\left(e^{c}\right.}{d}\right.}{}\right.}{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{C}}{4d}\right.}{a}$$

$$\Big\downarrow \; 2009$$

$$-\frac{b\left(-\frac{b\left(\frac{-f\left(\frac{2x\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e-}{}}{a}\right.}{a}\right.}{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{C}}{4d}\right.}{a}$$

$$\Big\downarrow \; 6107$$

3.500. $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(b-\frac{b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e-}{}}{a}\right)}{a}\right)}{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\operatorname{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\mathrm{C}}{4d}\right.}$$

$$a$$

$$\Big\downarrow \textbf{6107}$$

$$-\frac{b\left(b-\frac{b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e-}{}}{a}\right)}{}\right)}{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\operatorname{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\mathrm{C}}{4d}\right.}$$

$$a$$

$$\Big\downarrow \textbf{6095}$$

3.500. $\displaystyle\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{2x\,\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e-}{}}{a}\right.}{}\right.$$

$$-\frac{}{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\,\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\,\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{C}}{4d}\right.}$$

$$a$$

$$\Big\downarrow \; 2620$$

3.500. $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{b\left(b\left(-\frac{b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e\cdots}{}}{a}\cdots}{\cdots}\cdots\right)\cdots\right)}{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\operatorname{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\mathrm{c}\cdots}{4d}\cdots\right)}{a}$$

$\downarrow$ 2715

3.500. $\displaystyle\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e+\cdots}{}}{a}}{}\right.$$

$$-\frac{}{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\operatorname{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\mathrm{c}\cdots}{4d}\right.}$$

$$a$$

$$\downarrow 2838$$

3.500. $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{2x\,\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\text{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\frac{(e\cdots}{}}{a}\cdots\right)}{}\cdots\right)$$

$$-\frac{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\,\text{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\,\text{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\,\text{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\,\text{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\text{c}\cdots}{4d}\cdots\right.}{a}$$

$$\Big\downarrow \; 7293$$

3.500. $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$b\left(-\frac{b\left(\frac{-f\left(\frac{2x\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{2d^2}+\frac{\tanh(c+dx)}{2d^2}+\frac{x\log(\tanh(c+dx))}{d}-\frac{x}{2d}\right)-\frac{(e+fx)\tanh^2(c+dx)}{2d}+\cdots}{a}\right.}{\cdots}\right.$$

$$-\frac{8i\left(\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\operatorname{PolyLog}\left(2,e^{2c+2dx}\right)}{4d^2}\right)-\frac{if\operatorname{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\cdots}{4d}\cdots\right.}{a}$$

$\downarrow$ 2009

3.500. $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

$$-\frac{8i\left(-\frac{if\operatorname{csch}(2c+2dx)}{8d^2}-\frac{i(e+fx)\coth(2c+2dx)\operatorname{csch}(2c+2dx)}{4d}+\frac{1}{2}\left(\frac{i(e+fx)\operatorname{arctanh}\left(e^{2c+2dx}\right)}{d}+\frac{if\operatorname{PolyLog}\left(2,-e^{2c+2dx}\right)}{4d^2}-\frac{if\operatorname{Poly}}{}\right.\right.}{a}$$

$$b\left(\frac{\frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{2d}-\frac{3(e+fx)\arctan(\sinh(c+dx))}{2d}-\frac{3(e+fx)\operatorname{csch}(c+dx)}{2d}-f\left(\frac{3x\arctan\left(e^{c+dx}\right)}{d}-\frac{3x\arctan(\sinh(c+dx))}{2d}+\frac{\operatorname{arctanh}(\cosh}{d^2}\right.}{a}\right.$$

input
```
Int[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

3.500. $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

output
```
((-8*I)*(((-1/8*I)*f*Csch[2*c + 2*d*x])/d^2 - ((I/4)*(e + f*x)*Coth[2*c +
2*d*x]*Csch[2*c + 2*d*x])/d + ((I*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/d +
((I/4)*f*PolyLog[2, -E^(2*c + 2*d*x)])/d^2 - ((I/4)*f*PolyLog[2, E^(2*c +
2*d*x)])/d^2)/2))/a - (b*(((-3*(e + f*x)*ArcTan[Sinh[c + d*x]])/(2*d) - (3
*(e + f*x)*Csch[c + d*x])/(2*d) + ((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2
)/(2*d) - f*((3*x*ArcTan[E^(c + d*x)])/d - (3*x*ArcTan[Sinh[c + d*x]])/(2*
d) + ArcTanh[Cosh[c + d*x]]/d^2 - (((3*I)/2)*PolyLog[2, (-I)*E^(c + d*x)])
/d^2 + (((3*I)/2)*PolyLog[2, I*E^(c + d*x)])/d^2 + Sech[c + d*x]/(2*d^2)))
/a - (b*((((e + f*x)*Log[Tanh[c + d*x]])/d - ((e + f*x)*Tanh[c + d*x]^2)/(
2*d) - f*(-1/2*x/d + (2*x*ArcTanh[E^(2*c + 2*d*x)])/d + (x*Log[Tanh[c + d*
x]])/d + PolyLog[2, -E^(2*c + 2*d*x)]/(2*d^2) - PolyLog[2, E^(2*c + 2*d*x)
]/(2*d^2) + Tanh[c + d*x]/(2*d^2)))/a - (b*((b^2*((b^2*(-1/2*(e + f*x)^2/(
b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) +
((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (f*Poly
Log[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (f*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)))/(a^2 + b^2) + ((b*(e
+ f*x)^2)/(2*f) + (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/d - (b*(e + f*x)*Log
[1 + E^(2*(c + d*x))])/d - (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/d^2 + (I*a
*f*PolyLog[2, I*E^(c + d*x)])/d^2 - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*
d^2))/(a^2 + b^2)))/(a^2 + b^2) + ((a*(e + f*x)*ArcTan[E^(c + d*x)])/d ...
```

3.500.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.500. $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5984
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csch[2*a + 2*b*x
]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

rule 5985
```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u,
 x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n
, p]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

3.500. $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

rule 6107
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b^2/(a^2 + b^2)   Int[(e +
f*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Simp[1/(a^2
+ b^2)   Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0
]
```

rule 6123
```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> S
imp[1/a   Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Simp[b/
a   Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c +
d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
&& IGtQ[p, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.500.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3562 vs. $2(1045) = 2090$.

Time = 89.78 (sec) , antiderivative size = 3563, normalized size of antiderivative = 3.18

method	result	size
risch	Expression too large to display	3563

input
```
int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURN
VERBOSE)
```

output
```
10*I/(a^2+b^2)/d^2*b^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c+1/(a^2+b^2)^(5
/2)/d^2*a^2*b^2*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+12/(a^
2+b^2)/d*a*b^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x+12/(a^2+b^2)/d*a*b^2*f
/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x-12/(a^2+b^2)/d^2*a*c*b^2*f/(4*a^2+4*b^
2)*ln(1+exp(2*d*x+2*c))+1/2/(a^2+b^2)^(5/2)/d^2*a^2*c*b^2*f*arctanh(1/2*(2
*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+12/(a^2+b^2)/d^2*a*b^2*f/(4*a^2+4*b^2)
*ln(1+I*exp(d*x+c))*c+12/(a^2+b^2)/d^2*a*b^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*
x+c))*c+6*I/(a^2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))*b-6*I/
(a^2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))*b+10*I/(a^2+b^2)/d
*b^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x-10*I/(a^2+b^2)/d*b^3*f/(4*a^2+4*
b^2)*ln(1+I*exp(d*x+c))*x-10*I/(a^2+b^2)/d^2*b^3*f/(4*a^2+4*b^2)*ln(1+I*ex
p(d*x+c))*c-12/(a^2+b^2)/d^2*a^2*c*b*f/(4*a^2+4*b^2)*arctan(exp(d*x+c))-6*
I/(a^2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*c+6*I/(a^2+b^2)/d
^2*a^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*c+12/(a^2+b^2)/d^2*a*b^2*f/(4*
a^2+4*b^2)*dilog(1-I*exp(d*x+c))+1/(a^2+b^2)^(5/2)/d^2/a^2*b^6*f*arctanh(1
/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+12/(a^2+b^2)/d*a^2*b*e/(4*a^2+4*b
^2)*arctan(exp(d*x+c))+1/(a^2+b^2)^2/d^2/a^3*c*b^6*f*ln(b*exp(2*d*x+2*c)+2
*a*exp(d*x+c)-b)+8/(a^2+b^2)/d*a^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x-10
*I/(a^2+b^2)/d^2*b^3*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))+10*I/(a^2+b^2)/
d^2*b^3*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))-1/d^2/a^2*f*b^4/(a^2+b^2)...
```

3.500.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 16848 vs. $2(1020) = 2040$.

Time = 0.67 (sec) , antiderivative size = 16848, normalized size of antiderivative = 15.02

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Too large to include
```

3.500.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)**3*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.500.7 Maxima [F]

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{(fx+e)\,\text{csch}\,(dx+c)^3\,\text{sech}\,(dx+c)^3}{b\sinh\,(dx+c)+a}\,dx$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-(b^6*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^7 + 2*a^5*b^2 +
a^3*b^4)*d) + (3*a^2*b + 5*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b
^4)*d) - (2*a^3 + 3*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b
^4)*d) + (4*a*b^2*e^(-4*d*x - 4*c) - (3*a^2*b + 2*b^3)*e^(-d*x - c) + 2*(2
*a^3 + a*b^2)*e^(-2*d*x - 2*c) + (a^2*b - 2*b^3)*e^(-3*d*x - 3*c) - (a^2*b
 - 2*b^3)*e^(-5*d*x - 5*c) + 2*(2*a^3 + a*b^2)*e^(-6*d*x - 6*c) + (3*a^2*b
 + 2*b^3)*e^(-7*d*x - 7*c))/((a^4 + a^2*b^2 - 2*(a^4 + a^2*b^2)*e^(-4*d*x
- 4*c) + (a^4 + a^2*b^2)*e^(-8*d*x - 8*c))*d) + (2*a^2 - b^2)*log(e^(-d*x
- c) + 1)/(a^3*d) + (2*a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e + (128*
a^2*d*integrate(1/64*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 64*b^2*d*integrat
e(1/64*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 128*a^2*d*integrate(1/64*x/(a^3
*d*e^(d*x + c) - a^3*d), x) + 64*b^2*d*integrate(1/64*x/(a^3*d*e^(d*x + c)
 - a^3*d), x) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) + 1)/(a^3*d^2))
 + a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^3*d^2)) + (a*b^2 + (
a^2*b*e^(7*c) + (3*a^2*b*d*e^(7*c) + 2*b^3*d*e^(7*c))*x)*e^(7*d*x) - (2*a^
3*e^(6*c) + a*b^2*e^(6*c) + 2*(2*a^3*d*e^(6*c) + a*b^2*d*e^(6*c))*x)*e^(6*
d*x) - (a^2*b*e^(5*c) + (a^2*b*d*e^(5*c) - 2*b^3*d*e^(5*c))*x)*e^(5*d*x) -
 (4*a*b^2*d*x*e^(4*c) + a*b^2*e^(4*c))*e^(4*d*x) - (a^2*b*e^(3*c) - (a^2*b
*d*e^(3*c) - 2*b^3*d*e^(3*c))*x)*e^(3*d*x) + (2*a^3*e^(2*c) + a*b^2*e^(2*c
) - 2*(2*a^3*d*e^(2*c) + a*b^2*d*e^(2*c))*x)*e^(2*d*x) + (a^2*b*e^c - (...
```

3.500.8 Giac [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.500.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{e+f\,x}{\cosh\left(c+d\,x\right)^3\sinh\left(c+d\,x\right)^3\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.501 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.501.1 Optimal result

Integrand size = 29, antiderivative size = 211

$$\begin{aligned}\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{b\arctan(\sinh(c+dx))}{2(a^2+b^2)d}\\ &\quad+\frac{b(a^2+2b^2)\arctan(\sinh(c+dx))}{(a^2+b^2)^2 d}+\frac{b\operatorname{csch}(c+dx)}{a^2d}\\ &\quad-\frac{\operatorname{csch}^2(c+dx)}{2ad}+\frac{a(2a^2+3b^2)\log(\cosh(c+dx))}{(a^2+b^2)^2 d}\\ &\quad-\frac{(2a^2-b^2)\log(\sinh(c+dx))}{a^3d}\\ &\quad-\frac{b^6\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)^2 d}\\ &\quad-\frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d}\end{aligned}$$

output
```
1/2*b*arctan(sinh(d*x+c))/(a^2+b^2)/d+b*(a^2+2*b^2)*arctan(sinh(d*x+c))/(a
^2+b^2)^2/d+b*csch(d*x+c)/a^2/d-1/2*csch(d*x+c)^2/a/d+a*(2*a^2+3*b^2)*ln(c
osh(d*x+c))/(a^2+b^2)^2/d-(2*a^2-b^2)*ln(sinh(d*x+c))/a^3/d-b^6*ln(a+b*sin
h(d*x+c))/a^3/(a^2+b^2)^2/d-1/2*sech(d*x+c)^2*(a-b*sinh(d*x+c))/(a^2+b^2)/
d
```

3.501.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\frac{b\arctan(\sinh(c+dx))}{a^2+b^2}+\frac{2b\operatorname{csch}(c+dx)}{a^2}-\frac{\operatorname{csch}^2(c+dx)}{a}+\frac{(a-ib)(2a^2+iab+2b^2)\log(i-\sinh(c+dx))}{(a^2+b^2)^2}-\frac{2(2a^2-b^2)\log(\sinh(c+dx))}{a^3}+ (}{2d}$$

input
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
((b*ArcTan[Sinh[c + d*x]])/(a^2 + b^2) + (2*b*Csch[c + d*x])/a^2 - Csch[c
+ d*x]^2/a + ((a - I*b)*(2*a^2 + I*a*b + 2*b^2)*Log[I - Sinh[c + d*x]])/(a
^2 + b^2)^2 - (2*(2*a^2 - b^2)*Log[Sinh[c + d*x]])/a^3 + ((a + I*b)*(2*a^2
 - I*a*b + 2*b^2)*Log[I + Sinh[c + d*x]])/(a^2 + b^2)^2 - (2*b^6*Log[a + b
*Sinh[c + d*x]])/(a^3*(a^2 + b^2)^2) - (a*Sech[c + d*x]^2)/(a^2 + b^2) + (
b*Sech[c + d*x]*Tanh[c + d*x])/(a^2 + b^2))/(2*d)
```

3.501.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {3042, 26, 3316, 26, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{i}{\sin(ic+idx)^3\cos(ic+idx)^3(a-ib\sin(ic+idx))}dx$$
$$\Big\downarrow \; 26$$
$$-i\int \frac{1}{\cos(ic+idx)^3\sin(ic+idx)^3(a-ib\sin(ic+idx))}dx$$
$$\Big\downarrow \; 3316$$

$$-\frac{ib^3 \int \frac{i\operatorname{csch}^3(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{b^3 \int \frac{\operatorname{csch}^3(c+dx)}{(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^6 \int \frac{\operatorname{csch}^3(c+dx)}{b^3(a+b\sinh(c+dx))\left(\sinh^2(c+dx)b^2+b^2\right)^2}d(b\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{615}$$

$$\frac{b^6 \int \left(\frac{\operatorname{csch}^3(c+dx)}{ab^7}-\frac{\operatorname{csch}^2(c+dx)}{a^2b^6}+\frac{(b^2-2a^2)\operatorname{csch}(c+dx)}{a^3b^7}-\frac{1}{a^3(a^2+b^2)^2(a+b\sinh(c+dx))}+\frac{(a^2+2b^2)b^2+a(2a^2+3b^2)\sinh(c+dx)b}{b^6(a^2+b^2)^2(\sinh^2(c+dx)b^2+b^2)}-\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^6 \left(\frac{(a^2+2b^2)\arctan(\sinh(c+dx))}{b^5(a^2+b^2)^2}+\frac{\arctan(\sinh(c+dx))}{2b^5(a^2+b^2)}+\frac{\operatorname{csch}(c+dx)}{a^2b^5}+\frac{a(2a^2+3b^2)\log\left(b^2\sinh^2(c+dx)+b^2\right)}{2b^6(a^2+b^2)^2}-\frac{a-b\sinh(c+dx)}{2b^4(a^2+b^2)(b^2\sinh^2(c+\right.}}{d}$$

input
```
Int[(Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

output
```
(b^6*(ArcTan[Sinh[c + d*x]]/(2*b^5*(a^2 + b^2)) + ((a^2 + 2*b^2)*ArcTan[Si
nh[c + d*x]])/(b^5*(a^2 + b^2)^2) + Csch[c + d*x]/(a^2*b^5) - Csch[c + d*x
]^2/(2*a*b^6) - ((2*a^2 - b^2)*Log[b*Sinh[c + d*x]])/(a^3*b^6) - Log[a + b
*Sinh[c + d*x]]/(a^3*(a^2 + b^2)^2) + (a*(2*a^2 + 3*b^2)*Log[b^2 + b^2*Sin
h[c + d*x]^2])/(2*b^6*(a^2 + b^2)^2) - (a - b*Sinh[c + d*x])/(2*b^4*(a^2 +
 b^2)*(b^2 + b^2*Sinh[c + d*x]^2))))/d
```

3.501. $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}dx$

3.501.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

3.501.4 Maple [A] (verified)

Time = 54.09 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{b^6\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)^2 a^3}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-8a^2+4b^2\right)\ln\left(\tanh\left(\right.\right.}{4a^3}}{}$
default	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{b^6\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)^2 a^3}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-8a^2+4b^2\right)\ln\left(\tanh\left(\right.\right.}{4a^3}}{}$
risch	$-\frac{4d^2a^3x}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{4d\,a^3c}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{6a\,b^2d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{6a\,b^2dc}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{4x}{a}+\frac{4c}{da}-$

input
```
int(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4/a^2*(1/2*tanh(1/2*d*x+1/2*c)^2*a+2*b*tanh(1/2*d*x+1/2*c))-b^6/(a
^2+b^2)^2/a^3*ln(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a)-1/8/a/
tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(-8*a^2+4*b^2)*ln(tanh(1/2*d*x+1/2*c))+1/2*b
/a^2/tanh(1/2*d*x+1/2*c)+2/(a^2+b^2)^2*(((-1/2*a^2*b-1/2*b^3)*tanh(1/2*d*x
+1/2*c)^3+(a^3+a*b^2)*tanh(1/2*d*x+1/2*c)^2+(1/2*a^2*b+1/2*b^3)*tanh(1/2*d
*x+1/2*c))/(1+tanh(1/2*d*x+1/2*c)^2)^2+1/4*(4*a^3+6*a*b^2)*ln(1+tanh(1/2*d
*x+1/2*c)^2)+1/2*(3*a^2*b+5*b^3)*arctan(tanh(1/2*d*x+1/2*c))))
```

3.501.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3148 vs. $2(206) = 412$.

Time = 0.65 (sec) , antiderivative size = 3148, normalized size of antiderivative = 14.92

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fric
as")
```

output
```
((3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x + c)^7 + (3*a^5*b + 5*a^3*b^3 +
2*a*b^5)*sinh(d*x + c)^7 - 2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*cosh(d*x + c)^6
 - (4*a^6 + 6*a^4*b^2 + 2*a^2*b^4 - 7*(3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh
(d*x + c))*sinh(d*x + c)^6 - (a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c)^5 -
 (a^5*b - a^3*b^3 - 2*a*b^5 - 21*(3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x
+ c)^2 + 12*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 -
 4*(a^4*b^2 + a^2*b^4)*cosh(d*x + c)^4 - (4*a^4*b^2 + 4*a^2*b^4 - 35*(3*a^
5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x + c)^3 + 30*(2*a^6 + 3*a^4*b^2 + a^2*b
^4)*cosh(d*x + c)^2 + 5*(a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c))*sinh(d*
x + c)^4 + (a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c)^3 + (a^5*b - a^3*b^3
- 2*a*b^5 + 35*(3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x + c)^4 - 40*(2*a^6
 + 3*a^4*b^2 + a^2*b^4)*cosh(d*x + c)^3 - 10*(a^5*b - a^3*b^3 - 2*a*b^5)*c
osh(d*x + c)^2 - 16*(a^4*b^2 + a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 2
*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*cosh(d*x + c)^2 - (4*a^6 + 6*a^4*b^2 + 2*a^
2*b^4 - 21*(3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x + c)^5 + 30*(2*a^6 + 3
*a^4*b^2 + a^2*b^4)*cosh(d*x + c)^4 + 10*(a^5*b - a^3*b^3 - 2*a*b^5)*cosh(
d*x + c)^3 + 24*(a^4*b^2 + a^2*b^4)*cosh(d*x + c)^2 - 3*(a^5*b - a^3*b^3 -
 2*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^2 + ((3*a^5*b + 5*a^3*b^3)*cosh(d*x
 + c)^8 + 56*(3*a^5*b + 5*a^3*b^3)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(3
*a^5*b + 5*a^3*b^3)*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(3*a^5*b + 5*a^...
```

3.501.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.501.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(206) = 412$.

Time = 0.32 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.98

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b^6\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^7+2\,a^5b^2+a^3b^4)d}$$
$$-\frac{(3\,a^2b+5\,b^3)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d}+\frac{(2\,a^3+3\,ab^2)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d}$$
$$-\frac{4\,ab^2e^{(-4\,dx-4\,c)}-(3\,a^2b+2\,b^3)e^{(-dx-c)}+2\,(2\,a^3+ab^2)e^{(-2\,dx-2\,c)}+(a^2b-2\,b^3)e^{(-3\,dx-3\,c)}-(a^2b-2\,b^3}{(a^4+a^2b^2-2\,(a^4+a^2b^2)e^{(-4\,dx-4\,c)}+(a^4+a^2b^2)e}$$
$$-\frac{(2\,a^2-b^2)\log\left(e^{(-dx-c)}+1\right)}{a^3d}-\frac{(2\,a^2-b^2)\log\left(e^{(-dx-c)}-1\right)}{a^3d}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxi
ma")
```

output
```
-b^6*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^7 + 2*a^5*b^2 + a
^3*b^4)*d) - (3*a^2*b + 5*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^
4)*d) + (2*a^3 + 3*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^
4)*d) - (4*a*b^2*e^(-4*d*x - 4*c) - (3*a^2*b + 2*b^3)*e^(-d*x - c) + 2*(2*
a^3 + a*b^2)*e^(-2*d*x - 2*c) + (a^2*b - 2*b^3)*e^(-3*d*x - 3*c) - (a^2*b
- 2*b^3)*e^(-5*d*x - 5*c) + 2*(2*a^3 + a*b^2)*e^(-6*d*x - 6*c) + (3*a^2*b
+ 2*b^3)*e^(-7*d*x - 7*c))/((a^4 + a^2*b^2 - 2*(a^4 + a^2*b^2)*e^(-4*d*x -
 4*c) + (a^4 + a^2*b^2)*e^(-8*d*x - 8*c))*d) - (2*a^2 - b^2)*log(e^(-d*x -
 c) + 1)/(a^3*d) - (2*a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d)
```

3.501.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(206) = 412$.

Time = 0.30 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.20

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx =$$
$$-\frac{\frac{4\,b^7\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^7b+2\,a^5b^3+a^3b^5}-\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(3\,a^2b+5\,b^3)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(2\,a^3+3\,ab^2)\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right.\right.}{a^4+2\,a^2b^2+b^4}}{}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac
")
```

output
```
-1/4*(4*b^7*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^7*b + 2*a^5*
b^3 + a^3*b^5) - (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(
3*a^2*b + 5*b^3)/(a^4 + 2*a^2*b^2 + b^4) - 2*(2*a^3 + 3*a*b^2)*log((e^(d*x
 + c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*a^3*(e^(d*x +
c) - e^(-d*x - c))^2 + 3*a*b^2*(e^(d*x + c) - e^(-d*x - c))^2 - 2*a^2*b*(e
^(d*x + c) - e^(-d*x - c)) - 2*b^3*(e^(d*x + c) - e^(-d*x - c)) + 12*a^3 +
 16*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^(d*x + c) - e^(-d*x - c))^2 + 4))
+ 4*(2*a^2 - b^2)*log(abs(e^(d*x + c) - e^(-d*x - c)))/a^3 - 2*(6*a^2*(e^(
d*x + c) - e^(-d*x - c))^2 - 3*b^2*(e^(d*x + c) - e^(-d*x - c))^2 + 4*a*b*
(e^(d*x + c) - e^(-d*x - c)) - 4*a^2)/(a^3*(e^(d*x + c) - e^(-d*x - c))^2)
)/d
```

3.501.9 Mupad [B] (verification not implemented)

Time = 8.27 (sec) , antiderivative size = 554, normalized size of antiderivative = 2.63

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{\frac{4\,b^5}{a\,d\,(a^2\,b^3+b^5)}-\frac{4\,b^4\,\mathrm{e}^{3\,c+3\,d\,x}}{d\,(a^2\,b^3+b^5)}+\frac{4\,b^4\,\mathrm{e}^{c+d\,x}}{d\,(a^2\,b^3+b^5)}+\frac{4\,b^3\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(2\,a^2+b^2\right)}{a\,d\,(a^2\,b^3+b^5)}}{\mathrm{e}^{8\,c+8\,d\,x}-2\,\mathrm{e}^{4\,c+4\,d\,x}+1}$$
$$-\frac{\frac{4\,\left(a^2\,b^5+b^7\right)}{a\,d\,(a^2\,b^3+b^5)\,(a^2+b^2)}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(2\,a^4\,b^3+3\,a^2\,b^5+b^7\right)}{a\,d\,(a^2\,b^3+b^5)\,(a^2+b^2)}-\frac{\mathrm{e}^{3\,c+3\,d\,x}\,\left(3\,a^4\,b^4+5\,a^2\,b^6+2\,b^8\right)}{a^2\,d\,(a^2\,b^3+b^5)\,(a^2+b^2)}-\frac{b^4\,\mathrm{e}^{c+d\,x}\,\left(-a^4+a^2\,b^2+2\,b^4\right)}{a^2\,d\,(a^2\,b^3+b^5)\,(a^2+b^2)}}{\mathrm{e}^{4\,c+4\,d\,x}-1}$$
$$+\frac{\ln\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)\,\left(4\,a+b\,5\mathrm{i}\right)}{2\,\left(d\,a^2+2\mathrm{i}\,d\,a\,b-d\,b^2\right)}-\frac{b^6\,\ln\left(2\,a\,\mathrm{e}^{c+d\,x}-b+b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{d\,a^7+2\,d\,a^5\,b^2+d\,a^3\,b^4}$$
$$+\frac{\ln\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)\,\left(5\,b+a\,4\mathrm{i}\right)}{2\,\left(1\mathrm{i}\,d\,a^2+2\,d\,a\,b-1\mathrm{i}\,d\,b^2\right)}-\frac{\ln\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)\,\left(2\,a^2-b^2\right)}{a^3\,d}$$

input
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

output
```
(log(exp(c + d*x)*1i + 1)*(4*a + b*5i))/(2*(a^2*d - b^2*d + a*b*d*2i)) - (
(4*(b^7 + a^2*b^5))/(a*d*(b^5 + a^2*b^3)*(a^2 + b^2)) + (2*exp(2*c + 2*d*x
)*(b^7 + 3*a^2*b^5 + 2*a^4*b^3))/(a*d*(b^5 + a^2*b^3)*(a^2 + b^2)) - (exp(
3*c + 3*d*x)*(2*b^8 + 5*a^2*b^6 + 3*a^4*b^4))/(a^2*d*(b^5 + a^2*b^3)*(a^2
+ b^2)) - (b^4*exp(c + d*x)*(2*b^4 - a^4 + a^2*b^2))/(a^2*d*(b^5 + a^2*b^3
)*(a^2 + b^2)))/(exp(4*c + 4*d*x) - 1) - ((4*b^5)/(a*d*(b^5 + a^2*b^3)) -
(4*b^4*exp(3*c + 3*d*x))/(d*(b^5 + a^2*b^3)) + (4*b^4*exp(c + d*x))/(d*(b^
5 + a^2*b^3)) + (4*b^3*exp(2*c + 2*d*x)*(2*a^2 + b^2))/(a*d*(b^5 + a^2*b^3
)))/(exp(8*c + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - (b^6*log(2*a*exp(c + d*x
) - b + b*exp(2*c + 2*d*x)))/(a^7*d + a^3*b^4*d + 2*a^5*b^2*d) + (log(exp(
c + d*x) + 1i)*(a*4i + 5*b))/(2*(a^2*d*1i - b^2*d*1i + 2*a*b*d)) - (log(ex
p(2*c + 2*d*x) - 1)*(2*a^2 - b^2))/(a^3*d)
```

3.501. $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

3.502 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

3.502.1 Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

output
```
Unintegrable(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.502.2 Mathematica [N/A]

Not integrable

Time = 139.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

input
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])),x]
```

output
```
Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

3.502.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$
$$\Big\downarrow \text{6125}$$
$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}dx$$

input
```
Int[(Csch[c + d*x]^3*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

output
```
$Aborted
```

3.502.3.1 Defintions of rubi rules used

rule 6125
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> U
nintegrable[((e + f*x)^m*F[c + d*x]^n*G[c + d*x]^p)/(a + b*Sinh[c + d*x]),
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && HyperbolicQ[F] && Hyperbolic
Q[G]
```

3.502.4 Maple [N/A] (verified)

Not integrable

Time = 0.76 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(dx+c)^3\operatorname{sech}(dx+c)^3}{(fx+e)(a+b\sinh(dx+c))}dx$$

input
```
int(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
int(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

3.502.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="fricas")
```

output
```
Timed out
```

3.502.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

output
```
Timed out
```

3.502.7 Maxima [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 1950, normalized size of antiderivative = 54.17

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^3\operatorname{sech}\left(dx+c\right)^3}{(fx+e)(b\sinh\left(dx+c\right)+a)}\,dx$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="maxima")
```

output
```
-(a*b^2*f - (2*b^3*d*e*e^(7*c) + (3*d*e - f)*a^2*b*e^(7*c) + (3*a^2*b*d*f*
e^(7*c) + 2*b^3*d*f*e^(7*c))*x)*e^(7*d*x) + (2*(2*d*e - f)*a^3*e^(6*c) + (
2*d*e - f)*a*b^2*e^(6*c) + 2*(2*a^3*d*f*e^(6*c) + a*b^2*d*f*e^(6*c))*x)*e^
(6*d*x) - (2*b^3*d*e*e^(5*c) - (d*e - f)*a^2*b*e^(5*c) - (a^2*b*d*f*e^(5*c
) - 2*b^3*d*f*e^(5*c))*x)*e^(5*d*x) + (4*a*b^2*d*f*x*e^(4*c) + (4*d*e - f)
*a*b^2*e^(4*c))*e^(4*d*x) + (2*b^3*d*e*e^(3*c) - (d*e + f)*a^2*b*e^(3*c) -
 (a^2*b*d*f*e^(3*c) - 2*b^3*d*f*e^(3*c))*x)*e^(3*d*x) + (2*(2*d*e + f)*a^3
*e^(2*c) + (2*d*e + f)*a*b^2*e^(2*c) + 2*(2*a^3*d*f*e^(2*c) + a*b^2*d*f*e^
(2*c))*x)*e^(2*d*x) + (2*b^3*d*e*e^c + (3*d*e + f)*a^2*b*e^c + (3*a^2*b*d*
f*e^c + 2*b^3*d*f*e^c)*x)*e^(d*x))/(a^4*d^2*e^2 + a^2*b^2*d^2*e^2 + (a^4*d
^2*f^2 + a^2*b^2*d^2*f^2)*x^2 + 2*(a^4*d^2*e*f + a^2*b^2*d^2*e*f)*x + (a^4
*d^2*e^2*e^(8*c) + a^2*b^2*d^2*e^2*e^(8*c) + (a^4*d^2*f^2*e^(8*c) + a^2*b^
2*d^2*f^2*e^(8*c))*x^2 + 2*(a^4*d^2*e*f*e^(8*c) + a^2*b^2*d^2*e*f*e^(8*c))
*x)*e^(8*d*x) - 2*(a^4*d^2*e^2*e^(4*c) + a^2*b^2*d^2*e^2*e^(4*c) + (a^4*d^
2*f^2*e^(4*c) + a^2*b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^4*d^2*e*f*e^(4*c) + a^
2*b^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x)) + 64*integrate(-1/32*(a*b^6*e^(d*x +
c) - b^7)/(a^7*b*e + 2*a^5*b^3*e + a^3*b^5*e + (a^7*b*f + 2*a^5*b^3*f + a^
3*b^5*f)*x - (a^7*b*e*e^(2*c) + 2*a^5*b^3*e*e^(2*c) + a^3*b^5*e*e^(2*c) +
(a^7*b*f*e^(2*c) + 2*a^5*b^3*f*e^(2*c) + a^3*b^5*f*e^(2*c))*x)*e^(2*d*x) -
 2*(a^8*e*e^c + 2*a^6*b^2*e*e^c + a^4*b^4*e*e^c + (a^8*f*e^c + 2*a^6*b^...
```

3.502.8 Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorit
hm="giac")
```

output
```
Timed out
```

3.502.9 Mupad [N/A]

Not integrable

Time = 19.63 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$
$$=\int \frac{1}{\cosh\left(c+d\,x\right)^3\sinh\left(c+d\,x\right)^3\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

output
```
int(1/(cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))), x
)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```